EE 141 Controller Design for a Fast Tool Servo

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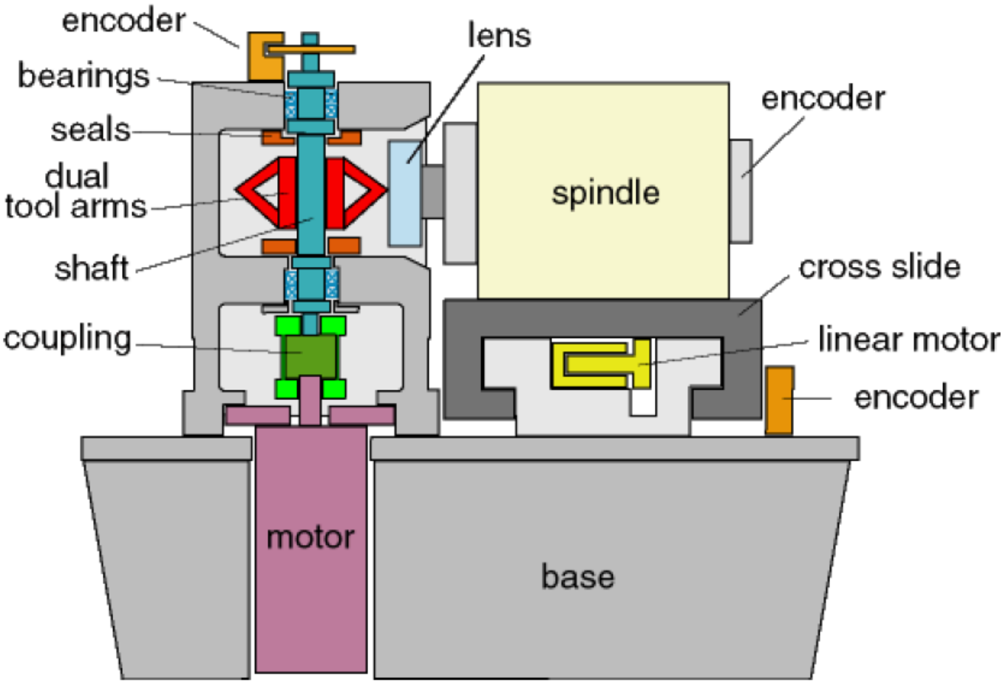


Figure 1: FTS Schematics

1 Introduction

The purpose of this project is to designe controllers for a Fast Tool Servo (FTS) system. The tasks of the project are as follows:

1. Design an OpAmp-based high-bandwidth controller for the current control loop to drive the actuator.
2. Identify the dynamics of the FTS electromechanical plant from the measured frequency response data
3. Design a position controller that minimizes tracking error for step and sinusoidal inputs at steady state in the presence of sensor noise.

The Fast Tool Servo schematics are given in Figure 1.

The overall control loop of the FTS system used in this project is given in Figure 2 and shows that the ultimate goal for the system is to control the position of the cutting tool x(t) such that it tracks the position trajectory $x_{ref}(t)$.

As can be seen in the entire control loop the system consists of the following parts:

- **FTS mechanical plant** $G_p(s)$: The actuator force results in the displacement of the cutting tool.

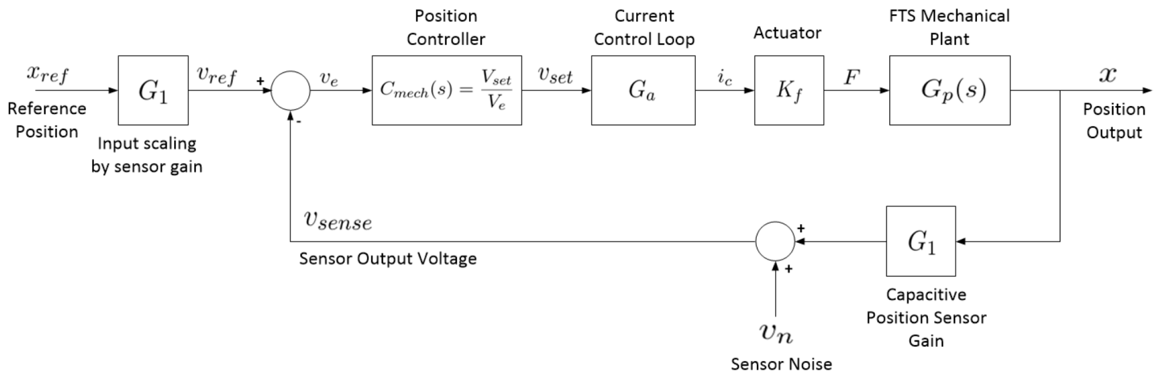


Figure 2: Fast Tool Servo Overall Control Loop

- **Actuator**: The motor where the current i_c is regulated to generate the required force for the mechanical plant. Can be described by a fixed gain of $K_f = 20N/A$.
- **Current Control Loop**: Controls the current i_c, which drives the actuator, based on the input voltage v_{set}. The loop is fast enough that the resulting dynamics are fast enough compared to the bandwidth that they can be ignored and the loop can be replaced with a constant gain $G_a = -0.5A/V$.
- **Position Controller** $C_{mech}(s)$: Gets the error between the reference voltage v_{ref} and the sensor output voltage v_{sense} and generates the voltage v_{set} to drive the control loop.
- **Position Sensor**: A capacitive displacement sensor that measures the position of the cutting tool. It can be modeled by a constant gain $G_1 = 5 * 10^5 V/m$ and the output is assumed to be corrupted by measurement noise v_n.

2 Current Control Loop Design

The first step is to design the current loop plant and current loop controller. The schematic for the current control loop is given in Figure 3. Another representation of the current control loop, in the form of a block diagram is given in Figure 4.

As can be seen in the schematics, the back electromotive force (back emf) from the actuator can be modeled as a dependent voltage source with voltage proportional to the velocity of the cutting tool: $v_{back\_emf} = K_f \dot{x}(t)$. Additionally, the force generated by the actuator is assumed to be proportional to the actuator current with the same gain: $F(t) = K_f i_c(t)$. A model describing the mechanical dynamics of the FTS is shown in Figure 5 where the actuator force F(t) is applied to a mass m_1 subject to resistance by a spring k_1 and a damper b_1.

1. Identifying $P_{elec}(s) = \frac{V_s(s)}{V_c(s)}$ with Back EMF

   The derivation for $P_{elec}(s)$ with back emf included is given below. In this case,

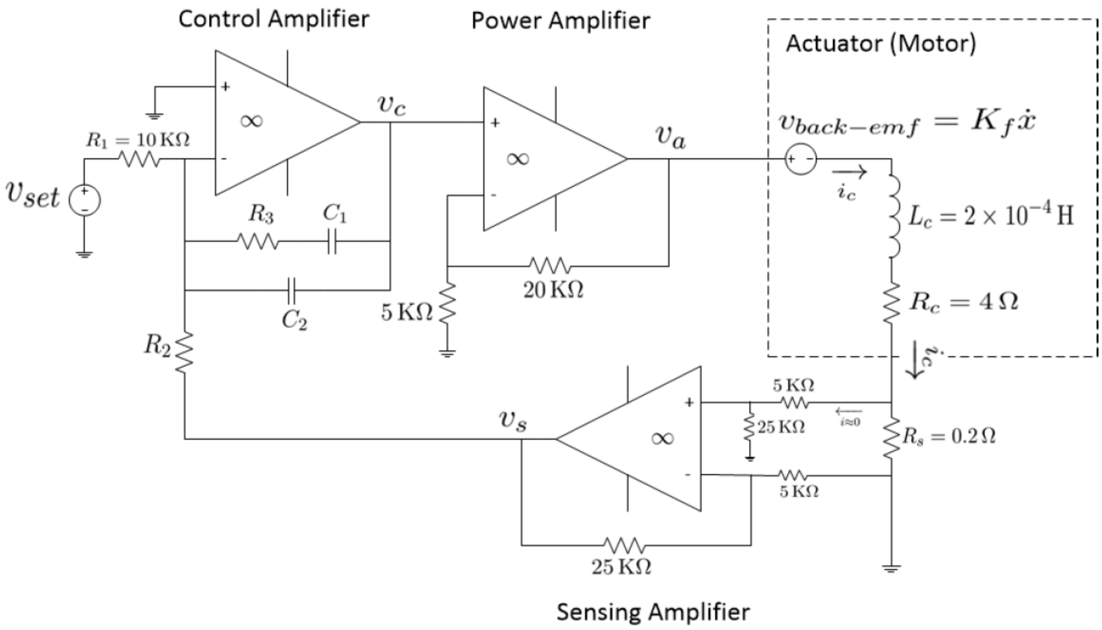


Figure 3: Current Control Loop Schematic

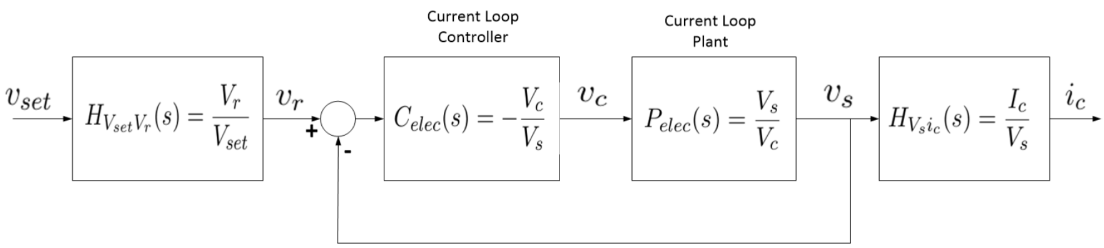


Figure 4: Current Control Loop Block Diagram

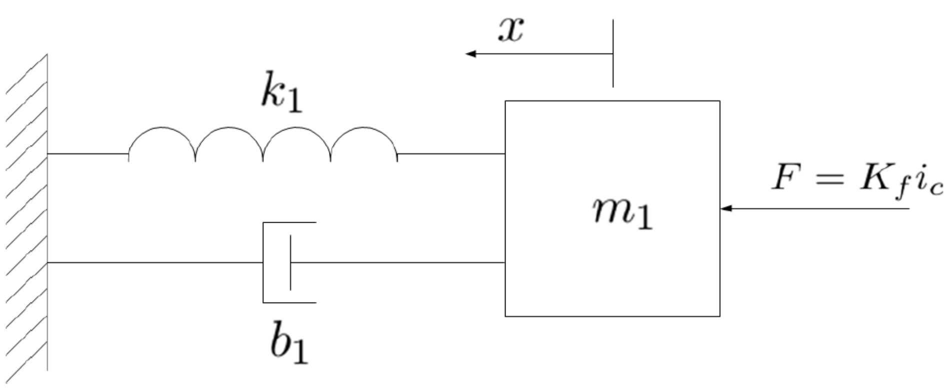


Figure 5: Model for mechanical dynamics of FTS

the FTS schematics given in Figure 1 are taken into account and reflected in the plant transfer function.

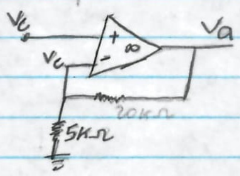
Good!

Current Control Loop Design

a. $P_{elec} = \frac{V_s(s)}{V_c(s)}$

$$\frac{V_s}{V_c} = \underbrace{\frac{V_a}{V_c}}_{\text{Power Amplifier}} \cdot \underbrace{\frac{V_x}{V_a}}_{\text{Actuator}} \cdot \underbrace{\frac{V_s}{V_x}}_{\text{Sensing Amplifier}}$$

power amplifier:

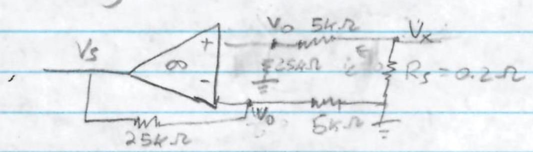


$$\frac{V_c}{5K} + \frac{V_c - V_a}{20K} = 0$$

$$4V_c + V_c - V_a = 0$$

$$5V_c - V_a = 0 \Rightarrow \frac{V_a}{V_c} = 5$$

sensing amplifier:



$$\frac{V_o - V_x}{5k} + \frac{V_o}{25k} = 0$$

$$\frac{V_o - V_s}{25k} + \frac{V_o}{5k} = 0 \Rightarrow V_o - V_s + 5V_o = 0$$

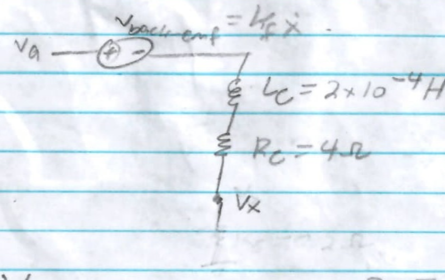
$$V_o = V_s/6$$

$$\frac{\left(\frac{V_s}{6}\right) - V_x}{5k} + \frac{\left(\frac{V_s}{6}\right)}{25k} = 0$$

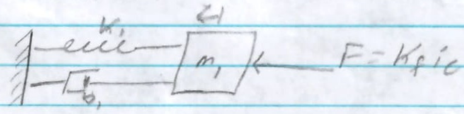
$$\frac{5V_s}{6} - 5V_x + \frac{V_s}{6} = 0$$

$$V_s = 5V_x \Rightarrow \frac{V_s}{V_x} = 5$$

Actuator:



$$V_a - V_{back\text{-}emf} - sL_c I_c - R_c I_c - R_s I_c = 0$$



$$K_f i_c - b_1 \dot{x} - k_1 x = m_1 \ddot{x}$$

$$m_1 \ddot{x} + b_1 \dot{x} + k_1 x = K_f i_c$$

$$\mathcal{L}\{\ \ m_1 s^2 X + b_1 s X + k_1 X = K_f I_c$$

$$X(m_1 s^2 + b_1 s + k_1) = K_f I_c$$

$$X(s) = \frac{I_c K_f}{(m_1 s^2 + b_1 s + k_1)}$$

$$\mathcal{L}\{\ \ V_{back\text{-}emf} = K_f \dot{x}(t)$$

$$V_{back\text{-}emf} = K_f \cdot s X(s)$$

$$= \frac{K_f^2 s I_c}{(m_1 s^2 + b_1 s + k_1)}$$

$$V_a - \frac{K_f^2 s I_c}{(m_1 s^2 + b_1 s + k_1)} - sL_c I_c - R_c I_c - R_s I_c = 0$$

$$V_a = I_c \left[\frac{K_f^2 s}{(m_1 s^2 + b_1 s + k_1)} + sL_c + R_c + R_s \right]$$

Note:

$$\frac{V_x}{V_a} = \frac{I_c R_s}{V_a} = \frac{R_s}{\left[\frac{k_f^2 s}{(m_1 s^2 + b_1 s + k_1)} + sL_c + R_c + R_s\right]}$$

$$P_{elec} = \frac{V_s}{V_c} = 5 \cdot 5 \cdot \frac{R_s}{\left[\frac{K_f^2 \cdot s}{(m_1 s^2 + b_1 s + k_1)} + sL_c + R_c + R_s\right]}$$

$$= \frac{25 R_s (m_1 s^2 + b_1 s + k_1)}{k_f^2 \cdot s + (sL_c + R_c + R_s)(m_1 s^2 + b_1 s + k_1)}$$

2. Identifying $P_{elec}(s) = \frac{V_s(s)}{V_c(s)}$ without Back EMF

The derivation for $P_{elec}(s)$ with back emf assumed to be zero is given below. In this case, the FTS dynamics given in Figure 1 have no effect on the transfer function.

b. $P_{elec} = \frac{V_s}{V_c}$

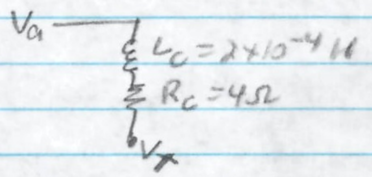
$$\frac{V_s}{V_c} = \underbrace{\frac{V_a}{V_c}}_{\text{power amplifier}} \cdot \underbrace{\frac{V_x}{V_a}}_{\text{Actuator}} \cdot \underbrace{\frac{V_s}{V_x}}_{\text{sensing amplifier}}$$

As found earlier,

power amplifier : $\frac{V_a}{V_c} = 5$

sensing amplifier : $\frac{V_s}{V_x} = 5$

Assuming $V_{back\text{-}emf} = 0$



$$V_a - sL_c I_c - R_c I_c - R_s I_c = 0$$

$$V_a = I_c \cdot (sL_c + R_c + R_s)$$

Note: $\frac{V_x}{V_a} = \frac{I_c R_s}{V_a} = \frac{R_s}{sL_c + R_c + R_s}$

$$P_{elec} = \frac{V_s}{V_c} = \frac{5 \cdot 5 \cdot R_s}{sL_c + R_c + R_s} = \boxed{\frac{25 R_s}{sL_c + R_c + R_s}}$$

3. Show that at high frequencies, both transfer functions are the same

   As can be seen in the analysis below, both versions of $P_{elec}(s)$, with and without back emf, are the same at high frequencies. Because of this fact, the back emf voltage can be assumed zero for the controller design portion.

c. P_{elec} w/ Back EMF

$$P_{elec} = \frac{25R_s(m_1 s^2 + b_1 s + K_1)}{K_f^2 \cdot s + (sL_c + R_c + R_s)(m_1 s^2 + b_1 s + K_1)}$$

$$= \frac{\cancel{(m_1 s^2 + b_1 s + K_1)}}{\cancel{(m_1 s^2 + b_1 s + K_1)}} \cdot \frac{25 R_s}{\left[\frac{K_f^2 \cdot s}{m_1 s^2 + b_1 s + K_1} + sL_c + R_c + R_s\right]}$$

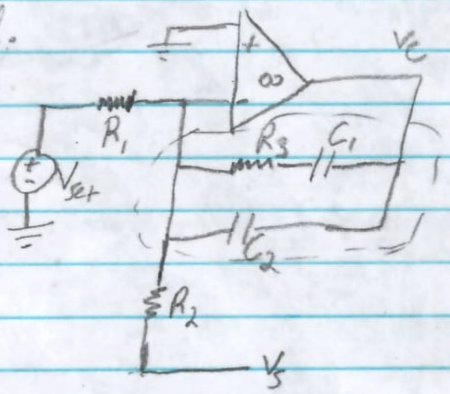
$s \approx \infty$

$$P_{elec} = \frac{25R_s}{sL_c + R_c + R_s} \checkmark$$

4. Design $C_{elec} = -\frac{V_c(s)}{V_s(s)}$

   The derivation of the current loop controller with a gain crossover frequency of $\omega_c = 6 * 10^5 rad/sec$ and a minimum phase margin of $PM \geq 60\circ$ is given below. Given that the input voltage of $v_{set} = 0$ results in a steady-state actuator coil current of $i_c = 5A$, it can be found that the full closed-loop current drive has a gain of $G_a = -0.5A/V$. Also, given that $R_1 = 10K\Omega$ and $R_s = 0.2\Omega$, we can find the resulting component values of R_1, R_3, C_1, and C_2.

d.

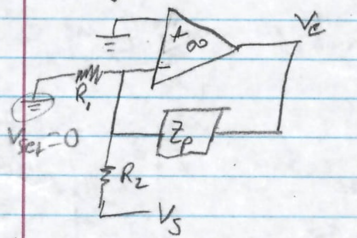


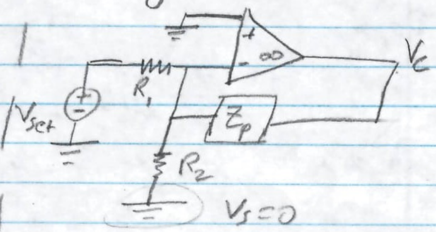
$$Z_p = \left(R_3 + \frac{1}{C_1 s}\right) \,||\, \frac{1}{C_2 s}$$

$$= \frac{\left(R_3 + \frac{1}{C_1 s}\right)\left(\frac{1}{C_2 s}\right)}{R_3 + \frac{1}{C_1 s} + \frac{1}{C_2 s}}$$

$$= \frac{R_3 C_1 s + 1}{s(C_1 + C_2 + R_3 C_1 C_2 s)}$$

Setting $V_{set} = 0$ | Setting $V_s = 0$





$$-\frac{V_c}{Z_p} - \frac{V_s}{R_2} = 0 \qquad -\frac{V_{set}}{R_1} - \frac{V_c}{Z_p} = 0$$

$$-\frac{V_c}{V_s} = \frac{Z_p}{R_2} \qquad -\frac{V_c}{V_{set}} = \frac{Z_p}{R_1}$$

$$V_c = -\frac{Z_p}{R_2} V_s - \frac{Z_p}{R_1} V_{set}$$

By block diagram:

$$V_c = C_{elec} \left(H_{vset} \cdot V_{set} - V_s \right)$$

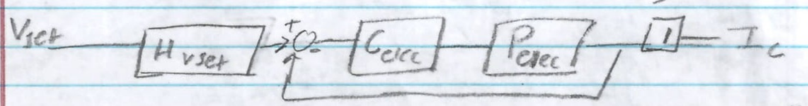
$$V_c = \underbrace{\frac{Z_p}{R_2}}_{C_{elec}} \left(\underbrace{-\frac{R_2}{R_1}}_{H_{vset}} V_{set} - V_s \right)$$

$$C_{elec}(s) = \frac{Z_p}{R_2} = \frac{R_3 C_1 s + 1}{s R_2 \left(C_1 + C_2 + R_3 C_1 C_2 s \right)}$$

$$= \frac{\cancel{R_3 C_1} \left[s + \frac{1}{R_3 C_1} \right]}{s R_2 \cancel{R_3 C_1} \left(\frac{C_1}{R_3 C_1} + \frac{C_2}{R_3 C_1} + C_2 s \right)}$$

$$= \frac{s + \frac{1}{R_3 C_1}}{R_2 C_2 s \left(\frac{1}{R_3 C_2} + \frac{1}{R_3 C_1} + s \right)} = \frac{\left[1 + \frac{1}{R_3 C_1 s} \right]}{R_2 C_2 \left[s + \left(\frac{1}{R_3 C_1} + \frac{1}{R_3 C_2} \right) \right]}$$

Gain of $\frac{I_c}{V_{set}} = -0.5$ $\quad * H_{V_{sic}} = 1$



$$\frac{I_c}{V_{set}} = \frac{H_{vset} \cdot C_{elec} \cdot P_{elec}}{1 + C_{elec} \cdot P_{elec}}$$

$$= \frac{\left(\frac{-5}{\frac{s}{5000}+4.2}\right)\left(\frac{R_3C_1s+1}{R_1(R_3C_1C_2s+C_1+C_2)}\right)}{5+\left(\frac{5}{\frac{s}{5000}+4.2}\right)\left(\frac{R_3C_1s+1}{R_2(R_3C_1C_2s+C_1+C_2)}\right)}$$

$$\lim_{s \to 0} \frac{I_c}{V_{set}} = \frac{\left(-\frac{5}{\cancel{4.2}}\right)\left(\frac{1}{R_1(C_1+C_2)}\right)}{\left(\frac{5}{\cancel{4.2}}\right)\left(\frac{1}{R_2(C_1+C_2)}\right)} = -0.5$$

$$= \underbrace{-\frac{R_2}{R_1}}_{H_{vset}} = -0.5$$

$\therefore H_{vset} = -0.5$

Note: $\frac{R_2}{R_1} = 0.5$ & $R_1 = 10k \Rightarrow R_2 = 5k$

$$C_{elec}(s) = \frac{k(1+\frac{k_i}{s})}{\tau s+1} = \frac{R_3 C_1 \left[1 + \frac{(\frac{1}{R_3 C_1})}{s}\right]}{R_2(C_1+C_2)\left(1+\frac{R_3 C_1 C_2}{C_1+C_2}s\right)}$$

pole @ $s = -\frac{1}{\tau}$

(1) $\tau = 1.667 * 10^{-7}$

zero @ $s = -k_i$

(2) $k_i = 6 * 10^4$

$k = \frac{R_3 C_1}{5000(C_1+C_2)}$

$\tau = \frac{R_3 C_1 C_2}{C_1+C_2}$

$k_i = \frac{1}{R_3 C_1}$

$$P_{elec}(s) = \frac{5}{s/5100 + 4.2}$$

$$|P_{elec}(s)| \cdot |C_{elec}(s)| \Big|_{s=j\omega_g} = 1$$

(3) $(0.041641) \cdot k = 1$

Using eqns. (1), (2) & (3)

(1) $\rightarrow \frac{R_3 C_1 C_2}{C_1+C_2} = 1.667 * 10^{-7}$

(2) $\rightarrow \frac{1}{R_3 C_1} = 6 * 10^4$

(3) $\rightarrow (0.047641) \cdot \frac{R_3 C_1}{5000(C_1+C_2)} = 1$

$C_1 = 1.37415 * 10^{-10}$ F

$C_2 = 1.38831 * 10^{-12}$ F

$R_3 = 121287\ \Omega$

5. Bode plot of Current Loop Transfer Function

   The bode plot in the attached Matlab results shows the current loop transfer function with the gain crossover frequency at $\omega_c = 6 * 10^5 rad/sec$ and a phase margin of 80.6 degrees.

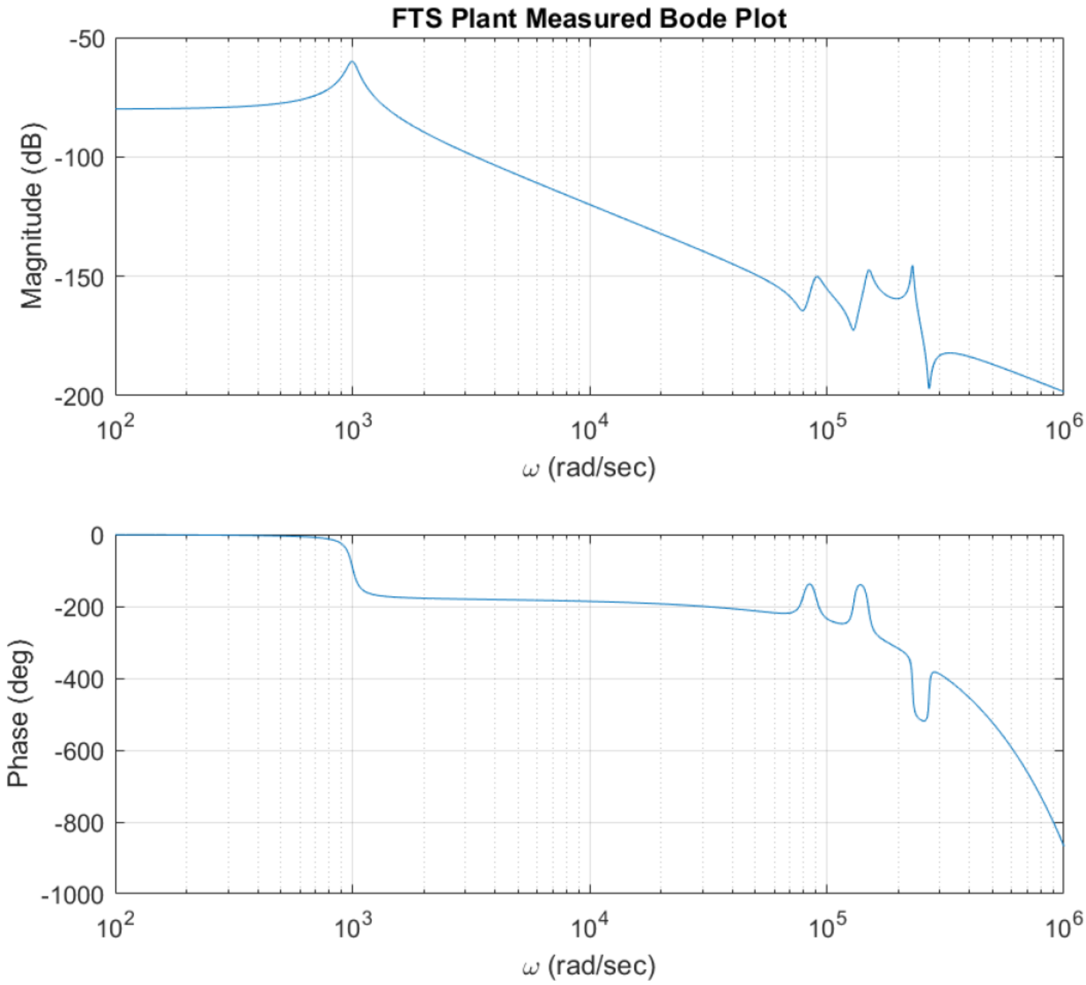


Figure 6: FTS Measured Data Bode Plots

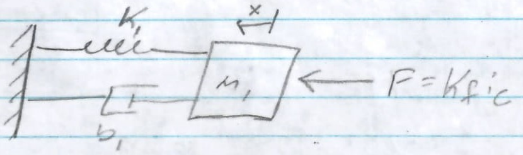
3 FTS Plant System Identification

The measured Bode plot for $G_p(s)$ is given in Figure 6. In this portion, we used the measured frequency response data to generate a transfer function that models the Bode dynamics as shown in Figure 6. As can be seen in this frequency response data, the mass/spring/damper model only accounts for the lowest frequency mode seen on the plot and this lowest mode is what we will focus on. In order to arrive at a transfer function that models the provided frequency data, we completed the following steps:

1. Find the values for the mass m_1, spring constant k_1, and the damper constant b_1 in the simplified FTS model given in Figure 1.
   The solutions for these values are given in the derivation below.

FTS Plant System Identification

a.



$$K_f i_c - K_1 x - b_1 \dot{x} = m_1 \ddot{x}$$

$$K_f I_c - K_1 X - b_1 s X = m_1 s^2 X$$

$$K_f I_c = X(m_1 s^2 + b_1 s + K_1)$$

$$\frac{X}{I_c} = \frac{K_f}{m_1 s^2 + b_1 s + K_1} = \frac{20}{m_1 s^2 + b_1 s + K_1}$$

$$20\log(M_p) = 20$$

$$M_p = 10$$

$$M_p = (2\zeta\sqrt{1-\zeta^2})^{-1} = 10$$

$$\zeta = 0.0501$$

$$\omega_r = 10^3$$

$$\omega_n = \frac{\omega_r}{\sqrt{1-2\zeta^2}} = 1002.52$$

$$\omega_n^2 \approx 10^6$$

$$2\zeta\omega_n = 100.452$$

$$\frac{X}{I_c} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{10^6 \cdot 10^{-4}}{s^2 + 100.452s + 10^6} = \frac{20}{m_1 s^2 + b_1 s + K_1}$$

$$10^2 = \frac{20}{m_1} \Rightarrow m_1 = 1/5$$

$$100.452 = \frac{b_1}{m_1} \Rightarrow b_1 = \frac{100.452}{5}$$

$$10^6 = \frac{K_1}{m_1} \Rightarrow \frac{10^6}{5}$$

$$G_p = \frac{100}{s^2 + 100.452s + 10^6}$$

2. Fit a dynamical model to the measured Bode plot.
   Using the formulas and derivations below, we were able to calculate a model for $G_p(s)$ along with an input time delay. We verified this result using the frd and tfest commands in Matlab as can be seen by the supplied code.

3. As can be seen in the Matlab results attached, the bode plots only match for the lower frequency mode.

4 Position Control Loop Design

The entire position control loop is given in Figure 2. Here, we designed a position controller $C_{mech}(s)$ which gets the error between the reference voltage v_{ref} and the sensor output voltage v_{sense} and generates the voltage v_{set} for the current control loop. Using the plant function $G_p(s)$ derived earlier along with the fact that the current control loop can be replaced by a constant gain of $G_a = -0.5A/V$, we can create $C_{mech}(s)$.
The controller will have two reference inputs as follows:

1. $1\mu * m$ Constant Position Trajectory: $x_{ref_1}(t) = 10^{-6}u(t)[m]$
2. Sinusoidal Position Trajectory: $x_{ref_2}(t) = 10^{-5}sin(3000t)[m]$

Additionally, the sensor measurement noise can be modeled as:

$$v_n(t) = 5 * 10^{-2}sin(10^5 t)$$

The specifications that $C_{mech}(s)$ must adhere to are as follows:

- The closed-loop FTS system is obviously stable
- The steady-state position tracking error for the step input is almost fully eliminated
- The steady-state position tracking error for the sinusoidal input trajectory is minimized
- The steady-state effect of the measurement noise on the output position is minimized
- The magnitude of the closed-loop Sensitivity Function stays below +10dB at all frequencies

Using these criteria, we designed $C_{mech}(s)$ using a PI-lead compensator, which takes the form of:

$$C_{mech}(s) = K * (1 + \frac{K_i}{s}) * (\frac{1 + \alpha\tau * s}{1 + \tau * s})$$

Because the noise was at a frequency of 10^5rad/s and the sinusoidal input was at a frequency of 3000rad/s, we decided to shape our loop transfer function to increase the gain at high frequency, which results in better reference tracking, and reduce the gain at low frequencies, which reduces the effect of noise.
To do this, we decided to set our crossover frequency, ω_c at 10^4rad/s. Also, we wanted to add a phase of roughly 55$\circ$ at that crossover frequency in order to increase stability and decrease the overshoot as much as possible, so we defined our α and τ using the following formulas:

$$\alpha = \frac{1 + sin(\phi_{max})}{1 - sin(\phi_{max})}$$

where ϕ_{max} is 55$\circ$ and

$$\tau = \frac{1}{\omega_c\sqrt{\alpha}}$$

Using these equations, we were able to get the form of our lead compensator. In order to find the gain values required for the PI portion of the compensator, we assumed K_i to be roughly $\frac{\omega_c}{5}$ and let K be a small negative number so as to allow the closed loop system to be stable. Then, we plotted our system in Matlab and manipulated the gain values until we obtained sufficiently small tracking error and a maximum sensitivity level under 10dB.

The final compensator was found to be:

$$C_{mech}(s) = -0.1(1 + \frac{\frac{3.38*10^4}{5}}{s})(\frac{1 + 3.1715 * 10^4 * s}{1 + 3.1529 * 10^5 * s})$$

These design choices gave us a strong negative sloped loop transfer function passing through the crossover frequency $1.58 * 10^4$ rad/s.

While making these choices we had to take into account the trade off between our closed loop transfer T(s) and sensitivity function S(s). We learned that T(s)+S(s)=1. This tells us that both T(s) and S(s) cannot be minimized, even though ideally that is what we hope to achieve. We saw this manifest itself while we attempted to accomplish the specification when the moment we achieved excellent results in the closed loop, the sensitivity of our system would go up substantially.