# *Managing congestion in dynamic matching markets*

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# *Introduction*

SEARCH FRICTIONS IN DYNAMIC MARKETS

## *Search frictions*

Search literature:

Diamond (1982), Mortensen (1982), Pissarides (1984)
Hosios (1990), Mortensen & Pissarides (1994), Moen (1997)
Acemoglu & Shimer (2000), Kennes, King & Julien (2001)
Guerrieri (2008), Kircher (2009), Lewis & Wang (2013)

Take-home messages:

- Costly search, coordination failure cause inefficiency
- Generally, as search costs vanish, efficiency is recovered

## *Dynamic online matching markets*

craigslist

airbnb

ebay

Zillow

Elance

oDesk

okcupid

lyft

UBER

eHarmony

These sites provide value by ***facilitating search***, allowing agents who can profitably transact to find each other.

## *Consequences of low frictions*

- Finding and contacting potential partners
  is easier than ever, but
  many inquiries *may not get a response*.

- If response rates are low,
  people may spend lots of time applying,
  even if each one is quick.

- This can lower response rates even further;
  a vicious cycle.

## *Anecdotal evidence*

oDesk worker:

"I applied to more than 100 jobs with no response... I have 4.8 star feedback, and I meet all the qualifications needed for the job."

oDesk website:

When clients send out invitations and freelancers don't reply, it's a frustrating experience that makes those clients less likely to hire anyone.

## *Our work*

Model a two-sided market in which:

- Matching is dynamic and asynchronous
- Learning preferences (screening) is costly
- Agents may be *unavailable* (this is unobservable)

Online labor markets, with *employers and applicants*

Elance
oDesk®

## *Key conclusions*

When application costs are low:

1. Applicants contact many employers.
2. Employers screen applicants who are *no longer available*.
3. This can *drive employer welfare to zero*

→ *Lower frictions may not be universally good.*

Intervening by limiting applications can:

- generate Pareto improvements, and
- boost employer welfare to a natural upper-bound

# *A dynamic matching market*

THE GAME THEORETIC MODEL

# *A dynamic matching market*

*Two-sided:*

- Applicants and employers

*Asynchronous:*

- Agents arrive and depart over time

*One-to-one:*

- Agents match to at most one other agent

*Homogeneous:*

- *Ex ante*, the agents look the same to each other

## *Overview*

1. Employers arrive, and live for unit lifetime
2. Applicants arrive, and
   apply to a subset of employers in the system
3. Upon leaving, employers screen and
   make offers to applicants
4. Upon receiving offers, applicants can accept or reject

## *Employer utility*

Agents are utility maximizers.

*Employers:*

- Want a *compatible* (i.e., qualified) applicant
- **P**(*s* compatible with *b*) = $\beta$ (i.i.d.)
- Value from a compatible match: $V$
- Must screen before hiring
  Screening cost per applicant: $C_S$

Overall utility:

$$V \times 1[\text{match to a compatible applicant}] \\ - C_S \times \# \text{ of screened applicants}$$

## *Applicant utility*

Agents are utility maximizers.

*Applicant:*

- Cost per application: $C_A$
- Value from *any* match: $W$

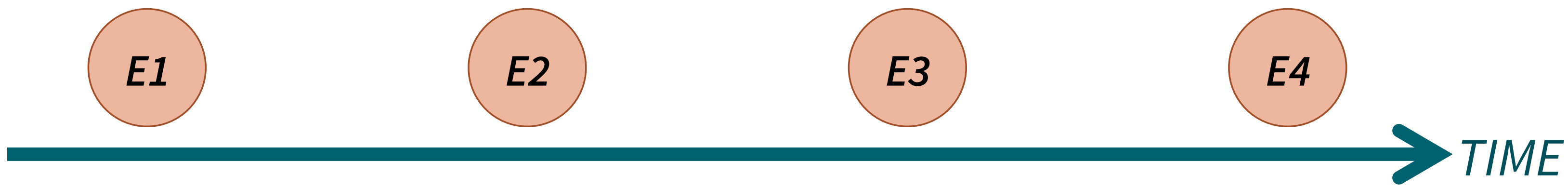
Overall utility:

$$
\begin{aligned}
&W \times 1[\text{matched to a employer}] \\
&\quad - \; C_A \times \# \text{ of applications sent}
\end{aligned}
$$

*Note:* As a result, “accept the first offer” is a (weak) dominant strategy.

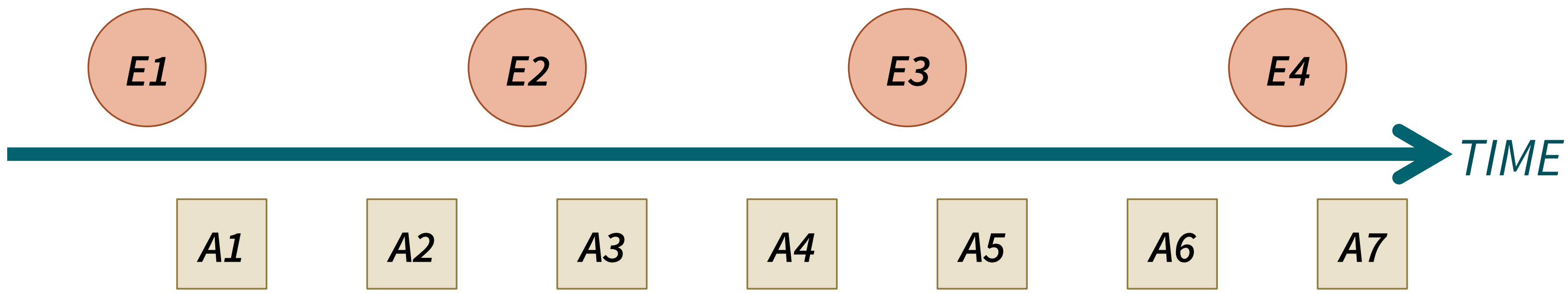
# *Dynamics: Employer arrivals*

Employers arrive at rate $n$.



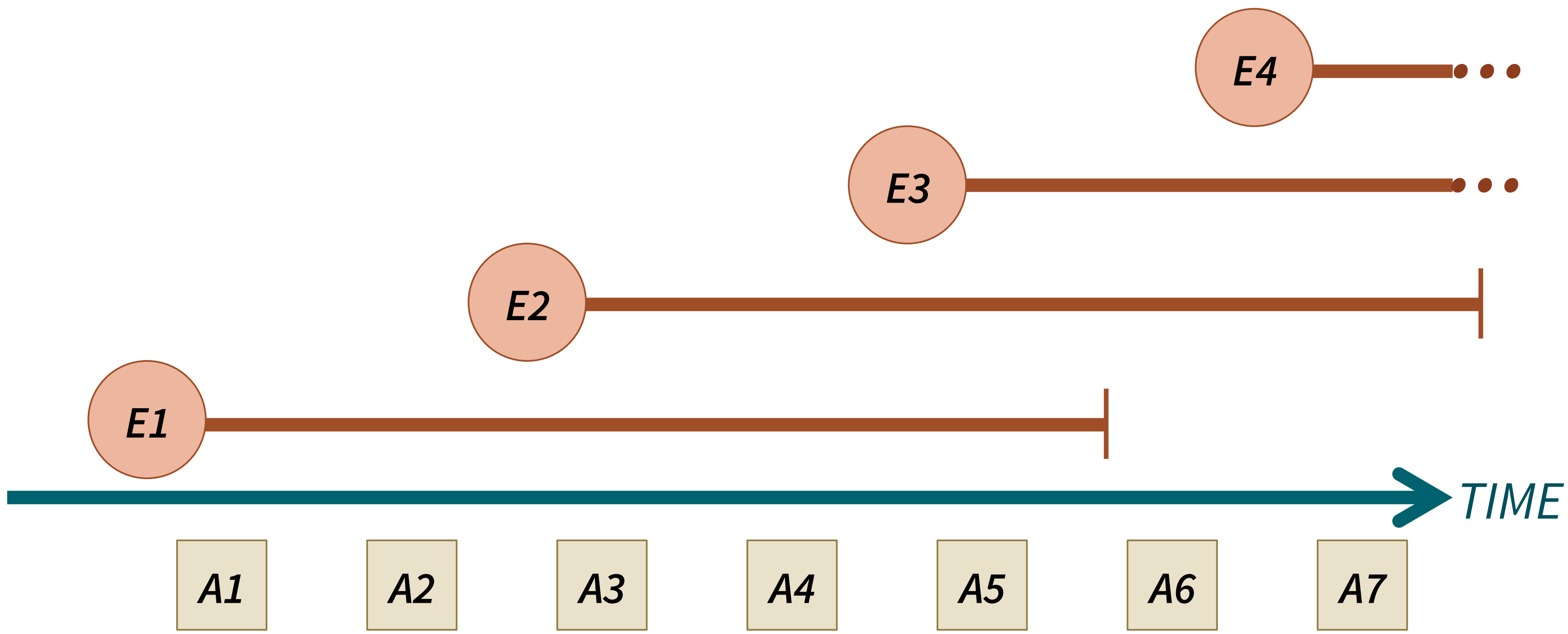
# *Dynamics: Applicant arrivals*

Applicants arrive at rate $Rn$. [ $R$ = market imbalance ]



# *Dynamics: Employer lifetimes*

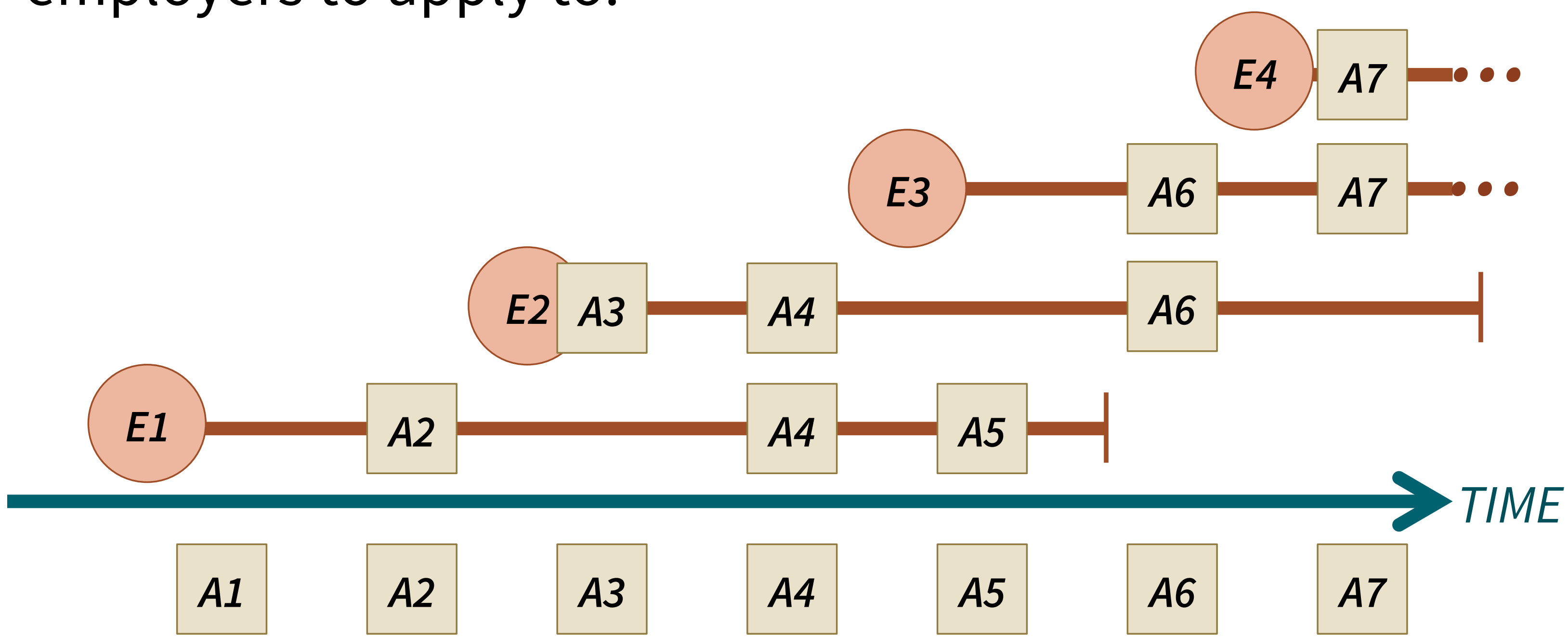
Employers live for unit lifetime.



# *Dynamics: Applications*

On arrival,

applicants choose a subset of employers to apply to.

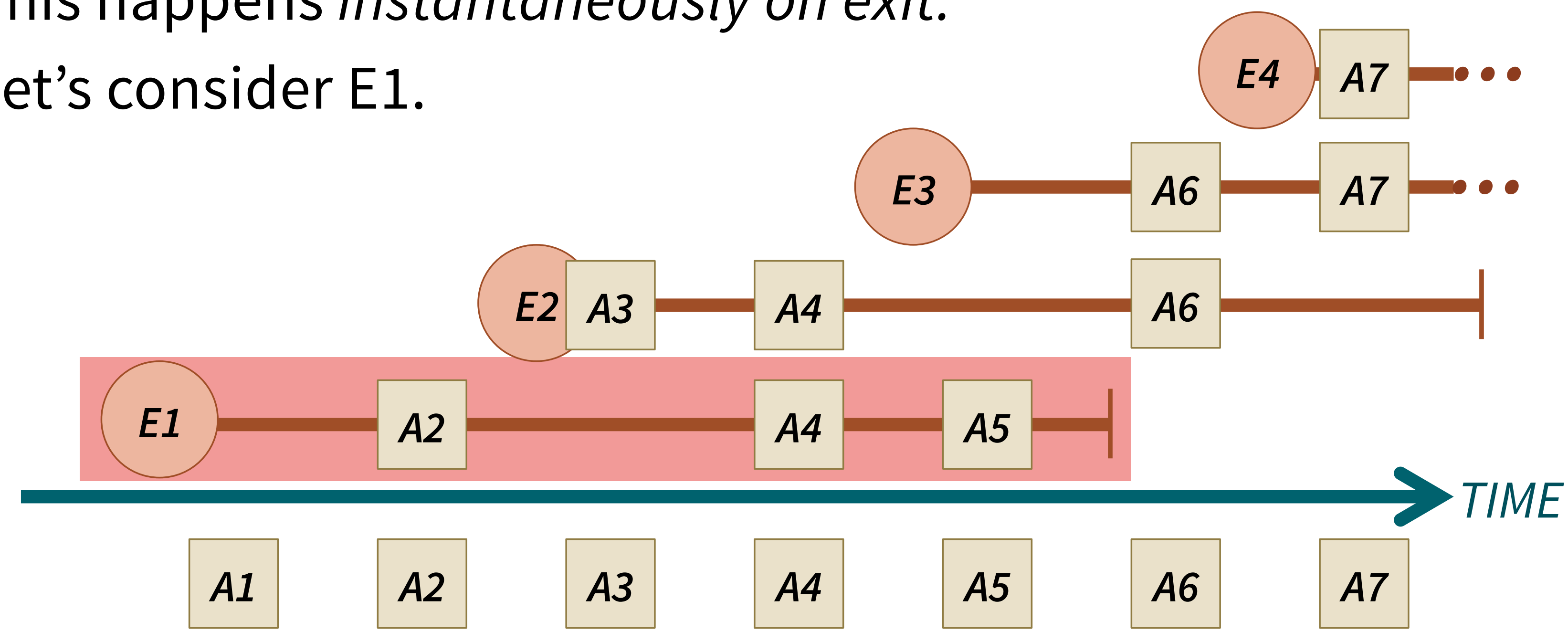


# *Dynamics: Employer screening and offers*

*On exit*, employers can screen for compatibility, and make offers to compatible applicants.

This happens *instantaneously on exit.*

Let's consider E1.



# *Dynamics: Employer screening and offers*

Let's consider E1.

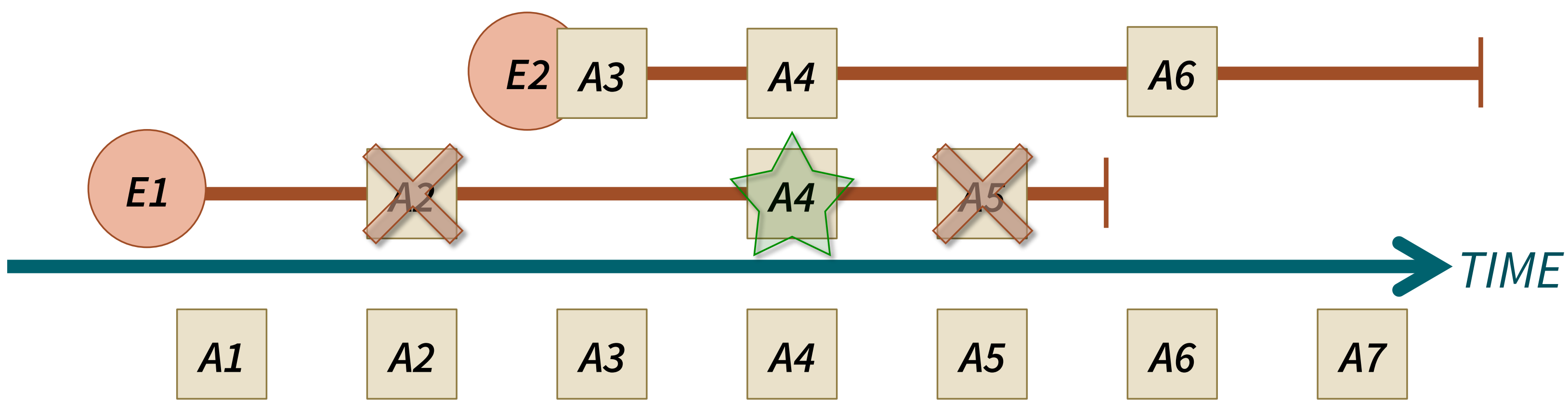
Suppose she screens A2 and A5,
and both are not compatible.

But A4 is compatible, so E1 makes her an offer.

A4 is available, and it is a weak dominant strategy
to accept, so she accepts.

# *Dynamics: Matching*

So E1 and A4 are matched.

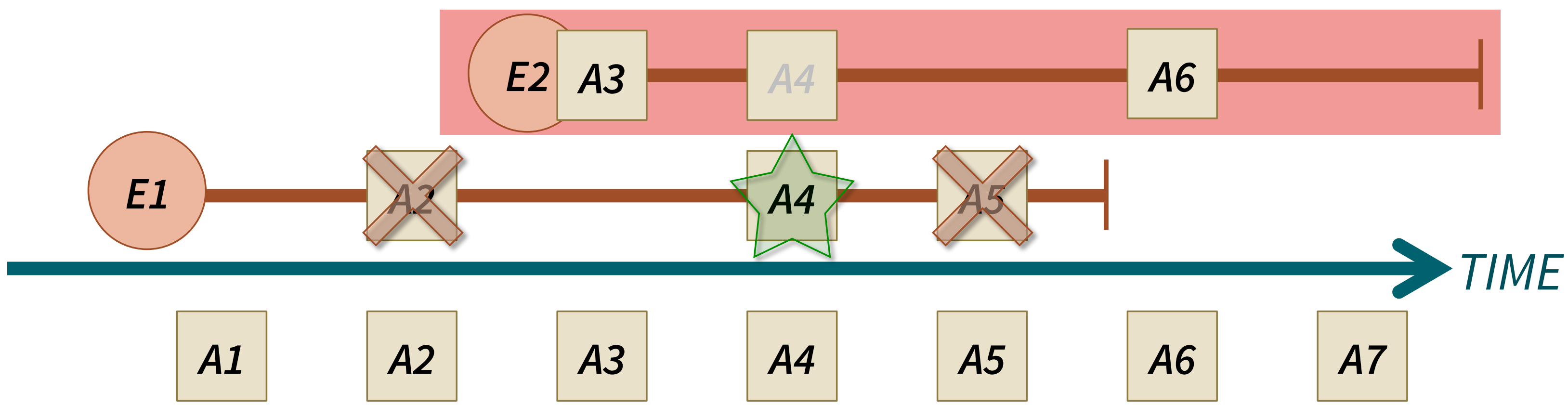


# *Dynamics: A lack of availability information*

So E1 and A4 are matched.

*Now A4 is no longer available…*
*but E2 doesn't know that when she exits!*

If E2 screens A4, this is *wasted effort* –
even if A4 turned out to be compatible.



# *Dynamics: A lack of availability information*

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even if A4 turns out to be compatible.


"Maybe that person had
**a limited number of hours** to give away,
**multiple interviews** set up, and
**somebody else just snatched him/her** before you."

## *The challenge*

Each employer chooses a *screening and offer strategy.*

Each applicant chooses an *application* strategy.

Agents may hold complex beliefs,
and play complex strategies as a result.

→ Characterizing PBE is challenging.

## *A mean field approach*

APPROXIMATING EQUILIBRIA VIA ASYMPTOTICS

## *A mean field model*

We simplify analysis by considering a *mean field model*:

Seems plausible that in markets with "many" agents:

1. For each applicant, whether an application yields an offer becomes *i.i.d* (say probability $p$)
2. For each employer, at time of exit, whether each applicant is available becomes *i.i.d.* (say probability $q$)

*Do these assumptions help analyze the game?*

*Do they hold as market size grows?*

***Answer to both is YES!***

## *Optimal responses*

Under previous assumptions,
given $p$ and $q$, agents have *simple optimal strategies.*

*Applicant optimal response:*

- Expected number of applications $m$

*Employer optimal response:* A (possibly) mixed strategy

- With probability $a$

  Simple sequential screening:
  screen, then make offer if compatible;
  repeat if needed

- With probability $1 - a$

  Exit without screening anyone

## *Mean field equilibrium*

A ***mean field equilibrium*** is
a pair of strategies (*m, a*) and
a pair (*p, q*)

such that

agents play *optimally*, given their beliefs, and
agent beliefs are *consistent* with agent strategies

**Theorem:**

MFE exist and are unique.

## *Mean field approximation*

**Theorem:**

As $n \to \infty$, the initial mean field assumptions hold.

In particular, any MFE is an approximate equilibrium for large enough $n$.

*(Proof via a stochastic contraction argument.)*

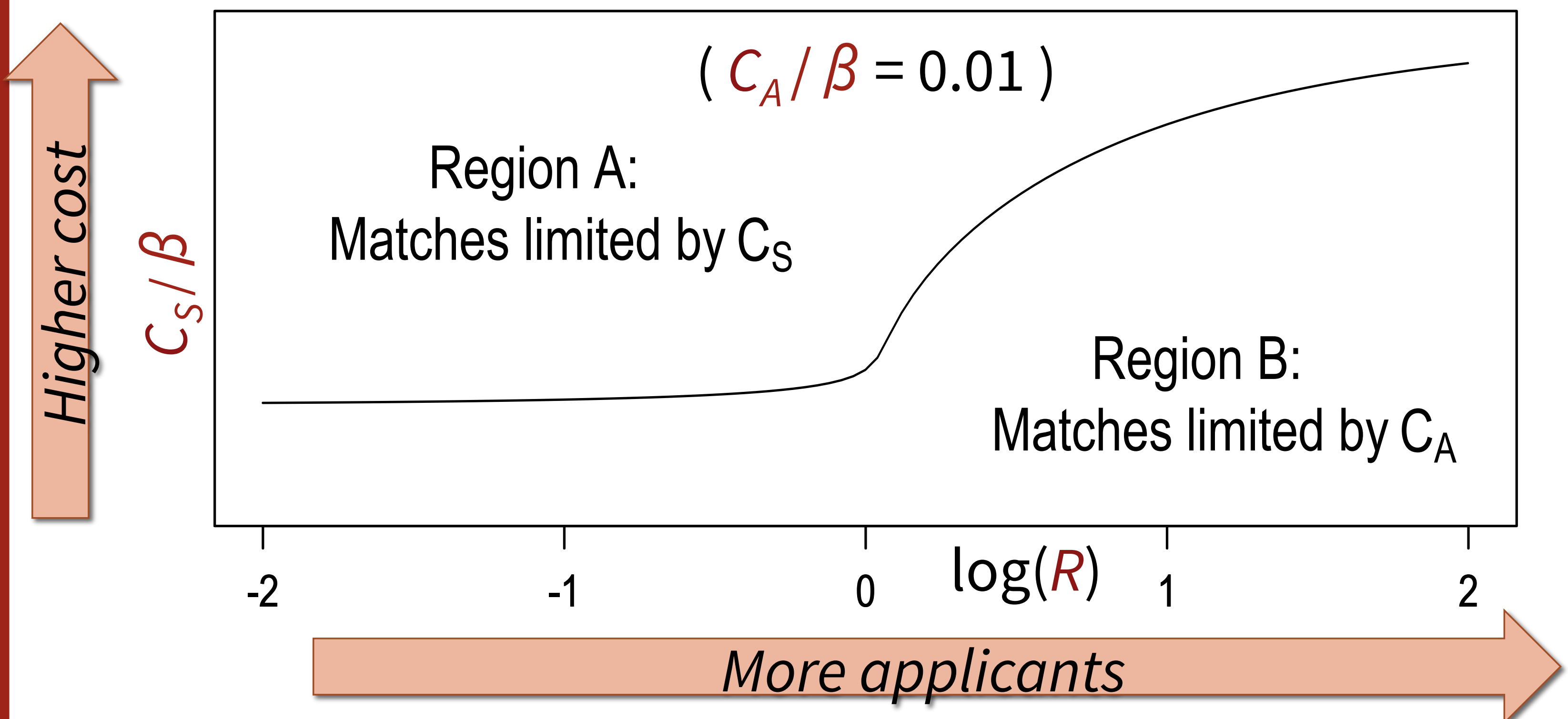
# *Model conclusions*

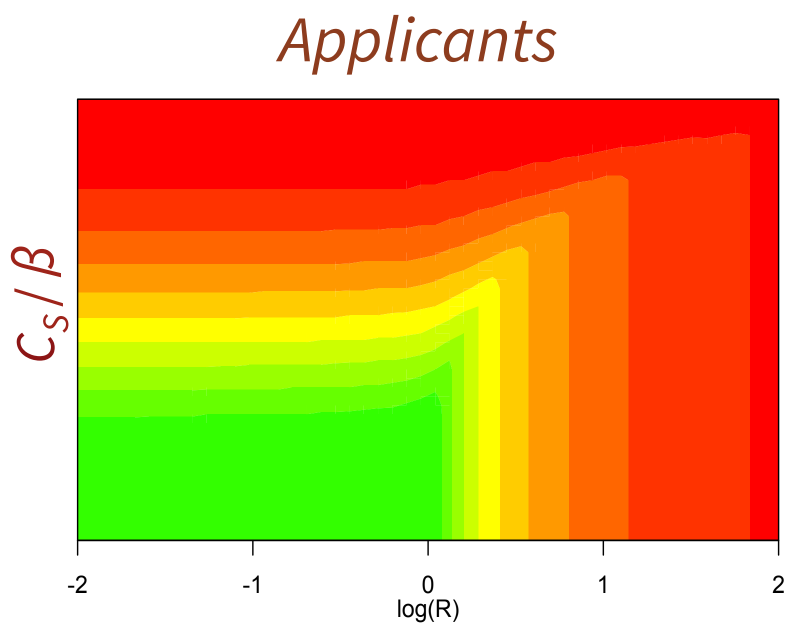
THE VALUE OF SIMPLE INTERVENTIONS

# *Preliminaries*

Market performance depends on three parameters:

1. # of applicants / # of employers: *$R$*
2. Normalized application cost: $C_A / \beta$
3. Normalized screening cost: $C_S / \beta$



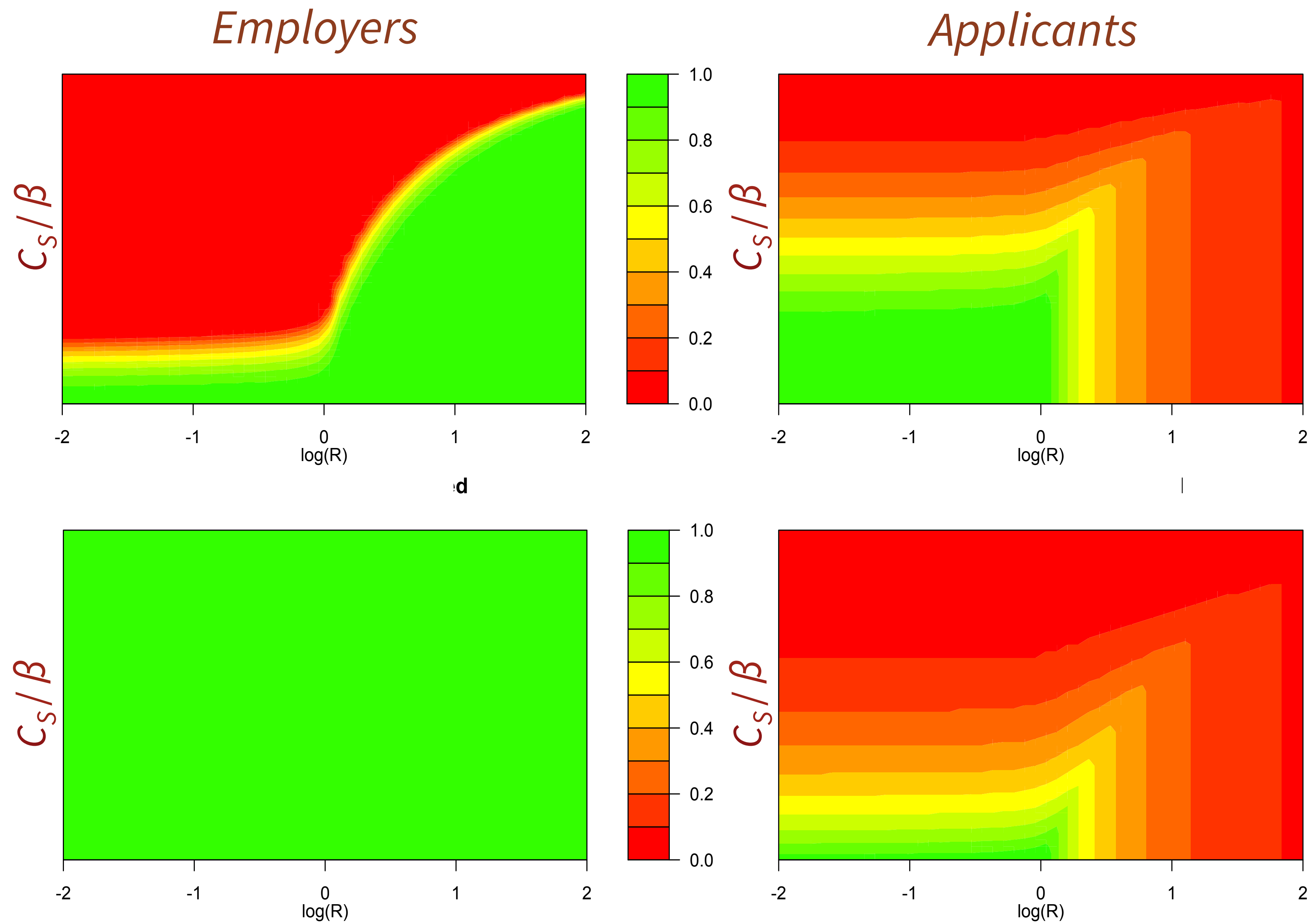




*Employers (Region A)*

- Applicants are unlikely to be available when screened.
- Employers get *zero surplus*, and some leave the market.
- Lowering $C_A$ expands Region A.

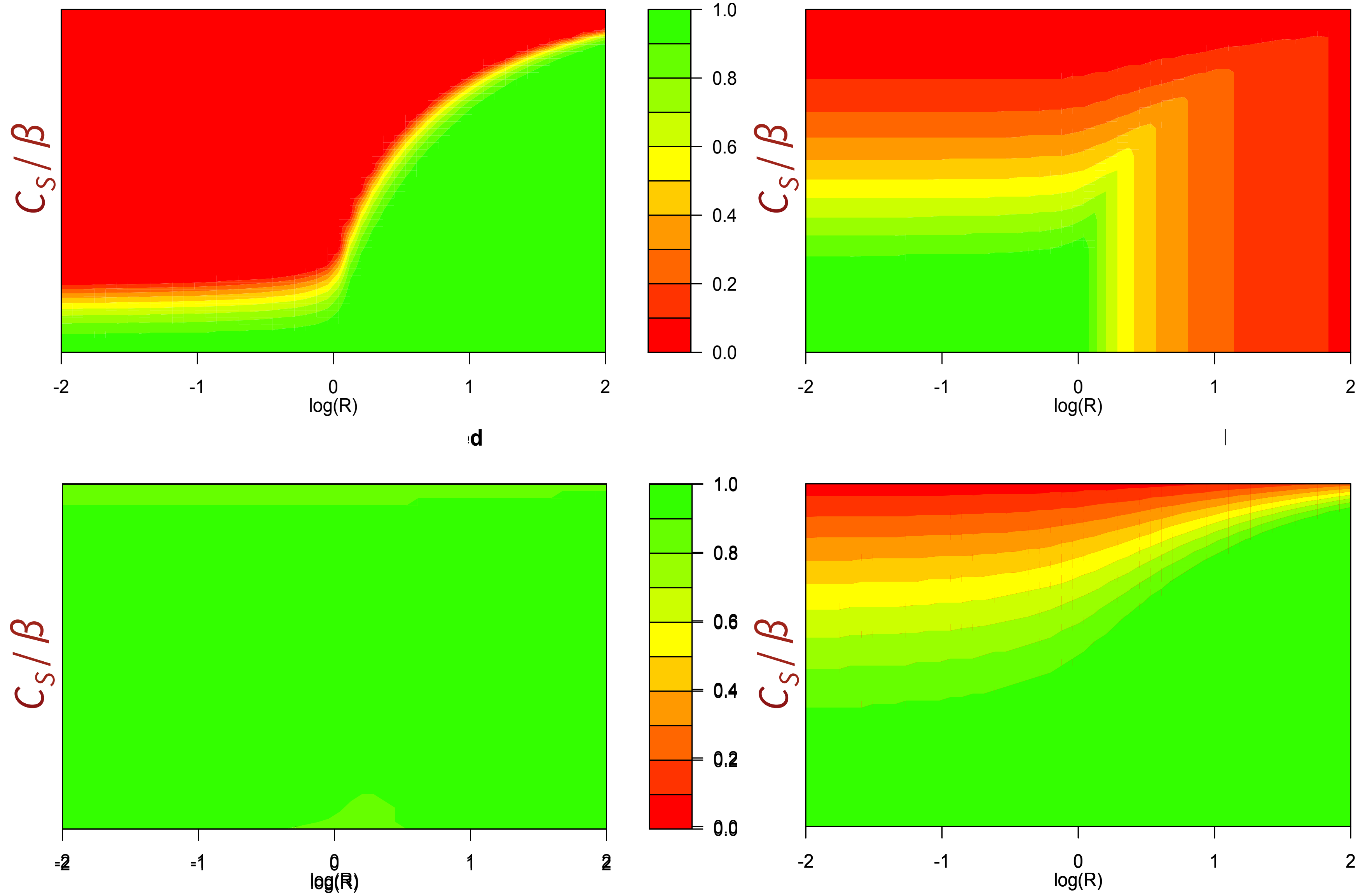
*Applicants*:

- Face "tragedy of the commons."
- Losses greatest when many applicants go unmatched.
- Losses persist as $C_A \rightarrow 0$.




What about raising $C_A$?

## *Employers* *Applicants*




What about limiting applications?

## *Helping applicants*

Define **A** as the *maximum applicant welfare* over all choices of $m$ and $a$.

**Theorem:**

Suppose we *restrict* applicants to choose $m \leq L$.

There always exists an $L$ that can strictly improve applicant welfare.

In Region B, the right choice of $L$ improves applicant welfare all the way to **A.**

**Theorem:**

Raising $C_A$ only reduces applicant welfare.

## *Helping employers*

Define **E** as the *maximum employer welfare*, over all choices of $m$ and $a$.

**Theorem:**

Suppose we *restrict* applicants to choose $m \leq L$.

In Region A, the right choice of $L$ improves employer welfare *from zero* all the way to **E.**

**Theorem:**

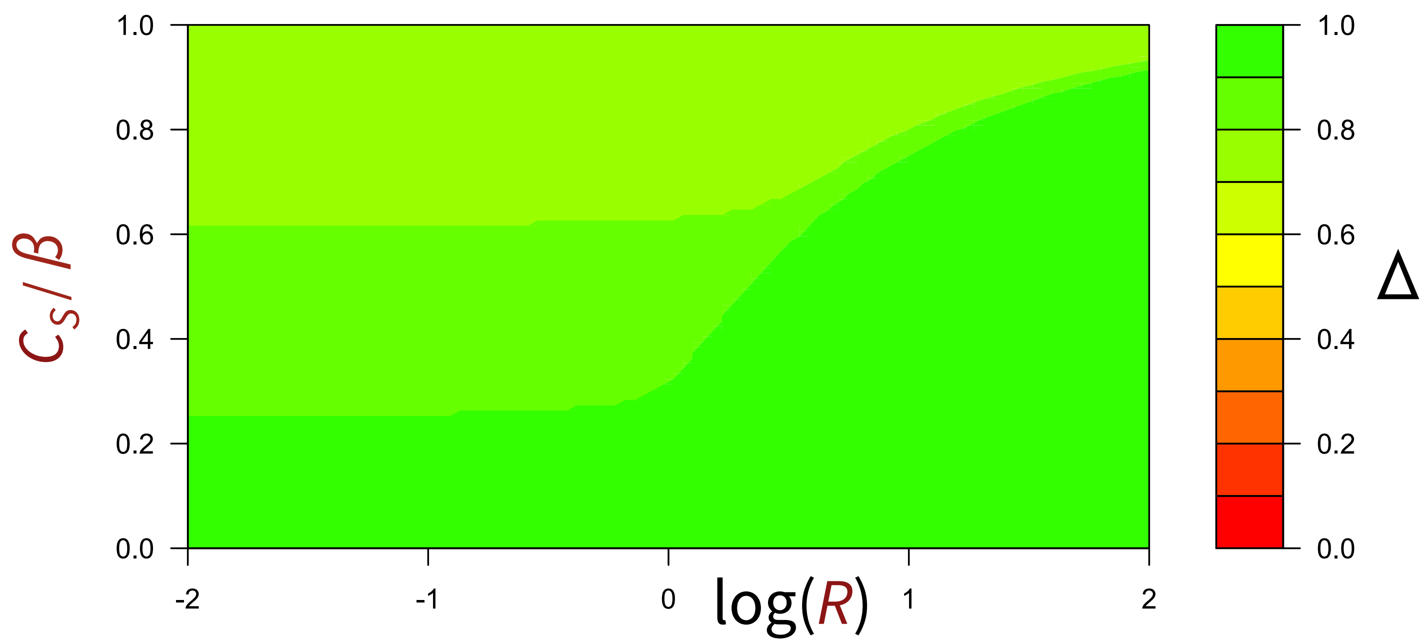
Raising $C_A$ has the same effect.

## *Fairness*

Let $\mathbf{A}^{\text{LIMIT}}$ (resp., $\mathbf{E}^{\text{LIMIT}}$ ) be the maximum welfare for applicants (resp., employers ) via an application limit.

Let $\Delta$ be the *largest fraction* such that we can *simultaneously achieve* $\Delta\,\mathbf{A}^{\text{LIMIT}}$ *and* $\Delta\,\mathbf{E}^{\text{LIMIT}}$.

How large is $\Delta$ ?



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How large is $\Delta$ ?

**Theorem:**

$\Delta \geq 3/4$.

# *Conclusion*

## *Conclusion*

When:

- application costs are low,
- screening is costly,
- and availability is unknown,

there can be a significant negative externality from applicants on employers.

Simple, *low-knowledge* interventions can benefit both sides.