Prob modelling, deep learning for . . .

ORIE 4742 - Info Theory and Bayesian ML

January 21, 2020

Semester: Spring 2020

essential course information

OH-TBA

- *instructor*: Sid Banerjee, sbanerjee@cornell.edu
- *TAs*: Cameron Ibrahim, cai29@cornell.edu
  Shengyuan Hu, sh797@cornell.edu
- *lectures*: TR 8:40-9:55am, Upson 216
- *website*
  https://piazza.com/cornell/spring2020/orie4742

the fine print

- *grading*
  50% homeworks, 20% prelim, 25% project,
  5% class participation
- *homeworks*
  6 homeworks (on average 2 weeks for each)
  teams of up to 3
  submit single Jupyter notebook, with theory answers in Markdown
  typically due Monday 5pm, on `https://cmsx.cs.cornell.edu`
  4 late days across homeworks, lowest grade dropped
- *prelim*
  in class, tentatively March 26 (before spring break)
  no final exam
- *project*
  use techniques learned in class on ML problem of your choosing
  teams of up to 3, report due on final exam date

what is this class about

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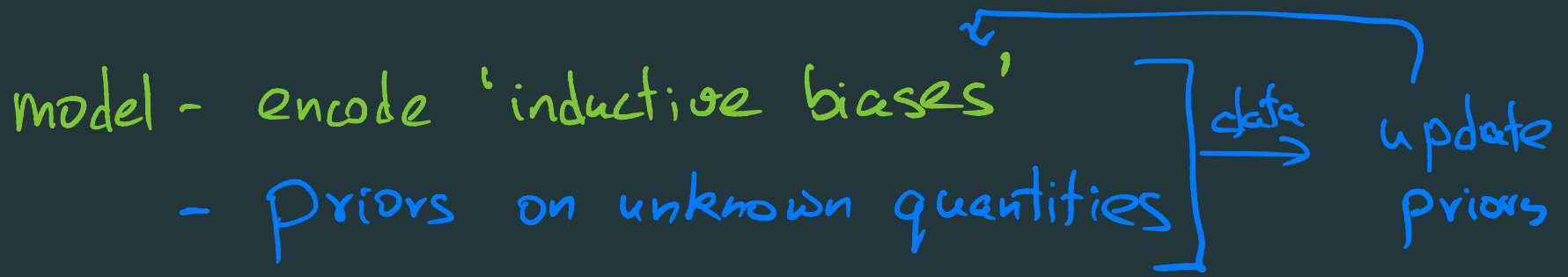
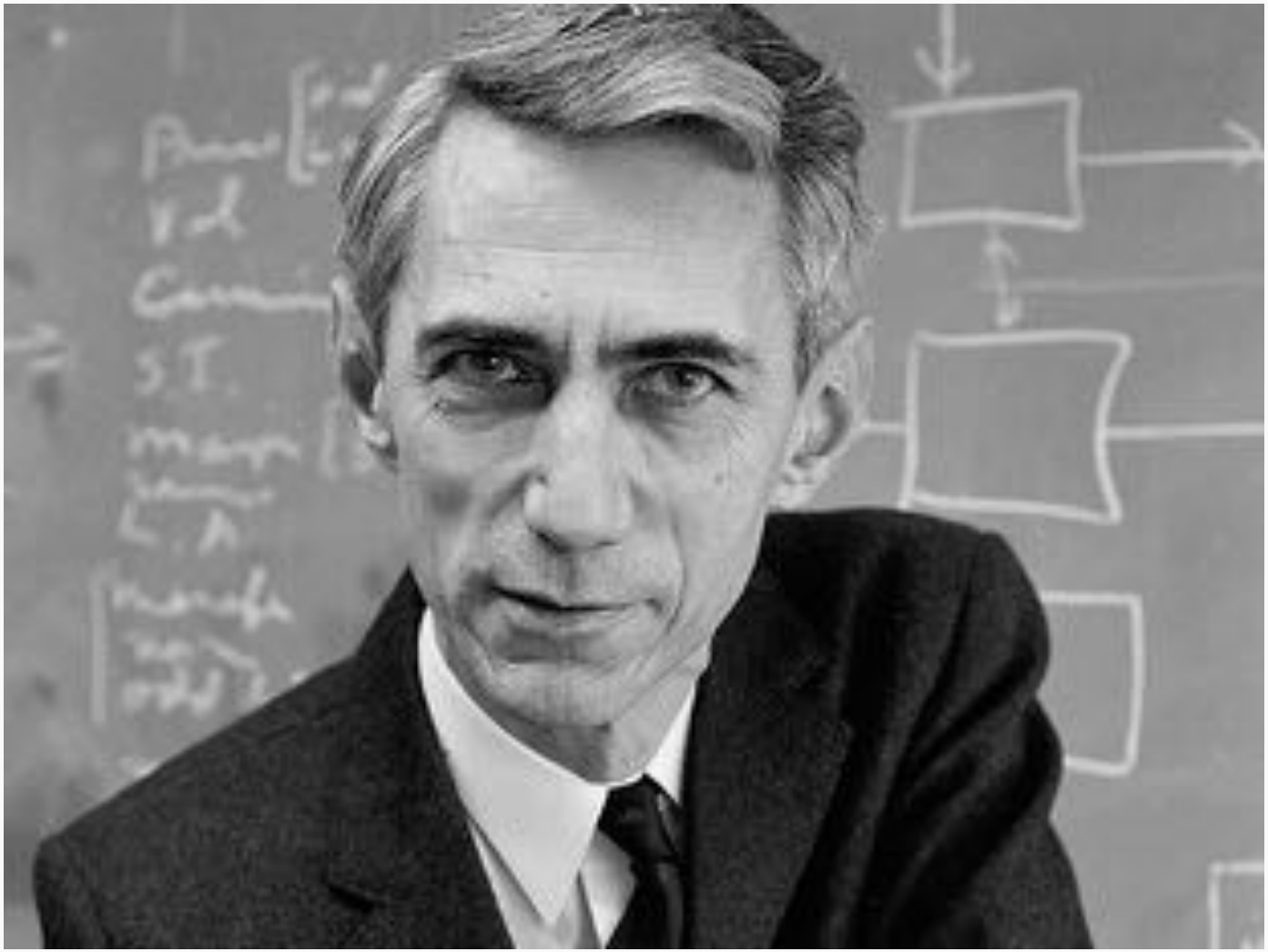
our approach in this course: probabilistic modeling

– bayesian inference: unified paradigm for learning and decision-making

– information theory: tool for designing and understanding data systems



problem: communicating over a noisy channel



reading assignment: chapter 1 of Mackay

communicating over channels

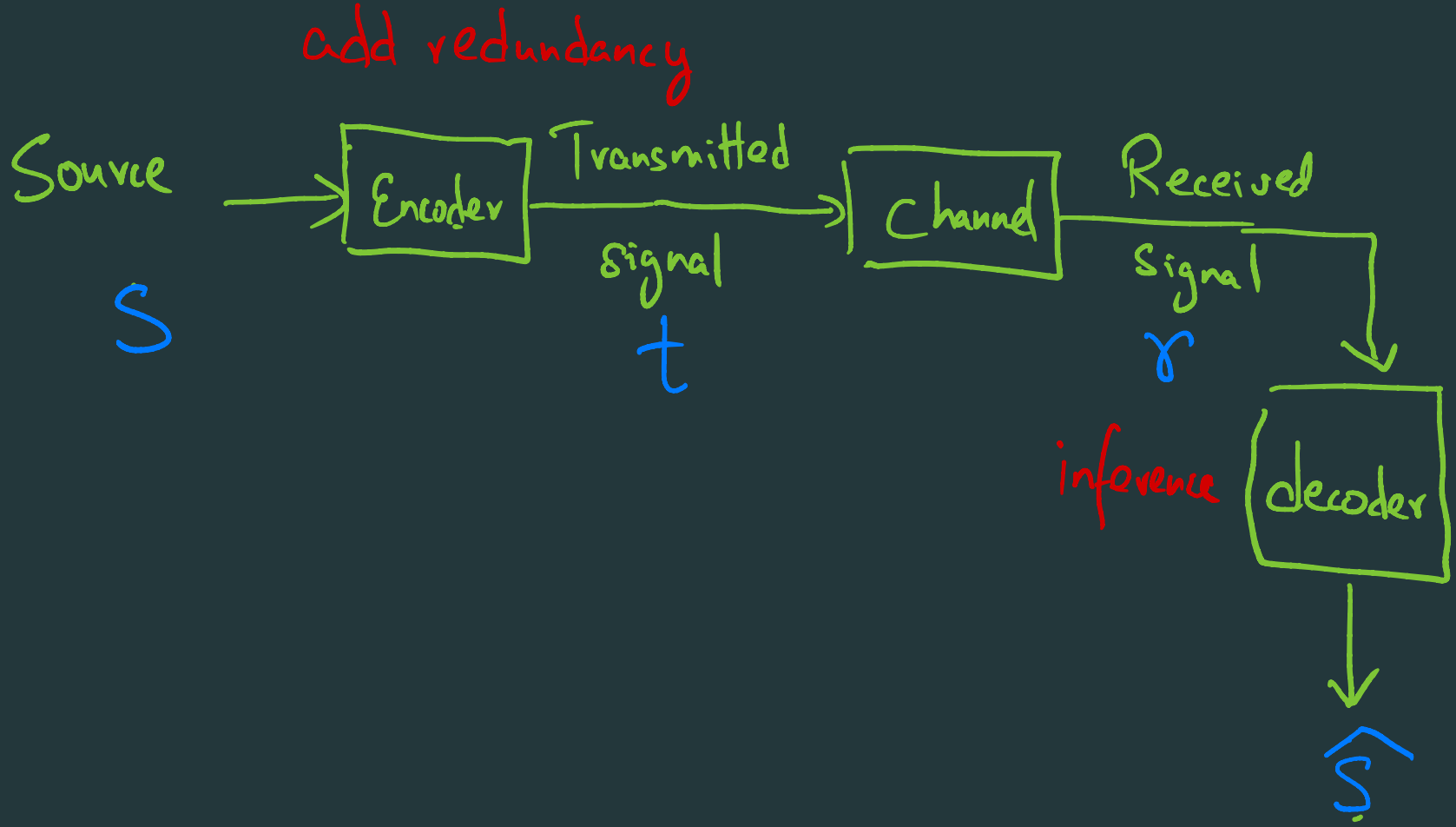
- mouth $\xrightarrow{\text{air}}$ ear
- ear $\xrightarrow{\text{retina, nerves}}$ brain
  ...
- dna, dna $\xrightarrow{\text{reproduction}}$ dna
- cellphone $\xleftrightarrow{\text{air}}$ basetower
- terrestrial antenna $\xleftrightarrow{\text{space}}$ Mars rover
- data $\xrightarrow{\text{storage}}$ data in future
- generative model $\xrightarrow{\text{data collection}}$ data

Signal = data + noise

Two approaches

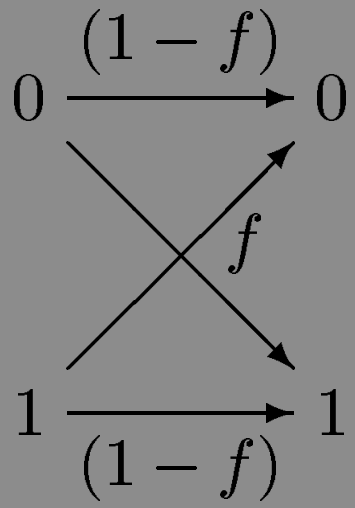
- physical approach
- systems approach

the system's solution



a toy model: the binary symmetric channel

$f \equiv \mathbb{P}[\text{bit flip}] \sim 0.1$



$\{0,1\}^n$ $\quad \mathbb{P}[r=0|t=0] = 1-f, \mathbb{P}[r=1|t=1] = 1-f$

$\mathbb{P}[r=1|t=0] = f, \mathbb{P}[r=0|t=1] = f$

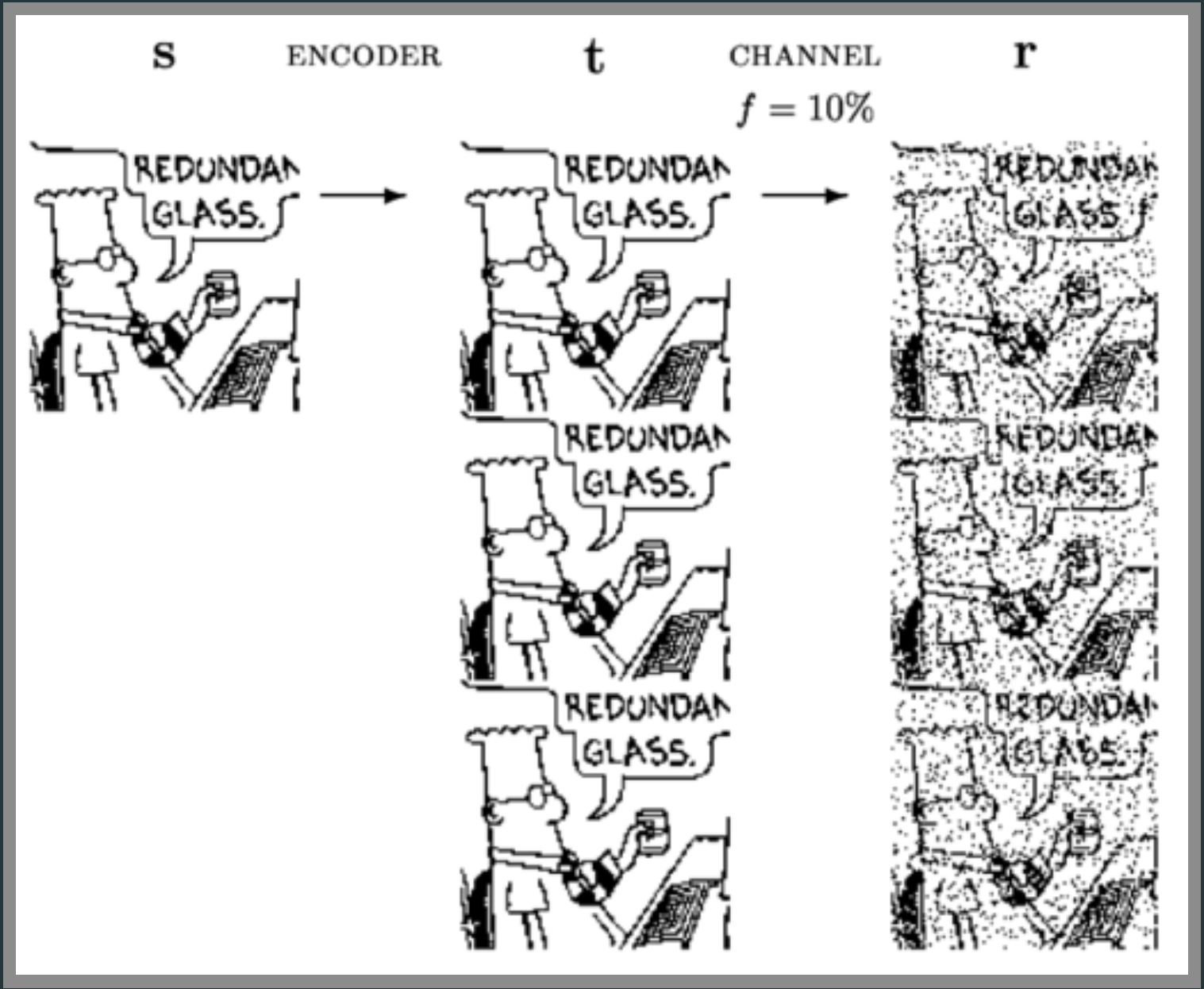
credit: David Mackay

ideas for encoding

$s = 011$, noise $= 00100001100$

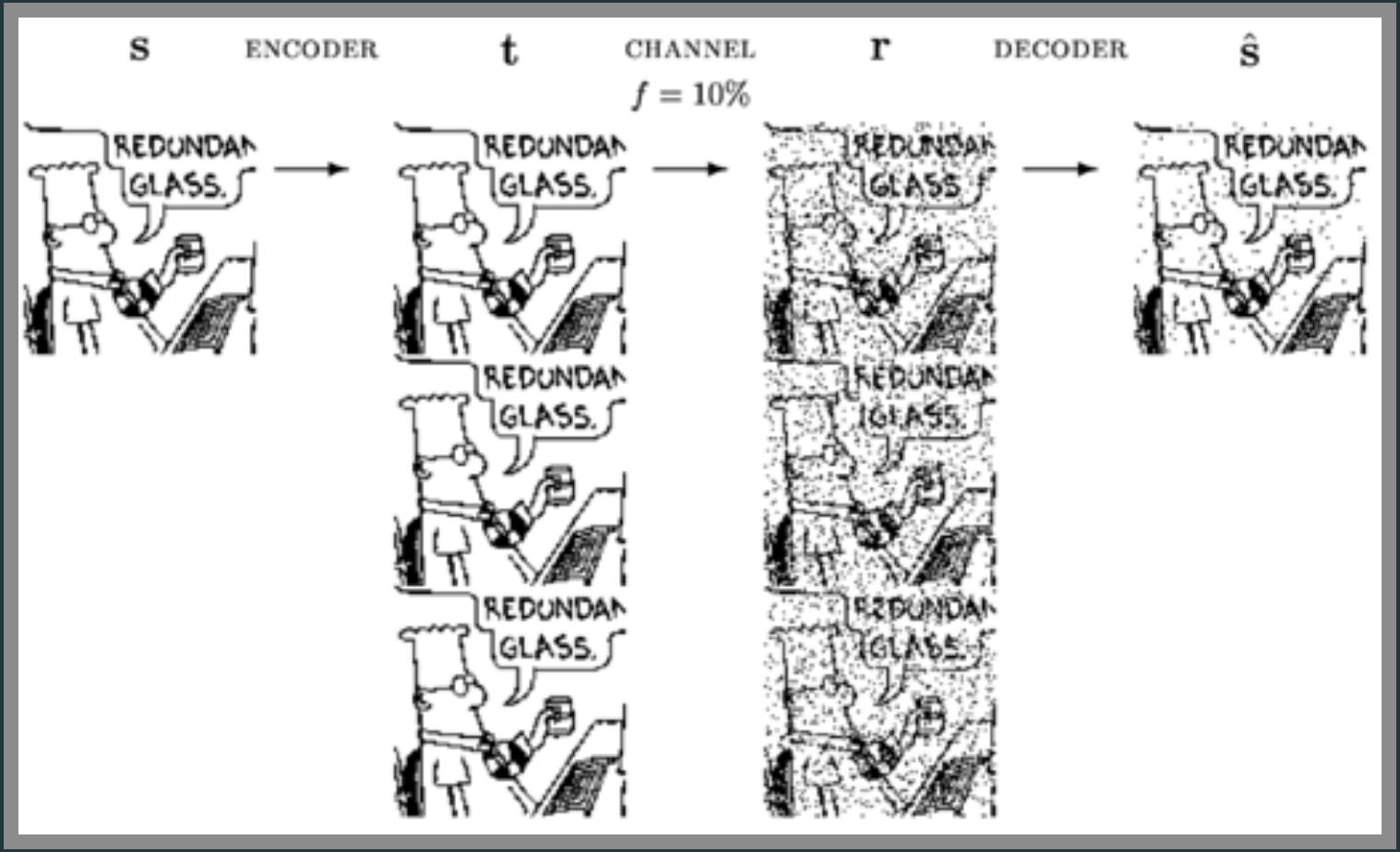
- $t = s \oplus n \bmod 2$
- no encoding - $r = 010$
- repeat 2 times - $t = 00\ 11\ 11$, $r = 00\ 01\ 11$
  3 times - $t = 000\ 111\ 111$, $r = 001\ 111\ 100$
  decoding - majority
- Parity bits - $t_1 = s_1, t_2 = s_2, t_3 = s_3, t_4 = s_1 + s_2 + s_3 \bmod 2$
  $t = 0110$, $r = 0100$

repetition codes: encoding



credit: David Mackay

repetition codes: decoding



credit: David Mackay

decoding rule ≡ majority

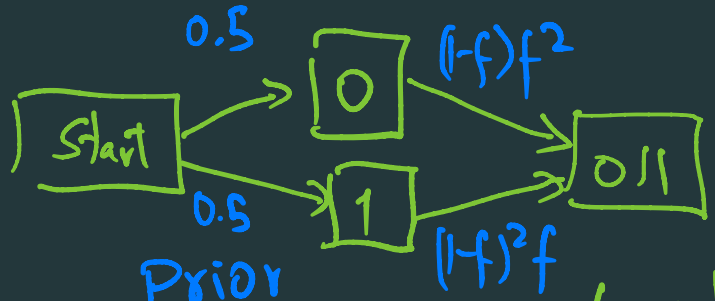
repetition codes: inference (inverse probability)

- 3-repititions, $r = 011$

$$p(r=011 \mid t=0) = (1-f)\cdot f \cdot f = (1-f) f^2 \quad \leftarrow \#\text{ of bit flips}$$

$$p(r=011 \mid t=1) = f \cdot (1-f)\cdot(1-f) = (1-f)^2 f^1$$

- Bayes thm: $p(t=0 \mid r=011) = \dfrac{p(t=0)\,p(r=011 \mid t=0)}{Z}$



$$Z = \sum_{i=0,1} p(t=i)\,p(r=011 \mid t=i)$$

$$p(t=1 \mid r=011) = \frac{p(t=1)\,p(r=011 \mid t=1)}{Z}$$

normalization constant

- Inference ≡ choosing signal that 'best explains' data
- If priors equal, optimal inference rule ≡ majority

repetition codes: performance

- If we use optimal inference rule, what is the probability of bit error (as a fn of # of reps R)
- $0 \xrightarrow{R=5} 00000 \xrightarrow{\text{BSC}} r \xrightarrow{\text{majority}} \hat{s}$

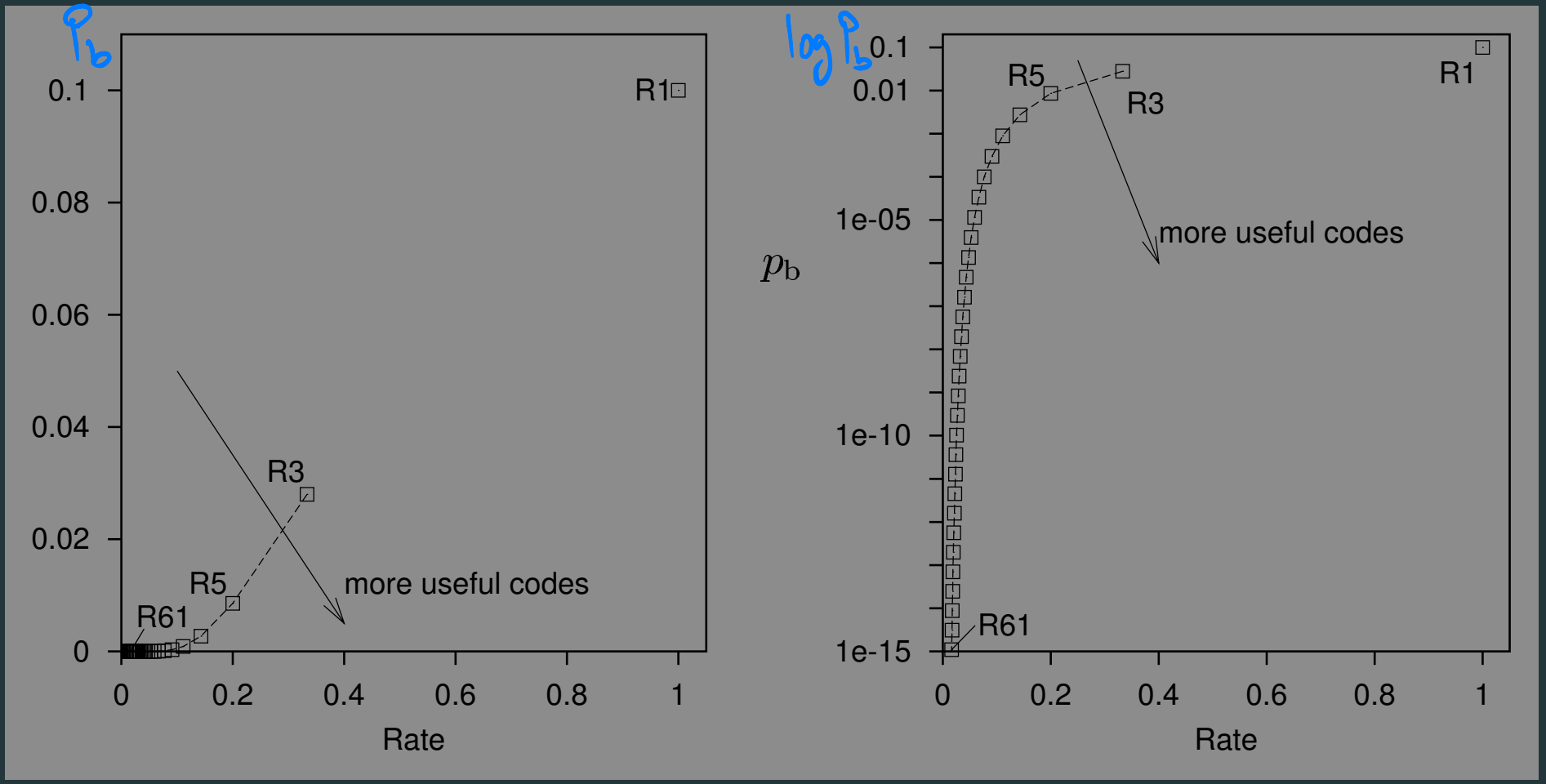
$$\mathbb{P}[\hat{s} \neq s] = \mathbb{P}[\#\text{ of flips} \geq 3]$$

$$= \binom{5}{3} f^3 (1-f)^2 + \binom{5}{4} f^4 (1-f) + f^5$$

$$\approx 10 f^3.$$

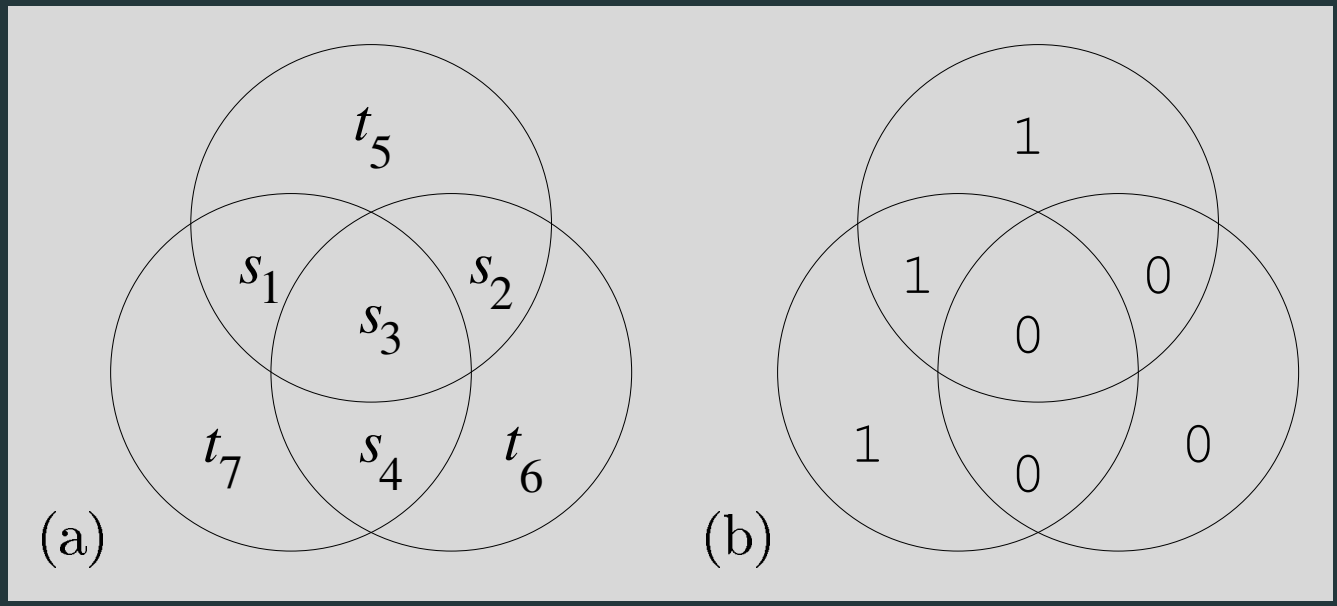
- In general $\mathbb{P}[\hat{s} \neq s] \approx c f^{\lfloor R/2 \rfloor}$

repetition codes: the rate-error plot



credit: David Mackay

the (7,4) Hamming code



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the (7,4) Hamming code: performance

s	t	s	t	s	t	s	t
0000	0000000	0100	0100110	1000	1000101	1100	1100011
0001	0001011	0101	0101101	1001	1001110	1101	1101000
0010	0010111	0110	0110001	1010	1010010	1110	1110100
0011	0011100	0111	0111010	1011	1011001	1111	1111111

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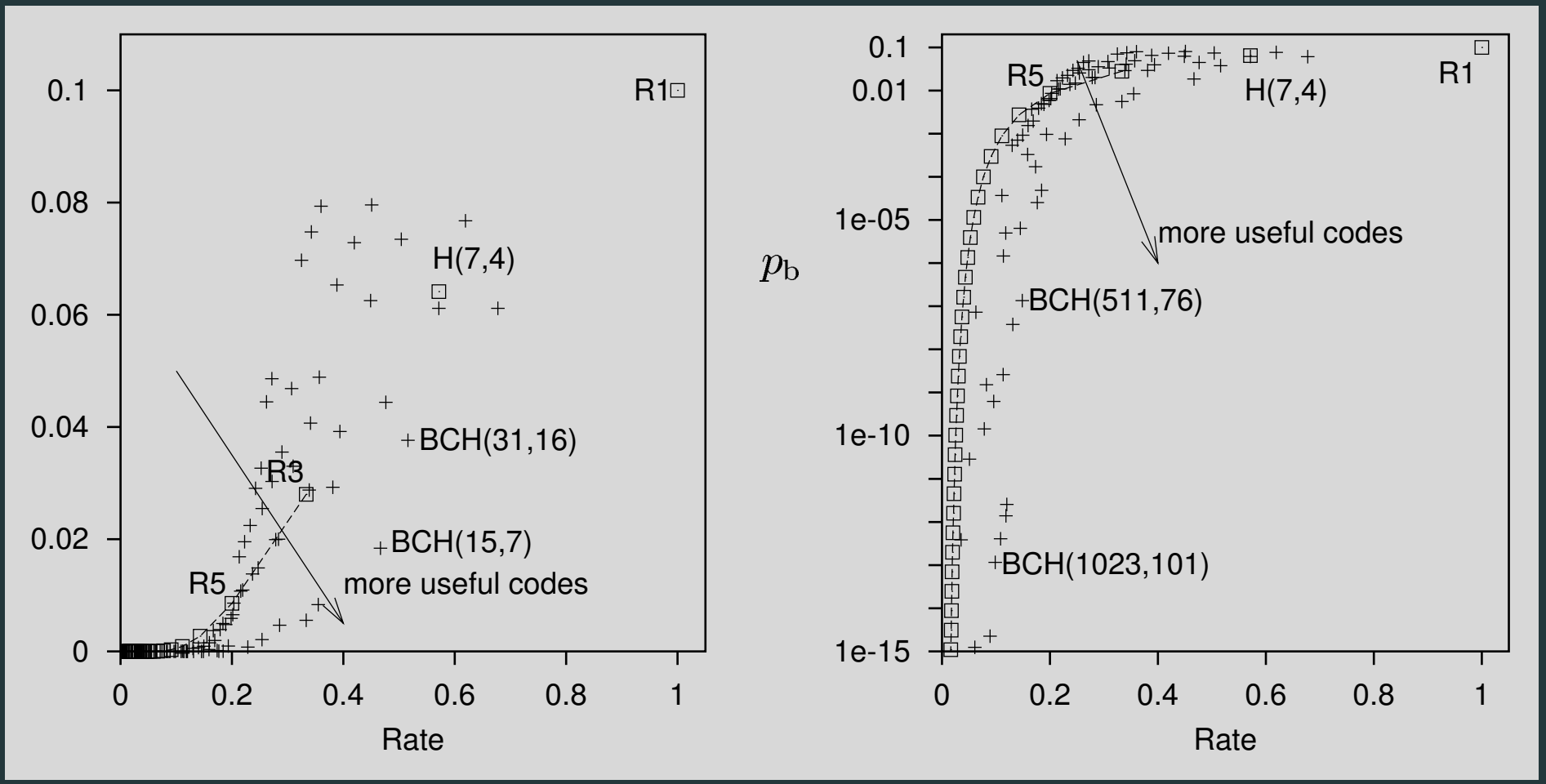
distance between codewords

the minimal Hamming distance between any two correct codewords is 3

Corrolary - (7,4) Hamming code 'fixes' 1 error

the rate-error plot



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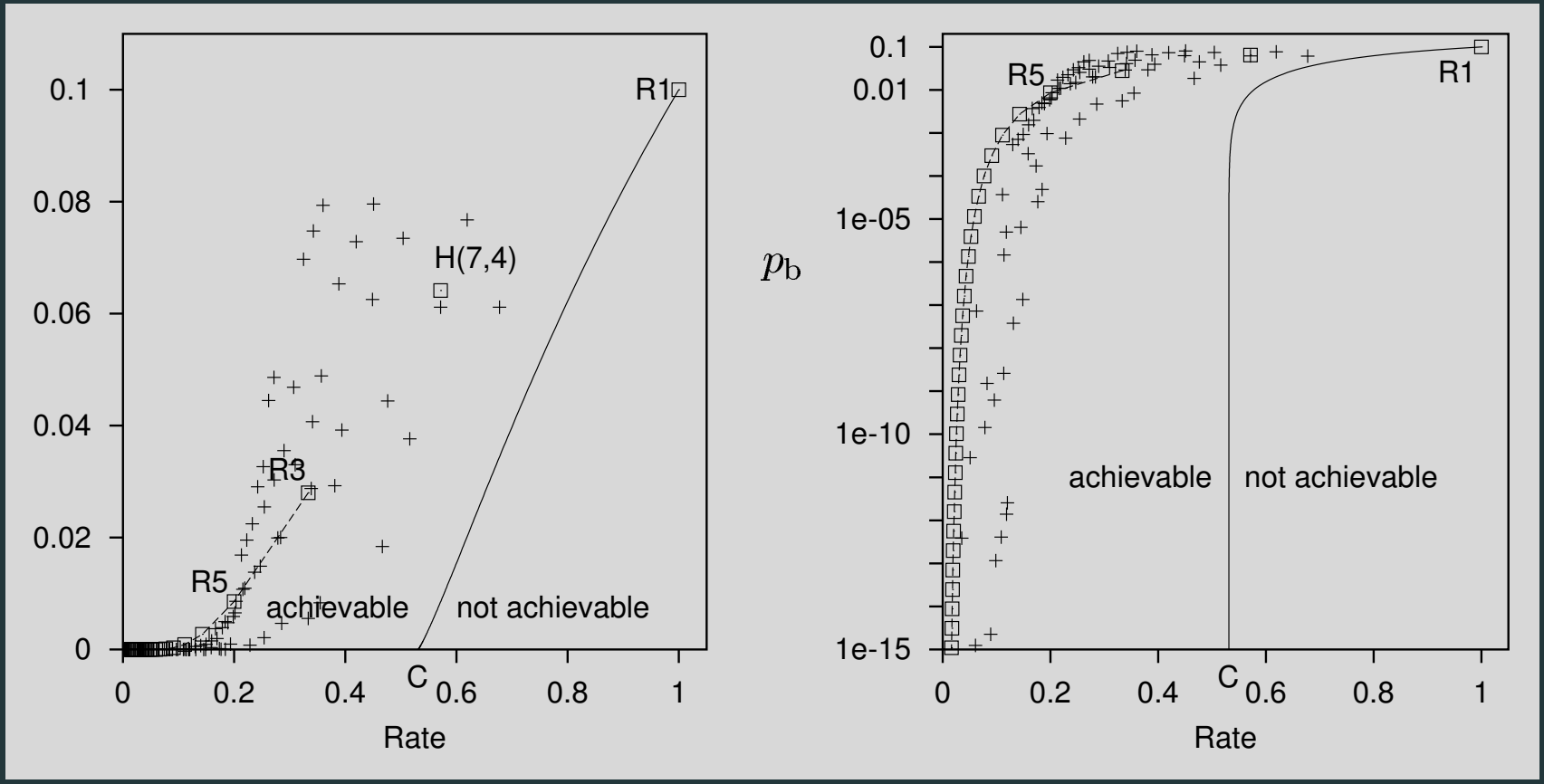
Shannon's channel coding theorem

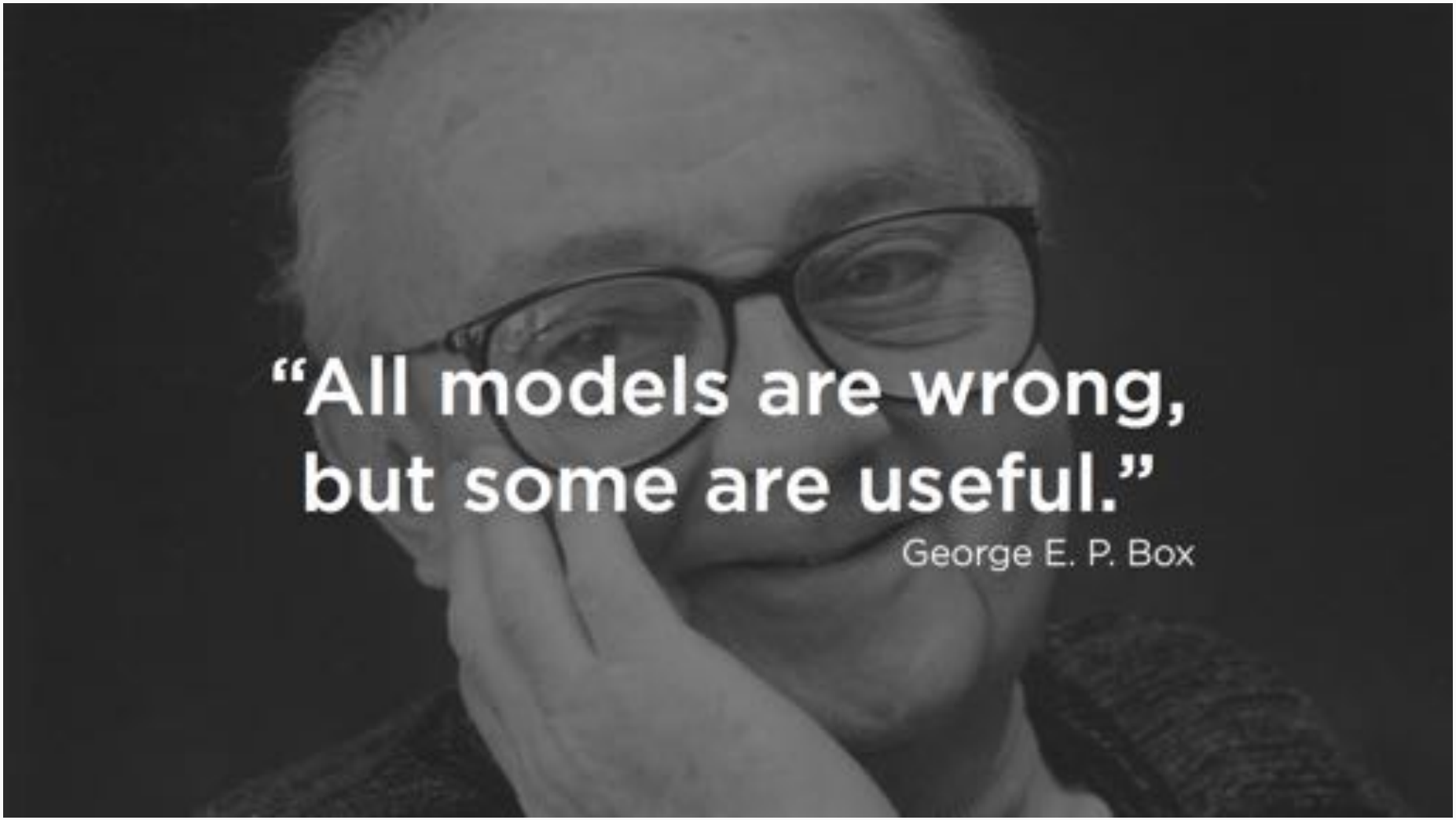


Theorem (Claude Shannon, 1948)

for any channel, 0-error communication is possible at a rate up to $C > 0$

For BSC - $C = H_2(f) = f \log_2\left(\frac{1}{f}\right) + (1-f)\log_2\left(\frac{1}{1-f}\right)$

noisy channel communication $\leftrightarrow$ machine learning



redundancy $\Rightarrow$ inference

```
Emma Woodh*use, hands*me, clever* and rich,*with a
comfortab*e home an* happy di*position,*seemed to*unite som*
of the b*st bless*ngs of e*istence;*and had *ived nea*ly
twenty *ne year* in the*world w*th very*little *o distr*ss
or vex*her.  *he was*the yo*ngest *f the *wo dau*hters *f a
most *ffect*onate* indu*gent *ather* and *ad, i* cons*quenc*
of h*r si*ter'* mar*iage* bee* mis*ress*of h*s ho*se f*om a
ver* ea*ly *eri*d. *er *oth*r h*d d*ed *oo *ong*ago*for*her
to*ha*e *or* t*an*an*in*is*in*t *em*mb*an*e *f *er*ca*es*es*
a*d*h*r*p*a*e*h*d*b*e* *u*p*i*d*b* *n*e*c*l*e*t*w*m*n*a*
g**e**e**,**h**h** **l**n**i**l**s**r**o**a**o**e**i*
a***c***n***S***e***y***s***d***s***a***r***e***n***
W****o****s****i****l****a****g****n****t****a****e****v***
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a*d*h*r*p*a*e*h*d*b*e* *u*p*i*d*b* *n*e*c*l*e*t*w*m*n*a*
g**e**e**,**h**h** **l**n**i**l**s**r**o**a**o**e**i*
a***c***n***S***e***y***s***d***s***a***r***e***n***
W****o****s****i****l****a****g****n****t****a****e****v***
```

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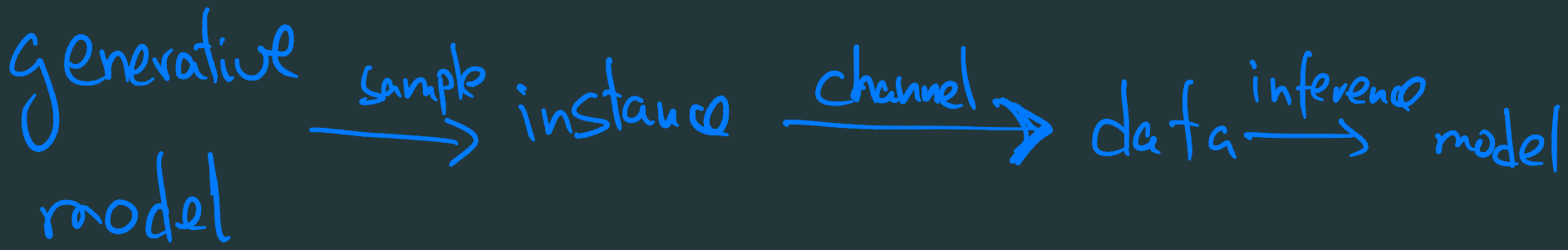
```
Emma Woodhouse, handsome, clever, and rich, with a
comfortable home and happy disposition, seemed to unite some
of the best blessings of existence; and had lived nearly
twenty one years in the world with very little to distress
or vex her.  She was the youngest of the two daughters of a
most affectionate, indulgent father; and had, in consequence
of her sister's marriage, been mistress of his house from a
very early period. Her mother had died too long ago for her
to have more than an indistinct remembrance of her caresses;
and her place had been supplied by an excellent woman as
governess, who had fallen little short of a mother in
affection.  Sixteen years had Miss Taylor been in Mr
Woodhouse's family, less as a governess than a friend, very
```

the noisy channel model in ML



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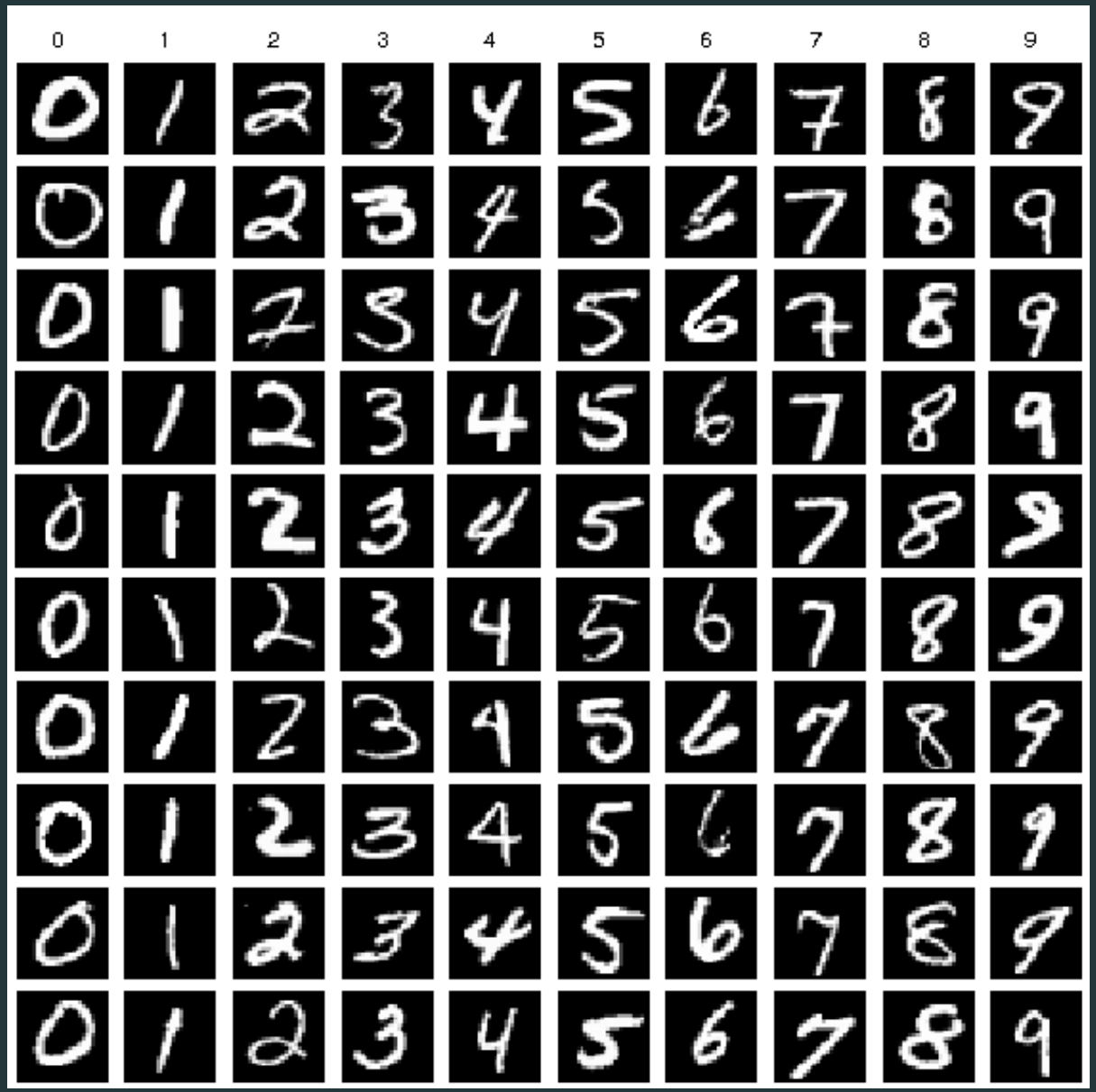
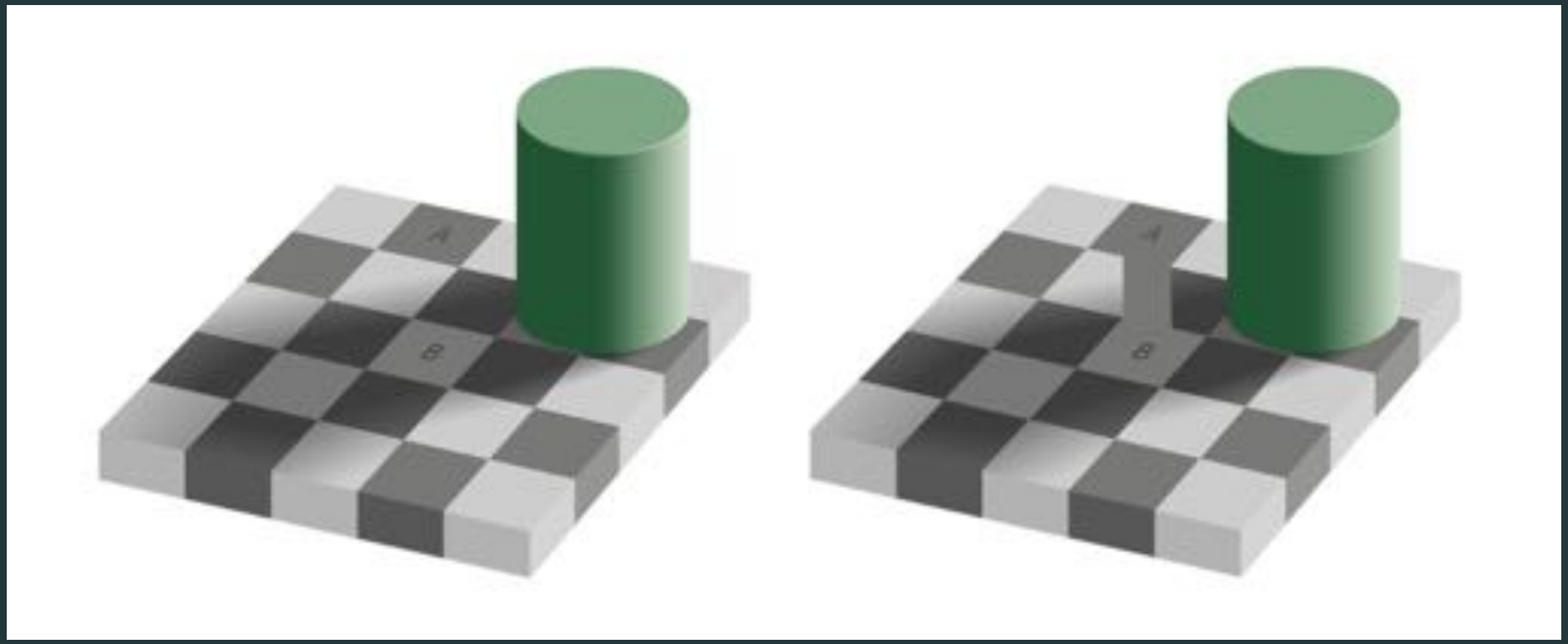


credit: MNIST dataset

we are inherently bayesian



credit: quantamagazine.org, original image by Edward Adelson

"Tile A looks darker than tile B, though they are both the same shade (connecting the squares makes this clearer). The brain uses coloring of nearby tiles and location of the shadow to make inferences about the tile colors. . . lead to the perception that A and B are shaded differently."

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– information theory: tool for designing and understanding data systems

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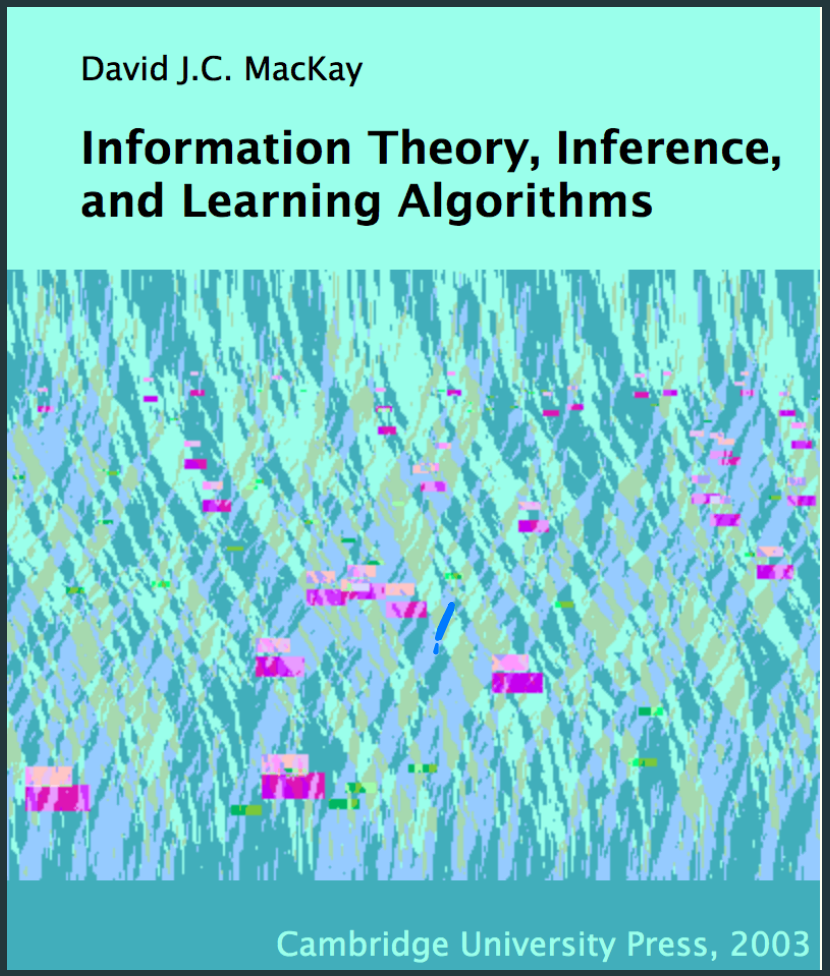
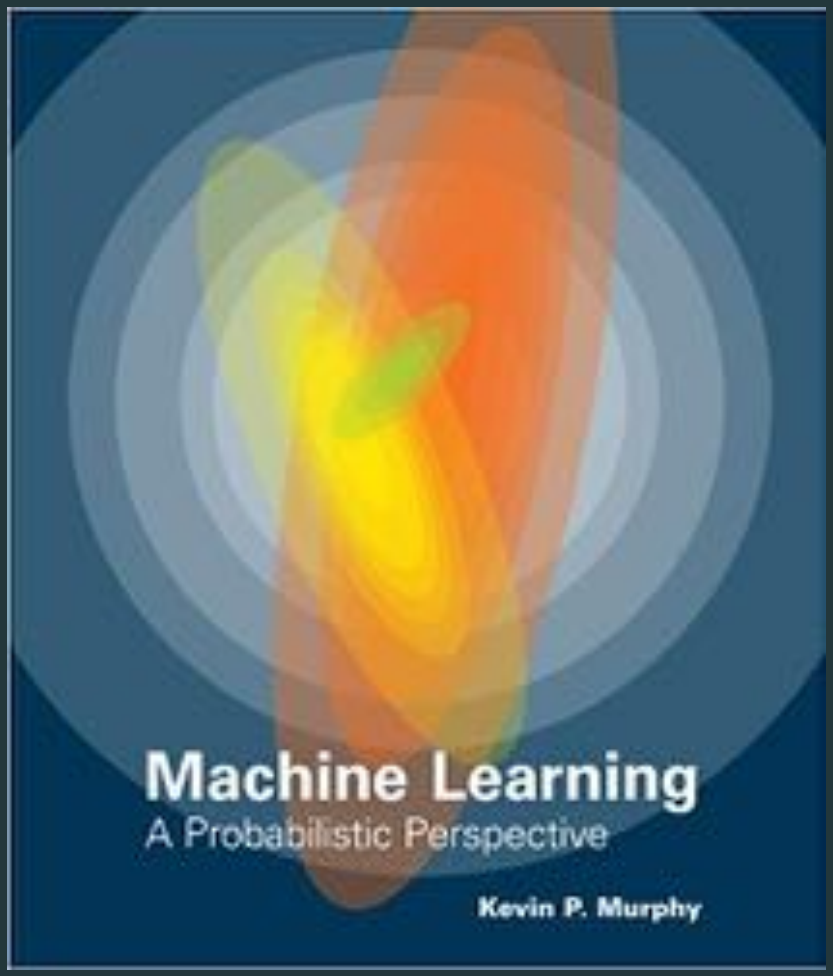
- model-based decision-making: bayesian optimization, causal inference, sequential decision-making and reinforcement learning

aids in learning

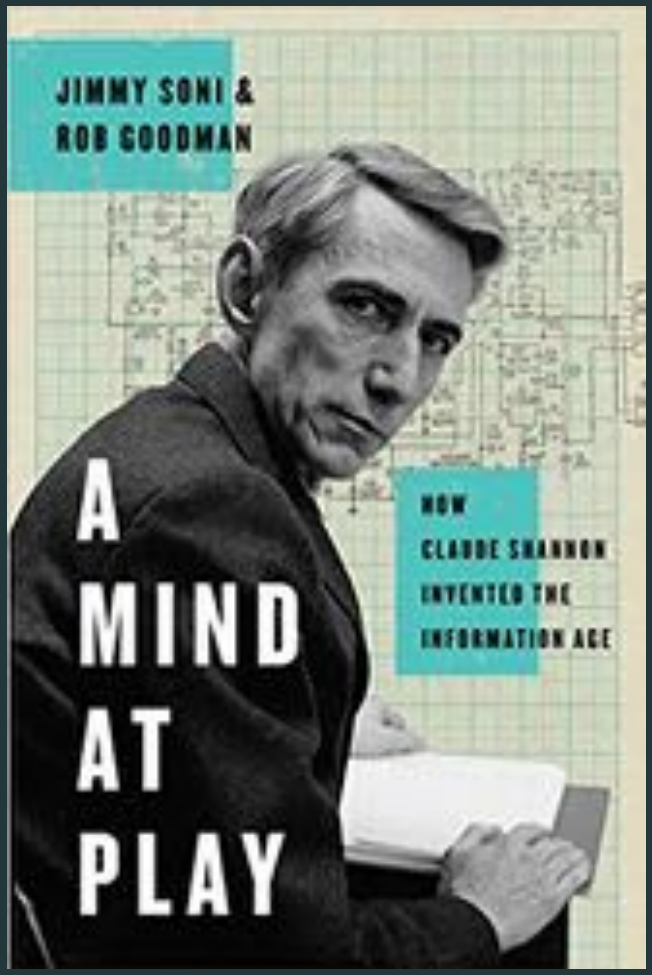
the following books are excellent references for most topics in the course

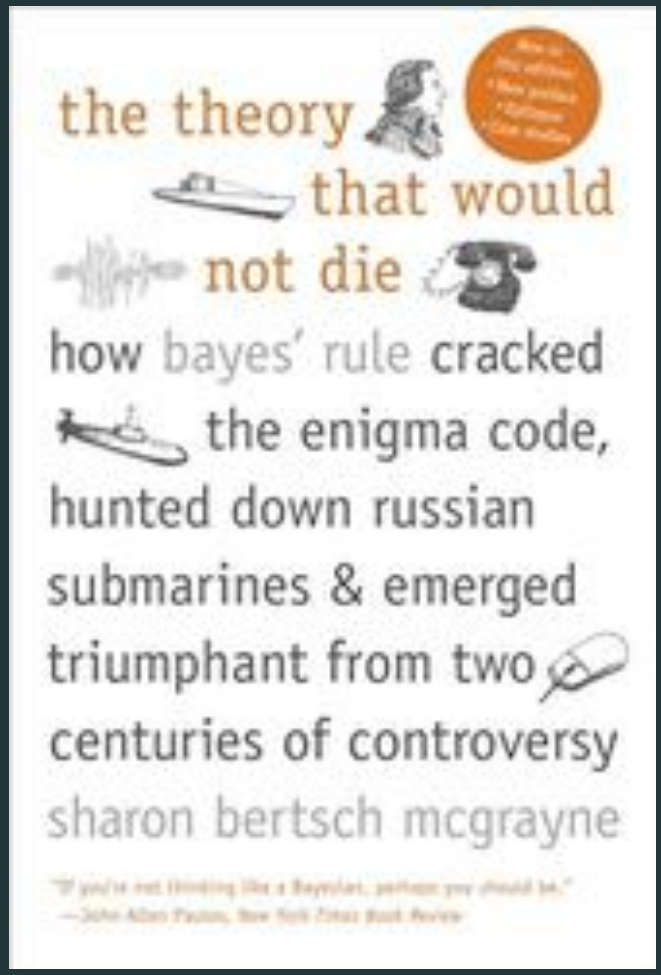




aids in getting excited about learning

the following help understand the larger context of what we will study





is this course right for you?

- prerequisites:
  - linear algebra, calculus
  - probability: ideally at the level of ORIE 3500
  - programming: python/julia

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- caveat emptor:
  - may not be ideal as a first course in ML
  - we will focus on Bayesian methods, and ignore alternate 'frequentist' methods
  - will involve a fair bit of additional reading and programming, and some 'Bayesian philosophy'