

Dynamic Pricing in Ridesharing Platforms

A Queueing Approach

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Ridesharing and Pricing

Ridesharing platforms



Examples of major platforms: Lyft, Uber, Sidecar

This talk: Pricing and ridesharing

Ridesharing is somewhat unique among online platforms:

The platform sets the transaction price.

Our goal: Understand optimal pricing strategy.

Our contributions

1. A model that combines:
 - ▶ Strategic behavior of passengers and drivers
 - ▶ Pricing behavior of the platform
 - ▶ Queueing behavior of the system
2. What are the advantages of *dynamic* pricing over *static* pricing?
 - ▶ *Static*: Constant over several hour periods
 - ▶ *Dynamic*: Pricing changes in response to system state; "surge", "prime time"

Related work

Our work sits at a nexus between several different lines of research:

1. *Matching queues* (cf. [Adan and Weiss 2012])
2. *Strategic queueing models* (cf. [Naor 1969])
3. *Two-sided platforms* (cf. [Rochet and Tirole 2003, 2006])
4. *Revenue management* (cf. [Talluri and van Ryzin 2006])
5. *Large-scale matching markets* (cf. [Azevedo and Budish 2013])
6. *Mean field equilibrium* (cf. [Weintraub et al. 2008])

Model

Two types: Strategic and queueing

We need a *strategic model* that captures:

1. Platform pricing
2. Passenger incentives
3. Driver incentives

We need a *queueing model* that captures:

1. Driver time spent idling vs. driving
2. Ride requests blocked vs. served

Preliminaries

1. Focus on a *block* of time (e.g., several hours) over which arrival rates are roughly stable
2. Focus on a single region (e.g., a single city neighborhood)
 - ▶ For technical simplicity
 - ▶ Insights generalize to networks of regions
3. Focus on throughput: rate of completed rides
 - ▶ For technical simplicity
 - ▶ Same results for profit, when system is supply-limited

Strategic modeling: Platform pricing

Platforms:

- ▶ Earn a (fixed) fraction γ of every dollar spent (e.g., 20%)
- ▶ Need *both* drivers (supply) and passengers (demand)
- ▶ Use pricing to align the two sides

Load-dependent pricing:

If # of available drivers = A , then price offered to ride = $P(A)$

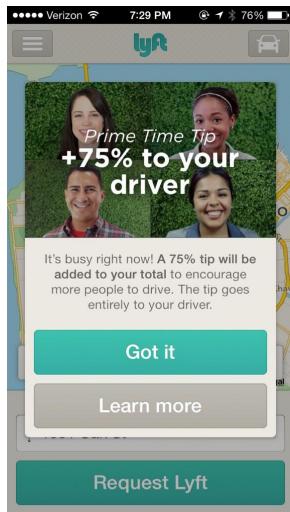
Strategic model: Platform pricing

In practice:

- ▶ Platforms charge a time- and distance-dependent *base price*
- ▶ Platforms manipulate price through a *multiplier*
- ▶ Base price typically is not varied

In our model:

price \equiv *multiplier*.



Strategic model: Passengers

How do passengers enter?

- ▶ Passenger \equiv one ride request
- ▶ Sees *instantaneous* ride price
- ▶ Enters if price $<$ reservation value V
- ▶ $V \sim F_V$, i.i.d. across ride requests

μ_0 = exogenous rate of "app opens".

μ = actual rate of rides requested.

Then when A available drivers present:

$$\mu = \mu_0 F_V(P(A)).$$

Strategic model: Drivers

How do drivers enter?

- ▶ Sensitive to *expected earnings over the block*
- ▶ Choose to enter if:
reservation earnings rate $C \times$
expected total time in system
< expected earnings while in system
- ▶ $C \sim F_C$, i.i.d. across drivers

Λ_0 = exogenous rate of driver arrival.

λ = actual rate at which drivers enter.

Then:

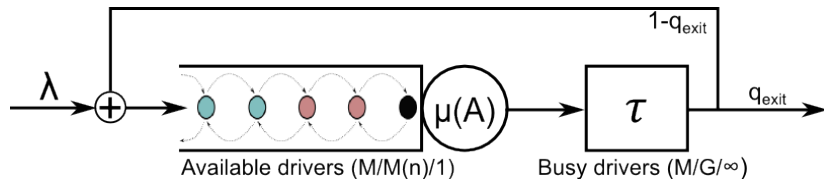
$$\lambda = \Lambda_0 F_C \left(\frac{\text{expected earnings in system}}{\text{expected time in system}} \right)$$

Queueing model

1. Drivers enter at rate λ .
2. When A drivers available, ride requests arrive at rate $\mu(A)$.
3. If a driver is available, ride is *served*; else *blocked*.
4. Rides last exponential time, mean τ .
5. After ride completion:
 - ▶ With probability q_{exit} : Driver signs out
 - ▶ With probability $1 - q_{\text{exit}}$: Driver becomes available

Queueing model: Steady state

Jackson network of two queues: $M/M(n)/1$ and $M/M/\infty$
 \Rightarrow product-form steady state distribution π .



Putting it together: Equilibrium

Given pricing policy $P(\cdot)$,
system equilibrium is $(\lambda, \mu, \pi, \iota, \eta)$ such that:

1. π is the steady state distribution, given λ and μ
2. η is the expected earnings per ride, given $P(\cdot)$ and π
3. ι is the expected idle time per ride, given π and λ
4. λ is the entry rate of drivers, given ι and η :

$$\lambda = \Lambda_0 \mathbf{F}_C \left(\frac{\eta}{\iota + \tau} \right)$$

5. $\mu(A)$ is the arrival rate of ride requests when A drivers are available, given $P(\cdot)$:

$$\mu = \mu_0 \bar{\mathbf{F}}_V(P(A)).$$

Putting it together: Equilibrium

If price increases when number of available drivers decreases:

- ▶ Equilibria always exist under appropriate continuity of F_C, F_V .
- ▶ Equilibria are unique under reasonable conditions

Large Market Limit

The challenge

- ▶ To understand optimal pricing, we need to characterize system equilibria.
- ▶ In particular, need *sensitivity* of equilibria to changes in pricing policy.
- ▶ Our approach: *asymptotics* to simplify analysis.

Large market asymptotics

Consider a sequence of systems indexed by n .

- ▶ In n 'th system, exogenous arrival rates are $n\Lambda_0, n\mu_0$.
- ▶ In n 'th system, pricing policy is $P_n(\cdot)$.
- ▶ In each system, this gives rise to a system equilibrium.

We analyze pricing by looking at asymptotics of equilibria.

Static Pricing

What is static pricing?

Static pricing means: *price policy is constant.*

Let $P(A) = p$ for all A .

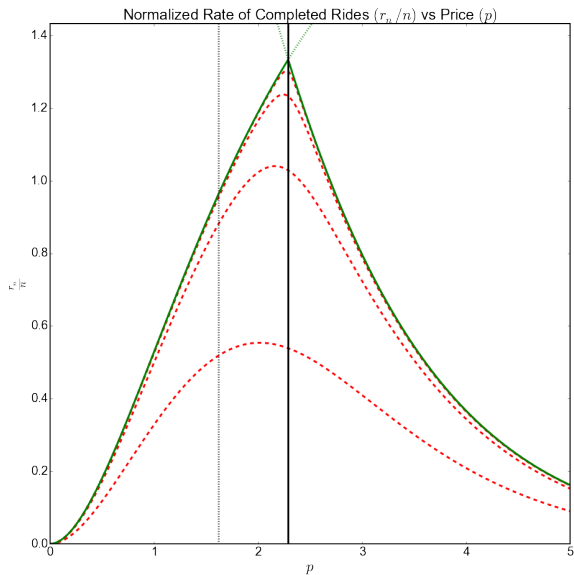
Theorem

Let $r_n(p)$ denote the equilibrium rate of completed rides in the n 'th system. Then:

$$r_n(p) \rightarrow \hat{r}(p) \triangleq \min\{\Lambda_0 F_C(\gamma p / \tau) / q_{\text{exit}}, \mu_0 \bar{F}_V(p)\}.$$

Throughput = min { available supply, available demand }

Static pricing: Illustration



Static pricing: Interpretation

Note that at *any price*, queueing system is always stable:

- ▶ When supply < demand:
Drivers become fully saturated
- ▶ When supply > demand:
Drivers forecast high idle times and don't enter

Balance price p_{bal} : Price where supply = demand

Corollary

The optimal static price is p_{bal} .

Dynamic pricing

What is dynamic pricing?

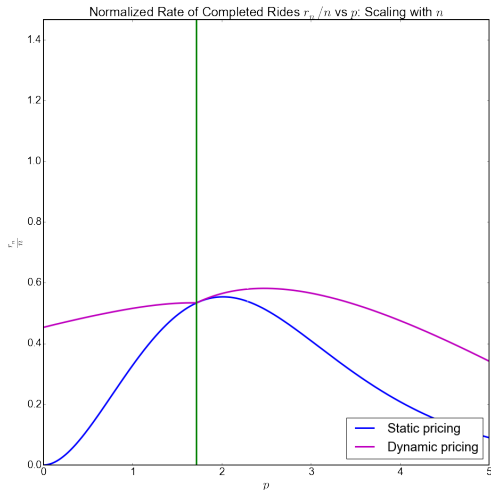
Meant to capture "surge" (Uber)
and "prime time" (Lyft) pricing strategies.

We focus on *threshold pricing*:

- ▶ Threshold θ
- ▶ High price p_h charged when
available drivers $< \theta$
- ▶ Low price $p_\ell < p_h$ charged when
available drivers $> \theta$

Dynamic pricing: Numerical investigation

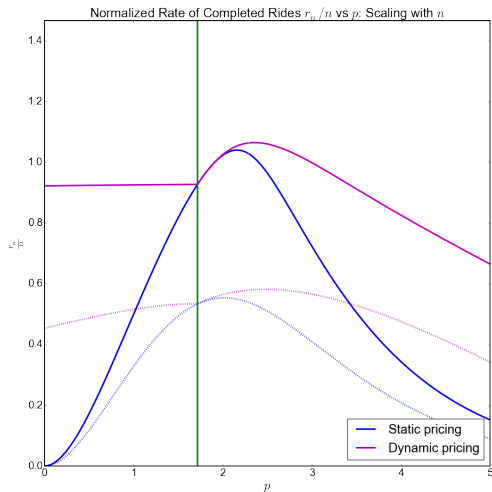
- Fix one price, and vary the other price.
- Compare to static pricing.



$n = 1$

Dynamic pricing: Numerical investigation

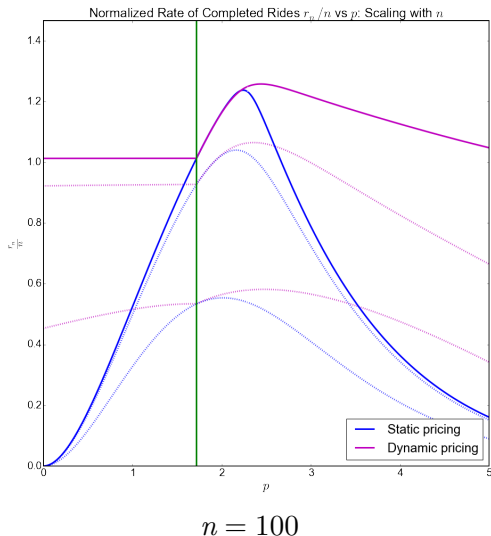
- Fix one price, and vary the other price.
- Compare to static pricing.



$n = 10$

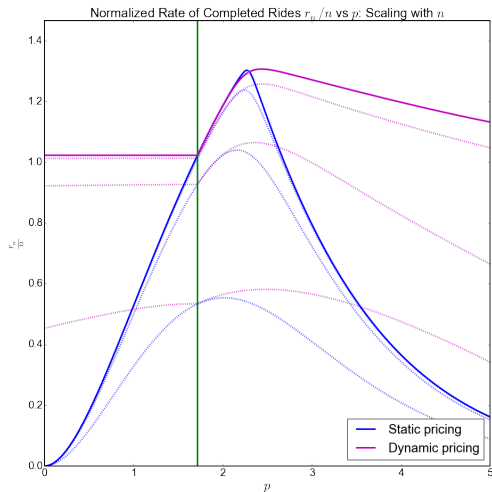
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.



Dynamic pricing: Numerical investigation

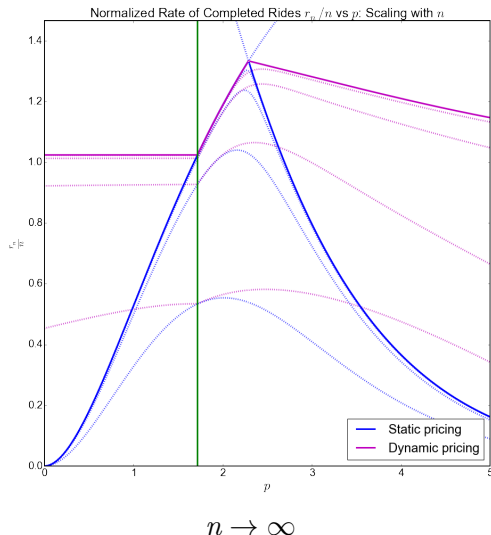
- Fix one price, and vary the other price.
- Compare to static pricing.



$n = 1000$

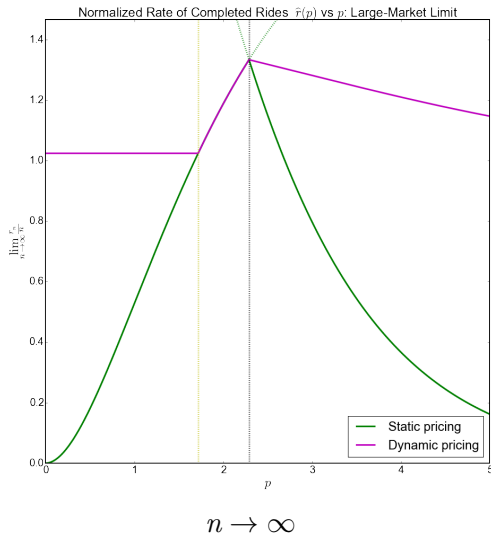
Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.



Dynamic pricing: Numerical investigation

- Fix one price, and vary the other price.
- Compare to static pricing.



Optimal dynamic pricing

Theorem

Let r_n^ be the rate of completed rides in the n 'th system, using the optimal static price.*

*Let r_n^{**} be the rate of completed rides in the n 'th system, using the optimal threshold pricing strategy.*

Then if F_V has monotone hazard rate,

$$\frac{r_n^* - r_n^{**}}{n} \rightarrow 0 \text{ as } n \rightarrow \infty.$$

Optimal dynamic pricing

In other words:

In the fluid limit, no dynamic pricing policy yields higher throughput than optimal static pricing.

Proof sketch

Under threshold pricing:

- ▶ Drivers are sensitive to *two* quantities: idle time, and price.
- ▶ Show that optimal $\theta_n^* \rightarrow \infty$, but chosen so that idle time $\rightarrow 0$ as $n \rightarrow \infty$.
- ▶ In this limit, drivers are sensitive to the *average* price per ride:

$$p_{\text{avg}} = \pi_h p_h + \pi_\ell p_\ell,$$

where π_h, π_ℓ are \approx probabilities of being below or above θ , respectively.

- ▶ If p_{avg} decreases, fewer drivers will enter.

Proof sketch (cont'd)

We note that:

1. If $p_\ell < p_h \leq p_{\text{bal}}$, then $p_{\text{avg}} = p_h$.
2. If $p_{\text{bal}} \leq p_\ell < p_h$, then $p_{\text{avg}} = p_\ell$.
3. If $p_\ell < p_{\text{bal}} < p_h$, then $\pi_\ell > 0, \pi_h > 0$.

In first two cases, *de facto* static pricing.

Proof sketch (cont'd)

We explore the third case.

Suppose that we start with $p_\ell < p_h = p_{\text{bal}}$ (so $p_{\text{avg}} = p_h$).

Now increase p_h :

- ▶ Before $\pi_\ell = 0$, but now $\pi_\ell > 0$, so some customers pay p_ℓ ; this lowers p_{avg} .
- ▶ p_h higher, so customers arriving when $A < \theta$ pay more; this increases p_{avg} .

When F_V is MHR, we show that the first effect dominates the second, so throughput falls.

Robustness

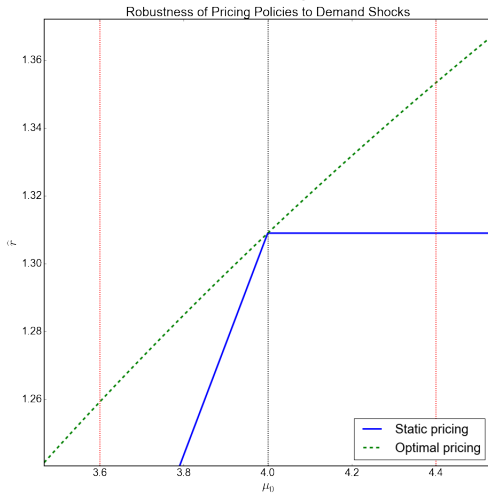
The value of dynamic pricing

How does dynamic pricing help?

- ▶ When system parameters are *known*, performance does not exceed static pricing.
- ▶ When system parameters are *unknown*, dynamic pricing naturally "learns" them.

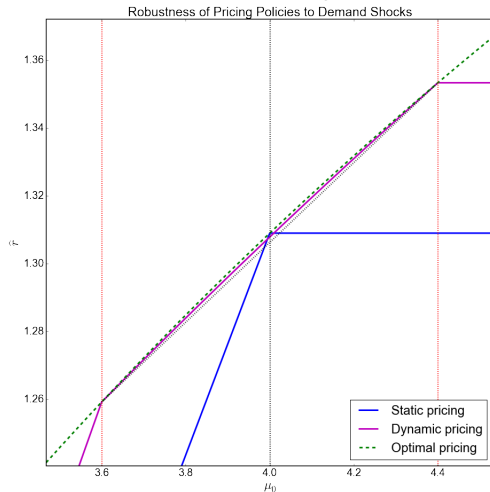
Robustness: Illustration

What happens to *static pricing* in a demand shock?



Robustness: Illustration

What happens to *dynamic pricing* in a demand shock?



Robustness: Dynamic pricing

We can formally establish the observation in the previous illustration:

- ▶ Suppose F_C is logconcave, and $\mu_0^{(1)} < \mu_0^{(2)}$ are fixed.
- ▶ Let $p_{\text{bal},n}^{(1)}, p_{\text{bal},n}^{(2)}$ = optimal static prices in the n 'th system.
- ▶ Let $r_n^{(1)}, r_n^{(2)}$ = optimal throughput in the n 'th system.
- ▶ Suppose now the true $\mu_0 \in [\mu_0^{(1)}, \mu_0^{(2)}]$.
- ▶ Using both prices $p_{\text{bal},n}^{(1)}, p_{\text{bal},n}^{(2)}$ is robust:
 - ▶ There exists a sequence of threshold pricing policies with throughput at any such μ_0 (in the fluid scaling) \geq the linear interpolation of $r_n^{(1)}$ and $r_n^{(2)}$.

(Same holds w.r.t. Λ_0 .)

Conclusion

Platform optimization

This work is an example of *platform optimization*:
Requires understanding *both* operations and economics.
Other topics under investigation:

1. Network modeling (multiple regions):
Our main insights generalize
2. Effect of pricing on aggregate welfare
3. Modeling driver heat maps
4. Fee structure: changing the percentage
5. Effect of changing the matching algorithm