Assignment - Parameter Estimation

Ques 1) Random Sample size = n

Given Population => Normal

mean = θ_1 , variance = θ_2

Max Likelihood estimation = ?

Soln) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

for θ_1 => Likelihood f^n

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

taking log on both sides

$$\log(L(\mu)) = -n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n} \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$$

diff. wrt θ_1 and put $= 0$

$$\Rightarrow \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) \Rightarrow \boxed{\frac{n\bar{x} - n\theta_1}{\sigma^2} = 0} \quad \text{— (I)}$$

simplifying (I) we get $\boxed{\theta = \bar{x}}$ Answer.

for θ_2

$$\boxed{Var(p) = \frac{1}{E\left[\frac{-\partial^2 \ln L(P)}{\partial p^2}\right]}} \quad \text{— (II)}$$

using above, we know that

$$\frac{\partial \log(L(\theta_1))}{\partial(\theta_1)} = \frac{n}{\sigma^2}(\bar{x}-\mu)$$

$$\frac{\partial^2 \log(L(\theta_1))}{\partial^2\theta_1^2} = \frac{-n}{\sigma^2} \quad \text{— (III)}$$

substituting (III) in (II) we get

$$\boxed{\text{variance} = \theta_2 = \frac{\sigma^2}{n}}$$

Answer

Ques 2)

$B(m,\theta)$ $\quad \theta \in [0,1]$

$$\boxed{pmf = P(X=x) = {}^{m}C_x \, \theta^{x} (1-\theta)^{m-x}}$$

x = No. of success in m trials

θ = probability of success

$1-\theta$ = probability of failure

likelihood $f^n \Rightarrow$ $L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$

$$\ln(L(\theta)) = \sum_{i=1}^{n} \ln({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \ln(\theta) + \sum_{i=1}^{n} (m-x_i)\ln(1-\theta)$$

$$\frac{\partial(\ln(L(\theta)))}{\partial\theta} = \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{(m-x_i)}{1-\theta} = 0$$

$$\therefore \frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{\sum_{i=1}^{n}(m-x_i)}{1-\theta}$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{mn - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\theta \cdot (nm - \Sigma x_i) = (1-\theta)\Sigma x_i$$

$$\theta \cdot nm = \Sigma x_i$$

$$\boxed{\theta = \frac{\Sigma x_i}{nm} = \frac{\bar{x}}{m}}$$

Answer

Siddharth Maithani

CO-28

102153035