

Magic number game

A magic application of Zeckendorf theorem

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Theorem: Zeckendorf theorem

Each positive integer is uniquely written as a sum of distinct non -adjacent terms of fibonnaci sequence .

Remainder: The fibonacci sequence defined as follow $F_{n+2} = F_{n+1} + F_n$, $\forall n \geq 0$ and $F_0 = 0$ and $F_1 = 1$.

First, there are 10 terms of the Fibonacci sequence that are strictly less than 100: (1, 2, 3, 5, 8, 13, 21, 34, 55, 89). By Zeckendorf's theorem, any positive integer smaller than 100 can be constructed from these numbers.

The Main Idea: How can we be sure to identify which Fibonacci numbers construct each positive integer between 1 and 100?

Card Design: We design cards containing numbers between 1 and 100 with the following conditions:

- The numbers appear in order.
- The first number of card i is the i^{th} Fibonacci term (among those less than 100).
- The other numbers on card i are all numbers less than 100 that contain the i^{th} Fibonacci term in their Zeckendorf representation.

How It Works: If the user chooses whether their number appears on each card (Yes/No), we can determine which Fibonacci terms appear in the unique Zeckendorf representation of their number. After 10 steps, we have identified all the Fibonacci terms that build this number.

Key Insight: The algorithm does not find your number—you construct your number through your choices.

For example when the user think in the number 60 , 60 appear just in the two following card:

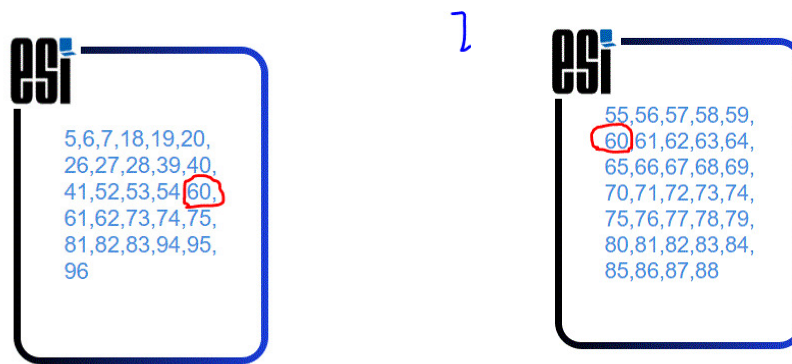


Figure 1: The game cards that have 60.

Because Figure 1 present all the cards that have 60 , so we Notice that $60=55+5$.(adding the first number of each card that is fibonnaci term also).