Kalkulus (1230012)

Pertemuan Ke-9
Turunan

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Sub Pokok Bahasan

- Turunan Fungsi Parameter
- Turunan Fungsi Tingkat Tinggi

Kompetensi Khusus

Mahasiswa mampu menyelesaikan berbagai turunan fungsi

TURUNAN FUNGSI PARAMETER

Jika : $y = f(t)$

$x = g(t)$

Maka : $y' = \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$ → Turunan ke -1

Dan :

$y'' = \dfrac{d^2 y}{dx^2} = \dfrac{d/dt\,(dy/dt)}{dx/dt}$ → Turunan ke-2

Contoh :

1)Diketahui : $y = 2t^2 + 3t \ dan \ x = 1 + 2t$

Maka

$$\frac{dy}{dt} = 4t + 3 \qquad \frac{dx}{dt} = 2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t+3}{2} = 2t + \frac{3}{2}$$

$$\frac{d^2 y}{dx^2} = \frac{d/d\text{?}(dy/dx)}{dx/dt} = \frac{d/dt(2t+\frac{3}{2})}{2} = \frac{2}{2} = 1$$

2) $y = \cos t \; dan \; x = \sin t$

$$\frac{dy}{dt} = -\sin t \qquad \frac{dx}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sin t}{\cos t} = -tg\, t$$

$$\frac{d^2 y}{dx^2} = \frac{d/dt(dy/dt)}{dx/dt} = \frac{d/dt(-tg\, t)}{\cos t}$$

$$= \frac{-sec^2 t}{\cos t}$$

$$= -\, sec^2 t.\sec t \qquad = -\, sec^3 t$$

TURUNAN FUNGSI TINGKAT TINGGI

Diberikan fungsi $y = f(x)$

-Turunan ke – 1 : $\dfrac{dy}{dx} = f'(x) = y'$

-Turunan ke – 2 : $\dfrac{d^2y}{dx^2} = f''(x) = y''$

-Turunan ke – 3 : $\dfrac{d^3y}{dx^3} = f'''(x) = y''' = f^3(x) = y^3$

-Turunan ke – n : $\dfrac{d^ny}{dx^n} = f^{(n)}(x) = y^{(n)}$

Contoh : 1) $f(x) = 4x^3 - 5x^2$

$$f\,'(x) = 12x^2 - 10x$$

$$f''(x) = 24x - 10$$

$$f'''(x) = 24$$

$$f^4(x) = 0$$

$$\vdots$$

$$f^n(x) = 0, n > 4$$

2) Tentukan turunan ke-4 dari :

$$y = \sin 2x$$

Jawab :

$$y' = 2.\cos 2x$$

$$y'' = -2^2 \sin 2x$$

$$y''' = -2^3 \cos 2x$$

$$y^{(4)} = 2^4 \sin 2x$$

3) Tentukan $\dfrac{d^3 y}{dx^3}$ dari $f(x) = (2x+3)^4$

Jawab :

$$\frac{dy}{dx} = 4 \cdot (2x+3)^3 \cdot 2 = 8(2x+3)^3$$

$$\frac{d^2 y}{dx^2} = 8 \cdot (2x+3)^3 \cdot 2 = 48(2x+3)^2$$

$$\frac{d^3 y}{dx^3} = 48.3(2x+3)^1 \cdot 2 = 192(2x+3)$$

4) Carilah rumus umum turunan ke – n dari fungsi

$$y = x^n$$

Jawab :

$$y' = n \cdot x^{n-1}$$

$$y'' = n(n-1) \cdot x^{n-2}$$

$$y''' = n(n-1) \cdot (n-2) \cdot x^{n-3}$$

$$\vdots$$

$$y^{(n)} = \underbrace{n(n-1) \cdot (n-2) \ldots 3.2.1}_{} \cdot x^{n-n}$$

$$y^{(n)} = n!$$

5) Carilah rumus umum turunan ke – n dari fungsi
$$y = \frac{1}{x} = x^{-1}$$

Jawab :

$$y' = (-1).x^{-2}$$

$$y'' = (-1)(-2).x^{-3}$$

$$y''' = (-1)(-2)(-3).x^{-4}$$

$$\vdots$$

$$y^{(n)} = (-1)(-2)(-3)\ldots(-n).x^{-n-1}$$

$$= (-1)1.(-1).2.(-1).3\ldots(-1).n.x^{-n-1}$$

$$y^{(n)} = (-1)^n.n!\ x^{-n-1}$$

5) Carilah rumus umum turunan ke – n dari fungsi
$$y = e^{1+2x}$$

Jawab :

$$y' = 2.e^{1+2x}$$

$$y'' = 2.2.e^{1+2x} = 2^2.e^{1+2x}$$

$$y''' = 2.2.2.e^{1+2x} = 2^3.e^{1+2x}$$

$$\vdots$$

$$y^{(n)} = 2^n.e^{1+2x}$$

Referensi

- Purcell, Varberg, *Kalkulus dan Geometri Analitis*, Penerbit Erlangga, 1993
- Frank Ayres, *Calculus*, Mc.Graw Hill, New York, 1972
- J.Salas and Hill, *Calculus One and Several Variables*, John Willey& Sons, NewYork, 1982