Kalkulus (1230012)

Bab V Turunan

Juwairiah, S.Si,M.T
(juwai_riah@yahoo.com)

Sub Pokok Bahasan

- Turunan Fungsi Trigonometri
- Turunan Fungsi Invers
- Turunan Fungsi Siklometri

Kompetensi Khusus

Mahasiswa mampu menyelesaikan berbagai turunan fungsi

TURUNAN FUNGSI TRIGONOMETRI

1) $\frac{d}{dx}(sinx) = cos\ x$

2) $\frac{d}{dx}(cosx) = -\sin x$

3) $\frac{d}{dx}(tg\ x) = sec^2 x$

4) $\frac{d}{dx}(cotg\ x) = -cosec^2 x$

5) $\frac{d}{dx}(\sec x) = sec\ x\ .\ tg\ x$

6) $\frac{d}{dx}(cscx) = -csc\ x.cot\ x$

Bukti: 3) $y = tg\ x = \frac{\sin x}{\cos x} \Rightarrow u = \sin x \rightarrow u' = \cos x$

$v = \cos x \rightarrow v' = -\sin x$

$$y' = \frac{u'v - uv'}{v^2}$$

$$= \frac{\cos x.\cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \frac{1}{(\cos x)^2}$$

$$= \sec^2 x$$

5) $y = sec\ x \rightarrow\ y' = ?$

$$y = \frac{1}{cos\ x} = \frac{u}{v}$$

$$y' = \frac{u'v - uv'}{v^2}$$

$$= \frac{0.cos\ x - 1.(-sin\ x)}{(cos\ x)^2}$$

$$= \frac{\sin x}{cos^2 x} = \frac{\sin x}{\cos x} . \frac{1}{\cos x}$$

$$= tg\ x\ .\sec\ x$$

TURUNAN FUNGSI INVERS

Teorema :

Jika y = f (x) mempunyai invers x = g (y), maka : $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

Contoh : 1. Tentukan $\frac{dy}{dx}$ jika diketahui $y = \sqrt{x}$

Jawab : $y = \sqrt{x}$ mempunyai invers : $x = y^2$

Maka : $\frac{dx}{dy} = 2y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{2y} = \frac{1}{2\sqrt{x}}$$

TURUNAN FUNGSI SIKLOMETRI

$$y = arc\sin x \rightarrow y' = \frac{1}{\sqrt{1-x^2}}$$

$$y = arc\cos x \rightarrow y' = \frac{-1}{\sqrt{1-x^2}}$$

$$y = arc\,tg\,x \rightarrow y' = \frac{1}{1+x^2}$$

$$y = arc\,cotg\,x \rightarrow y' = \frac{-1}{1+x^2}$$

$$y = arc\,sec\,x \rightarrow y' = \frac{1}{x\sqrt{x^2-1}}$$

$$y = arc\,cosec\,x \rightarrow y' = \frac{-1}{x\sqrt{x^2-1}}$$

Contoh :

1) $y = arc\sin x$

$$x = \sin x$$

$$x' = \frac{dx}{dy} = \cos x$$

$$= \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\sqrt{1-x^2}}$$

2) $y = arc\sin 2x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} . 2 = \frac{2}{\sqrt{1-4x^2}}$$

3) $y = arc\ tg\ x^2$

$$\frac{dy}{dx} = y' = \frac{1}{1+(x^2)^2} . 2x = \frac{2x}{1+x^4}$$

Referensi

- Purcell, Varberg, *Kalkulus dan Geometri Analitis*, Penerbit Erlangga, 1993
- Frank Ayres, *Calculus*, Mc.Graw Hill, New York, 1972
- J.Salas and Hill, *Calculus One and Several Variables*, John Willey& Sons, NewYork, 1982