

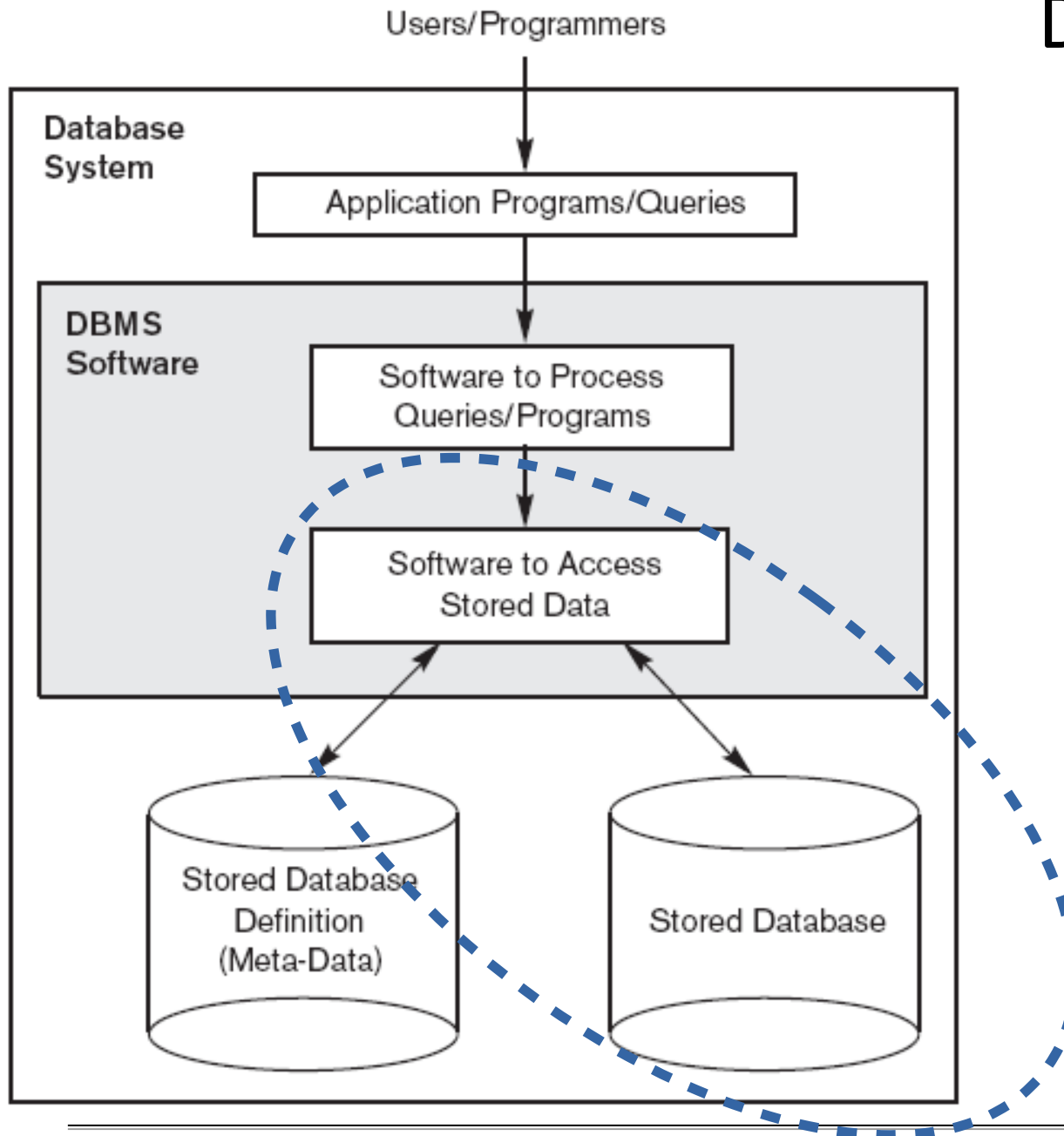
# Database Technology

## Topic 7: Data Structures for Databases

Olaf Hartig

[olaf.hartig@liu.se](mailto:olaf.hartig@liu.se)

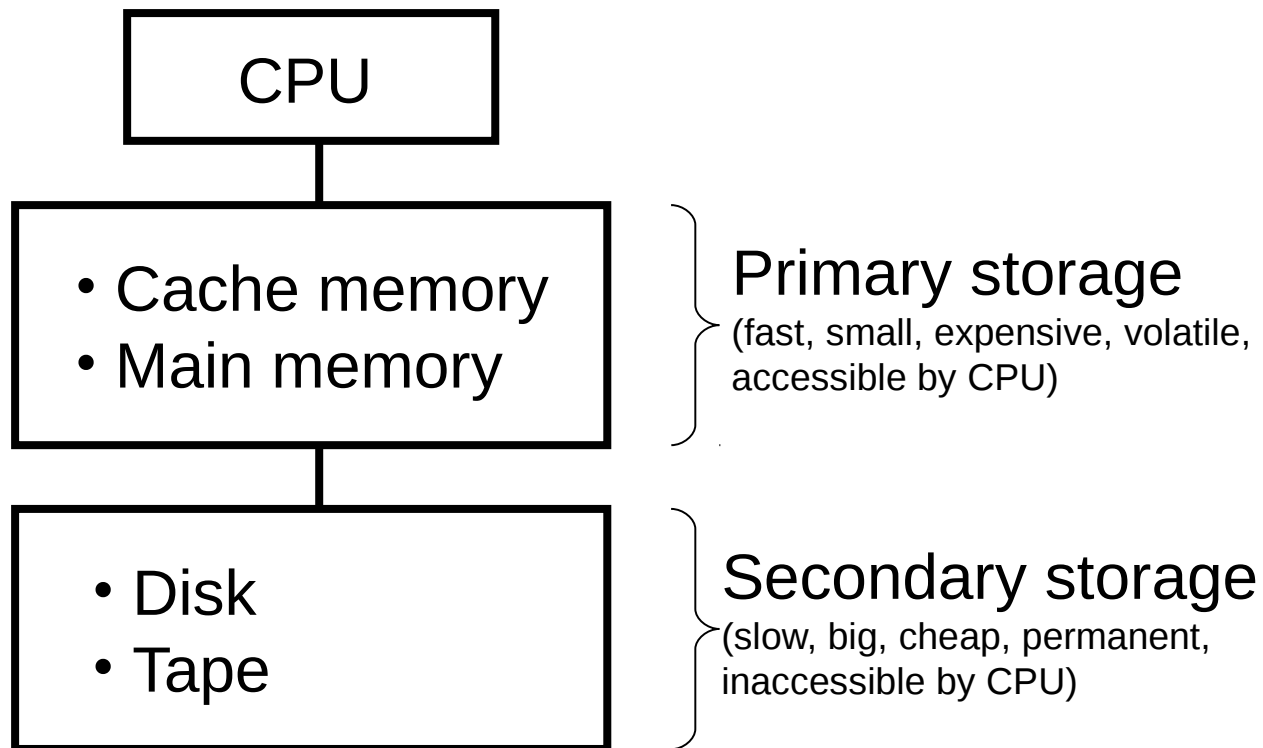
# Database System



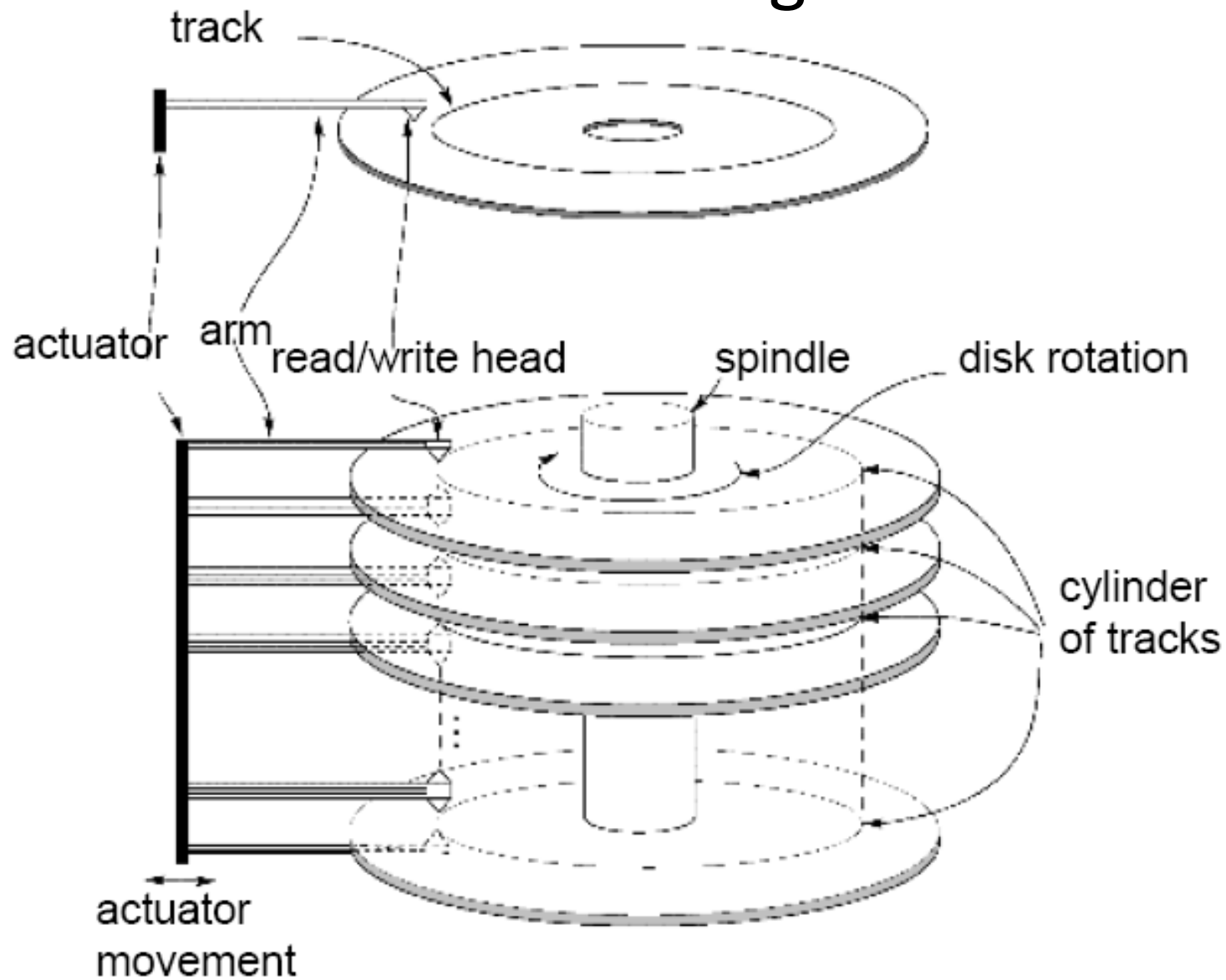
**Figure 1.1**  
A simplified database  
system environment.

# Storage Hierarchy

# Traditional Storage Hierarchy



# Magnetic Disk



# Properties of Using Magnetic Disks

- Formatting divides the hard-coded sector into equal-sized **blocks**
  - Block is the **unit of data transfer** between disk and main memory
  - Typical block sizes: 512 – 8192 bytes
- Read/write from/to disk is a major **bottleneck!**

$$R/W \text{ time} = \underbrace{\text{seek time} + \text{rotational delay}}_{(search \ track) \quad (search \ block)} + \text{block transfer time}$$

*12–60 ms*

- CPU instruction: ca. 1 ns ( $10^{-9}$  secs)
- Main memory access: ca. 10 ns ( $10^{-8}$  secs)
- Disk access: ca. 1 ms (1M ns,  $10^{-3}$  secs)

# Files and Records

# Terminology

- Data stored in files
- File is a sequence of records
- Records are allocated to file blocks
- Record is a set of field values
- For instance,
  - File = relation
  - Record = row
  - Field = attribute value



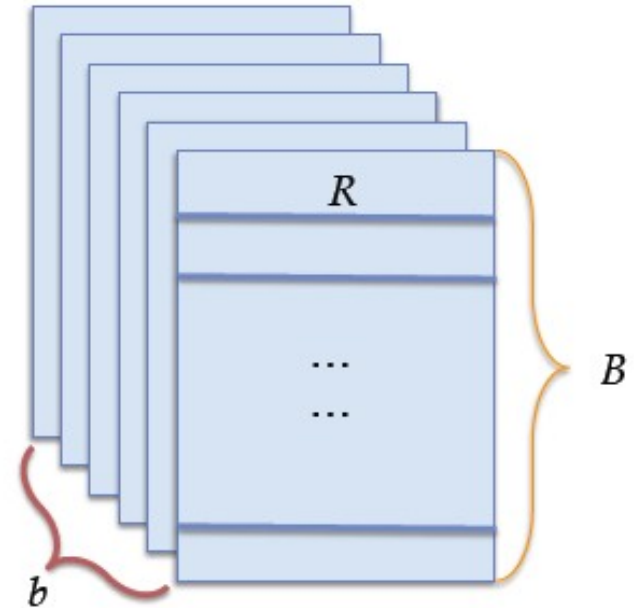


# Blocking Factor

- Blocking factor ( $bfr$ ) is the number of records per block
- Assume
  - $r$  is the number of records in a file,
  - $R$  is the size of a record, and
  - $B$  is the block size in bytes,

then:

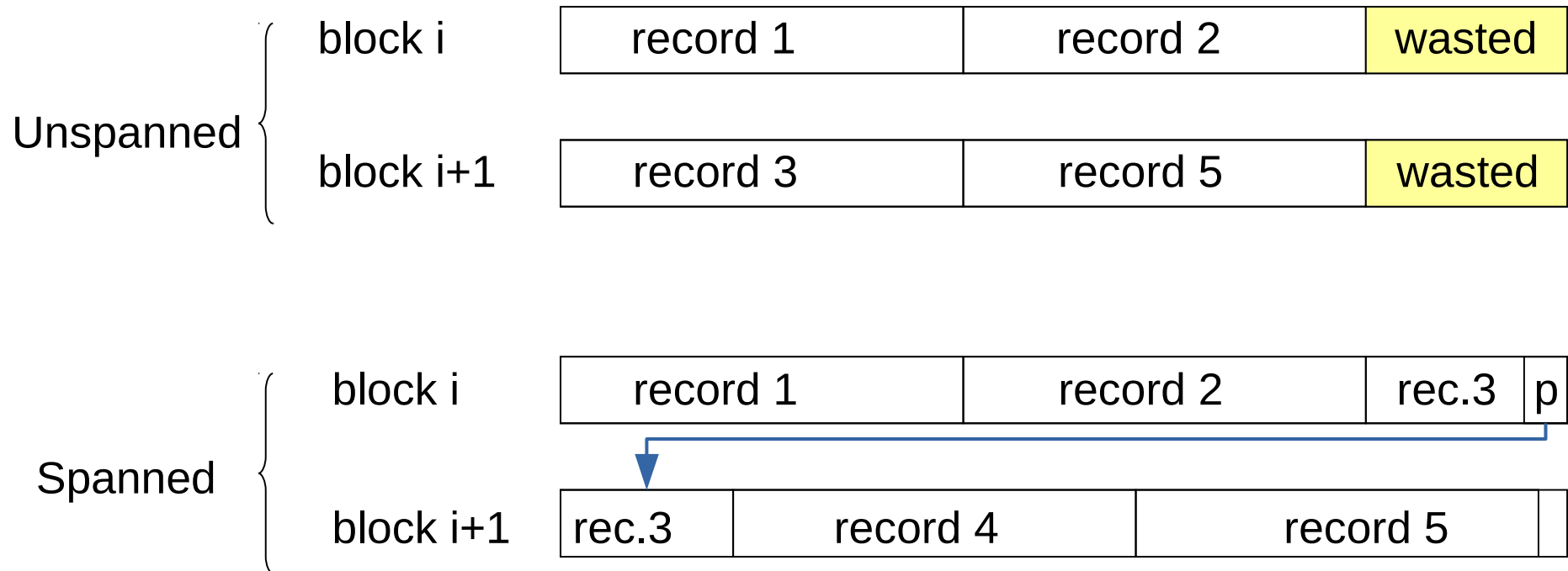
$$bfr = \left\lfloor \frac{B}{R} \right\rfloor$$



- Blocks needed to store the file:  $b = \left\lceil \frac{r}{bfr} \right\rceil$
- Space wasted per block =  $B - bfr * R$

# Spanned Records

... avoid wasting space



# Allocating File Blocks on Disk

- **Contiguous allocation:** file blocks allocated consecutively (one after another)
  - Fast sequential access, but expanding is difficult
- **Linked allocation:** each file block contains a pointer to the next one
  - Expanding the file is easy, but reading is slower
- **Linked clusters allocation:** hybrid of the two above
  - i.e., linked clusters of consecutive blocks
- **Indexed allocation:** index blocks contain pointers to the actual file blocks

# File Organization

(Organizing Records in Files)

# Heap Files

- Records are added to the end of the file
- Adding a record is cheap
- Retrieving, removing, and updating a record is expensive because it implies *linear search*
  - Average case:  $\lceil \frac{b}{2} \rceil$  block accesses
  - Worst case:  $b$  block accesses

(recall,  $b$  is the number of blocks of the file)
- Record removal also implies waste of space
  - Periodic reorganization

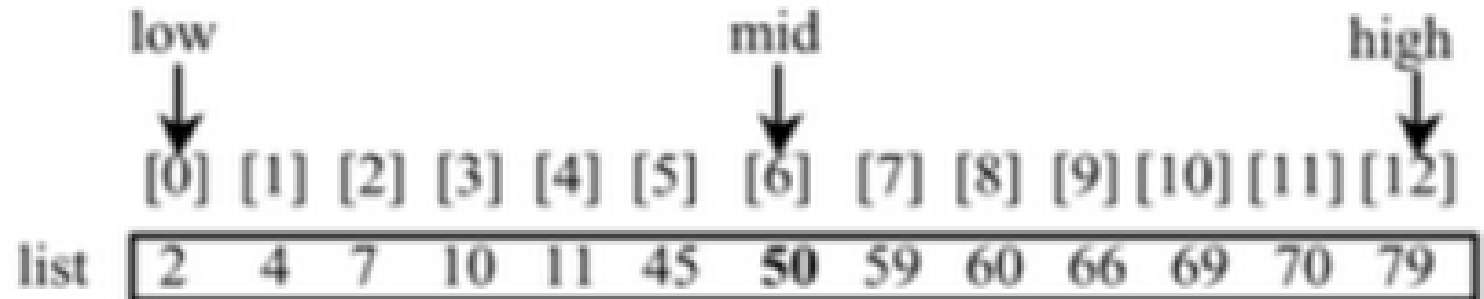
# Sorted Files

- Records ordered according to some field
- Ordered record retrieval is cheap (i.e., on the ordering field, otherwise expensive)
  - All the records: access the blocks sequentially
  - Next record: probably in the same block
  - Random record: *binary search*; hence,  $\lceil \log_2 b \rceil$  block accesses in the worst case
- Adding a record is expensive, but removing is less expensive (deletion markers and periodic reorganization)

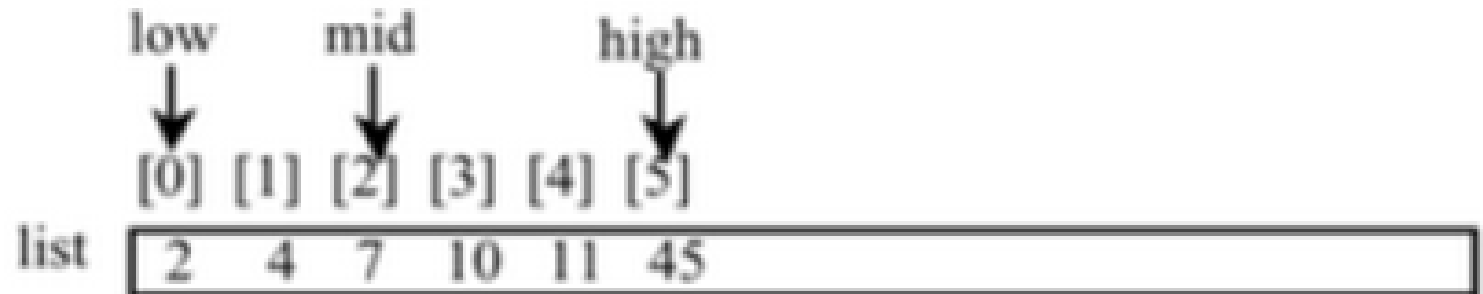
# Binary Search

key is 11

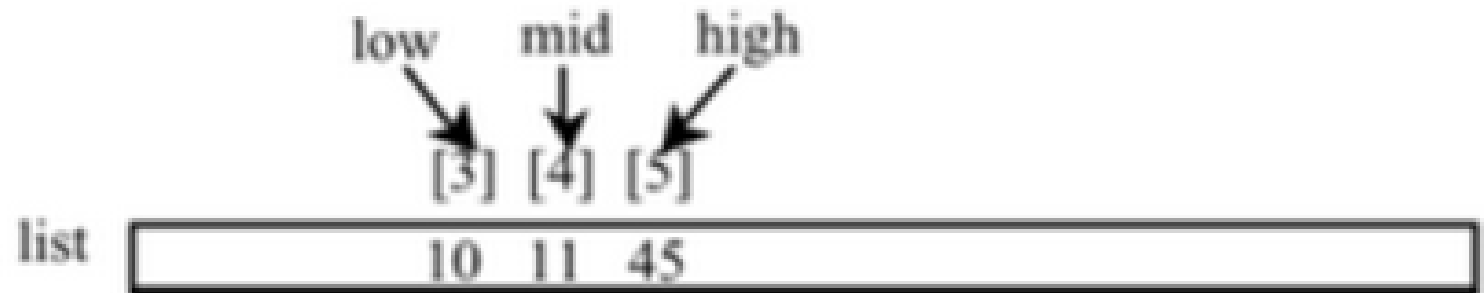
key < 50



key > 7



key == 11



# Internal Hashing

- Choose a field of the records to be the *hash field*
- Applying hash function  $h$  to the value  $x$  of the hash field returns the position of the record in the file
  - e.g.,  $h(x) = x \bmod r$   
(recall,  $r$  is the number of records in the file)
- Collision: different field values hash to same position
- Solutions to deal with collisions (*collision resolution*):
  - Check subsequent positions until one is empty
  - Use a second hash function
  - Put the record in an overflow area and link it



# External Hashing

- Hashing for disk files
- Applying hash function to the value of the hash field returns a bucket number (instead of a position)
  - Bucket: one or several contiguous disk blocks
  - Table converts bucket number into address of block
- Collisions are typically resolved via overflow area
- Cheapest random retrieval  
(when searching for equality)
- Ordered record retrieval is expensive

# Indexes

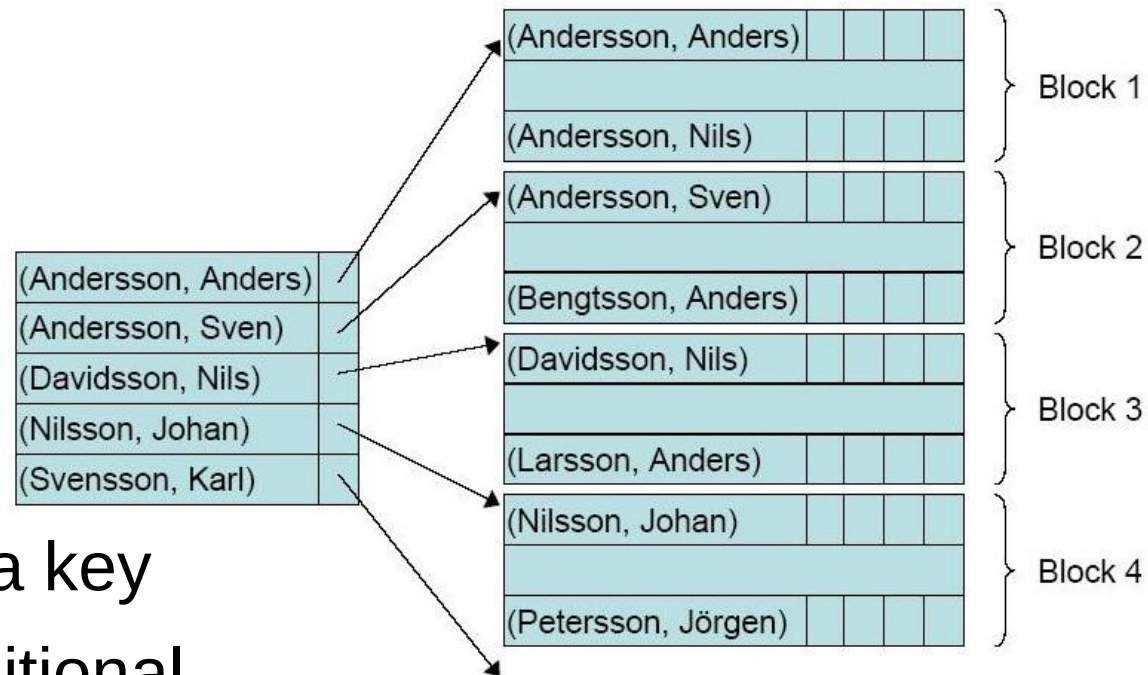
(Secondary Access Methods)

# Overview

- Seen so far: file organization
  - Analogous to organization of books into chapters, sections, etc.
  - Determines primary method to access data in a file
    - e.g., sequential search, binary search
- Now: index structures
  - Allow for secondary access methods
  - Analogous to the index of a book
  - Goal: speed up access under specific conditions
  - Outline:
    - 1) Single-level ordered indexes (primary, secondary, and clustering indexes)
    - 2) Multilevel indexes
    - 3) Dynamic multilevel indexes (B+-trees)

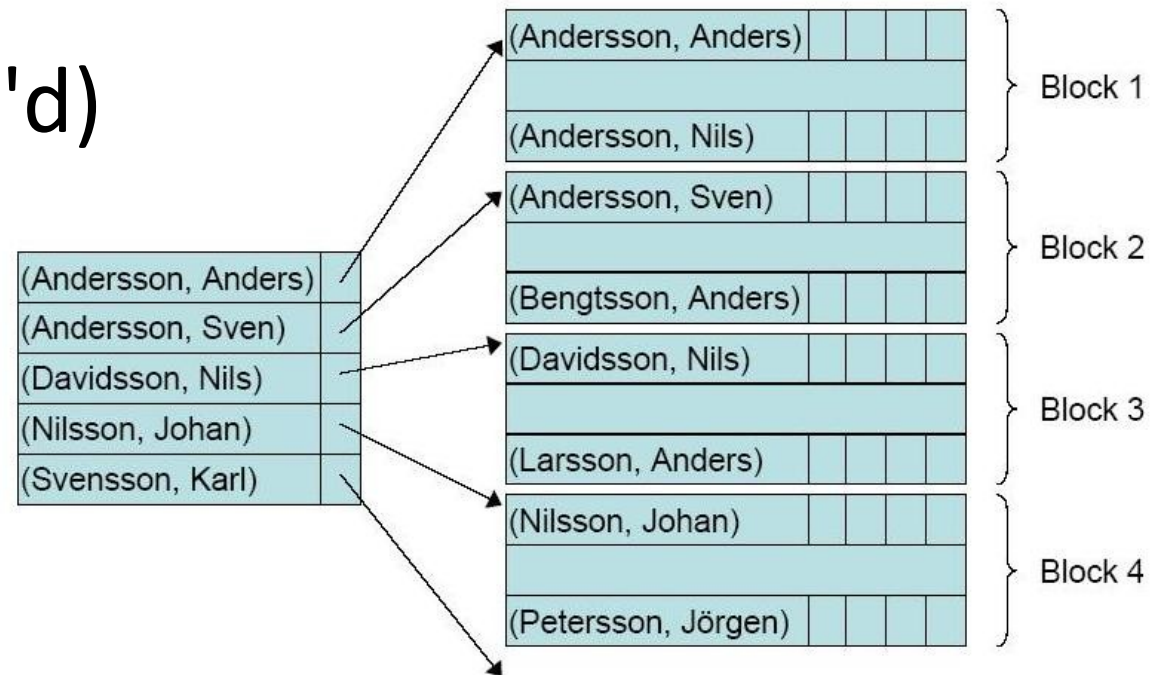
# Single-Level Ordered Indexes

# Primary Index



- Assumptions:
  - Data file is sorted
  - Ordering field  $F$  is a key
- Primary index: an additional *sorted file* whose records contain two fields:
  - $V$  - one of the values of  $F$
  - $P$  - pointer to a disk block of the data file
- One index record  $(V, P)$  per data block such that the first data record in the data block pointed to by  $P$  has  $V$  as the value of the ordering key  $F$

# Primary Index (cont'd)



- Why is it faster to access a random record via a binary search in the index than in the data file?
  - Number of index records  $\ll$  number of data records
  - Index records smaller than data records (i.e., higher blocking factor for the index file than for the data file)
- What is the cost of maintaining a primary index? (if the order of the data records changes)

# Clustering Index

- Assumptions:
  - Data file is sorted
  - Ordering field  $F$  is **not** a key (hence, we cannot assume distinct values)

Index Data File

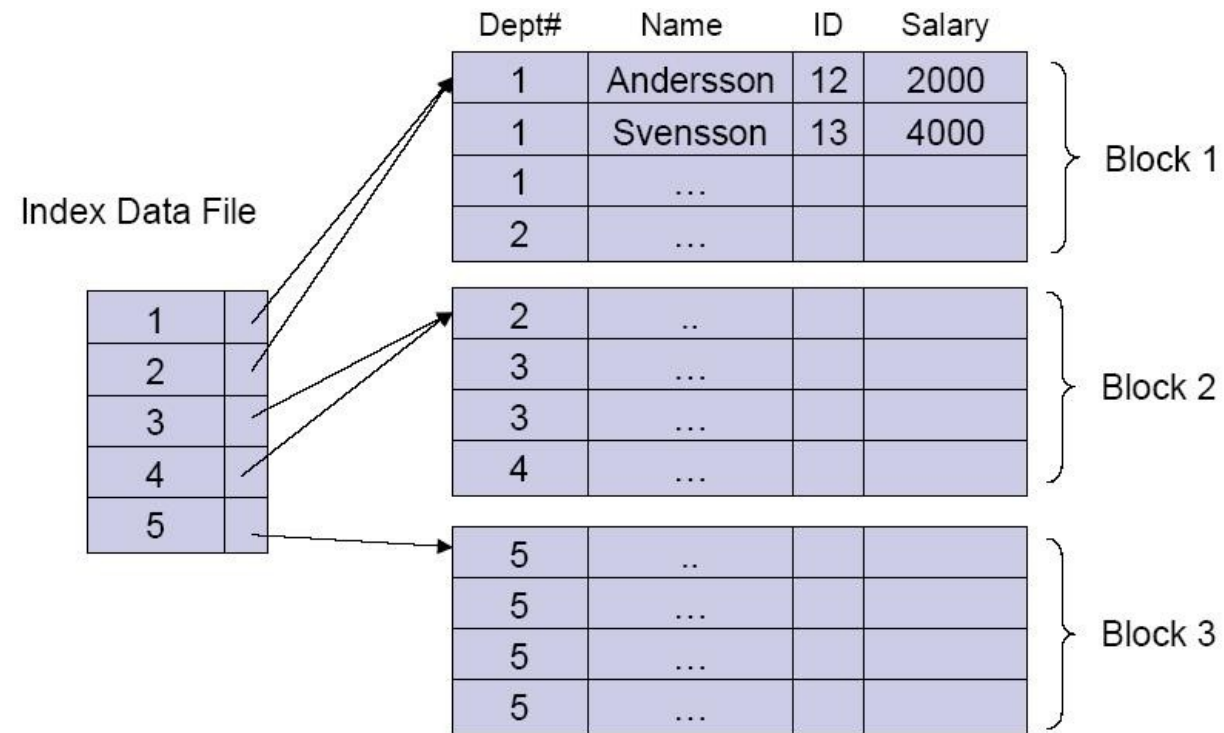
1	
2	
3	
4	
5	

Dept#	Name	ID	Salary	
1	Andersson	12	2000	Block 1
1	Svensson	13	4000	
1	...			
2	...			
2	..			Block 2
3	...			
3	...			
4	...			
5	..			Block 3
5	...			
5	...			
5	...			

- Clustering index: additional *sorted file* whose records contain two fields:
  - $V$  - one of the values of  $F$
  - $P$  - pointer to a disk block of the data file
- One index record  $(V,P)$  for each distinct value  $V$  of the ordering field  $F$  such that  $P$  points to the first data block in which  $V$  appears

# Clustering Index

- Efficiency gain?
- Maintenance cost?

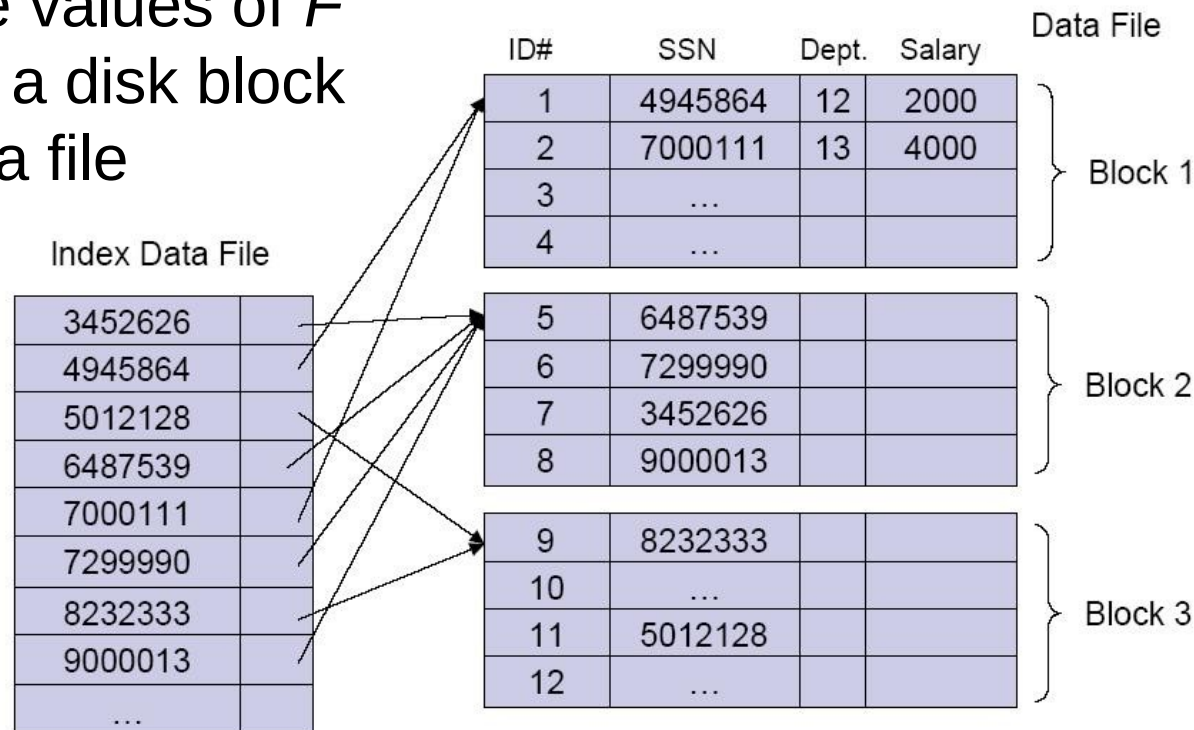




# Secondary Indexes on Key Field

- Index on a *non-ordering key* field  $F$ 
  - Data file may be sorted or not
- Secondary index: additional *sorted* file whose records contain two fields:
  - $V$  - one of the values of  $F$
  - $P$  - pointer to a disk block of the data file

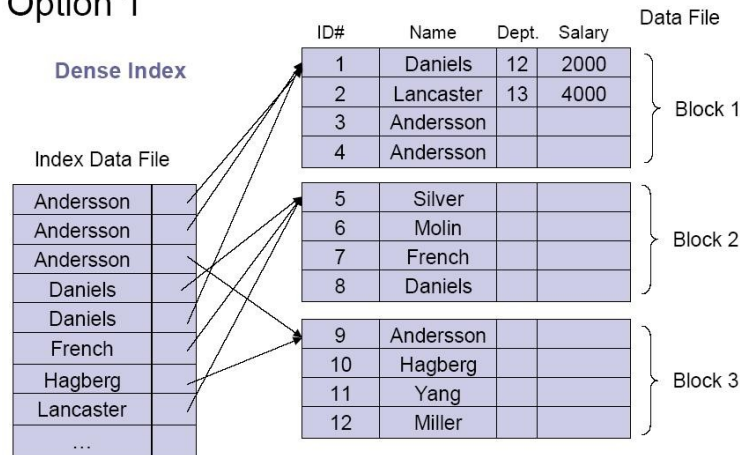
- One index record per data record



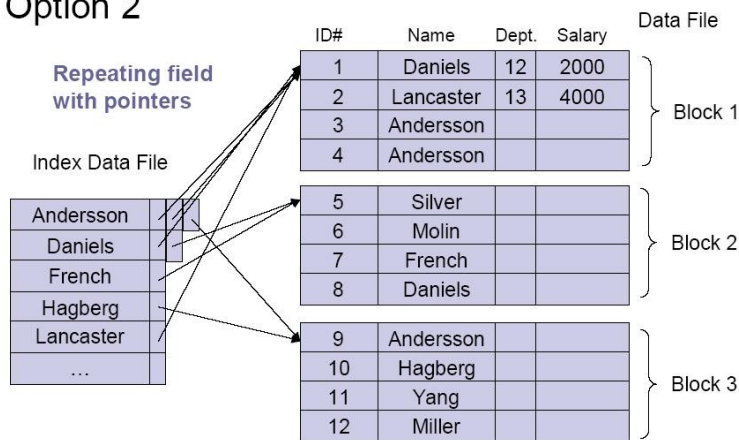
# Secondary Indexes on Non-Key

- Index on a *non-ordering non-key* field

## Option 1



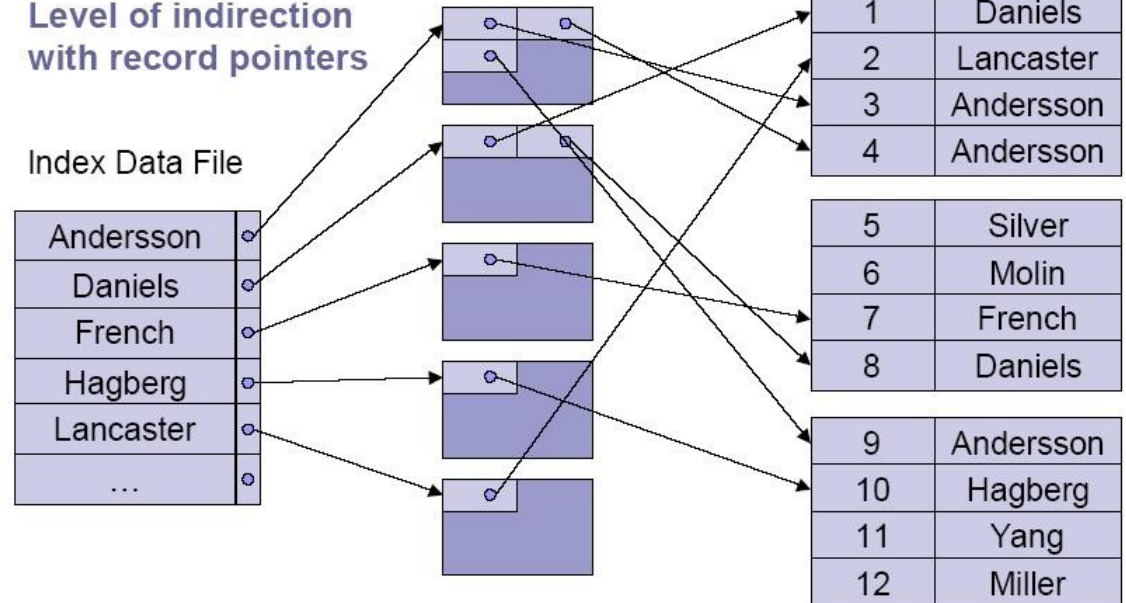
## Option 2



## Option 3

Level of indirection  
with record pointers

Index Data File



# Summary of Single-Level Indexes

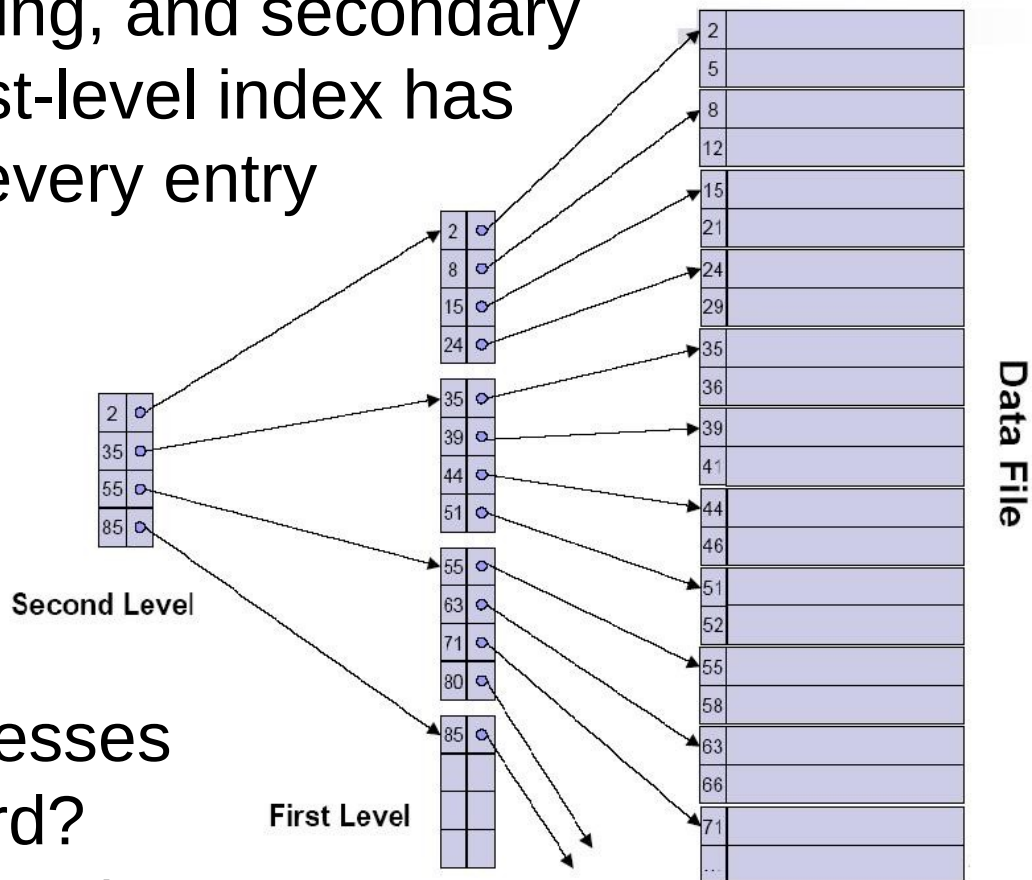
	Index field used for ordering the data file	Index field <i>not</i> used for ordering the data file
Index field is key	Primary index	Secondary index (key)
Index field is not key	Clustering index	Secondary index (non-key)

Type of index	Number of index entries
Primary	Number of blocks in data file
Clustering	Number of distinct index field values
Secondary (key)	Number of record in data file
Secondary (non-key)	Number of records or number of distinct index field values

# Multilevel Indexes

# Multilevel Indexes

- Index on index (first level, second level, etc.)
- Works for primary, clustering, and secondary indexes as long as the first-level index has a distinct index value for every entry
- How many levels?
  - Until the last level fits into a single disk block
- How many disk block accesses to retrieve a random record?
  - Number of index levels + 1



# Multilevel Indexes (cont'd)

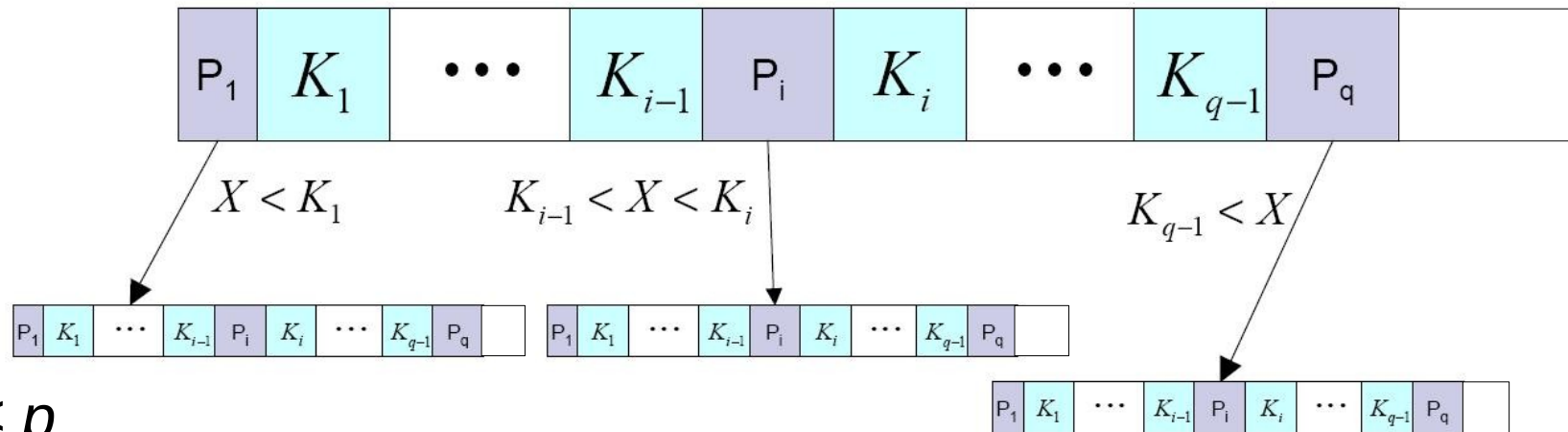
- When using a (static) multilevel index, record insertion, deletion, and update may be expensive because all the index levels are *sorted* files
- Solutions:
  - Overflow area + periodic reorganization
  - Dynamic multilevel indexes that leave some space in index blocks for new entries (e.g., B-trees and B+-trees)

# Dynamic Multilevel Indexes

(B-Trees and B+-Trees)

# Search Trees

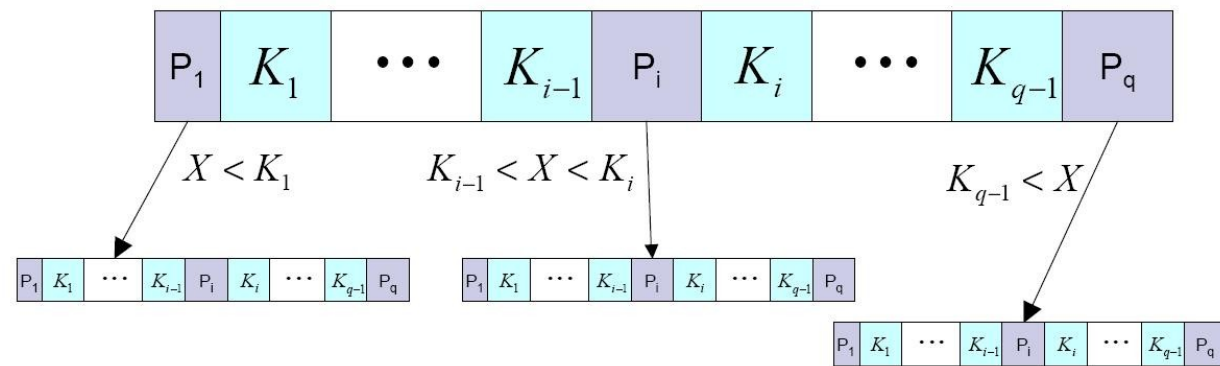
- Used to guide the search for a record
  - Generalization of binary search
- Nodes of a search tree of **order  $p$**  look like:



- $q \leq p$
- Every  $K_i$  is a **key value**
- Every  $P_i$  is a **tree pointer** to a subtree (or a null pointer)
- Within each node:  $K_1 < K_2 < \dots < K_{q-1}$
- For every value  $X$  in the subtree:  $K_{i-1} < X < K_i$



# B-Trees

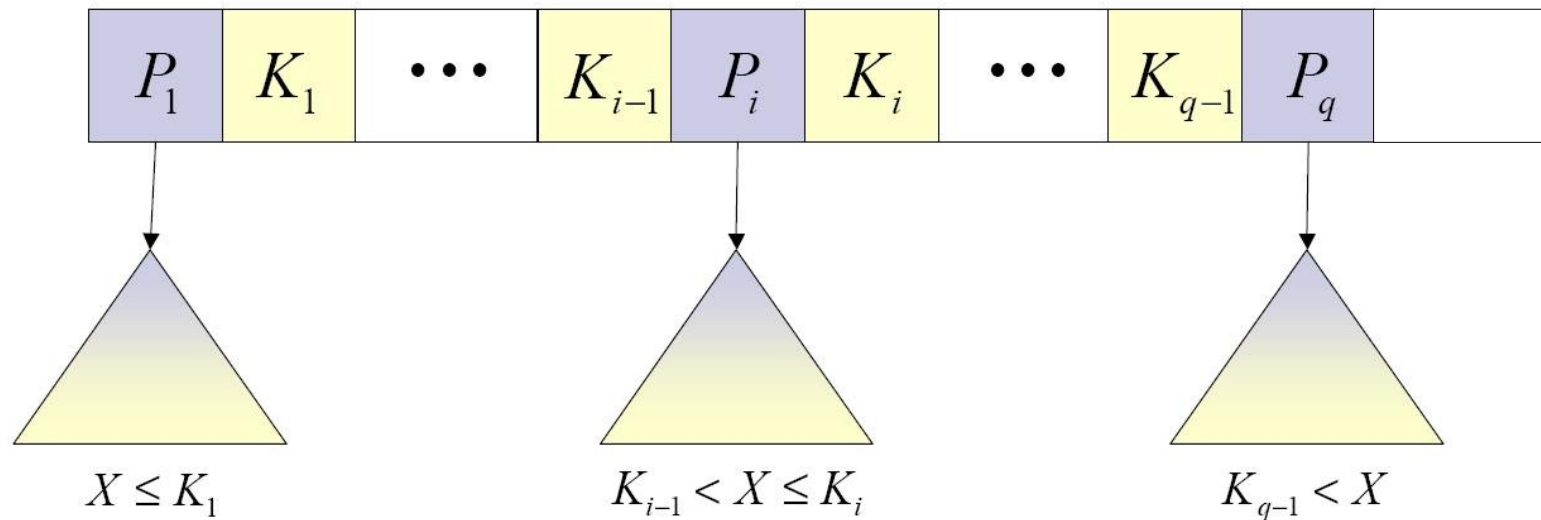


- B-tree is a variant of a *balanced* search tree
  - **Balanced**: all leaf nodes are at the same level (Why is this good?)
- Additional constraints:
  - In addition to a tree pointer  $P_i$ , each key value  $K_i$  is associated with a data pointer  $Pr_i$  to the record with value  $K_i$
  - Each internal node must have at least  $\lceil \frac{P}{2} \rceil$  tree pointers (i.e., is at least half full)

# B+-Trees

- Variation of B-trees, most commonly used
- In contrast to a B-tree, in a B+tree the leaf nodes are different from the internal nodes; that is:
  - Internal nodes have key values and tree pointers only (no data pointers)
  - Leaves have key values and data pointers
  - Usually, each leaf node additionally has a pointer to the next leaf to allow for ordered access (much like a linked list)
- Every key value is present in one of the leaves
- Of course, B+-trees are balanced

# Internal Nodes of a B+-Tree



- $q \leq p$  (where  $p$  is the order of the B+-tree)
- Every  $K_i$  is a key value, every  $P_i$  is a tree pointer
- Within each node:  $K_1 < K_2 < \dots < K_{q-1}$
- For every value  $X$  in the subtree:  $K_{i-1} < X \leq K_i$
- Every internal node (except the root) has at least  $\lceil \frac{p}{2} \rceil$  tree pointers

# Leaf Nodes of a B+-Tree

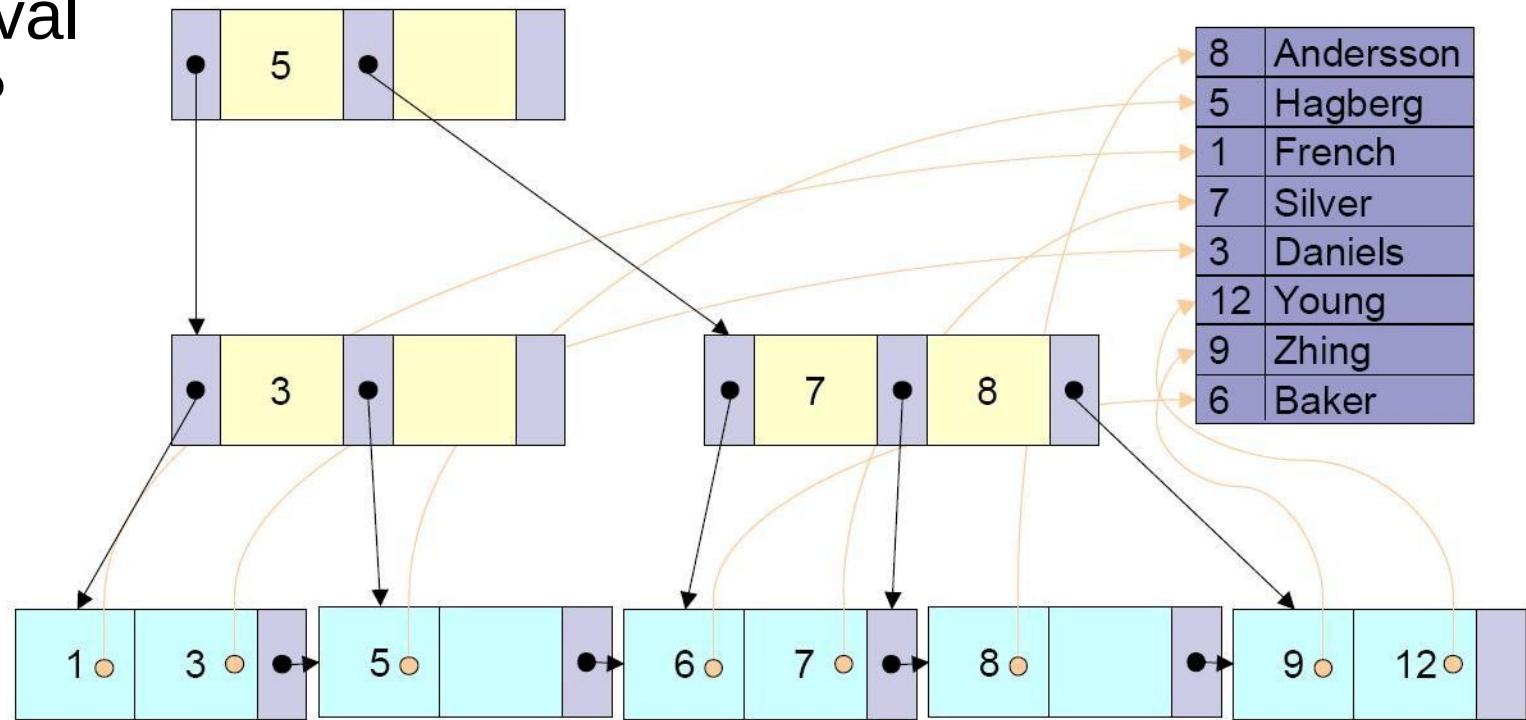
$K_1$	$Pr_1$	...	$K_i$	$Pr_i$	...	$K_q$	$Pr_q$	$P$
-------	--------	-----	-------	--------	-----	-------	--------	-----

- $q \leq p$  (where  $p$  is the order for leaf nodes of the B+-tree)
- Every  $K_i$  is a key value
- Every  $Pr_i$  is a data pointer to the record with key value  $K_i$
- $P$  is a pointer to the next leaf node
- Within each node:  $K_1 < K_2 < \dots < K_q$
- Every leaf node has at least  $\lceil \frac{p}{2} \rceil$  key values

# Retrieval of Records in a B+-Tree

- Very fast retrieval of a random record, at worst:  $\lceil \log_{\lceil \frac{p}{2} \rceil} N \rceil + 1$ 
  - $p$  is the order of the internal nodes
  - $N$  is the number of leaf nodes

- How would the retrieval proceed?

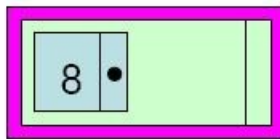


# B+-Tree Insertion



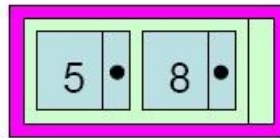
Insert: 8

# B+-Tree Insertion



Insert: 5

# B+-Tree Insertion

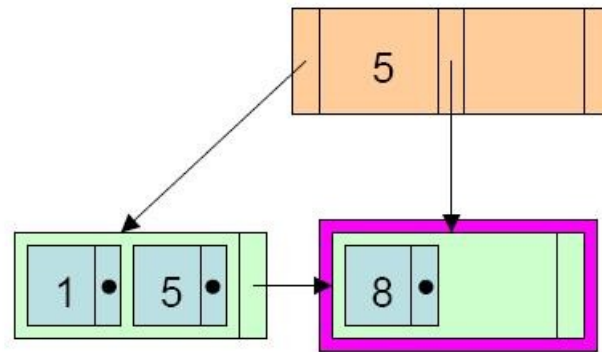


Overflow – create a new level

Insert: 1

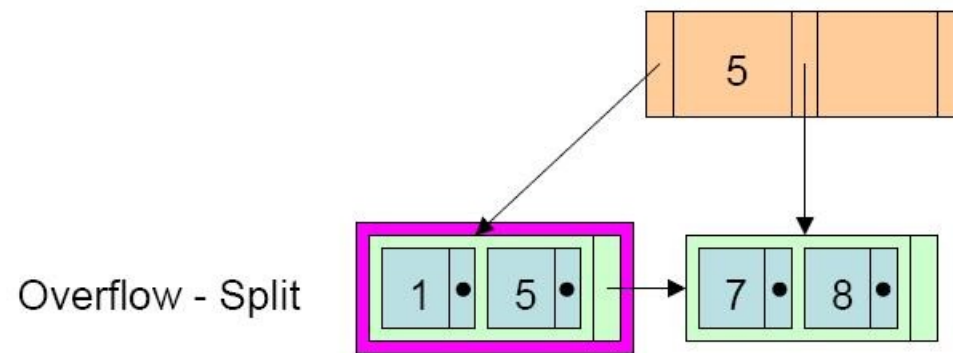


# B+-Tree Insertion



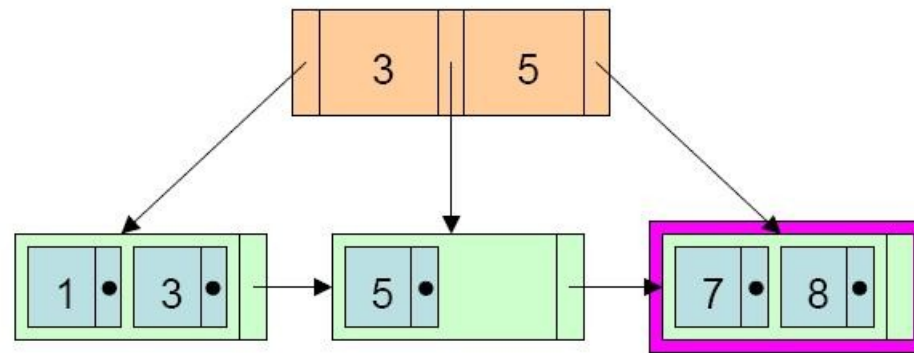
Insert: 7

# B+-Tree Insertion



Insert: 3

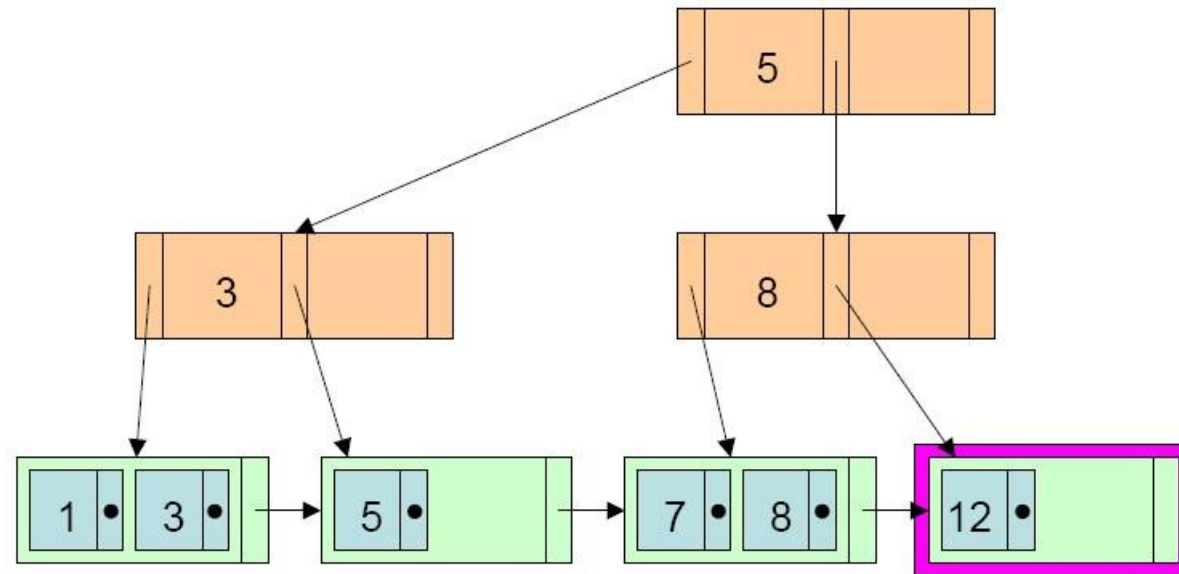
# B+-Tree Insertion



Overflow - Split  
Propagates a new level

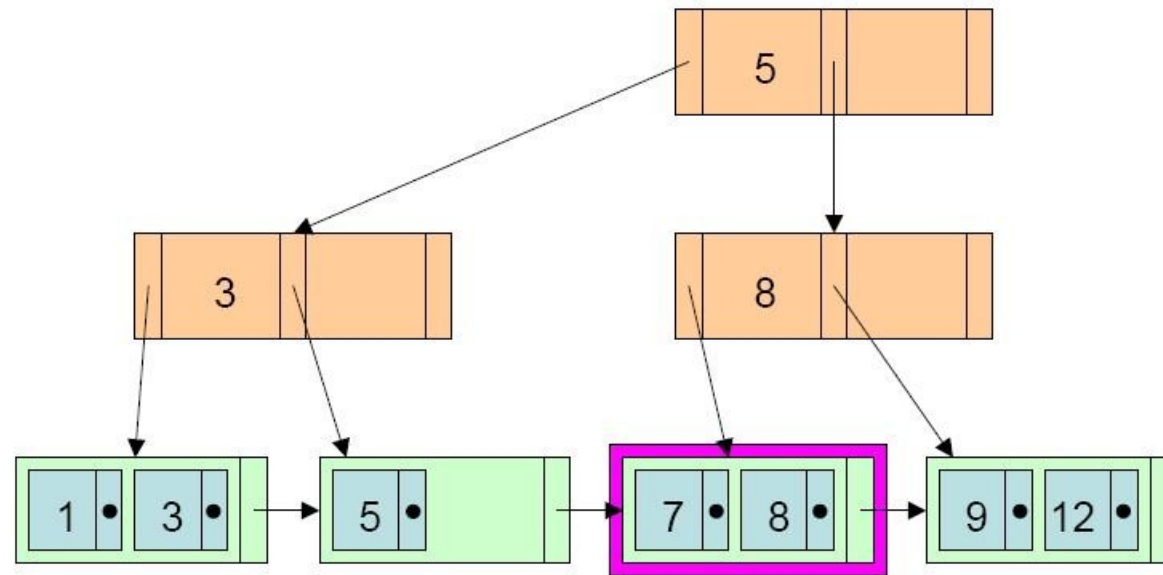
Insert: 12

# B+-Tree Insertion



Insert: 9

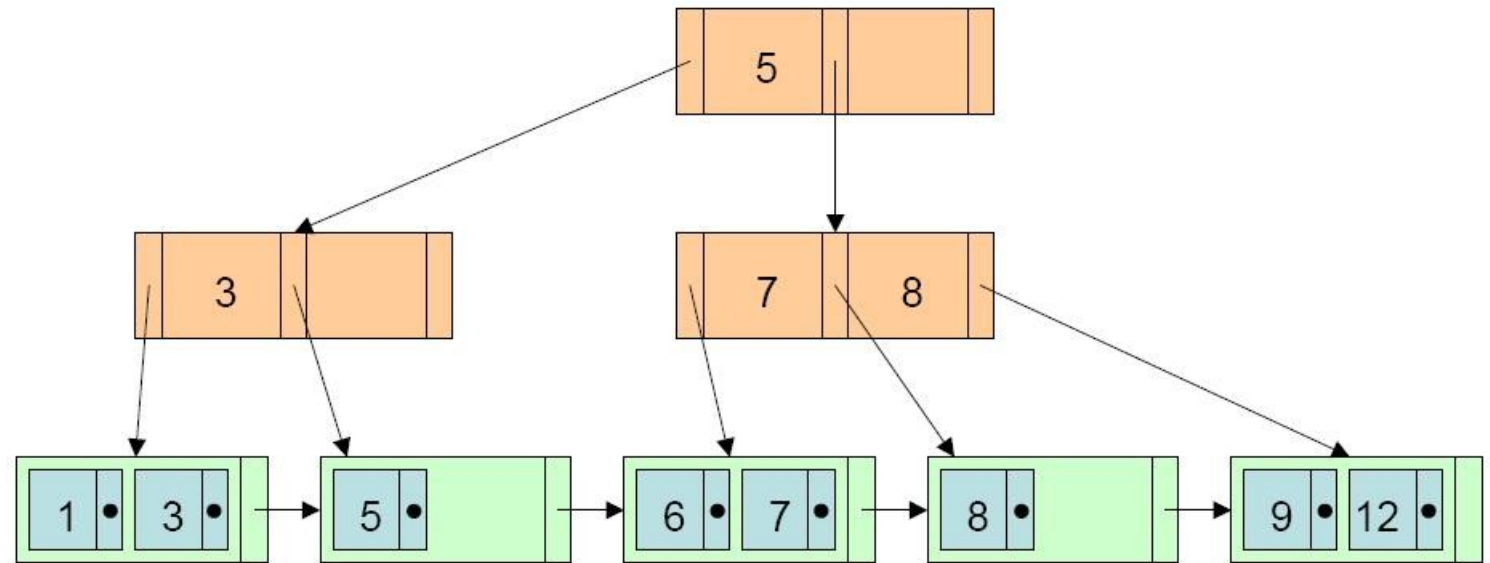
# B+-Tree Insertion



Overflow – Split, propagates

Insert: 6

# B+-Tree Insertion



Resulting B+-tree

# Summary

# Summary

- Storage hierarchy
  - Accessing disk is major bottleneck
- Organizing records in files
  - Heap files, sorted files, hash files
- Indexes
  - Additional sorted files that provide efficient secondary access methods
- Primary, secondary, and clustering indexes
- Multilevel indexes
  - Retrieval requires reading fewer blocks
- Dynamic multilevel indexes
  - Leave some space in index blocks for new entries
  - B-tree and B+-tree