Kalkulus

BAB V TURUNAN

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Sub Pokok Bahasan

- Rumus Dasar Turunan
- Turunan Fungsi Aljabar
- Aturan Dasar Turunan
- Aturan Rantai
- Turunan Fungsi Trigonometri

Kompetensi Khusus

Mahasiswa mampu memahami konsep turunan dari limit, mengetahui rumus dasar turunan, dan mampu menyelesaikan turunan dengan aturan rantai

TURUNAN / DERIVATIF

$$f(x_2) = y_2$$

f (x)

$$B(x_2, y_2)$$

$$f(x_1) = y_1$$

$$A(x_1, y_1)$$

$$x_1 \qquad x_2$$

$$h = \Delta x = x_2 - x_1$$
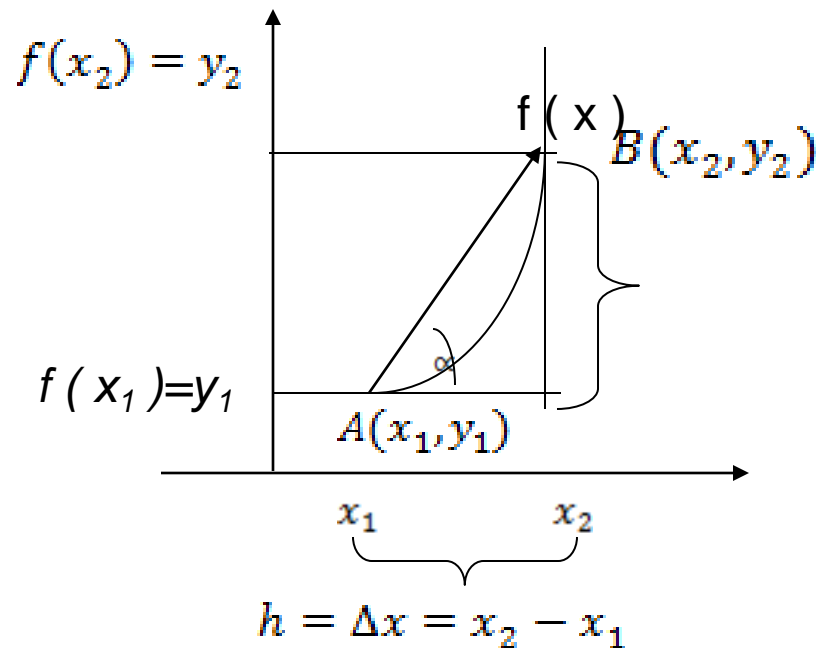
Gradien garis AB =

$$m_{AB} = \frac{\Delta y}{\Delta x} = tg \propto$$

$$\Delta y = y_2 - y_1$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{\Delta x \to 0} \frac{f(x_2) - f(x_1)}{\Delta x}$$

Sehingga :

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Atau :

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Contoh 1 : Carilah dy/dx dari y = k

- $y' = \dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

  $= \lim\limits_{h \to 0} \dfrac{k-k}{h}$

  $= 0$

Contoh 2 : Carilah dy/dx dari y = x

□ $y' = \dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\qquad\qquad = \lim\limits_{h \to 0} \dfrac{(x+h) - (x)}{h}$

$\qquad\qquad = \lim\limits_{h \to 0} \dfrac{\cancel{h}}{\cancel{h}}$

$\qquad\qquad = 1$

Contoh 2 : Carilah dy/dx dari $y = x^2$

□ $y' = \dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{(x^2 + 2xh + h^2) - x^2}{h}$

$\qquad = \lim\limits_{h \to 0} \dfrac{\cancel{h}(2x+h)}{\cancel{h}}$

$\qquad = \lim\limits_{h \to 0} 2x + h$

$\qquad = 2x$

Contoh 2 : Carilah dy/dx dari $y = x^3$

Contoh 2 : Carilah dy/dx dari $y = 1/x$

Carilah dy/dx dari $y = x^n$

□

□ $y' = \dfrac{dy}{dx} = n.x^{n-1}$

Rumus Dasar

1. $y = k \rightarrow y' = 0$
2. $y = x \rightarrow y' = 1$
3. $y = x^2 \rightarrow y' = 2x$
4. $y = x^3 \rightarrow y' = 3x^2$
5. $y = 1/x = x^{-1} \rightarrow x = -x^{-2}$

Contoh : Carilah $\dfrac{dy}{dx}$ dari :

1) $f(x) = 2x+3$

$$\dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \dfrac{2(x+h) + 3 - (2x+3)}{h}$$

$$= \lim_{h \to 0} \dfrac{2x + 2h + 3 - 2x - 3}{h}$$

$$= \lim_{h \to 0} \dfrac{2h}{h} = 2$$

$$2)\,f(x) = x^3 + 7x$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(x+h)^3 + 7(x+h)\right] - (x^3 + 7x)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 h + 3xh^2 + h^3 + 7h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^3 + 7$$

$$= 3x^2 + 7$$

Definisi :

Turunan fungsi f di a, dinyatakan oleh $f'(a)$ dengan :

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- □ Contoh :

Tentukan turunan fungsi f(x) = x^2 − 3x di x= 1

Jawab :

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 3(1+h) - (1^2 - 3 \cdot 1)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 - h}{h} = \lim_{h \to 0} \frac{h(h-1)}{h}$$

$$= \lim_{h \to 0} h - 1 = -1$$

Teorema :

Diketahui y = f (x)

Jika f'(a) ada maka berarti f kontinu di
  x =a

ATURAN TURUNAN :

1) $y = k \cdot u \Rightarrow$ $\quad y' = k.u', \qquad k = konstanta$

2) $y = u + v \Rightarrow$ $\quad y' = u' + v', \qquad u, v = fungsi\ x$

3) $y = u - v \Rightarrow$ $\quad y' = u' - v'$

4) $y = u \cdot v \Rightarrow$ $\quad y' = u'.v + uv'$

5) $y = \dfrac{u}{v}$ $\quad \Rightarrow y' = \dfrac{u'v - uv'}{v^2}$

Contoh :

1) Diketahui : u = 3x - 5 dan v = x^2 + 7

a) y = 2u = 2(3x-5) → y'=2.u'=2.3=6

 b) y = u + v = (3x − 5) + (x^2 + 7)

$$y' = u' + v' = 3 + 2x$$

c)　y = u − v = (3x − 5) − (x^2 + 7)

$$y' = u' - v' = (3) - 2x$$

d)　y = u . v = (3x − 5) (x^2 + 7)

$$y' = u'v + uv'$$

$= 3(x^2 + 7) + (3x - 5).2x$
$= 3x^2 + 21 + 6x^2 - 10x$
$= 9x^2 - 10x + 21$

e) $y = \dfrac{u}{v} = \dfrac{3x-5}{x^2+7}$

$y' = \dfrac{u'v - uv'}{v^2}$

$\quad = \dfrac{3\left(x^2+7\right) - (3x-5).2x}{(x^2+7)^2}$

$\quad = \dfrac{3\,x^2 + 21 - 6x^2 + 10x}{(x^2+7)^2}$

$\quad = \dfrac{-3x^2 + 10x + 21}{(x^2+7)^2}$

□ **Contoh :**

1) $y = x \rightarrow y' = 1$

$y = x^2 \rightarrow y' = 2x$

$y = x^3 \rightarrow y' = 3x^2$

2) $y = x^3 - 3x^2 + 7x$

$$\frac{dy}{dx} = 3x^2 - 6x + 7$$

3) Jika y = $\dfrac{3t}{t^2+1}$ maka :

$$y' = \frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

$$= \frac{3(t^2+1) - 3t.2t}{(t^2+1)^2} = \frac{3 - 3t^2}{(t^2+1)^2}$$

ATURAN RANTAI

Misalkan y = f(u) dan u = g(x), maka:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Contoh :

1) Diketahui $y = (x^3+3)^5$. Hitunglah $\frac{dy}{dx}$

substitusi ; $u = x^3+3 \rightarrow \frac{du}{dx} = 2x$

$y = u^5 \rightarrow \frac{dy}{du} = 5u^4$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4 \cdot 2x = 5(x^2+3)^4 2x$$

$$= 10x \cdot (x^2+3)^4$$

Contoh 2

2. Diketahui $y = \left(\dfrac{2x-3}{3x+1}\right)^7$

Turunan Fungsi Trigonometri

1. y = sin x

2. y = cos x

Tentukan turunan dengan menggunakan cara:

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

sin (a+b) = sin a.cos b + cos a.sin b

cos(a+b) = cos a.cos b − sin a.sin b

Rumus Turunan Trigonometri

1. $y = \sin x \rightarrow y' = \cos x$

2. $y = \cos x \rightarrow y' = -\sin x$

3. $y = \tan x$

4. $y = \cot x$

5. $y = \sec x$

6. $y = \cosec x$

Soal (aturan rantai)

1. $y = \sin(x^3)$
2. $y = \sin^3 x$
3. $y = \tan(x^2 + 4x)$
4. $y = \cos(x^2 - 7x)^6$
5. $y = \cos^6(x^2 - 7x)$

Referensi

- Purcell, Varberg, *Kalkulus dan Geometri Analitis*, Penerbit Erlangga, 1993
- Frank Ayres, *Calculus*, Mc.Graw Hill, New York, 1972
- J.Salas and Hill, *Calculus One and Several Variables*, John Willey& Sons, NewYork, 1982