Kalkulus

Bab IV
Limit Fungsi dan Kontinuitas (1)

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Sub Pokok Bahasan

- Limit Fungsi

Kompetensi Khusus

Mahasiswa mampu memahami konsep limit dan cara menyelesaikan limit fungsi

Pemahaman secara intuisi

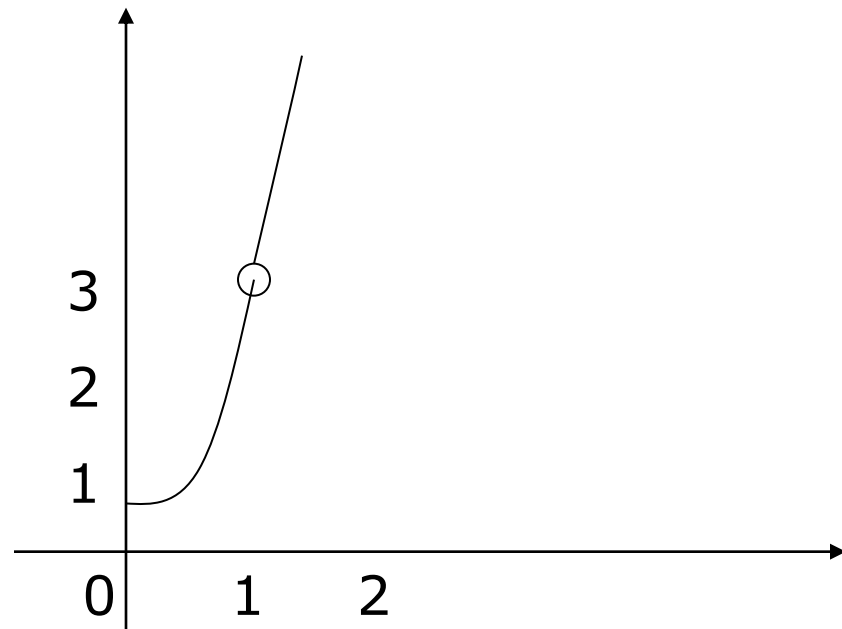- Perhatikan fungsi : $f(x) = \dfrac{x^3 - 1}{x - 1}$

- Fungsi f (x) tidak terdefinisi di x = 1, karena $f(1) = \dfrac{0}{0}$

LIMIT FUNGSI

Tabel untuk beberapa nilai f (x) untuk x mendekati 1

x	y
0	1
0,9	2,710
0,99	2,970
0,999	2,997
1	?
1,001	3,003
1,01	3,030
1,1	3,310
2	7

LIMIT FUNGSI

❑ **Dari tabel dan grafik di dapat kesimpulan :**
  f (x) mendekati 3 jika x mendekati 1

❑ **Dengan notasi matematis :**

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$

Secara aljabar :

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)}$$

$$\lim_{x \to 1} (x^2 + x + 1) = 1^2 + 1 + 1 = 3$$

Maple

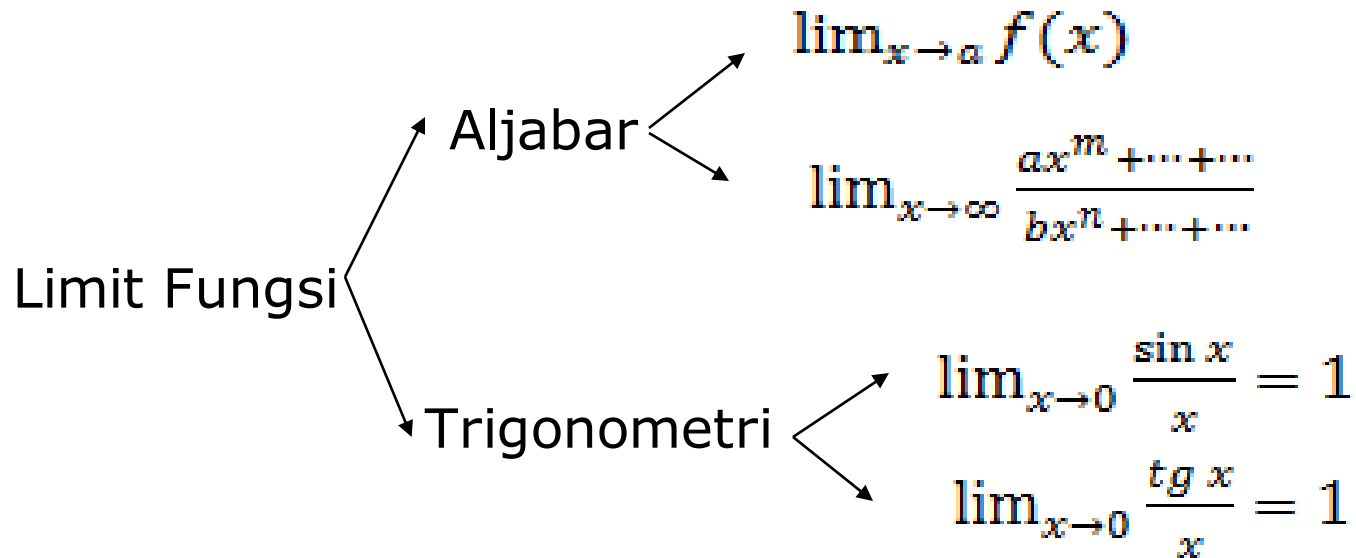> **f:=x->(x^3-1)/(x-1);**

>limit (f(x), x = 1)

Atau :

> **y:= (x^3-1)/(x-1);**

> limit (y, x=1)

LIMIT FUNGSI

- Definisi : $\lim_{x \to c} f(x) = L$
  berarti jika x mendekati c maka f(x) mendekati L.

- Limit fungsi hanya merupakan nilai pendekatan dari fungsi untuk x tertentu, bukan nilai fungsi yang sebenarnya

Macam – macam limit fungsi

$$\lim_{x \to a} f(x)$$

Aljabar

$$\lim_{x \to \infty} \frac{ax^m + \cdots + \cdots}{bx^n + \cdots + \cdots}$$

Limit Fungsi

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Trigonometri

$$\lim_{x \to 0} \frac{tg\, x}{x} = 1$$

Langkah–langkah penyelesaian :

1). Masukkan nilai x pada fungsi, jika hasilnya riil (ada) maka selesai.

2). Jika tak tentu / tak ada, maka sederhanakan dengan cara :
  - Dijabarkan
  - Difaktorkan
  - Disamakan penyebut
  - Dirasionalkan → dikalikan dengan sekawannya. (berlawanan tanda)

3) Gunakan limit kiri dan limit kanan

Contoh :

1) $\lim_{x \to 3}(4x - 5)$ = 4 . 3 – 5 = 7 (langsung dimasukkan, hasilnya riil → selesai)

2) Difaktorkan

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+3)}{(x-2)} = 2+3 = 5$$

3) Dikalikan dengan sekawannya

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(\sqrt{x}-1)}$$

$$= \lim_{x \to 1} \sqrt{x} + 1$$

$$= \sqrt{1} + 1 = 2$$

4) Disamakan penyebut :

$$\lim_{x \to 2} \frac{1 - \dfrac{2}{x}}{x^2 - x - 2}$$

$$= \lim_{x \to 2} \frac{\dfrac{x}{x} - \dfrac{2}{x}}{x^2 - x - 2} = \lim_{x \to 2} \frac{\dfrac{x - 2}{x}}{(x - 2)(x + 1)}$$

$$= \lim_{x \to 2} \frac{1}{x(x + 1)} = \frac{1}{2.3} = \frac{1}{6}$$

Maple :

1. > **y:=4*x-5;**
   > **limit(y,x=3);**

2. > **y:=(x^2+x-6)/(x-2);**
   > **limit(y,x=2);**

3. > **y:=(x-1)/(x^(1/2)-1);**
   > **limit(y,x=1);**

4. > **y:=(1-2/x)/(x^2-x-2);**
   > **limit(y,x=2);**

Contoh

5. $\displaystyle\lim_{x\to 4}\frac{\sqrt{4+x}-\sqrt{2x}}{x-4}$

6. $\displaystyle\lim_{x\to 3}\frac{9-x^2}{4-\sqrt{x^2+7}}$

7. $\displaystyle\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4}$

8. $\displaystyle\lim_{x\to 4}\frac{2x^2-6x-8}{x^2-16}$

9. $\displaystyle\lim_{x\to 3}\frac{\sqrt{x+2}-\sqrt{2x-1}}{\sqrt{2x-3}-\sqrt{x}}$

10. $\displaystyle\lim_{x\to 2}\left[\frac{6-x}{x^2-4}-\frac{1}{x-2}\right]$

Contoh

1. $\displaystyle\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$

2. $\displaystyle\lim_{x\to 25}\frac{5-\sqrt{x}}{x-25}$

3. $\displaystyle\lim_{x\to 4}\frac{\sqrt{4+x}-\sqrt{2x}}{x-4}$

4. $\displaystyle\lim_{x\to -2}\frac{\sqrt{x^2+12}-4}{4-x^2}$

5. $\displaystyle\lim_{x\to 4}\frac{2x^2-6x-8}{x^2-16}$

6. $\displaystyle\lim_{x\to 2}\left[\frac{6-x}{x^2-4}-\frac{1}{x-2}\right]$

7. $\displaystyle\mathit{Lim}_{x\to 3}\frac{\sqrt{x+2}-\sqrt{2x-1}}{\sqrt{2x-3}-\sqrt{x}}$

8. $\displaystyle\lim_{x\to 3}\frac{9-x^2}{4-\sqrt{x^2+7}}$

9. $\displaystyle\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4}$

10. $\displaystyle\lim_{x\to 3}\frac{1-3/x}{x^2-2x-3}$

Contoh :

11. $\displaystyle \lim_{x \to 2} \frac{x^2-4}{3-\sqrt{x^2+5}}$

12. $\displaystyle \lim_{x \to 3} \frac{1/x-1/3}{x-3}$

13. $\displaystyle \lim_{x \to 2} \frac{\sqrt{x+2}-\sqrt{3x-2}}{x-2}$

14. $\displaystyle \lim_{x \to 1} \left[\frac{2x^2-x-1}{3x^2-x-2} \right]$

15. $\displaystyle \lim_{x \to -2} \frac{x^3+8}{x^2-4}$

LIMIT TAK HINGGA

- $\lim\limits_{x\to\infty} \dfrac{1}{x} = 0$ dan $\lim\limits_{x\to-\infty} \dfrac{1}{x} = 0$ → $\lim\limits_{x\to\pm\infty} \dfrac{1}{x^n} = 0$

- $\lim\limits_{x\to\infty} x^n = \infty$

Cara penyelesaian $\lim\limits_{x\to\infty} \dfrac{ax^n + \cdots}{bx^m + \cdots}$ → dibagi pangkat tertinggi dari penyebut

1. $\lim\limits_{x\to\infty} \dfrac{x^2 - 2x - 3}{2x^2 + 4x + 7}$

2. $\lim\limits_{x\to\infty} \dfrac{x^3 + 7x - 6}{3x^5 + 2x^3 + 10}$

3. $\lim\limits_{x\to-\infty} \dfrac{x^6 - 3x^2}{x^4 + 5x - 3}$

Soal

1. $\lim\limits_{x \to \infty} (\sqrt{x^2 - 1} - \sqrt{x^2 + 2x})$

2. $\lim\limits_{x \to \infty} (\sqrt{x^2 + 2x} + 5x)$

3. $\lim\limits_{x \to \infty} (x - \sqrt{x^2 - 2x})$

4. $\lim\limits_{x \to \infty} \dfrac{x - 7}{\sqrt{x^2 - 7x + 5}}$

5. $\lim\limits_{x \to \infty} \left(\dfrac{x^2}{2x - 1} - \dfrac{x^2}{2x + 1} \right)$

Teorema Limit Utama

1. $\lim\limits_{x \to c} k = k$

2. $\lim\limits_{x \to c} x = c$

3. $\lim\limits_{x \to c} k.f(x) = k.\lim\limits_{x \to c} f(x)$

4. $\lim\limits_{x \to c} (f(x) + g(x)) = \lim\limits_{x \to c} f(x) + \lim\limits_{x \to c} g(x)$

5. $\lim\limits_{x \to c} (f(x) - g(x)) = \lim\limits_{x \to c} f(x) - \lim\limits_{x \to c} g(x)$

6. $\lim\limits_{x \to c} (f(x).g(x)) = \lim\limits_{x \to c} f(x).\lim\limits_{x \to c} g(x)$

Teorema Limit Utama (lanjutan)

$$7.\ \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$$

$$8.\ \lim_{x \to c}[f(x)]^n = [\lim_{x \to c} f(x)]^n$$

$$9.\ \lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$$

Contoh (penerapan Teorema limit utama)

1. Jika $\lim\limits_{x \to 1} f(x) = 3$ dan $\lim\limits_{x \to 1} g(x) = -1$

   a. $\lim\limits_{x \to 1} 2.f(x) = 2.\lim\limits_{x \to 1} f(x) = 2.3 = 6$

   b. $\lim\limits_{x \to 1}[f(x) + g(x)] = \lim\limits_{x \to 1} f(x) + \lim\limits_{x \to 1} g(x) = 3 + (-1) = 2$

   c. $\lim\limits_{x \to 1}[f(x) - g(x)] = \lim\limits_{x \to 1} f(x) - \lim\limits_{x \to 1} g(x) = 3 - (-1) = 4$

Contoh (penerapan Teorema limit utama)

d) $\lim_{x \to 1} f(x).g(x) = \lim_{x \to 1} f(x).\lim_{x \to 1} g(x) = 3.(-1) = -3$

e) $\lim_{x \to 1} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to 1} f(x)}{\lim_{x \to 1} g(x)} = \dfrac{3}{-1} = -3$

f) $\lim_{x \to 1}[f(x)]^2 = [\lim_{x \to 1} f(x)]^2 = [3]^2 = 9$

g) $\lim_{x \to 1} \sqrt[3]{g(x)} = \sqrt[3]{\lim_{x \to 1} g(x)} = \sqrt[3]{-1} = -1$

Limit Trigonometri

1. $\lim\limits_{x \to 0} \dfrac{x}{sinx} = \lim\limits_{x \to 0} \dfrac{sinx}{x} = 1 \rightarrow \lim\limits_{x \to 0} \dfrac{ax}{sinbx} = \lim\limits_{x \to 0} \dfrac{sinax}{bx} = \dfrac{a}{b}$

2. $\lim\limits_{x \to 0} \dfrac{x}{tanx} = \lim\limits_{x \to 0} \dfrac{tanx}{x} = 1 \rightarrow \lim\limits_{x \to 0} \dfrac{ax}{tanbx} = \lim\limits_{x \to 0} \dfrac{tanax}{bx} = \dfrac{a}{b}$

Contoh :

1. $\lim\limits_{x \to 0} \dfrac{sin5x}{tan4x} = \lim\limits_{x \to 0} \dfrac{sin5x}{x} \cdot \dfrac{x}{tan4x} = \dfrac{5.1}{4} = 5/4$

2. $\lim\limits_{x \to 0} \dfrac{1-cosx}{x^2}$

3. $\lim\limits_{x \to 0} \dfrac{x^3}{3.sin^2 2x}$

4. $\lim\limits_{x \to 0} \dfrac{cos^2 x}{1+sinx}$

5. $\lim\limits_{x \to 0} \dfrac{sin^2 4x}{x.tan3x}$

6. $\lim\limits_{x \to 0} \dfrac{cos2x-1}{x.sin3x}$

7. $\lim\limits_{x \to 0} \dfrac{3x+sin4x}{5x-tan2x}$

8. $\lim\limits_{x \to \pi/2} \sec^2 x - \sec x . \tan x$

9. $\lim\limits_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{x.cosx} \right)$

10. $\lim\limits_{x \to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x} \right)$

Ringkasan Materi

- Limit hanya merupakan nilai pendekatan, bukan nilai sebenarnya

- Cara menyelesaikan limit : Masukkan nilai x pada fungsi, jika hasilnya riil (ada) maka selesai, tetapi jika tak tentu / tak ada, maka sederhanakan dengan cara : dijabarkan, difaktorkan, disamakan penyebut, atau dirasionalkan (dikalikan sekawannya). Jika tidak ada nilainya juga, gunakan limit kiri dan limit kanan.

SOAL – SOAL

❑ **Hitunglah limit berikut :**

1). $\lim_{x \to 1} \dfrac{5x - x^2}{x^2 + 2x - 4}$

2). $\lim_{x \to 3} \dfrac{2x^2 + 5x - 3}{x + 3}$

3). $\lim_{x \to 0} \dfrac{x^3 - 16x}{x^2 + 4x}$

4). $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

5). $\lim_{x \to 2} \dfrac{\sqrt{2 + x} - \sqrt{2x}}{2 - x}$

6). $\lim_{x \to 3} \dfrac{x^3 - 27}{x^2 - 9}$

7). $\lim_{x \to \infty} \dfrac{(x + 2)(x^2 - x - 6)}{x^2 + 4x + 4}$

8). $\lim_{x \to \infty} \dfrac{(2x - 6)(4x - 5)(3x + 2)}{3x^2 + x - 1}$

9). $\lim_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 + x - 6}$

10). $\lim_{x \to 0} \dfrac{tg\ 2x}{3^x}$

11). $\lim_{x \to 0} \dfrac{1 - c0s\ x}{x^2}$

Referensi

- Purcell, Varberg, *Kalkulus dan Geometri Analitis*, Penerbit Erlangga, 1993

- Frank Ayres, *Calculus*, Mc.Graw Hill, New York, 1972

- J.Salas and Hill, *Calculus One and Several Variables*, John Willey& Sons, NewYork, 1982