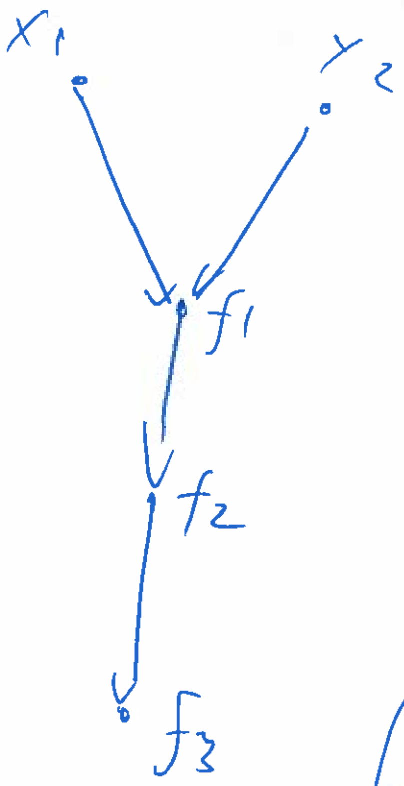
$f_1(x_1,x_2) = x_1 x_2$

$f_2(x) = \sin(x)$

$f_3(x) = \cos(x)$

$f_3(f_2(f_1(x_1,x_2)))$



$\frac{df_3}{dv_1}$

Forward Diff (v_1, v_2)

1) $\underline{t_{x_1} = 1 \, , \, t_{x_2} = 0}$

2) $t_{f_1} = \underbrace{\frac{\partial f_1}{\partial x_1}}\Big|_{v_1,v_2} t_{x_1} + \frac{\partial f_1}{\partial x_2}\Big|_{v_2,v_1} t_{x_2}$

$= v_2 \cdot 1 + v_1 \cdot 0 = \underline{v_2}$

$v_{f_1} = f_1(v_1, v_2) = v_1 v_2$

3) $t_{f_2} = \frac{\partial f_2}{\partial f_1}\Big|_{v_{f_1}} \cdot t_{f_1} = \cos(v_{f_1}) v_2 = \cos(v_1 v_2) v_2$

$v_{f_2} = \sin(v_1 v_2)$

4) $t_{f_3} = \frac{\partial f_3}{\partial f_2}\Big|_{v_{f_2}} \cdot t_{f_2} = -\sin(v_{f_2}) \cdot \cos(v_1 v_2) v_2$

$= \underline{-\sin(\sin(v_1 v_2)) \cos(v_1 v_2) v_2}$

$v_{f_3} = \cos(\sin(v_1 v_2))$

$(\alpha v_2 + \beta v_1)$

1) $$\left|\begin{array}{l} v_{f_1} = v_1 v_2 \\ v_{f_2} = \sin(v_1 v_2) \\ v_{f_3} = \cos(\sin(v_1 v_2)) \end{array}\right.$$

2) $$\underline{\underline{t_{f_3} = 1}}$$

3) $$t_{f_2} = \frac{\partial f_3}{\partial f_2}\bigg|_{v_{f_2}} \cdot t_{f_3} = -\sin(v_{f_2})$$

4) $$t_{f_1} = \frac{\partial f_2}{\partial f_1}\bigg|_{v_{f_1}} \cdot t_{f_2} = \cos(v_{f_1})(-\sin(v_{f_2}))$$
$$= -\sin(v_{f_2})\cos(v_{f_1})$$

5) $$t_{x_1} = \frac{\partial f_1}{\partial x_1}\bigg|_{v_1, v_2} \cdot t_{f_1} = \underline{v_2}\,(-\sin(v_{f_2})\cos(v_{f_1}))$$

$$\mathcal{L}(f, x, v) = J_x^{f\,'}\, v$$

$$\tilde{f} = f^T v$$

$$\mathcal{L}(\tilde{f}, x, 1) = J_x^{\tilde{f}\,T}\, \underline{1} =$$

$$= \Big(\underbrace{J_f^{\tilde{f}}}_{v^T} J_x^{f}\Big)^T = J_x^{f\,T}\, \underline{v}$$

$\underline{y = Wx}$ $\qquad\qquad$ $v^T x$

$\underline{J_x^y = W}$ $\qquad\qquad$ $J = \underline{v^T}$

$$J_x^{\tilde{f}} = J_f^{\tilde{f}} J_x^{f} + J_v^{\tilde{f}} J_x^{v}$$

Assumed to be $\underline{0}$.

$$Gv = J_\theta^{y\,T} H_2 J_\theta^{y} v$$

$$V_1 = J_\theta^{y} v = Rop(y, \theta, v)$$

$$\Rightarrow Gv = J_\theta^{y\,T} \underbrace{H_2 V_1}_{V_2}$$

$$Gv = J_\theta^{y\,T} V_2 = L(y, \theta, V_2)$$