Forecasting series with a trend and/or seasonality

ADVANCED EXPONENTIAL SMOOTHING

Holt’s Method
(“double exponential” in XLMiner)

Idea: augment simple exp smoothing by capturing a **trend** component

Forecasts = most recent *estimated level* + trend

$$\mathbf{F_{t+k} = L_t + k\, T_t}$$

Equation #1: Estimating the Level

$$\mathbf{L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})}$$

Adjust the previous level by adding trend

Equation #2: Estimating the Trend

$$\mathbf{T_t = \beta\ (L_t - L_{t-1}) + (1 - \beta)\ T_{t-1}}$$

Update previous trend using the difference between the most recent level values.

Local trend: trend is allowed to vary adaptively over time!
(compare to regression-based models)

Choosing α and β

Default values

Minimize RMSE or MAPE (or other goodness-of-fit criterion) for training set

- Danger of over-fitting!
- What to do: make sure chosen values are reasonable

DoubleExponentialOutput1

XLMiner: Holt's on Coca Cola series

With default $\alpha=0.2$, $\beta=0.15$

(Optimized RMS values $\alpha=0.29$, $\beta=0$)

Error Measures (Training)

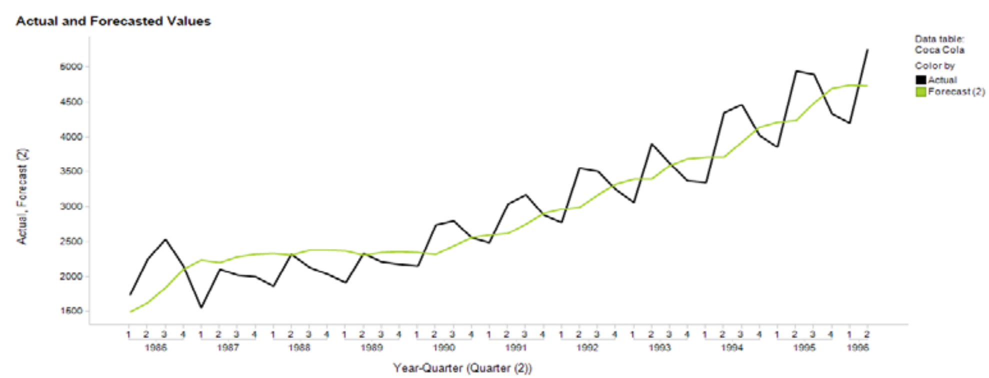
MAPE	11.549713
MAD	326.24241
MSE	147869.36

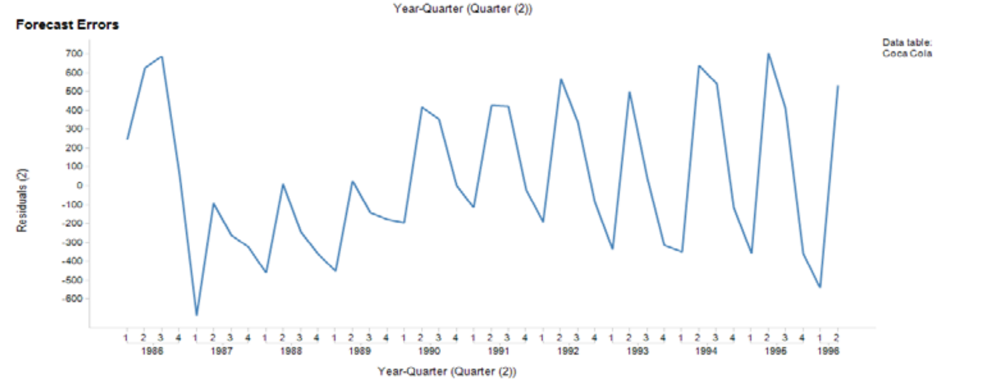
Error Measures (Validation)

MAPE	10.3673
MAD	443.79552
MSE	369187.87

$$F_{t+k} = L_t + k\,T_t$$

Time	Actual	Forecast	Residuals
Q1_86	1734.827	1558.80586	176.021141
Q2_86	2244.961	1667.36073	577.600269
Q3_86	2533.80499	1873.55943	660.245558
Q4_86	2154.963	2116.09456	38.8684329
Q1_87	1547.819	2235.52032	-687.701326
Q2_87	2104.41199	2189.00109	-84.5890932
Q3_87	2014.363	2260.56663	-246.203629
Q4_87	1991.747	2292.42315	-300.676154
Q1_88	1869.05	2304.36489	-435.314887
Q2_88	2313.632	2276.31943	37.3125694
Q3_88	2128.32	2343.91884	-215.598836
Q4_88	2026.829	2354.468	-327.639001
Q1_89	1910.604	2332.77996	-422.175963
Q2_89	2331.16499	2279.51925	51.6457452
Q3_89	2206.55	2322.57225	-116.022255
Q4_89	2173.96799	2328.61099	-154.642991



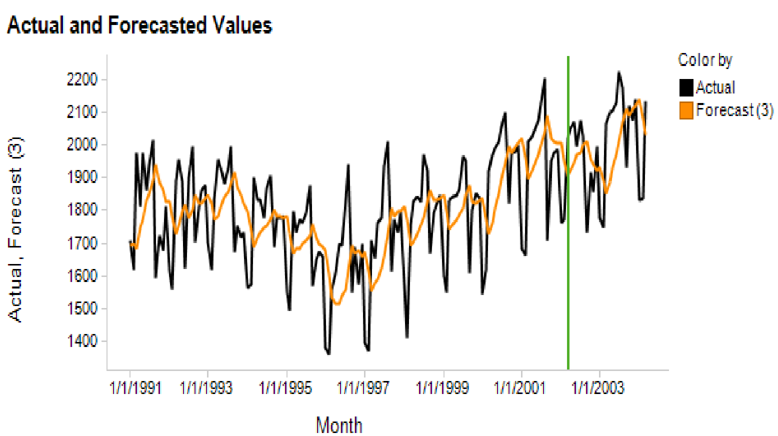


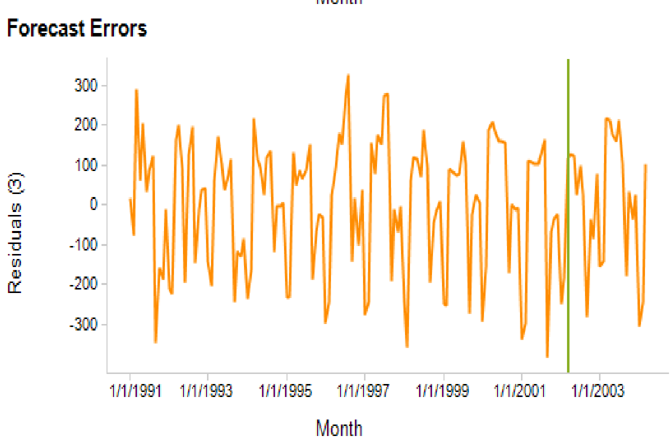
Amtrak Ridership

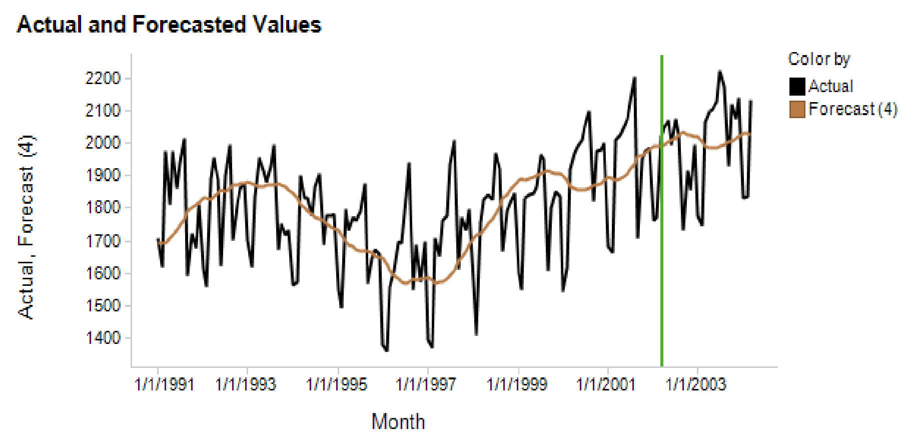
Default (α=0.2, β=0.15)

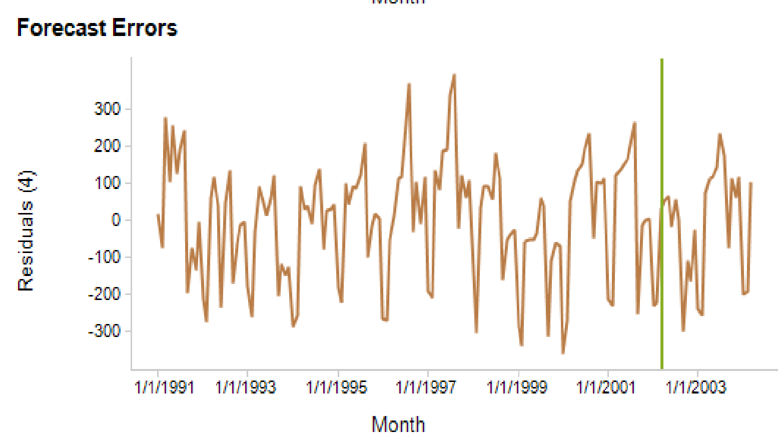
Optimized

Parameters/Options	
Optimization Selected	Yes
Alpha (Level)	0.030350724
Beta (Trend)	0.432623792









Winter's Method

Augments Holt's method by capturing a ***seasonal*** component

Winter's Method

Forecasts take into account level, trend, and seasonality:

$$\mathbf{F_{t+k} = (L_t + k\,T_t)\,S_{t+k-M}}$$

number of seasons

S_t = seasonal index of period t

This formula: *multiplicative* seasonality. *Additive* seasonality also available

If forecast horizon > one cycle of seasons, use the last set of estimated seasonal indexes

Three Equations: for Level, Trend, and Seasonality

Level:

$$L_t = \alpha \frac{Y_t}{S_{t-M}} + (1-\alpha)(L_{t-1} + T_{t-1})$$

Trend (same as Holt's):

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

Seasonality (multiplicative):

$$S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)S_{t-M}$$

Technical Notes

All three smoothing constants are between 0-1.

To obtain forecasts (in XLMiner) you must re-run the exp smoothing on the **non-partitioned data**! Otherwise you lose the info in the most recent observations.

Initialization (technical):

$L_1 = Y_1$ or L_1=a from estimated model $\mathbf{Y_t = a + bt}$

$T_1 = Y_2 - Y_1$ or $T_1 = (Y_T - Y_1) / T$ (avg overall trend)

Initial seasonal indexes = MA indexes (that we saw earlier)

HoltWinterOutput1

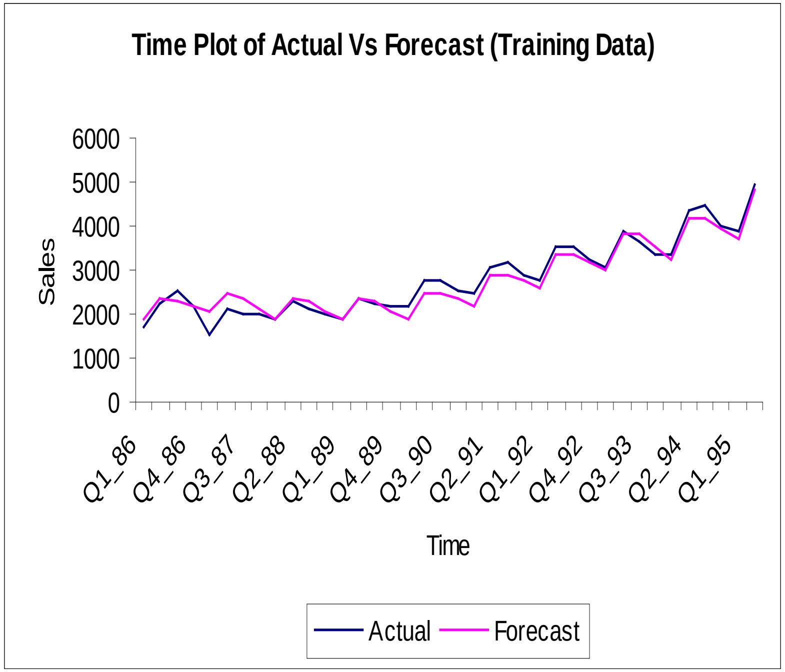
XLMiner Output for Holt-Winter's method, Coca-Cola Sales

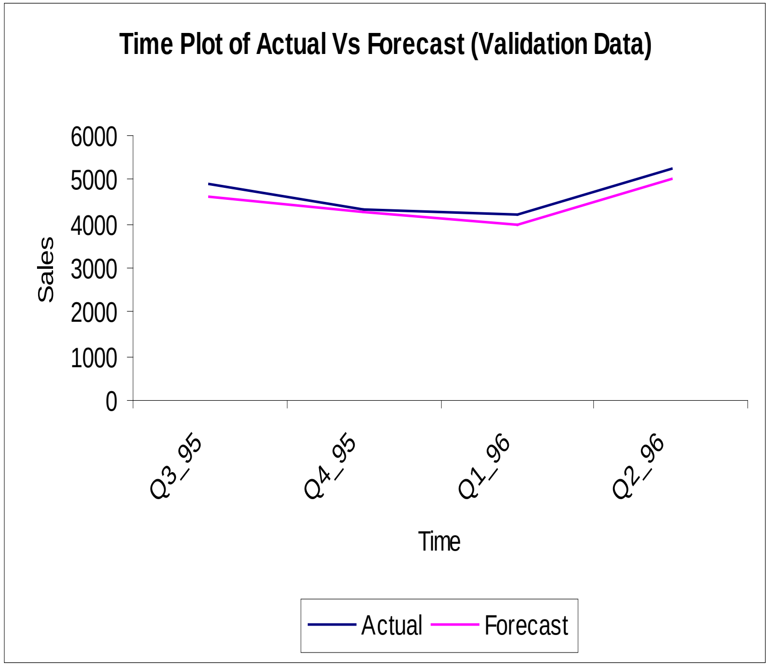
With default $\alpha=0.2$, $\beta=0.15$, $\gamma=0.05$

(Optimized RMS values: $\alpha=1$, $\beta=0$, $\gamma=0.246$)

Error Measures (Validation)

MAPE	4.28216685
MAD	201.637077
MSE	47071.0568





Comparing forecast accuracy for Coca-Cola Sales: validation set performance

MA(4)

MAPE	8.23209427
MAD	413.25
MSE	306391.25

Time	Actual	Forecast
Q3_95	4895	4317
Q4_95	4333	4317
Q1_96	4194	4317
Q2_96	5253	4317

SimpleExpSmooth(.2)

MAPE	13.1712597
MAD	648.344129
MSE	603013.297

Time	Actual	Forecast
Q3_95	4895	4020.40587
Q4_95	4333	4020.40587
Q1_96	4194	4020.40587
Q2_96	5253	4020.40587

Holt’s

MAPE	8.74607892
MAD	405.25
MSE	172378.647

Time	Actual	Forecast
Q3_95	4895	4486.10574
Q4_95	4333	4598.40464
Q1_96	4194	4710.70354
Q2_96	5253	4823.00244

Winter’s

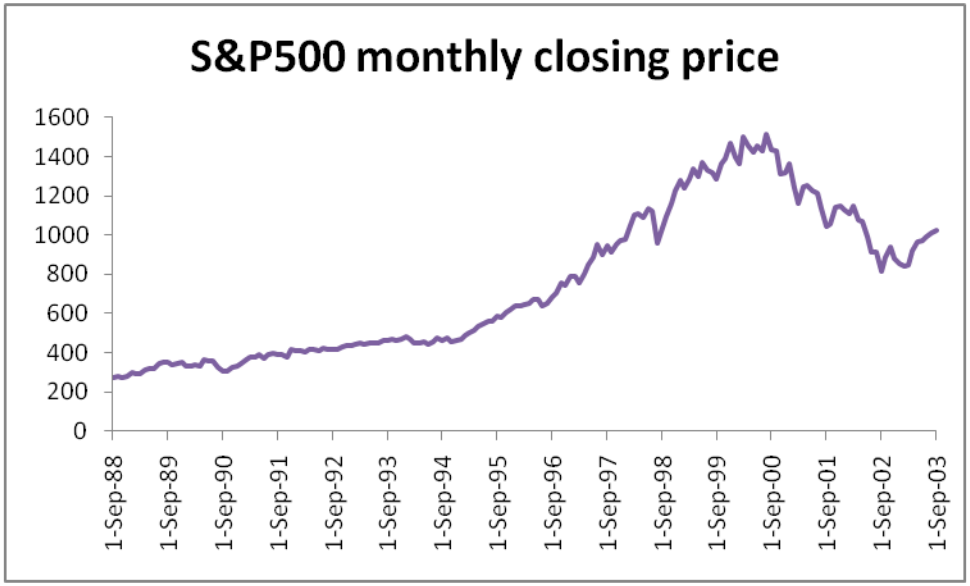
MAPE	4.28216685
MAD	201.637077
MSE	47071.0568

Time	Actual	Forecast
Q3_95	4895	4607.37068
Q4_95	4333	4262.60355
Q1_96	4194	3967.31917
Q2_96	5253	5031.15829

Another Example: S&P500

The S&P500 index in the last 15 years (finance.yahoo.com) – see S&P500.xls.

Monthly closing values from 9/1988-8/2003



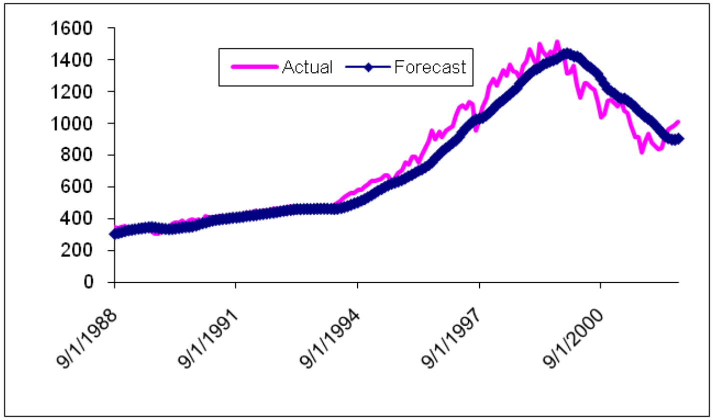
Goal: Forecast Sept 2003

MA12, MA4

Moving Average (W=12)

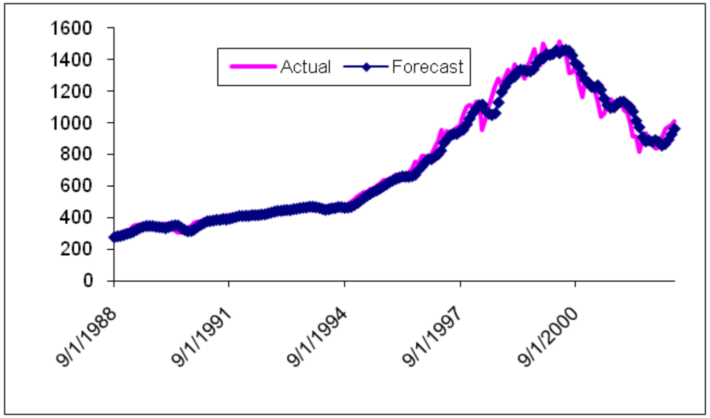
W=12

MAPE	8.41172486
MAD	70.4965972
MSE	8403.06674



W=4

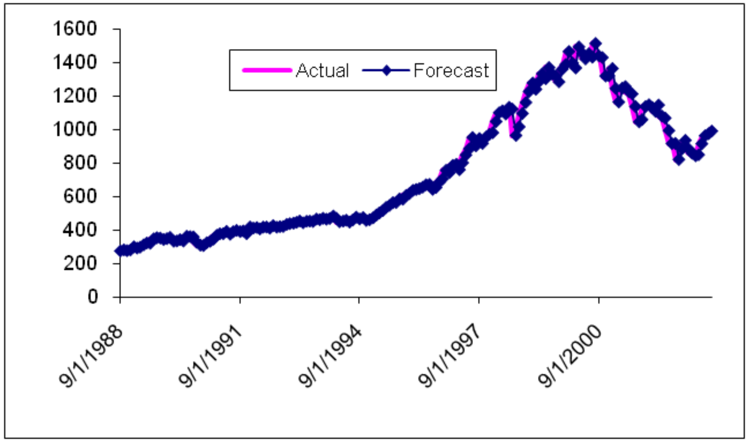
MAPE	4.68738538
MAD	37.155767
MSE	2899.21905



Simple Exponential Smoothing

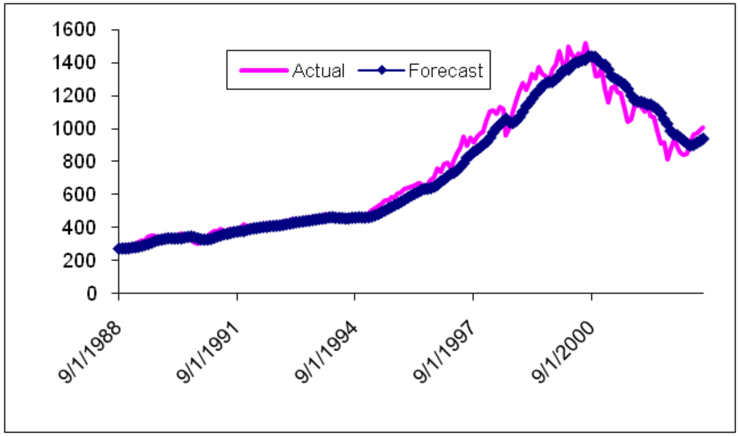
Optimal α=0.961

MAPE	3.4444871
MAD	27.9766304
MSE	1627.15726



α=0.2

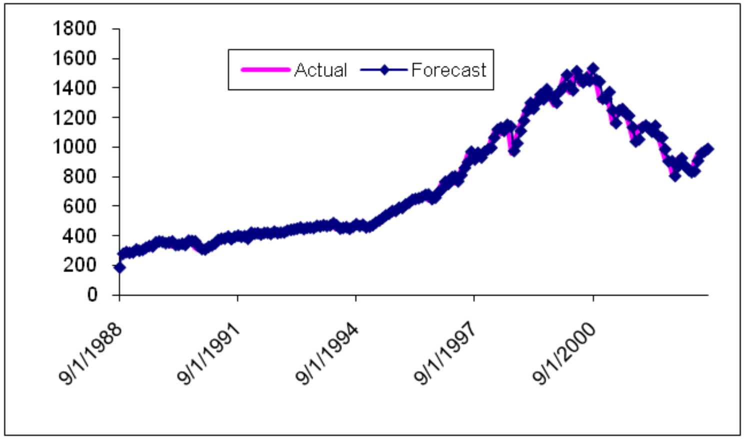
MAPE	6.53493028
MAD	52.4019141
MSE	5034.2055



Holt's Method

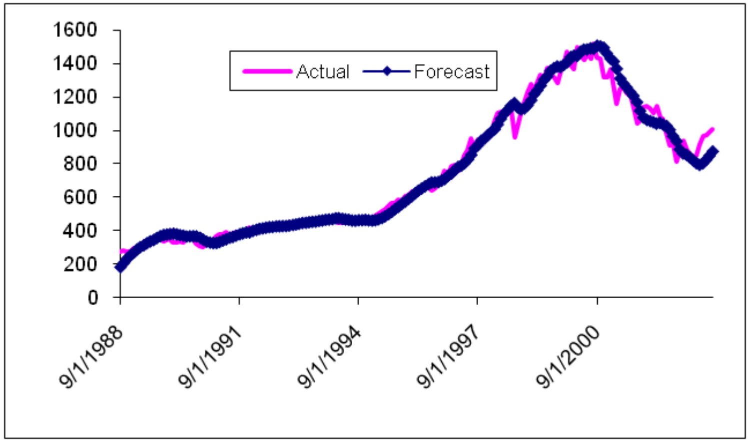
Optimal α=0.954, β=0.05

MAPE	3.54238963
MAD	27.9064492
MSE	1661.23489



α=0.2, β=0.15

MAPE	5.21029776
MAD	38.0573344
MSE	3110.82211

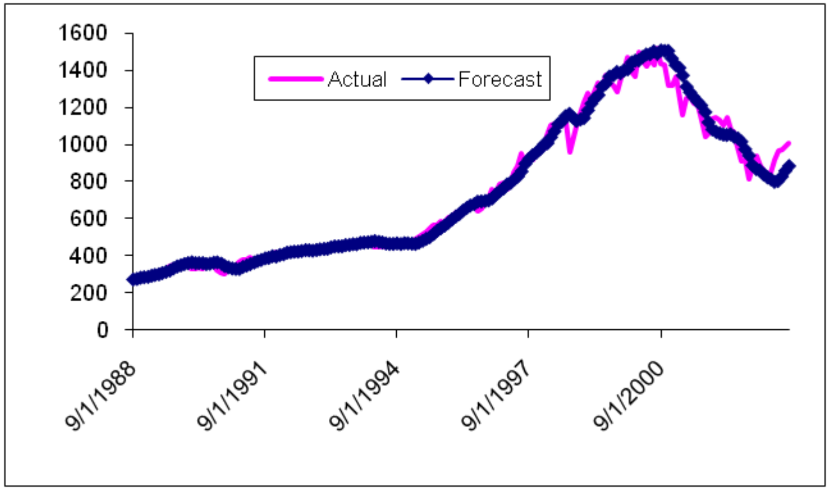


HoltWinterOutput1

Holt-Winter's Method (4 seasons)

α=0.2, β=0.15, γ=0.05

MAPE	4.49048677
MAD	35.9005141
MSE	2976.22705



Which model would you choose?

1. MA(12)
2. MA(4)
3. Simple exp smoothing
4. Holt exp smoothing
5. Holt-Winter’s exp smoothing

Application: Forecasting Firm Demand

Goal: “model average demand, taking into account seasonality and the progression of time”

The time-dependence of demand is derived using a time-series analysis; for the purpose of our analysis, we are using CB Predictor from Decision Engineering. The time series is comprised of both a base demand dependent on time, and a seasonal component that takes into account factors like weather, holidays, and other factors that repeat on a routine basis.

Using the base data indication to the right, CB Predictor, when analyzing the data set, produces the results for periods 29 through 32.

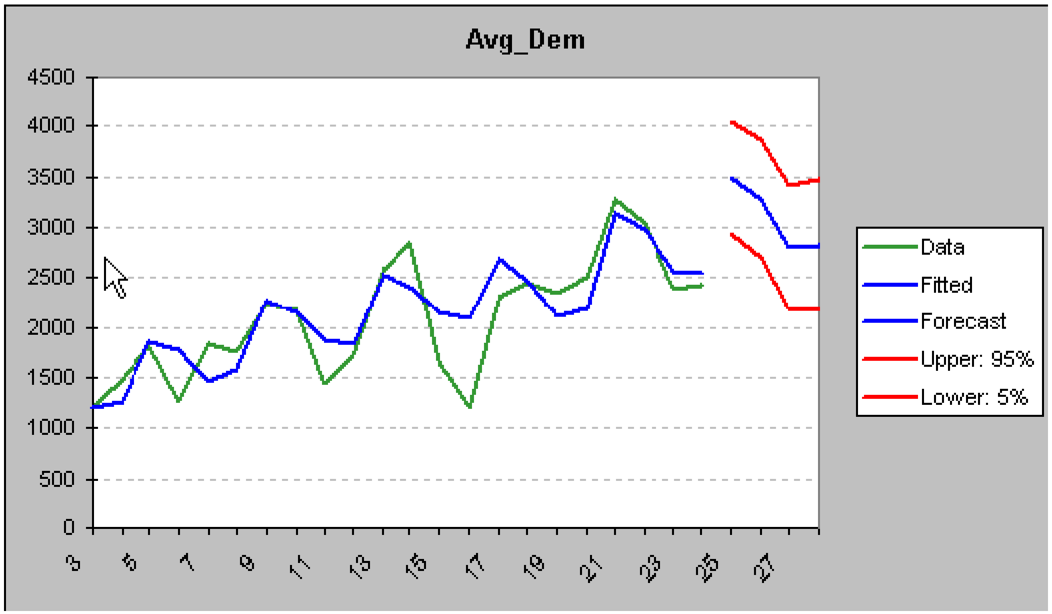
Period	Season	Forecast
29	1	3502.5
30	2	3281.6
31	3	2802.6
32	4	2823.9

Methods	Rank	RMSE	MAD	MAPE	Durbin-Watson	Theil's U	Periods	Alpha	Beta	Gamma
Double Exponential Smoothing	7	559.72	482.75	25.086	1.488	0.98		0.429	0.151	
Double Moving Average	8	644.42	581.9	27.46	1.259	0.849	5			
Holt-Winters' Additive	3	348.76	293.46	16.517	1.26	0.674		0.093	0.044	0.001
Holt-Winters' Multiplicative	**1**	**320.9**	**236.87**	**13.892**	**1.445**	**0.587**		**0.19**	**0.001**	**0.001**
Seasonal Additive	4	382.43	320.7	17.913	1.817	0.748		0.802		0.001
Seasonal Multiplicative	2	346.19	269.21	14.596	1.693	0.652		0.565		0.001
Single Exponential Smoothing	5	515.55	444.83	22.263	1.565	0.875		0.395		
Single Moving Average	6	545.51	495.69	23.975	1.279	0.836	4			

The best method, as determined by CB Predictor was the Holt-Winters' Multiplicative, derived from lower Root Mean Square Error (RMSE) in the table below. It is also worth noting that the H-WM method also had the lowest MAD (Mean Absolute Deviation) and MAPE (Mean Absolute Percentage Error) as well. The chart shows the graphical representation of the predicted values for periods 20 through 32, as well as the fit for the historical data given the model. The Holt-Winters' model assumes that seasonality is multiplicative, or growing (or decreasing) over time.

Forecasting firm demand – cont.

http://members.tripod.com/dsc8240_ppc/Forecast/ForecastDemand.htm#VariandMeasure_AFD



Once the seasonality and time component of the average firm demand has been determined, we examine the residuals to predict what impact the industry effects have on the firm demand. The remaining variables include average price, average advertising for current and two previous periods, and average R&D expenditures for two previous periods.

Forecasting series with industry-strength software

BEYOND EXCEL: R

Exponential Smoothing in R

- R is an open-source, free statistical computing software.
- Many firms, such as Google, are heavy R users.
- Use RStudio to run R; download instructions at https://www.otexts.org/fpp/using-r.
- Load the **`forecast`** package in R.
- Use the **`ets`** function to run exponential smoothing models.

ETS in R

- ETS stands for Error, Trend, and Seasonality.
- Errors can be additive (A) or multiplicative (M).

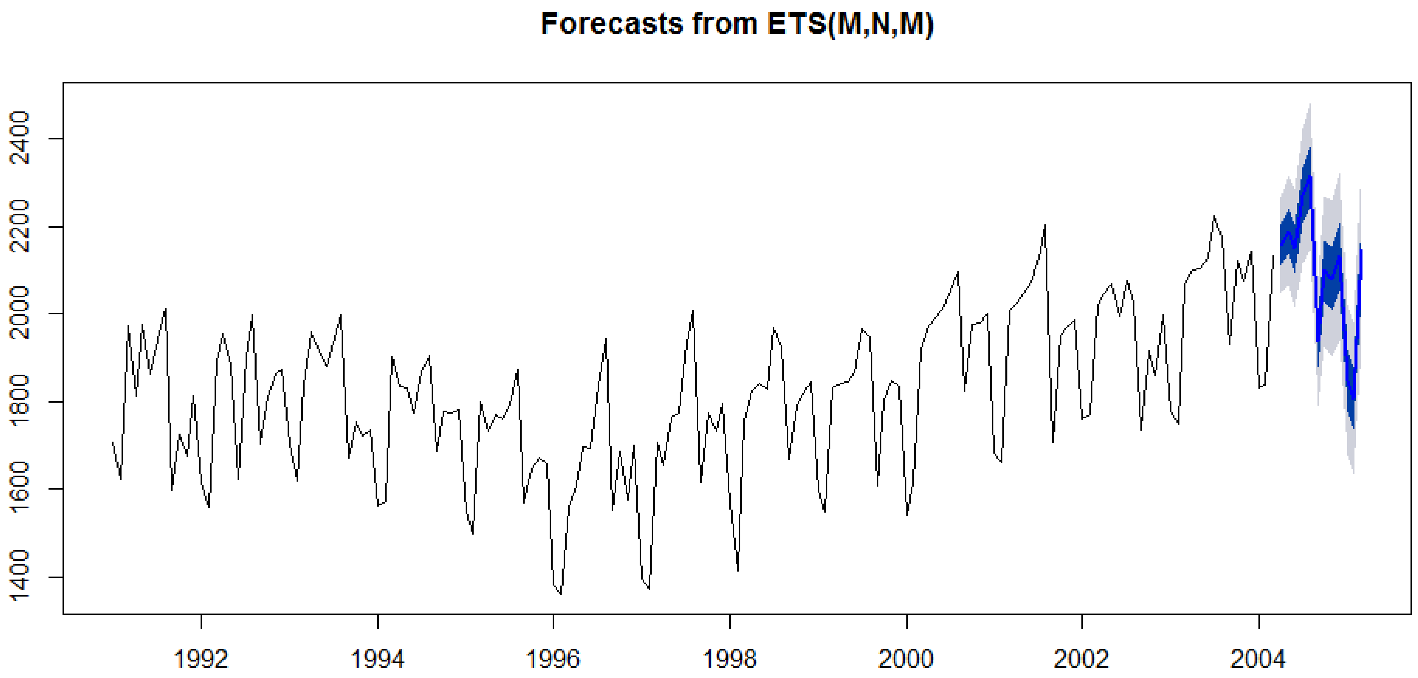
	Seasonality		
Trend	None (N)	Additive (A)	Multiplicative (M)
None (N)	(N,N) Simple Exponential Smoothing	(N,A)	(N,M) Holt Winter’s No Trend
Additive (A)	(A,N) Double Exponential Smoothing	(A,A)	(A,M) Holt Winter’s Additive
Multiplicative (M)	(M,N)	(M,A)	(M,M) Holt Winter’s Multiplicative

Seven Steps in Seven Lines…

Step 1: Install package
Step 2: Load package
Step 3: Read in data
Step 4: Create a time series
Step 5: Fit the ets model
Step 6: Make forecasts
Step 7: Visualize the forecasts

```
install.packages("forecast") # Run only once on your machine.
library("forecast")
Amtrak.data <- read.csv("Amtrak.csv")
ridership.ts <- ts(Amtrak.data$Ridership, start = c(1991,1), freq = 12)
ridership.ets <- ets(ridership.ts, model = "ANM", restrict = FALSE)
ridership.ets.pred <- forecast(ridership.ets, h = 12, level = c(0.5, 0.9))
plot(ridership.ets.pred)
```

The Results




```
Smoothing parameters:
  alpha = 0.5058
  gamma = 1e-04

Initial states:
  l = 1903.2691
  s=1.0185 0.9935 1.0022 0.9252 1.1053 1.0837
         1.0259 1.0454 1.03 1.0251 0.8621 0.883
```

From January's seasonal index backwards

Summary

Smoothing-based methods are more adaptive than regression-based methods; suitable for local patterns

Can be used not only for forecasting but also for visualization and creating seasonal indexes

When smoothing constants are selected to minimize goodness-of-fit to training set, beware of over-fitting!

Fast, memory-light, easy to automate

Next class

Try out smoothing with your project

Assignment #2

Chap 6: Problem #8 Instead of creating an ACF plot, plot the residuals themselves. Also, create a plot that highlights the seasonality.

Chap 9.1: Tips & Suggested Steps #5-6 How can exponential smoothing be fitted to the series, given the inter-day and intra-day cycles?
Write the type of method(s), its components and the number of seasons to use.