Smoothing Methods

Forecasting Analytics

Smoothing methods are useful for

Data visualization

Removing seasonality and computing seasonal indexes

Forecasting

Example: Coca Cola Sales

Coca Cola smoothing.xls contains quarterly sales of Coca Cola (in millions of $) from Q1-86 to Q2-96

Possible Goals:

- Time series forecasting: *create forecasts* for the next 4 quarters
- Time series analysis: *quantify* the components (patterns and noise)

Components of a Time Series - Recap

1. **Level** (always present)
2. **Trend**: steady increase/decrease over time.
3. **Seasonality**: pattern that repeats itself every season
4. Random **noise** (always present)

Reminder: Modeling Principles

Time series Analysis

- Reasonableness and parsimony
- Goodness of fit (residual analysis)

Time series Forecasting

- Forecast accuracy
- Parsimony and reasonableness

Moving Averages

Visualizing, Forecasting &
Computing seasonal indexes

The Moving Average Method

The Idea:

Forecast future points by using an average of several past points

Uses

- Time series visualization
- Computing seasonal indexes
- Forecasting

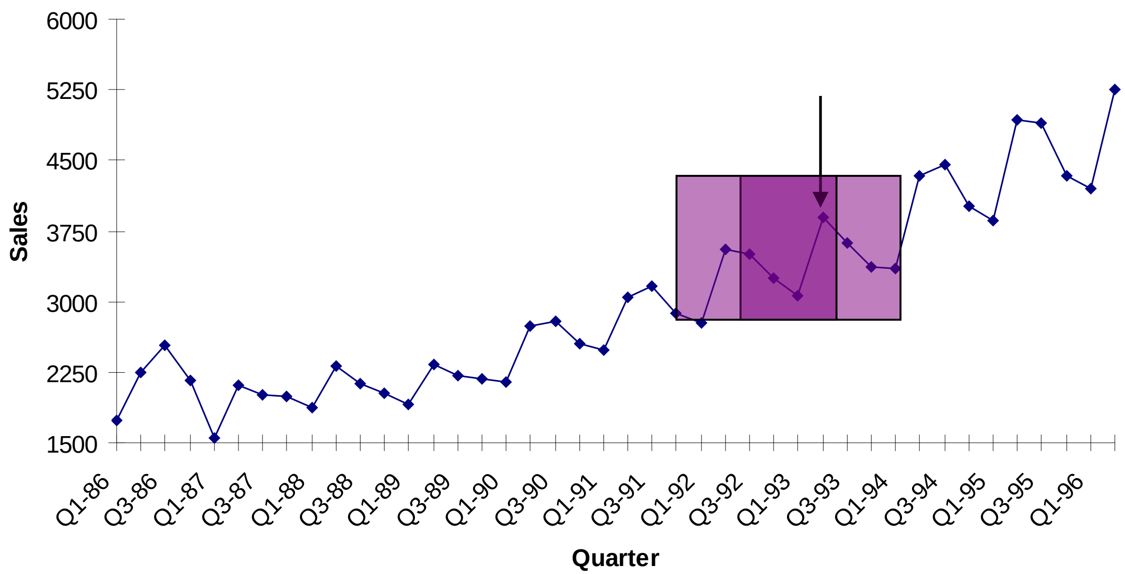
Advantages: Simple, popular

Key concept: width of window

Two types of windows

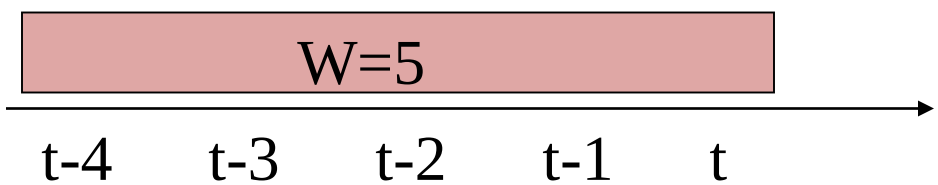
Centered moving average: based on a window centered around time *t*

Trailing moving average: based on a window from time *t* and backwards



Computing a Trailing MA

1. Choose window width (W)
2. For MA at time t, place window on time points *t-W+1,…,t*



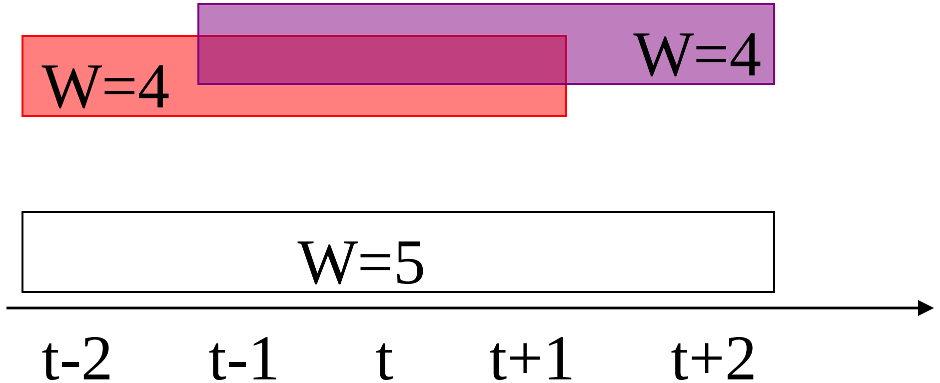
3. Compute average of values in the window:

$$MA_t = \frac{y_{t-W+1} + y_{t-W+2} + \cdots + y_{t-1} + y_t}{W}$$

Computing a Centered MA

Compute average of values in window (of width *W*), which is centered at *t*

- **Odd width**: center window on time *t* and average the values in the window
- **Even width**: take the two "almost centered" windows and average the values in them



$$MA_t = \left(\frac{y_{t-2} + y_{t-1} + y_t + y_{t+1}}{4} + \frac{y_{t-1} + y_t + y_{t+1} + y_{t+2}}{4} \right) / 2$$

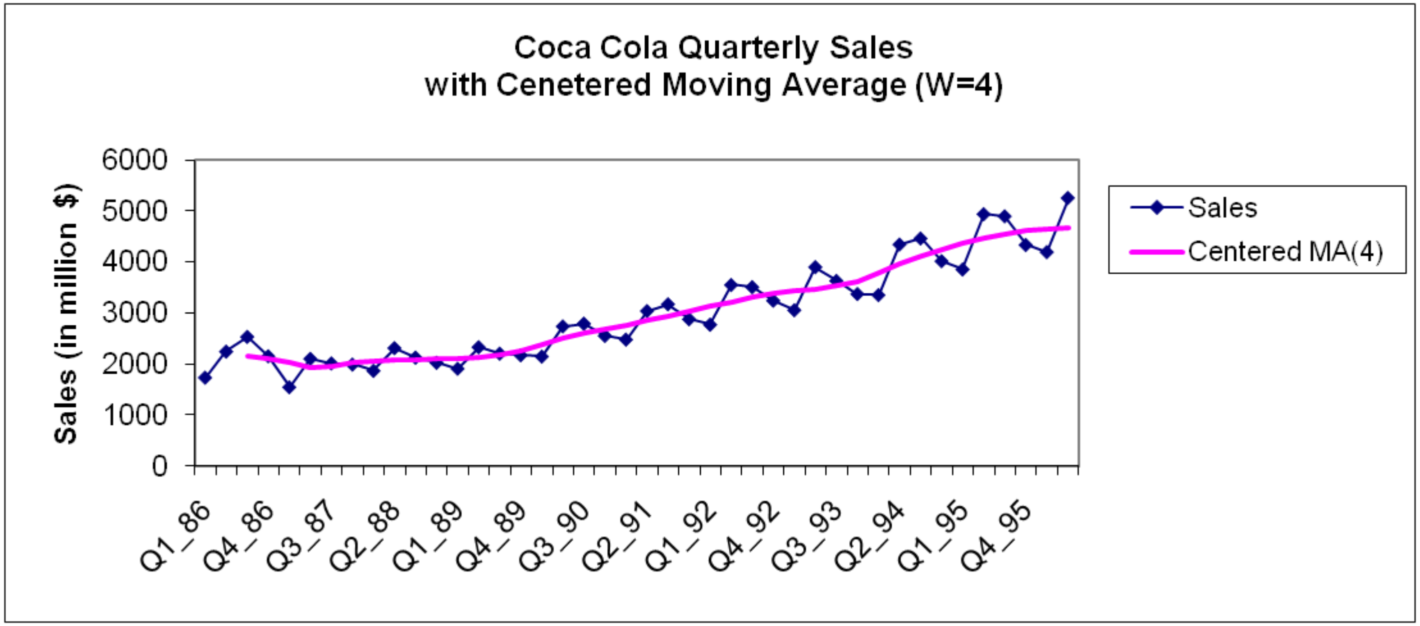
$$MA_t = \frac{y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2}}{5}$$

MOVING AVERAGES FOR VISUALIZING TIME SERIES

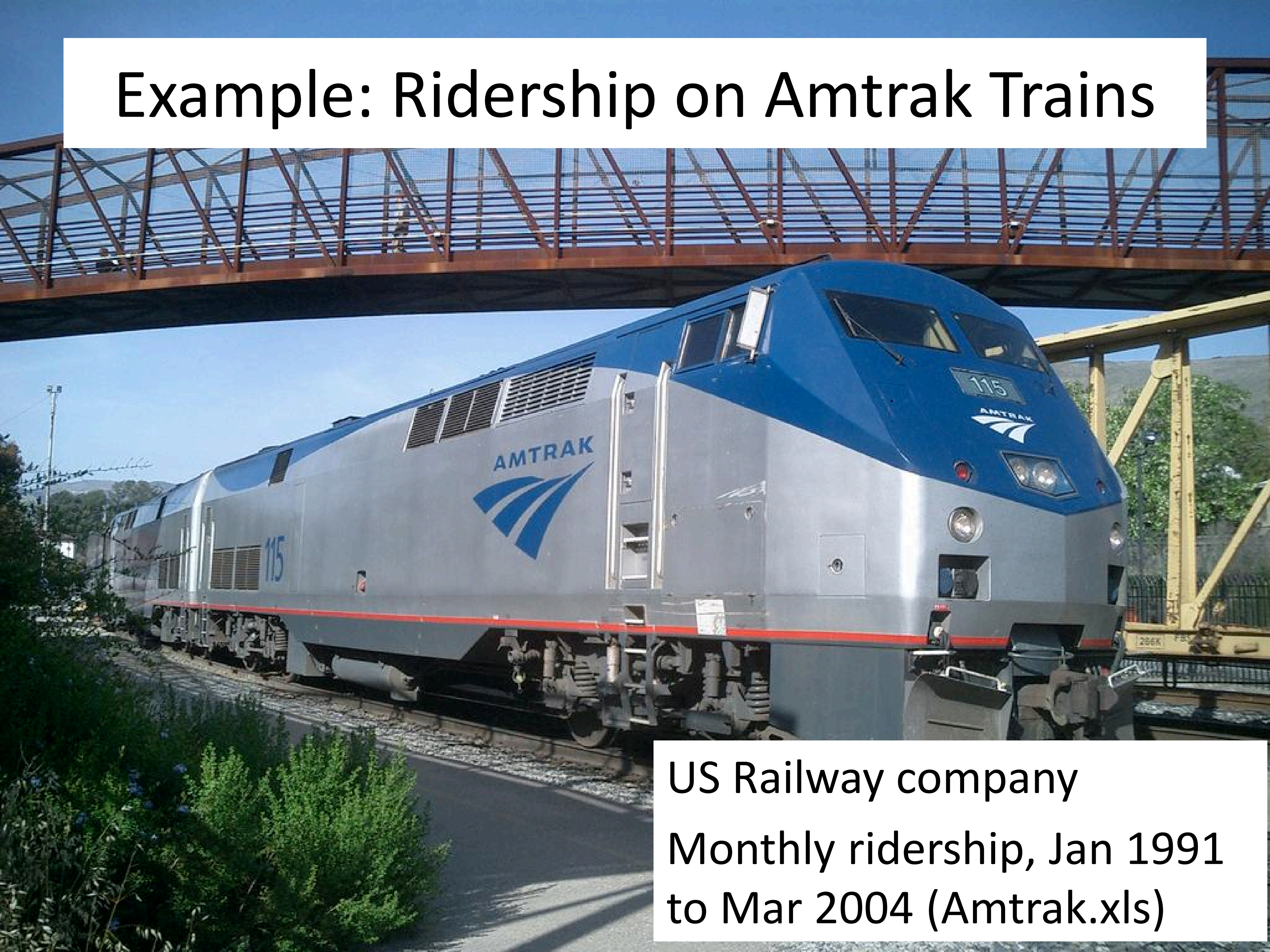
A time-plot of the **moving averages** can help reveal the LEVEL and TREND of a series, by filtering out the seasonal and random components

Visualizing Coca Cola Sales via MA chart (W=4)

Excel: *Chart Tools > Design > Add Chart Element > Trendline*



Example: Ridership on Amtrak Trains

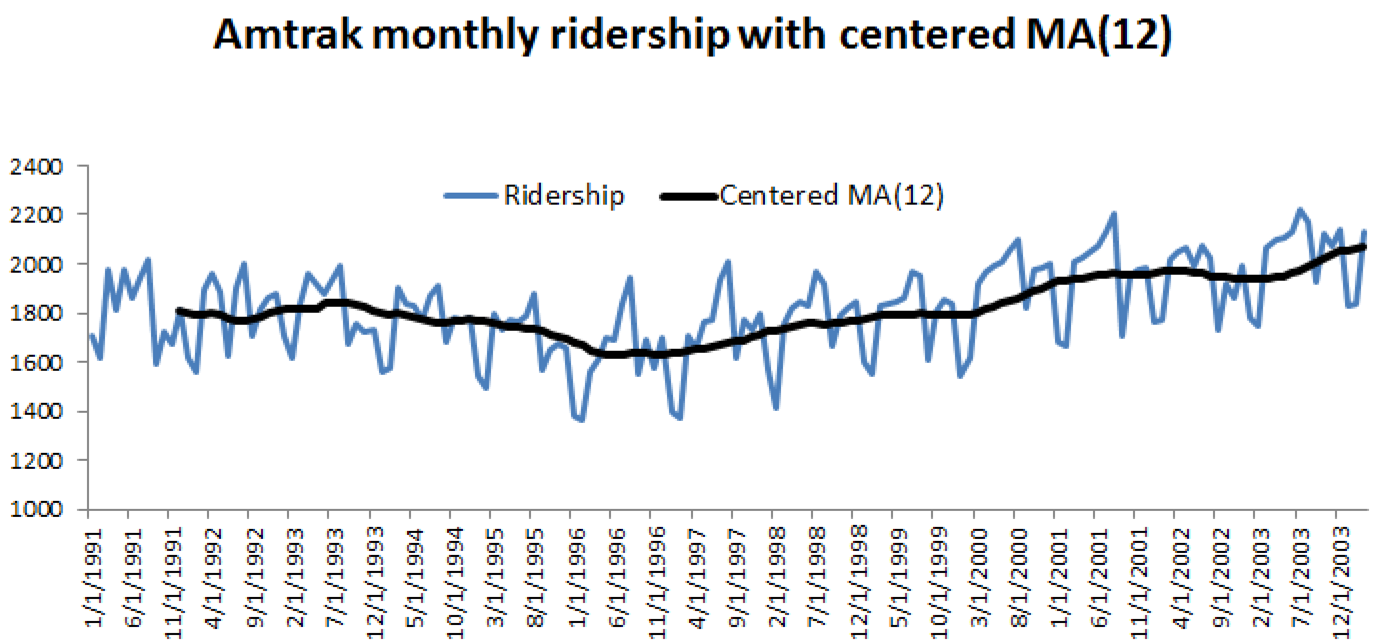


US Railway company

Monthly ridership, Jan 1991 to Mar 2004 (Amtrak.xls)

Visualizing Amtrak Ridership via MA chart (W=12)

Excel: *Chart Tools > Design > Add Chart Element > Trendline*



Choosing Window Width (W)

Balance over- and under-smoothing

If no seasonality, use narrow window (under-smoothing)

FORECASTING WITH MOVING AVERAGES

To forecast a series at time *t+k*, use a trailing MA that ends at time *t*:

$$F_{t+k} = \frac{y_{t-W+1} + y_{t-W+2} + \cdots + y_{t-1} + y_t}{W}$$

MASmoothingOutput1

Quarter	Actual	Forecast	Residuals
Q1_86	1734.827	*	*
Q2_86	2244.961	*	*
Q3_86	2533.805	*	*
Q4_86	2154.963	*	*
Q1_87	1547.819	2167.139	-619.32
Q2_87	2104.412	2120.387	-15.975001
Q3_87	2014.363	2085.2497	-70.886746
Q4_87	1991.747	1955.3892	36.357751
Q1_88	1869.05	1914.5852	-45.535248
Q2_88	2313.632	1994.893	318.739
Q3_88	2128.32	2047.198	81.122002
Q4_88	2026.829	2075.6872	-48.85825
Q1_89	1910.604	2084.4577	-173.85375
Q2_89	2331.165	2094.8462	236.31875
Q3_89	2206.55	2099.2295	107.3205
Q4_89	2173.968	2118.787	55.180999
Q1_90	2148.278	2155.5717	-7.293745
Q2_90	2739.308	2214.9902	524.31775
Q3_90	2792.754	2317.026	475.728
Q4_90	2556.01	2463.577	92.432997
Q1_91	2480.974	2559.0875	-78.113499
Q2_91	3039.523	2642.2615	397.2615
Q3_91	3172.116	2717.3152	454.80075
Q4_91	2879.001	2812.1557	66.845253
Q1_92	2772	2892.9035	-120.9035
Q2_92	3550	2965.66	584.34
Q3_92	3508	3093.2792	414.72075
Q4_92	3243.86	3177.2502	66.609743
Q1_93	3056	3268.465	-212.465
Q2_93	3899	3339.465	559.535
Q3_93	3629	3426.715	202.285
Q4_93	3373	3456.965	-83.964998
Q1_94	3352	3489.25	-137.25
Q2_94	4342	3563.25	778.75
Q3_94	4461	3674	787
Q4_94	4017	3882	135
Q1_95	3854	4043	-189
Q2_95	4936	4168.5	767.5

Example: Coca Cola Sales with W=4

Time Series> Smoothing> Moving Average

Error Measures (Training)

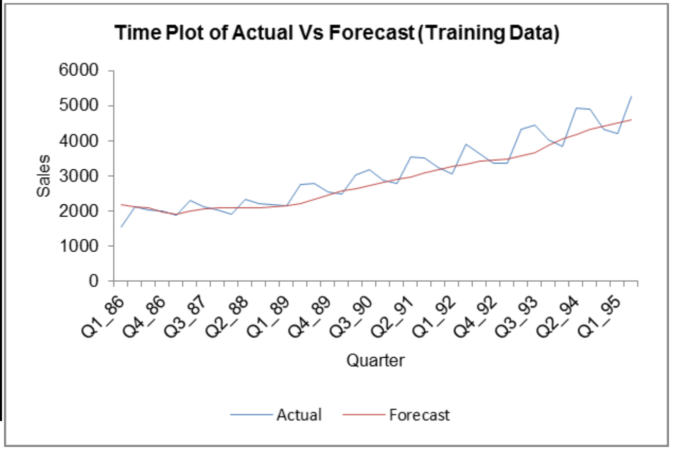
MAPE	8.6984558
MAD	263.10546
MSE	126906.91

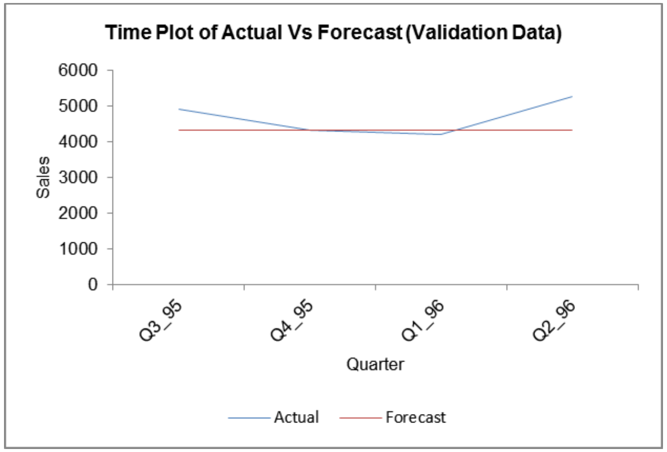
Forecast

Quarter	Actual	Forecast	Error	LCI	UCI
Q3_95	4895	4317	578	3656.5402	4977.4598
Q4_95	4333	4317	16	3656.5402	4977.4598
Q1_96	4194	4317	-123	3656.5402	4977.4598
Q2_96	5253	4317	936	3656.5402	4977.4598

Error Measures (Validation)

MAPE	8.2320943
MAD	413.25
MSE	306391.25





Example: Amtrak Ridership with W=12

Validation = last 3 years

	Actual	Forecast	Residuals
1/1/1991	1709	*	*
2/1/1991	1621	*	*
3/1/1991	1973	*	*
4/1/1991	1812	*	*
5/1/1991	1975	*	*
6/1/1991	1862	*	*
7/1/1991	1940	*	*
8/1/1991	2013	*	*
9/1/1991	1596	*	*
10/1/1991	1725	*	*
11/1/1991	1676	*	*
12/1/1991	1814	*	*
1/1/1992	1615	1809.6667	-194.66667
2/1/1992	1557	1801.8333	-244.83333
3/1/1992	1891	1796.5	94.5
4/1/1992	1956	1789.6667	166.33333
5/1/1992	1885	1801.6667	83.333333
6/1/1992	1623	1794.1667	-171.16667
7/1/1992	1903	1774.25	128.75
8/1/1992	1997	1771.1667	225.83333
9/1/1992	1704	1769.8333	-65.833333
10/1/1992	1810	1778.8333	31.166667
11/1/1992	1862	1785.9167	76.083333
12/1/1992	1875	1801.4167	73.583333
1/1/1993	1705	1806.5	-101.5
2/1/1993	1619	1814	-195
3/1/1993	1837	1819.1667	17.833333
4/1/1993	1957	1814.6667	142.33333
5/1/1993	1917	1814.75	102.25
6/1/1993	1882	1817.4167	64.583333
7/1/1993	1933	1839	94
8/1/1993	1996	1841.5	154.5
9/1/1993	1673	1841.4167	-168.41667
10/1/1993	1753	1838.8333	-85.833333
11/1/1993	1720	1834.0833	-114.08333
12/1/1993	1734	1822.25	-88.25
1/1/1994	1563	1810.5	-247.5
2/1/1994	1574	1798.6667	-224.66667

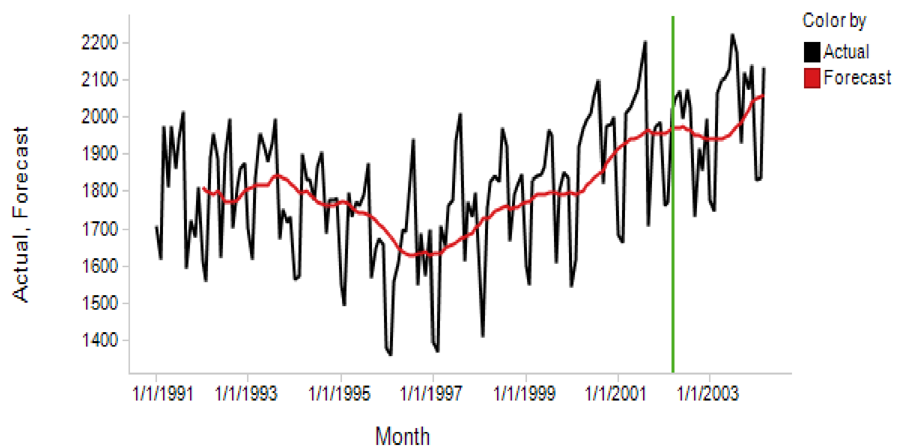
Error Measures (Training)

MAPE	6.8894421
MAD	122.28685
MSE	21531.894

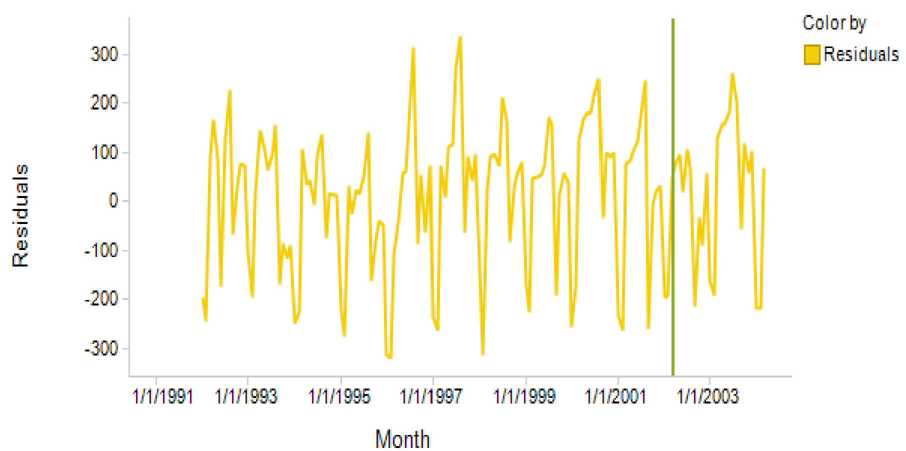
Error Measures (Validation)

MAPE	6.333254
MAD	118.74074
MSE	25568.824

Actual and Forecasted Values



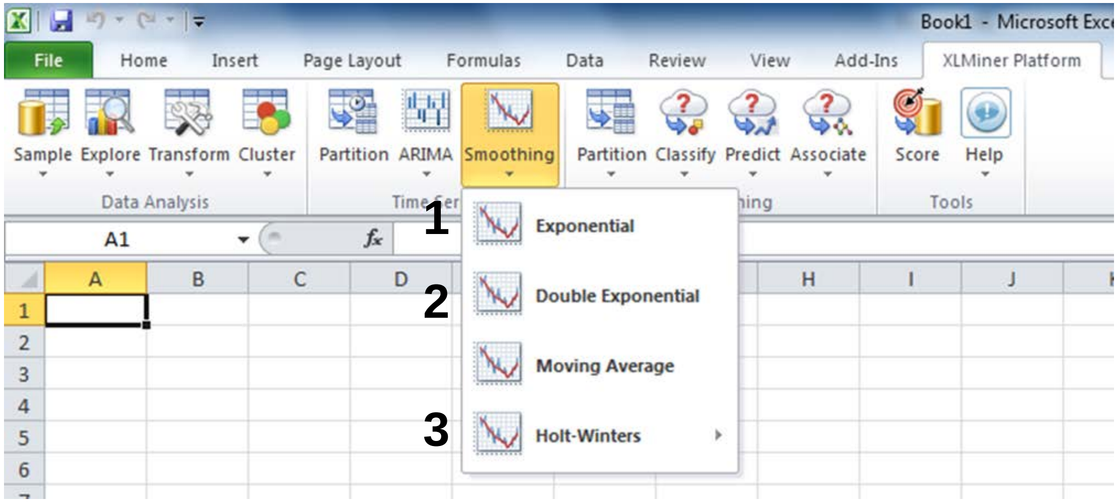
Forecast Errors



FORECASTING WITH EXPONENTIAL SMOOTHING

Types of Exponential Smoothing

1. **Simple exponential smoothing** (for series with no trend or seasonality)
2. **Holt's method** (with trend, no seasonality)
3. **Winter's method** (with trend & seasonality)



Simple Exponential Smoothing

Assumption: Series has only *level* (L_t) and *noise*.

Noise is unpredictable ; Level will "stay put"

Forecasts = *estimated level* at most recent time point: $\mathbf{F_{t+k} = L_t}$

Adaptive algorithm: adjusts most recent forecast (or level) based on the actual data: $\mathbf{L_t = \alpha Y_t + (1-\alpha) L_{t-1}}$

α = the *smoothing constant* ($0<\alpha \leq 1$)

Initialization: $F_1 = L_1 = Y_1$

Why is it called "Exponential Smoothing"?

The formula: $\mathbf{L_t = \alpha Y_t + (1\text{-}\alpha)\, L_{t\text{-}1}}$

Substitute L_t with its own formula:

$$
\begin{aligned}
\mathbf{L_t} &= \mathbf{\alpha Y_t + (1\text{-}\alpha)[\, \alpha Y_{t\text{-}1} + (1\text{-}\alpha)\, L_{t\text{-}2}] =} \\
&= \mathbf{\alpha Y_t + \alpha\, (1\text{-}\alpha) Y_{t\text{-}1} + (1\text{-}\alpha)^2\, L_{t\text{-}2} = \ldots} \\
&= \mathbf{\alpha Y_t + \alpha\, (1\text{-}\alpha) Y_{t\text{-}1} + \alpha\, (1\text{-}\alpha)^2\, Y_{t\text{-}2} + \ldots}
\end{aligned}
$$

Weights decrease exponentially into the past!
(compare to MA)

Another way of viewing it: An adaptive learning process

The formula: $\mathbf{L_t = \alpha Y_t + (1-\alpha) L_{t-1}}$

$$\begin{aligned} F_{t+1} = L_t &= L_{t-1} + \alpha\,(Y_t - L_{t-1}) \\ &= F_t + \alpha\,(Y_t - F_t) \\ &= F_t + \alpha\, E_t \end{aligned}$$

update previous forecast

By an amount that depends on the error in the previous forecast

α controls the degree of "learning"

The Smoothing Constant α

α determines how much weight is given to the past

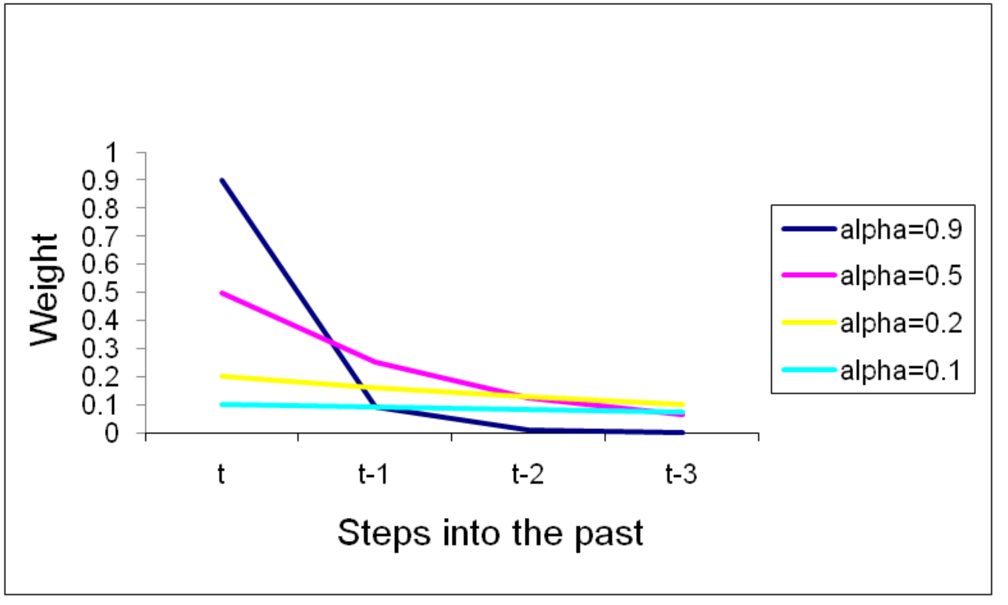
$\alpha=1$: past observations have <u>no</u> influence over forecasts (under-smoothing)

$\alpha\to 0$: past observations have large influence on forecasts (over-smoothing)

Selecting α

- “Typical” values: 0.1, 0.2.
- Trial & error: effect on visualization
- Minimize RMSE or MAPE of training data – beware!
- Trailing MA with window W $\cong$ simple exp smoothing with α = 2 / (W+1), or equivalently W = 2/α -1.

“Feeling” the Effect of α



α	$\alpha(1-\alpha)$	$\alpha(1-\alpha)^2$	$\alpha(1-\alpha)^3$
0.9	0.09	0.009	0.0009
0.5	0.25	0.125	0.0625
0.2	0.16	0.128	0.1024
0.1	0.09	0.081	0.0729

The Beauty of the Method

Although forecasts take into account all previous observations, we only need to store **the last forecast** and **most recent observation**!

Unlike MA, gives more weight to more recent observations

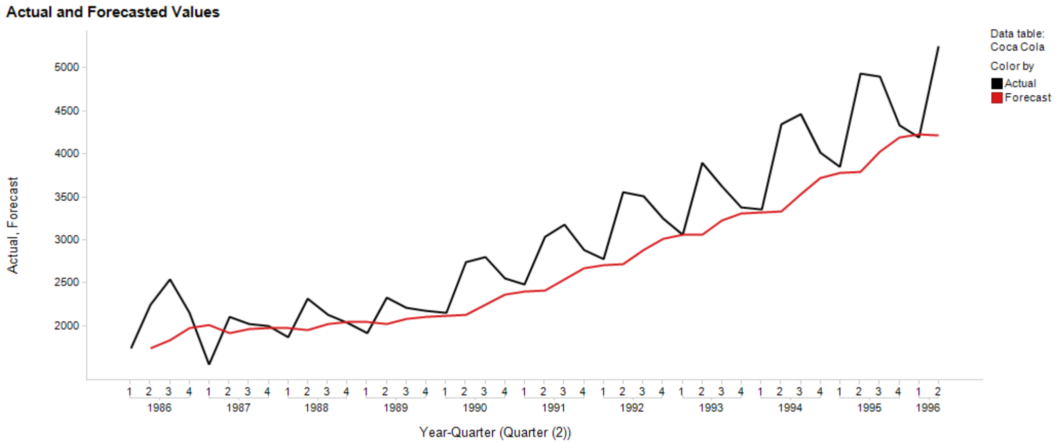
Simple to understand

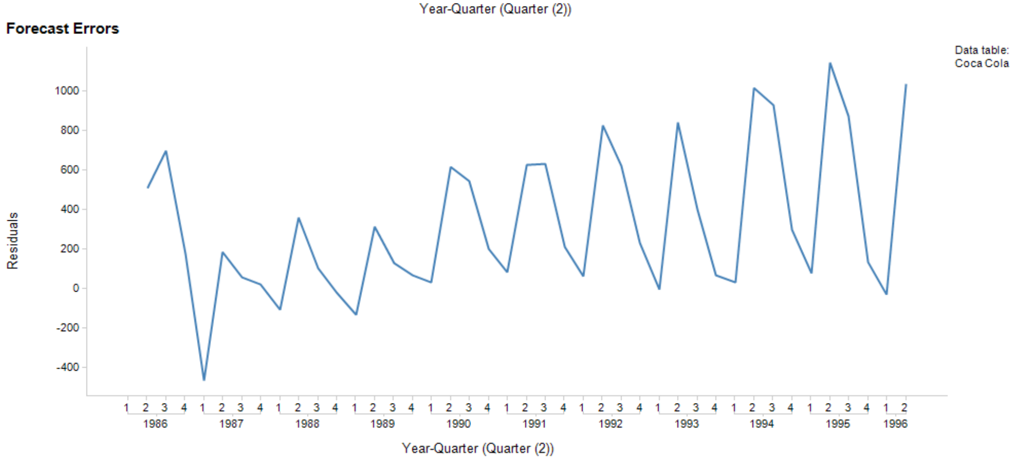
ExponentialOutput1

Example: Coca Cola Sales

XLMiner: Time Series >Smoothing > Exponential smoothing

Default: $\alpha=0.2$





$$F_{t+1} = F_t + \alpha E_t$$

Quarter	Actual	Forecast	Residuals
Q1_86	1734.827	*	*
Q2_86	2244.961	1734.827	510.134
Q3_86	2533.805	1836.8538	696.95119
Q4_86	2154.963	1976.244	178.71896
Q1_87	1547.819	2011.9878	-464.16883
Q2_87	2104.412	1919.1541	185.25793
Q3_87	2014.363	1956.2056	58.157349
Q4_87	1991.747	1967.8371	23.909878
Q1_88	1869.05	1972.6191	-103.5691
Q2_88	2313.632	1951.9053	361.72672

Example: Amtrak Ridership (α=0.2)

Error Measures (Training)

MAPE	7.3126507
MAD	129.76168
MSE	23249.994

Error Measures (Validation)

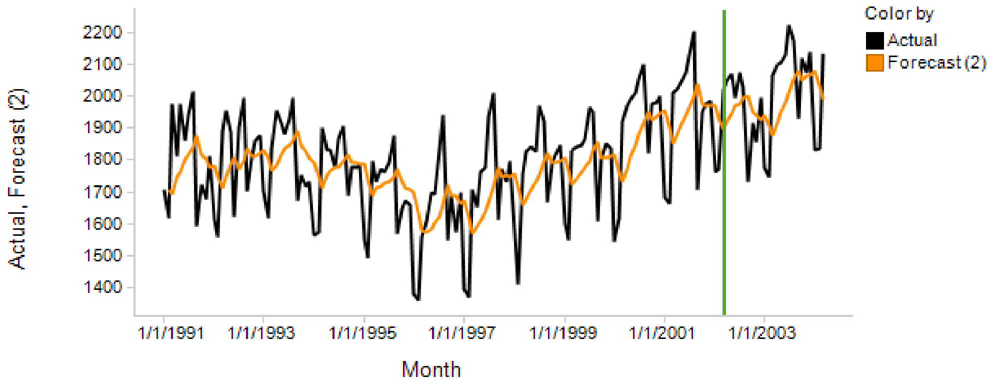
MAPE	5.9893487
MAD	114.54186
MSE	20880.544

$$F_{t+1} = F_t + \alpha E_t$$

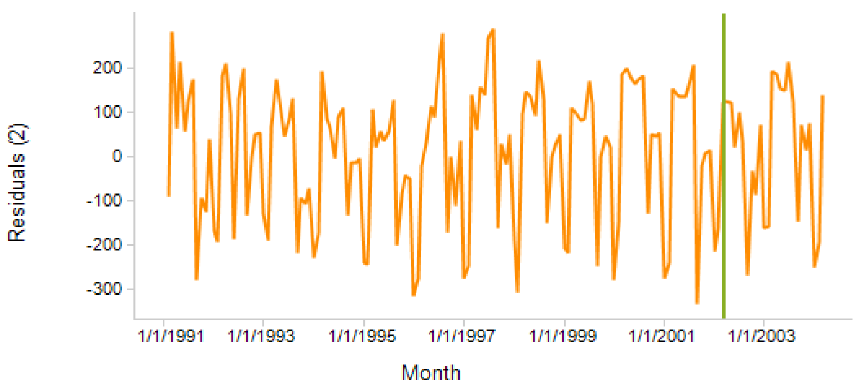
Fitted Model

Month	Actual	Forecast	Residuals
1/1/1991	1709	*	*
2/1/1991	1621	1709	-88
3/1/1991	1973	1691.4	281.6
4/1/1991	1812	1747.72	64.28
5/1/1991	1975	1760.576	214.424
6/1/1991	1862	1803.4608	58.5392
7/1/1991	1940	1815.1686	124.83136
8/1/1991	2013	1840.1349	172.86509
9/1/1991	1596	1874.7079	-278.70793
10/1/1991	1725	1818.9663	-93.966344
11/1/1991	1676	1800.1731	-124.17307

Actual and Forecasted Values



Forecast Errors



Moving averages and exponential smoothing for series with trend and/or seasonality…

REMOVING TREND AND/OR SEASONALITY

Approach 1: Regression

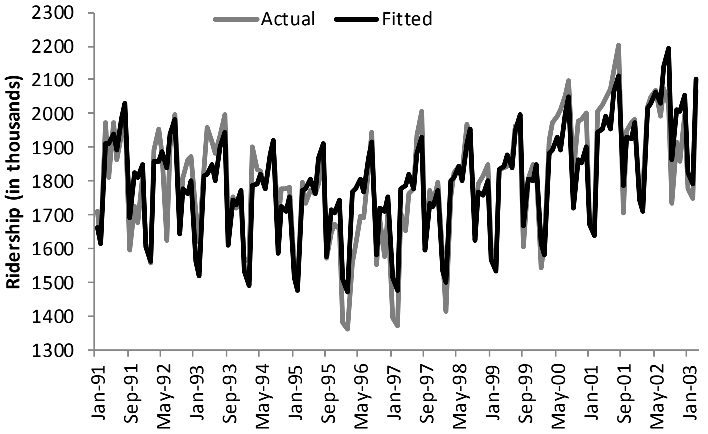
To remove trend and/or seasonality, fit a regression model with trend and/or seasonality (Note: we will see how to do this in Class #5)

The series of forecast errors should be de-trended and de-seasonalized

Recall Amtrak example

The Regression Model

Input variables	Coefficient	Std. Error	p-value	SS
Constant term	1932.998779	27.85863113	0	477456500
season_Aug	135.1726227	30.52143288	0.00001955	483721.3125
season_Dec	-29.6587296	30.53801155	0.33320817	33314.77734
season_Feb	-306.307831	29.94875526	0	665331.9375
season_Jan	-267.444458	29.94642067	0	598841.0625
season_Jul	91.31225586	30.5189991	0.00330446	187691.7656
season_Jun	-12.0447455	30.51724434	0.69370645	11869.09277
season_Mar	-7.04482555	29.95207596	0.81441271	48930.94922
season_May	30.31717491	30.51618195	0.32228076	114420.9141
season_Nov	-72.2664108	30.53282547	0.01938256	3121.062012
season_Oct	-60.9804916	30.52834129	0.04781064	14579.31641
season_Sep	-199.128098	30.52454758	0	224972.1094
t	-5.246521	0.58674908	0	398979.7188
t^2	0.0437566	0.00384071	0	725213.9375



Amtrak-Regression.xlsx, worksheet *MLR_Output5*

Approach 2: Differencing

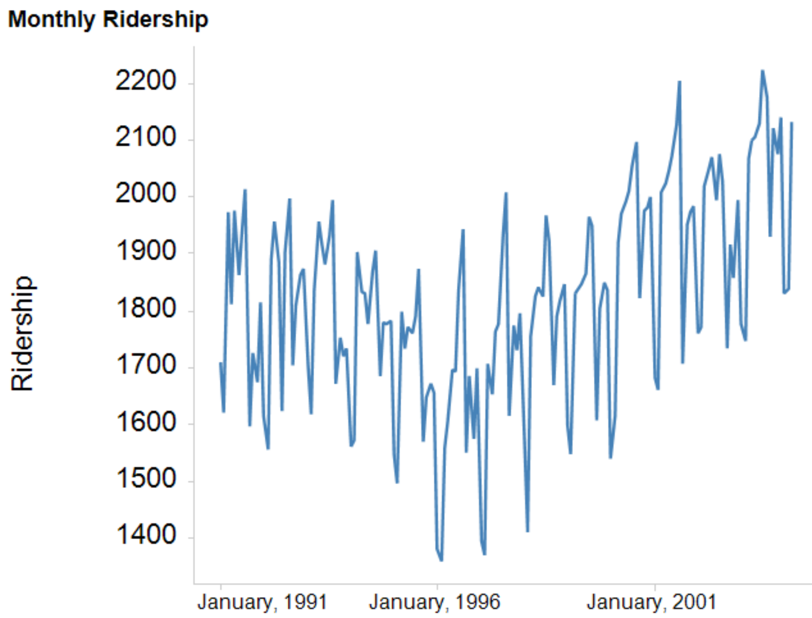
Differencing means taking the difference between two observations

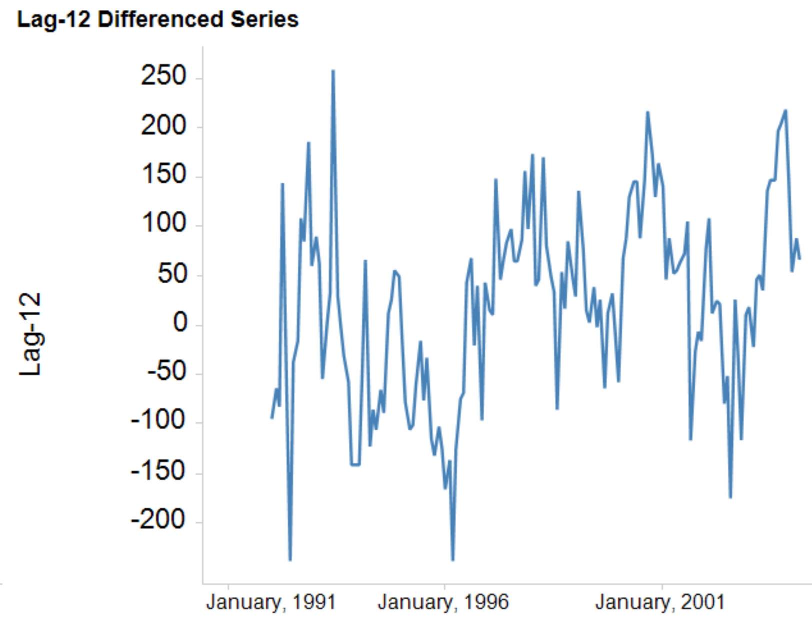
Differencing is a simple and popular operation for removing a trend and/or seasonality from a time series

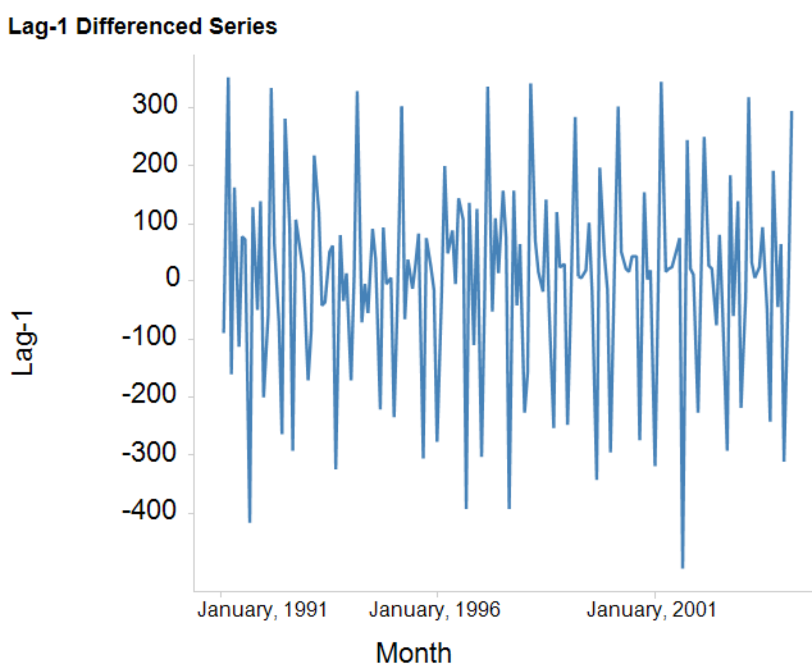
Lag-1 difference: $y_t - y_{t-1}$
for removing trend

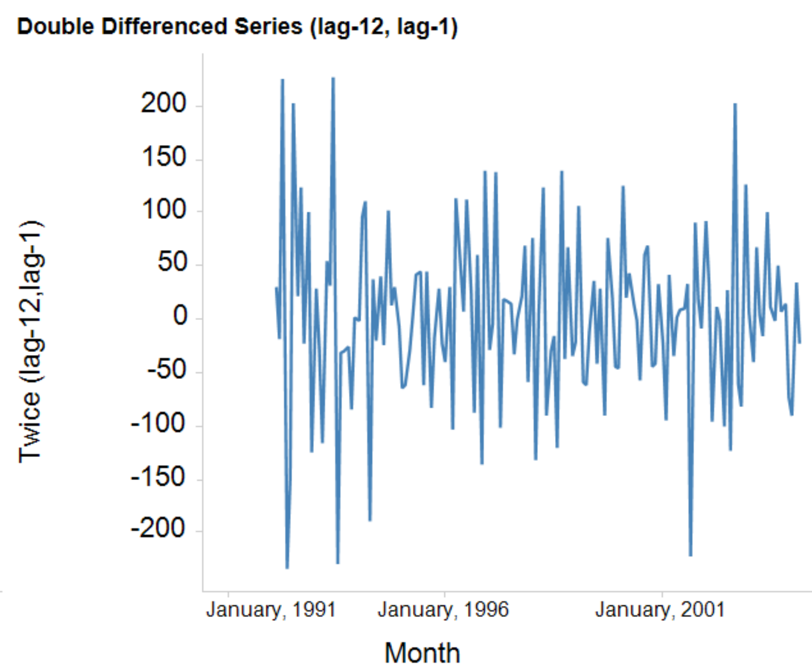
Lag-M difference: $y_t - y_{t-M}$
for removing seasonality with M seasons

Double-differencing: difference the differenced series


Monthly Ridership
Ridership
2200
2100
2000
1900
1800
1700
1600
1500
1400
January, 1991
January, 1996
January, 2001


Lag-12 Differenced Series
Lag-12
250
200
150
100
50
0
-50
-100
-150
-200
January, 1991
January, 1996
January, 2001


Lag-1 Differenced Series
Lag-1
300
200
100
0
-100
-200
-300
-400
January, 1991
January, 1996
January, 2001
Month


Double Differenced Series (lag-12, lag-1)
Twice (lag-12,lag-1)
200
150
100
50
0
-50
-100
-150
-200
January, 1991
January, 1996
January, 2001
Month

Compared to residuals from regression with trend -- **differencing** is useful for removing a local/changing trend shape

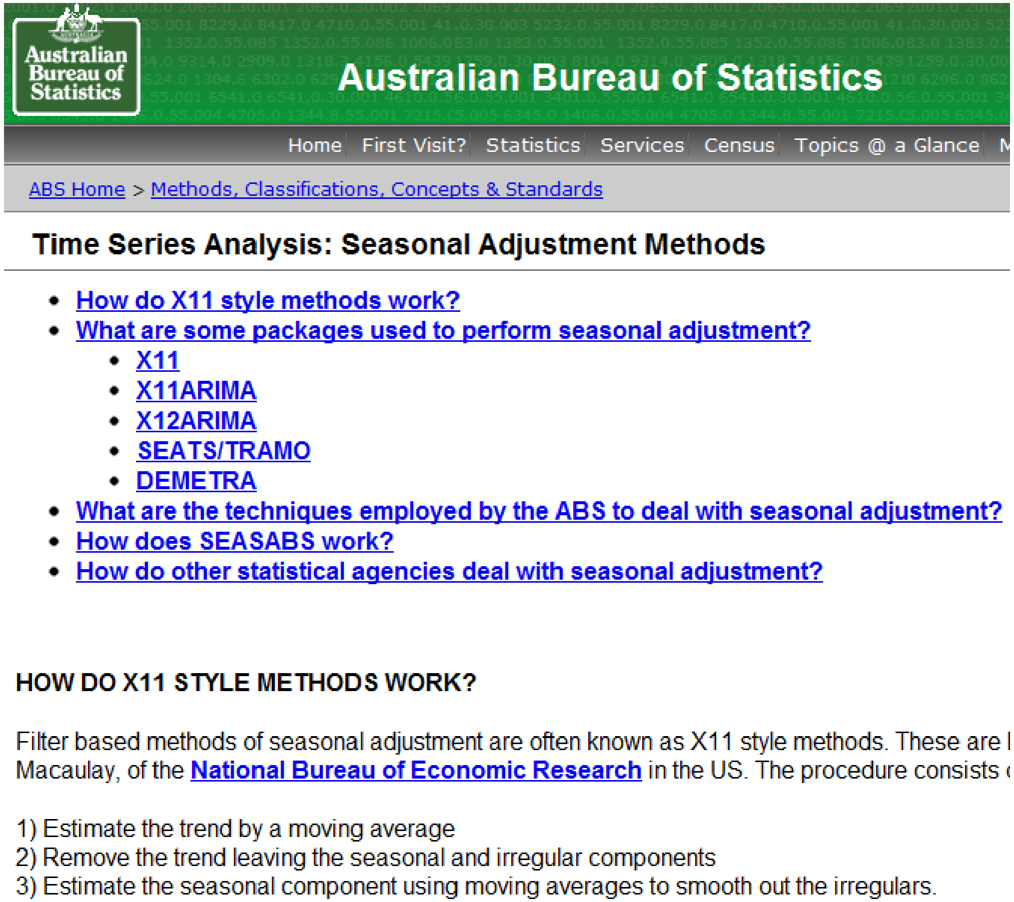
Differencing is a simple and popular operation for removing a trend and/or seasonality from a time series

REMOVING SEASONALITY VIA MOVING AVERAGES
(AND COMPUTING SEASONAL INDEXES)

Approach 3:
Ratio-to-moving-average method

Uses moving average to remove seasonality

Also generates *seasonal indexes* as a byproduct



Australian Bureau of Statistics

Home | First Visit? | Statistics | Services | Census | Topics @ a Glance

ABS Home > Methods, Classifications, Concepts & Standards

Time Series Analysis: Seasonal Adjustment Methods

- How do X11 style methods work?
- What are some packages used to perform seasonal adjustment?
  - X11
  - X11ARIMA
  - X12ARIMA
  - SEATS/TRAMO
  - DEMETRA
- What are the techniques employed by the ABS to deal with seasonal adjustment?
- How does SEASABS work?
- How do other statistical agencies deal with seasonal adjustment?

HOW DO X11 STYLE METHODS WORK?

Filter based methods of seasonal adjustment are often known as X11 style methods. These are l
Macaulay, of the National Bureau of Economic Research in the US. The procedure consists (

1) Estimate the trend by a moving average
2) Remove the trend leaving the seasonal and irregular components
3) Estimate the seasonal component using moving averages to smooth out the irregulars.

What are seasonal indexes?

For a series with M seasons:

***S_j* = seasonal index for the *j^{th}* season**

indicates the exceedance of Y on season *j* above/below the average of Y in a complete cycle of seasons

Example: Daily sales at ISB café show that Friday has a seasonal index of 1.20 and Monday has an index of 0.70.

Meaning: Friday sales _20_% higher than the weekly average, and Monday sales _30_% lower than the weekly average sales.

Average of the M seasonal indexes is _1_ (they must sum to _M_).

An Algorithm for Computing the Seasonal Indexes ($S_1, S_2, \ldots, S_M$)

1. Construct the series of *centered* moving averages of span *M.*
2. For each *t*, compute the *raw seasonals* = Y_t / MA_t
3. S_j = average of raw seasonals belonging to season *j* (normalize to ensure that seasonal indexes have average=1)

Centered MA with W=4

Quarter	Sales		raw seasonal	s(j)	seasonal index
Q1_86	1734.83				
Q2_86	2244.96				
Q3_86	2533.80	2143.76	1.18194269	1.063872992	**1.062660535**
Q4_86	2154.96	2102.82	1.02479749	0.96423378	**0.963134878**
Q1_87	1547.82	2020.32	0.76612585	0.879530967	**0.878528598**
Q2_87	2104.41	1934.99	1.08755859	1.096926116	**1.09567599**
Q3_87	2014.36	1954.74	1.03050222		
Q4_87	1991.75	2021.05	0.9855033		
Q1_88	1869.05	2061.44	0.90667088		
Q2_88	2313.63	2080.07	1.11228431		
Q3_88	2128.32	2089.65	1.01850452		
Q4_88	2026.83	2097.04	0.96651998		
Q1_89	1910.60	2109.01	0.90592533		
Q2_89	2331.16	2137.18	1.09076712		

Using Seasonal Indexes to De- and Re-Seasonalize

De-seasonalized (=seasonally-adjusted) series:

$$DSY_t = Y_t \text{ / appropriate seasonal index}$$

- If done appropriately, de-seasonalized series will not exhibit seasonality
- If so, examine for trend and fit a model
- This model will yield **de-seasonalized forecasts**
- Convert forecasts by re-seasonalizing, i.e. multiply them by the appropriate seasonal index

Next Class

Forecasting series with a trend and/or seasonality

ADVANCED EXPONENTIAL SMOOTHING