

# i D D E X

Name: Siddharth Bhatiyalwar Subject: Exam notes

Standard: XII Division: C Roll No.: 8

① → Use in normal scientific calc.

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**NOTE:**

'1' formulae are to be used in the normal scientific calculator.

'X' formulae are to be used in the ME calculator

'?' formulae are yet to be decided, or to be left for other members to figure out.

\*PHYSICS\*Kinematics

→ Equations of motion →

(a)  $v = u + at$

(b)  $s = ut + \frac{1}{2}at^2$

(c)  $v^2 = u^2 + 2as$

(d)  $a = \frac{v-u}{t}$

→ Differentials in kinematics →

(a)  $v = \frac{dx}{dt}, a = \cancel{\frac{dv}{dt}}$

(b)  $a = \frac{dv}{dt} = v \frac{dv}{dx}$

→ Projectile motion →

(a)  $u_x = u \cos \theta, u_y = u \sin \theta$

(b)  $a_{x2} = 0, a_y = -g$

(c)  $S_x = \text{length of projectile}, S_y = 0$ 

(d) Time of flight =  $2u \sin \theta$

Flight time

(e) Maximum height =  $\frac{u^2 \sin^2 \theta}{2g}$

Height

(f) Horizontal range =  $\frac{u^2 \sin 2\theta}{g}$

range

(g) Equation of trajectory =  $x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$

Trajectory (y)

→ Gravitational acceleration on inclined plane →

(a)  ~~$g \cos \theta$~~  = Perpendicular to plane(b)  $g \sin \theta$  = Along the plane.

P.T.O

→ Relative motion →

(a)  $V_{A/B} = V_A - V_B$

$a_{A/B} = a_A - a_B$

(Acceleration)

(b) Relative velocity velocity is:

(i) Maximum when  $\theta = 180^\circ$  (Anti-parallel)

(ii) Minimum when  $\theta = 0^\circ$  (Parallel)

(c) Rain - Person situations:

Velocity (Rain w.r.t person)

(d) River - Boat situations:

(i) Upstream  $\rightarrow V_{rel} = (-)ve$

(ii) Downstream  $\rightarrow V_{rel} = (+)ve$

(iii) Shortest possible path  $\rightarrow \theta = 90^\circ$

(iv)  $[S = v \times t]$

(e) Wind - Aeroplane situation

Velocity (of Aeroplane w.r.t wind)

• Heat and Thermodynamics:-

→  $0 \text{ Kelvin} = -273^\circ C = -459.4^\circ F$

→ Heat capacity & specific heat capacity →

(a) Heat capacity =  $\frac{\Delta Q}{\Delta T}$

(b) Specific heat capacity =  $\frac{\Delta Q}{m \Delta T}$

(c) Molar specific heat capacity =  $\frac{\Delta Q}{n \Delta T}$

→ Co-efficient of expansion →

Linear expansion =  $\frac{\Delta L}{L \Delta T}$

? (b) Area thermal =  $\frac{\Delta A}{\Delta T}$   
 expansion

(c) Volume thermal =  $\frac{\Delta V}{\Delta T}$   
 expansion

\* Heat and latent of fusion / vapourization →

(a)  $\Delta Q = m s \Delta T$  ( $s \rightarrow$  specific heat)

(b)  $\Delta Q = m s \Delta T$

(c)  $L_{vf} = \frac{\Delta Q}{m}$  ( $L \rightarrow$  latent heat of vaporization / fusion)

\*  $V_{rms} = \sqrt{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}$

$V_{rms} \rightarrow$  Root mean square velocity.

\* Laws of thermodynamics →

(a) Zeroeth law →  $A \geq B, B \geq C, \therefore A \geq C$

? (b) First law →  $\Delta Q = \Delta U + \Delta W$

(c) Second law → Efficiency (engine)  $< 100\%$  (Always)

\* First law of thermodynamics →

(a) Isochoric process ( $\Delta V = 0$ ):

$$\Delta Q = n C_V \Delta T = \Delta U \quad (C_V \rightarrow \text{heat capacity const. volume})$$

(b) Isobaric process ( $\Delta P = 0$ ):

$$C_P = C_V + R \quad (C_P \rightarrow \text{heat capacity constant pressure})$$

(c) Isothermal process ( $\Delta U = 0$ ):

$$\Delta Q = \Delta W = P \Delta V$$

(d) Adiabatic process ( $\Delta Q = 0$ ):

$$\Delta U = -\Delta W$$

$$\frac{C_P}{C_V} = \gamma \quad (1.67 \rightarrow \text{Mono}, 1.4 \rightarrow \text{Dia}, 1.33 \rightarrow \text{Tri})$$

\* Degrees of freedom →

(a) Monatomic  $\rightarrow \frac{3}{2} n R T$   
 gases

(b) Diatomic gases =  $\frac{5}{2} nRT$

(c) Triatomic gases =  $\frac{6}{2} nRT$

(d) Equipartition  $\rightarrow \frac{1}{2}$  of energy

→ Second law of Thermodynamics →

(a) Carnot engine:

$$\text{Efficiency} (\eta) = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \quad (Q_2 < Q_1) \quad (T_2 < T_1)$$

(b) Heat pump:

$$\text{Efficiency} = \frac{1}{(COP_{HP})} \eta$$

→ Heat Transfer →

? (a)  $\Delta Q = KA(T_1 - T_2)$  ( $K \rightarrow$  Thermal conductance)  
 $\Delta t$   $L$

(b) Thermal Resistance =  $\frac{1}{k} = \frac{L}{KA}$

(c) Rods in series  $\rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$

(d) Rods in parallel  $\rightarrow R_{eq} = R_1 + R_2 + R_3 + \dots + R_n$

• Elasticity-

→ Stress  $\Rightarrow \frac{F}{A}$

(a) Longitudinal strain stress

(b) Shear stress

(c) Volume stress

→ Strain =  $\frac{\Delta L}{L}$

(a) Longitudinal strain

(b) Shear strain

(c) Volume strain

→ Modulus of rigidity →

$$(a) \text{Young's (Y)} = \frac{\text{Longitudinal stress}}{\text{modulus}} = \frac{F L}{A \Delta L}$$

$$(b) \text{Bulk's (B)} = \frac{\text{Volume stress}}{\text{modulus}} = \frac{\sigma P V}{\Delta V}$$

→ Hooke's law → [Stress & Strain]

→ Spring combination →

$$(a) k = \frac{YA}{L}$$

$$(b) \text{In series: } \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n}$$

$$(c) \text{In parallel: } k_{eq} = k_1 + k_2 + k_3 + \dots + k_n$$

$$\rightarrow \text{Elastic potential (U)} = \frac{1}{2} k x^2$$

[energy]

Fluid mechanics:-

$$P = P_0 + \rho g h$$

$$\rightarrow \text{In barometer, } h = \frac{P_0}{\rho g}$$

$$\rightarrow \text{In accelerating fluid, } P = P_0 + \rho A x$$

$a \rightarrow$  acceleration of fluid.

$$\rightarrow \text{In rotating fluid, } g = \omega^2 r$$

→ Important information →

$$(a) 1 \text{ atm} = 76 \text{ cm of Hg. (Barometer)}$$

$$(b) \rho \text{ of water} = 1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3$$

$$(c) \rho_{\text{ice}} = 900 \text{ kg/m}^3 = 0.9 \text{ g/cm}^3$$

$$(d) \rho_{\text{mercury}} = 13600 \text{ kg/m}^3 = 13.6 \text{ g/cm}^3$$

\* Buoyancy force →

Thrust of liquid = weight displaced by object

Direction → opposite to gravitational force

It is a pseudo force.

\* Continuity equation →

$$A_1 v_1 = A_2 v_2$$

$v_i$  = velocity of flowing fluid

This equation is an application of Kirchhoff's current/junction law.

\* Bernoulli's equation →

$$\rho gh + \frac{1}{2} mv^2 + \rho gh = \text{constant}$$

\* Application of fluid mechanics →

(a) Hydraulic lift

(b) To determine pressure exerted by a fluid on the walls of a container

\* Viscosity:

\* Viscous force -

$$(a) F_v = \eta A \frac{v}{h}$$

$$(b) F_v = 6\eta \pi r v \quad (\text{For a spherical body})$$

$\eta$  → co-efficient of viscosity

$F_v$  → Viscous force

\* Stokes' law and terminal velocity →

$$V_T = \frac{2 r^2 (\rho_s - \rho_l) g}{9 \eta}$$

$\rho_s$  → Density of spherical body

$\rho_l$  → Density of liquid

$V_T$  → Terminal velocity / Velocity at  $a = 0$

\* Rate of flow of fluid through a pipe →

- (a) Velocity at the edges of a pipe is 0 whereas pressure exerted at the centre of the pipe is maximum

$$V = \frac{(P_1 - P_2)}{4\eta L} \pi r^2$$

$$(b) Q = \frac{\pi r^4}{8\eta L} (P_1 - P_2)$$

$Q \rightarrow$  Volume flow rate.

\* Efflux velocity →

$$v_e = \sqrt{2gh \times a}$$

$a \rightarrow$  cross-sectional area

- (b) Velocity of efflux from a height 'h' of a container of height 'H':

$$v = \sqrt{2gh + 2g(H-h)}$$

### • Surface tension -

\* Definition of surface tension ( $\tau$ ) →

$$\tau = \frac{F}{L}$$

\* Angle of contact →

- (a) When ~~Fractional~~  $F_{\text{adhesive}} < F_{\text{cohesive}}$ ,  $\theta > 90^\circ$

Eg: water on grass

- (b) When  $F_{\text{adhesive}} = F_{\text{cohesive}}$ ,  $\theta = 90^\circ$

The substance is wetted by the liquid

- (c) When  $F_{\text{adhesive}} > F_{\text{cohesive}}$ ,  $\theta < 90^\circ$

The substance is not wetted by the liquid

\* Excessive pressure inside drops →

- (a) For water drops,  $P_f - P_i = \Delta P = \frac{2\tau}{R}$

(b) For soap bubbles,  $P_f - P_i = \Delta P = \frac{4T}{R}$

(c) Work done ( $w$ ) =  $4\pi R^2 T (n-1)$  (coalesce into one)

(d) Work done ( $w$ ) =  $4\pi R^2 T (1/n-1)$  (Formation of many bubbles)

→ Capillary action →

(a)  $h = \frac{2T}{\gamma g R}$  (height of liquid column)

(b) At an angle  $\theta$ ,

$$h = \frac{2T \cos \theta}{\gamma g R}$$

→ The contact angles on toothpaste are made low so as to increase spread of toothpaste into our mouths.

### • Simple Harmonic motion:-

? → Definition of SHM →

$$\vec{a} \propto -\vec{x}$$

$$\vec{a} = -\omega^2 \vec{x}$$

$\omega$  → Angular Frequency (rad/s)

→ Equations of SHM →

(a)  $v = \omega \sqrt{A^2 - x^2}$

(b)  $v^2 + \dot{x}^2 = \frac{\omega^2 A^2}{A^2 - x^2}$

(c)  $x_{eq} = A \sin(\omega t + \phi)$

(d)  $v = A \omega \cos(\omega t + \phi)$ , where

$\phi$  → Phase difference

$\omega A$  → Maximum/optimal velocity

$A$  → Amplitude of SHM

$x$  → Displacement from mean position

\* Energy of SHM →

(a)  $PE = \frac{1}{2} k A^2 \sin^2 \omega t = \frac{1}{2} k x^2$

(b)  $KE = \frac{1}{2} k A^2 \cos^2 \omega t = \frac{1}{2} k (A^2 - x^2)$

(c)  $TE = \frac{1}{2} k A^2 (\sin^2(\omega t) + \cos^2(\omega t)) = \frac{1}{2} k A^2$

\* Phase difference of SHM →

(a) When  $\phi = 0^\circ$ , PE & A = 0 and KE = maximum

(b) When  $\phi = 90^\circ$ , PE & A = Maximum and KE = 0

(c) When  $\phi = 180^\circ$ , PE & A = 0 and KE = maximum

Same sequence follows for  $270^\circ$  and  $360^\circ$  and so on.

\* Time period of SHM →

(a)  $T = \frac{2\pi}{\omega}, T = \frac{1}{f}$  (F → Frequency)

(b)  $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{g}}$

\* Combination of springs →

(a) Series →

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$$

(b) Parallel →

$$k_{eq} = k_1 + k_2 + \dots$$

\* Reduced mass =  $m = \frac{m_1 m_2}{m_1 + m_2}$

\* Waves and sound :-

\* Description of a sound wave →

3-D, longitudinal and mechanical

\* Equation of progressive waves  $\rightarrow$

(a)  $y = A \sin(kx - wt)$ , where

$$k\lambda = 2\pi \text{ (OR)} \quad k = \frac{2\pi}{\lambda}$$

\* Super-position of two progressive waves  $\rightarrow$

This results in the formation of a standing wave

$$A_{\text{res}} = y_1 + y_2$$

\* Standing waves from different supports  $\rightarrow$

(a) Fixed support:

Wave formed at other end is water image.

$$y = B \cdot A \sin(wt - kx + \pi)$$

(b) Free ends:

Wave formed at other end is mirror reflection

$$y = A \sin(wt - kx)$$

? \* Velocity of wave and particle  $\rightarrow$

$$u = -v \left( \frac{dy}{dx} \right)$$

$u \rightarrow$  Particle speed

$v \rightarrow$  Wave speed

$$\frac{1}{v^2} \frac{d^2y}{dt^2} = \frac{1}{k^2} \frac{dy}{dx}$$

\* Velocity and fundamental frequency  $\rightarrow$

$$v = f\lambda = \sqrt{\frac{T}{\mu}}$$

$\mu \rightarrow$  Linear mass density

$$f = \frac{v}{2L}$$

$$f = \frac{v}{4L}$$

, for closed organ pipe

, for open organ pipe

Successions of frequencies are called harmonics.

\* Beats Frequency →

$$(a) \text{ Beats} = f_2 - f_1$$

(b) Waxing, frequency decreases

Fading, frequency increases.

\* Loudness ( $\beta$ ) =  $10 \log_{10} \left( \frac{I}{I_0} \right)$

$$I_0 = 10^{-12} \text{ units.}$$

$I \rightarrow$  Intensity of sound wave

\* Doppler effect →

$$f = f_0 \left( \frac{v \pm v_o}{v \mp v_s} \right)$$

$v_o \rightarrow$  velocity of observer

$v_s \rightarrow$  Velocity of source

\* Ray Optics :-

\* Plane mirrors →

(a) Distance, shape, size of the object are same in magnitude.

(b)  $|V_{\text{image}}| = 2 V_{\text{mirror}} - V_{\text{object}}$  (X-axis)

In Y-axis, parallel component of velocity remains the same.

(c) Field of view:

Take reflection of two ends of a mirror dropped from the source to find field of view.

\* Spherical mirrors →

(a)  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

(b) magnification =  $-\frac{v}{u} = \frac{|h_{\text{image}}|}{|h_{\text{object}}|}$

(c) (b) Rays that are very close to the pole of the mirror are called paraxial rays.

$$x_{\text{paraxial}} = R - R \cdot \sec \theta$$

(d) Velocity of image and object:

$$\frac{1}{u^2} V_{\text{obj}} = \frac{1}{v^2} V_{\text{im}}$$

\* Refraction →

(a) Snell's law:

$$n_1 \sin i = n_2 \sin r$$

$$\frac{v_1}{n_1} = \frac{v_2}{n_2}$$

$$(b) A = r_1 + r_2$$

$$(c) S_m = i + e - A$$

$A \rightarrow$  Angle of prism

$e \rightarrow$  Angle of emergence

\* Total internal reflection →

$$n_1 \sin i = n_2 \sin q$$

$\sin i = \sin c$ ; where  $c \rightarrow$  critical angle.

\* Lateral shift in a glass slab →

$$x = t \left( 1 - \frac{n_{\text{surrounding}}}{n_{\text{slab}}} \right)$$

$t \rightarrow$  Thickness of slab

\* Apparent depth →

$$\frac{n_1}{n_2} = \frac{d}{d'}, \text{ where } d' \rightarrow \text{Apparent depth.}$$

\* Parallel slabs of different indices →

$$n_i \sin(i) = n_f \sin(r_f)$$

\* Spherical reflection →

$$\frac{n_1 - n_2}{u} = \frac{n_2 - n_1}{R}$$

→ Lenses →

(a)  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

(b) Power of lens =  $\frac{1}{f}$ , measured in dioptres

(c)  $P_{eq} = P_1 + P_2 + P_3 + \dots$

→ Angular dispersion →

(a)  $n_g = \frac{D_v + D_r}{2}$

(b) Angle of dispersion ( $\theta$ ) = Deviation -  $S_{red}$

(c) Dispersive power ( $w$ ) =  $\frac{\text{Angular dispersion}}{\text{Mean Deviation}}$

### • Wave Optics:-

→ Amplitude and intensity →

$y_1 = A_1 \sin(\omega t)$

$y_2 = A_2 \sin(\omega t + \alpha)$

$|A_{res}|^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \Delta\phi$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos \Delta\phi}$

$I \propto A^2$

$I_{total} = A_1 \sin \alpha$

$A_1 + A_2 \cos \alpha$

→ Huygen's principle and wavefronts →

(a) Point source → Spherical wavefronts

(b) Infinite source → Planar wavefront

(c) Secondary wavelet → A particle acting as a source

(d) Condition for locus of wavefronts → Same phase

→ Interference patterns →

(a) Constructive interference:

Waves or intensities get added (same phase).  
Condition: Multiple of  $n\lambda$ .

(b) Destructive interference:

Waves of opposite phase get cancelled out.  
Condition: multiple of  $(2n+1)\frac{\lambda}{2}$

(c) Maximum intensity (constructive interference) =  $4I$

Minimum intensity (destructive interference) = 0

→ Young's Double-Slit experiment →

(a) Brightest spot at centre → central maxima

(b) Fringe width ( $\alpha$ ) =  $\frac{\lambda D}{2d}$  (How?)

$$\boxed{\alpha = \frac{n\lambda D}{2Bd}}, \text{ where}$$

$d$  → Distance between two slits/sources.

$D$  → Distance between slit/source and screen.

$m$  → Fringewidth / distance between corresponding minima/maxima

$\lambda$  → Wavelength of monochromatic light.

→ Optical path length →  $LS + RT - LR - RT$

$$\boxed{\text{Phase } (\Delta\phi) = \frac{2\pi}{\lambda} nt}$$

Difference

$nt$  → Optical path length.

$\mu$  → Refractive index.

$$\boxed{\frac{\lambda_{air}}{\lambda_{medium}} = \mu} \quad (\text{When other medium is air})$$

$$\boxed{\Delta x_{eff} = (\mu - 1)t}$$

$t$  → Thickness of medium.

$\Delta x$  → Path differences.

→ Fresnel Biprism →

P.T.O

$$TS = a(u-1)A$$

$S \rightarrow$  Distance between sources.

$a \rightarrow$  Distance between source and deviation.

\* Thin film interference  $\rightarrow$

(a)  $2n_{\text{eff}}t = \frac{\lambda}{2}$

(b) For maxima, we have,  $n_{\text{eff}}t = (m + \frac{1}{2})\lambda$

$$2nt = (n + \frac{1}{2})\lambda$$

(c) For minima,

$$2nt = n\lambda$$

\* Modern Physics:-

\* Photo-electric effect  $\rightarrow$

Ejection of  $e^-$  from metal surface by light of suitable frequency and wavelength.

\* Work function ( $\phi$ )  $\rightarrow$

Minimum energy required to remove an  $e^-$ .

$$\text{Energy (E)} = h\nu = \frac{hc}{\lambda}$$

(a) Threshold frequency ( $\nu_0$ )  $\leq \nu_{\text{required}}$

(b) Threshold wavelength ( $\lambda_0$ )  $\geq \lambda_{\text{required}}$

\* Kinetic energy of photon  $\rightarrow$

(a)  $K_E = h\nu - h\nu_0 = h(\nu - \nu_0)$

(b)  $K_E = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$

\* Photocurrent  $\rightarrow$

Current flowing from anode to cathode where photo-electric effect takes place on cathode.

Cathode  $\rightarrow$  Emitter of  $e^-$

Anode  $\rightarrow$  Collector of  $e^-$

Electric field produced from cathode to anode to cathode.

Maximum current (photo) flowing is called as saturation current.

$\rightarrow$  Stopping potential ( $V_s$ )  $\rightarrow$

Potential required to stop photocurrent.

$$eV_s = h\nu = \frac{hc}{\lambda}$$

$e \rightarrow$  charge on  $e^-$ .

$V_s \rightarrow$  Stopping potential.

$\rightarrow$  Intensity of  $e^-$  on a surface  $\rightarrow$

$$(a) \text{ No. of photons} = \frac{\text{Energy incident}}{\text{incident } (s/A \text{ area})} = \frac{I_0}{\text{Energy on photon}} = \frac{I_0}{hc/\lambda}$$

$$\text{No. of photons} = \frac{\text{Energy incident}}{\text{incident per second}} = \frac{I_0 A}{\text{Energy on photon}} = \frac{I_0 A}{hc/\lambda}$$

$\rightarrow$  Radiation pressure  $\rightarrow$

$$(a) \text{ Pressure on reflecting surface} = \frac{2I}{c}$$

$$(b) \text{ Pressure on absorbing surface} = \frac{I}{c}$$

$$(c) \text{ Pressure exerted on absorbing reflecting surface} = \frac{I}{c}(a+2r)$$

$c \rightarrow$  Speed of light

$a \rightarrow$  Absorptivity

$r \rightarrow$  Reflectivity

$\rightarrow$  Bohr's model  $\rightarrow$

(a)  $e^-$  move in special stationary orbits.

$$\text{Momentum} = \frac{h}{\lambda}$$

$$\boxed{L_n = \frac{nh}{2\pi}} \Rightarrow mv_n r_n = \frac{nh}{2\pi}$$

In  $n^{\text{th}}$  orbit.

(b)  $r_n = (2.529 \text{ Å}) \frac{n^2}{Z}$

(c)  $v_n = (2.18 \times 10^6) \times \frac{Z}{n}$

(d) Energy =  $(-13.6 \text{ eV}) \frac{Z^2}{n^2}$

\* De-Broglie's wavelength  $\rightarrow$

$$\boxed{\lambda_D = \frac{h}{mv}}$$

$\lambda_D \rightarrow$  De-Broglie wavelength.

\* X-rays  $\rightarrow$

EM waves of short wavelength ( $0.1 \text{ Å}^{\circ}$  to  $100 \text{ Å}^{\circ}$ ) strike a metal of atomic mass to form X-rays.

It is a reverse photo-electric effect.

Continuous X-rays  $\rightarrow$

~~X~~  $e^-$  collides multiple times in target material and finally comes to rest.

$$\boxed{\lambda = \frac{hc}{\Delta E}}$$

(b) Characteristic X-rays  $\rightarrow$

$\gamma^-$  removes innermost  $e^-$  of material and  $e^-$  from higher energy shells fill gap.

(i)  $e^-$  coming from ultimate shell w.r.t shell  $\rightarrow \alpha - e^-$

(ii)  $e^-$  coming from penultimate shell w.r.t shell  $\rightarrow \beta - e^-$ .

\* Nuclear physics  $\rightarrow$

$\alpha$ -decay  $\rightarrow$   ${}^4_2\text{He}$  particle ( $\alpha$ -particle)

$\beta$ -decay  $\rightarrow$   $e^-$  + neutrino ( $\bar{\nu}$ ) (OR) positron + anti-neutrino

$\gamma$ -decay  $\rightarrow$   $\gamma$ -particles.

- Newton's laws of motion →
- \* Newton's laws →
  - (a) First law →  
Body remains at rest or in continuous motion (uniform) unless acted upon by external force.
  - (b) Second law →  
$$F = \frac{dp}{dt} = ma$$
  - (c) Third law →  
$$\vec{F}_1 = -\vec{F}_2$$
- \* Free-body diagram of an object →  
Free-body diagram (FBD) involves writing external and internal forces on an object, so as to keep it in equilibrium.
- \* Different types of external forces →
  - (a) Weight of object =  $mg$
  - (b) Normal reaction contact force ( $N$ ) =  $N$ .
  - (c) Tension force in a string =  $T$
- \* Body in acceleration and pseudo forces →
  - (a) When body's in acceleration upward or downward, we equate FBD of object to  $ma$ .  
Greater magnitude force is in dir" of acceleration.
  - (b) Frame of reference:
    - Internal frame →  $a_{net} = 0$
    - Non-inertial frame →  $a_{net} \neq 0$
  - (c) When in non-inertial frame, a force acts opposite to acceleration to balance and keep body in equilibrium.  
This force is called pseudo force.
- \* Constraints →
  - (a) Pulley constraint:

- (b) If pulley strings attached to a constraint pulley which is in turn attached to a third string attached to the body is equal to :

$$V_{3^{\text{rd}} \text{ string}} = V_{\text{irr string}} - (-V_{2^{\text{nd}} \text{ string}})$$

## → Friction →

Forces that oppose motion of an object.

Direction of frictional force is opposite to that of motion of body.

(a) Static friction force ( $F_{fs}$ ) =  $\mu_s N$

(b) Kinetic friction force ( $f_k$ ) =  $\mu_k N$

- Circular motion :- An object not free moves

Angular Displacement = length of arc  $= r\theta$ ,

$$\Rightarrow \text{Angular velocity} = \frac{\text{Angular Displacement}}{\text{Time}} = \omega$$

Total time taken

$\rightarrow \ddot{\theta} = \text{Angular acceleration} = \frac{d\omega}{dt} = \text{Angular velocity}$

\* Relations between circular and linear terms →

$$(a) \quad \boxed{F = r\omega} \quad v \rightarrow \text{linear velocity}$$

(b)  $\alpha = \frac{dv}{dt}$   $\alpha \rightarrow$  linear acceleration.

\* Uniform circular motion →

w = constant In which case we may say

$$a_{\text{net}} = a_r = \frac{v^2}{r} \quad \text{for } r \rightarrow \text{Radial acceleration.}$$

$\rightarrow$  In non-uniform circular motion,

$$A_{net} = \sqrt{a_{radial}^2 + a_{tangential}^2}$$

→ Banking of roads →

$$v = \sqrt{Rg}$$

$$F_s = \frac{mv^2}{r}$$

Maximum speed to be driven on road:

$F_s \rightarrow$  frictional force

$$\text{Angle of banking} = \tan \theta = \frac{v^2}{rg}$$

→ Centripetal and centrifugal force →

$$F_{\text{centripetal}} = -F_{\text{centrifugal}} = \frac{mv^2}{r}$$

Centripetal force → Real

Centrifugal force → Pseudoforce

→ Vertical circular motion →

(a) Velocity at top point =  $\sqrt{gl}$

(b) Velocity at mid-point =  $\sqrt{3gl}$

(c) Velocity at bottom-most point =  $\sqrt{5gl}$

Systems of Particles:-

→ Centre of mass →

$$(a) x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$(b) y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots + m_ny_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$(c) z_{\text{com}} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots + m_nz_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$x_{\text{com}}, y_{\text{com}}, z_{\text{com}} \rightarrow$  Center of mass about its axes.

This is the point where the whole mass of the body seems to be concentrated.

→ COM of some different shapes →

$$(a) \text{ COM of semi-circular ring} = \frac{2R}{\pi} \text{ (in Y-direction only)}$$

(b) COM of semi-circular disc =  $\frac{4R}{3\pi}$  (Only Y-direction)

\* Linear momentum  $\rightarrow$  mass  $\times$  velocity  
Body having a velocity and moves in a particular direction.

$$\vec{P} = m\vec{v}$$

$$KE = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

\* Conservation of linear momentum  $\rightarrow$

When external force,  $F_{net} = 0$ , then

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$u \rightarrow$  Initial velocity

$v \rightarrow$  Final velocity

\* Collisions  $\rightarrow$

The incident wherein two bodies come in contact and collide with each other.

Here, linear momentum is always conserved (3)

(a) Elastic collision  $\rightarrow$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Here, final velocity and initial velocity of one body in motion are same.

KE is also conserved in this type of collision.

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

(b) Inelastic collision  $\rightarrow$

Where linear momentum is conserved but not kinetic energy.

\* Co-efficient of restitution  $\rightarrow$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$e = 1$  (for perfectly elastic)

$e = 0$  (Perfectly inelastic)

$0 < e \leq 1$  (Inelastic collisions)

## Rotational motion:-

\* Moment of inertia →

$I_T = m_1 r_1^2, I_S = m_2 r_2^2$  and so on.

$$I = \sum_{i=1}^n m_i r_i^2$$

It is the angular equivalent for inertia.

\* MoI of different-shaped objects →

(a) Rod → Centre =  $\frac{ML^2}{3}$ , Ends =  $\frac{ML^2}{12}$

(b) Square → Centre =  $\frac{Ma^2}{6}$

(c) Rectangle → Centre =  $M(a^2 + b^2)$

(d) Ring (Circular) → Centre =  $MR^2$ , Ends =  $MR^2$

(e) Circular disc → Centre =  $\frac{MR^2}{2}$ , Ends =  $3\frac{MR^2}{2}$

(f) Hollow cylinder → Centre =  $MR^2$ , Ends =  $2MR^2$ .

(g) Solid cylinder → Centre =  $\frac{MR^2}{2}$ , Ends =  $3\frac{MR^2}{2}$

(h) Hollow sphere → Centre =  $\frac{2}{3}MR^2$ , Ends =  $5\frac{MR^2}{3}$

(i) Solid sphere → Centre =  $\frac{2}{5}MR^2$ , Ends =  $7\frac{MR^2}{5}$ .

\* Theorems on rotation → rigid rotation

(a) Theorem of parallel axes:

$$I = I_{cm} + mr^2$$

r → Perpendicular from centre of mass to point.

(b) Theorem of perpendicular axes:

$$I_z = I_x + I_y$$

\* Radius of gyration ( $k$ ) →

$$I = m k^2 (\text{or}) \quad k = \sqrt{\frac{I}{m}}$$

We can calculate ' $k$ ' using MoI for different shapes

\* Torque ( $\tau$ ) →

$\tau$  is the force equivalent in rotation.

$$\tau = F \times r = I \alpha$$

\* Angular momentum and its conservation →

(a)  $\vec{L}$  (Angular momentum) =  $\vec{r} \times \vec{p} = I \omega$

(b) If  $I_{\text{ext}} = 0$ , then

$$I_{\text{initial}} = I_{\text{final}}$$

(c)  $KE_{\text{rotational}} = \frac{1}{2} I \omega^2 = \frac{I^2}{2M}$

\* Combined translational and rotational motion (CTRIM) →

(a) When a performs linear as well as circular/rotational motion, then it will be considered under CTRIM.

In a wheel,

Northern-most ( $v_A$ ) =  $(V + \omega R)\hat{i}$

point

Eastern-most ( $v_B$ ) =  $V\hat{i} - \omega R\hat{j}$

point

Southernmost ( $v_C$ ) =  $(V - \omega R)\hat{i}$

point

Western-most ( $v_D$ ) =  $V\hat{i} + \omega R\hat{j}$

Point

(b) Pure rolling →

It is a condition when the rolling body does not slip on the surface.

\* (i) Here, at point A,  $v = \omega R$ .

- Point B,  $v - \omega R < 0$ .

Point C,  $v = 0$

Point D,  $v_{towR} < 0$

✓ (iii) 'fs' will start acting, and at bottom-most point,

$$f_s = \text{Work done} = 0$$

(c)  $T_{\text{eg}} = kE_T + kG_R$

→ Instantaneous axis of rotation ( $\rightarrow$ ) support  
We can find IAOR by finding the point where velocity seems to take place, drop a perpendicular from it, and find intersection.

→ Toppling  $\rightarrow$  If width of the body is  $L$ , then if  $\frac{L}{2}$  of

If torque applied, which is counter-balanced by a normal reaction torque, is exceeding  $\frac{L}{2}$  of

length, then body will topple.

∴ If the surface is not rigid, it will start rotating.

Gravitation: Attraction between all matter particles.

→ Newton's law of gravitation  $\rightarrow$

$$F = k \frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2$$

$$\text{kg}^2$$

→ Gravitational field  $\rightarrow$

This is exerted by every body having a mass.

$$E = \frac{F}{m} = \frac{Gm}{r^2}$$

→ G-field in different shapes  $\rightarrow$

(a) Ring  $\rightarrow \frac{GL}{R}$  (quarter),  $\frac{2GL}{R}$  (semi-circular)

$\lambda \rightarrow$  linear mass density  $= \frac{M}{L}$

(b) Hollow sphere  $\rightarrow$  Internal  $\rightarrow 0$ , External  $\rightarrow \frac{GM}{R^2}$

(c) Solid sphere  $\rightarrow$  Internal  $\rightarrow 0$ , External  $\rightarrow \frac{GM}{R^2}$

\* Gravitational potential  $\rightarrow$

$$GP \text{ Energy} (U) = -\frac{GMm}{r} \leftarrow \text{Work done} \rightarrow$$

$$G \cdot \text{Potential} (V) = \frac{1}{m} \frac{dU}{dr} = -\frac{GM}{R} \rightarrow V \text{ is constant}$$

(a) Ring  $\rightarrow -\frac{GM}{R}$

(b) Hollow sphere  $\rightarrow$  Internal = External  $= -\frac{GM}{R}$

(c) Solid sphere  $\rightarrow$  Internal  $= -\frac{GM(3R^2 - x^2)}{R^3}$ , External  $= -\frac{GM}{R}$

$x \rightarrow$  Radius of sphere.

\* Kepler's laws of planetary motion  $\rightarrow$

(a) Planets move around the sun with the latter at one of its foci.

$$\frac{dA}{dt} = \text{constant} = \frac{1}{2m}$$

$dA \rightarrow$  Area vector.

$$T^2 = \frac{4\pi^2}{GM} R^3, \therefore T^2 \propto R^3$$

\* Acceleration due to gravity ( $g$ )  $\rightarrow$

$$(a) g = \frac{GM}{R^2}$$

$$(b) g = \frac{GM}{(R+h)^2} = GM \left( \frac{1-2h}{R} \right)$$

$$(c) g = GM \left( 1 - \frac{1}{R} \right)$$

\* Orbital and escape velocity  $\rightarrow$  velocity of satellite (Q)

$$\boxed{V_{\text{orbital}} = \frac{GM}{R}}$$

$$\boxed{V_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R+h}}}$$

$V < V_0 \rightarrow$  Elliptical

$V = V_0 \rightarrow$  Circular

$\Rightarrow V_0 < V < V_e \rightarrow$  Elliptical

$V = V_e \rightarrow$  Parabola

$V > V_e \rightarrow$  Hyperbola.

\* Energy of satellite  $\rightarrow$

$$\boxed{PE = -\frac{GMm}{R}}$$

$$\boxed{KE = \frac{1}{2} \frac{GMm}{2R}}$$

$$\boxed{TE = -\frac{GMm}{R} = -BE}$$

\* Electrostatics:-

\* Coulomb's law:-

$$\boxed{F = \frac{kq_1 q_2}{r^2}}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

\* Electric field  $\rightarrow$

$$\boxed{F = q\vec{E}}$$

$$\boxed{E = \frac{kq}{r^2}}$$

\* Electric field of different shapes  $\rightarrow$

(a) Uniform wire (long) at a point  $\rightarrow E_x = \frac{k\lambda}{r^2} (\sin\theta_1 + \sin\theta_2)$

$$E_y = \frac{k\lambda}{r^2} (\cos\theta_1 - \cos\theta_2)$$

$\lambda \rightarrow$  Linear charge density.

(b) One end of a long uniformly charged wire  $\rightarrow E_x = \frac{k\lambda}{r^2}, E_y = 0$

$$\theta_1 = 90^\circ, \theta_2 = 0^\circ$$

(c) Throughout the uniformly charged wire  $\rightarrow E_x = \frac{2k\lambda}{r^2}, E_y = 0$

$$\theta_1 = 90^\circ, \theta_2 = 90^\circ$$

(d) Uniformly charged quarter ring  $\rightarrow E_x = \frac{k\lambda}{r^2}$

Uniformly charged semi-circular ring  $\rightarrow E_x = \frac{k\lambda}{r^2}$

(e) Uniformly charged hollow sphere  $\rightarrow$  External  $= \frac{kq}{r^2}$ , Internal  $= 0$

(f) Non-conducting solid sphere  $\rightarrow$  External  $= \frac{kq}{r^2}$ , Internal  $= 0$

$\rightarrow$  Electric potential energy  $\rightarrow$

$$U = \frac{kq_1 q_2}{r}$$

Negative of work done from bringing unit (+ve) charge from infinity to the electric field

$\rightarrow$  Electric potential  $\rightarrow$

$$V = \frac{du}{q} = \frac{kq}{r}$$

$$\nabla V = -\vec{E} \cdot d\vec{r}$$

$\rightarrow$  Electric potential of different shapes  $\rightarrow$

Uniformly charged ring  $= \frac{kq}{\sqrt{R^2 + r^2}}$

$r \rightarrow$  Radius of ring

P.T.O

(b) Hollow sphere  $\rightarrow$  External = Internal =  $\frac{kq}{r}$  (a)

(c) Solid sphere  $\rightarrow$  External  $\geq \frac{kq}{r}$ , Internal  $= \frac{kq}{r} (3R^2 - r^2)$

$$\delta V = -\vec{E} \cdot d\vec{r}$$

$\rightarrow$  Electric dipole  $\rightarrow$  Flows from (+ve charge) to (-ve charge).

$$\vec{P} = \vec{q} \times \vec{2r}$$

Cosine component  $\rightarrow 2k(p \cos \theta)$  (a)

Sine component  $\rightarrow k(p \sin \theta)$  (a)

$\propto 1/r^2$  (a)

$\rightarrow$  Electric flux ( $\Phi$ )  $\rightarrow$

$$\Phi = \int \vec{E} \cdot d\vec{s}$$
 (a)

Scalar quantity.

$$\oint \vec{E} \cdot d\vec{s} = Q / (\text{charge enclosed in gaussian surface})$$

This is called as Gauss' theorem. (a)

$\rightarrow$  Conductors, in non-conducting objects  $\rightarrow$

(a) When no cavity is present, charge is ~~present~~ present at the closed Gaussian surface.

(b) When a cavity is present in the middle, charge is present at outermost surface and zero in innermost surface. (a)

\* Capacitors :-

$\rightarrow$  Definition  $\rightarrow$

$$Q = CV$$

C  $\rightarrow$  capacitance

$\Rightarrow$  S.I unit is Farad.

\* Energy stored in capacitors  $\rightarrow$

$$\text{Energy} = \frac{q^2}{2c} = \frac{1}{2} CV^2 = \frac{1}{2} \Delta V$$

\* Types of capacitors  $\rightarrow$

(a) Parallel-plate  $\rightarrow C = \frac{\kappa A\epsilon_0}{d}$

(b) Spherical capacitor  $\rightarrow C = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$

(c) Cylindrical capacitor  $\rightarrow C = \frac{2\pi\epsilon_0 L}{\ln(R_2/R_1)}$

\* Introduction of dielectrics  $\rightarrow$

? (a) Disconnected  $\rightarrow$  Charge, voltage, P.d., energy  $\rightarrow$  Decreases

Charge  $\rightarrow$  constant

Capacitance  $\rightarrow$  Increases

? (b) Battery connected  $\rightarrow$  Charge, energy, capacitance  $\rightarrow$  Increases

P.d./voltage, electric field  $\rightarrow$  constant.

\* Current Electricity :-

\* Electric current  $\rightarrow$

$$I = \frac{\Delta Q}{\Delta t} = nAeV_d$$

$V_d$   $\rightarrow$  Drift speed ( $\equiv$  Average Speed)

$$V_d = \frac{eEt}{2m}$$

\* Current density ( $J$ )  $\rightarrow$

$$\text{Current } (J) = \sigma \bar{E} = \frac{I}{A}$$

$$\sigma \rightarrow \text{conductivity} = \frac{ne^2 T}{2m}$$

\* Kirchoff laws  $\rightarrow$

(a) First law (current law):  $\text{Sum of current entering a junction} = \text{Sum of current leaving a junction}$   
Based on conservation of charge.

(b) Second law (loop law):

Sum of all potential across circuit is zero.

\* Current measuring devices  $\rightarrow$

(a) Galvanometer:  $\text{Current sensitivity device. } R = \text{few } \Omega.$

(b) Ammeter:  $\text{Connect shunt resistance parallel to galvanometer.}$   
Ideal Ammeter resistance = 0.

(c) Voltmeter:

Connect high resistance series to galvanometer.

Ideal voltmeter resistance =  $\infty$ .

\* Metre-Bridge  $\rightarrow$

$$R_x = R_0 \left( \frac{L_x}{L} \right)$$

\* Potentiometer  $\rightarrow$

$$E_1 + E_2 = E = \frac{V_o}{L} X_l$$

\* R-C circuit  $\rightarrow$

$$i = \frac{dq}{dt} = \frac{Q_0}{C} e^{-t/R}$$

$t_0 \rightarrow \text{time constant} = t$

63% of charging and also discharging of capacitor takes place (in) one time constant

$$q = Q_0 e^{-t_0/RC}$$

\* Magnetism:-

→ Biot-Savart law →

$$\boxed{B = \frac{\mu_0 I}{4\pi} \frac{dx}{r^2}} \quad \boxed{B = \frac{\mu_0 I d \sin \theta}{4\pi r^2}}$$

\* Magnetic field of different shapes →

(a) Infinitely long straight wire =  $\frac{\mu_0 I}{2\pi r}$

(b) Centre of coil carrying current  $= \frac{\mu_0 I \times n}{2\pi r}$

(c) Axis of current carrying coil  $= \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}$

\* Definition of Ampere →

$$\boxed{\frac{F}{l} = 2 \times 10^{-7} \text{ N/m}}$$

F → Force applied

l → length of conductor

\* Fleming's left-hand rule →

Thumb → force (external)

Index finger → magnetic field

Middle finger → current.

\* Ampere's law →

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

\* Lorentz force →

$$\vec{F} = \vec{F}_{\text{electric}} + \vec{F}_{\text{magnetic}}$$

\* Force on external applied magnetic field →

$$\vec{F} = q \vec{v} \times \vec{B}$$

$$\boxed{F = \frac{mv}{qB}}$$

$$\boxed{\text{Time } (T) = \frac{2\pi f_{\text{pp}}}{\text{Period}} = \frac{2\pi m}{qB}}$$

\* Magnetic field of coils  $\rightarrow$

$$(a) \text{ Solenoid} \rightarrow B = \mu_0 N I$$

$$(b) \text{ Toroid} \rightarrow B = \frac{\mu_0 N I}{2\pi R}$$

$$(c) \text{ Infinite sheet} \rightarrow B = \frac{\mu_0 k}{2}$$

~~$k \rightarrow$  linear charge density~~  $\rightarrow$  ~~the origin~~  $\rightarrow$

\* Moving-coil Galvanometer  $\rightarrow$

$$\text{Current sensitivity (CS)} = \frac{nBA}{\text{of Galvanometer}}$$

$$F = I(\vec{I} \times \vec{B})$$

## Electromagnetic Induction

\* Magnetic flux  $\rightarrow$

$$\oint \vec{B} \cdot d\vec{s} = \Phi$$

\* Faraday's laws of EMF  $\rightarrow$

(a) First Law:

EMF is induced by changing magnetic flux.

$$\text{emf induced} = -\frac{d\Phi}{dt} = -\frac{(\Phi_f - \Phi_i)}{t}$$

$$I_{\text{induced}} = \frac{\text{emf induced}}{R}$$

$$(b) Q = -(\Phi_f - \Phi_i)$$

$$(c) \text{emf} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

\* Self-induction ( $L$ )  $\rightarrow$

$$\boxed{\Phi = L_{\text{self}} \times i}$$

$$\text{emf} = -L \frac{di}{dt}$$

$$L_{\text{self}} \text{ of solenoid} = \mu_0 n^2 (\pi r^2 l)$$

$$I_{\text{eq}} = I_1 + I_2 + \dots \quad (\text{Series})$$

$M \rightarrow$  Mutual inductance

$$I = I_1 + I_2 + \dots \quad (\text{Parallel})$$

$$\text{Energy stored} = \frac{1}{2} L I^2$$

in inductor

$$\text{Energy} = \frac{\text{Magnetic energy}}{\text{Volume}} = \frac{B^2}{2\mu_0}$$

$$\text{Density} = \frac{\text{Volume}}{(\text{Area} \cdot 2\pi r)}$$

$\mu \rightarrow$  Permittivity of medium.

\* Mutual induction  $\rightarrow$  ~~definition of flux~~

$$\Phi_{\text{coil 2}} = M i_{\text{coil 1}}$$

$$\Phi_{\text{small coil}} = \frac{M \mu_0 r^2 \times i}{2R} \quad (\text{In long and small coil})$$

$R \rightarrow$  Larger radius

normal component

$r \rightarrow$  Smaller radius

normal component

$$M = \frac{M \mu_0 r^2}{2R} = \text{constant with respect to } r$$

$$M = \alpha \sqrt{L_1 L_2}$$

$\alpha \rightarrow$  Coefficient of coupling

\* Different circuits with DC Source

(a) L-R circuit:

$$i = \frac{E_0}{R} (1 - e^{-\frac{Rt}{L}})$$

$$\frac{di}{dt} = \frac{E_0}{L}$$

$$i = i_{\text{max}} = \frac{E_0}{R}, \text{ at } t = 0$$

$$\text{time constant} \rightarrow t_0 = \frac{L}{R} \quad (\text{in seconds})$$

$$i = 0.63 i_{\max} \quad (\text{at } t = L/R)$$

(b) When battery is removed, what happens?

$$\frac{i}{i_0} = e^{-Rt/L}$$

$$\text{Heat} = \frac{1}{2} L i_0^2$$

(c) L-C oscillations: initial current is zero

$$i = i_0 \sin(\omega t)$$

\* Errors & experiment 51 - methods, limitations

\* Types of errors →  $i.e. \Delta R, \Delta L, \Delta m, \Delta I_0, \Delta \omega$

(a) Systematic error

(b) Random error

(c) Absolute error

(d) Mean Absolute error =  $|\Delta a_1 - a_m| + |\Delta a_2 - a_m| + \dots$

$a_m \rightarrow$  Mean error of observations

(e) Percentage =  $\frac{\Delta R}{R} \times 100$

\* Significant figures → Numerical digits after the integer.

1.6 → 2 significant figures

0.002 → 1 SF

\* Zero error →

(+)ve zero error = Calculated value - (+)ve zero error

(-)ve zero error = Calculated value + (-)ve zero error

• Alternating current :-

$E_0 = BA\omega = \text{Peak inverse voltage}$ .

$$\text{Band width} = \frac{1}{2\pi} = \frac{1}{2\omega} \quad (\text{Frequency})$$

→ Different cases between  $E_0$  and  $I$  →

(a)  $E = E_0 \sin(\omega t)$ ,  $I = I_0 \sin(\omega t + \phi)$

$I$  leads  $E$  by  $\phi$

(b)  $E = E_0 \sin(\omega t + \phi)$ ,  $I = I_0 \sin(\omega t)$

$E = E_0 \sin(\omega t)$ ,  $I = I_0 \sin(\omega t - \phi)$

$I$  lags behind  $E$

→ Different types of electric circuits with AC source →

(a) Purely resistive :

$$I = \frac{E_0}{R}$$

(b) Purely inductive :

$$I = \frac{E_0}{(wL)} \quad w \rightarrow \text{Angular frequency}$$

(c) Purely capacitive :  $R = \frac{V}{I}$

$$I = \frac{E_0}{(1/wC)} \quad R = \frac{V}{I}$$

(d) R-C circuit :  $I = \frac{E_0}{(R^2 + (1/wC)^2)^{1/2}}$

$$I = \frac{E_0}{\sqrt{R^2 + (1/wC)^2}} \quad R = \frac{V}{I}$$

(e) R-L circuit :  $I = \frac{E_0}{\sqrt{R^2 + (wL)^2}}$

$$I = \frac{E_0}{\sqrt{R^2 + (wL)^2}} \quad L = \frac{V}{I}$$

(f) L-C circuit :  $I = \frac{E_0}{\sqrt{(wL)^2 + (1/wC)^2}}$

$$I = \frac{E_0}{\sqrt{(wL)^2 + (1/wC)^2}} \quad C = \frac{V}{I}$$

(g) R-L-C circuit:

$$I = \frac{E_0}{|Z|}$$

$$\text{Impedance } |Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

→ Resonant frequency →

$$\omega R < \frac{1}{\sqrt{LC}}, I \text{ leads } V \Rightarrow f < \frac{1}{2\pi\sqrt{LC}}$$

2. Similarly,

$$\omega R > \frac{1}{2\pi\sqrt{LC}}, I \text{ lags } V$$

$$f = \frac{1}{2\pi\sqrt{LC}}, I \text{ and } V \text{ are in phase}$$

Resonant frequency

• Semi-conductors :-

→ Band theory →  $E_F$  between  $V_B$  and  $cB$

$cB$  → Conduction Band (Permitted energy level above  $V_B$ )

$V_B$  → Valence Band (Energy lvl of valence  $e^-$ )

Distance b/w  $V_B$  and  $cB$  → Band Gap

(a) Band Gap negligible → Conductors

(b) Band Gap  $< 3\text{eV}$  → Semi-conductors

(c) Band Gap  $> 3\text{eV}$  → Insulators.

→ Types of semi-conductors

Intrinsic : Pure semi-conductors

Extrinsic : Doped semi-conductors (added impurities)

→ p-type, n-type and p-n junction

(a) Doped with trivalent impurities → p-type

(b) Doped with pentavalent impurities → n-type

(c) Doped with both impurities → p-n junction

- Creation of E-field due to transfer of charges: Potential Barrier
  - When  $n_e \approx n_h$ , it is forward biased (p region is +ve)
  - When  $n_e > n_h$ , it is reverse biased (p region is -ve)
  - \* Rectifier → Converts AC signal to DC.
  - \* Oscillator → Converts DC into AC signals without any input required. When output of amplifier is coupled with its input, it is called feedback.
    - Negative Feedback → Out of phase.
    - Positive Feedback → In phase.
  - \* Diodes →
    - (a) Zener diode: Maintains constant voltage even after large change in current.
    - (b) LED
    - (c) Photodiode.
  - \* Transistor → It has 3 junctions (Base, emitter, collector) and 3 regions (Active, saturated, cut-off).
 
$$\alpha = \frac{I_c}{I_E}, \beta = \frac{I_C}{I_E}$$
- Active region  $\rightarrow V_{BE} > V_{BE}$  (forward bias)
- Saturated region  $\rightarrow V_B > V_{BE} + V_{CE}$  (saturation)

## X. Optical Instruments and Diffraction:-

## CHEMISTRY

• Mole Concept :- number of moles = mass / molecular mass

→ Avogadro's number →

$$N_A = \frac{\text{Given mass}}{\text{Molecular mass}} = \frac{\text{Given volume}}{22.4 \text{ L}}$$

→ Ideal gas equation →

$$PV = nRT$$

$$R = 0.0821 \text{ atm} \cdot \text{L mol}^{-1} \text{K}^{-1}$$

$$= 0.0833 \text{ bar L mol}^{-1} \text{K}^{-1}$$

$$= 8.314 \text{ J L mol}^{-1} \text{K}^{-1}$$

$$= 2 \text{ cal L mol}^{-1} \text{K}^{-1}$$

→ Empirical and molecular formulae →

$$\text{Molecular formula} = (\text{Empirical formula})^n$$

$$\text{Relative atomic mass} = \frac{\% \text{ composition}}{\text{Atomic mass}}$$

→ Stoichiometry and balanced equation →  
A chemical equation where no. of moles of reactants is equal to moles of products.

Other than moles, even no. of molecules and volume (in case of gases) can be considered but, not their mass.

→ Excess and limiting reagents →

When more than one reactants are present, one present in limited quantity governs reaction.

$$\text{Moles of L.R} = \text{Moles of products}$$

→ Redox reactions →

Oxidation and reduction processes are collectively

called as redox reactions.

- (a) Neutral compounds  $\rightarrow$  charge = 0.
- (b) (+ve compounds / ions  $\rightarrow$  charge = +n ( $0, 1, 2, 3, 4, \dots$ ))
- (c) (-ve compounds / ions  $\rightarrow$  charge = -n ( $1, 2, 3, 4, \dots$ ))

### \* Atomic Structure:-

#### \* Discovery of sub-atomic particles $\rightarrow$

JJ Thompson  $\rightarrow$  electron (1899)

Rutherford  $\rightarrow$  nucleus (1908)

Chadwick  $\rightarrow$  neutron (1932)

#### \* Photo-electric effect $\rightarrow$

Discovered by Maxwell and Einstein in 1906.

$$KE_{\max} = h\nu - \phi$$

$\phi \rightarrow$  work function.

#### \* Hydrogen spectrum $\rightarrow$

$e^-$  can jump from shell  $n=\infty$  to:

(a)  $n=1$  (Lyman series)

(b)  $n=2$  (Balmer series)

(c)  $n=3$  (Paschen series)

(d)  $n=4$  (Brackett series)

(e)  $n=5$  (Pfund series)

$$\frac{1}{\lambda} = Rz^2 \left( \frac{1}{n_1} - \frac{1}{n_2} \right), \text{ where}$$

$$E = \frac{hc}{\lambda} = h\nu$$

$$R \rightarrow \text{Rydberg's constant} = 1.097 \times 10^7 \text{ cm}^{-1}$$

#### \* Bohr's model $\rightarrow$

In 1913, Niels Bohr successfully explained atomic structure for single  $e^-$  elements.

~~Radius of  $(r_n) = 0.529 \text{ Å}^0 \frac{n^2}{z}$   
 in  $n^{\text{th}}$  orbit~~

~~Velocity of  $e^- (v) = 2.18 \times 10^6 \text{ m/s} \times \frac{z}{n}$   
 in  $n^{\text{th}}$  orbit~~

~~Energy of  $e^- (E_n) = (-13.6 \text{ eV}) \frac{z^2}{n^2}$   
 in  $n^{\text{th}}$  shell~~

~~$E_1 = -13.6 \text{ eV}, E_2 = -3.4 \text{ eV}, E_3 = -1.51 \text{ eV}$~~

→ Quantum numbers →

Principle ( $n$ ) → Shell in which  $e^-$  is present  
 quantum no.

Azimuthal ( $l$ ) → 0 to  $n-1$

quantum no. Decides shape of orbital.

Magnetic ( $m$ ) → Decides orientation of  
 quantum no. orbital

SPIN quantum ( $s$ ) →  $+\frac{1}{2}$  or  $-\frac{1}{2}$ .  
 number

→ Shape and orientation of orbitals →

S-orbital → spherical

$|1L|$

P-orbital → dumb-bell

$|1L|1L|3L|$

D-orbital → Double-dumbbell

$|3L|1L|1L|1L|1L|$

F-orbital → complex structure  $|1L|1L|1L|1L|1L|1L|1L|$

→ Rules governing filling of  $e^-$  in orbitals →

(a) Aufbau principle →

Filling of  $e^-$  are based on filling lower energy shells first.

(b) Pauli's exclusion principle →

No 2  $e^-$  can have the same set of quantum number set.

(c) Hund's rule for multiplicity →

In degenerate orbitals (having similar energy), 2  $e^-$  can be filled in 1 orbital only when all  $e^-$  of

same spin fills all shells in all orbitals of a sub-shell.

- \* Relation between nodes and orbitals →  
Node → 0% probability of finding an  $e^-$   
orbital → Almost 100% probability of finding an  $e^-$ .

- Gaseous State:-

- \* Definition →

The state where speed of molecules is dependent on the temperature

- \* Gas Laws →

- (a) Boyle's Law:

$$P \propto \frac{1}{V}, P \cdot V = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

- (b) Charles' law →

$$P \propto \frac{V}{T}, \frac{V}{T} = \text{constant}, \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- (c) Gay-Lussac's law →

$$P \propto \frac{1}{T}, \frac{P}{T} = \text{constant}, \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- (d) Avogadro's law →

$$V \propto n, \frac{V}{n} = \text{constant}, \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

- (e) Ideal Gas equation:

$$PV = nRT$$

$$\rho M = \frac{P}{R} T$$

- \* Dalton's Law of partial pressure:

The total pressure = Sum of their  
exerted by mixture gases Partial pressures

$$P = X_i \times P_{\text{TOTAL}}$$

$P \rightarrow$  Partial pressure

$X \rightarrow$  Mole fraction

→ Graham's law of diffusion and effusion →

(a) Diffusion →

Movement from higher to lower concentration.

(b) Effusion →

Escape of gas (Diffusion) through a small hole.

$$\text{Rate } (r) = \frac{P}{M} \quad M \rightarrow \text{Molecular mass}$$

$$\text{Rate} = \frac{\text{Volume}}{\text{Time}} = \frac{\text{Pressure}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Temperature}}{\text{Time}}$$

→ Exploding container →

$$\frac{P_i V_i}{n_i T_i} = \frac{P_{\text{max}} V_{\text{max}}}{n_{\text{max}} T_{\text{max}}}$$

→ Deviations from b. ideal behaviour →

$$(a) \text{Compressibility } (\gamma) = \frac{P_i V_i}{R T_i}$$

$$\gamma = \frac{V_m^{\text{real}}}{V_m^{\text{ideal}}} \quad (V_m \rightarrow \text{Molar volume})$$

(i) At low pressures,  $V_m^{\text{real}} < V_m^{\text{ideal}}$

(ii) At high temperatures,  $V_m^{\text{ideal}} < V_m^{\text{real}}$

(iii) At low pressure and high temperature,  $V_m^{\text{ideal}} \approx V_m^{\text{real}}$

(b) → Van Der Waal's equation:

$$\left( P + \frac{a n^2}{V^2} \right) (V - n b) = n R T$$

$a, b \rightarrow$  Van Der Waal's constants.

(c) At low pressure,

$$\left( P + \frac{a n^2}{V^2} \right) V = n R T$$

## • Chemical Thermodynamics:-

### \* Systems →

- Open System: Exchange of matter and energy
- Closed system: Exchange of energy only.
- Isolated system: No exchange takes place.

### \* Functions and properties →

- Intensive properties: Non-additive in nature
- Extensive properties: Additive in nature
- State functions: Depends on initial and final states
- Path functions: Depends on path taken.

### \* Types of processes →

- Reversible: Driving force  $\approx$  Opposing force
- Irreversible: Driving force  $>>$  opposing force

### \* Constant parameter processes →

- Isochoric process:  $V = \text{constant}$ ,  $\Delta V = 0$
- Isobasic process:  $P = \text{constant}$ ,  $\Delta P = 0$
- Isothermal process:  $T = \text{constant}$ ,  $\Delta T = 0$
- Adiabatic process:  $Q = \text{constant}$ ,  $\Delta Q = 0$

### \* First law of Thermodynamics →

$$\boxed{\Delta U = \Delta Q + \Delta W}$$

~~At Isochoric~~ →  $\boxed{\Delta U = \Delta Q}$ ,  $\boxed{\Delta W = 0}$

Isobasic → Stays the same

~~At Isothermal~~ →  $\boxed{\Delta Q = -\Delta W}$ ,  $\boxed{\Delta U = 0}$

~~At Adiabatic~~ →  $\boxed{\Delta U = \Delta W}$ ,  $\boxed{\Delta Q = 0}$

### \* Enthalpy →

Heat content stored at constant pressure.

$$\boxed{U = H + PV}, \boxed{H = U + PV}$$

### \* Relation of first law with $C_p$ and $C_v$ →

$$\boxed{C_v = \frac{\Delta U}{\Delta T}}, \boxed{C_p = \frac{\Delta H}{\Delta T}}$$

\* Work done at different constant processes  $\rightarrow$

(a) Isochoric  $\rightarrow W = P \text{ constant}$

(b) Isobaric  $\rightarrow W = P \text{ ext. } V$

(c) Isothermal  $\rightarrow W = nRT \ln \frac{(V_2)}{(V_1)}$

(d) Adiabatic  $\rightarrow W = \frac{nR}{\gamma - 1} (T_2 - T_1)$

\* Thermodynamics  $\rightarrow$

$$\Delta H = \Delta U + P\Delta V$$

(a) Heat of formations

Enthalpy when 1 mole of substance is obtained from its constituents by heating.

(b) Heat of combustion:

Heat evolved when substance (1 mole) burnt in air.

(c) Calorific value:

Heat evolved by burning in presence of air in terms of calories or Joules.

(d) Enthalpy of neutralisation:

Heat evolved/absorbed when 1 mole acid reacts with 1 mole base.

(e) SA and SB  $\rightarrow -13.7 \text{ kcal}$

WA and WB  $\rightarrow < -13.7 \text{ kcal}$

(f) Enthalpy of hydration  $\rightarrow$

Heat evolved for formation of 1 mole hydrate.

(g) Ionisation enthalpy  $\rightarrow$

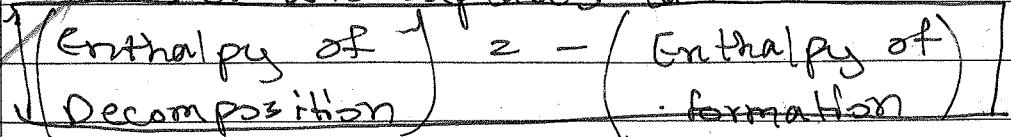
Heat absorbed to break 1 mole substance into its respective ions.

(h) Bond dissociation enthalpy  $\rightarrow$

Heat absorbed when 1 bonds of 1 mole compound are broken.

\* Laws of thermochemistry →

(a) Lavoisier and Laplace's law:



(b) Hess' law for constant heat summation:

Sum of individual changes is equal to the overall heat change provided initial and final states are same.

\* Entropy (S) →

Entropy is the degree of randomness.

$\Delta S$  (+ve) → Breaking of bonds, more no. of products

$\Delta S$  (-ve) → Formation of bonds, less no. of products

\* Second law of thermodynamics →

$$\text{No engine } \Delta G = \Delta H - T\Delta S$$

$\Delta G \rightarrow$  Gibb's Free energy.

$\Delta G < 0 \rightarrow$  Spontaneous

$\Delta G = 0 \rightarrow$  Equilibrium

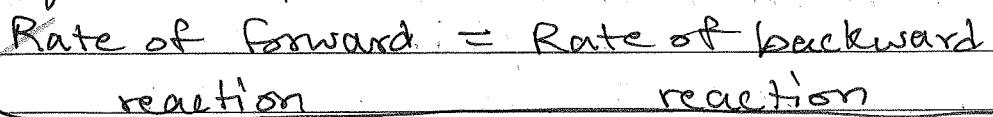
$\Delta G > 0 \rightarrow$  Non-spontaneous

\* Third law of thermodynamics →

Absolute zero temperature.

- Chemical equilibrium :-

\* Equilibrium situation →



\* Equilibrium constants →

$$(a) K_c (\text{concentrations}) = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

$[C]^c [D]^d \rightarrow$  Concentrations of product raised to S.R

$[A]^a, [B]^b \rightarrow$  Conc. of reactants raised to stoichiometric Ratios (S.R).

$$\boxed{\cancel{K_p = \frac{P^d P^c}{P^a P^b}}}$$

$P$  → Partial pressure

\* Relation between  $K_p$  and  $K_c$  →

$$\boxed{\cancel{K_p = K_c R T^{Dng}}}$$

$D_{ng} \rightarrow$  moles of products - Moles of reactants.

\* Dependence of equilibrium on different factors →

(a) Temperature:

Since  $[C] \propto T$ , thus  $K_c \propto T$

(b) Addition of inert gas:

Does not really affect, but increases constant.

(c) Catalyst addition →

Catalyst increases rate of reaction

\* Reaction quotient →

Value of constant at time other than equilibrium.

$Q < K \rightarrow$  Forward reaction

$Q = K \rightarrow$  Already in equilibrium

$Q > K \rightarrow$  Backward direction

\* Equilibrium at exothermic and endothermic →

Exothermic →  $T \downarrow$ , Forward direction

Endothermic →  $T \uparrow$ , Backward direction.

\* Add " and sub " of reversible processes →

(a) ~~Addition of their multiplication of their processes equilibrium constants~~

(b) Subtraction of  $\rightarrow$  Division of their

processes equilibrium constants

\* Le Chatelier's principle →

When a reaction is extensively moving in a specific direction, the reaction tries to lower the change and minimise it as much as possible.

## • Ionic Equilibrium:-

### \* Electrolytes →

- (a) Strong electrolyte → 100% dissociation (OR)  $\alpha = 1$   
 (b) Weak electrolyte → 0% < diss. < 100%. (OR)  $\alpha < 1$ .

If  $\alpha < 0.08$ , then ' $\alpha$ ' is negligible.

### \* Factors affecting degree of dissociation ( $\alpha$ ) →

- (a) Nature of solute and solvent.  
 (b) Temperature  
 (c) Concentration.

### \* Ostwald's law of dilution →

$$K(1-\alpha) = C\alpha^2$$

$$K = \frac{C\alpha^2}{1-\alpha}$$

When  $\alpha < 0.08$ ,

$$K = C\alpha^2$$

### \* Common-ion effect →

When a common ion is present in dissociation of two different compounds, the concentrations get added.

~~Strong electrolytes →  $C_1\alpha + C_2\alpha$  and  $C_1\alpha + C_2$~~

~~Weak electrolytes →  $x + y$~~

### \* Ionic product of water →

$$K_w = [H^+][OH^-]$$

$K_w = 10^{-14}$  at  $25^\circ C$ .

### \* Calculation of pH →

- (a) Strong acid → Use formula given by dilution law.

$$pK_a = -\log K_a$$

- (b) Similar is the case for other acids & bases.

~~weak acids →  $pK_a = -\log K_a$~~  {  $[H^+]$  value, not

~~weak bases →  $pK_b = -\log K_b$~~  {  $pH$  value

$$pH = \frac{1}{2} (pK_a - \log c)$$

$$pOH = \frac{1}{2} (pK_b - \log c)$$

\* Salt hydrolysis →

(a) SA and SB → No hydrolysis.

(b) SA and WB → ~~pH = 7~~  $pH = 7 - \frac{1}{2} (pK_b + \log c)$

(c) WA and SB →  $pH = 7 + \frac{1}{2} (pK_a + \log c)$

(d) WA and WB →  $pH = \frac{pK_1 + pK_2}{2}$  (OR)

$$pH = 7 + \frac{1}{2} (pK_a - pK_b)$$

\* Buffer solutions →

(a) Simple buffers (salts of WA and WB)

(b) Mixed buffers:

(i) Acidic buffer: WA with conjugate base (CB)

(ii) Basic buffer: WB with conjugate acid (CA).

(i) →  $pH = pK_a + \log \frac{[\text{salt}]}{[\text{acid}]}$

(ii) →  $pOH = pK_b + \log \frac{[\text{salt}]}{[\text{base}]}$

\* Solubility →

$L \cdot E > H \cdot E \rightarrow$  Insoluble salts

$L \cdot E < H \cdot E \rightarrow$  Soluble salts

$L \cdot E = H \cdot E \rightarrow$  Slightly soluble salts.

$$K_{sp} = x^y s^{z+y}$$

x, y → moles of products.



$$S \rightleftharpoons K_{sp} = 1S)(2S)^2 = 4S^3$$

We also use common ion principle on solubility  
 → Ionic Product ( $Q$ ) →

Same as reaction quotient concept. Solubility at any moment except equilibrium.

$Q < K_{sp}$  → unsaturated, no precipitation

$Q = K_{sp}$  → saturated, precipitation just began.

$Q > K_{sp}$  → super saturated, precipitation occurs.

→ Acid-Base indicator theory →

(a) Phenolphthalein (HPh):

Acidic indicator

$$pH = pK_a + \log \frac{[Ph^-]}{[HPh]}$$

(b) Methyl orange (MeOH):

Basic indicator

$$pOH = pK_b + \log \frac{[Me^+]}{[MeOH]}$$

• Chemical kinetics:-

→ Rate of Reaction (R or R) →

$$\text{Rate of disintegration of reactants} = -\frac{1}{n} \frac{\Delta [n]}{\Delta t}$$

$$\text{Rate of formation of products} = \frac{1}{m} \frac{\Delta [m]}{\Delta t}$$

$$\frac{-1}{n} \frac{\Delta [n]}{\Delta t} = +\frac{1}{m} \frac{\Delta [m]}{\Delta t}$$

$$R_oR = \frac{\text{Change in pressure}}{\text{time}}$$

→ Rate law →

It is based on concentration of its reactants.

$$R_o R = k [A]^x [B]^y$$

\* Order of a reaction →

$x, y \in a, b \rightarrow$  Elementary reactions

$x, y \notin a, b \rightarrow$  Complex reactions

$a, b$  are s.c.s of A and B respectively.

$x+y = n \rightarrow$  order.

Purely experimental value

In complex reactions, order can be determined from r.d.s.

\* Molecularity →

Purely theoretical value

$n = a, y = b$ .

\* Pseudo unimolecular reactions →

$n = 1$  but  $m \neq 2$

\* Integrated rate law →

(a) zero order : 
$$K = \frac{A_0 - A_t}{t}$$

(b) First order : 
$$A_t = A_0 e^{-kt}$$

→ & half-life ( $t_{1/2}$ ) →

(c) zero order : 
$$t_{1/2} = \frac{A_0}{2k}$$

(d) First orders 
$$t_{1/2} = \frac{0.693}{k}$$

\* Collision theory for reaction rates →

(a) Effective collisions are the one that give products.

(b) No. of effective collisions is very less as compared to total no. of collisions.

(c) Minimum amt. of energy required to start a reaction is called activation energy ( $E_a$ ).

$$E_{\text{threshold}} = E_{\text{normal}} + E_a$$

(d) ROR & collision & Fraction of  $\alpha$  orientation frequency eff. collision factor

$\rightarrow$  Arrhenius' equation  $\rightarrow$

$$k = A e^{-E_a/RT}$$

$A \rightarrow$  Frequency Factor.

$$\log k = \log A - \frac{E_a}{2.303 RT}$$

- Nuclear chemistry:-

$\rightarrow$  Radioactivity  $\rightarrow$

Conversion of radioactive nuclide to stable non-radioactive element.

(a)  $\alpha$ -particles ( ${}^4_2\text{He}$ )

(b)  $\beta$ -particles (high speed  $e^- \rightarrow {}^0_1e$ )

(c)  $\gamma$ -particles (EM radiation)

$\rightarrow$  Rate of disintegration (R<sub>d</sub>D)  $\rightarrow$

Total no. of atoms disintegrating per unit time.

R<sub>d</sub>D & no. of atoms

Independent of temperature, pressure and nature of radioactive substance

$\rightarrow$  Average (T<sub>avg</sub>) =  $\frac{\sum \text{lifetime of all radioactive atoms}}{\text{Total no. of atoms}}$

$\rightarrow$  Activity  $\rightarrow$

No. of atoms disintegrating from 1g Radium (Ra).

$$1\text{g Ra} \rightarrow \frac{1}{226} \text{ moles 'Ra'} \rightarrow \frac{N_A}{226} \text{ molecules 'Ra'}$$

$$\text{Activity (A)} = 3.7 \times 10^{10} \text{ dps} = 1 \text{ curie}$$

$\rightarrow$  C-14 carbon dating technique  $\rightarrow$

$$\frac{\text{C-14}}{\text{C-12}} = \text{constant}$$

→ Mass Defect ( $\Delta m$ ) =  $M_{\text{He}} - M_{\text{H}_2\text{e}}$

→ Binding (BE) =  $9.345 \times 10^8 \text{ eV}$  (for  $p^+$ )  
energy

### • Electrochemistry:-

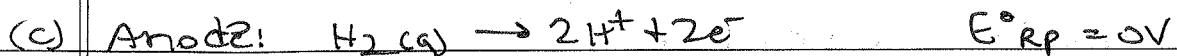
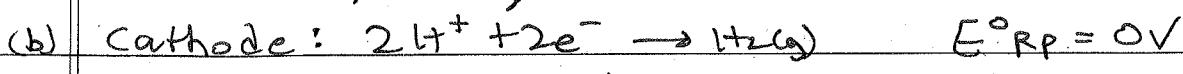
→ Galvanic cell →

Spontaneous reaction,  $E_{\text{Chemical}} \rightarrow E_{\text{Electrical}}$

- (a) Cell notation: [Anode | salt || cathode | salt | cathode]
- (b)  $E_{\text{RP}} | E_{\text{MF}} = E_{\text{RP}}(\text{anode}) - E_{\text{RP}}(\text{cathode})$
- (c)  $E_{\text{RP}} = -E_{\text{RP}}$  (According to IUPAC)

→ (at) Standard hydrogen electrode (SHE) →

- (a) Hydrogen is used for reference at either cathode or anode, along with Pt.



Cathode → Reduction

Anode → Oxidation.

(d)  $E_{\text{MF}} = E_{\text{RP}}(\text{cathode}) - E_{\text{RP}}(\text{anode})$

→ Electrochemical series →

Arrangement of various electrodes according to the reduction potentials.

(a) Most (+)ve  $E_{\text{RP}} \rightarrow$  Strongest O.A / Weakest t.RA

(b) Most (-)ve  $E_{\text{RP}} \rightarrow$  Weakest O.A / Strongest t.RA

→ Nernst equation →

(a)  $\Delta G^\circ = -RT \ln k = -nFE^\circ$

(b)  $\Delta G = \Delta G^\circ + RT \ln k$

1 Faraday = 96,500 C

(c)  $\Delta G = E^\circ - 0.059 \log_{10} \frac{\text{Products}}{\text{Reactants}}$

(d)  $E_{\text{RP}}^\circ = -0.059 \times p^t$

(c)  $\log_{10} K = \frac{0.059}{0.059} n \times E^\circ$

\* Concentration cells →

Non-spontaneous reaction, Electrical → Chemical

(a) Electrolytic cell →

Both electrodes are made of same metal/non-metal

(b) Electrode concentration cell →

Same as electrolytic cell, additional property being that electrolytic solution is same for both.

\* Calomel electrode →

Secondary reference electrode used only for concentration cells.

$$E_{op} = E^\circ_{op} - \frac{0.059}{2} \log_{10} \frac{I}{[Cl^-]^2}$$

\* Generalization of decomposition in electrolysis →

(a)  $F^-; Cl^-; Br^-; I^-; RCOO^- > OH^-$

$NO_3^-; SO_4^{2-}; PO_4^{3-} < OH^-$

(b)  $Al^{3+}; Li^+; Ba^{2+}; Zn^{2+}; Na^+; Sr^{2+}; Ca^{2+}; H^+; Ni^{2+}; Mg^{2+}; Cu^{2+}; Fe^{2+}$

$Cu^{2+}; Au^{3+}; Hg^{2+}; Ag^+ \rightarrow H^+$

\* Faraday's laws of electrolysis →

(a) First Law →

$$\text{W}_{\text{eq}} = e I t$$

$$e = \frac{E}{96500}$$

$E \rightarrow$  Equivalent weight.

(b) Second law →

$$\frac{w_1}{w_2} = \frac{E_1}{E_2}$$

$w$  &  $f$  have same meaning as first law.

\* Resistance ( $R$ ) and conductance ( $G$ )  $\leftrightarrow$

$$R = \frac{\rho L}{A}, \quad G = \frac{1}{R}$$

$$G = \frac{K A}{L}$$

$K \rightarrow$  conductivity

$$\Lambda_{eq} = \frac{1000 \times K}{N}$$

$N \rightarrow$  Normality

$\Lambda_{eq} \rightarrow$  Molar conductivity

$$\Lambda_m = \frac{\text{no. of equivalents of solute}}{\text{Volume of solution (L)}}$$

$$\Lambda_m = \frac{1000 K}{M}$$

$$\Lambda_m = \Lambda_{eq} \times n_f$$

\* Kohlrausch's law  $\rightarrow$

$$\Lambda_{eq} = -A S C + \Lambda^{\circ}_{eq}$$

$\Lambda^{\circ}_{eq} \rightarrow$  Molar conductivity at zero concentration  
or infinite dilution

$$\Lambda_{eq} = \Lambda^{\circ}(+) + \Lambda^{\circ}(-)$$

$\Lambda^{\circ} \rightarrow$  Ionic conductance

### \* Solutions & colligative properties!:-

\* Solution  $\rightarrow$

Homogeneous mixture of 2 or more components.

Small quantity  $\rightarrow$  Solute

Large quantity  $\rightarrow$  Solvents.

\* Colligative properties  $\rightarrow$

Property which depends on number of solutes particles

(i) Lowering in vapour pressure (VP):

(ii) Raoult's law:

$$\frac{P_o - P_s}{P_o} = \frac{x_{\text{solute}}}{n_{\text{solute}} + n_{\text{solvent}}}$$

Rate of evaporation area  $\propto$  Surface area  $\propto$  Temperature  $\propto$  Volatility  $\propto$  flow of air of liquid column  
 Vapour Pressure  $\propto$  Temperature  $\propto$  Volatility  
 Pressure of liquid

(ii) Experimental determination of MW by Ostwald-Walker method  $\rightarrow$   
 (MW  $\rightarrow$  Molecular weight)

Loss in mass of  $\Delta P_s$  solution container

Loss in mass of  $\Delta P_o$  solvent container

Gain in mass of  $\Delta P_o$  solute container

Loss in mass of solvent container  $= \frac{P_o - P_s}{P_o} = \frac{a/m}{b/M}$

(b) Elevation in boiling point ( $\Delta T_b$ )  $\rightarrow$

$$[T_s - T_o] = \Delta T_b = K_b \times \text{molality}$$

$K_b \rightarrow$  Ebullioscopic constant.

(c) Depression in freezing point  $\rightarrow$

$$[T_o - T_f] = \Delta T_f = K_f \times \text{molality}$$

$-K_f \rightarrow$  Cryoscopic constant.

$$K_b = \frac{R T_o^2}{1000 \times L_v}$$

$$K_f = \frac{R \times T_o^2}{1000 \times L_f}$$

(d) Osmotic pressure ( $\pi$ )  $\rightarrow$

$\pi \propto$  no. of solute particles

$$\pi_1 < \pi_2,$$

$\pi_1 \rightarrow$  Hypotonic solution

$\pi_2 \rightarrow$  Hyper tonic solution

$\pi_1 = \pi_2 \rightarrow$  Isotonic solution.

$$\Pi = h \rho g$$

$h \rightarrow$  Rise in height of liquid column

$\rho \rightarrow$  Density of solution.

\* Van't Hoff factor  $\rightarrow$  ( $i$ )  $\rightarrow$

Any colligative & no. of solute  
property particles

$i =$  Experimental C.P

Theoretical C.P

Abnormal C.P & No. of solute particles

after dissociation/Association

(a) Van't Hoff - Bouyols law  $\rightarrow$

$$\Pi \propto c, \Pi \propto 1/V$$

(b) Van't Hoff - Charles' law  $\rightarrow$

$$\Pi = cst$$

\* Dissociation and association  $\rightarrow$

Solute =  $1 + (n-1)\alpha$  | (After dissociation)  
Particles

No. of solute particles  $\propto 1 + \left( \frac{1}{n} - 1 \right) \alpha$   
after association

$$\Pi = cst \times i$$

\* Factors affecting dissolution of gases in liquid  $\rightarrow$

(a) Temperature:  $T \propto P_{\text{gas}}$ ,  $m = k P_{\text{gas}}$

$P_{\text{gas}} \rightarrow$  Partial pressure.

$k \rightarrow$  Henry's constant (g/atm)

(b) Solubility:  $P_{\text{gas}} \propto \text{Solubility}$

$$S = k_H P_{\text{gas}}$$

$k_H \rightarrow$  Henry's constant (mol L<sup>-1</sup> atm<sup>-1</sup>)

(c)  $P_{\text{gas}} \propto X_{\text{gas}}$ ,  $P_{\text{gas}} = k_H X_{\text{gas}}$

$\$ k_H \rightarrow$  Henry constant (atm).

- Solid State -

→ Types of solids →

(a) Crystalline: Definite shape, Fixed M.P and B.P

(b) Amorphous: Indefinite shape, Range of MP.

→ Types of crystalline solids →

(a) Ionic → Electrostatic force of attraction

(b) Covalent → Covalent bonding

(c) Molecular → Van Der Waal's forces.

(d) Metallic → Metal ions / metallic bonds.

→ Crystal lattice →

Definite arrangement of atoms in a solid

Particles can be arranged in 15 different types of lattices (Bravais lattices).

→ Cubic unit cells →

(a) Simple cubic (OR) primitive cubic:

$$\text{No. of} = 8 \Rightarrow \text{Contribution} = \frac{1}{8}$$

positions of each atom

$$\text{Packing (pf)} = \frac{\text{Volume occupied by atoms}}{\text{Total volume}} = 0.52 = 52\%$$

fraction

(b) Body centred cubic (Bcc):

$$\text{No. of} = 1 \Rightarrow \text{Contribution} = \frac{1}{8}$$

positions of each atom

$$\text{P.F.} = 0.68 = 68\% = 32\% \text{ void}$$

space

(c) Face-centred cubic (Fcc):

$$\text{No. of} = 6 \Rightarrow \text{Contribution} = \frac{1}{2}$$

positions of each atom

$$\text{P.F.} = 0.74 = 74\% = 26\% \text{ void}$$

space

$$\text{Density of } (g) = \frac{Z \times M}{N_A \times a^3}$$

unit cell

$Z \rightarrow$  Number of atoms.

$M/N_A \rightarrow$  mass of 1 g atoms.

$a \rightarrow$  cube edge length

\* Voids  $\rightarrow$

$$\begin{array}{c} \cancel{\text{Octahedral}} \times 2 = \text{Tetrahedral} \\ \cancel{\text{voids}} \qquad \qquad \qquad \text{voids} \end{array}$$

$$\text{Bcc} \rightarrow 1 \underset{\text{void}}{\text{octahedral}} (\text{centre}) \rightarrow 2 \underset{\text{voids}}{\text{tetrahedral}}$$

$$\text{Fcc} \rightarrow 3 \underset{\text{voids}}{\text{octa}} \rightarrow 6 \underset{\text{voids}}{\text{tetra}}$$

\* Limiting radius ratio (LRR)  $\rightarrow$

$$LRR = \frac{r_{ct.}}{r_A^-}$$

$c^+ \rightarrow$  cation

$A^- \rightarrow$  Anion

$LRR < 0.155 \rightarrow$  linear void  $\rightarrow$  Co-ordination no. = 2

$LRR (0.155 - 0.224) \rightarrow$  triangular  $\rightarrow$  Co-ordination no. = 3

$LRR (0.224 - 0.414) \rightarrow$  tetrahedral  $\rightarrow$  Co-ordination no. = 4

$LRR (0.414 - 0.732) \rightarrow$  octahedral  $\rightarrow$  6

$LRR (0.732 - 0.999) \rightarrow$  cubic  $\rightarrow$  8.

\* Hexagonal cubic lattice (hcp)  $\rightarrow$

(a) Corner  $\rightarrow$  contribution =  $\frac{1}{8} \Rightarrow$  No. of atoms = 3

(b) Face-centred  $\rightarrow$  contribution =  $\frac{1}{2} \Rightarrow$  No. of atoms = 1

(c) Body-centred  $\rightarrow$  contribution = 1  $\Rightarrow$  No. of atoms = 1

→ Crystal defects →

(a) Ions are missing which leads to increase in density.  
(~~st~~ Schottkey defect)

(b) Frenkel defect:

Cations are missing, when they are much smaller than the anions. Can be seen in crystals having high co-ordination numbers.

(c) Metal excess defects:

Excess of metal gets accumulated due to its vapour being passed.

- Surface Chemistry:-

→ Adsorption →

Absorption but on surface level.

(a) Absorbent: Where adsorption takes place.

(b) Absorbate: Substance that creates adsorption.  
Process is spontaneous.  $\therefore \Delta G = (-)ve$ .

(c) Desorption: Process of escaping of molecules.

(d) Osculation: Accumulation of gas particle at surface of metals.

→ Factors affecting adsorption →

(a) Nature of gas.

(b) Surface area of adsorption.

(c) Extent of pressure.

→ Types of adsorption →

(a) Physisorption:

Weak inter-molecular forces, less energy, not specific.

(b) Chemisorption:

Strong intermolecular forces, more energy, specific, strong forces.

→ Colloidal state →

Substances b/w true solutions and suspensions

Size:  $0.1 - 100 \text{ \AA}$

(a) Dispersed phase (dp), solute

(b) Dispersion medium : solvent.

Colloidal solutions are also called as sols.

→ Classification of colloidal sols →

(a) Lysophilic: More affinity b/w dp & dm, stable, reversible colloid, more SA and high mol. wt.

(b) Lysophobic: Less affinity b/w dp & dm, unstable, irreversible colloid, less SA and low mol. wt.

→ Properties of colloidal sols →

(a) Electrical:

Process of migration of charged colloidal particles towards one of electrodes when PD is applied across electrodes is called electrophoresis.

Movement of particles towards other electrode is called as electro-osmosis.

Coagulation d /

value

coagulation power

Protective power d /

of colloid

gold number

(b) Gold number:

~~Any~~ ~~mass~~ m-grams of lysophilic sol which must be added to 100 ml gold sol in order to prevent precipitation by addn of 1ml of 10% NaCl is called gold number (lysophilic).

(c) Optical properties:

Fryndall → Illumination of path of light radiation by effect scattering of light by colloidal particles

(d) Kinetic properties:

Brownian  $\rightarrow$  zig-zag motion of colloidal particle due to movement continuous collisions w/ dm particles

$\rightarrow$  Emulsions  $\rightarrow$

liquid in liquid colloidal sols.

Eg: Milk, vanishing cream, butter, cod liver oil.

Emulsions are generally unstable due to immiscible nature of dp and dm.

$\rightarrow$  Cleansing action of soaps  $\rightarrow$

Soaps are sodium/potassium salts of higher fatty acids.

Critical micelle  $\alpha$  Temperature

temperature

CMC  $\alpha$

length of carbon chain

kraft temp. is temp. above which micelle formation takes place.

$\rightarrow$  Gels  $\rightarrow$

Liquids  $\rightarrow$  dp and solids  $\rightarrow$  dm.

Gels are unstable. Can be stabilized by add<sup>n</sup> of reagents called gelatines.

- Periodic Properties:-

$\rightarrow$  Theories regarding classification of elements  $\rightarrow$

(a) Lavoisier's classification:

He classified into metals & non-metals which soon faded away after discovery of metalloids.

(b) Dobriner's triad triads:

He grouped elements into three elements each group, with similar properties. Average mass number of second element equalled mean of remaining two elements.

(d) Newlands' law of octaves:

He arranged elements in increasing order of their atomic weight and predicted that properties of 4<sup>th</sup> and 8<sup>th</sup> are same.

(d) mendeleev's periodic table:

be arranged elements in increasing order of their molecular weights. However, there were many drawbacks in this arrangement.

(e) Modern periodic table →

It solved many of the problems Mendeleev's periodic table possessed. Elements were arranged based on increasing atomic number (no. of  $p^+$ ).

→ Different block elements →

(a) S-block : Configuration  $ns^1$  or  $ns^2$ .

(b) p-block: configuration  $n\text{p}'$  to  $n\text{p}^6$ .

(c) d-block : configuration  $(n-1)d^1$  to  $(n-1)d^{10}$ .

(d) f-block: configuration  $(n-1)f' \rightarrow (n-1)f''$ .

→ Effective nuclear charge →

Shielding of outer  $e^-$  by inner  $e^-$  from nuclear attraction.

Effective ( $z^*$ ) = Atomic ( $z$ ) - Shielding of ( $\sigma$ )

→ Calculation of shielding effect →

$n_s, n_p \rightarrow 0.35$  (upto) each

Trend:

~~shells~~  $(n-1)$  shells  $\rightarrow 0.85$  each

(n-2) shells  $\rightarrow$  1 each.

↓ Group  $z^+$  almost  
the same

## Atomic radius

$$\text{Covalent radius} = \frac{d_A - A}{2}$$

(b) Heteroatomic:

$$(i) \quad X_A = X_B, \text{ then } \left\{ \begin{array}{l} r_B - r_A = \frac{d_{A-B}}{2} \end{array} \right.$$

(i)  $X_A > X_B$ , then  $d_{A-B} = r_A + r_B - 0.9(X_A - X_B)$

$X \rightarrow$  Electronegativity

(b) Metallic radius =  $\frac{D}{2}$

$\rightarrow$  Distance b/w 2 metal atoms that touch each other.

(c) Van-Der Waal Radius =  $\frac{d_{A-B}}{2}$

$\rightarrow d_{A-B} \rightarrow$  Dist. b/w atoms of 2 different molecules

order of radius - Van der waal > Metallic > Covalent

(d) Ionic radius:  $r_A^+ < r_A < r_A^-$

$Z^* \uparrow$ , (Atomic radius)  $\downarrow$

(No. of bonds)  $\uparrow$ , (Atomic radius)  $\downarrow$

\* Lanthanoid contraction  $\rightarrow$

Effect of decreases small decreasing in atomic radius due to poor shield of  $f$  sub-shell.

\* Ionisation energy  $\rightarrow$

Energy required to remove  $e^-$  from outermost shell.

$I_E^3 > I_E^2 > I_E^1$

Atomic size  $\uparrow$ ,  $I_E \downarrow$

$Z_{eff} \uparrow$ ,  $I_E \uparrow$

\* Penetration power -  $s > p > d > f$

Half-filled and completely filled orbitals are exceptionally stable.

$e^-$  affinity  $\rightarrow$

Tendency to accept  $e^-$  in valence shell.

$\Delta H_{e\text{-gain}} = - E^- \text{ Affinity}$

$E_{A1} + E_{A2} < 0$

Atomic size  $\uparrow$ ,  $E_A \downarrow$

L  $\rightarrow$  R EAT, Group EA  $\downarrow$

\* Electronegativity →

Tendency to attract e<sup>-</sup> pair (shared) in a covalent bond.

Atomic size ↑, E.N ↓

$Z_{\text{eff}}' \uparrow$ , E.N ↑

Charge on anion ↑ E.N ↓

$$\boxed{\text{E.N} \rightarrow \text{sp} > \text{sp}^2 > \text{sp}^3}$$

L → R E.N ↑, 1 group E.N ↓

\* Scales of Electronegativity →

(a) Pauling scale:  $\boxed{\chi_A - \chi_B = 0.208 \sqrt{E_{AB} - \sqrt{E_{AA} \times E_{BB}}}}$

$\chi \rightarrow$  Electronegativity.

(b) Mulliken scale:  $\boxed{\text{E.N} = \frac{I + E.A}{2}}$

$$\boxed{\chi_p = \frac{x_m}{2.8}}$$

$\chi_p \rightarrow$  Pauling E.N

$x_m \rightarrow$  Mulliken E.N

(c) Alfred Rohr scale:  $\boxed{\chi = \frac{Z_{\text{eff}} \cdot e^2}{r^2}}$

$\boxed{\chi_A - \chi_B > 1.7 \rightarrow \text{Ionic bond}}$

$\boxed{\chi_A - \chi_B < 1.7 \rightarrow \text{Covalent bond.}}$

R → F<sup>-</sup> = 4, O<sup>2-</sup> = 3.5, N<sup>3-</sup> = 3, C = 2.5, P = 2.1, S = 2.8

Cl<sup>-</sup> = 3, Br = 2.8, I = 2.5

\* Chemical Bonding:-

\* Lattice energy →

Energy required to break 1 mole of crystal lattice.

It is formed energy released when 1 mole of crystal lattice is formed from constituent gas ions.

$$\boxed{\Delta H_F = \Delta H_{\text{sub.E}} + \Delta H_{\text{IE}} + \Delta H_{\text{Ois.E}} + \Delta H_{\text{regain}} + LE}$$

\* Properties of ionic bonds →

(a) Forms crystal lattice.

(b) High BP and M.P.

(c) Very high solubility in water.

(d) Brittle

→ \* Covalent bonds →

Formed by sharing pair of e<sup>-</sup>.

(a) E.N difference is large → Polar bonds

(b) E.N difference is similar → Non-polar bonds.

→ \* Calculation of formal charge (F.C) →

$$F.C = \text{Valence } e^- - \left( \frac{\text{No. of lone pairs}}{\text{on that atom}} \right) - \frac{1}{2} \times \left( \frac{\text{No. of bonding electrons}}{2} \right)$$

→ \* Properties of covalent bonds →

(a) Low MP and BP (exception of 3-D structure).

(b) Electrical conductivity: Some conduct in aqueous form.

(c) More soluble in non-polar solvents.

→ \* Hybridization →

Mixing of orbitals of almost same energy.

(a) Neutral compounds:  $x = \frac{1}{2} [V + G]$

$x \rightarrow$  Hybridization

$V \rightarrow$  Valence e<sup>-</sup> present on central atom.

$G \rightarrow$  No. of monovalent side atoms.

~~E.G~~ Add ' $\rightarrow C$ ' or ' $+A$ ' for cationic and anionic compounds respectively.

→ \* Types of covalent bonds →

(a) Head-on overlap: sigma ( $\sigma$ ) bonds

Between s-orbitals

(b) Side-ways overlap: pi ( $\pi$ ) bonds

s and p-orbitals or b/w p-orbitals.

→ \* Geometry and shape of covalent bonds →

(a)  $sp^2$  → trigonal planar

(b)  $sp^3d$  → Trigonal bi-pyramidal

(c)  $sp^3 d^2 \rightarrow$  Octahedral, 1LP  $\rightarrow$  Sq. pyramidal, 2LP  $\rightarrow$  Sq. planar.

(d)  $sp^3 \rightarrow$  Tetrahedral

\* Valence shell e<sup>-</sup> pair repulsion theory  $\rightarrow$

Lone-pair - lone pair > LP - Bond pair > B-P-BP

Shape is determined by repulsion present b/w all e<sup>-</sup> present in valence shell

L.P. takes <sup>space</sup> more than BP.

Because of angle repulsions, BA decreases

BA  $\rightarrow$  Bond angle.

BA ↑, No. of lone pairs ↓

BA ↑, E.N. of side atom ↓

\* Molecular orbital theory  $\rightarrow$

Atoms have atomic orbitals, molecules have molecular orbitals.

$\sigma, \pi \rightarrow$  Bonding molecular orbitals

$\sigma^*, \pi^* \rightarrow$  Anti-bonding molecular orbitals.

Order of :  $\sigma$  is  $\sigma^*$  is  $\sigma_{2s}$   $\sigma^*_{2s}$  [  $2\pi_{2p_z} = \pi_{2p_y}$  ]  $\sigma_{2p_z}$  [  $2\pi_{2p_z} = \pi_{2p_y}$  ]  $\sigma^*_{2p_z}$

Filling e<sup>-</sup>

Bond = e<sup>-</sup> in BMO - e<sup>-</sup> in ABMO

order 2

B.O  $\propto$  Bond strength  $\propto$  Bond energy  $\propto$  stability

B.O  $\propto$  1

Bond length

\* Dipole moment  $\rightarrow$

Product of magnitude of charge and dist b/w atoms

$$|\vec{\mu}| = q \times d$$

(a) Homo-atomic,  $M_{net} = 0$

(b) Hetero-atomic,  $M_{net} \neq 0$

% ionic =  $\frac{\text{Experimental } 'u' \times 100}{\text{Theoretical } 'u'}$

character

- Hydrogen and its compounds:-

- \* Preparation of  $H_2 \rightarrow$

(a) Reaction with mineral acids and bases.

(b) Bosch's process  $\rightarrow \text{C}_O + H_2 \rightarrow H_2 + CO_2$   
Water gas

(c) Lane's process  $\rightarrow Fe + H_2O \rightarrow Fe_3O_4 + H_2$

(d) Electrolysis of water  $\rightarrow$

(i) Acidic water: Cathode -  $2H^+ + 2e^- \rightarrow H_2$

(ii) Basic water:  $2H_2O + 2e^- \rightarrow 2H_2 + 2OH^-$  (Cathode)  
 $4OH^- \rightarrow O_2 + 2H_2O + 4e^-$  (Anode)

- \* Properties of  $H_2 \rightarrow$

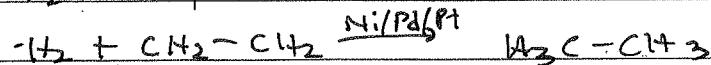
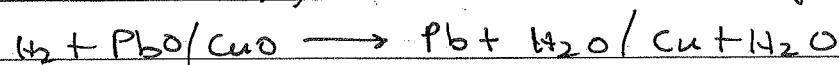
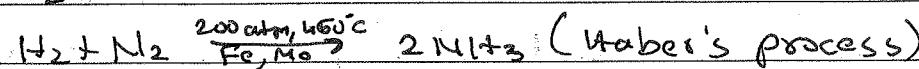
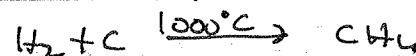
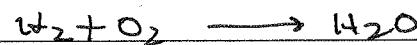
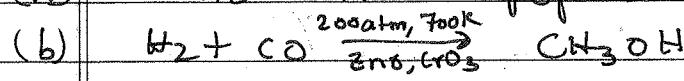
(a) Physical properties:

(i) Colourless, odourless, tasteless

(ii) Slightly soluble in  $H_2O$ .

(iii)  $\rho_{H_2} = 0.09 \text{ g/cm}^3$ .

(iv) Burns with a pop sound.



- \* Isotopes of hydrogen  $\rightarrow$

Protium ( ${}^1H$ ), Deuterium ( ${}^2H$ ), Tritium ( ${}^3H$ )

Existence  $\rightarrow H_2 : D_2 = (6000 : 1)$

$| H_2 < D_2 \text{ (Bond strength)} | (H_2 > D_2 \text{ (Reactivity)}) |$

$[H] \ggg H$

$[H] \rightarrow$  Nascent hydrogen.

$H \rightarrow$  atomic ' $H$ '

\* Water ( $H_2O$ ) →

(a) Physical properties:

- (i) Tasteless, colourless, odourless.
- (ii) Neutral in nature.
- (iii) Dielectric constant = 79.

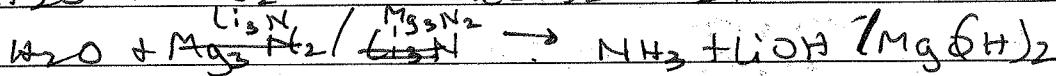
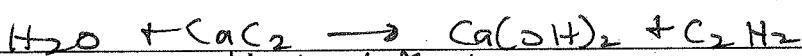
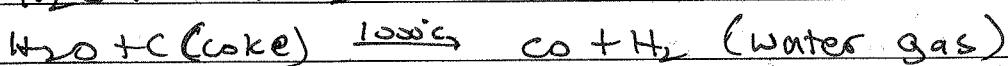
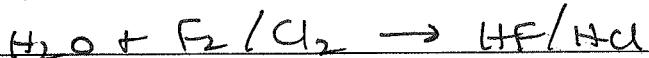
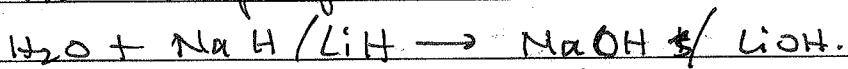
Dielectric constant is measure of solubility in a particular solvent.

$$\delta H_{\text{solvation/hydration}} = \frac{k z_+ z_- e^2}{2r} \left( 1 + \frac{1}{k} \right)$$

$z_+ / z_- \rightarrow$  Charge on cation/anion

$k \rightarrow$  Dielectric constant

(b) Chemical properties:



\* Hardness of water →

Water does not form lather, or is unfit for drinking.

(a) Temporary hardness: Bicarbonate salts of  $Ca^{2+}$  &  $Mg^{2+}$ .

(b) Permanent hardness:  $Cl^-$ ,  $SO_4^{2-}$  salts of  $Ca^{2+}$  &  $Mg^{2+}$ .

We can remove permanent hardness by add<sup>n</sup> of:

(i)  $Na_2CO_3$  (Soda Ash)

(ii) Calgon ( $Na_3(Po_3)_6$ )

(iii) Perlite (zeolite -  $Na_2Al_2Si_2O_8 \cdot 2H_2O$ )

\* S-block :-

→ Group - I (Alkali metals) →

(a) Physical properties:

$e^-$  configuration  $\rightarrow ns^1$

Size  $\rightarrow Li < Na < K < Rb < Cs$

Density  $\rightarrow Li < K < Na < Rb < Cs$

Oxidation state  $\rightarrow +1$

Reduction (gas phase)  $\rightarrow Li < Na < K < Rb < Cs$

Reduction (aqueous phase)  $\rightarrow Li > Na > K > Rb > Cs$

I.E  $\rightarrow Li > Na > K > Rb > Cs$

(b) Chemical preparation:

(i) Reaction with  $O_2$  (excess):

$[Li \rightarrow \text{oxide}, Na \rightarrow \text{peroxide}, \text{others} \rightarrow \text{superoxide}]$

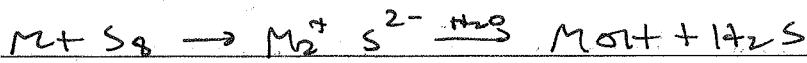
(ii) Reaction with  $N_2$ :

$[Li \rightarrow \text{nitrile } (Li_3N); Na, K, Rb, Cs \rightarrow \text{Azide}]$

(iii) Reaction with  $H_2O$ :

Li reacts slowly, others react vigorously.

(iv) Reaction with  $P_4$  and  $S_8$ :



(v) Reaction w/  $NH_3$ :

$Li \rightarrow$  Amide Nitride;  $Na, K, Rb, Cs \rightarrow$  Amide ( $MNH_2$ )

$\rightarrow$  Compounds of sodium (Na)  $\rightarrow$

(a)  $Na_2O$  (sodium oxide):

(i) Preparation:  $Na + O_2 \xrightarrow{180^\circ C} Na_2O$

(Excess)

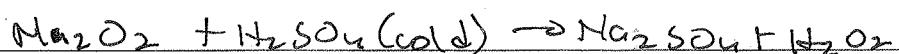


(ii) Properties:  $Na_2O + H_2O \rightarrow NaOH$

(b)  $Na_2O_2$  (sodium peroxide):

(i) Preparation:  $Na + O_2$  (excess)  $\xrightarrow{\Delta} Na_2O_2$

(ii) Properties:  $Na_2O_2 + H_2O$  (warm)  $\rightarrow NaOH + O_2$ .



(c) ~~(e)~~  $\text{NaOH}$  (sodium hydroxide):

(i) Preparation:  $\text{Ca(OH)}_2 + \text{NaCl} \rightarrow \text{NaOH} + \text{CaCl}_2$   
 (caustification) (Yossage reaction)

(ii) Properties:  $\text{NaOH} + \text{Cl}_2 \rightarrow \text{NaCl} + \text{NaOCl}(\text{solid})/\text{NaClO}_3(\text{hot})$   
 $\text{NaOH} + \text{S}_8 \rightarrow \text{Na}_2\text{S} + \text{Na}_2\text{SO}_3$   
 $\text{NaOH} + \text{Si} + \text{H}_2\text{O} \xrightarrow{\text{heat}} \text{Na}_2\text{SiO}_3$

(d)  $\text{Na}_2\text{CO}_3$  (sodium carbonate):

(i) Preparation:  $2\text{NaCl} + \text{H}_2\text{SO}_4 \rightarrow \text{Na}_2\text{SO}_4$   
 $\text{Na}_2\text{SO}_4 + \text{C} + \text{CaCO}_3 \rightarrow \text{CaS} + \text{Na}_2\text{CO}_3 + \text{CO} \uparrow$

All S  $\rightarrow$  Black, CaS  $\rightarrow$  Yellow, ZnS  $\rightarrow$  White, MnS  $\rightarrow$  Buff pink  
 $[\text{NH}_3 + \text{CO}_2 + \text{H}_2\text{O} \rightarrow \text{NH}_4\text{NO}_3]$  (Solvay's process)

Baking = Baking +  $\text{K}^+$  salt of  
 powder            soda            tartaric acid

(ii) Properties:  $\text{Na}_2\text{CO}_3 + \text{Fe}_2(\text{SO}_4)_3 \rightarrow \text{Fe}_2(\text{CO}_3)_3 + \text{Na}_2\text{SO}_4$   
 $\text{Na}_2\text{CO}_3 + \text{CaCl}_2 \rightarrow \text{CaCO}_3 + 2\text{NaCl}$

(e)  $\text{Na}_2\text{SO}_4$  (sodium sulphate)  $\rightarrow$

(i) ~~(e)~~ Preparation:  $2\text{NaCl} + \text{H}_2\text{SO}_4 \xrightarrow{\text{heat}} \text{Na}_2\text{SO}_4 + 2\text{HCl}$   
 (Le-Blanc process)

(ii) Properties:  $\text{Na}_2\text{SO}_4 + \text{C} \rightarrow \text{Na}_2\text{S} + \text{CO}$ .

(f) sodium thiosulphate ( $\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O}$ ):

(i) Preparation:  $\text{Na}_2\text{SO}_3 + \text{S} \xrightarrow{\Delta} \text{Na}_2\text{S}_2\text{O}_3$ .

(ii) Properties:  $\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O} \xrightarrow{215^\circ\text{C}} \text{Na}_2\text{S}_2\text{O}_3 + 5\text{H}_2\text{O}$   
 $\text{Na}_2\text{S}_2\text{O}_3 \xrightarrow{T > 223^\circ\text{C}} \text{Na}_2\text{SO}_4 + \text{Na}_2\text{S}_2\text{S}$

$\rightarrow$  Compounds of potassium(K)S  $\rightarrow$

(a) ~~(e)~~  $\text{K}_2\text{O} \rightarrow$

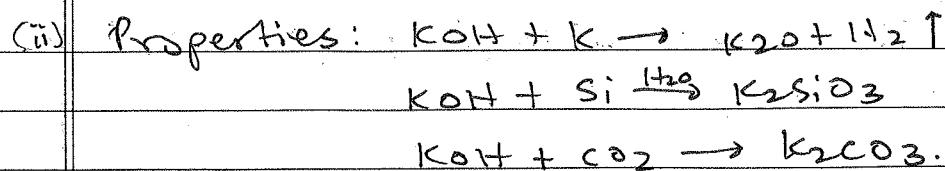
(i) Preparation:  $4\text{K} + \text{O}_2 \xrightarrow{190^\circ\text{C}} \text{K}_2\text{O}$

$\Rightarrow \text{KNO}_3 + \text{K} \rightarrow \text{K}_2\text{O} + \text{KNO}_2$

(ii) Properties:  $\text{K}_2\text{O} + \text{H}_2\text{O} \rightarrow 2\text{KOH}$

(b)  $\text{KOH}$ :

(i) Preparation: Electrolysis of  $\text{KCl}$  and  $\text{H}_2\text{O}$ .



$CO_2$  ( $SO_2$ ) absorbed by  $KOH$ .

$CO_2$  is absorbed by turpentine oil

(c)  $K_2CO_3$ :

(i) Preparation = properties,



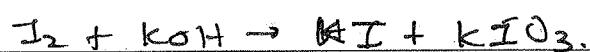
(d)  $K_2SO_4$ :

(i) Preparation:  $2KCl + H_2SO_4 \rightarrow K_2SO_4 + 2HCl$

(ii) Properties:  $K_2SO_4 + HCl \rightarrow K_2S + H_2O$ .

(e)  $KI$ :

(i) Preparation:  $KOH + HI \rightarrow KI + H_2O$ .



(ii) Properties:  $KI + I_2 \rightarrow KI_3$ .



Solubility of  $I_2$  increases in the presence of  $KI$  due to formation of  $KI_3$ .



\* Second group elements  $\rightarrow$

(a) Size:  $Be < Mg < Ca < Sr < Ba$

(b)  $e^-$  configuration:  $ns^2$

(c) Density:  $Be > Mg > Ca > Sr > Ba$

(d) Hydration:  $Be^{2+} > Mg^{2+} > Ca^{2+} > Sr^{2+} > Ba^{2+}$   
energy

(e) Chemical properties:

(i)  $Be \rightarrow$  No reaction;  $Mg, Ca, Sr, Ba \rightarrow MO_2$  (with  $O_2$ )

(ii)  $Be \rightarrow$  No reaction;  $Mg \rightarrow MgO$  (cold)  $Mg(OH)_2$  (hot);  $Ca, Sr, Ba \rightarrow MO_2$  (with  $H_2O$ )

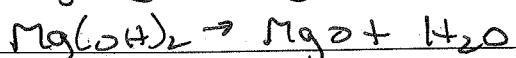
(iii) All form hydrides.

(d)  $\text{Be}_2\text{Be} \rightarrow$  carbide ( $\text{MC}_2$ );  $\text{Mg} \rightarrow$  Alkylide;  $\text{Ca} \rightarrow$  Acetylidyce

$\rightarrow$  Compounds of Mg  $\rightarrow$

(a)  $\text{MgO}$ :

(i) Preparation:  $\text{MgCO}_3 \rightarrow \text{MgO} + \text{CO}_2 \uparrow$

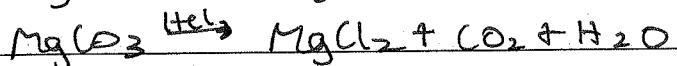


(ii) Properties:  $\text{MgO} \xrightarrow{\text{H}_2\text{O}(\text{cold})} \text{MgO}$   
 $\xrightarrow{\text{H}_2\text{O}(\text{hot})} \text{Mg(OH)}_2$

(b)  $\text{MgCO}_3$ :

(i) Preparation:  $\text{Mg(OH)}_2 + \text{CO}_2 \rightarrow \text{MgCO}_3 + \text{H}_2\text{O}$

(ii) Properties:  $\text{MgCO}_3 \xrightarrow{\Delta} \text{MgO} + \text{CO}_2$

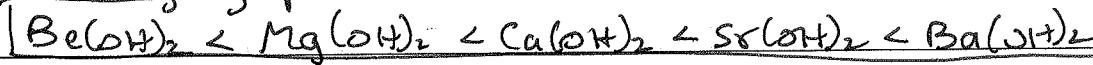


(c)  $\text{Mg(OH)}_2$ :

(i) Preparation:  $\text{MgSO}_4 + \text{NaOH} \rightarrow \text{MgSO}_4 \cdot \text{Mg(OH)}_2 + \text{Na}_2\text{SO}_4$

(ii) Properties:  $\text{MgO} + \text{C} \rightarrow \text{Mg}_2\text{C}_3 + \text{CO} \uparrow$

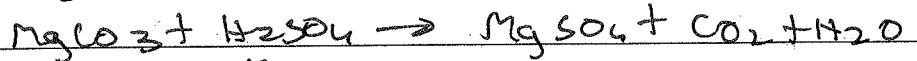
(Transitory process)



(Basicity)

(d)  $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$ :

(i) Preparation:  $\text{Mg(OH)}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{MgSO}_4 + \text{H}_2\text{O}$



(ii) Properties:  $\text{MgSO}_4 \cdot 7\text{H}_2\text{O} \xrightarrow[60^\circ\text{C}]{\text{Heat}} \text{MgSO}_4 \cdot 6\text{H}_2\text{O} \xrightarrow[200^\circ\text{C}]{\text{Heat}} \text{MgSO}_4$

(e)  $\text{MgCl}_2 \cdot 6\text{H}_2\text{O}$ :

(i) Preparation: Fractional crystallization of carnallite  
 $(\text{KCl} \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O})$

(ii) Properties:  $\text{MgCl}_2 + \text{MgO} \rightarrow \text{MgCl}_2 \cdot 5\text{MgO} \cdot 2\text{H}_2\text{O}$

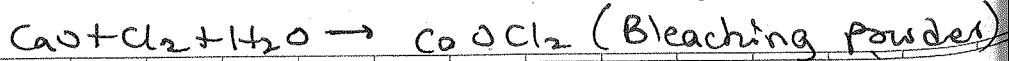
Sorel cement / magnesium cement

$\rightarrow$  Compounds of 'Ca'  $\rightarrow$

(a)  $\text{Calcium oxide} (\text{CaO})$ :

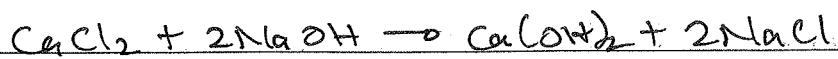
(i) Preparation:  $\text{CaCO}_3 \xrightarrow{T>800^\circ\text{C}} \text{CaO} + \text{CO}_2$

(ii) Properties:  $\text{CaO} + \text{SiO}_2 \rightarrow \text{CaSiO}_3$  (Slag)



(b)  $\text{Ca(OH)}_2$ :

(i) Preparation:  $\text{CaO} + \text{H}_2\text{O} \rightarrow \text{Ca(OH)}_2$



(ii) Properties:  $\text{Ca(OH)}_2 \xrightarrow{\text{CO}_2} \text{CaCO}_3 \downarrow + \text{CO}_2$

(c)  $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ :

(i) Preparation:  $\text{CaCl}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{CaSO}_4 + 2\text{HCl}$

(ii) Properties:  $\text{CaSO}_4 \cdot 2\text{H}_2\text{O} \rightarrow \text{CaSO}_4 \cdot \frac{1}{2}\text{H}_2\text{O} + \text{H}_2\text{O}$  (setting)

(d)  $\text{CaH}_2$ :

(i) Preparation:  $\text{Ca} + \text{H}_2 \rightarrow \text{CaH}_2$

(ii) Properties:  $\text{CaH}_2 \xrightarrow{\text{H}_2\text{O}} \text{Ca(OH)}_2 + \text{H}_2$

→ Thermal stability (T.S) →

T.S is determined by Fajan's rule.

size of cation ↑, ionic character ↑ (Down the group)

ionic character ↑, covalent character ↓

" ↑, T.S ↑

T.S  $\propto$  L.E

L.E  $\propto$   $q_1 q_2 \propto \frac{1}{r_+ + r_-}$

• Co-ordination compounds:

→ Definition →

Compounds wherein shared  $e^-$  pair comes from a single atom. They consist of:

(a) central atom; (b) ligand; (c) Co-ordination sphere.

→ Adduct compounds →

$\text{A}^0 \rightarrow \text{B}^-$  (Sharing of  $e^-$  pair)

→ Properties of co-ordinate compounds →

(a) covalent < co-ordinate < ionic : MP and BP

(b) Can exist in all three states.

(c) Sparingly soluble in  $\text{H}_2\text{O}$  but readily soluble in non-polar

Solvents.

(d) Bad conductors of electricity.

No. of ions & Conductivity

No. of ions & Depression in F.P.

\* Co-ordination Number →

No. of ligands surrounding central atom.

Every metal has fixed co-ordination number.

Order of donors: ( $> N > O > X$ )

\* Ligands and its types →

$e^-$  pair donors. (Lewis bases). No. of sites are represented by denticity.

(a) Monodentate: Only one donor site present. Eg:  $Cl^-$ ,  $Br^-$ .

(b) Bidentate: 2 donor sites present. Eg:  $NH_2CH_2CH_2NH_2$ .

Glycinate ( $H_2C-C(O^-)-NH_2$ ) the  $\alpha$ -amino acid ligand.

(c) Pentadentate: 5 donor sites present.

(d) Hexadentate: 6 donor sites present.

(e) Flexidentate: Upon the need of the central atom, the number of donor sites can vary.

(f) Ambidentate: Exhibit linkage isomerism.

In  $CN^-$ , both 'C' & 'N' are donors.

(g)  $\pi$  e<sup>-</sup> donor:  $\pi$  Te<sup>-</sup> donation ↑, Resonance ↓

\* Classification of ligands →

(a) Neutral ligands:  $H_2O$ ,  $NH_3$ ,  $R-NH_2$ , [Oxid<sup>n</sup> number = 0]

(b) Negatively charged ligands: [Oxid<sup>n</sup> number = -  $\frac{n}{2}$ ]

(c) Positively charged ligands: [Oxid<sup>n</sup> number = +n]

\* Effective atomic number (EAN) (sidwick theory) →

Elements tend to acquire configuration of nearest noble gas.

[ $E.A.N = Z - \text{Oxid}^n \text{ no. of } CA + 2 \times \text{no. of ligands}$ ]

\* Back bonding →

'CO' becomes  $\pi$ -acceptor ligand and is also known as  $\pi$ -acidic ligand.

Bond length in metal carbonyls increase.

Strength  $\uparrow$ , density  $\uparrow$ , bond length  $\downarrow$ .

$\text{CN}^-$ ,  $\text{NO}$  also exhibit back-bonding.

Back-bonding ability:  $\text{CO} > \text{NO}^- > \text{CN}^-$

Back-bonding ability  $\uparrow$ ,  $\sigma$ -bonding ability  $\downarrow$

\* Nomenclature of co-ordination compounds →

(a) metal: name of metal + 'ate' / Latin name + 'ate'

(b) Neutral ligands:

$\text{NH}_3 \rightarrow$  Ammine,  $\text{H}_2\text{O} \rightarrow$  Aqua,  $\text{CO} \rightarrow$  carbonyl,  $\text{en} \rightarrow$  ethylene diamine,  $\text{PPh}_3 \rightarrow$  triphenyl phosphine.

(c) Charged ligands:

ate  $\rightarrow$  ato, ite  $\rightarrow$  ito, ide  $\rightarrow$  ido.

\* Werner theory of complexes →

(a) Every metal atom has fixed co-ordination number.

(b) Complexes have two types of valencies:

(i)  $1^\circ$  valency  $\rightarrow$  Present outside co-ordination sphere.

(ii)  $2^\circ$  valency  $\rightarrow$  Present within co-ordination sphere.

\* Valence Bond theory →

(a) Co-ordination number = 4  $\rightarrow$   $\text{sp}^3$  and  $\text{dsp}^2$ .

(b) Co-ordination number = 6  $\rightarrow$   $\text{d}^2\text{sp}^3\text{d}^2$  and  $\text{d}^2\text{sp}^3$ .

Strength: Carbon > Nitrogen > Oxygen > Halogen

of Ligand	donor	donor	donor	donor
-----------	-------	-------	-------	-------

\* Spectrochemical series →

Arrangement of ligands in order of field strength. When a strong ligand is present, pairing of unpaired  $e^-$  takes place.

Co-ordination number can tell us no. of orbitals present in reaction for central atom.

→ Crystal - field theory →

Complexes in which central atom surround by ligands are considered as point charges.

$d_{x^2-y^2}, d_{z^2} \rightarrow e_g$  orbital (Higher energy)

$d_x, d_y, d_z \rightarrow t_{2g}$  orbital (lower energy)

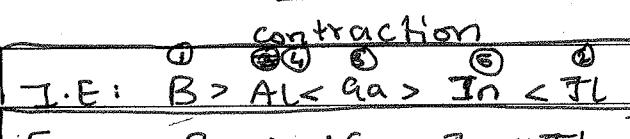
In strong field ligands, a particular orbital is filled first

In weak field ligands, all vacant orbitals are filled.

- P-block (I and II) :-

→ Group-13 (Boron family) →

• Size:  $B < Al \leq Ga < In < Tl$



Stable oxid<sup>n</sup> states:  $B, Al, Ga, In \rightarrow 3; Tl \rightarrow +1$

Chemical properties:

- (a) Reaction with H<sub>2</sub>:

B → Nido-borane ( $B_nH_{n+6}$ ), Archano boranes ( $B_nH_{n+8}$ )

Al → Alanes ( $AlH_3$ )<sub>n</sub>

Ga → Di-gallanes ( $Ga_2H_5$ )

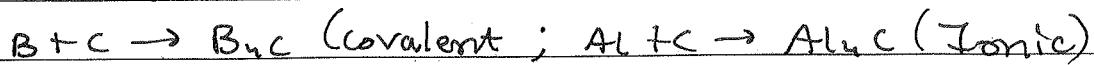
In → Indanes ( $InH_3$ )<sub>n</sub>

Tl → No reaction due to large size.

- (b) Reaction with O<sub>2</sub> and N<sub>2</sub>:



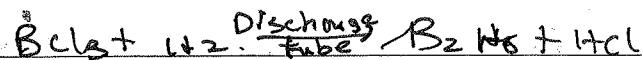
- (c) Reaction with C:



→ Compounds of Boron (B) →

- (a)  $B_2H_6$ :

(b) Preparation:  $BF_3 \cdot OEt_2 + LiAlH_4 \xrightarrow{OEt_2} B_2H_6 + LiAlH_4 + OEt_2$



(ii) Properties:  $\text{B}_2\text{H}_6 + \text{H}_2\text{O} \rightarrow \text{H}_3\text{BO}_3 + \text{H}_2$

Diborane can be used as alternative rocket fuel.

Cal. value = 75 kJ/KJ (gm.)

With  $\text{N}_1\text{H}_3$ ,

$T < 450\text{ K} \rightarrow (\text{B}_2\text{H}_6 \cdot 2\text{N}_1\text{H}_3)$ ;  $T = 450\text{ K} \rightarrow \text{B}_3\text{N}_3\text{H}_6$ ;  $T > 450\text{ K} \rightarrow (\text{BN})$

### Inorganic Benzene

(b)  $\text{H}_3\text{BO}_3$ :

(i) Preparation:  $\text{Na}_2\text{B}_4\text{O}_7 + \text{HCl} + 2\text{H}_2\text{O} \rightarrow \text{H}_3\text{BO}_3 + \text{NaCl}$ .

(ii) Properties:  $\text{H}_3\text{BO}_3 + \text{H}_2\text{O} \rightarrow [\text{B}(\text{OH}_3)_3]^\ominus + \text{H}_3^+$ .

Weak monobasic acid, moderately soluble in  $\text{H}_2\text{O}$

Exists in solid powder form.

(c)  $\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$  (Borax):

(i) Preparation: From colemanite ore ( $\text{Ca}_2\text{B}_6\text{O}_4$ )



(ii) Properties:  $\text{Na}_2\text{B}_4\text{O}_7 \xrightarrow{\Delta} \text{NaBO}_2 + \text{B}_2\text{O}_3$  (Borax Bead)

\* Bridge-bonding →

Also known as banana bonding or 3-centre-2 $e^-$  bonding

2 'B' atoms are used as a bridge for sharing pair of  $e^-$  between 2 'B' atoms.

Eg:  $\text{B}_2\text{H}_6$ ,  $\text{Al}_2\text{H}_6$ ,  $\text{Al}_2(\text{GeH}_6)_6$ .

On the other hand,  $\text{Al}_2\text{Cl}_6$  show 3-centre 4 $e^-$  bonding due to presence of LP on chlorine atom.

\* Nomenclature of oxyacids →

Replaceable  $\text{H}^+$  ion attached to oxygen atom.

'ie' suffix for higher oxid<sup>n</sup> state, 'ous' for lower oxid<sup>n</sup> state

For haloxyacids, 'per' comes above 'ic' and 'hypo' below 'ous'

(a) 'meta' suffix added to that oxyacids which loses  $(-\text{H}_2\text{O})$ .

[1 mole parent acid  $\xrightarrow{-\text{H}_2\text{O}}$  'meta' oxyacid]

Meta boric acid is present in trimeric form.

- (b) 'Ortho' suffix added when 'meta' derivative is possible.  
used on parent oxyacid.
- (c) 'Pyro' suffix added when  $\text{H}_2\text{O}$  is removed from 2 molecules of parent acid.
- (d) 'Per' suffix added when  $\text{O}$  is added to 'ic' acid. It does not give any assurance about the existence of Peroxyl linkage in a compound.
- (e) 'Hypo' suffix added when 'o' is removed from 'ous' acid.  
Salt formation: 'ic'  $\rightarrow$  'ate'; 'ous'  $\rightarrow$  'ite'.

$\rightarrow$  14<sup>th</sup> group (carbon family)  $\rightarrow$

C, Si  $\rightarrow$  Non-metals; Ge  $\rightarrow$  Metalloid; Sn, Pb  $\rightarrow$  Metals

(a) (i) Allotropes of carbon:

(i) Diamond: Bond-length (C-C)  $\approx 1.54 \text{ \AA}^\circ$

3-D structure,  $\text{sp}^3$  hybridised.

Does not conduct electricity

(ii) Graphite: Bond-length (C-C)  $\approx 1.414 \text{ \AA}^\circ$

2-D sheet-like structure,  $\text{sp}^2$  hybridised

Each layer is attracted to each other by weak Van Der Waal's forces

(b)  $\rightarrow$  Properties of carbon  $\rightarrow$

(i) Allotropic forms, burn in air to give  $\text{CO}$  and  $\text{CO}_2$ .

(ii) Reducing nature:  $\text{Fe}_2\text{O}_3 + \text{C} \rightarrow \text{Fe} + \text{CO} \uparrow$



(iii)  $\text{C} + \text{Al}_2\text{O}_3 \xrightarrow{2000^\circ\text{C}} \text{Al}_2\text{C}$



$\text{SiC} \rightarrow$  Carborundum (2<sup>nd</sup> hardest substance known)



$\rightarrow$  Compounds of carbon (C)  $\rightarrow$

(a)  $\text{CO}$ :

P.T.O

(i) Preparation:  $\text{ZnO} + \text{C} \rightarrow \text{Zn} + \text{CO} \uparrow$   
 (Incomplete combustion).

(ii) Properties:  $\text{CO} + \frac{1}{2}\text{O}_2 \rightarrow \text{CO}_2 \uparrow$   
 $\text{CuO} + \text{CO} \rightarrow \text{Cu} + \text{CO}_2 \uparrow$

(iii) Test for CO:

Burns with a blue colour flame.

When CO is ~~step~~ passed through filter paper dipped in  $\text{PdCl}_2$ , it turns black due to deposition of 'Pd' metal.

(b)  $\text{CO}_2$ :

(i) Preparation:  $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$

$\text{Fe}_3\text{O}_4 \xrightarrow{\Delta} \text{FeO} + \text{CO}_2 \uparrow$  (Complete combustion)

(ii) Properties:  $\text{Ca(OH)}_2 + \text{CO}_2 \rightarrow \text{CaCO}_3 \xrightleftharpoons{\text{H}^+} \text{Ca(HCO}_3)_2$  (Elder)

(c) Carbides:

(i) Ionic (salt carbides): Methanide, Acetylides, Allylides.

(ii) Covalent carbides: Carbides of non-metals (B & Si)

(iii) Interstitial carbide: Carbides with metal of d-block.

Carbon increases Thermal Stability by increasing Metallic bond strength.

\* Compounds of silicon  $\rightarrow$

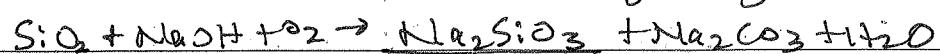
(a) Silicon:

(i) Amorphous  $\rightarrow$  Brownish powder, Crystalline  $\rightarrow$  solid

(i)  $\text{SiO}_2 + 2\text{Mg} \rightarrow \text{Si} + 2\text{MgO}$  (Amorphous 'Si') Preparation

$\text{SiO}_2 + \text{C} \xrightarrow{\text{little Fe to prevent SiC}} \text{Si} + \text{CO} \uparrow$

(ii) Properties:  $\text{Si} + \text{F}_2 \rightarrow \text{SiF}_4$  (Itching of glass)



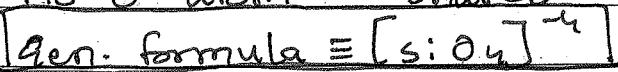
water glass

(b) Silicates:

3-D tetrahedral compounds in which 'Si' atom is connected to 4 ' $\text{O}^-$ ' atoms via single bonds. They are non-reacting and highly non-corrosive on reaction with acid and base.

### Types of silicates:

(i) Ortho-silicates: No 'O' atom is shared.

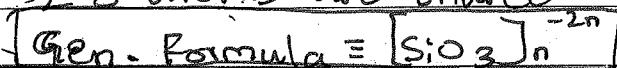


Exceptions:  $\text{Zr}(\text{SiO}_4) \rightarrow$  Zircon;  $\text{Zn}_2(\text{SiO}_4)$  (Wollastonite)

(ii) Pyrosilicates: One 'O' atom is shared by each unit.



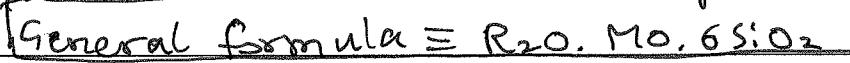
(iii) Single-chain silicates: 2 'O' atoms are shared.



(iv) Ring silicate: Gen. Formula  $\equiv [\text{SiO}_3]_n^{-2n}$

### Glass:

Transparent/translucent supercooled solid solution of silicates and borates. Amorphous in nature.



Raw materials used to manufacture glass:

(i) Acidic  $\rightarrow \text{SiO}_2, \text{B}_2\text{O}_3, \text{P}_2\text{O}_5$

(ii) Basic  $\rightarrow \text{CaCO}_3, \text{BaCO}_3, \text{PbO}, \text{ZnO}$ .

Colouring materials used for glass:

Oxides of d-block elements from  $Z = 24$  to  $29 + \text{'Se' metal}$

\* Hydrolysis  $\rightarrow$

Breaking of covalent bond using  $\text{H}_2\text{O}$  as solvent ( $\text{N}^{\ddagger}$ ).

It is a non-redox reaction (except a few).

1 atom in a molecule should have partial (+)ve charge.

Atom having partial (+)ve charge must have vacant orbitals.

Mag. of (+)ve charge  $\uparrow$ ; hydrolysis  $\uparrow$ .

If, F-N side atoms  $\uparrow$ ; hydrolysis ease  $\downarrow$ .

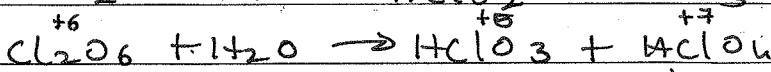
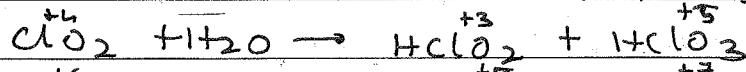
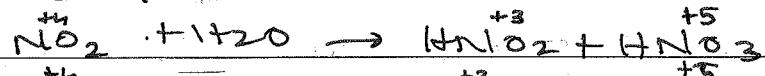
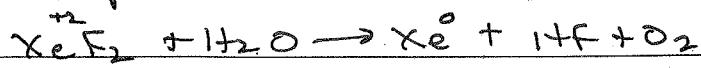
Mechanisms used for hydrolysis of compounds:

(i)  $\text{SN}' \rightarrow \text{CCl}_4, \text{NF}_3, \text{SF}_6$

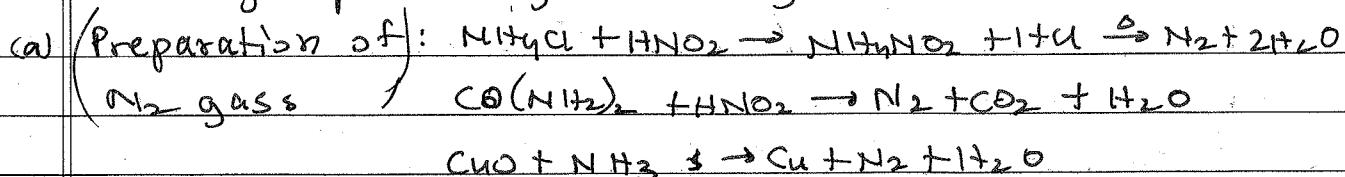
(ii)  $\text{SN}^2 \rightarrow \text{SiCl}_4, \text{BCl}_3, \text{PCl}_5, \text{PCl}_3, \text{AsCl}_3$

Due to large size of 'Bi' and 'Sb', partial (+)ve charge gets de-localized due to large volume.

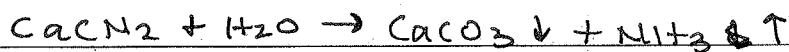
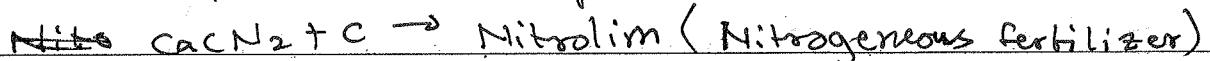
### Exceptions:



→ 15<sup>th</sup> group (Nitrogen family) →



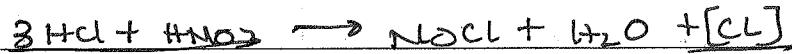
(b) Fixation of N<sub>2</sub> in atmosphere:



→ Compounds of N<sub>2</sub> →

(a) HNO<sub>3</sub>:

(i) Properties:  $\text{HNO}_3 + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{NO}_3^-$



Aqua regia

Nascent chlorine

(b) NH<sub>3</sub>:

(i) Preparation: Haber's process

(ii) Properties:  $\text{NH}_3 + \text{Cl}_2 / \text{Cl}_2 (\text{excess}) \rightarrow \text{NH}_4\text{Cl} + \text{H}_2 / \text{NCl}_3 + 3\text{HCl}$

→ Compounds of Phosphorous →

(a) Ores:

(i) Phosphonite:  $\text{Ca}_3(\text{PO}_4)_2$  (bone ash)

(ii) Chlorapatite:  $3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaCl}_2$

(iii) Fluorapatite:  $3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_3$

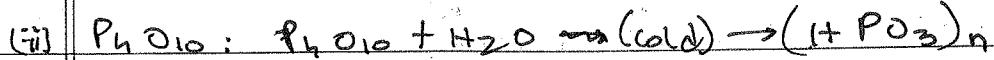
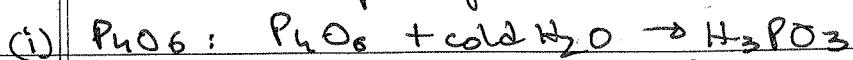
(b) Allotropes:

White, Red, Black

(i) White (Yellow): Ignition temp. = 29°C.

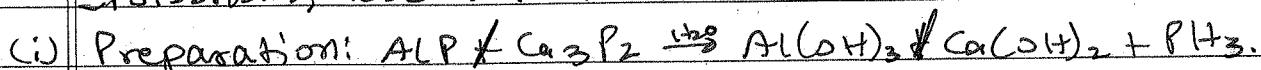
Catches fire at room temperature.

(b) Oxides of phosphorus:



(c)  $\text{P}_4\text{H}_3$ :

Poisonous, less soluble in  $\text{H}_2\text{O}$ .



→ Group - 16 (Chalcogens) →

(a) Preparation of  $\text{O}_2$  gas:

(i) Electrolysis of  $\text{H}_2\text{O}$

(ii) Fractional distillation of liquid air.



→ Compounds of  $\text{O}_2$  →

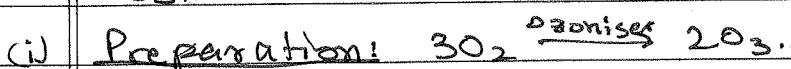
(a) Oxides:

(i) Acidic → Non-metals +  $\text{H}_2\text{O}$

(ii) Basic → Metals +  $\text{H}_2\text{O}$

(iii) Amphoteric → Can React w/ acids and bases.

(b)  $\text{O}_3$ :

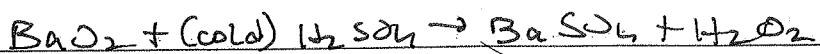
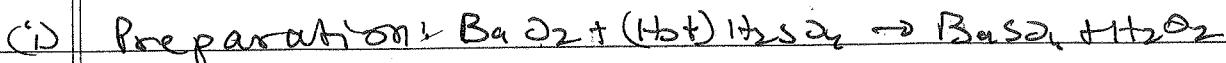


Prepared by passing silent discharge through dry, pure and cold  $\text{O}_2$  in a spiral tube called ozoneizer.

(ii) Properties:

Pale colour, fishy smell, explosive, can damage body tissue

(c)  $\text{H}_2\text{O}_2$ :



(Merk's process)

(ii) Properties: Viscous blue liquid, 30%  $\text{H}_2\text{O}_2 \rightarrow$  Peroxydrol  
Decomposition of  $\text{H}_2\text{O}_2$  is exothermic.

- D-block and F-block :-

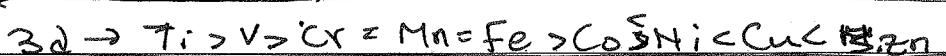
→ d-block elements →

$$\text{General configuration} = (n-1) d^{1-10} ns^{1-2}$$

Elements where penultimate d-shell is incomplete are called transition elements.

Zn, Cd, Hg are not transition elements. Their penultimate d' shells are completely filled.

(a) Atomic radius:



4d and 5d have similar radii; therefore - cannot be separated in a mixture.

(b) Oxidation state (O.S.):

Min. O.S. → No. of valence shells (e⁻ in)

Max. O.S. → valence e⁻ + no. of unpaired d' orbitals

3d → General O.S. (+2); (+1) → Cr, Cu; (+7) → Mn

4d → General O.S. (+2); (+8) → Ru, Os

[Lower (+) ve O.S. < higher (+) ve O.S.]

Stable O.S. compounds do not DPP and are stable at room temperature.

(c) Magnetic properties:

(i) Diamagnetic → no unpaired e⁻

(ii) Paramagnetic → Unpaired e⁻

(iii) Ferromagnetic → Permanently magnetic (Fe, Co, Ni)

$$\mu = \sqrt{n(n+2)} BM$$

(d) Melting Point:

Max MP → Cr, Mo, W

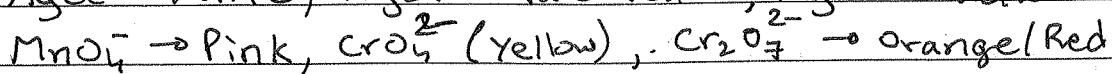
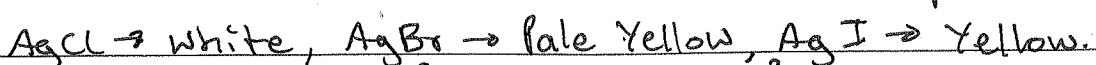
Min. MP → Zn, Cd, Hg

(e) Colour:

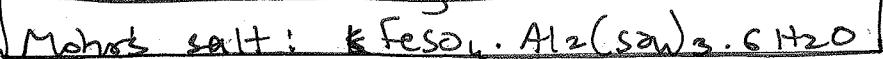
(i) polarization

(ii) Charge transfer phenomenon

(iii) d-d transition (used when there are unpaired e<sup>-</sup>)

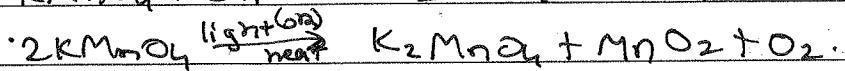
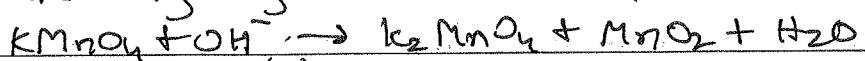


\* Reactions involving d-block elements:



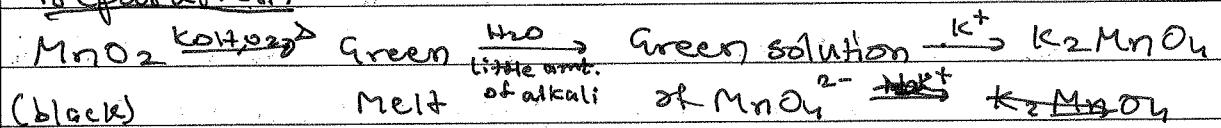
(i) For acidification of  $\text{KMnO}_4$ , only  $\text{H}_2\text{SO}_4$  could be used but not  $\text{HCl}$  because if  $\text{HCl}$  is used then  $\text{Cl}_2$  is liberated.

(ii) For  $\text{K}_2\text{Cr}_2\text{O}_7$ ,  $\text{H}_2\text{SO}_4$  and  $\text{HCl}$  are used since it is a milder oxidising agent than  $\text{KMnO}_4$ .



\*  $\text{KMnO}_4 \rightarrow$

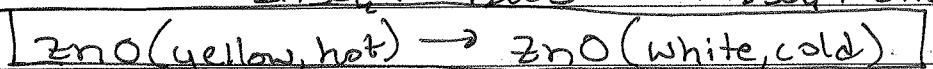
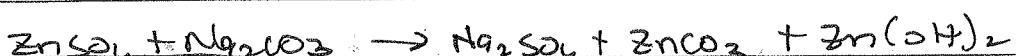
(a) Preparation:



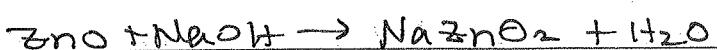
\* Compounds of Zinc →

(a)  $\text{ZnO}$  (Philosopher's wool):

(i) Preparation:  $\text{ZnCO}_3 \xrightarrow{\Delta} \text{ZnO} + \text{CO}_2$

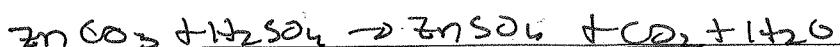


(ii) Properties:  $\text{ZnO} + \text{C} \rightarrow \text{Zn} + \text{CO}$



(b)  $\text{ZnSO}_4$ :

(i) Preparation:  $\text{ZnO} + \text{H}_2\text{SO}_4 \rightarrow \text{ZnSO}_4 + \text{H}_2\text{O}$

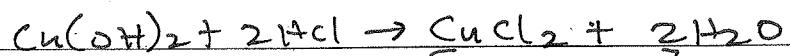


(ii) Properties: Isomorphous w/  $\text{MgSO}_4 \cdot 2\text{H}_2\text{O}$

\* Compounds of copper ( $\text{Cu}$ ) →

(a)  $\text{CuCl}_2 \cdot 2\text{H}_2\text{O}$

(i) Preparation:  $\text{CuO} + 2\text{HCl} \rightarrow \text{CuCl}_2 + \text{H}_2\text{O}$



(ii) Properties:  $\text{CuCl}_2 \cdot 2\text{H}_2\text{O} \xrightarrow[\text{dilution}]{\text{H}_2\text{O}} [\text{Cu(H}_2\text{O)}_4] \text{Cl}_2$

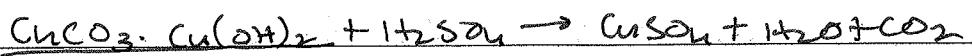
green

blue



(b)  $\text{CuSO}_4$ :

(i) Preparation:  $\text{Cu}(\text{scrap}) + \text{H}_2\text{SO}_4 \xrightarrow{\text{conc.}} \text{CuSO}_4 + \text{H}_2 \uparrow$  No reaction



Malachite green

(ii) Properties:  $\text{CuSO}_4 \xrightarrow{4\text{N}^{+3}} \text{CuSO}_4 \cdot 4\text{N}^{+3}$

white

(Dark Blue)

\* Compounds of Fe →

(a)  $\text{FeO}$ :

(i) Preparation:  $\text{FeS} + \text{O}_2 \xrightarrow{\text{roasting}} \text{FeO} + \text{SO}_2$



(ii) Properties:  $\text{FeO} + \text{O}_2 \rightarrow \text{Fe}_2\text{O}_3$ .

(b)  $\text{FeSO}_4$ :

Used for absorption of NO:

$\text{FeSO}_4 \cdot \text{NO} \rightarrow$  Nitroso ferrous sulphate.

(i) Preparation:  $\text{FeO} + \text{H}_2\text{SO}_4 \rightarrow \text{FeSO}_4 + \text{H}_2\text{O}$



(ii) Properties:  $\text{FeSO}_4 + \text{NaOH} \rightarrow \text{Fe(OH)}_2 \downarrow + \text{Na}_2\text{SO}_4$

only amphoteric hydroxides can dissolve as ppt.

Eg: Be, Al, Zn, Sn, Pb.

$\text{MgSO}_4 \cdot 7\text{H}_2\text{O} \rightarrow$  Epsom salt.

- General Organic Chemistry:-

- Isomerism →

Same molecular formula, Different structural formula.

- Chain isomer → Differ in carbon chain length.
- Position isomers → Differ in position of substituent group.
- Ring-chain " → Cyclic and acyclic isomer
- Functional " → Differ in functional group.
- Metamers → Differ in alkyl group.
- Tautomers → Alka atom, enol to ketone

- Resonance →

Molecules in which electronic arrangement differs.

All resonance structures are imaginary.

Structure which represent all other resonating / canonical structures is called as a resonance hybrid

[P.E ↓, Resonance stability ↑]

- Mesomeric effect →

The effect where groups contribute to resonance in organic molecules. This is also known as resonance effect.

- $e^-$  releasing groups:  $-OH^-$ ,  $-OR$ ,  ~~$\text{O}^-\text{OCOR}$~~ ,  $-\text{NH}_2$ ,  $-\text{NHR}$ ,  $-\text{NR}_2$
- $e^-$  withdrawing groups:  $-\text{COOH}$ ,  $-\text{CHO}$ ,  $>\text{C=O}$ ,  $-\text{CN}$ ,  $-\text{NO}_2$

- Electromeric effect →

Transfer of  $e^-$  pair ( $\pi e^-$ ) to one of the atoms joined by a multiple bond on the demand of an attacking reagent. It is a temporary effect.

Organic compounds having double or triple bond show this effect.

- E effect:  $\pi e^-$  are transferred to that atom where attacking reagent does not get attached.
- +E effect:  $\pi e^-$  are transferred to the atom where attacking reagent gets attached.

→ Hyperconjugation →

Presence of 'H' atom on  $sp^3$  carbon which is attached to  $sp^2$  carbon.

No. of  $H_2$  structures determines extent of the hyperconjugation effect.

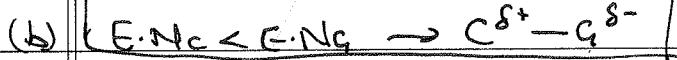
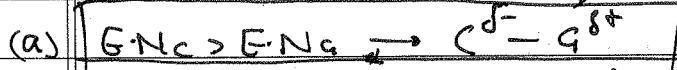
Total no. of resonating structures =  $(n+1)$

$n \rightarrow$  no. of hyperconjugation structures.

Stability ↑, No. of  $H_2$  structures ↑, Heat of hydrogenation ↑

→ Inductive effect →

In a bond  $C-Q$ , ( $Q \rightarrow$  substituent group.)



Inductive effect groups:

+I effect:

e<sup>-</sup> WDG:  $X^-$ ,  $NO_2$ ,  $-CN$ ,  $-COOH$ ,  $-COOR$ ,  $-OAr$ ,  $-OC_6H_5$ .

e<sup>-</sup> RG:  $-CH_3$ ,  $-C_2H_5$ .

→ Bond-dissociation in a covalent bond →

(a) Homolytic: Results in formation of radicals.

Used in reactions involving  $h\nu$ ,  $\Delta$ ,  $R_2O_2$ .

Stability of free radical  $\propto$  Resonance

Radical stability:  $3^\circ > 2^\circ > 1^\circ > -CH_3$

(b) Heterolytic: Unequal bond distribution.

Results in formation of (+)ve and (-)ve charges

Depends on E.N of carbon or group.

→ Carbocation and carbanion →

(a) Carbocation:

6e<sup>-</sup> species, e<sup>-</sup> deficient, behaves like electrophile.

Vacant 'p' or 'd' orbitals act as electrophile.

Can be formed by compounds with multiple bonds.

Stability: Butyl > Propyl > Ethyl > Methyl

Stability:  $3^\circ > 2^\circ > 1^\circ$

$e^-$  R.G ↑ stability, W.D.G ↓ stability

When  $C^+$  is formed, re-arrangements are possible

### (B) Carbanions:

Acts as  $\bar{N}u^-$ ,  $e^-$  rich,  $\delta e^-$  species.

W.D.G ↑ stability, R.G ↓ stability

Resonance ↑, carbanion stability

$e^-$  stability is exact opposite of  $C^+$  stability orders.

\* Acidity of molecules →

Solvents that can act as acid and bases are called as amphotropic solvents.

Aprotic → Do not donate  $p^+$ .

Protic → Donates  $p^+$ .

$H_2O$  increases strength of acid to  $H_3O^+$  ion.

Acidity ↑,  $pK_a \downarrow$

W.D.G ↑ acidity, R.G ↓ acidity

\* Weak C.B.s are good leaving groups.

$p^+$  molecules have increased acidity.

De-localization & Acidity

$C_6H_5COOH > CH_3COOH$  → Acidic strength.

\* Basicity →

$H_2O$  increases strength of bases to that of  $OH^-$  ion.

Basicity ↑,  $pK_b \downarrow$

De-localization ↑, basicity ↓

Molecular size ↑, basicity ↑

R.G ↑ basicity, W.D.G ↓ basicity

Density & L.P space & basicity

\* Dipole moment →

$$\vec{\mu} = q \times \vec{d}$$

Unit → Debye.

D.M ≡ ortho > para > meta

→ H-bonding →

Attaching of one hydrogen to N, O, F.

- (a) (i) Inter-molecular H-bonding:
- (ii) Between molecules
- (iii) Dimerisation
- (iv) High solubility, high BP, less volatility
- (b) (i) Intra-molecular H-bonding: →
- (ii) Within the molecule
- (iii) Chelation
- (iv) less soluble, less BP, More volatile

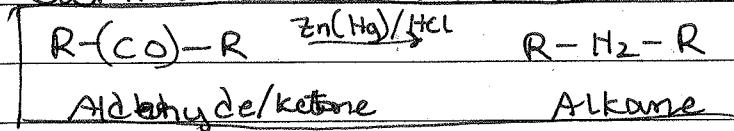
→ Boiling point →

[Boiling point (B.P) ∝ Surface area]  
[B.P ∝ Molecular weight]

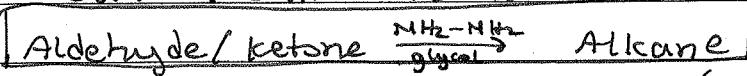
• Alkanes:-

→ Preparation of alkanes →

(a) (a) Clemmenson's reduction:

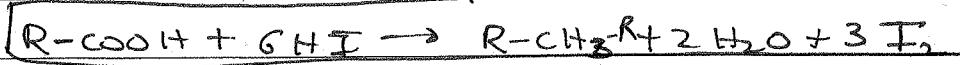


(b) Wolff-Kishner reduction:

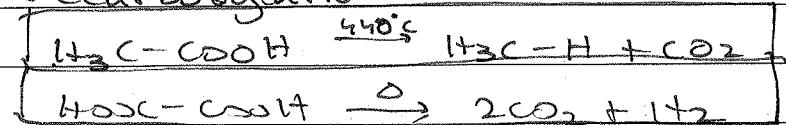


Glycol → Polar protic solvent ( $\text{HO}-\text{CH}_2-\text{CH}_2-\text{OH}$ )

(c) Reaction with  $\text{LiI}/\text{red Pb}$ :



(d) Decarboxylation:

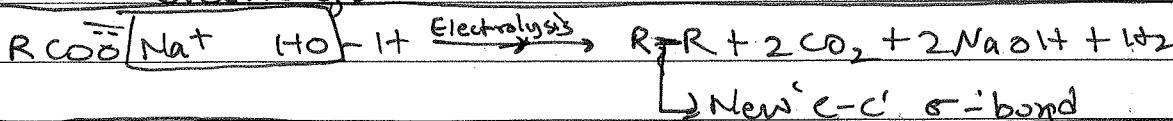


Mono carboxylic acids cannot be decarboxylated easily.

Di-carboxylic acids can be decarboxylated, but with limited number of carbons.

Soda lime  $\rightarrow \text{NaOH} + \text{CaO}$

(e) Kolbe electrolysis:



Cathode  $\rightarrow$  Metal, Anode  $\rightarrow$  Non-metal

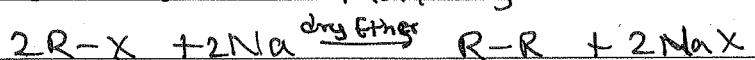
Solid salts  $\rightarrow$  Cannot be electrolysed

Molten/Fused/Aqueous  $\rightarrow$  Can be electrolysed.

(f) With LiAlH<sub>4</sub>:



(g) Reaction with Na/Dry ether (Wurtz reaction):



Formation of radicals takes place.

Methane cannot be prepared by this method.

Carbon radicals  $\rightarrow$  Head-to-head

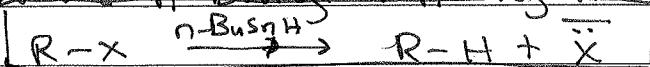
C-H radicals  $\rightarrow$  Head-to-tail.

$\therefore$  product formed is very less.

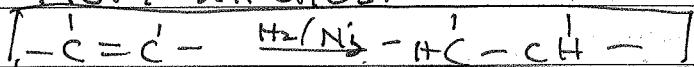
(h) Frankland reaction:



(i) With n-butyl tin hydride:



(j) From alkenes:



\* Properties of alkanes  $\rightarrow$

(a) Radical initiations:

This is the characteristic reaction of alkanes.

Reagents commonly:  $\text{hv}$ ;  $\text{Cl}_2$  at  $500^\circ\text{C}$ ;  $\text{SO}_2\text{Cl}_2$ ;  $(\text{Bu})_3\text{C}-\text{oCl}$ ;  $\text{H}_2\text{O}_2$

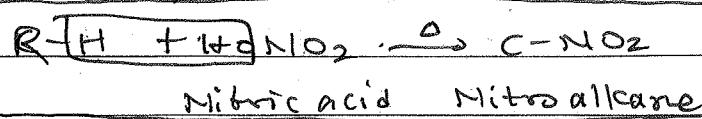
Rate of reaction in X':  $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$

Carbon stability:  $3^\circ > 2^\circ > 1^\circ$

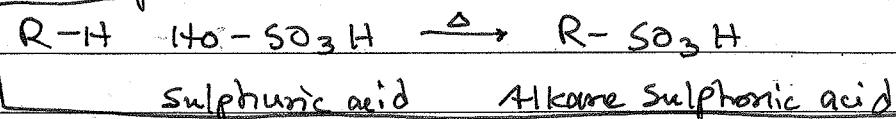
[B.E  $\propto$  ROR],  $[\text{C-D} \text{ B-E} > \text{C-H} \text{ B-F}]$

Alkanes are called as paraffins.

(b) Nitration:

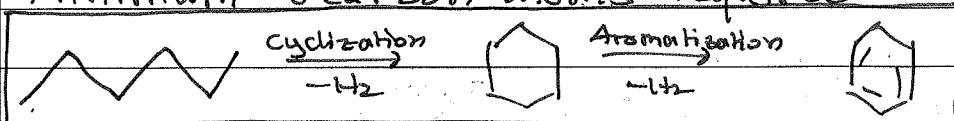


(c) Sulphonation:

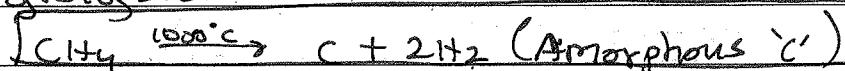


(d) Aromatization:

Minimum 6 carbon atoms required



(e) Pyrolysis:



- Alkenes:-

→ \* Isomerism →

(a) Geometrical isomerism:

(i) cis-trans. (OR) based on priority groups (Mol. wt)

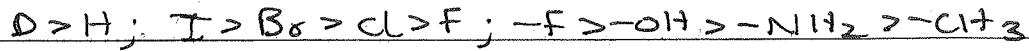
(b) Positional isomerism:

Differ in position of carbon.

(c) Ring-chain isomerism.

(d) Chain isomerism.

→ \* Priority order →



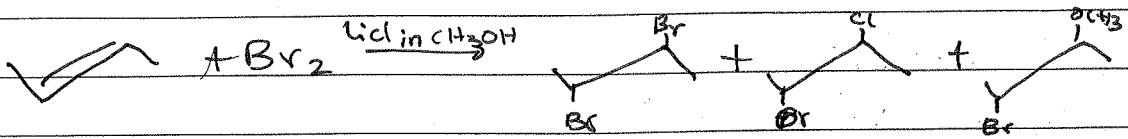
A doubly-bonded atom is considered twice.

→ Test for alkenes →

(a) Bromine test:

Reddish-Brown solution turns colourless

Exception: Benzene



Cis ~~as~~ reactants  $\rightarrow$  Anti-addition

Trans compounds  $\rightarrow$  Syn-addition.

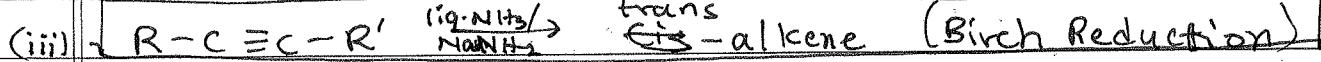
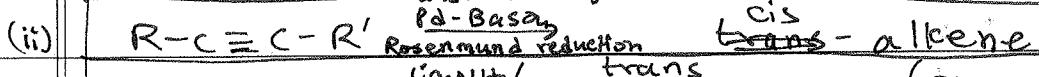
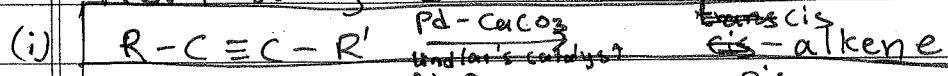
(b) Reaction with cold alkaline  $KMnO_4$

Pink colour of  $KMnO_4$  becomes colourless.

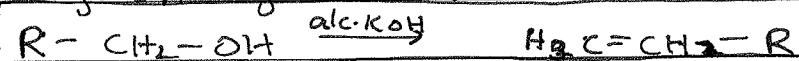
(c) Baeyer's reagent  $\rightarrow KMnO_4, OH^-$

\* Preparation of alkenes  $\rightarrow$

(a) From alkynes:



(b) Dehydrohalogenation:



(c) Boration (oxidation):

\* Properties of alkenes  $\rightarrow$

(a) Ozonolysis:

Breakdown of a double bond in a compound using  $O_3$ .

(i) Terminal carbon double bond  $\rightarrow$  formaldehyde.

(ii) Diene is present  $\rightarrow CO_2$  forms.

(iii)  $2^\circ / 3^\circ$  carbon has double bond  $\rightarrow$  ketone.

(iv) Conjugated diene  $\rightarrow$  di-carbonyl.

If strong oxidizing agents like  $H_2O_2$  are present, then aldehyde turns to carboxylic acids and this process is called as oxidative ozonolysis.

(b) Addition of hydro halogen ( $H-X$ ) compounds:

(i) Markownikoff's rule:

Addition of  $H^+ / p^+$   $\rightarrow$  More number of  $H_2$ .

Nucleophile  $\rightarrow$  Added to less number of "H" atoms

( $\rightarrow$  one part of the addendum acts to the more stable  $^+$ ).

(ii) Peroxide effect:

This is opposite to that of Markownikoff's rule.



However, this effect is observed only in  $\text{HBr}$ .

(iii) Normal addition:



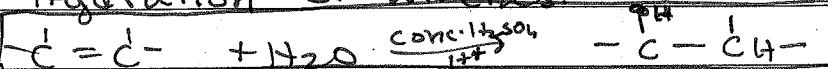
Re-arrangements are possible after formation of  $\text{C}^+$ .

$e^- \text{ RG}$  increase ROR in  $\alpha$ -electrophiles.

Ring expansion:  $3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6$

Ring contraction:  $7 \rightarrow 6, 6 \rightarrow 5$

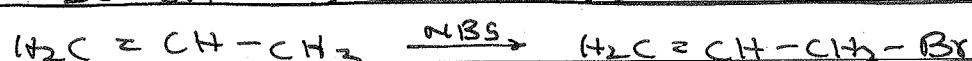
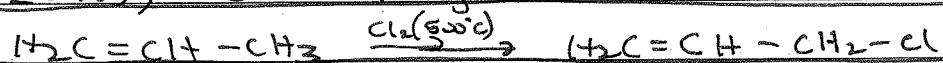
(c) Hydration of alkenes:



(d) Allylic-halogenation:

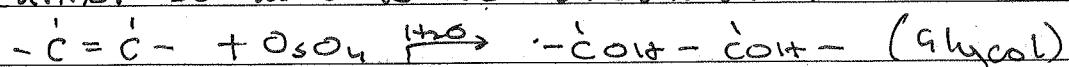
$\text{Cl}_2$  at  $50^\circ\text{C}$   $\rightarrow$  Allylic chlorination

$\text{Br}_2$  (nrt), NBS  $\rightarrow$  Allylic Bromination.



3. (e) Reaction with  $\text{OsO}_4$ :

Cannot be used to test unsaturation.

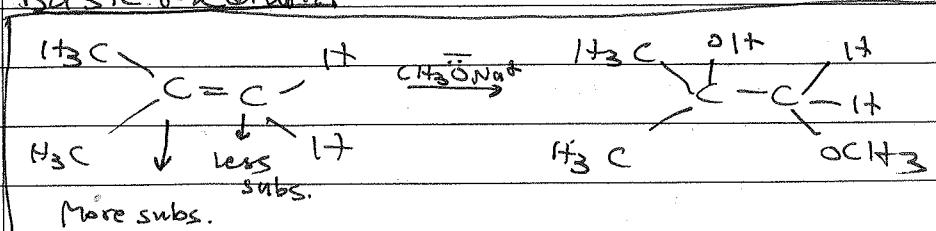


(f) Reaction with  $\text{R}-\text{CO}_2\text{H}$

Carboxylic acid  $\xrightarrow{\text{CO}_2}$  Per-carboxylic acid

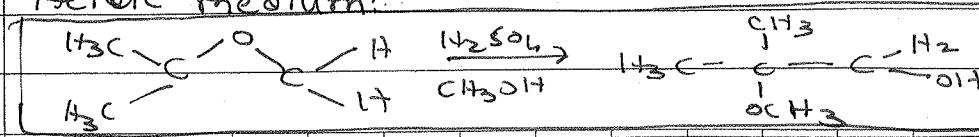
(g) Ring openings in epoxides:

(i) Basic medium:



More subs.

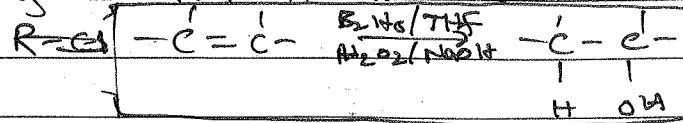
(ii) Acidic medium:



Opp. to that  
in basic medium

(h)

Hydroboration/oxidation:



Trialkyl borane is resistant to reaction with acid hydrolysis. It decomposes only with mild acids like  $\text{CH}_3\text{COOH}$ .

(i)

oxy-mercuration ( $\text{Hg}-\text{OAc}$ ):

Product and mechanism is same as that of hydroboration

(j)

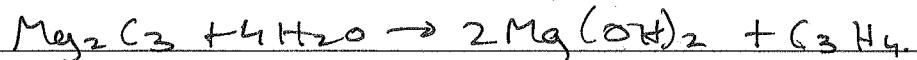
Elimination reactions:

(i)  $\alpha$ -elimination  $\rightarrow$  Removes  $\text{H}\alpha$ (ii)  $\beta$ -elimination  $\rightarrow$  Removes  $\text{H}\beta$ 

• Alkynes:-

→ Preparation of alkynes  $\rightarrow$ 

(a) Metal carbides:

'Acetylide ( $\text{Ca}(\text{C}_2)$ ):  $\Rightarrow \text{CaH}_2$  gives  $(\text{C}_2\text{H}_2)$ .'Butylide ( $\text{Mg}(\text{C}_3)$ ) gives  $(\text{C}_3\text{H}_4)$ .

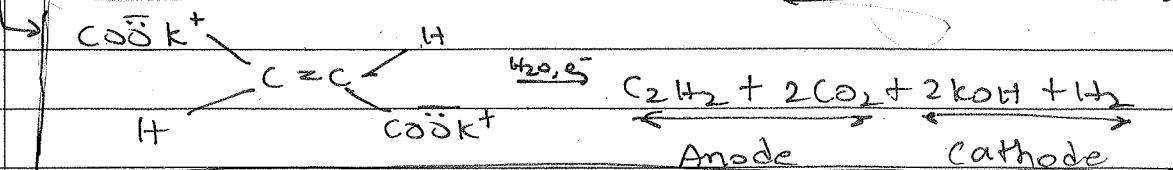
(b)

From trihalogen derivatives:



(c)

Kolbe electrolysis:



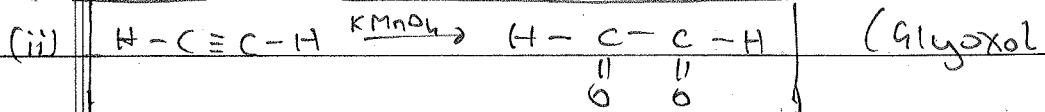
(d)

Dehydrohalogenation:

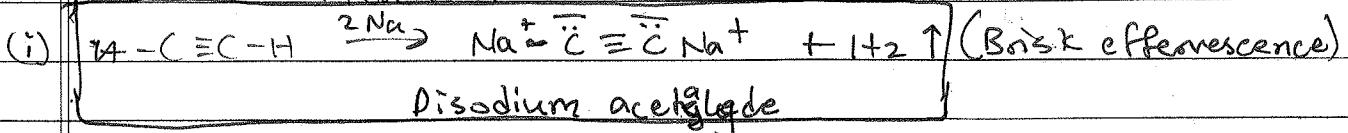


## Properties of alkynes →

### (a) R-c Identification test:



(b) Test to differentiate from alkenes:

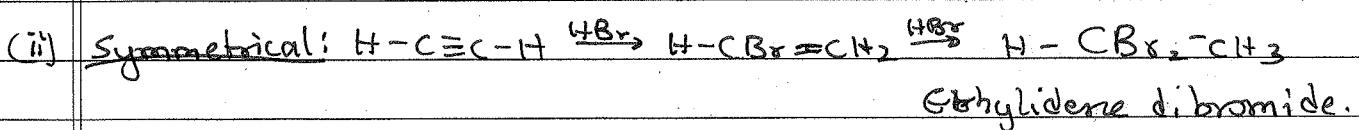
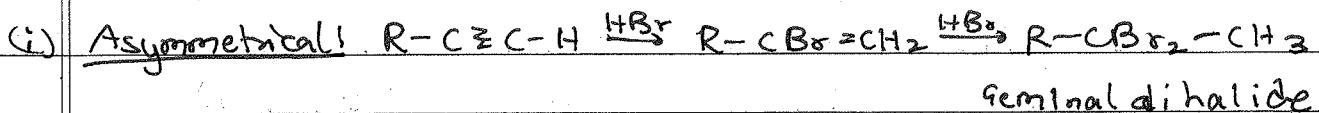


(c) Reaction with Tollen's reagent:

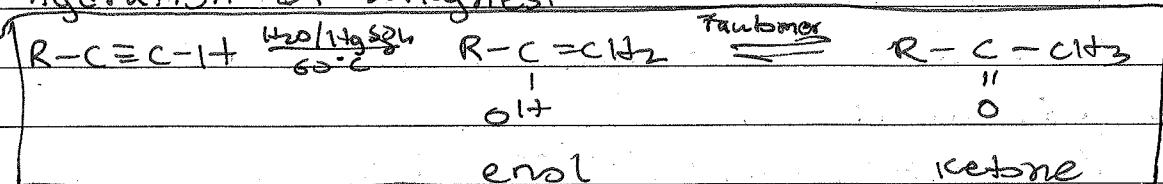
Alkenes show (→)ne Tollen's test.

$\text{Mg(OAc)}_2$  shows (+)ve Tollens's test along with aldehydes.

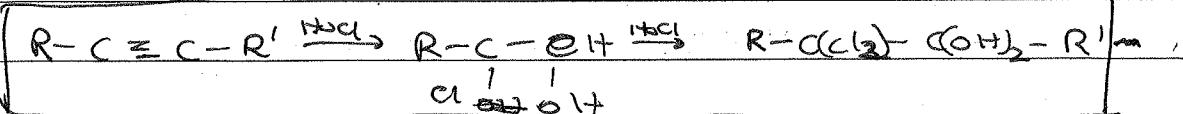
(d) Reaction with HX:



### (e) Hydration of alkynes:



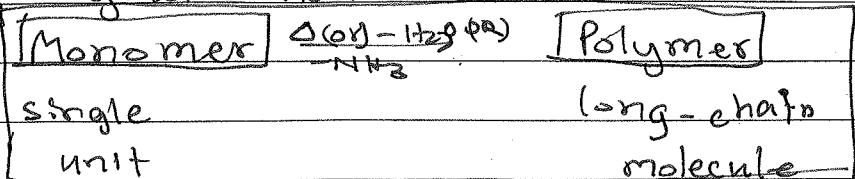
### (f) Addition of NaCl



For alkene: partial reduction

For alkane: complete reduction.

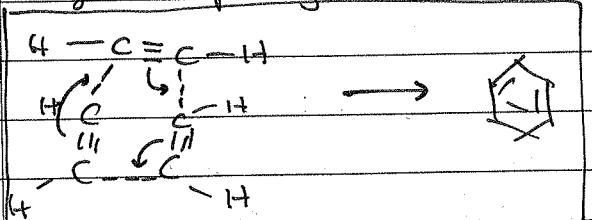
## (g) Polymerisation reaction:



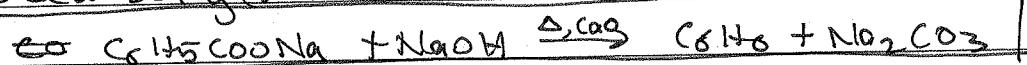
- Aromatic compounds:-

→ Preparation of benzene →

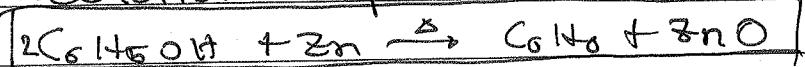
(a) Cyclic polymerisation of ethyne:



(b) Dearborylation of aromatic acids:



(c) Reduction of phenol:



→ Aromaticity →

A cyclic compound can be called aromatic if the following conditions are to be satisfied.

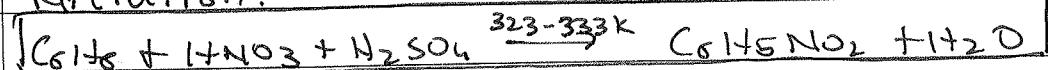
(a) Planarity.

(b) Complete delocalization of  $\pi e^-$  in the ring

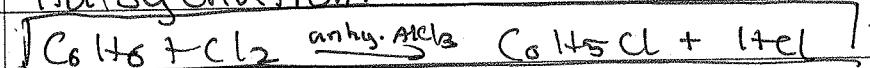
(c) Follows Hückel's rule (Presence of  $(4n+2)\pi e^-$ )

→ Properties of aromatic compounds →

(a) Nitration:



(b) Halogenation:

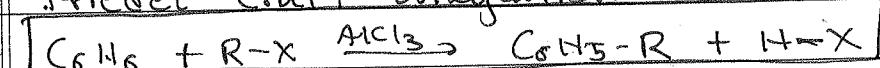


other reagents like  $\text{FeCl}_3$  and  $\text{FeBr}_3$  can also be used

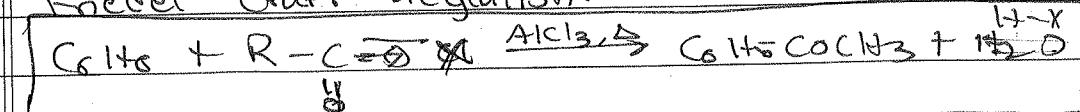
(c) Sulphonation:

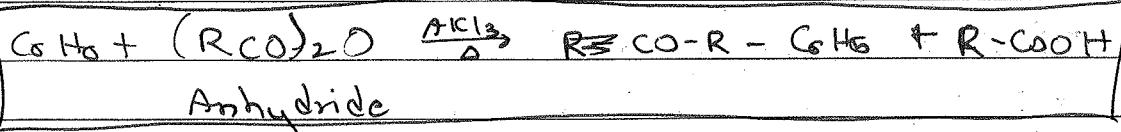


(d) Friedel-Crafts alkylation:

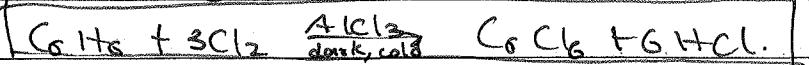


(e) Friedel-Crafts acylation:





(f) Excessive chlorination:



\* Ortho and para-directing groups  $\rightarrow$

Shown by:  $-\text{NH}_2$ ,  $-\text{NHR}$ ,  $-\text{OH}$ ,  $-\text{OCH}_3$ ,  $-\text{NHCOCH}_3$ ,  $-\text{CH}_3$ ,  $-\text{C}_2\text{H}_5$ , etc.

\* Meta-directing groups  $\rightarrow$

Shown by:  $-\text{NO}_2$ ,  $-\text{CN}$ ,  $-\text{CHO}$ ,  $-\text{COR}$ ,  $-\text{COOH}$ ,  $-\text{COOR}$ ,  $-\text{SO}_3\text{H}$ , etc.

\* Stereochemistry:-

\* Stere

\* Conformational isomers  $\rightarrow$

Free rotation of C-C bond is present (only  $\sigma$ -bond).

They are also known as conformers/rotamers.

Dipole moment is used to distinguish b/w every  $60^\circ$  turn.

$0^\circ$ ,  $360^\circ$   $\rightarrow$  Eclipsed

$60^\circ$ ,  $300^\circ$   $\rightarrow$  Gauche

$120^\circ$ ,  $240^\circ$   $\rightarrow$  Partially eclipsed

$180^\circ$   $\rightarrow$  Anti

Anti  $>$  Gauche  $>$  P-Eclipsed  $>$  Eclipsed (Stability)

Stability ↑, P.E ↓.

C-C with 2 'c' atoms  $\rightarrow$  Gauche  $>$  Anti  $>$  Partially Eclipsed

with O/N/F on adj. atoms  $\rightarrow$  Eclipsed

(Stability)

\* Geometrical isomerism  $\rightarrow$

$\text{C}=\text{C}$ : 1,2-disubs. molecules exhibits geometrical isomerism.

$\text{C}=\text{C}$  will not exhibit geometrical isomerism.  
 $\text{C}=\overset{\text{a}}{\underset{\text{a}}{\text{C}}} \text{C}$

(a) 'P' E-Z' configuration:

(i) Priority order on same side  $\rightarrow$  Z

(ii) Priority order on opp. side  $\rightarrow$  E.

Priority order is based on atomic weight of directly bonded atoms.

\* Optical isomerism  $\rightarrow$

These can deviate plane polarised light by an angle.

(+)ve angle: Dextrorotatory (D)

(-)ve angle: Laevorotatory (L)

Non-superimposable mirror images  $\rightarrow$  Optical isomers

Pair of (D) and (L) compounds are called enantiomers.

(a) Racemic mixture:

Contains equal amounts of D & L in mixture.

Separation of racemic mixture is called resolution

(b) Chiral carbon:

Carbon bonded to 4 different groups.

Chiral carbon is always optically active.

(c) Plane of symmetry:

Can be divided into 2 equal groups.

Plane of symmetry  $\rightarrow$  optically inactive (meso compounds)

\* Diastereomers  $\rightarrow$

Molecules which are not mirror images.

'cis' and 'trans' are a pair of diastereomers.

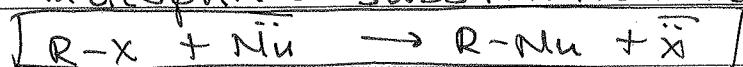
$R-S$  configuration  $\rightarrow$

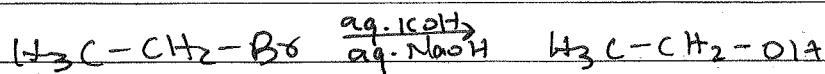
(a) R  $\rightarrow$  Priority molecules are arranged anti-clockwise

S  $\rightarrow$  Priority molecules are arranged clockwise

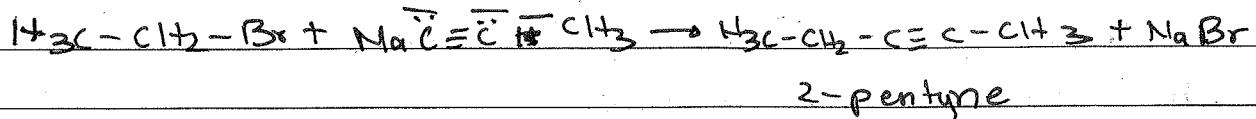
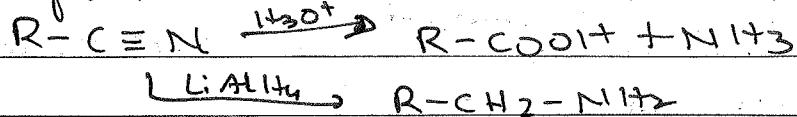
• Alkyl halides and aryl halides:-

$\rightarrow$  Nucleophilic substitution reaction  $\rightarrow$

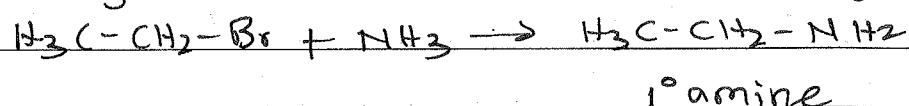




concentration of base is strong, then elimination is preferred over substitution.



Isocyanides are also called as carbylamines.

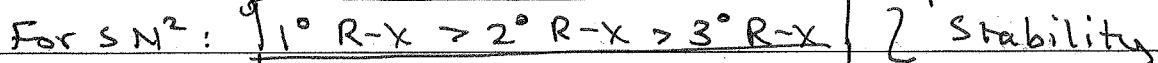


It keeps on going to create quaternary amine.

→ Mechanism of Nucleophilic substitution reactions →

$\text{SN}^1$ : Two-step,  $\ddagger$  formation, Retention and inversion of configuration, unimolecular  
 $\ddagger$  is added after removal of L.G.

$\text{SN}^2$ : Bimolecular, one-step, No  $\ddagger$ , only inversion of configuration, add $\ddagger$  and removal take place simultaneously

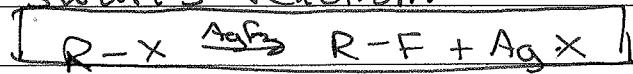


$2^\circ \text{R-X}$  can undergo both mechanisms readily.

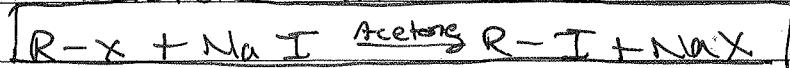
Stability of : Cyclopentyl > Cyclohexyl > Cyclobutyl >  
 Allylic compounds      Cyclopropyl

→ Properties of alkyl halides →

(a) Swart's reaction:

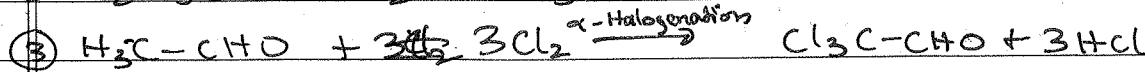
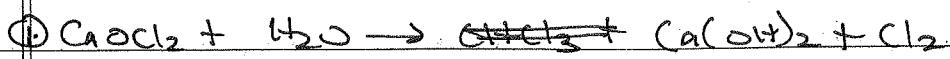
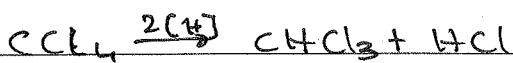


(b) Finkelstein reaction:



→ Chloroform ( $\text{CHCl}_3$ ) →

(a) Preparation of Chloroform:

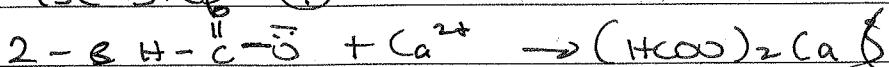


Chlorinating agent



(b) Formation of  $CHCl_3$ :

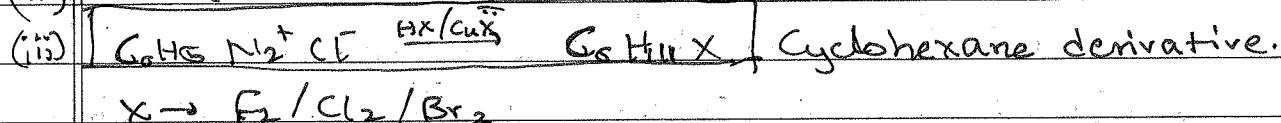
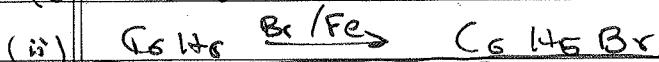
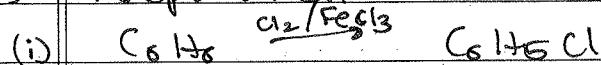
Use step - (4).



Calcium acetate

\* Aryl halides  $\rightarrow$

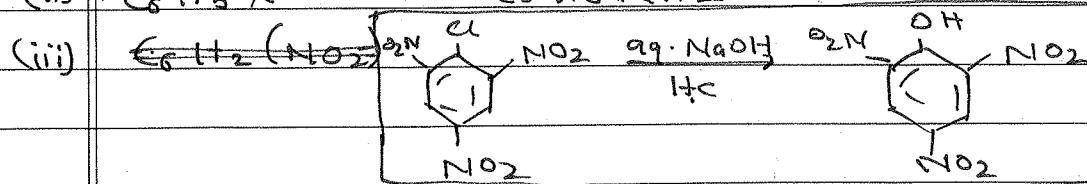
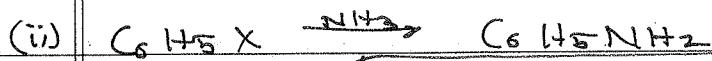
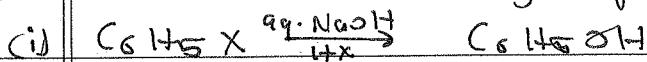
(a) Preparation:



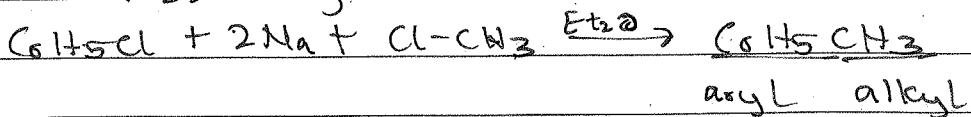
(b) Properties:

Nu substitution:  $C_6H_5D > C_6H_5 > C_6H_5A$

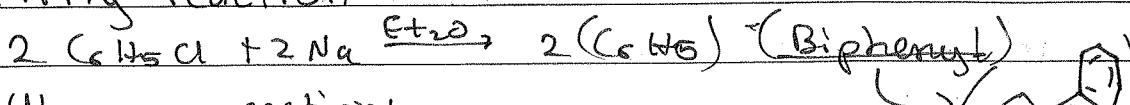
A  $\rightarrow$  substituent group



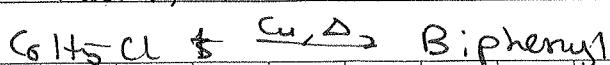
(iv) Wurtz  $\rightarrow$  Fittig reaction:



(v) Fittig reaction:

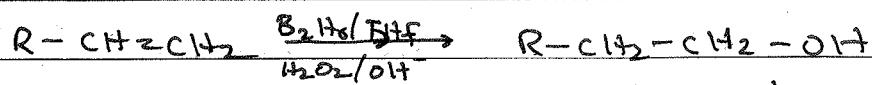
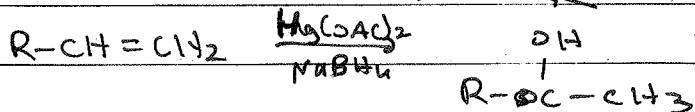
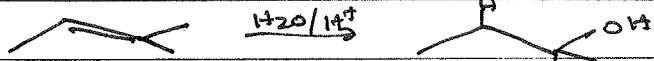
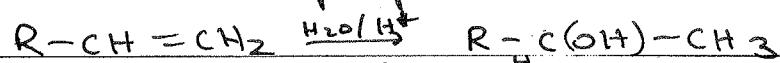


(vi) Ullmann reaction:



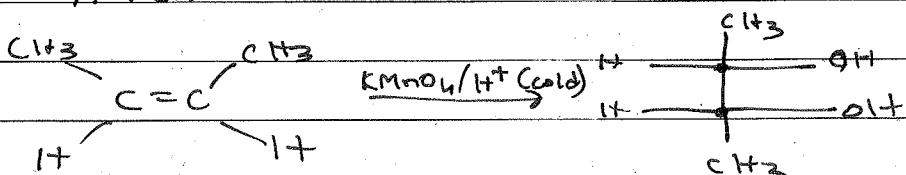
- Alcohols and ethers:-

\* Methods of preparation of monohydric alcohols →



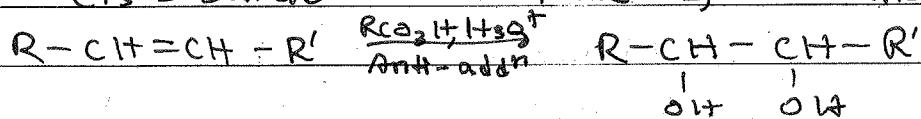
Alkene

Alcohol



cis-2-butene

Meso - 2,2 - butane diol



\* Industrial method of preparation of  $\text{C}_2\text{H}_5\text{OH} \rightarrow$

(i) Molasses  $\xrightarrow{\text{dilution}}$  dil. sugar  $\xrightarrow[\text{24-72 hrs}]{\text{(NH}_4\text{)}_2\text{PO}_4 = \text{Yeast}} \text{CO}_2 \uparrow + \text{C}_2\text{H}_5\text{OH}$   
(Fermentation)

(ii)  $\text{C}_6\text{H}_{12}\text{O}_6 \xrightarrow{\text{zymase}} 2\text{CO}_2 + 2\text{C}_2\text{H}_5\text{OH}$ .

It is formed in dilute quantities.

30%  $\text{C}_2\text{H}_5\text{OH} \rightarrow$  Wash (Wort).

30%  $\text{C}_2\text{H}_5\text{OH}$   $\xrightarrow{\text{distillation}}$  70% Sugar  $\xrightarrow[\text{vacuum}]{\text{distillation}}$  90%  $\text{C}_2\text{H}_5\text{OH}$   
conc.  $\text{H}_2\text{SO}_4$

Absolute alcohol

(i) Distillation 95% (Rectified) (ii) Distillation  
(iii) Add  $\text{BF}_3\text{Hg}(\text{OAc})_2$

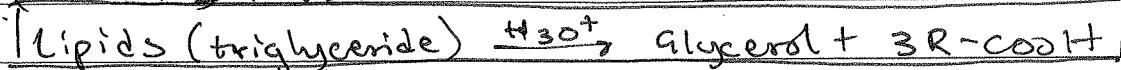
(iii) From starch

Potatoes  $\rightarrow$  Mash  $\xrightarrow{\text{Germinated}} \text{Malt + Mash} \xrightarrow[24-72 hrs]{\text{ferment}} \text{CO}_2 \uparrow + \text{C}_2\text{H}_5\text{OH}$

(iv) Ester hydrolysis

Ester  $\xrightarrow{\text{H}_2\text{O}^+}$  carboxylic acid + alcohol

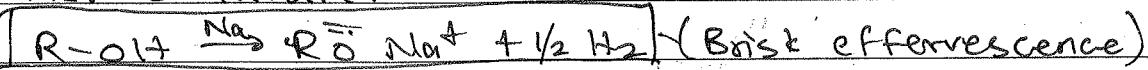
\* Preparation of methanol →



Glycerol is base unit of lipids.

\* Tests for alcohols →

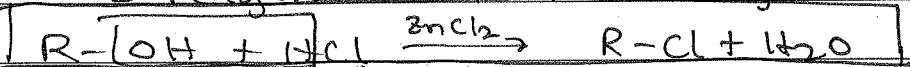
(a) Acidic nature:



(b) Lucas test:

This test is conducted to differentiate b/w  $1^\circ, 2^\circ, 3^\circ$  alcohols

Lucas reagent:  $HCl/ZnCl_2$  (anhydrous)

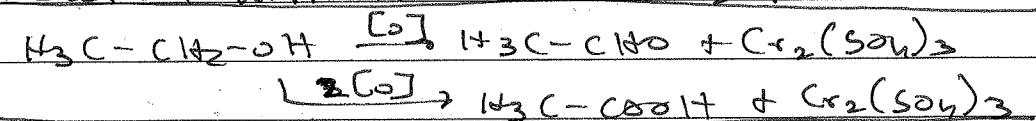


$1^\circ$  alcohol → No turbidity at room temperature.

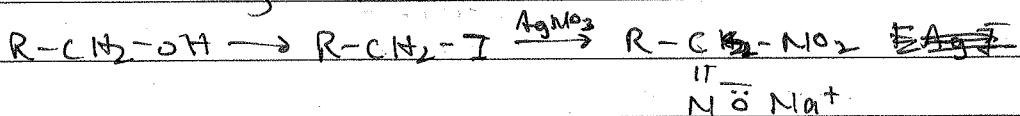
$2^\circ$  alcohol → Turbidity after 3–6 minutes.

$3^\circ$  alcohol → Turbidity is formed.

(c) Reaction with acidified  $K_2Cr_2O_7$ :



(d) Victor-Meyer method:



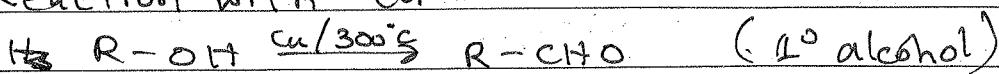
Sodium nitrolate

$1^\circ$  alcohol → Red due to sodium nitrolate

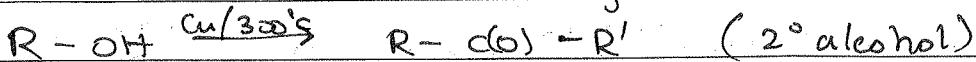
$2^\circ$  alcohol → Blue (does not react with  $NaOH$ )

$3^\circ$  alcohol → Colourless

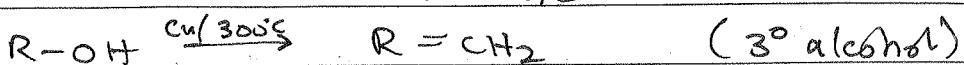
(e) Reaction with 'Cu':



Aldehyde



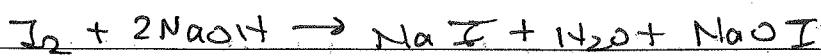
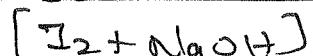
Ketone



Alkene (Iso)

(f)

Iodoform reaction:



sodium hypiodite

Products formed are carboxylates.

(g)

Per Iodide acid test:

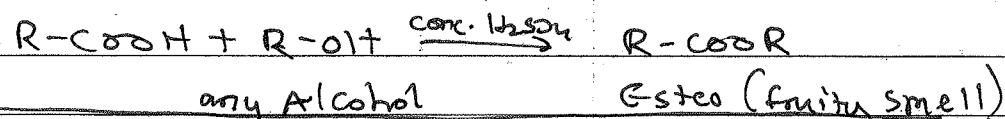
(i) Vicinal diols:



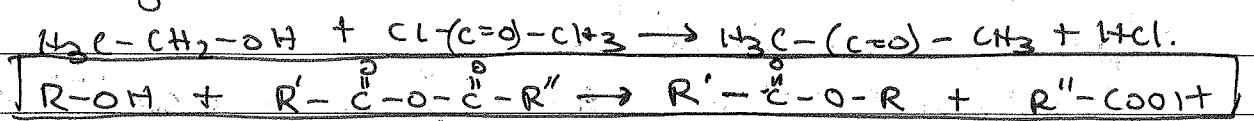
Based on products formed, structure determination of polyhydroxic alcohols or vicinal diols can be done.

Properties of alcohols →

(ii) Esterification:

"Ni add": MeOH > EtOH > PrOH > BuOH

(b) Acetylation:



Alcohols can be methylated by using Dimethyl sulphate in basic medium.

• Aldehydes and ketones:-

→ Identification of carbonyl functional group →

(a) 2,4-DNP test:

2,4-Dinitro phenyl hydrazine

Red colour indicates presence of functional group (carbonyl).

(b) Tollen's test:Tollen's reagent: Ammonical  $\text{AgNO}_3 \rightarrow [\text{Ag}(\text{NH}_3)_2]^+$

Silver lining is formed on walls of test tube.

Aldehydes give (+)ve test.

Ketones give (-)ve test (Exception: Fructose)

In acetone, no oxidation takes place.

Formic acid gives +ve test with Tollen's reagent.

(c) Fehling's test:

Fehling's solution A: alkaline  $\text{CuSO}_4$

Fehling's solution B: sodium potassium tartarate (Rochelle salt)

Aliphatic aldehydes  $\rightarrow$  (+)ve test

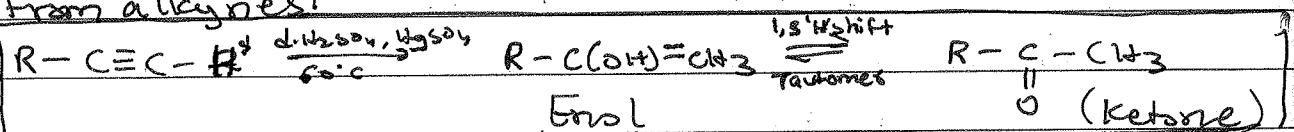
Aromatic aldehydes  $\rightarrow$  (-)ve test

Ketones also give (-)ve Fehling's test

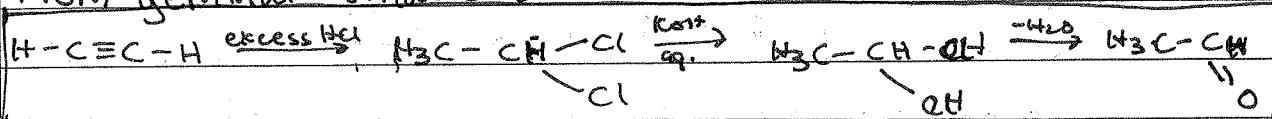
\* Preparation of aldehydes and ketones -

(a) From hydrocarbons (=,  $\equiv$ ):

(i) From alkynes:

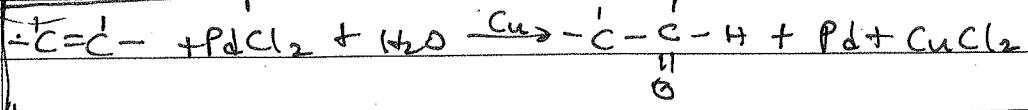


(ii) From geminal dihalides:



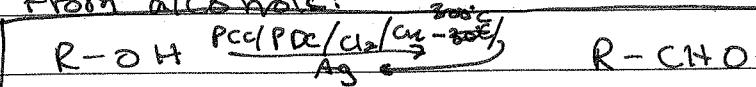
(iii) (a) From alkenes:

(b) Wacker's process:



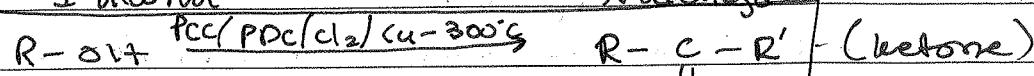
(b)

From alcohols:



1° alcohol

Aldehyde



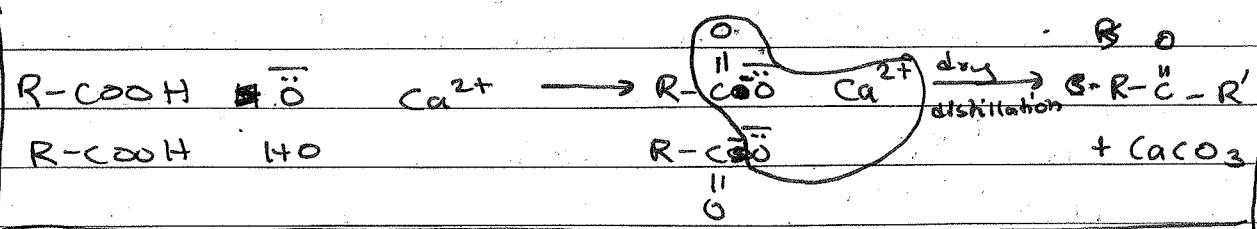
2° alcohol

O

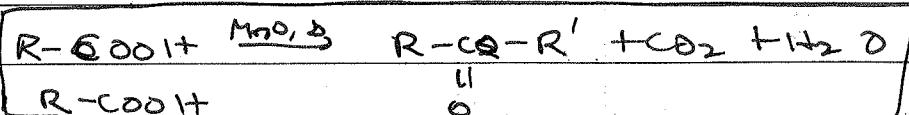
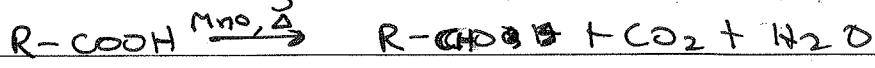
(c)

From carboxylic acid and its derivatives:

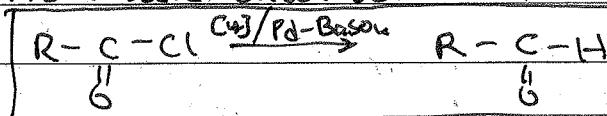
(i) From 'Ca' salts or carboxylic acids:



(ii) From carboxylic acids:



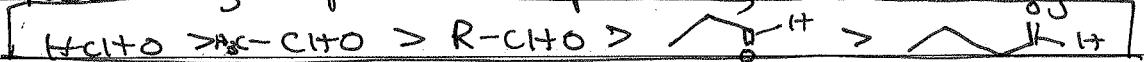
(iii) From acid chloride:



\* Nucleophilic addition reactions →

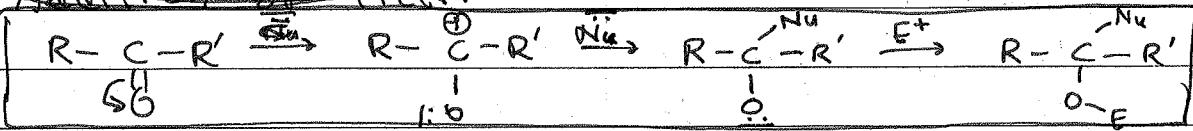
(a) Characteristic reactions:

Reactivity depends on polarity of  $\text{C}=\text{O}$  group.



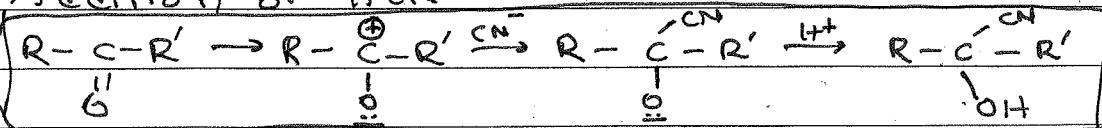
(b)

Addition of  $\text{HCO}^+$ :



(b)

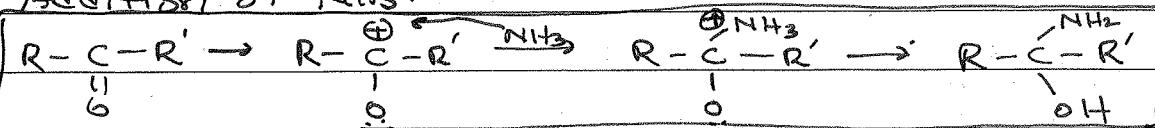
Addition of  $\text{HCN}$ :



Product formed is called as cyanohydrin.

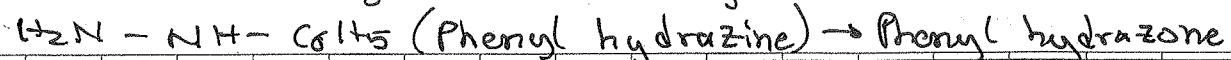
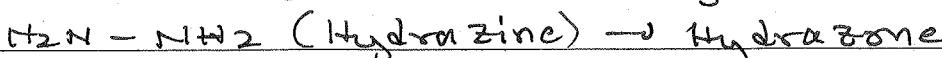
(c)

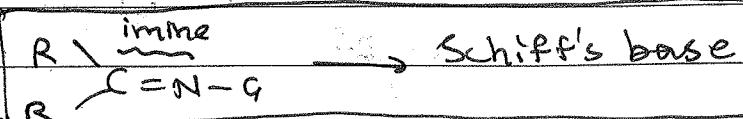
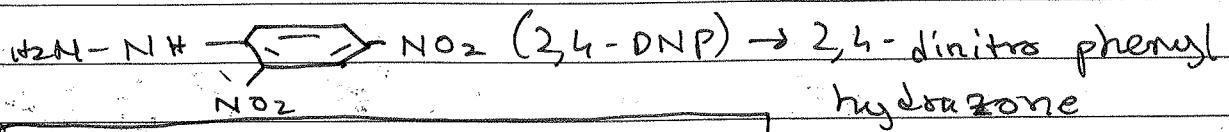
Addition of  $\text{NH}_3$ :



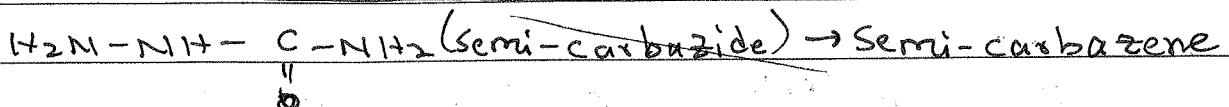
(d)

Addition with derivatives of ammonia:



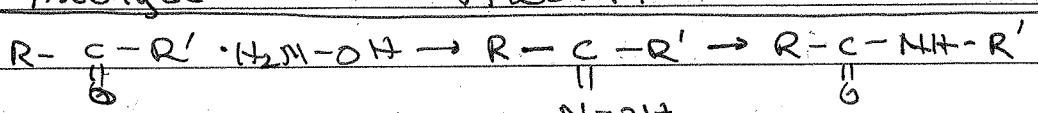
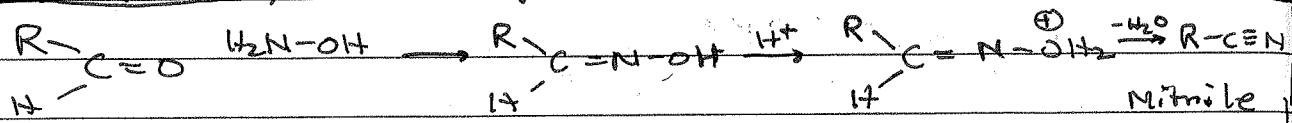


$\text{G} = -\text{R}, -\text{OR}, -\text{NH}_2, \text{NH-Ph}, \text{NH-C}_6\text{H}_5(\text{NO}_2)_2$

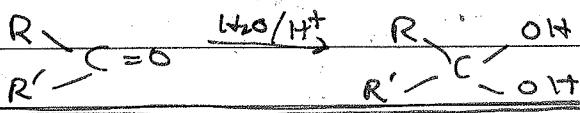


(e) Beckmann re-arrangement:

$\text{H}_2\text{SO}_4, \text{PCl}_5, \text{SOCl}_2 \rightarrow$  provides acidic medium.

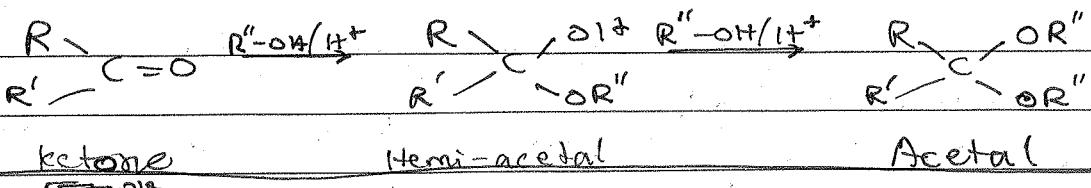


(f) Addition of  $\text{H}_2\text{O}$ :



Distance b/w  $\text{O}^+$  groups  $\uparrow$ ; stability  $\downarrow$ ; strain  $\uparrow$   
strained

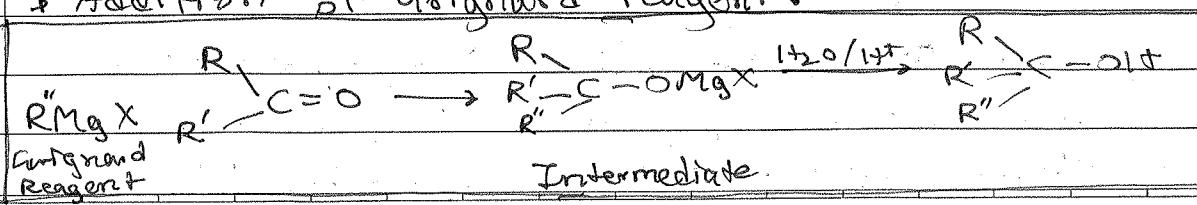
(g) Addition of alcohols:

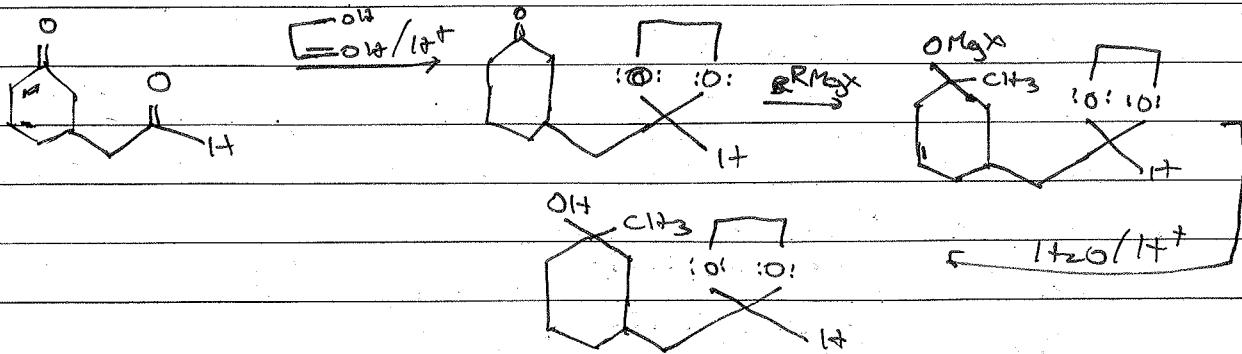


$\text{---O}^+$   $\rightarrow$  ethylene glycol  $\rightarrow$  A good protecting group.

Forms cyclic acetals/cyclic ethers with ketones & Aldehydes

(h) & Addition of Grignard reagent:





Since aldehydes are more reactive than ketones, ethylene glycol protects aldehydes and  $\text{RMgX}$  reacts with ketones.

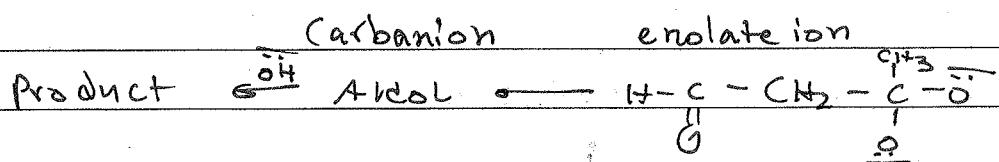
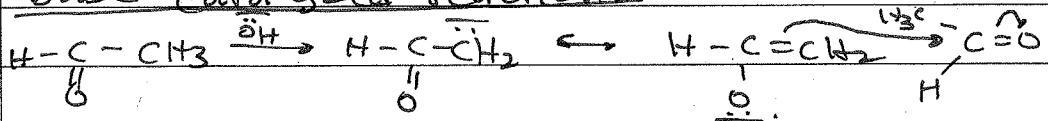
(i) Aldo condensation!

Aldol  $\rightarrow$   $\beta$ -hydroxy aldehyde, containing  $\text{H}_2\text{O}$

Ketol  $\rightarrow$   $\beta$ -hydroxy ketone.

Product:  $\alpha, \beta$ -unsaturated compound.

Base catalyzed reaction:



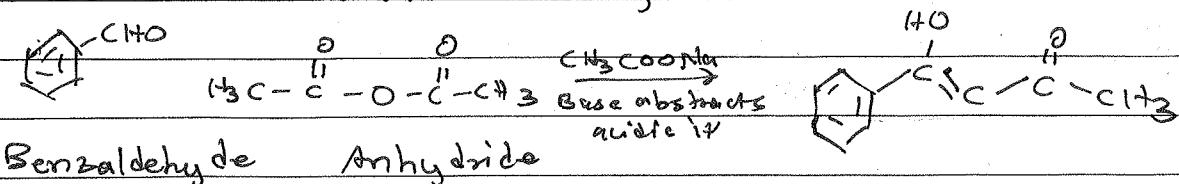
(j) Cannizaro reaction:

Base catalyzed reaction:

No  $\text{H}_2\text{O}$  should be present on Aldehyde / ketone  
Products are  $\text{CO}_2$  and  $1^\circ$  alcohol.

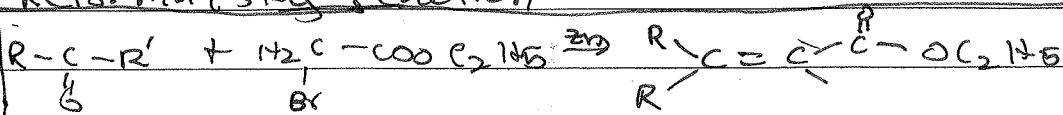
(k) Perkin reaction:

Used for aromatic aldehydes.

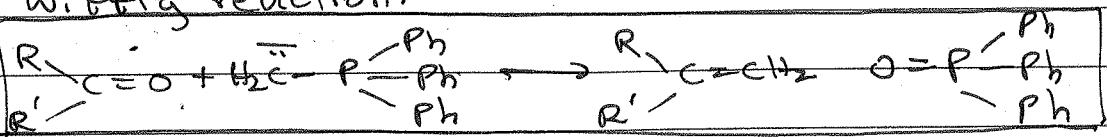


$\alpha, \beta$  unsaturated.

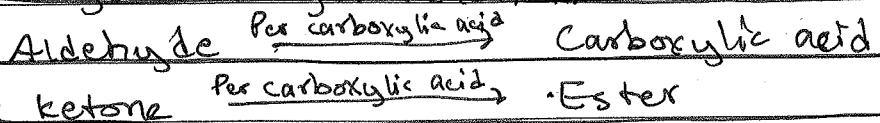
(l) Reformatsky reaction:



(m) Wittig reaction:



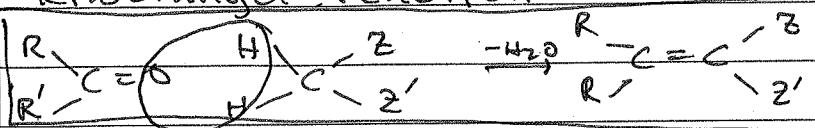
(n) Bayer - Villiger reaction:



mCPBA gives same effect as that of per-acid.

mCPBA  $\rightarrow$  meta-chloro perbenzoic acid.

(o) Knoevenagel reaction:



Z  $\rightarrow$  Strong WDG ( $-CN, -NO_2, -OC_2H_5, -OC_6H_5, -OH$ )

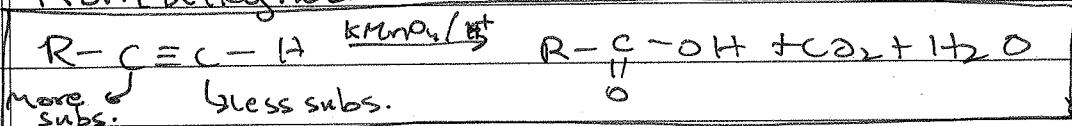
Strength of WDG & Reactivity

- Carboxylic acids:-

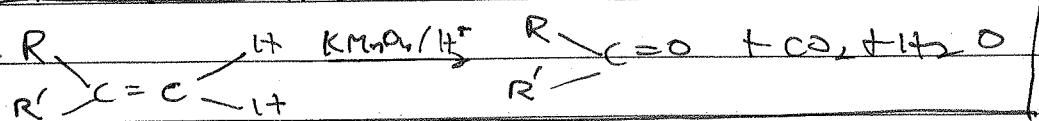
\* Methods of preparation  $\rightarrow$

(a) From hydrocarbons:

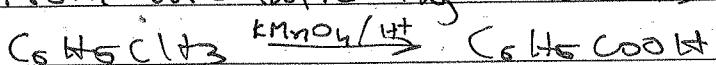
(i) From alkynes:



(ii) From alkenes:



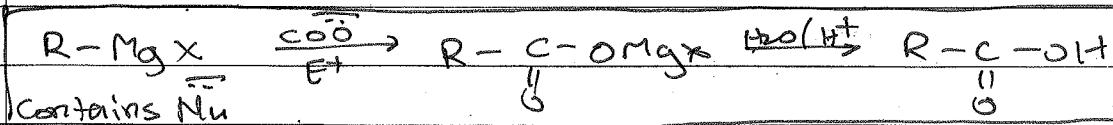
(iii) From aromatic hydrocarbons:



Benzyllic  $H^+$

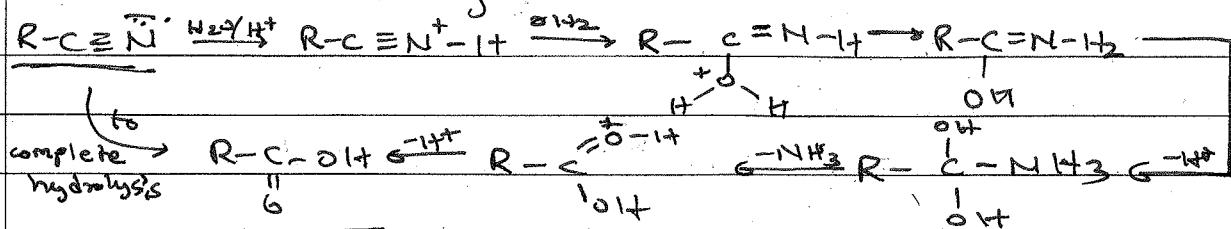
As long as there is benzyllic  $H^+$ , product formed will always be benzoic acid.

(b) From  $RMgX$  and  $CO_2$ :

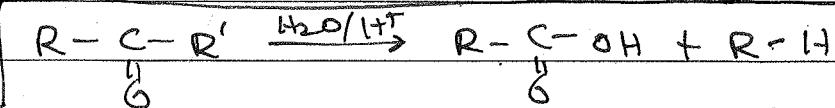
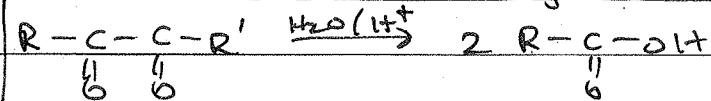


Formic acid cannot be prepared

### (c) Reaction with cyanide's:

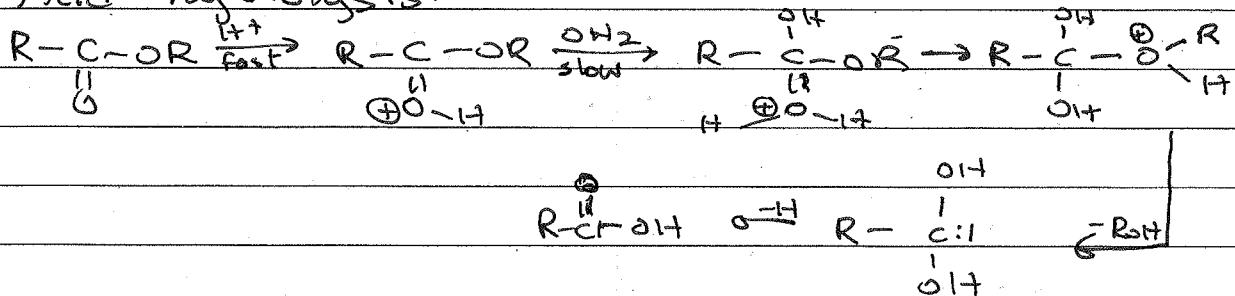


### (d) Reaction with carboxylic acid derivatives



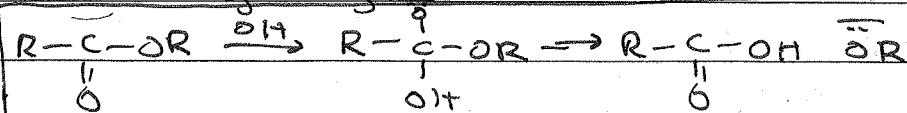
$$R' = (-N1+3, -Cl, -OR)$$

(e) Acid hydrolysis:



$R'$  is chiral, configuration is retained, if C-O bond cleavage takes place.

(f) Base hydrolysis:

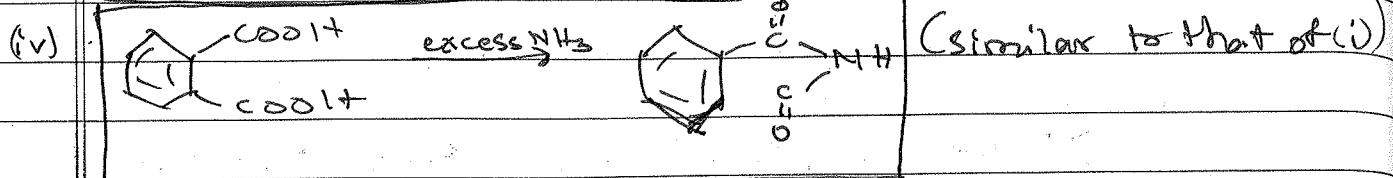
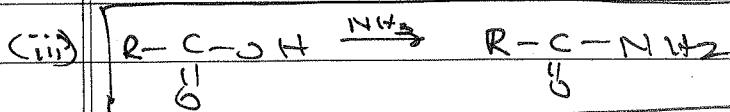


## Properties of carboxylic acids

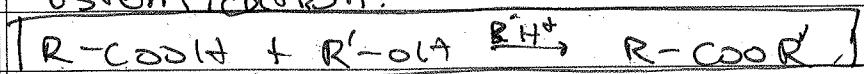
$$(i) \quad R-COOH + NaHCO_3 \xrightarrow{G} R-CO + CO_2 + H_2O$$

(ii)  $R-COOH \xrightarrow{PbO_2}$   $R-C(=O)O + H_2O$

$R-COOH$   $\xrightarrow[\text{Powerful dehydrating agent}]{\text{PbO}_2}$   $R-C(=O)O + H_2O$



(v) Esterification:



$\text{HCOO}^+$ ,  $\text{CH}_3\text{COO}^+$  react faster.

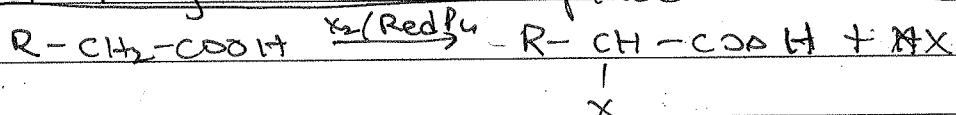
(vi) Trans-esterification:



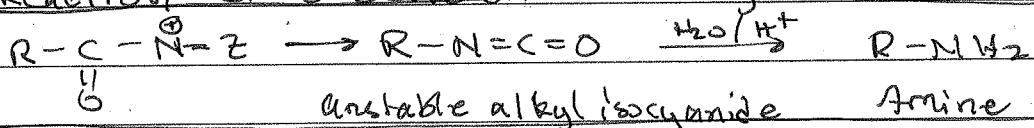
(vii)  $\text{IVZ}$  reaction:

Reagent  $\rightarrow \text{X}_2 / \text{Red P}_4$ .

$\alpha$ -halogenation takes place.



(viii) Reaction of e<sup>-</sup>-deficient nitrogen:



Amine

Z  $\rightarrow$  leaving group

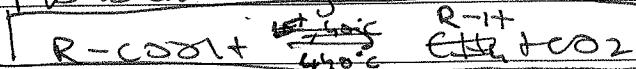
Z =  $-\text{Br}(\text{OR})-\text{Cl} \rightarrow$  Hoffmann Bromamide reaction

=  $-\text{N}_2^+$   $\rightarrow$  Curtius reaction, Schmidt reaction

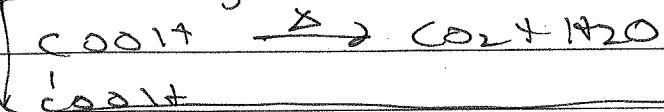
=  $\text{O}-\text{C}-\text{R} \rightarrow$  Lossen re-arrangement

(ix) Decarboxylation:

(a) Monocarboxylic acids:



(b) Dicarboxylic acids:

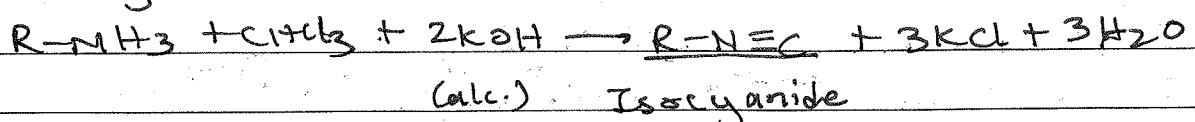


$\beta$ -keto carboxylic acids decarboxylate easily except  $-\text{COOH}$  present on bridge-head carbon ( $n_c=7$ ).

\* Amines:-

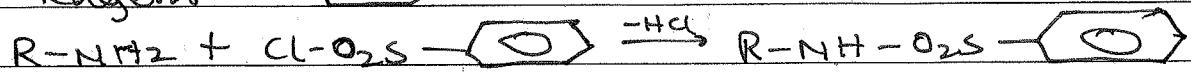
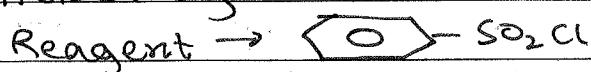
\*) Test for amines →

(a) Carbylaminies test:



Presence of isocyanide confirms presence of amines.

(b) Heisenberg test:

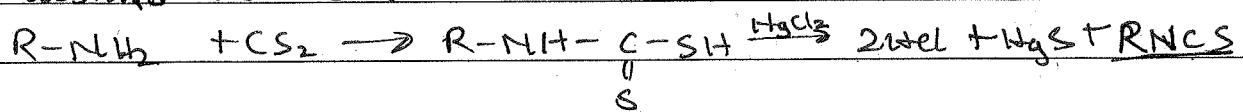


1° amines → Water soluble

2° amines → Water insoluble

3° amines → Get regenerated so, no reaction.

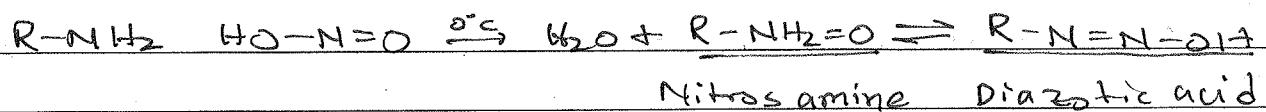
(c) Mustard oil test:



Only in case of 1° amine.

No reaction in case of 2° and 3° amines.

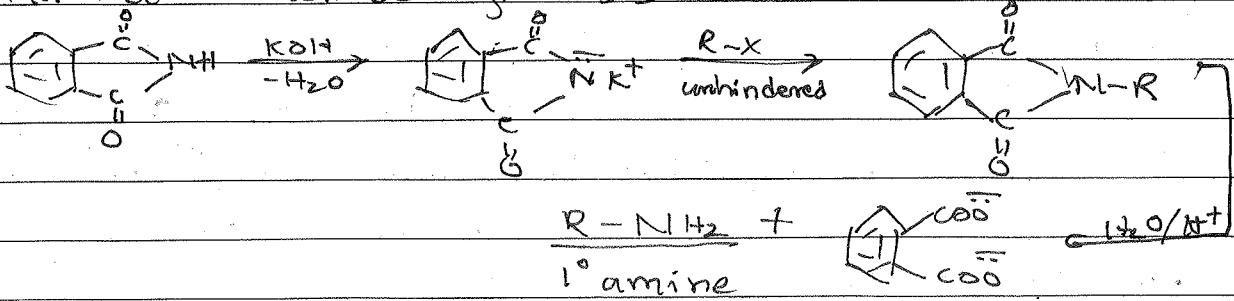
(d) Reaction with  $HNO_2$ :



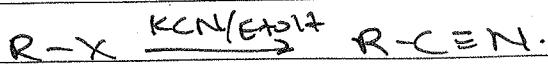
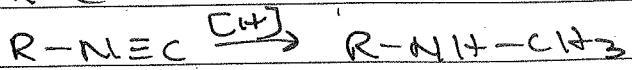
B.R<sub>3</sub>-N HO-N=O  $\xrightarrow{0^\circ C}$  No nitrosoamine formation.

\* Preparation of amines →

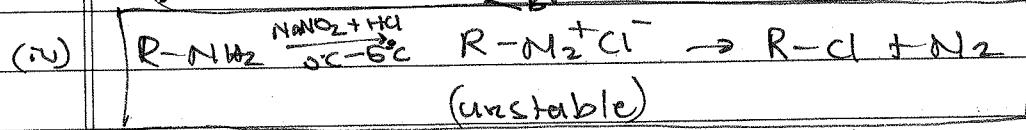
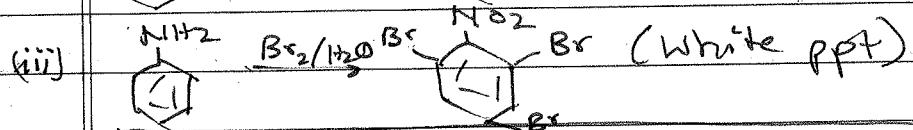
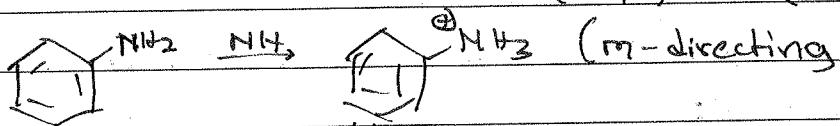
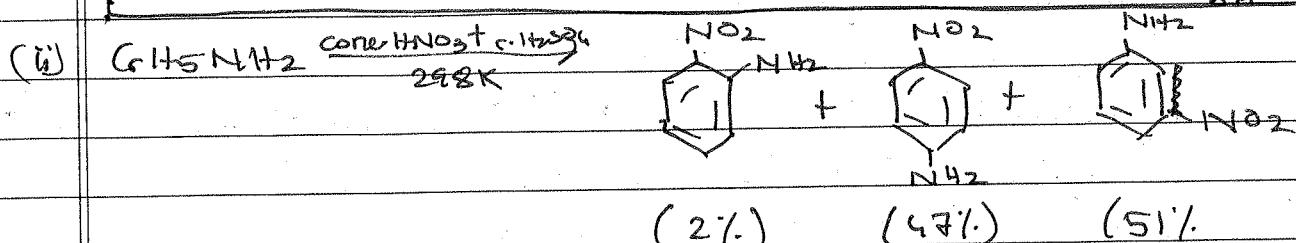
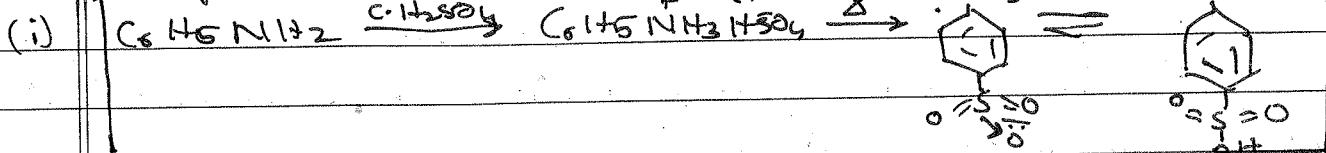
(a) Gabriel Phthalamide synthesis:



(b) Reduction:



\* Properties of amines →



\* Biomolecules :-

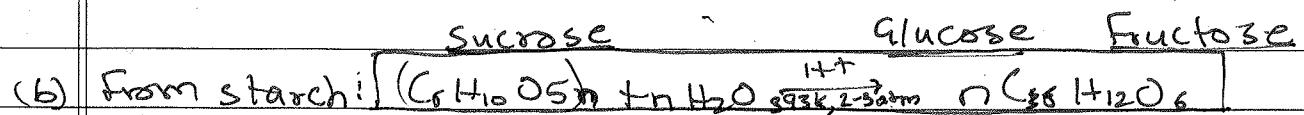
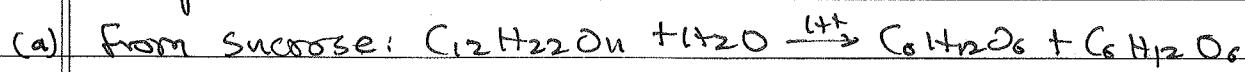
\* Classification of carbohydrates →

- (i) Monosaccharides: Cannot be hydrolysed to simpler unit.
- (ii) Disaccharides: Can hydrolyse two to ten simpler units.
- (iii) Polysaccharides: Can hydrolyse more than ten simple units.
- (iv) Reducing sugars: Can & reduce Fehling's and Tollen's reagent. Eg: Sugars containing  $(-\text{CH}_2\text{OH})$ .
- (v) Non-reducing sugars: Don't reduce Fehling's and Tollen's reagent. Eg: Sugar containing  $(>\text{C}=\text{O})$

\* Glucose →

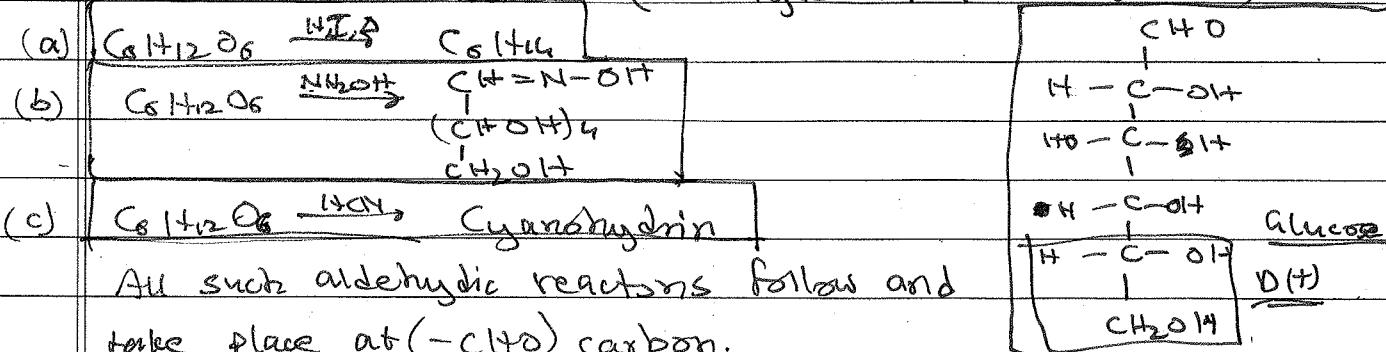
P $\rightarrow$ O

(i) Preparation:



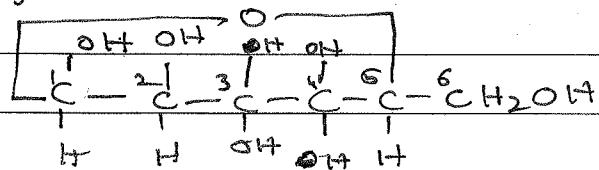
(ii) Structure:

It is an aldohexose (aldehyde with 6-'c' atoms)

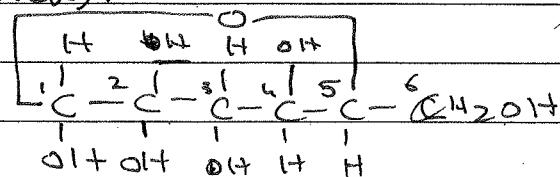


All such aldehydic reactions follow and take place at (-C<sub>1</sub>H<sub>2</sub>O) carbon.

(iii) Cyclic Structure (horizontal view):



$\alpha-(D)-(+)-\text{Glucose}$

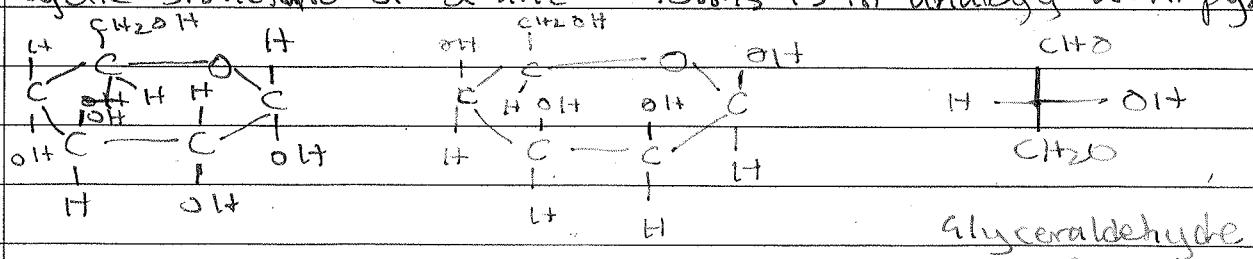


$\beta-(D)-(-)-\text{Glucose}$

Difference in: C-2 subs. pos.; C-1 subs. pos.

$\alpha$ - and  $\beta$ - Forms: anomers.

Cyclic structure of  $\alpha$ - and  $\beta$ - forms is in analogy with pyran.

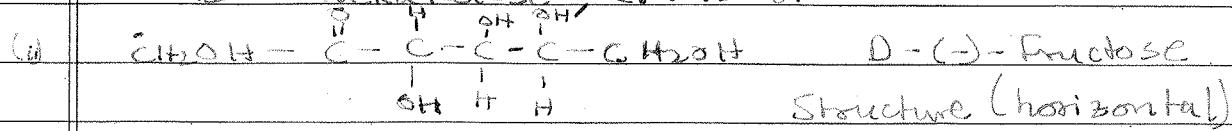


$\alpha-(D)-(+)-\text{Glucopyranose}$        $\beta-(D)-(-)-\text{Glycopyranose}$

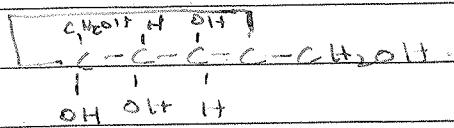
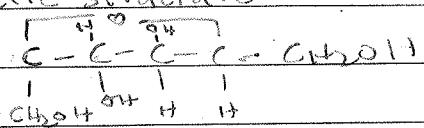


\* Fructose  $\rightarrow$

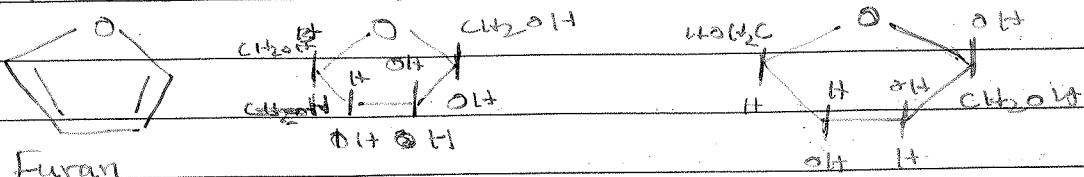
It is a hexaketose  $C_6H_{12}O_6$ .



(ii) cyclic structure:



$\alpha$ -(D)-(-) = Fructofuranose;  $\beta$ -(D)-(-) = Fructofuranose



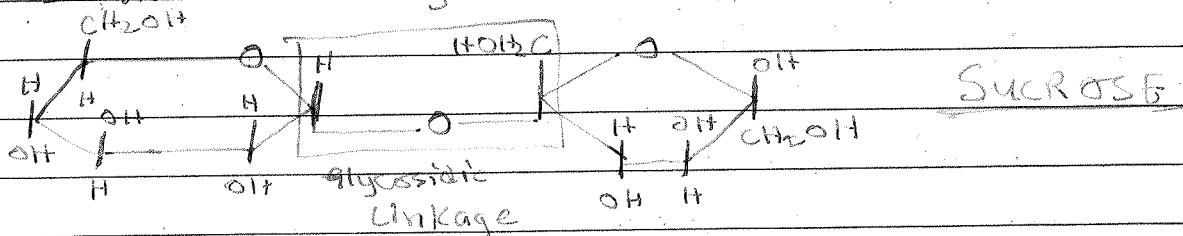
$\alpha$ -(D)-Furanose       $\beta$ -(D)-Fructofuranose

## → Disaccharides →

(a) Sucrose:

Sucrose gives equimolar mixture of Glucose and Fructose.

Initially, sucrose is dextrorotatory and but after hydrolysis, it becomes levorotatory. It is thus called as an invert sugar.



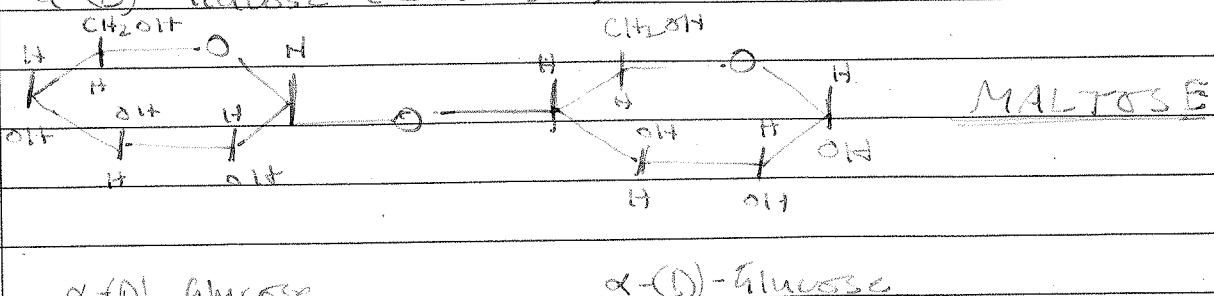
## $\alpha$ -D-Glucose

$\beta$ -(D)-Fructose.

6

## Maltese:

It is formed by glycosidic linkage between C<sub>1</sub> and C<sub>6</sub> of  $\alpha$ -(D)-Glucose (2 of them).

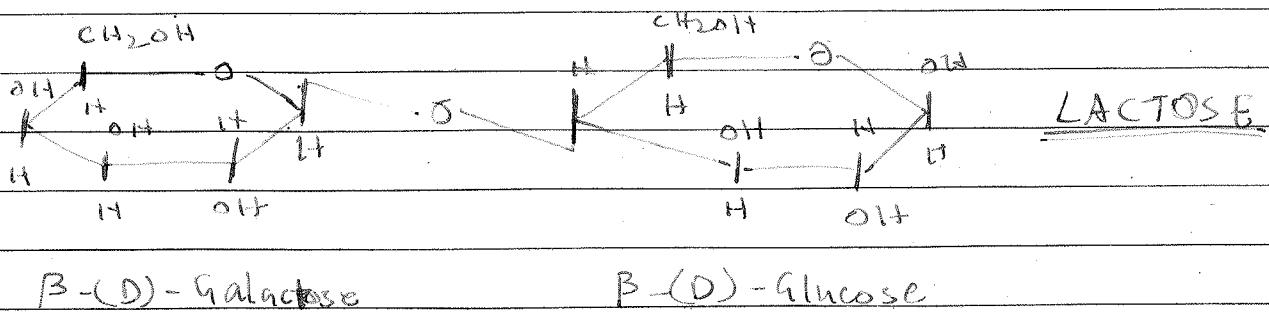


11

## Maltose & Lactose:

Commonly known as milk. It is a reducing sugar.

Structure is: glycosidic linkage between C-1 of  $\beta$ -(D)-Galactose and C-4 of  $\beta$ -(D)-Glucose.



\* Polysaccharides →

(a) Starch:

$(\text{C}_6\text{H}_{10}\text{O}_5)_n$ . Consists of  $\alpha$ -Amylose and Amylopectin.  
Amylopectin constitutes about 80-85% starch. It is water insoluble.

Amylose is water soluble, constitutes 15-20% starch.

(b) Cellulose:

occurs in plants, most abundant organic substance in plants.  
It is a straight chain polysaccharide consisting of only  $\beta$ -(D)-Glucose units, joined by glycosidic linkage.

\* Proteins →

Most abundant biomolecules of the living system.

They are polymers of  $\alpha$ -amino acid

\*  $\alpha$ -Amino acids →

These compounds contain  $(-\text{NH}_2)$  and  $(-\text{COOH})$  groups.

$\alpha, \beta, \gamma, \delta$  are given relative to their position, from  $(-\text{COOH})$ .

(a) Classification of  $\alpha$ -amino acids:

(i) Neutral → Have equal no. of  $(-\text{COOH})$  &  $(-\text{NH}_2)$  ions. E.g.,  
also contain zwitter ions

(ii) Acidic →  $[-\text{COOH} > -\text{NH}_2]$

(iii) Basic →  $[-\text{NH}_2 > -\text{COOH}]$

Proteins are polymers of  $\alpha$ -amino acids and are connected to each other by peptide linkages.

Peptide linkage is an amide formed between  $-\text{COOH}$  and  $-\text{NH}_2$ .

2 amino acids → Dipeptide, 3 → tripeptide,  $> 10$  → polypeptide.

\* levels of studying protein structure →

- Primary structure → No. of polypeptide chains.
- Secondary structure →  $\alpha$ -helix and  $\beta$ -pleated sheet.
- Tertiary structure → Overall folding of polypeptide chain
- Quaternary structure → Spatial arrangement b/w sub-units (polypeptide chains b/w proteins but more than 1 chain)

Loss of biological activity and uncoiling of protein helix when is subjected to chemical or physical changes is called as denaturation of proteins.

\* Enzymes →

Biological catalysts that increase the rate of consumption of carbohydrates and proteins.

Enzymes are named based on the sugar on which they decompose.

\* Vitamins →

Organic compounds that are consumed in small quantities but important to maintain health.

They are required in small diets to perform specific biological functions for normal maintenance of optimum growth and health of the organism.

(a) Classification of vitamins:

(i) Fat soluble → vitamins A, E, D, K.

(ii) Water soluble → vitamins B & C.

Except B12, all other vitamins need to be supplied regularly.

\* Nucleic acids →

(a) DNA: Adenine, Guanine, Thymine, Cytosine.

(b) RNA: First three same, fourth → Uracil.

## • Polymers:-

### → Classification of polymers →

#### (a) Based on source:

- (i) Natural polymers → obtained from plants and animals.
- (ii) Semi-synthetic → Cellulose and its derivatives.
- (iii) Synthetic → Artificially prepared, such as plastics.

#### (b) Based on structure:

- (i) Branched → contain linear chains having some branches.
- (ii) Linear → consist of long and straight chains.
- (iii) Cross-linked → formed from bi and tri-functional monomers.

#### (c) Based on mode of polymerisation:

- (i) Addition polymers: Formed by repeated addition of monomer molecules possessing double (or) triple bonds.
- (ii) Condensation polymers: Repeated condensation b/w 2 bi and tri-functional monomeric units.

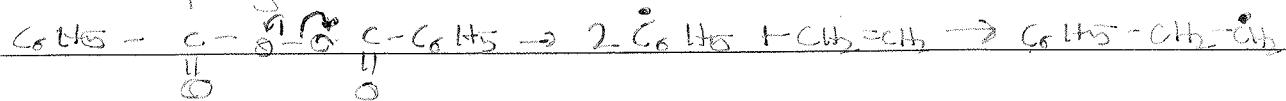
#### (d) Based on molecular forces:

- (i) Elastomers: Rubber-like solids with elastic properties.  
Held by weak inter-molecular forces. (IM)
- (ii) Fibres: Thread forming solids possessing high tensile strength and high modulus. Strong IM forces.
- (iii) Thermoplastics: Can be remoulded after heating.
- (iv) Thermosetting plastics: Cannot be remoulded after heating.

### → Types of polymerisations and polymers formed →

#### (a) Addition polymerisation:

This polymerisation process is taken place by using free radical polymerisation.



Benzoyl peroxide

The chain then continues

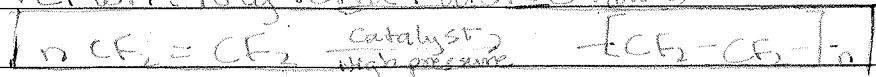
(i) Polythene:

~~(a)~~ LDPE: Benzoyl peroxide catalyst. 100-200 atm pressure,

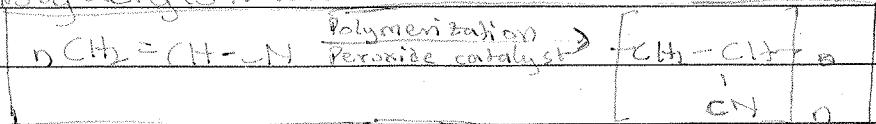
~~HDPE~~: temperature of 350 K to 370 K. O<sub>2</sub> is initiator.

(b) HDPE: Ziegler-Natta catalyst [TiCl<sub>3</sub>(C<sub>2</sub>H<sub>5</sub>)<sub>3</sub>Al], 6-7 atm pressure and 333 K-343 K temperature.

(ii) Teflon (Polytetrafluoroethylene):



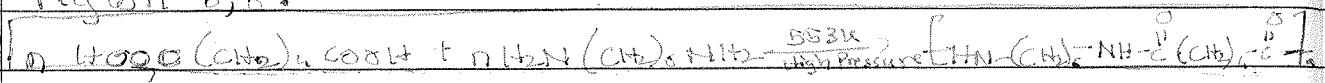
(iii) Polyacrylonitrile:



Used as a substitute for wool

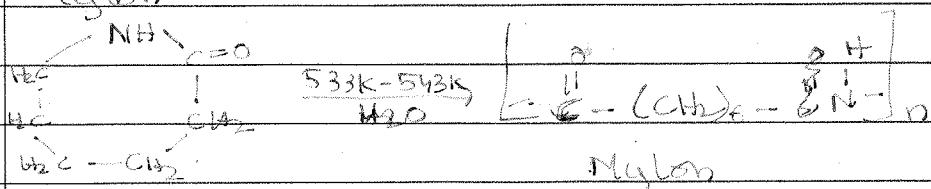
(b) Condensation polymerization:

(i) Nylon 6,6s



Poly adipic acid hexamethylene diamine

(ii) Nylon-6:



Caprolactum

(iii) Polyesters:

Polycondensation products of dicarboxylic acids and diols.

Eg: Dacron, terylene

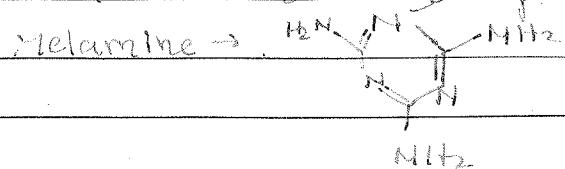
Manufactured by heating mixture of ethylene glycol and terephthalic acid at 420 K to 460 K in the presence of zinc acetate-antimony trioxide catalyst.

(iv) Phenol-formaldehyde polymers:

Usual polymers formed are bakelite and novolac.

- Novolac is first prepared in presence of either base or acid catalyst, which is then heated with HCHO to undergo cross-linking and form an infusible solid called bakelite.

(v) Melamine-formaldehyde polymer



\* Co-polymerisation →

Reaction in which a mixture of more than one monomer species is allowed to polymerise and form a co-polymer.

(a) Natural rubber:

It is an elastomer manufactured from rubber latex.

It is a linear polymer of isoprene (2-methyl 1,3-butadiene).

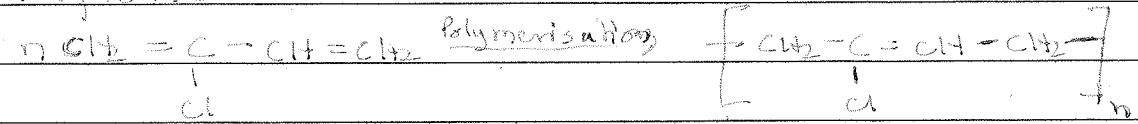
Vulcanisation of rubber is the process to improve the physical properties of natural rubber. 5% of sulphur is used as cross-linking agent.

(b) Synthetic rubbers:

↳ Vulcanisable rubbers like polymer which is capable of getting stretched to about twice its length.

\* Preparation of synthetic rubber →

(a) Neofrene:

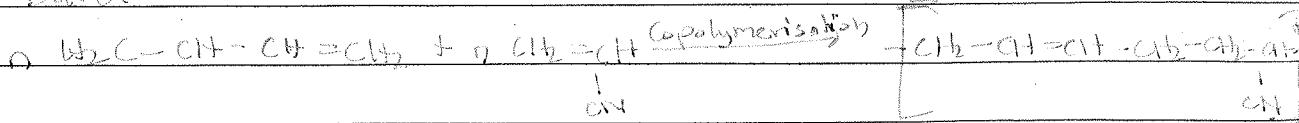


Chloroprene

Polychloroprene / Neoprene

Superior resistance to mineral and vegetable oils. Used for manufacturing conveyor belts, gaskets and hoses.

(b) Buna-N



1,3-butadiene

Acrylonitrile

Buna-N

Resistant to the action of petrol, lubricating oil and organic solvents. Used in making of seals, tank lining, etc.

## MATHEMATICS

### Trigonometry:-

\* Trigonometric ratios →

- (a)  $\sin \theta = \text{opp.}/\text{hypotenuse}$ ,  $\theta \in \mathbb{R}$ ,  $\sin \theta \in [-1, 1]$
- (b)  $\cos \theta = \text{Adj.}/\text{hypotenuse}$ ,  $\cos \theta \in [-1, 1]$
- (c)  $\tan \theta = \sin \theta / \cos \theta$ ,  $\tan \theta \in (-\infty, \infty)$
- (d)  $\operatorname{cosec} \theta = 1 / \sin \theta$ ,  $\operatorname{cosec} \theta \in (-\infty, -1] \cup [1, \infty)$
- (e)  $\sec \theta = 1 / \cos \theta$ ,  $\sec \theta \in (\infty, -1] \cup [1, \infty)$
- (f)  $\cot \theta = 1 / \tan \theta$ ,  $\cot \theta \in (-\infty, \infty)$

\* Identities →

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \sin \theta = \cos(90^\circ - \theta)$$

$$\sec^2 \theta - \tan^2 \theta = 1, \quad \cot \theta = \tan(90^\circ - \theta)$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1, \quad \sec \theta = \operatorname{cosec}(90^\circ - \theta)$$

\* Identities on compound angles →

$$\sin(\theta + \alpha) = \sin \theta \cos \alpha + \sin \alpha \cos \theta$$

$$\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$$

$$\cos(\theta + \alpha) = \cos \theta \cos \alpha - \sin \theta \sin \alpha$$

$$\cos(\theta - \alpha) = \cos \theta \cos \alpha + \sin \theta \sin \alpha$$

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \tan \theta \\ 1 + \tan^2 \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = \cos^2 \theta - 1 = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan(\theta + \alpha) = \frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha}$$

$$\tan(\theta - \alpha) = \frac{\tan \theta - \tan \alpha}{1 + \tan \theta \tan \alpha}$$

$$\begin{aligned}\sin 3\theta &= 3\sin \theta - 4\sin^3 \theta \\ \cos 3\theta &= 4\cos^3 \theta - 3\cos \theta\end{aligned}$$

\* Principal values of trigonometric ratios.  $\rightarrow$

$$\sin \theta = \sin \alpha, \quad \alpha = n\pi + (-1)^n \theta$$

$$\cos \theta = \cos \alpha, \quad \alpha = 2n\pi \pm \theta$$

$$\tan \theta = \tan \alpha, \quad \alpha = n\pi + \theta$$

$$\text{When } \sin \theta = 1, \quad \alpha = \frac{(4n+1)\pi}{2} \quad \text{and when } \sin \theta = -1, \quad \alpha = \frac{(4n-1)\pi}{2}$$

$$\cos \theta = 0, \quad \alpha = \frac{(2n+1)\pi}{2}$$

$$\text{If } \sin \theta = \tan \theta = 0, \quad \alpha = n\pi$$

$$\text{If } \cos \theta = 1, \quad \alpha = 2n\pi, \quad \text{and if } \cos \theta = -1, \quad \alpha = (2n+1)\pi$$

### • Quadratic Equations:

\* Definition  $\rightarrow$

An expression of the form  $ax^2 + bx + c$ , where  $a \neq 0$  and  $a, b, c \in \mathbb{R}$   
is known as a quadratic equation.

$$\text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$D > 0 \rightarrow$  there are 2 distinct roots

$D = 0 \rightarrow$  There is only 1 root / both roots are equal.

$D < 0 \rightarrow$  Roots are imaginary

$$d + Bi = \frac{-b}{a}; \quad dBi = \frac{c}{a} \quad (d, B \rightarrow \text{Roots of the equation})$$

If  $p + \sqrt{q}$  is an irrational root of the quadratic equation, then  
 $p - \sqrt{q}$  is also a root of the equation, provided that all  
co-efficients are rational.

If  $a+b+e=0$ , then 1 is a root of the equation, the other  
being  $(c/a)$ .

If an equation  $ax^2+bx+c=0$  is satisfied by more than 2 distinct numbers, then it is an identity and  $a=b=c=0$ .

\* Transformation of equations →

$$x^2+bx+1 \Rightarrow x=a \Rightarrow x^2=1^2$$

Let  $x^2 = y$ , then  $x = \pm \sqrt{y}$ .

$$y \pm \sqrt{y} + 1 = 0$$

$$y + 1 = \mp \sqrt{y}$$

$$y^2 + 2y + 1 = 0$$

$$y^2 + 2y + 1 = 0$$

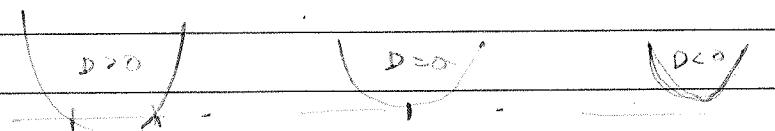
Solve further and substitute  $y = x^2$ .

\* Graphical representation of quadratic equations →

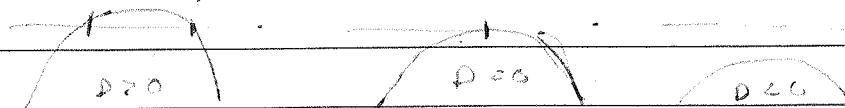
Minimum value =  $-\frac{b}{2a}$

Maximum value =  $-\frac{D}{4a}$

When  $a > 0$ ,



When  $a < 0$ ,



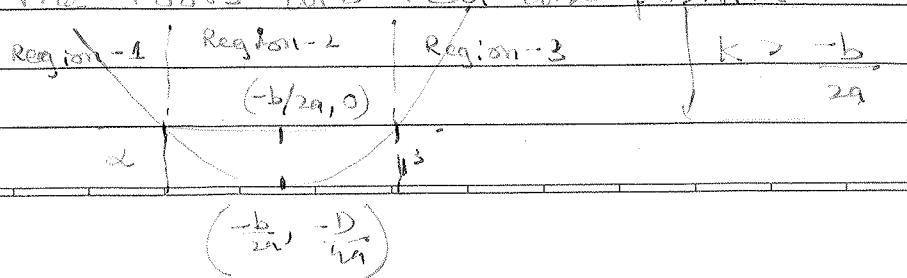
\* Remainder theorem →

If we divide a polynomial  $p(x)$  by  $x-a$ , remainder is  $p(a)$ .

$a \rightarrow$  one of its roots.

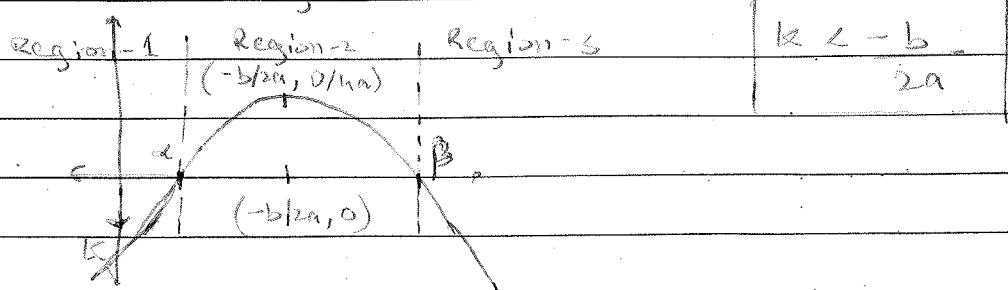
\* Position of roots →

(a) When  $D > 0$  (when roots are less than given number  $b$ ), both the roots are real and positive.



(b)  $\Rightarrow$  When roots are more than a given number  $k$ ;  $D \geq 0$

'k' lies in Region  $\rightarrow$  I



### • Sequence and Series:-

\* Arithmetic Progression (AP)  $\rightarrow$

$$a_n = a + (n-1)d$$

$$S_n = n \left[ \frac{2a + (n-1)d}{2} \right]$$

If  $t_m = a + (m-1)d$ , then series formed is an AP.

If  $S_n = a n^2 + b n + c$ , then series is an AP provided  $c=0$

(a) Middle term term:

(i) 'n' is odd  $\rightarrow$  mid-term =  $\frac{n+1}{2}$

(ii) 'n' is even  $\rightarrow$  mid-term lies b/w  $\frac{n}{2}$  and  $\frac{n+1}{2}$

\* G.P  $\rightarrow$  Geometric progressions (G.P)  $\rightarrow$

$$a r^n = a r^{n-1}$$

$$S_n = a \left( \frac{r^n - 1}{r - 1} \right), r > 1 \text{ and } S_n = a \left( \frac{1 - r^n}{1 - r} \right), r < 1$$

$$\text{Infinite G.P} \rightarrow \left\{ \begin{array}{l} S_\infty = a \\ 1 - r \end{array} \right.$$

\* Harmonic Progression (HP)  $\rightarrow$

If  $a, b, c$  are in HP, then  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are in AP.

\* Relation b/w AM, GM and HM  $\rightarrow$

AM  $\rightarrow$  Arithmetic mean

GM  $\rightarrow$  Geometric mean

HM  $\rightarrow$  Harmonic mean

$$AM = \text{arithmetic mean} = \frac{a_1 + a_2 + \dots + a_n}{n}, \quad GM = \sqrt[n]{a_1 a_2 \dots a_n}$$

$a, A_1, A_2, A_3, \dots, A_n, b \rightarrow A.P. \quad (\text{Terms} = n+2)$

$$b = a + (n+2-1) d$$

$$[b = a + (n+1) d]$$

$$d = \frac{b-a}{n+1}$$

$a, A_1, A_2, A_3, \dots, A_n \rightarrow G.P. \quad (\text{Terms} = n+2)$

$$a_{n+1} = a \cdot b = ad^{n+1}$$

$$[b = ad^{n+1}]$$

\* Sum of miscellaneous series  $\rightarrow$

$$\sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i = \sum_{i=1}^n a_i \pm b_i$$

$$\sum_{i=1}^n i = n(n+1) \quad \sum_{i=1}^n i^2 = (n+1)(n+2) \quad \sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

\* A.G.P  $\rightarrow$

A.P  $\rightarrow a, a+d, a+2d, a+3d, \dots, a+(n-1)d$

G.P  $\rightarrow 1, r, r^2, r^3, \dots, r^{n-1}$

A.G.P  $\rightarrow a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \dots, (a+(n-1)d)r^{n-1}$

$$S = a + (a+d)r + (a+2d)r^2 + \dots$$

$$rs = ar + (a+d)r^2 + (a+2d)r^3 + \dots$$

Subtracting  $-rs$  from  $S$ , we get

$$(1-r)S = a + dr + d(r^2 + r^3 + \dots)$$

$$= a + d(r + r^2 + r^3 + \dots)$$

$$(1-r)S = a + d \cdot r \cdot (1 - r^{n-1}) - [a + (n-1)d]r^n$$

- Binomial theorem:-

\* Definition  $\rightarrow$

$$(a+by)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_n b^n$$

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

$T_{r+1} \rightarrow (r+1)^{\text{th}}$  (greatest) term.

$r^{\text{th}}$  term from end is  $(n-r+2)^{\text{th}}$  term from beginning.

$n$  is even  $\rightarrow$  middle term =  $\binom{n}{2}^{\text{th}}$  term

$n$  is odd  $\rightarrow$  middle  $k^{\text{th}}$  term =  $\binom{n+1}{2}^{\text{th}}$  and  $\binom{n+3}{2}^{\text{th}}$  term

$$\left| \frac{T_{r+1}}{T_r} \right| \geq 1$$

\* Binomial identities  $\rightarrow$

$$(a+b)^n = {}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + {}^n C_n$$

$$a = {}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + (-1)^n C_n$$

$$1. [a^{m+n} + (a-m)]^n = {}^n C_0 + {}^n C_1 + {}^n C_2 + \dots$$

$$2. [(a+m) - (a-n)]^n = {}^n C_0 + {}^n C_1 + {}^n C_2 + \dots$$

Differentiation of  $(1+x)^n = n(1+x)^{n-1}$

Integration of  $(1+x)^n = \frac{(1+x)^{n+1}}{n+1}$

$$\ln(2)^{n+1} = {}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n$$

$$a = {}^n C_1 - {}^n C_2 + {}^n C_3 - {}^n C_4 + \dots$$

\* Binomial expansion for rational index  $\rightarrow$

$$(1+xy)^n = 1 + n! + \frac{n!}{1!(n-1)!} x + \dots$$

$$\text{No. of terms in } (a+by)^n = n+1 \text{ (e.s.)}$$

\* Applications of binomial theorems :-

(a) Finding remainder of a big number

\* Permutations and combinations :-

→ Principle of counting :-

(a) Addition rule:

When two different operations are done in ' $m$ ' and ' $n$ ' different ways respectively, then total no. of ways is  $[m+n]$ .

(b) Multiplication rule:

When two simultaneous (same operation(s)) are done in ' $m$ ' and ' $n$ ' ways respectively, then the total no. of ways is  $[m \times n]$ .

\* Factorials and their properties :-

$$[n!] = n(n-1)(n-2)(n-3) \dots [3 \times 2 \times 1]$$

$$(i) 0! = 1$$

$$(ii) n! = n(n-1)!$$

$$(iii) (2n)! = 2^n [1 \times 2 \times 3 \times 4 \times 5 \dots \times n] [1 \times 3 \times 5 \times 7 \dots (2n+1)!]$$

\* Principle of permutation:-

If ' $n$ ' number of objects are arranged in ' $r$ ' distinct ways, then total no. of ways of arranging is represented by  $[n^P_r]$ .

$$[n^P_r = \frac{n!}{(n-r)!}]$$

No. of permutations of ' $n$ ' distinct objects out of which ' $p$ ' are alike, ' $q$ ' are alike and ' $r$ ' are alike are :-

$$\frac{n!}{p!q!r!}$$

\* Combinations :-

${}^n P_r = (\text{no. of ways of selecting}) \times r!$

$$\therefore {}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

Properties:

$${}^n C_0 = 1 = {}^n C_n$$

$${}^n C_1 = n$$

$${}^n C_r = {}^n C_{n-r}$$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$${}^n C_0 + {}^n C_1 + {}^n C_2 + \dots + {}^n C_n = 2^n$$

If 'n' is odd, then greatest value of  ${}^n C_r$  is either:

$$[{}^n C_{(n+1)/2} \text{ OR } {}^n C_{(n-1)/2}]$$

If 'n' is even  $\rightarrow$  greatest value =  ${}^n C_{n/2}$

No. of ways of selecting 'r' things out of 'n' identical things is 1.

No. of ways of selecting at least 1 object out of 'n' different objects is  $2^n - 1$ .

\* Circular permutations -

No. of circular = no. of linear permutations  $- (n-1)!$

Permutations

Arrangement of a board or  $= (n-1)!$

Flower around a necklace

\* Division in 2 groups -

No. of ways in which 'n' distinct objects can be split into

2 groups containing  $S_1, S_2, S_3, \dots, S_k$  where  $S_i \neq S_j$ , is:

$$n!$$

$$S_1! S_2! S_3! \dots S_k!$$

If 'k' no. of groups are equal, then  $\frac{1}{k!} \times n! = \text{Ways}$

$$k! \quad S_1! S_2! \dots S_k!$$

\* Distribution among persons -

No. of ways in which 'n' objects can be divided into 'r' groups to be distributed among persons / (No. of ways of dividing)  $\left( \begin{matrix} n \\ r \end{matrix} \right)$

→ Number of divisors?

(ii) No. of  $= B_1(2,1)B_2(2,1)\dots B_n(2,1)$   
 i.e.  $= 2^n$

$\lambda, \mu, \nu$  > Prime numbers' occurrence

$$(b) \text{Sum of divisors} = (P_1^{e_1} + 1)(P_2^{e_2} + 1) \cdots (P_n^{e_n} + 1)$$

P  $\rightarrow$  Distinct states.

Ex:  $2^5$ ,  $2 \rightarrow P=2$  and  $d=5$ .

Noise source of putting  $\mathbf{y} = \frac{1}{2} (\text{dist}) (\text{dist}) \dots (\text{dist})$   
as a product of natural nos.

→ Original number (dividend)

$\rightarrow$  Multinomial theorem -

$$(\text{C}_6\text{H}_5)_2\text{P} = -\eta_1 \text{Co}^{2+} + \eta_2 \text{Cr}^{2+} + \eta_3 \text{Fe}^{2+} + \eta_4 \text{Mn}^{2+}$$

$$(1+xt+x^2+t^3+xt^5-t^7)(1+x+x^2+x^3+x^5+x^7+x^8+x^9)$$

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7 36  
7 44

$$n^{\beta} > x^{\gamma}$$

3 3 3 3

三

$$\int x^k \rightarrow x^{k+1} / k + C_k \rightarrow \text{(General term)}$$

## Complex numbers!

→ Definition →

A number that can be represented in the form of  $a+bi$ , where  $a, b \in \mathbb{R}$  and  $i = \sqrt{-1}$ .

\* Identities of complex numbers  $\rightarrow$

- $(a+ib) + (c+id) = (a+c) + i(b+d)$
- $(a+ib) - (c+id) = (a-c) + i(b-d)$
- $(a+ib)(c+id) = ac-bd + i(ad+bc)$
- $a+ib = \frac{ac+bd}{c^2+d^2} + i\frac{ad+bc}{c^2+d^2}$
- $|a+ib| = \sqrt{a^2+b^2}$

$$(v) |k(a+ib)| = |ka+kb|$$

\* Integrals powers of  $i \rightarrow$

$$\begin{cases} i^{4k+1} = i & ; i^{4k+2} = -1 \\ i^{4k+3} = -i & ; i^{4k+4} = 1 \end{cases}$$

Also,  $i^{4m} + i^{4n} = 0$  and  $i^{4m+2} + i^{4n+2} = 0$

\* Geometrical representation of complex numbers  $\rightarrow$

$$z = r(\cos \theta + i \sin \theta)$$

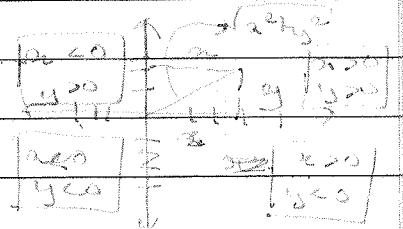
$\tan \theta = y \Rightarrow \text{Imaginary}(z)$   
 $r \cos \theta = x \Rightarrow \text{Real}(z)$

$riy \in I^{\text{st}} \Rightarrow \theta$

$riy \in II^{\text{nd}} \Rightarrow \pi - \theta$

$riy \in III^{\text{rd}} \Rightarrow \pi + \theta$

$riy \in IV^{\text{th}} \Rightarrow 2\pi - \theta$



\* Conjugate of a complex number  $\rightarrow$

If  $z = a+ib$ , then  $\bar{z} = a-ib$

$\bar{z} \Rightarrow$  conjugate of complex number.

Properties:

$$(i) |z| = |\bar{z}| \geq 0$$

$$(ii) z + \bar{z} = |z|^2$$

$$(iii) \text{Real}(z) = \frac{1}{2}(z + \bar{z})$$

$$(iv) \text{Imaginary}(z) = \frac{1}{2}(z - \bar{z})$$

$$(v) x=0 \Rightarrow z \text{ is imaginary}; y=0 \Rightarrow z \text{ is real}$$

- (vi)  $|(z_1 + z_2)|^2 = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$
- (vii)  $|(z_1 - z_2)|^2 = |z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z_1 \bar{z}_2)$
- (viii)  $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 + z_2| \geq |z_1| - |z_2|$

\* Properties of argument of a complex number  $\rightarrow$

- (i)  $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2k\pi$
- (ii)  $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2k\pi$

Same goes till 'n' terms in argument

- (iii)  $\arg(i/b)$
- If  $b > 0$ , then  $\theta \in \pi/2$
- If  $b < 0$ , then  $\theta \in -\pi/2$
- (iv)  $\arg(-b) = -\pi + \arg(b)$
- (v)  $\arg(b^2) = 2\arg(b)$
- (vi)  $\arg(z) \in [0, \pi] \rightarrow \text{Real}$   
 $\in (\pi, 2\pi) \rightarrow \text{Imaginary}$
- (vii)  $\arg(\bar{z}) = \arg\left(\frac{1}{z}\right) = -\arg(z)$

\* Polar form of a complex no.  $\rightarrow$

$$z = |z| e^{i\theta}$$

$$\theta = \cos^{-1} \frac{x}{r} + i \sin^{-1} \frac{y}{r} = \operatorname{Arg}(z)$$

Properties

\* Cube root of unity  $\rightarrow$

$$z^3 = 1$$

Roots  $\rightarrow 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i$  and  $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$$\begin{aligned} w &= e^{i(2\pi/3)} \\ w^2 &= e^{i(4\pi/3)} \end{aligned}$$

Properties:

(i)  $1, \omega, \omega^2$  are vertices of an equilateral triangle inscribed in a circle of radius 1 unit.

$$(ii) |1 + \omega + \omega^2| = ?$$

$$(iii) \omega^{3n} = ?$$

$$(iv) |1 + \omega^n + \omega^{2n}| = ?$$

$$(v) |1 + \omega^{3n} + (\omega^2)^{3n}| = ?$$

\* De-Moivre's theorem  $\rightarrow$

$$z^n = (\cos \theta + i \sin \theta)^n$$

$$= e^{in\theta}$$

$$z^n = \cos(n\theta) + i \sin(n\theta) = (r^n)(\cos \theta + i \sin \theta)$$

$$z^n = r^n e^{in\theta} = r e^{i(n\theta)}$$

\*  $n^{\text{th}}$  roots of unity?

$$z^n = 1$$

$$\left| z = \text{cis}\left(\frac{2k\pi i}{n}\right) \right|$$

\* Straight line:-

\* Introduction to co-ordinate geometry?

$$(i) \text{Distance formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(ii) Section formula:

$$\text{External} \rightarrow x = \frac{mx_1 + nx_2}{m+n}, y = \frac{my_1 + ny_2}{m+n}$$

$$\text{Internal} \rightarrow x = \frac{mx_2 - mx_1}{m-n}, y = \frac{my_2 - my_1}{m-n}$$

(iii) Where medians intersect  $\rightarrow$  Centroid.

PoC of altitudes  $\rightarrow$  orthocentre

PoC of angle bisector  $\rightarrow$  circumcentre

$$\text{Centroid}(G) = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

In equilateral  $\Delta$ , they coincide, otherwise they are collinear.

\* Equations of a line ?

- slope-intercept form  $\rightarrow y = mx + c$
- slope-point form  $\rightarrow (y - y_1) = m(x - x_1)$
- two-point form  $\rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
- double-intercept form  $\rightarrow \frac{x}{a} + \frac{y}{b} = 1$

(v) Normal form  $\rightarrow ax + by + c = 0$

$a \rightarrow$  distance from origin.

Slope of line =  $-c/a$ .

- parametric form  $x = a + t \cos \theta$   
 $y = b + t \sin \theta$

\* Slope of a line ?

$$\text{Slope } m = \tan \theta$$

$$\text{Angle } \theta \text{ b/w } = \cos \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

b/w two lines

\* Distance between 2 lines ?

Normally we talk about 2 parallel lines.

$$d = \frac{|a_1 x_1 + b_1 y_1 + c_1|}{\sqrt{a_1^2 + b_1^2}}$$

$$d = \frac{|a_2 x_1 + b_2 y_1 + c_2|}{\sqrt{a_2^2 + b_2^2}}$$

\* Line at a point ?

This is a set of points satisfying a given equation  
(say,  $ax + by + c = 0$ )

\* Family of lines ?

$$\text{When slopes are fixed, } \left\{ \begin{array}{l} a_1 = -A_1, \dots, A_3 = -a_3 \\ b_1 = -B_1, \dots, B_3 = -b_3 \end{array} \right.$$

$A_1, B_1$  are fixed lines, from family of lines:

$$A_1 x + B_1 y = a_1 x + b_1 y = 0$$

If we use concept of family of lines, we do not actually

Need to find point of lines.

\* Rotation :-

$x = r \cos \theta$  and  $y = r \sin \theta$

$$(x) = (\cos \theta \ - \sin \theta) (r)$$

$$(y) = (\sin \theta \ \cos \theta) (r)$$

$x, y \rightarrow$  New co-ordinates.

\* Pair of lines :-

Combination of 2 lines.

Homogeneous equation [ $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ]

Not all equations represent pair of lines. It may a circle, ellipse, parabola or hyperbola.

(Condition for equation to be a pair of lines)  $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$

We can use concept of quadratic roots here assuming slopes to be the roots of the equation

$$\tan \theta = \frac{\sqrt{h^2 - ab}}{a+b} \quad (\text{angle b/w pair of lines})$$

\* Circles :-

\* Definition :-

A locus of points in a plane where points are equidistant from a given point.

$$\text{If centre } = (a, b) \rightarrow [(x-a)^2 + (y-b)^2 = r^2]$$

$$\text{If centre } = (0,0) \rightarrow x^2 + y^2 = r^2$$

$r \rightarrow$  Radius of the circle.

$$r^2 = x^2 + y^2 - r^2$$

Set a point on the circle,  $|S=0|$

Set a point inside the circle,  $|S<0|$

Set a point outside the circle,  $|S>0|$ .

'S' is called as power of the circle point w.r.t. circle.

$$S = x^2 + y^2 + 2gx + 2fy + c$$

Special case / equation of PNSL.

$$\text{Centre} = (a, b) \quad \text{and radius} = \sqrt{a^2 + b^2 - c}$$

In case of director's circle,  $[S = r^2]$

\* Different equation forms of a circle  $\rightarrow$

(i) Standard form  $\rightarrow x^2 + y^2 = r^2$

(ii) Parametric form  $\rightarrow x = r \cos \theta, y = r \sin \theta$

Add x, y if centre becomes ss.

(iii) Diametric form  $\rightarrow (x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

\* Equation of tangent  $\rightarrow$

Slope of  $\ell$  is  $y_1$ , Slope of  $= -x_1$   
Hence  $m_1$  tangent  $\ell_1$

$$\text{Equation of } \ell_1: y - y_1 = -x_1$$

tangent	$x - x_1$	$y - y_1$
---------	-----------	-----------

(i)  $x^2 + y^2 + 2gx_1 = r^2$

(ii) Parametric form  $\rightarrow x \cos \theta + y \sin \theta = r$

(iii) Slope intercept  $\rightarrow y = mx + r$

$$T_C = r^2(1 + m^2)$$

\* Finding equation of pair of tangents  $\rightarrow$

$$T = 2x_1 + 2y_1 - r^2$$

$$S_1 = x^2 + y^2 - r^2$$

$$T^2 = S_1 \quad (S \text{ is mentioned above})$$

Diameter  $\Rightarrow$  Distance of chord from Centre

equation from line

\* Chord of contact  $\equiv (CSC) \rightarrow$

In this, we see that  $l_1 T_2 l_2$

i.e. the chord formed by joining points of intersection

of pair of tangents from a point on the circle

$$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$$

$\rightarrow$  In a chord with a given mid-point

(T.S.)

$$S_1 = ax_1^2 + 2h_1x_1y_1 + b_1y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1$$

\* Radical axis of two circles  $\rightarrow$

$$2(a_1 - a_2)x_1 + 2(h_1 - h_2)y_1 + c_1 - c_2 = 0$$

It is the locus of point whose power of point to the 2 circles is same.

Position of radical axis:-

(i)  $C_1C_2 > r_1 + r_2 \rightarrow$  In b/w centres  $\perp$  to line joining centre closer to centre of smaller circle.

(ii)  $C_1C_2 = r_1 + r_2 \rightarrow$  Circles touching each other externally.

(iii)  $C_1C_2 < r_1 + r_2 \rightarrow$  Common secant

(iv)  $C_1C_2 = (r_1 - r_2) \rightarrow$  Radial axis will be common tangent

(v)  $C_1C_2 > (r_1 - r_2) \rightarrow$  It lies outside the two circles.

(vi)  $C_1C_2 = 0 \rightarrow$  No common tangents or secants.

\* Family of circles  $\rightarrow$

Equation of family of circles passing through the intersection of 2 circles  $\rightarrow$  given by

$$(S + kS') = 0$$

• Functions:-

\* Set  $\rightarrow$

It is a collection of well-defined objects.

(a) set-builder form:

$$A = \{1, 2, 3, 4, 5\}, C_N = 5$$

Cardinal numbers ( $C_N$ ) is defined as the number of elements present in a set.

(b) Roster form:

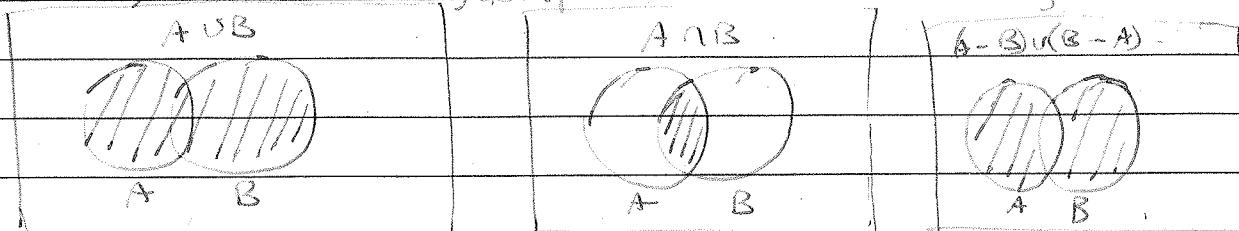
$$A = \{x : x \in N, x \in S\}$$

- (iii) Null set  $\rightarrow \{\emptyset\}$  nothing is present
- (iv) Singleton set  $\rightarrow \{\{x\}\}$  Only one element present
- (v) Finite set  $\rightarrow$  Finite CN
- Infinite set  $\rightarrow$  Infinite CN
- (vi) Subset  $\rightarrow$

No. of subsets possible =  $2^n$

$n \rightarrow$  No. of cardinal numbers present in the set.

\* Unions, intersections, compliments and Venn Diagram  $\rightarrow$



$$(i) (A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

$$(iii) A - B' = A \cap B'$$

$$(iv) B - A = B \cap A'$$

$$(v) n(A') = R - n(A)$$

R  $\rightarrow$  Universal set

$$n(A' \cap B') = n(A \cup B)' = R - n(A \cup B)$$

$$n(A' \cup B') = n(A \cap B)' = R - n(A \cap B)$$

\* Relations  $\rightarrow$

(i) Product of 2 sets:

$$A, B \rightarrow A \times B$$

One element of a set is multiplied with one element of another set, at a time.

Relation  $\rightarrow A \rightarrow B$

No. of relations possible =  $2^{m^n}$

$m \rightarrow$  No. of elements in domain

$n \rightarrow$  No. of elements in co-domain.

\* Types of relations:

- (a) Reflexive  $\rightarrow$  In a set A, if  $a \in A$ , then  $(a, a) \in R$ .
- (b) Symmetric  $\rightarrow$  If  $a_1, a_2 \in A$ , then  $(a_1, a_2) \in R$  as well as  $(a_2, a_1) \in R$ .
- (c) Transitive  $\rightarrow$  If  $a_1, a_2, a_3 \in A$ , then  $(a_1, a_2) \in R$ ,  $(a_2, a_3) \in R$  and  $(a_1, a_3) \in R$ .
- (d) Equivalence  $\rightarrow$  A combination of the first three relations.

### \* Functions $\rightarrow$

These are special types of relations.

Types of functions:

#### (i) onto function:

$f: X \rightarrow Y$ , then every element of Y is the image of some element under X under f i.e. for every  $y \in Y$

#### (ii) One-one Function:

$f: X \rightarrow Y$ , then function is one-one if images of distinct elements of X under f are distinct, i.e. for every  $x_1, x_2 \in X$

$$f(x_1) = f(x_2), \text{ implying } x_1 = x_2$$

If function is not onto  $\rightarrow$  Into function

If function is not one-one  $\rightarrow$  Many-one function

One-one (injective) + onto (surjective)  $\rightarrow$  Bijective

#### (iii) Composite functions:

$f \circ g$  and  $g \circ f$ .

They are always bijective in nature.

#### (iv) Invertible functions:

$f: X \rightarrow Y$  is invertible if there exists a function  $f: Y \rightarrow X$ ,

such that  $g \circ f = I_X$  and  $f \circ g = I_Y$ . Denoted by  $f^{-1}$ .

### \* Graph Transformations $\rightarrow$

(i)  $f(x) + a \rightarrow$  Shifts upwards 'a' distance above origin.

(ii)  $f(x) - a \rightarrow$  Shifts downwards 'a' distance

(iii)  $f(x+a) \rightarrow$  Along (+)ve x-axis.

(iv)  $f(x-a) \rightarrow$  Along (-)ve x-axis.

(v)  $y = -f(x) \rightarrow$  Take of image of  $f(x)$  along x-axis.

(vi)  $y = f(-x) \rightarrow$  Take image of  $f(x)$  along y-axis.

- (vii) (~~iv~~)  $y = |f(x)| \rightarrow$  graph below  $x$ -axis (i.e.  $y < 0$ ) will get inverted in the opposite, i.e. same direction.
- (viii)  $y = f(-x) \rightarrow$  Identical in 1<sup>st</sup> and 4<sup>th</sup> quadrant and symmetrical about  $y$ -axis.
- (ix)  $y = af(x) \rightarrow$  The corresponding points would have their ordinates in the ratio 1:a.
- (x)  $y = f(ax) \rightarrow$  Stretch ' $1/a$ ' units along  $y$ -axis.  
Compress 'a' units along  $x$ -axis.
- (xi)  $|y| = f(x) \rightarrow f(x) \geq 0$  would be reflected along  $x$ -axis  
 $f(x) < 0$  would get omitted.
- (xii)  $y = [f(x)] \rightarrow$  Mark each point of new graph and plot accordingly.
- (xiii)  $y = f[n] \rightarrow$  Follow same procedure as (xi).
- \* ~~(xiv)~~ Even and odd functions  $\rightarrow$
- (i) Even Functions: Functions where  $f(-x) = f(x)$ .
  - (ii) Odd Functions: Functions where  $f(-x) = -f(x)$ .

### \* Limits, continuity, Differentiability:-

#### \* Limits $\Rightarrow$

It's that value obtained when the function is approaching or tending to a certain value.

$$\text{LHL} = \text{RHL}$$

Indeterminate forms:

$$\left[ \frac{\infty}{\infty}, \frac{\infty}{0}, \frac{0}{0}, \infty - \infty, (\infty)^{\infty}, (0)^{\infty} \right]$$

#### \* Evaluation of limits $\rightarrow$

##### (a) Rationalization $\Rightarrow$

##### (b) Factorization

(c) If  $\lim_{n \rightarrow \infty} \frac{a_0 + a_1 n + a_2 n^2 + \dots + a_m n^m}{b_0 + b_1 n + b_2 n^2 + \dots + b_m n^m}$

It will be based on the largest powers of numerator and denominator.

If  $\text{num} = \text{den} \rightarrow \lim_{n \rightarrow \infty} = \infty$ ,

If  $\text{num} < \text{den} \rightarrow 0$

If  $\text{num} > \text{den} \rightarrow \infty$

#### (iv) L-H Rule:

If  $f(x)$  is a limit tending to zero and it comes of form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ , then

$$\lim_{x \rightarrow 0} \frac{\text{f}(x)}{\text{g}(x)} = \frac{\lim_{x \rightarrow 0} \text{f}(x)}{\lim_{x \rightarrow 0} \text{g}(x)} = \frac{\lim_{x \rightarrow 0} \text{f}'(x)}{\lim_{x \rightarrow 0} \text{g}'(x)}$$

$\rightarrow$  Using standard limits  $\rightarrow$

$$(a) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \tan x = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin^{-1} x} = \lim_{x \rightarrow 0} \frac{x}{\tan^{-1} x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$$

$$(c) \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$(d) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$$

$$(e) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$(f) \lim_{x \rightarrow 0} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

$$(g) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{y \rightarrow 0} \frac{y \sin 1}{y} = 1$$

$$(h) \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} \frac{[f(x)-f(a)]}{[g(x)-g(a)]} g(x) \quad (\text{Form: } (-\infty), (\infty))$$

#### (i) Sandwich theorem:

$$\text{If } g(x) \leq f(x) \leq h(x), \text{ then } \lim_{x \rightarrow a} g(x) \leq \lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} h(x)$$

\* Continuity and its types  $\rightarrow$

$$\text{LHL} = \text{RHL} = f(a) \rightarrow \text{Condition for continuity}$$

(Graph is smooth and symmetric at a particular point.)

Types of discontinuity:

(a) Removable discontinuity  $\rightarrow \text{LHL} = \text{RHL} \neq f(a)$

(b) Jump discontinuity  $\rightarrow \text{LHL} = f(a) \neq \text{RHL}$  (left continuous)

$$\text{RHL} = f(a) + \text{LHL}$$
 (Right continuous)

(c) Infinite discontinuity  $\rightarrow \text{LHL} = \text{RHL} = \text{Infinite}$

(d) End point discontinuity  $\rightarrow x=a$ , LHL does not exist, RHL = 0

\* Functional equations  $\rightarrow$

(a)  $f(ax) = f(a) + f(x) \rightarrow$  e.g. logarithmic function

(b)  $f(a+xy) = f(x) + f(y) \rightarrow$  e.g. Any norm multiplication function.

(c)  $f(a^xy) = f(x). f(y) \rightarrow$  e.g. Exponential function

(d)  $f(xy) = f(x). f(y) \rightarrow$  e.g. Any function.

(e)  $f(a). f(1/x) = f(a) + f(1/x)$

\* Differentiability  $\rightarrow$

$\lim_{x \rightarrow a} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \rightarrow$  derivative

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}, \text{RHD} = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{h}$$

If LHD = RHD, then  $f(x)$  is differentiable.

\* Application of Derivatives

\* Derivatives as rate of change  $\rightarrow$

Rate of change is always given by

$$y = \frac{dx}{dt}$$

$dt \rightarrow$  change in time.

This is normally applied in kinematics.

\* Errors and approximations ->

Let the function given be  $f(a+h)$ ,  $f(a)$ .

If it is close to a known value, then  $f(a)$  can be re-written as  $f(a+h) = f(a) + h \cdot f'(a)$ , where value of  $f(a)$  is known to us.

$$\therefore f(a) = h \cdot f'(a+h) - f(a+h)$$

$f(a+h) \rightarrow$  known value.

$f(a) \rightarrow$  to be found.

\* Equation of tangent and normal ->

$$y - y_1 = m(x - x_1)$$

$$\text{for tangent} \rightarrow m = f'(x_1)$$

$$\text{for normal} \rightarrow m = -\frac{1}{f'(x_1)}$$

$$\text{Angle b/w curves } \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Always remember to find point of intersection, except in case of orthogonal curves.

\* Length of tangent, sub-tangent, normal and sub-normal ->

tangent  $\rightarrow$   $y \cos \theta$

normal  $\rightarrow$   $y \sin \theta$

sub-tangent  $\rightarrow$   $y \cot \theta$

sub-normal  $\rightarrow$   $y \tan \theta$

\* Monotonicity ->

(a) Increasing Functions:

$f'(x) \geq 0 \rightarrow$  Input increases, output increases

(b) Decreasing Functions:

$f'(x) \leq 0 \rightarrow$  Input decreases, output decreases

(c) Strictly increasing and decreasing functions:

Strictly inc.  $\rightarrow f'(x) > 0$ ; strictly dec.  $\rightarrow f'(x) < 0$

If  $f'(x) = 0$  / Not defined  $\rightarrow$  critical point

If  $f'(x) = \infty \rightarrow$  stationary point

\* Rolle's theorem  $\rightarrow$

$$f(x) \rightarrow x \in [a, b]$$

(i)  $f(a)$  should be continuous between  $[a, b]$ .

(ii)  $f'(x)$  should be differentiable in  $(a, b)$ .

$$f(a) = f(b)$$

$$f'(c) = 0, c \in (a, b)$$

There is at least one value of 'c' being satisfied.

\* Lagrange's mean value theorem (LMVT)  $\rightarrow$

(i)  $f(x)$  should be continuous in  $[a, b]$ .

(ii)  $f'(x)$  should be differentiable in  $(a, b)$ .

$$\frac{f(b) - f(a)}{b - a}$$

\* Local maxima and minima  $\rightarrow$

(a) Steps to find local minima/maxima:

(i) Find critical point of the function.

(ii) Make sure the critical point lies in the domain of  $f(x)$ .

(iii) If  $f'(x)$  goes from +ve to -ve  $\rightarrow$  local minima

If  $f'(x)$  goes from -ve to +ve  $\rightarrow$  local maxima.

(b) Second derivative test:

(i) Find double derivative of given function.

(ii) If  $f''(x_{cp}) < 0 \rightarrow$  local maxima

If  $f''(x_{cp}) > 0 \rightarrow$  local minima.

(c)  $n^{th}$  derivative test:

(i) Find the  $n^{th}$  derivative of the function.

(ii) Odd derivatives:

$f^n(x_{cp}) = 0 \rightarrow$  (exists)  $f^{n+1}(x_{cp}) > 0 \rightarrow$  local minima

$f^n(x_{cp}) \neq 0 \quad f^{n+1}(x_{cp}) < 0 \rightarrow$  local maxima

(iii) Even derivatives:

Maxima/minima does not exist

- Conics:-

→ Parabola →

$SP = PM$ ,  $e = 1$ , P is a point on parabola,  
 'P' → point on the directrix.

(a) Standard equation  $\Rightarrow y^2 = 4ax$

Axis → Line passing through focus and l' to parabola.

Focus  $\Rightarrow (a, 0)$

Latus rectum → Length of chord l' to axis and passing through the focus.

Chord l' to axis of parabola is called double ordinate.

If  $y^2 > 4ax \rightarrow$  Point is outside the parabola.

If  $y^2 < 4ax \rightarrow$  Point is inside the parabola.

(b) Parametric equations:

$$x = at^2, y = 2at$$

Any point on parabola  $y^2 = 4ax$  is  $(at^2, 2at)$  and we refer to it as point 'P', where 't' is a parameter.

(c) Equation of tangent:

(i) Point Form  $\rightarrow ly = 2a(x + a)$

(ii) Parametric Form  $\rightarrow ty - x = at^2$   
 (Taking  $x = at^2$  and  $y = 2at$ )

(iii) Condition of tangency  $\rightarrow y^2 = 4ax, y = mx$

$$\left. \begin{array}{l} C = a \\ m \end{array} \right\} ; \text{tangent's slope } = \frac{1}{t}; \text{General } \equiv \left( a, \frac{2a}{m^2} \right), \text{ Point}$$

(iv) Point of intersection of points at  $t_1$  and  $t_2$ :

$$x = at, t_1, t_2; y = 2a(t_1 + t_2)$$

$$y = 4a, x = 6a$$

(d) Equation of normal parabola:

(i) Point Form  $\equiv \left[ y - y_1 = \frac{-x_1 - a}{2a}(y - m) \right]$

(ii) Parametric Form  $\equiv y = at^2 + 2at - ta$

(iii)  $f_1 = -t$

$$f_2 = ma - 2am - am^3$$

(iv) Point of intersection  $\equiv (am^2, -2am)$

(v) If a normal at the point  $t_1$  meets the parabola again at  $t_2$ , then:

$$\begin{cases} f_1 = -t_1 - 2 \\ t_1 \end{cases}$$

(vi) Equation of chord of contact:

(i)  $f_2 = 2yy_1 + 2x_1 - a^2$

(ii) For chord of contact with given mid point  $T(x_1, y_1)$

$$f_2 = y_1^2 + x_1^2 - a^2$$

(ii) Pair of tangents from a given point:

$$f_2 = ss_1$$

\* Ellipse  $\rightarrow$

(i) Standard form:

Equation  $\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a \rightarrow$  length of major axis  
 $b \rightarrow$  length of minor axis

Focus  $\equiv (ae, 0)$ , Directrix  $\equiv \left(\frac{a}{e}, 0\right)$

$$f_2 = a^2(1-e^2), [ecc], \text{ latus rectum} = \frac{2b^2}{a}$$

Vertices  $\equiv (a, 0)$  and  $(-a, 0)$

(b) General equation of an ellipse:

$$(ii) \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = e^2 \cdot (ax+by+c)^2$$

$ax+by+c \rightarrow$  equation of directrix.

$(h, k) \rightarrow$  point of intersection.

$$(iii) \frac{\text{Distance from minor axis}}{\text{Length of semi-minor axis}} + \frac{\text{Distance from major axis}}{\text{Length of semi-major axis}} = 1$$

(iii)  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ , it represents an ellipse if:

$$h^2 < ab \quad |$$

$\Delta \rightarrow$  Delta of all values.

(e) Parametric co-ordinates:

$$(i) P \equiv (a \cos \theta, b \sin \theta)$$

$$Q \equiv (a \cos \alpha, b \sin \alpha)$$

$$\left. \begin{array}{l} m = -\frac{b}{a} \cot(\theta + \alpha) \\ z = \dots \end{array} \right|$$

(ii) Equation of chord passing through  $\Theta$  and  $\alpha$ :

$$\left. \begin{array}{l} x \cos(\theta + \alpha) + y \sin(\theta + \alpha) = \cos(\alpha - \theta) \\ \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = \cos(\alpha - \theta) \end{array} \right|$$

(d) Equation of tangent:

$$(i) \left. \begin{array}{l} \frac{x^2}{a^2} + \frac{(y+mt)^2}{b^2} = 1 \\ m^2 + (y+mt)^2 = 1 \end{array} \right\} \Rightarrow y = mx \pm \sqrt{a^2m^2 + b^2}, \text{ where } m^2 = \frac{b^2 - a^2}{a^2}$$

$$\left. \begin{array}{l} c^2 = a^2m^2 + b^2 \end{array} \right|$$

(ii) Parametric and point form:

$$\left. \begin{array}{l} x \cos \theta + y \sin \theta = 1 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \end{array} \right\} \rightarrow \text{Point form}$$

In parametric form,  $(x_1 = a \cos \theta, y_1 = b \sin \theta)$

(e) Chord of contact equation:

$$(i) \left. \begin{array}{l} \text{tangents} \Rightarrow \frac{x_1}{a^2} + \frac{y_1}{b^2} = 1 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \end{array} \right\} \rightarrow \text{Standard equation of conic}$$

(ii) Equation with given mid-point:

$$\left. \begin{array}{l} S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 \\ z = \frac{x_1 + x}{2}, \frac{y_1 + y}{2} \end{array} \right\} \rightarrow \text{tangents} \equiv S_1$$

\* Hyperbola  $\rightarrow$

$$(a) (i) Standard equation  $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  |  $a \rightarrow$  length of transverse axis  
 $b \rightarrow$  length of conjugate axis$$

$$\left. \begin{array}{l} \text{Latus rectum} = \frac{b^2}{a^2} \cdot 2a \\ \text{Focus} = (c, 0), \text{ Directrix} = (a, 0) \end{array} \right|$$

$$\left. \begin{array}{l} c > a \\ b^2 = a^2(c^2 - 1) \end{array} \right|$$

(ii) Parametric co-ordinates:

$$\left[ x = a \sec \theta, y = b \tan \theta \right]$$

(b) Conjugate hyperbola:

$$\left| \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \right|$$

(c) Rectangular hyperbola:

A hyperbola whose transverse and conjugate axis are interchanged is called rectangular hyperbola.

$$\left[ xy = a^2 \right] \rightarrow \text{General equation.}$$

$$[\text{Radius rectangle} = 2a]$$

(d) Asymptotes:

Tangents extended till infinity.

$$\left[ \text{If } Ax + By + C = 0 \text{ is an asymptote if: } \frac{A^2}{a^2} + \frac{B^2}{b^2} = 0 \right]$$

Rotating  $xy$  about  $60^\circ$  on origin, we get  $[xy = c^2]$  from  $x^2 - y^2 = a^2$

$$\left[ \text{where } c^2 = a^2 \right]$$

(e) 1st Equation of tangent  $\equiv \left| \frac{ax_1}{a^2} - \frac{by_1}{b^2} = -1 \right|$ (f) Parametric equation  $\equiv [x_1 = a \sec \theta, y_1 = b \tan \theta]$ 

(g) Slope form and condition for tangency:

$$\left[ \text{Condition: } \left| \frac{c^2}{a^2} - \frac{m^2}{b^2} - 1 \right| \right]$$

(h) Equation of normal:

$$\left( i) \text{ Point form: } \frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 b^2 \right)$$

$$\left[ \text{Alternatively, } y - y_1 = -\frac{a^2}{b^2} m (x - x_1), \text{ where } m = -\frac{a^2}{b^2} \right]$$

(i) Parametric co-ordinates:

$$\left[ x_1 = a \sec \theta, y_1 = b \tan \theta \right]$$

$$\left[ a \cos \theta + b \sin \theta \cot \theta = a^2 c^2 \right]$$

(g) (g) Chord of contact:

$$(i) \left[ \begin{matrix} xy \\ xy \end{matrix} \right] \rightarrow \left[ \begin{matrix} x_1y_1 - y_1x_1 \\ x_2y_2 - y_2x_2 \end{matrix} \right] = \left[ \begin{matrix} 1 \\ 1 \end{matrix} \right]$$

$$(ii) \text{ with given mid-point } \left[ \begin{matrix} x_3 \\ y_3 \end{matrix} \right]$$

$$\left[ \begin{matrix} x^2 \\ y^2 \end{matrix} \right] \rightarrow \left[ \begin{matrix} x_1^2 - y_1^2 \\ x_2^2 - y_2^2 \end{matrix} \right] = \left[ \begin{matrix} 1 \\ 1 \end{matrix} \right]$$

(iii) Pair of tangents drawn from a given point,

$$PT^2 = SS_1$$

(h) Director circles:

$$x^2 + y^2 = a^2 + b^2 \rightarrow \text{Equation (standard)}$$

### Determinants and Matrices:-

\* Matrix  $\rightarrow$

A set of 'mn' elements arranged in a rectangular array having 'm' rows and 'n' columns.

(i) Some terms regarding matrix:

(i) Sub-matrix  $\rightarrow$  Just like set and subset.

(ii) Diagonals  $\rightarrow$  The elements  $a_{ij}$  where  $i=j$  ( $i \rightarrow$  rows,  $j \rightarrow$  columns)

(iii) Trace  $\rightarrow$   $\text{Tr}(A) = a_{11} + a_{22} + a_{33} + \dots + a_{nn}$

(b) Types of matrices:

(i) Square  $\rightarrow$  Rows and columns have same elements.

(ii) Horizontal  $\rightarrow$  Rows  $>$  Columns

(iii) Vertical  $\rightarrow$  Rows  $<$  Columns

(iv) Row  $\rightarrow$  Consists only of rows

(v) Column  $\rightarrow$  Only columns.

(vi) Zero matrix  $\rightarrow$  All m.n-elements are zero

(vii) Diagonal matrix  $\rightarrow$   $a_{ij} = 0, i \neq j$

(viii) Scalar matrix  $\rightarrow$  Diagonals are equal  $a_{ij} = 0, i \neq j$

$$\text{No. of non-zero elements} = \left\{ \begin{matrix} n^2 - n(n-1) & n \rightarrow \text{No. of rows/columns} \end{matrix} \right.$$

\* Addition of 2 matrices →

We add the corresponding elements. In other words, if we have a matrix of ' $a_{ij}$ ' elements and another matrix of ' $b_{ij}$ ' elements, then  $(a_1+b_1, a_2+b_2, a_3+b_3, \dots)$  will be the addition of the 2 matrices.

\* Multiplication of a matrix →

(i) with a scalar →  $[k \times a_{ij}] = k \times [a_{ij}]$

(ii) with another matrix → columns

we will multiply by taking first row for finding first element and multiplying them with first column/raw respectively to get first element

\* Transpose of a matrix →

In this method, if  $A = [a_{ij}]_{m \times n}$ , then  $B = A^T = [a_{ji}]_{n \times m}$

Properties:

? (i)  $(A^T)^T = A$

? (ii)  $(A+B)^T = A^T + B^T$

? (iii)  $kA^T = k[(a_{ij})^T]^T$  where  $A = [a_{ij}]$

? (iv)  $(AB)^T = B^T A^T$

\* Symmetric and skew-symmetric matrix →

(i) Symmetric →  $a_{ij} = a_{ji}$

(ii) Skew-symmetric →  $a_{ij} = -a_{ji}$

In skew-symmetric;  $A^T = -A$ .

Diagonals in a skew-symmetric matrix are equal to zero.

Properties:

? (i)  $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

Symmetric      Skew-symmetric

(ii) Sum of skew-symmetric matrices is skew-symmetric.

(iii) Cube of skew-symmetric matrix is skew-symmetric

(iv) Square of skew-symmetric is not skew-symmetric.

(v) Product of skew-symmetric and symmetric is skew-symmetric if it is commutative (product).

\* Different types of matrices :

(i) Orthogonal  $\rightarrow A \times A^T = I$

(ii) Idempotent  $\rightarrow A^2 = A$ , only for square matrix

(iii) If  $AB = A$  and  $BA = B$ , then both  $A$  &  $B$  are idempotent.

If  $A$  is idempotent then  $B = I - A$  is idempotent, and  $AB = BA = 0$ .  
A product of 2 idempotent matrices if one of the matrices commute in multiplication.

(iv) Involutory  $\rightarrow A^2 = I$  only  $(A + I)(A - I) = 0$

(v) Nilpotent  $\rightarrow A^m = 0$ , then it is called periodic

(vi) Periodic  $\rightarrow (A^{km} = A)$ ,  $k \rightarrow$  positive integers / period of  $A$ .  
For idempotent matrices,  $k = 1$ .

\* Determinants :

It is a matrix that gives the value of elements present.  
Properties:

(i) Determinant value remains unchanged if rows & columns are rearranged.

(ii) If all elements in a row or column is zero, then the value of the determinant becomes zero.

(iii) If any two rows or columns have same elements, then value of the determinant becomes zero.

(iv) Interchanging of two rows/ column changes the sign of determinant.

(v) If the elements get multiplied by a non-zero number, then all the elements get multiplied by that number.

\* (vi) Multiplication of 2 determinants  $\rightarrow$

Can be done using the following methods:

R Row x Row, Column x Column, Row x Column, Column x Rows.

\* Calculation of differentiation of a determinant  $\rightarrow$

If we need to do so, then we differentiate each row separately and then add the three determinants.

\* Special Determinants  $\rightarrow$

(i) Skew-symmetric  $\rightarrow [a_{ij} = -a_{ji}]$

(ii) Circular  $\rightarrow$  rows/columns are cyclic shifts of first row/column.

(a)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

(b)

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

(c)

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(d)

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ac)$$

\* Inverse trigonometric functions:

\* Domain and range of inverse functions  $\rightarrow$

(i)  $\sin^{-1}x \rightarrow$  Domain:  $[-1, 1] \rightarrow$  Range  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii)  $\cos^{-1}x \rightarrow [0, \pi] \rightarrow [0, \infty]$

(iii)  $\tan^{-1}x \rightarrow \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) Similar will be done for  $\csc^{-1}x$ ,  $\sec^{-1}x$  and  $\cot^{-1}x$ .  
 Graph of inverse function will be symmetric about  $y=x$ .  
 & Inverse function are not periodic. Thus, they are one-one functions.

\* Properties of inverse functions  $\rightarrow$

(a)  $\sin^{-1}x + \cos^{-1}x = \pi/2$

(b)  $\csc^{-1}x + \sec^{-1}x = \pi/2$

- (c)  $\sin^{-1}x + \cot^{-1}x = \pi/2$   
 (d)  $\sin^{-1}(-x) = -\sin^{-1}x$   
 (e)  $\cos^{-1}(-x) = \pi - \cos^{-1}x$   
 (f)  $\tan^{-1}(-x) = -\tan^{-1}x$   
 (g)  $\cot^{-1}(-x) = \pi - \cot^{-1}x$   
 (h)  $\sec^{-1}(-x) = \pi - \sec^{-1}x$   
 (i)  $\cosec^{-1}(-x) = -\cosec^{-1}x$   
 (j)  $\sin(\sin^{-1}x) = \sin^{-1}(\sin x) = x$   
 (k)  $\cos^{-1}(\cos x) = \cos(\cos^{-1}x) = x$   
 (l)  $\tan^{-1}(\tan x) = \tan(\tan^{-1}x) = x$   
 (m)  $\cot^{-1}(\cot x) = \cot(\cot^{-1}x) = x$   
 (n)  $\sec^{-1}(\sec x) = \sec(\sec^{-1}x) = x$   
 (o)  $\cosec^{-1}(\cosec x) = \cosec(\cosec^{-1}x) = x$

\* Inverse trigonometric identities →

$$(a) \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(\sqrt{1-x^2} + y\sqrt{1-y^2}\right)$$

$$(b) \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(\sqrt{1-x^2} - y\sqrt{1-y^2}\right)$$

$$(c) \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(\sqrt{1-x^2}(1-y^2)\right)$$

$$(d) \cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{(1-x^2)(1-y^2)}\right)$$

$$(e) \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy > 1$$

$$= \pi/2, \quad , xy < 1$$

$$= \pi + \tan^{-1}\left(\frac{xy+y}{1-xy}\right), xy < 1$$

$$(f) \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right).$$

### • Indefinite Integration :-

\* Definition →

$$\frac{d}{dx} f(x) = p(x)$$

$$\int f(x) dx = p(x) dx$$

$$\boxed{\int p(x) dx = f(x)}$$

\* Methods to integrate a function ->

(a) Integration by substitution:

When we have to substitute integrate a function using substitution, we substitute such a value such that its differential is also substituted as another variable.

(b) Integration by parts:

When we have a function where two different functions have been multiplied with each other, then we differentiate the more complex function and integrate the least complex function in the following manner:

$$f(x) \rightarrow g(x)$$

$$f'(x) dx \rightarrow \int g(x) dx$$

We can also use the ILATE rule.

(c) Integration by partial fractions:

We can also use partial fractions by splitting the denominator, using two or more variables, finding them and substituting them into the integral.

\* Standard Substitutions ->

$$(a) \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}(x) + C$$

$$(b) \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$(c) \int \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$(d) \int \frac{1}{\sqrt{a^2+x^2}} = \ln \left( x + \sqrt{x^2+a^2} \right) + C$$

$$(e) \int \frac{1}{\sqrt{x^2-a^2}} = \frac{1}{2} \ln \left( \frac{\sqrt{x^2-a^2}+a^2}{\sqrt{x^2-a^2}-a^2} \right) + C$$

$$(e) \int \sqrt{a^2 - x^2} dx = x \frac{\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$(g) \int \sqrt{a^2 + x^2} dx = x \frac{\sqrt{a^2 + x^2}}{2} + \frac{a^2}{2} \left( \ln\left(x + \sqrt{a^2 + x^2}\right) \right) + C$$

\* Various types of integrations  $\rightarrow$

$$(a) \int \frac{1}{(ax^2 + bx + c)^{1/2}} dx ; \int (ax^2 + bx + c)^{1/2} dx \rightarrow \text{try getting it in the form } (ax^2 + bx + c)^{1/2} \text{ or } (ax^2 + bx + c)^{1/2} \text{ and substitute.}$$

$$(b) \int \frac{ax+b}{cx+d} dx \rightarrow ax^2 + bdx = t, \quad ax+b = A(cx+d) + B$$

$$(c) \int \frac{dx}{(ax^2 + bx + c)^{1/2}} \rightarrow \det ax^2 + bx + c = 1, \quad \text{so that we get} \quad t$$

$$(d) \int \frac{px \sin x + qx \cos x}{a \cos x + b \sin x} dx \rightarrow \text{Num} = A(\text{den}) + B(\text{den})' + C$$

$$(e) \begin{cases} f(-\sin x, \cos x) \rightarrow \cos x = t \\ f(-\sin x, -\cos x) \rightarrow \sin x = t \\ f(\sin x, -\cos x) \rightarrow \tan x = t. \end{cases}$$

### Definite Integration:

$$\int_a^b f(x) dx = [F(b) + C] - [F(a) + C] \\ = g(b) - g(a)$$

If  $\int f(x) dx = 0$  then  $f(x)$  will have at least one root in  $a, b$  provided  $f(x)$  is continuous in  $(a, b)$ .

\* Properties of definite integrals  $\rightarrow$

$$(i) \int_a^b f(x) f'(x) dx = \int_{f(a)}^{f(b)} t dt$$

$$(ii) \int_a^b f(x) dx = \int_a^b f(t) dt = \int_a^b f(y) dy \quad (\text{Dummy variable})$$

$$(iii) \int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$(iv) \int_a^b f(bx) dx = - \int_b^a f(x) dx$$

$$(v) \int_a^b f(x) dx = \int_a^b at f(at+b-x) dx$$

$$(vi) \int_a^{an} f(x) dx = n \int_0^1 f(x) dx$$

$$(vii) \int_{-\frac{a}{2}}^{\frac{a}{2}} f(x) dx = 2 \int_0^{\frac{a}{2}} f(x) dx \quad (\text{If } f(x) \text{ is even})$$

$$= 0 \quad (\text{If } f(x) \text{ is odd})$$

$$(viii) \int_{-T}^{mT} f(x) dx = (n-m) \int_0^T f(x) dx$$

\* Differentiation of definite integration  $\rightarrow$

This is also known as Leibnitz rule.

$$P(a) = \left[ \begin{array}{l} f(x) \\ g(x) \end{array} \right] dx$$

$$P'(x) = g(f_2(x)) f'_2(x) - g(f_1(x)) f'_1(x)$$

$f'(x, t) \rightarrow$  Differentiation of  $f(x, t)$  w.r.t keeping  $x$  constant.

$$F(t) = \int_a^b P(x, t) dx$$

\* Approximation in definite integrals  $\rightarrow$

[If  $f_1(x) \geq f_2(x) \geq f_3(x)$ ]

$$\int_a^b f_1(x) dx \geq \int_a^b f_2(x) dx \geq \int_a^b f_3(x) dx$$

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

$m(b-a) \rightarrow$  Minimum of  $f(x)$

$M(b-a) \rightarrow$  Maximum value of  $f(x)$ .

\* Area under curves  $\rightarrow$

(a) Area under simple curves:

(i) For taking vertical strips  $\rightarrow \int_a^b y dx$ .

(ii) For taking horizontal strips  $\rightarrow \int_a^b x dy$ .

When given equation of a curve, we substitute in terms of  $x$  and  $y$  depending on the method we are using.

(b) Area of the region bounded by a curve and a line:

Here, when we will be given equations of lines and ~~curves~~ curves, we will take a common variable and substitute in the integral accordingly.

(c) Area between two curves:

Suppose  $y = f(x)$ ,  $y = g(x)$  are equations of two different curves. Then, we will use the formula:

$$A = \int [f(x) - g(x)] dx$$

### \* Vectors:

\* Terms regarding vectors  $\rightarrow$

(i) Representation -  $\vec{a}/\vec{a}$

(ii) Magnitude  $\rightarrow |\vec{a}|/\|\vec{a}\|$

(iii) Unit Vector  $\rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|}$

(iv) Zero vector  $\rightarrow$  Sum of vectors is 0.

(v) Equality of vector  $\rightarrow$  same magnitude, same direction.

(vi) Like vectors  $\rightarrow$  same direction.

(vii) Unlike vectors  $\rightarrow$  opposite direction.

$$\vec{AB} = \vec{B} - \vec{A}$$

$$\text{In a } \triangle ABC, \text{ centroid } G = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$$\text{Incentre } I = (\vec{c} - \vec{b})\vec{a} + (\vec{a} - \vec{c})\vec{b} + (\vec{b} - \vec{a})\vec{c}$$

$$= \frac{\vec{a}}{|\vec{a}|} + |\vec{c} - \vec{b}| + |\vec{b} - \vec{a}|$$

\* Linear combination of scalars and vectors  $\rightarrow$

(a) Fundamental theorem:

If  $\vec{a}$  and  $\vec{b}$  are two non-collinear vectors, then the combination of  $a$  and  $b$  gives a vector lying on a co-plane containing  $\vec{a}$  and  $\vec{b}$ .  $[\vec{a} = k\vec{b}]$

(ii) Fundamental theorem - 2:

If  $\vec{a}$  and  $\vec{b}$  and  $\vec{c}$  are non-coplanar vectors then the linear combination of  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  gives any vector in 3D

\* Orthogonal system of unit vectors  $\rightarrow$

$x \rightarrow$  Angle made with X-axis

$y \rightarrow$  Angle made with Y-axis

$z \rightarrow$  Angle made with Z-axis.

$$l = \cos \alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$m = \cos \beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$n = \cos \gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\vec{a} = l\hat{i} + m\hat{j} + n\hat{k}$$

since  $|\vec{a}| = 1$ ,  $[l^2 + m^2 + n^2 = 1]$

$l, m, n$  are called direction cosines.

\* Multiplication of 2 vectors  $\rightarrow$

(a) Dot Product:

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$i \cdot i = j \cdot j = k \cdot k = 1, \text{ rest combinations} = 0.$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} \Rightarrow |\vec{a}| \cos \theta = \vec{a} \cdot \vec{b}.$$

$$(\vec{a} \cdot \vec{b})_{\max} = (\vec{a} || \vec{b}) ; (\vec{a} \cdot \vec{b})_{\min} = (\vec{a} || \vec{b})$$

(b) Cross Product:

$$\vec{a} \times \vec{b} = (\vec{a} || \vec{b} | \sin \theta|)$$

Direction can be decided with the help of thumb rule.

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}, \vec{a} \times \vec{a} = 0 \quad (\text{since } \theta = 0, \sin \theta = 0)$$

$$\begin{array}{|c|} \hline \mathbf{i} \times \mathbf{j} = \mathbf{k}; \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}; \quad \mathbf{k} \times \mathbf{i} = \mathbf{j} \\ \mathbf{j} \times \mathbf{i} = -\mathbf{k}; \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}; \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \\ \hline \end{array}$$

Area of quadrilateral =  $\bar{a} \times \bar{b}$

\* Scalar triple product (STP)  $\rightarrow$

$$STP \rightarrow \bar{a} \cdot (\bar{b} \times \bar{c}) = |a||b||c| \sin \theta \cos \phi$$

$\theta \rightarrow$  angle b/w  $\bar{a}$  and  $\bar{b} \times \bar{c}$

$\phi \rightarrow$  angle b/w  $\bar{b}$  and  $\bar{c}$ .

$$STP = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

= Volume of parallelepiped

$$Volume of tetrahedron = (1/6) (STP)$$

STP is commutative by nature.

• 3-D Geometry:

\* Octant  $\rightarrow$

It is a division of x, y, z into equal parts, like a quadrant.

Shifting of origin:

$$x = x - a, \quad y = y - b, \quad z = z - c$$

\* 3-D co-ordinate geometry  $\rightarrow$

$$(i) \text{ Distance}(a) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

formula

(ii)

Two vectors  $\Rightarrow a_1 = a_2 = a_3$ ,

are parallel  $b_1 = b_2 = b_3$

(iii)

Two vectors  $\Rightarrow a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$

are perpendicular

(iv)

Section  $= \left( m_1 x_1 + m_2 x_2, m_1 y_1 + m_2 y_2, m_1 z_1 + m_2 z_2 \right)$

Formula  $m_1 = \frac{m}{m+n}, \quad m_2 = \frac{n}{m+n}$

$m_1 m_2 = mn$

\* Equations of line  $\rightarrow$

(i) Vector and cartesian equations

$P_1 P_2 = 0$

$$\vec{r} = \vec{a} + \lambda \vec{b} \quad (\text{vector equation})$$

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} \quad (\text{cartesian eqn.})$$

(ii) Two-point form  $\rightarrow$   $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

(iii) Parametric form:

$$x = x_1 + r \cos \alpha$$

$$y = y_1 + r \cos \beta$$

$$z = z_1 + r \cos \gamma$$

(iv) Angle b/w  $= \cos \theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$   
2 lines

(v) Foot of perpendicular:

If  $\vec{AB}$  is  $\perp$  to  $\vec{c}$ , then  $[\vec{AB} \cdot \vec{c}] = 0$

\* Different cases of lines:

(i) Intersecting lines  $\rightarrow [a_2 - a_1 \ b_1 \ b_2] = 0$

(ii) Parallel lines  $\rightarrow d = [(a_2 - a_1) \times \vec{b}]$   $a_1, a_2 \rightarrow \text{pos. vector}$   
 $|\vec{b}|$

(iii) Skew lines:

Distance = projection of  $\vec{a}_2 - \vec{a}_1$  on  $\vec{b}_1 \times \vec{b}_2$ .

$$d = |(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|$$

\* Plane  $\rightarrow$

(i)  $\Rightarrow$  vector and cartesian form

$$\text{vector form} \rightarrow [\vec{r} \cdot \hat{n} = p]$$

$\hat{n} \rightarrow$  normal unit vector

$$\text{Cartesian form} \rightarrow [ax + by + cz + d = 0] \quad (d = -p)$$

$a, b, c \rightarrow$  direction ratios.

We can also write it in the form of its dir<sup>n</sup> cosines.

(ii) Angle between 2 planes:

$$\cos \theta = a_1 a_2 + b_1 b_2 + c_1 c_2$$

$$\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}$$

(iii) Family of planes  $\rightarrow P = P_1 + \lambda P_2$

(When a plane is passing through 2 other planes.)

(iv) Distance of a point from the plane:

$$P = ax_1 + by_1 + cz_1 + d$$

$$\sqrt{a^2 + b^2 + c^2}$$

From origin,  $P = d$

$$\sqrt{a^2 + b^2 + c^2}$$

Between 2 parallel planes;  $P = \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}$

\* Differential equations:-

\* Order and degree of a differential equation  $\rightarrow$

Order  $\rightarrow$  Power of highest  $n^{th}$  derivative ('n' is max.)

Degree  $\rightarrow$  Power (exponential) on the order.

\* Properties of differential equations  $\rightarrow$

$$(i) (1+x^2)y^2 + x^2y^2 dx + xy dy = 0$$

$$(ii) \sqrt{1+x^2+xy^2} (1+x^2) dx = -xy dy$$

$$(iii) \sqrt{1+x^2} dx = -xy dy$$

$$\sqrt{1+y^2}$$

$$\frac{d(1+x^2) dx}{x^2} = -\frac{y dy}{\sqrt{1+y^2}}$$

Set  $y$  &  $x$  on separate sides.

\* Types of differential equations:

$$(i) \frac{dy}{dx} = \frac{(ax+by+c)}{dx} \rightarrow$$

Substitute for 'z' and solve further.

$$(ii) x \frac{d(xy)}{dx} = x dy \rightarrow (d(xy)) = ady - y dx$$

$$(iii) \frac{d(x/y)}{y^2} = y dx - x dy$$

\* Solving using homogeneous equations →

Make substitutions, such as  $[y = vx]$ .

\* Equation reducible to homogeneous form →

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

$$[x = \lambda t; y = \lambda t k; dx = d\lambda t; dy = \lambda dt]$$

$$\text{Then, } [Y = vX]$$

\* Linear differential equations →

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$\text{Reduce it to the form: } yI = \int q(x)I dx$$

$I \rightarrow$  Integrating factor.

• Statistics:-

\* Mean →

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \bar{x}$$

$$\text{Mean} = f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n = \frac{\sum f_i x_i}{N}$$

\* Median →

(i) If  $N$  is even → Median =  $(x_{\frac{N}{2}})^{th}$  and  $(\frac{N}{2}+1)^{th}$

(ii) If  $N$  is odd → Median =  $(\frac{N+1}{2})^{th}$

For a median class,

$$\text{Median} = \begin{cases} l + \left( \frac{N}{2} - c \right) \times h \\ f \end{cases}$$

$l \rightarrow$  lower limit

$h \rightarrow$  median class width

$f \rightarrow$  frequency of median class

$\rightarrow$  cumulative frequency.

\* Mode  $\rightarrow$  Mode

$$\text{Mode} = \left[ L_1 + \frac{(f_1 - f_{0.5}) \times h}{2f_1 - f_0 - f_2} \right]$$

$L_1 \rightarrow$  lower limit of modal class

$f_1 \rightarrow$  frequency of modal class

$f_0 \rightarrow$  frequency of previous class.

$$\text{Mode} = 3 \text{Median} - 2 \text{Mean}$$

\* Types of distribution.

(i) Symmetric  $\rightarrow$  Mean = Median = Mode

(ii) Skew-symmetric  $\rightarrow$  Positive: Mean  $>$  Median  $>$  Mode

Negative: Mean  $<$  Median  $<$  Mode

\* Measure of spread.

(i) Range:

$$\text{Coefficient of range} = \frac{\text{Maximum} - \text{Minimum}}{\text{Maximum} + \text{Minimum}}$$

$$(ii) \text{Mean Deviation} = \frac{1}{n} \times \sum_{i=1}^n |x_i - M| \quad [M \rightarrow \text{Mean/Median/Mode}]$$

$$(iii) \text{Variance} = \frac{1}{n} \sum_{i=1}^n x_i^2 - (\text{mean})^2$$

$$(iv) \text{Standard deviation (SD)} = \sqrt{\text{Variance}}$$

$$(v) \text{Co-efficient} = \frac{\text{S.D.}}{\text{Mean}} \times 100$$

$$(vi) \text{Mean square deviation} = \frac{\sum (x_i - M)^2}{n}$$

$$\text{Variance} = \frac{1}{n} \sum x_i^2 - \left( \frac{1}{n} \sum x_i \right)^2$$