1. $K = (a^T b + r)^d$

<u>given</u> :

$r = 0$

$d = 3$

$K = (a^T b + 0)^3$

$= (a^T b)^3$

$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$; $b = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$

$K = (a^T b)(a^T b)(a^T b)$

$K = (a_1^2 b_1^2 + 2 a_1 a_2 b_1 b_2 + a_2^2 b_2^2) \cdot (a_1 b_1 + a_2 b_2 + \cdots)$

$= (a_1^3 b_1^3 + 3 a_1^2 b_1^2 a_2 b_2 + 3 a_1 b_1 a_2^2 b_2^2 + a_2^3 b_2^3)$

$$\boxed{K = (a_1^3 b_1^3 + 3 a_1^2 b_1^2 a_2 b_2 + 3 a_1 b_1 a_2^2 b_2^2 + a_2^3 b_2^3)}$$

Q2 Dot product :

we know the value of k.

$$k = a_1^3 b_1^3 + 3a_1^2 b_1^2 a_2 b_2 + 3a_1 b_1 a_2^2 b_2^2 + a_2^3 b_2^3$$

$$= (a^T b)^3$$

The above equation can be expressed as a dot product of two vectors 'a' and 'b'.

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix} \quad ; \quad b = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$$

The above expression of 'k' can be written in terms of 'a' and 'b' by splitting the terms.

$$K = a_1^3 \cdot b_1^3 + \sqrt{3}\, a_1^2 a_2 \cdot \sqrt{3}\, b_1^2 b_2 + \sqrt{3}\, a_1 a_2^2 \cdot \sqrt{3}\, b_1 b_2^2 + a_2^3 \cdot b_2^3$$

$$a = \begin{bmatrix} a_1^3 \\ \sqrt{3}\, a_1^2 a_2 \\ \sqrt{3}\, a_1 a_2^2 \\ a_2^3 \end{bmatrix} \qquad b = \begin{bmatrix} b_1^3 & \sqrt{3}\, b_1^2 b_2 & \sqrt{3}\, b_1 b_2^2 & b_2^3 \end{bmatrix}$$

The dot product of the above two vectors will result in k.

Q3

point : $(2, 4)$

$$K = (a^T b)^3$$

writing the vector in kernel space:

$$K = \begin{bmatrix} a_1^3 \\ \sqrt{3}\, a_1^2\, a_2 \\ \sqrt{3}\, a_1\, a_2^2 \\ a_2^3 \end{bmatrix}$$

$a_1 = 2$; $a_2 = 4$

$$x = \begin{bmatrix} (2)^3 \\ \sqrt{3}\,(2)^2\,(4) \\ \sqrt{3}\,(2)\,(4)^2 \\ (4)^3 \end{bmatrix} = \begin{bmatrix} 8 \\ \sqrt{3}\,(4)(4) \\ \sqrt{3}\,(2)(16) \\ 64 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 16\sqrt{3} \\ 32\sqrt{3} \\ 64 \end{bmatrix}$$

For a higher Dimensional space:

considering vectors a, and b

Point : a (4, 2)

b (3, 6)

$a_1 = 4$; $a_2 = 2$

$b_1 = 3$; $b_2 = 6$

$$K = a_1^3 b_1^3 + 3a_1^2 a_2 b_1^2 b_2 + 3a_1 a_2^2 b_1 b_2^2 + a_2^3 b_2^3$$

$$= (4)^3 (3)^3 + 3(4)^2 (2)(3)^2 (6) + 3(4)(2)^2 (3)(6)^2 + (2)^3 (6)^3$$

$$= (64)(27) + (3)(16)(2)(9)(6) + (3)(4)(4)(3)(36) + (8)(216)$$

$$= 1728 + 5184 + 5184 + 1728$$

$$= 13824$$

$$\boxed{K = 13824}$$

Q2 : Dot product :

we know the value of K :

$$K = (a_1^3 b_1^3 + 3a_1^2 b_1^2 a_2 b_2 + 3a_1 b_1 a_2^2 b_2^2 + a_2^3 b_2^3)$$

$$K = (a^T b)^3$$

let $c = a^T b$.

$$K = (c)^3$$

$$c = a^T b$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad ; \quad b = [b_1 \; b_2 \cdots b_n]$$

can be rewritten as:

$$a = \begin{bmatrix} a_1^3 \\ 3a_1^2 a_2 \\ 3a_1 a_2^2 \\ a_2^3 \end{bmatrix} \quad ; \quad b = [b_1^3 \quad b_1^2 b_2 \quad b_1 b^2 \quad b_2^3]$$

$$a^T \cdot b = \begin{bmatrix} a_1^3 \\ 3a_1^2 a_2 \\ 3a_1 a_2^2 \\ a_2^3 \end{bmatrix} \cdot [b_1^3 \quad b_1^2 b_2 \quad b_1 b^2 \quad b_2^3]$$

$$K = a_1^3 b_1^3 + 3a_1^2 a_2 b_1^2 b_2 + 3a_1 a_2^2 b_1 b_2^2 + a_2^3 b_2^3$$