Regression

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Content Questions

Linear Regression Predictions

dimension	weight
b	1
w_1	2.0
w_2	-1.0
σ	1.0

1. $\mathbf{x}_1 = \{0.0, 0.0\}$; y_1=
2. $\mathbf{x}_2 = \{1.0, 1.0\}$; y_2=
3. $\mathbf{x}_3 = \{.5, 2\}$; y_3=

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Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	-1.0
σ	1.0

$$p(y\,|\,x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y\,|\,x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1\,|\,\mathbf{x}_1 = \{0.0, 0.0\}) =$
2. $p(y_2 = 3\,|\,\mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1\,|\,\mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
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$$p(y \mid x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

$$p(y \mid x) = \frac{\exp\left\{-\frac{(y-\hat{y})^2}{2}\right\}}{\sqrt{2\pi}}$$

1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) =$ 0.399
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) =$
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
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1. $p(y_1 = 1 \mid \mathbf{x}_1 = \{0.0, 0.0\}) =$ 0.399
2. $p(y_2 = 3 \mid \mathbf{x}_2 = \{1.0, 1.0\}) =$ 0.242
3. $p(y_3 = -1 \mid \mathbf{x}_3 = \{.5, 2\}) =$

Probabilities

dimension	weight
w_0	1
w_1	2.0
w_2	-1.0
σ	1.0

$$p(y\,|\,x) = y \sim N\left(b + \sum_{j=1}^{p} w_j x_j, \sigma^2\right)$$

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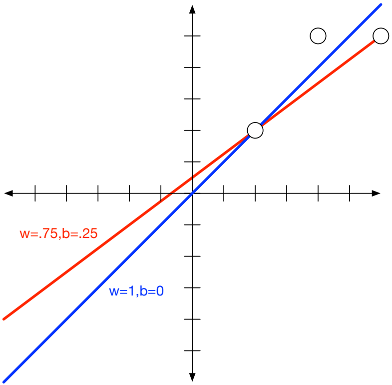
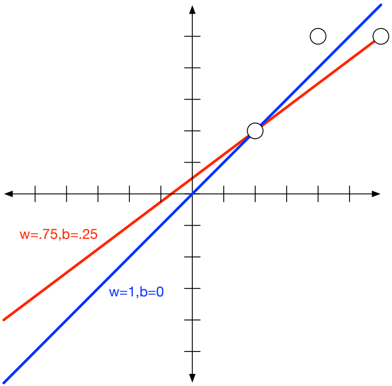
2. $p(y_2 = 3\,|\,\mathbf{x}_2 = \{1.0, 1.0\}) =$ 0.242
3. $p(y_3 = -1\,|\,\mathbf{x}_3 = \{.5, 2\}) =$ 0.242

Consider these points and data

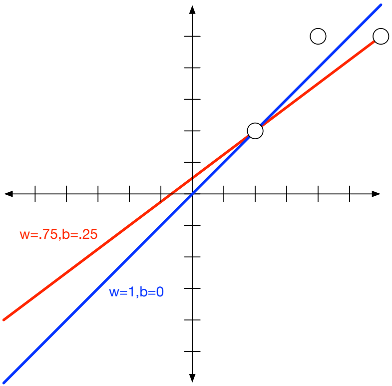


Consider these points and data



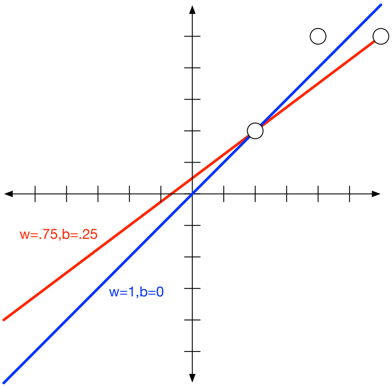
Which is the better OLS solution?

Consider these points and data



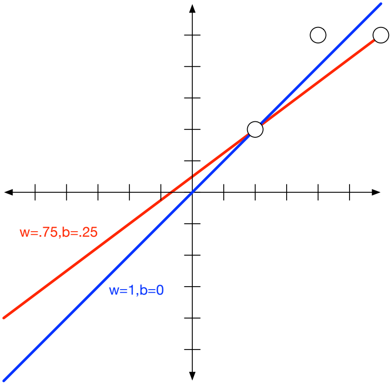
Blue! It has lower RSS.

Consider these points and data



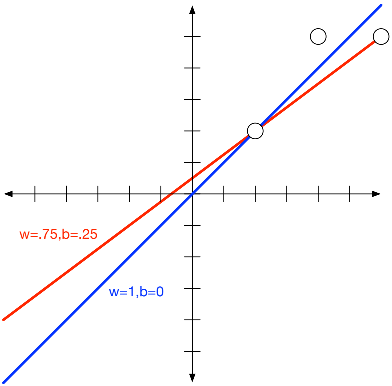
What is the RSS of the better solution?

Consider these points and data



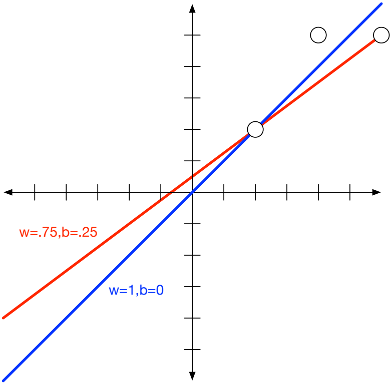
$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + (2.5-2)^2 + (2.5-3)^2\right) = \frac{1}{4}$$

Consider these points and data



What is the RSS of the red line?

Consider these points and data



$$\frac{1}{2}\sum_i r_i^2 = \frac{1}{2}\left((1-1)^2 + (2.5-1.75)^2 + (2.5-2.5)^2\right) = \frac{3}{8}$$