Computational Linguistics

Natural Language Processing

University of Maryland

Classification Examples

Reminder: Logistic Regression

$$P(Y=0|X) = \frac{1}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{1}$$

$$P(Y=1|X) = \frac{\exp\left[\beta_0 + \sum_i \beta_i X_i\right]}{1+\exp\left[\beta_0 + \sum_i \beta_i X_i\right]} \tag{2}$$

- Discriminative prediction: $p(y|x)$
- Classification uses: ad placement, spam detection
- What we didn't talk about is how to learn β from data

Logistic Regression: Objective Function

$$\mathscr{L} \equiv \ln p(Y \mid X, \beta) = \sum_j \ln p(y^{(j)} \mid x^{(j)}, \beta) \tag{3}$$

$$= \sum_j y^{(j)} \left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) - \ln \left[1 + \exp\left(\beta_0 + \sum_i \beta_i x_i^{(j)} \right) \right] \tag{4}$$

Algorithm

1. Initialize a vector B to be all zeros
2. For $t = 1, \ldots, T$
   - For each example $\vec{x}_i, y_i$ and feature j:
     - Compute $\pi_i \equiv \Pr(y_i = 1 \mid \vec{x}_i)$
     - Set $\beta[j] = \beta[j]' + \lambda(y_i - \pi_i)x_{i,j}$
3. Output the parameters $\beta_1, \ldots, \beta_d$.

Example Documents

$$\beta[j] = \beta[j] + \lambda(y_i - \pi_i)x_i$$
$$\vec{\beta} = \langle \beta_{bias} = 0, \beta_A = 0, \beta_B = 0, \beta_C = 0, \beta_D = 0 \rangle$$

$y_1 = 1$

A A A A B B B C

$y_2 = 0$

B C C C D D D D

(Assume step size $\lambda = 1.0$.)

You first see the positive example. First, compute π_1

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$\pi_1 = \Pr(y_1 = 1 \mid \vec{x}_1) = \frac{\exp \beta^T x_i}{1 + \exp \beta^T x_i} = \frac{\exp 0}{\exp 0 + 1} = 0.5$

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$\pi_1 = 0.5$ What's the update for β_{bias}?

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Now you see the negative example. What's π_2?

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Different Activation Function

Your boss demands that you replace the sigmoid function in logistic regression with the trigonometric sin function because it looks the same and he has a sin button on his calculator.

1. Plot both between -1 and 1. What would a choice of constants A and B be that would make $s(z) = A\sin(Bz) + C$ look as much like the logistic function?
2. What would be the update for an example? How is it different?
3. Would there be any other problems with using this formulation?

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- Same `min` and `max`
- Derivative at 0 should match
- Same value at 0

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- Same `min` and `max` : $s(\pi) = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$
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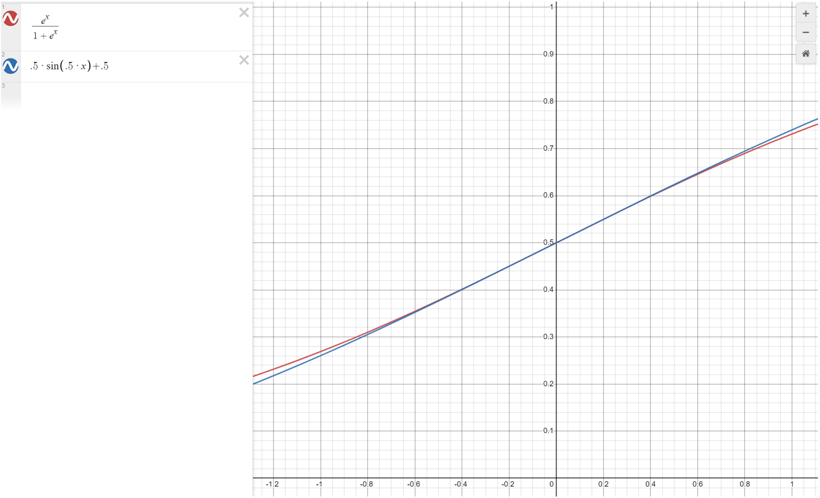
- Derivative at 0 should match : $s'(0) = \frac{1}{4}\cos 0 = \frac{1}{4}$
- Same value at 0 : $s(0) = \frac{1}{2}\sin 0 + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$

$$s(z) \equiv \frac{1}{2}\sin\left(\frac{z}{2}\right) + \frac{1}{2} \tag{5}$$

Plot



Perfect match!

Update

The update becomes a function of sine and cosine.
If the input the function is near zero, everything is fine, but beyond that, as the gradient is a periodic function, the gradient is possible to be zero or inf.

If you are interested ...

Let us define:

$$\pi_i = \frac{1}{2}\sin\left(\frac{\beta^T x_i}{2}\right) + \frac{1}{2}$$

Thus,

$$\frac{\partial \pi_i}{\partial \beta_j} = \frac{1}{4}\cos\left(\frac{\beta^T x_i}{2}\right) x_{i,j} \tag{6}$$

If you are interested ...

To ease notation, let us further define:

$$z' = \frac{\beta^T x_i}{2}$$

Thus,

$$\frac{\partial \mathscr{L}_i}{\partial \beta_j} = \begin{cases} \frac{1}{\pi_i}\frac{\partial \pi_i}{\partial \beta_j} & \text{if } y_i = 1 \\ \frac{1}{1-\pi_i}\left(-\frac{\partial \pi_i}{\partial \beta_j}\right) & \text{if } y_i = 0 \end{cases} = \begin{cases} \frac{\cos z'}{2(\sin z'+1)} x_{i,j} & \text{if } y_i = 1 \\ \frac{\cos z'}{2(\sin z'-1)} x_{i,j} & \text{if } y_i = 0 \end{cases} \tag{7}$$