Math Review

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Bayes Rule

There's a test for Boogie Woogie Fever (BWF). The probability of geting a positive test result given that you have BWF is 0.8, and the probability of getting a positive result given that you do not have BWF is 0.01. The overall incidence of BWF is 0.01.

1. What is the marginal probability of getting a positive test result?
2. What is the probability of having BWF given that you got a positive test result?

Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) =$
- $P(D = \top \mid T = \top) =$

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Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 =$
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Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
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- $P(D=\top \mid T=\top)=\frac{P(T=\top \mid D=\top)P(D=\top)}{P(T=\top)}=\frac{0.8\cdot 0.01}{0.02}=$

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- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D = \top \mid T = \top) = \frac{P(T=\top \mid D=\top) P(D=\top)}{P(T=\top)} = \frac{0.8 \cdot 0.01}{0.02} = 0.4$

Dot Product

What is the dot product of $\vec{a} \cdot \vec{b}$

$$a \equiv \langle 1, 2, 3 \rangle b \equiv \qquad \langle 4, -5, 6 \rangle \tag{1}$$

Is this less than 90 degrees, greater than 90 degress, or exactly 90 degrees?

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Is this less than 90 degrees, greater than 90 degress, or exactly 90 degrees?

$$a\dot{b} = 1(4) + 2(-5) + 3(6) = 4 - 10 + 18 = 12 \tag{2}$$

Derivative

Consider the function describing the height of the ball?

$$f(t) \equiv h = 3 + 14t - 5t^2 \tag{3}$$

What's the derivative when:

- $t = 2$?
- $t = 1.4$

What is the maximum height of the ball?

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What's the derivative when:

- $t = 2$?
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$$\frac{\partial f}{\partial t} = 14 - 5(2t) \tag{4}$$

$$\frac{\partial f}{\partial t}(t = 2) 14 - 20 = -6 \tag{5}$$

$$\frac{\partial f}{\partial t} 14 - 14 = 0 \tag{6}$$

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What's the derivative when:

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What is the maximum height of the ball?

$$f(1.4) = 3 + 19.6 - 9.8 = 12.8 \tag{4}$$

Dice

A die is rolled twice

what is the probability that the sum of the faces is greater than 7, given that

- the first outcome was 4?
- the first outcome was greater than 4?
- the first outcome was a 1?
- the first outcome was less than 5?

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) =$
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- $p(X_1 + X_2 > 7 | X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1+X_2>7 \wedge X_1=4)}{p(X_1=4)} =$
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- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
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- $p(X_1 + X_2 > 7 \,|\, X_1 < 5) =$

Dice

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- $p(X_1 + X_2 > 7 \,|\, X_1 < 5) = \frac{p(X_1 + X_2 > 7 \wedge X_1 < 5)}{p(X_1 < 5)} = \frac{6/36}{2/3} = \frac{18}{64} = \frac{1}{4}$

Children

What is the probability a family of two children has two boys

- given that it has at least one boy?
- given that the first child is a boy?

Children

- $P(X_1 = \top, X_2 = \top \mid X_1 = \top \vee X_2 = \top) =$
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Conditional Probabilities

One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from the lot and then tossed, turns up heads 6 times in a row, what is the probability that it is the two-headed coin?

Conditional Probabilities

- Let C be the coin chose ($\top$ for fake)
- Let H be the number of heads out of six

$$P(C = \top \,|\, H = 6) = \tag{5}$$

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$$P(C = \top \mid H = 6) = \frac{P(C = \top \wedge H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = \tag{5}$$

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