Computing the Equitable Domatic Number of Graphs

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Abstract

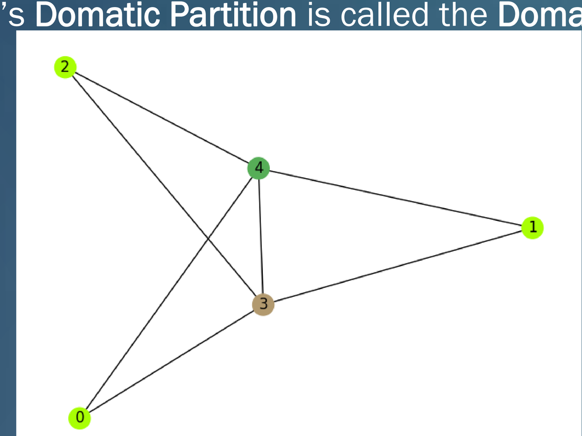
In this study we are taking a closer look at **Domatic partitions** and a subset of them called **Equitable Domatic partitions**. To do this we utilized a Python package called NetworkX which allows us to generate several graphs fitting certain conditions. We found that the **Equitable Domatic Number** is usually equal to or 1 less than the **Domatic number**. However, if there is 1 or 2 central nodes then the **Equitable Domatic Number** is farther from the **Domatic Number**. In relation to tree graphs, if the number of leaves on any node exceeds half of the total number of nodes, then the $d_e(G) = 1$. In conclusion, the degree of centralized nodes has a drastic impact on the **Equitable Domatic Number.**

Important terms

Nodes – The points from which edges extend from
Edges – Connections between the various nodes of the graph
Leaf – nodes that connect to only one other node
Leaf Degree – highest number of leaves on a given node
Domatic Partition – A graph in which every color is connected to every other color
Equitable Domatic partition – When all the colors in the **Domatic Partition** are equal

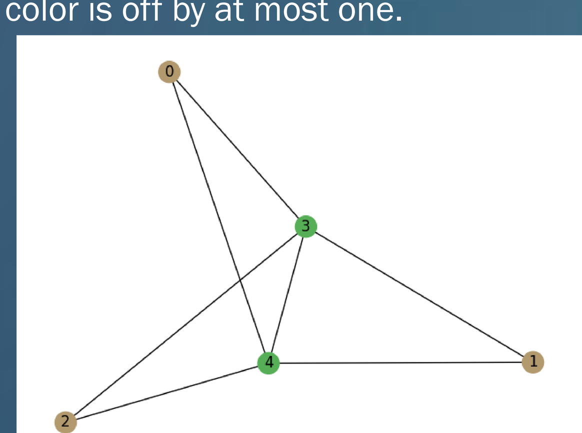
Introduction

Throughout the experiment we focused on a subset of **Domatic partitions** known as the **Equitable Domatic Partition**. A **Domatic partition** is an arrangement of the colored nodes in a graph such that every color is adjacent to every other color. The highest number of colors possible in a graph's Domatic Partition is called the Domatic Number.



$d(G) = 3$

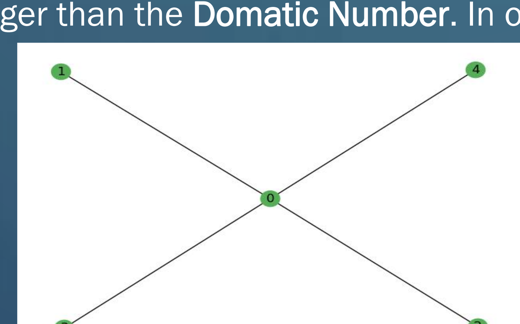
As shown in the figure above, every color is adjacent to every other color by connecting edges. In this graph, the **Domatic Number** is 3 since the largest number of colors we can have and still have a **Domatic Partition** is 3. In the case of an **Equitable Domatic Partition**, the total number of each color is off by at most one.



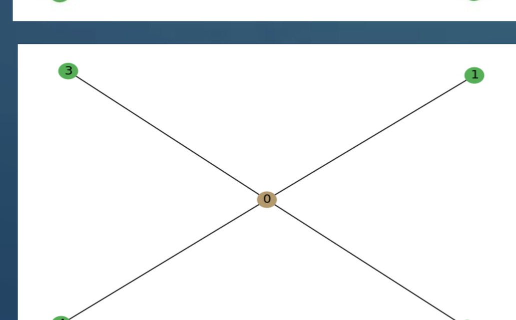
$d_e(G) = 2$

As shown in the figure above, every color is not only adjacent to every other color but also all total amount of each color is equal (3 brown nodes and 3 green nodes). Since some graphs have an odd number of nodes, making it impossible to distribute colors perfectly evenly, the amount of each color is allowed to be ± 1 from other colors. Since the **Equitable Domatic Partition** is a subset of the **Domatic Partitions** of a graph, the **Equitable Domatic Number** can't be larger than the **Domatic Number**. In other words: $d(G) \geq d_e(G)$.

Every graph with at least 2 nodes and one edge has at least one Domatic and Equitable Domatic Partition. The Domatic Partition consists of 2 colors touching each other and the Equitable Domatic Partition consists of all nodes being the same color.



$d_e(G) = 1$



$d(G) = 2$

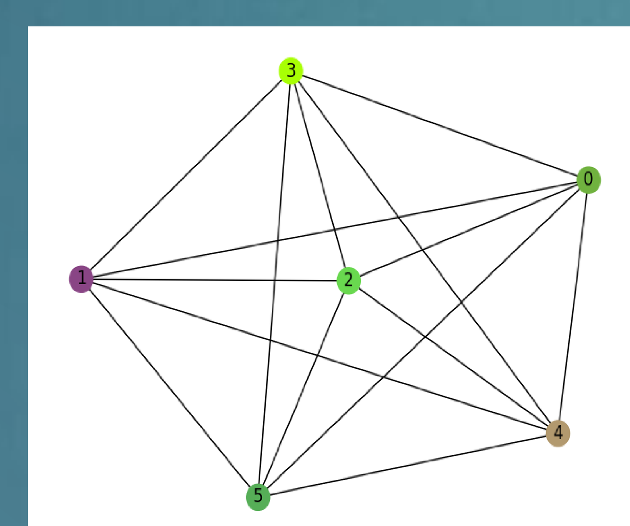
How did we go about proving our statement?

In order to prove our statement, we created a python program to generate random graphs given a desired number of nodes. Linked is the GitHub repository for the code used (https://github.com/Siddarth-Kota/DomaticNumberResearch). For graph generation we made use of the graph managing program, network X. This allowed us to create the network of nodes and edges that could be automatically drawn for easy visualization

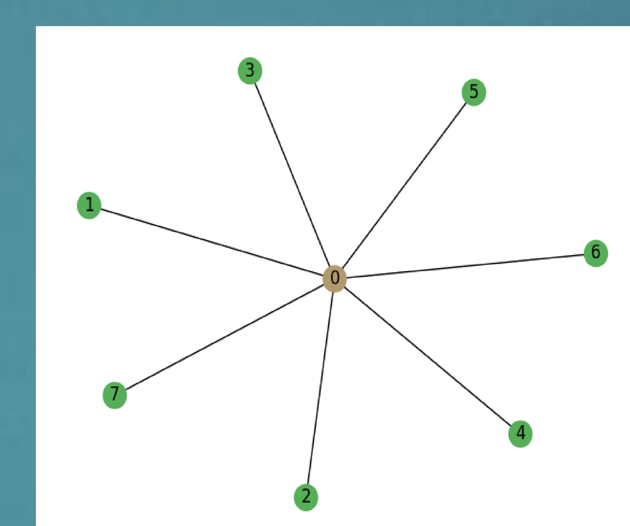
Below is the process by which the code works:

1. The code starts off by generating a random graph by finding all possible edges within a net of nodes
2. It then chooses randomly from the list of edges to create a graph with nodes and edges
3. Then the code utilizes this graph and finds all the possible ways to color the nodes of the graph
4. It then checks each of these colorings for possible **Domatic Partitions.**
5. The code filters the list of **Domatic Partitions** to those that are also equitable.
6. Using this information, the program colors the randomly generated graph with the coloring with the largest **Domatic Numbers**
7. In order to learn patterns about these graphs, the program searches for graphs in which the **Domatic Number** isn't equal to the **Equitable Domatic Number**

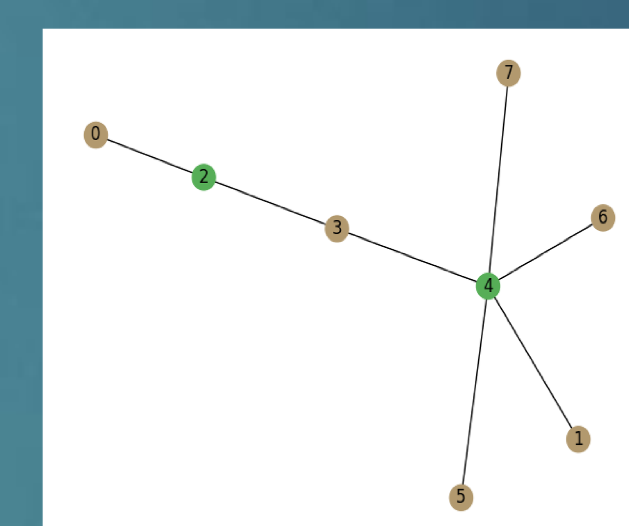
Types of graphs



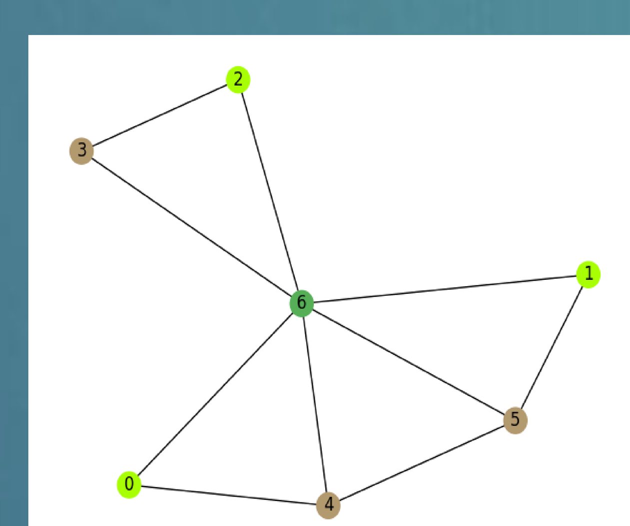
Complete graph
$d(G) = n = d_e(G)$
Where n = number of nodes



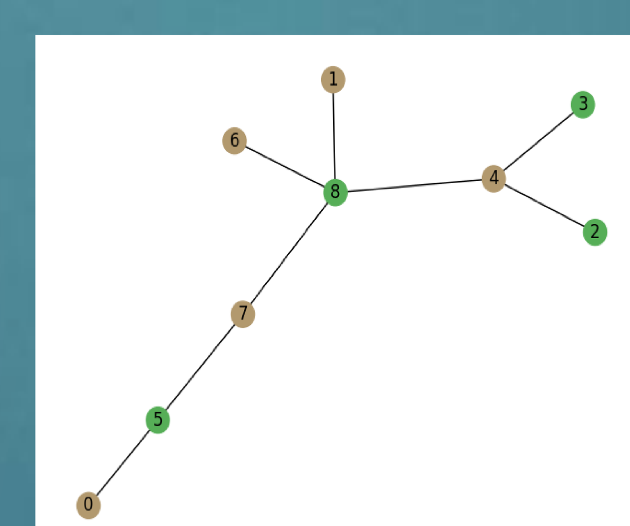
Dominating Set
In this graph, all nodes only touch one node. When the central node's degree is greater than 3,
$d(G) = 2$ & $d_e(G) = 1$



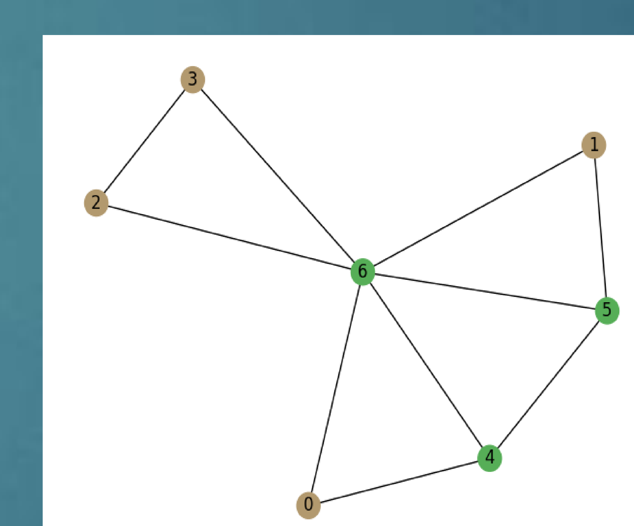
Tree graph
In this graph $d(G)$ is always 2 assuming there are at least 2 nodes.
$d_e(G) \leq 2$



Domatic Partition
$d(G) = 3$



Repeating graph
In this graph, colors are alternating during the line segments, $d(G) = 2$



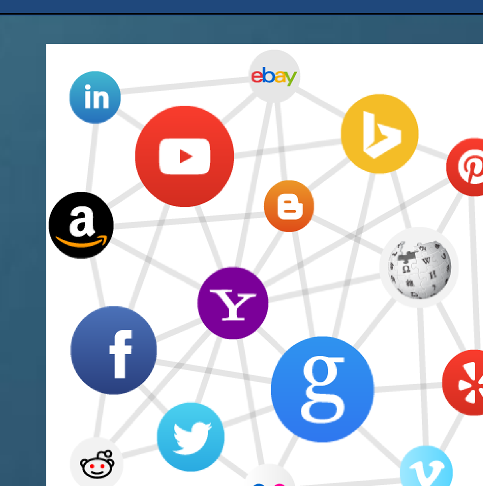
Equitable Domatic Partition
$d(G) = 3$

Real World Applications

Graph theory has several applications in real life ranging from mapping out complicated social networks to finding efficient paths in quantum computing systems.

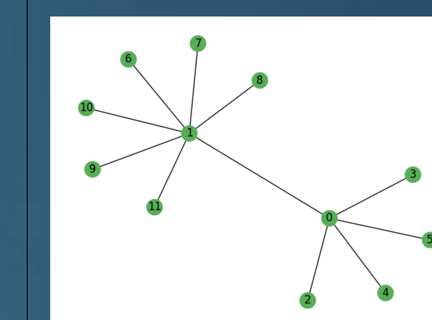
Domatic partitions are particularly important for computer organization. An example of this is drop box. Drop box keeps user data on multiple locations and multiple hard drives within these locations. Suppose they organize all hard drives dedicated to user into 3 categories: Photos, Videos, Apps. In order to allow the user to access all the data seamlessly, all hard drives must be able to access the other types of hard drives.

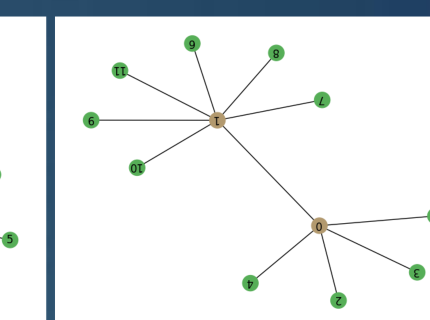
Equitable Domatic Partitions are helpful to streamline the process. In the same example as above, we need to be able to save equal space for each category. This is where the **Equitable Domatic partition** comes in handy to find an arrangement of categories such that there is equal distribution among all the drives.

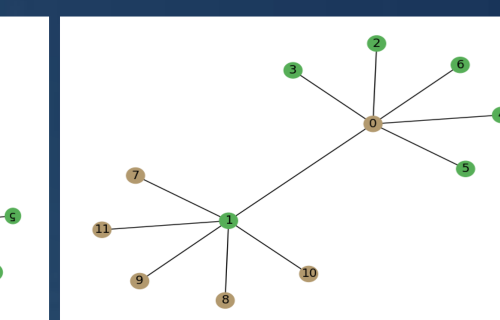


Findings

While exploring the Equitable Domatic Number we started by exploring Tree graphs. Within these graphs we were trying to find a connection between leaf number and the Equitable Domatic Number. Specifically, the following graphs allowed us to find useful patterns.



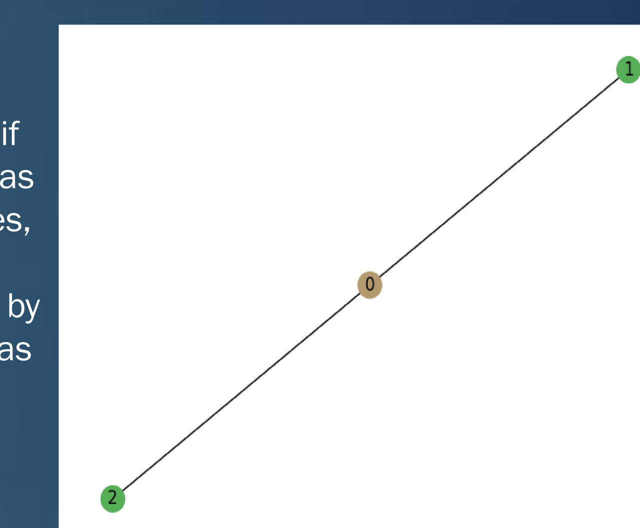




Leaf degree: 6
Total number of Nodes: 12
When one node has more leaves or equal number of leaves connected to it than half the number of nodes, then $d_e(G) = 1$ & $d(G) = 2$ In other words:
If $m \geq \frac{n}{2}$ then $d_e(G) = 1$

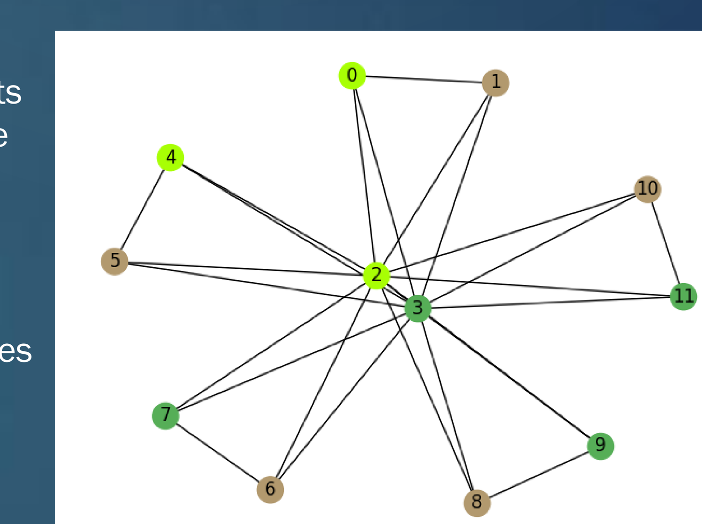
- Leaf degree: 5
- Total number of nodes: 12
When the nodes with the largest leaf degree have an equal number of leaves
$d_e(G) = 2$

We then tested the converse, that being if the highest leaf degree of all the nodes was less than half of the total number of nodes, is an Equitable Domatic Number of 2 guaranteed? This theory was proven false by a graph with 3 nodes. The central node has a leaf degree of 2 yet $d_e(G) = 2$.



Originally thought that the **Equitable Domatic number** could only be at most 1 less than the **Domatic Number**. This was proven false, however.

This graph consists of sets of 2 nodes on the outside forming boxes with 2 central nodes. The two central nodes are connected to every other node while the outer nodes connect to their partner node, and the 2 central nodes



$d(G) = 4$
$d_e(G) = 2$

Future Directions

Given more time on this project, we noticed that graphs where there was a large difference between the minimum degree and maximum degree were the ones that mainly resulted in different **Equitable Domatic** and **Domatic Numbers**.

We would also like to utilize new technology and implement a machine learning algorithm to upgrade the python program to be able to detect any pattens that the human eye cannot see.

Acknowledgments

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