**Date: January 19, 2021**
**Time: 9:00 – 09:45 hrs.**

# Indian institute of technology Indore

MA 204: Numerical Methods

**Examination: Quiz-1** **Total Marks: 15**

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**This question paper contains Multiple Select Type Questions.**
**Choose the appropriate answer(s) of all the following questions:**

1. How one can form a Partial Differential Equations? **[2]**
   (a) By eliminating arbitrary functions from a given relation between the dependent and independent variables.
   (b) By eliminating the arbitrary constants that occur in the functional relation between the dependent and independent variables.
   (c) Only can define in a boundary domain.
   (d) Partial and ordinary derivative may be involved.

2. The explicit method of solution $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ is convergent if $r = \frac{\alpha^2 \Delta t}{(\Delta x)^2}$ must be **[2]**
   (a) $r > 1$
   (b) $r < 1$
   (c) $r \leq 0.5$
   (d) $r = 2$

3. Using Crank-Nicholson formula solve the following PDE $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$. Given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 200t$. Compute $u(x, t)$ for one step in $t$ division. **[3]**
   (a) 3.171, 14.586, 53.271
   (b) 3.251, 14.379, 53.832
   (c) 3.571, 14.286, 53.571
   (d) None of the above

4. Which of the following is (are) NOT diagonally dominant matrices? **[1]**

   (a) $\begin{bmatrix} 15 & -2 & 2 \\ 1 & 7 & -3 \\ -2 & 1 & 4 \end{bmatrix}$,  (b) $\begin{bmatrix} 8 & -3 & 4 \\ 3 & 4 & -1 \\ 3 & 8 & 10 \end{bmatrix}$,

   (c) $\begin{bmatrix} 5 & -1 & 6 \\ 3 & 8 & 4 \\ 0 & -3 & 5 \end{bmatrix}$,  (d) $\begin{bmatrix} 9 & 3 & 5 \\ 7 & 11 & -3 \\ -3 & 3 & 7 \end{bmatrix}$.

5. Which of the following statements is (are) correct? **[2]**

   (a) The number of significant digits in the number 502.076090 is 8.

   (b) The number of operations in transforming a $3 \times 3$ real matrix to an upper triangular matrix through the Gauss elimination is about 18.

   (c) The condition number of a real non-singular square matrix is always greater than or equal to one.

   (d) The matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is diagonalizable.

   (e) The maximum value of round-off error in 5-digits rounding is $0.5 \times 10^{-5}$.

6. Let $A(\alpha) = \begin{bmatrix} 0.1\alpha & 0.1\alpha \\ 1 & 1.5 \end{bmatrix}$ be a real matrix, where $\alpha$ is a positive number. If $\alpha_0$ is the value of $\alpha$ such that the conditional number of $A(\alpha)$ is minimum, then the value of $\alpha_0$ is (**Hint:** Use the $L_\infty$ norm). **[2]**

   (a) 2.5 (b) 7.5 (c) 12.5 (d) 15

7. The system

$$\begin{aligned} 2x_1 + 2x_2 + x_3 &= 1 \\ 4x_1 + 2x_2 + 3x_3 &= 2 \\ x_1 + x_2 + x_3 &= 3 \end{aligned}$$

   has a unique (exact) solution $\boldsymbol{x} = [-4.5, 2.5, 5]^\top$. Starting with an initial guess $\boldsymbol{x}^{(0)} = [1, 1, 1]^\top$, solve the system using the Gauss–Seidel methods. Which of the following is the 4th iteration solution of the system with its relative error (with respect to $L_\infty$ norm)? **[2]**

   (a) $\boldsymbol{x}^{(4)} = (-3, 1.75, 4.25)$; relative error=0.3333

   (b) $\boldsymbol{x}^{(4)} = (-2.25, 1.75, 3.5)$; relative error=0.5

   (c) $\boldsymbol{x}^{(4)} = (-3.375, 1.375, 5)$; relative error=0.25

   (d) $\boldsymbol{x}^{(4)} = (-3.375, 0.25, 6.125)$; relative error=0.7778

8. The spectral radius $\rho(A)$ of the matrix $A = \begin{bmatrix} 4 & -3 & 2 \\ 2 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$ is **[1]**

   (a) 7 (b) 2 (c) 3 (d) 4 (e) 8