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INDIAN INSTITUTE OF TECHNOLOGY INDORE

MA 204 NUMERICAL METHODS

Tutorial Sheet I

(Instructor: Dr. Swadesh Kumar Sahoo)

1. Using the trapezoidal rule, find the approximate values of the following integrals. Also obtain a bound for the error in each case. Compute up to five decimal places using your calculator.

   (a) $\displaystyle\int_0^{0.4} \ln(1+x^4)\,dx$

   (b) $\displaystyle\int_1^5 \sqrt{1+x^2}\,dx$

   (c) $\displaystyle\int_{0.8}^1 e^{-x^2}\,dx$

   Ans: (a) 0.00506, 0.00965; (b) 13.02647, 1.88562; (c) 0.08952, 0.00049

2. Approximate the area under the curve $y = f(x)$ between $x = a$ and $x = b$ using the trapezoidal rule, where the function is given in the table of values.

   (a)

x	0	4
$f(x)$	2	5

   (b)

x	-4	2
$f(x)$	0	2

   (c)

x	1.6	2
$f(x)$	-3	-6.5

   Ans: (a) 14, (b) 6, (c) -1.9

3. Using the trapezoidal rule, approximate the area under the curve $y = f(x)$ between $x = a$ and $x = b$:

   (a) $f(x) = \sin^2 x$ with $a = 0$ and $b = \pi/2$;

   (b) $f(x) = 1/x$ with $a = 1$ and $b = 5$;

   (c) $f(x) = 2^x$ with $a = -1$ and $b = 3$.

   Ans: (a) 0.78540, (b) 2.4, (c) 17

4. Choose the correct answer. Suppose that $|f''(x)| \le 1$ for $0 \le x \le 2$. If E is the error in the trapezoidal rule, then the absolute error $|E|$ is less than

   (a) 1/2

   (b) 2/3

   (c) 3/2

   (d) 2

5. Derive the approximation formula for $\displaystyle\int_a^b f(x)\,dx$ from the Newton-Cotes method with $n = 2$.

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