TIME DOMAIN SIMULATION USING EULER INTEGRATION

Arjun Yeravdekar[1], Sagar Paradkar[2], Siddhesh Shailesh Rajput[3], and Swagat Shah[4]

University of North Carolina at Charlotte
Charlotte, North Carolina, 28223
[1]ayeravde@uncc.edu, [2]sparadka@uncc.edu, [3]srajput1@uncc.edu, [4]sshah81@uncc.edu

December 5, 2018

Abstract

A time domain simulation will be developed to solve the equation of motion for a single degree of freedom forced vibration system using Euler Integration approach and time domain solution is used to identify the magnitude versus frequency behavior of the system. A plot of FRF magnitude in the frequency domain and in the time domain will be compared.

Keywords : Forced Vibration, Euler Integration, Numerical Integration, Frequency Response Function, Sine Sweep Test

Nomenclature

m : Mass
k : Stiffness
c : Viscous Damping
ω : Forcing Frequency
ω_n : Natural Frequency
F_o : Force Magnitude
T : Time
dt : Time Step Size
FRF : Frequency Response Function

1 Introduction

The topic of mechanical vibrations deal with the oscillating response of elastic bodies to disturbances. Any body that possess mass and stiffness can vibrate. Mechanical vibrations are categorized into 3 types-free vibrations, forced vibrations and self-excited vibration. In forced vibrations, a continuous periodic force is applied to the system and the oscillations are produced [1]. In this project, we focus on single degree of freedom forced vibration system. The project aims to obtain a time domain solution which is developed using Euler integration method for a equation of motion of a single degree of freedom forced vibration system. This solution is used to identify the magnitude versus frequency behaviour of system.

The forcing frequency is varied to identify the magnitude of steady-state solution and the magnitude of time domain solution. At each frequency, the solution is obtained. Plots are developed for the magnitude of steady-state solution, the magnitude of time domain solution versus the forcing frequency.

1.1 Euler Integration

The Euler integration method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size.
For a single degree of freedom, lumped parameter, viscously damped system under forced vibration, the corresponding equation of motion is:

$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t \tag{1}$$

For time domain simulation we use the Euler Integration method to calculate the displacement, velocity, and acceleration. The equation of motion is numerically integrated to find the current values of displacement, velocity, and acceleration using the previous values. The simulation is carried out in small time steps, 'dt'. Generally 'dt' is set to be 10 to 50 times smaller than the period corresponding to the highest

natural frequency in the system's dynamic model. We assume the initial conditions to be zero i.e., displacement ($x_0 = 0$) and velocity ($dx_0 = 0$).
The acceleration in the current time step is determined by rewriting the previous equation;

$$\ddot{x} = \frac{F_o \sin \omega t - c\dot{x} - kx}{m} \tag{2}$$

The velocity for the current time step is then determined by numerical (Euler) integration;

$$\dot{x} = \dot{x} + \ddot{x}.dt \tag{3}$$

where the velocity on the right hand side of the equation is retained from the previous time step and is used to update the current value.
The displacement for the current time step is then determined by numerical (Euler) integration using the velocity calculated above:

$$x = x + \dot{x}.dt \tag{4}$$

Again, the displacement on the right hand side of the equation is retained from the previous time step. Finally, the time-dependent displacement can be written to a vector, y, as:

$$y_n = x \tag{5}$$

where the n subscript on y indicates the time step. The corresponding time is $t_n = n.dt$

1.2 Objective

The objective of the project is to find magnitude versus frequency behavior of the system using time-domain simulation to solve the differential equation of motion using the Euler integration. Then use that time-domain solution to identify the magnitude of the vibration response

2 Description of the System

We are using a Single degree freedom lumped parameter model supported by mass less spring and damper forced vibration system for the purpose of this project. Single degree of Freedom system means the system can move in one direction only as marked in the figure given below. Lumped parameter model means that all the mass is concentrated at a single co-ordinate. This mass is supported by a spring and damper assumed to have no accountable mass. The forced vibration system means that is system isn't freely excited. It is excited by a sinusoidal force input at the mass coordinate.
The system parameters are:

$$\begin{aligned} m &= 10Kg \\ k &= 1e6N/m \\ c &= 300Ns/m \\ F_o &= 100N \end{aligned}$$

where M is the mass, K is the spring constant, c is the viscous damping coefficient and F_o is the force magnitude.
The equation of motion is

$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t \tag{6}$$

The unit of frequency is rad/s.

The purpose of this project is to solve this system to get the time domain solution at each forcing frequency from 1 rad/s to 3 ω_n rad/s using numerical integration method and plot the magnitude of displacement normalized to the magnitude of forcing vs the forcing frequency in Hz. To get the solution, we used the Euler integration technique to get the time domain response at each frequency. We also got the forcing function vs time plot at each frequency. In order to get the frequency response function, we normalized the displacement to the forcing function at the corresponding frequency, picked a max value after attaining the steady state. Repeated the same for each frequency from 1rad/s

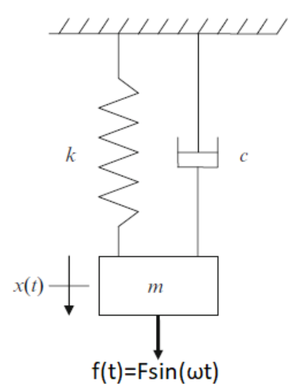

Figure 1: Vibration System

The application of sinusoidal force input to get a time domain solution resembles to something known as a sine sweep test In the sine sweep the frequency response function is determined using one frequency at a time. At each frequency the sinusoidal force is applied and the time domain response to the input force is calculated and averaged over a short time interval. In this test a vibration tone is ramped up and down through a range of frequencies and for a specified rate and duration. It is primarily used to identify the resonant conditions within a test item.

Example of sine sweep test: On electronic circuit board, the boards themselves are resonant structures and typically their fatigue conditions are most important to understand. The components mounted on the PCB have their own unique natural frequencies. Using sine vibration test we can easily identify the mechanical vibrations that propagate through the device and help the designers to stiffen or dampen the elements to reduce the fatigue failure[2].

3 Simulation Description

We use numerical integration to solve the single degree of freedom forced vibration equation of motion and then use that information to identify the magnitude versus frequency behavior of the system. We have developed a time domain simulation to solve the differential equation of motion using the Euler integration approach and then used that time domain solution to identify the magnitude of the vibration response.

The magnitude of the steady state response is obtained from the time domain solution of the equation of motion as each frequency. The time domain simulation is carried out in small time steps (dt), if dt value is too large inaccurate results are obtained. Thus as a rule of thumb, it is generally acceptable to set dt to be 10 to 50 times smaller than the period corresponding to the highest natural frequency in the system's dynamic model. We use 2,3,4 and 5 to calculate the magnitude of the vibration response at each frequency. Thus if we plot the displacement versus a time graph we can see that the waveform comprises of a transient section as well as a steady state section. Since we are interested in the steady state section of the waveform we select the later half of the waveform where the transient has decayed. Thus for different frequencies we will get different amplitudes of the steady state waveform and when we plot the ratio of these amplitudes with respect to the Force magnitude (given) by varying the forcing frequency from 1 rad/s to 3 times the natural frequency with a resolution of 2 rad/s, we will get a graph which will be an exact replica of the graph of the frequency response function of a single degree of freedom system under forced vibration, In this case we have varied the forcing frequency from 1 rad/s to 3 times the natural frequency with a resolution of 1 rad/s.

4 Results

Using Euler integration approach, we have found the time domain solution of the system, normalized to the force magnitude at steady state condition and plotted it against frequency from 1 Hz to $3\omega_n$. We have also plotted the FRF magnitude in the frequency domain with a resolution of 1 rad/s. Compared to the time domain solution with a resolution of 2 rad/s. It cab be seen from the Figure 2 that both the solutions overlap as expected.

5 Conclusion

The time domain solution for a single degree freedom forced vibration system was calculated using the Euler Integration method to get the magnitude versus frequency behaviour of the system. It was compared to the frequency response function obtained using the frequency domain equation. It was found that both the methods give same results.

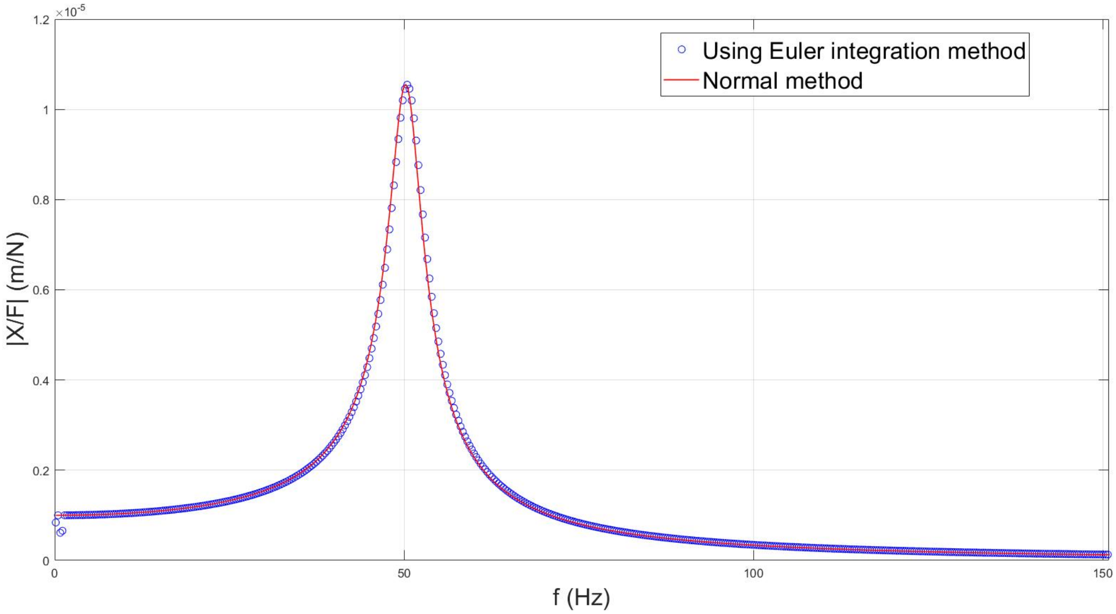

Figure 2: Frequency Behavior of the System

6 Refrences

[1] Tony L Schmitz and K Scott Smith. Mechanical Vibrations. Springer, 2012.
[2] Eric Sauther. Sine Sweep Vibration Testing for Modal Response Primer. Department of Optical Sciences. 2013.
[3] Matlab version 9.2.0 (R2017a), the MathWorks Inc. Naick Massachusetts. 2017.

7 Appendix

7.1 Matlab code

[3]
article graphicx color
lightgraygray0.5

Contents

- given

```
clc;
clear all;
close all;
```

given

```
k = 1e6;                                     %stiffness, N/m
m = 10;                                      %mass, Kg
c = 300;                                     %viscous damping coefficients, N-s/m
Fo = 100;                                    %force magnitude, N
```

```
wn = sqrt(k/m);                        %natural frequency
w = 1:3*wn;                            %varying the forcing frequency
                                       %between 1rad/s and 3*natrual
                                       %frequency
f_max = wn/(2*pi);                     %highest natural frequency
T = 1/f_max;                           %Period corresponding to the
                                       %highest natural frequency
dt = T/40;                             %dt = 40times smaller than the period
                                       %corresponding to the highest natural
                                       %frequency
t = 0:dt:1;
dx = 0;                                %initial condition for velocity
x = 0;                                 %initial condition for displacement
y = zeros(size(w,2),size(t,2));
for i = 1:size(w,2)
    for j = 1:size(t,2)
        ddx = (Fo*sin(w(i)*t(j)) - c*dx - k*x)/m;
        dx = dx + ddx*dt;
        x = x + dx*dt;
        y(i,j) = x;
    end
end
r = w/wn;
zeta = c/(2*(sqrt(k*m)));
mag = 1/k*(1./((1-r.^2).^2 + (2*zeta*r).^2)).^0.5;
X = zeros(size(w,2),size(t,2)-999);
X_max = zeros(size(w,2),1);
 for k = 1:size(w,2)
     X(k,:) = y(k,1000:size(t,2));
     X_max(k,1) = max(X(k,:));
 end
figure();
plot(w(1:2:end)/(2*pi),X_max(1:2:end)/Fo,'bo');
hold on;
plot(w(1:1:end)/(2*pi),mag(1:1:end),'r','Linewidth',1);
lgd = legend('Using Euler integration method','Normal method');
lgd.FontSize = 20;
xlabel('f (Hz)','FontSize',20)
ylabel('|X/F| (m/N)','FontSize',20)
xlim([0 max(w/(2*pi))])
grid on;
```