day

Monday

Date : 12/08/24

Module 3 : Greedy Method Approach

(Fractional)
* Knapsack Problem (Maximum Optimization) optimize optimize

$n = 7$; $M = 15$

$P\{1:7\} = \{10, 5, 15, 7, 6, 18, 3\}$

$W\{1:7\} = \{2, 3, 5, 7, 1, 4, 1\}$

→

object	P	W	P/W
1	10	2	5
2	5	3	1.67
3	15	5	3
4	7	7	1
5	6	1	6
6	18	4	4.5
7	3	1	3

(descending order)

→

object		P	W	P/W
~~1~~	5	6	1	6
~~2~~	1	10	2	5
~~3~~	6	18	4	4.5
~~4~~	3	15	5	3
~~5~~	7	3	1	3
~~6~~	2	5	3	1.67
~~7~~	4	7	7	1

Date :

$= \{1 + 2 + 4 + 5 + 1 + (3\times 2/3)\}$

$= (6 + 10 + 18 + 15 + 3 + (5\times 2/3))\}$

$= 55.34$

$= \{1, (1\times 2/3), 1, 0, 1, 1, 1\}$

Q) $n = 5$ $W = 60kg$

$P\{1:5\} = \{30, 40, 45, 77, 90\}$

$w\{1:5\} = \{5, 10, 15, 22, 25\}$

object	P	W	P/W
1	30	5	6
2	40	10	4
3	45	15	3
4	77	22	3.5
5	90	25	3.6

Descending order

object	P	W	P/W
1	30	5	6
2	40	10	4
5	90	25	3.6
4	77	22	3.5
3	45	15	3

$\cancel{(1+2+} \Rightarrow \left(5+10+25+\left(\frac{22\times 20}{22}\right)\right)$

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$= 80 + 40 + 90 + \left(77 \times \frac{20}{22}\right)$

MAX profit $= 230$

weight $= \left(1, 1, 0, \frac{10}{11}, 1\right)$

Date :

* Job Sequencing with Deadlines

$n = 7$

$I\{1:7\} = \{1, 2, 3, 4, 5, 6, 7\}$ I → Job no.

$P\{1:7\} = \{3, 5, 20, 18, 1, 6, 30\}$ p: profit of each job

$D\{1:7\} = \{1, 3, 4, 3, 2, 1, 2\}$ D = Deadline of each job.

As maximum waiting time is not given in given problem Take maximum waiting time that is Job which has more deadline

i.e. $n = 4$

Job	7 ✓	3 ✓	4 ✓	6 ✓	2	1	5
Profit	30	20	18	6	5	3	1
Deadline	2	4	3	1	3 ×	1 ×	2 ×

0 1 2 3 4

6	7	4	3
6	30	18	20
1	2	3	4

Max Profit = $\{6 + 30 + 18 + 20\} = 74$

$= \{0, 0, 1, 1, 0, 1, 1\}$

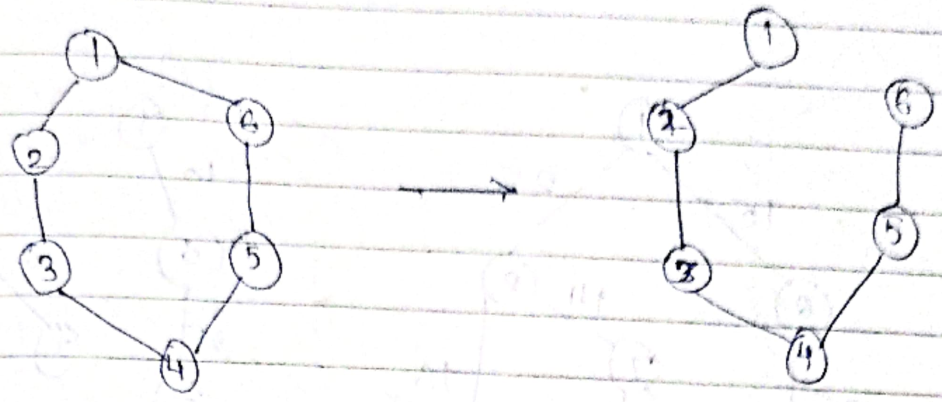
Date : 19/08/24

Module 3

(Minimum Optimalization)

* Spanning Tree

Not: Form loop/cycle.



$|V| \; \& \; |E|$

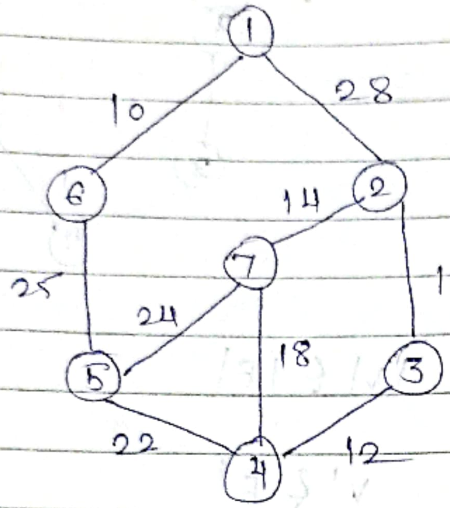
$V' \; \& \; E'$

$V' = |V| \; \& \; E' = |E| - 1$

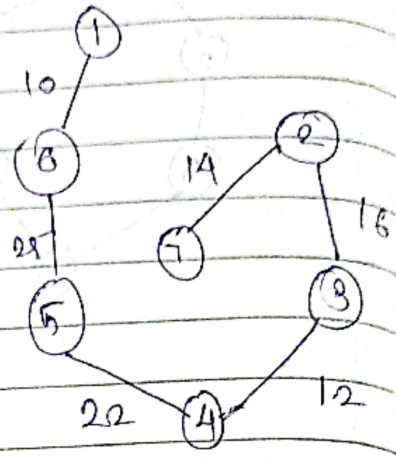
it vary as it depend on the given spanning tree

Date :

* Prim Algorithm (node by node)



$\rightarrow$



V = 7 , E = 9

Final
Minimum Spanning Tree
Cost = 10+25+22+12+16+14=99

* Analysis (Complexity)

$V' = |V|$ & $E' = |E| - 3$; $E' \approx |E|$

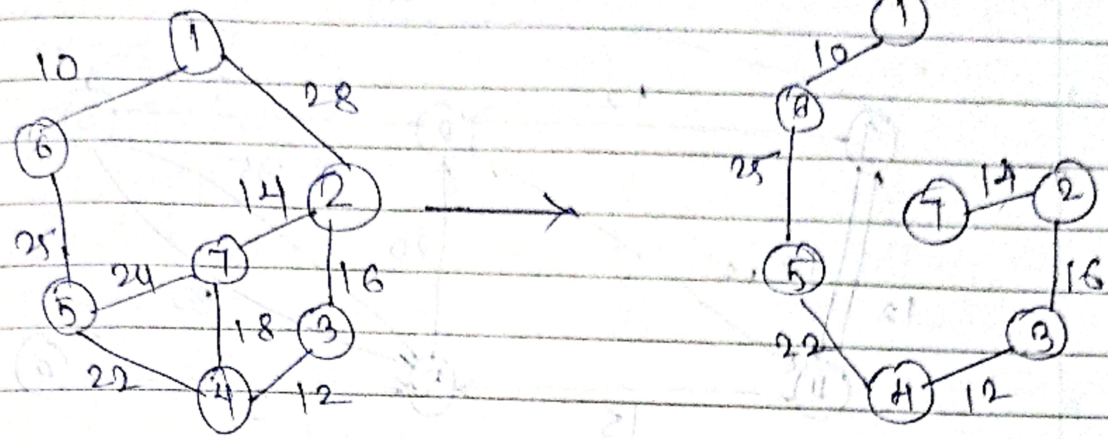
$(V' \times E') = (n \times n) = \underline{O(n^2)}$

$|V| = n$;

$|E| = n$;

Date :

* Kruskal Algorithm: (edge by-edge)



Minimum Spanning Tree

Cost = 10+14+16+12+22+25

= 99

* Analysis (Complexity)

|V| & |E|

Minimum

V' ⊆ E'

V' = |V| => n;

E' ≈ |E| = n;

Complexity

$T(n) = O(n^2)$

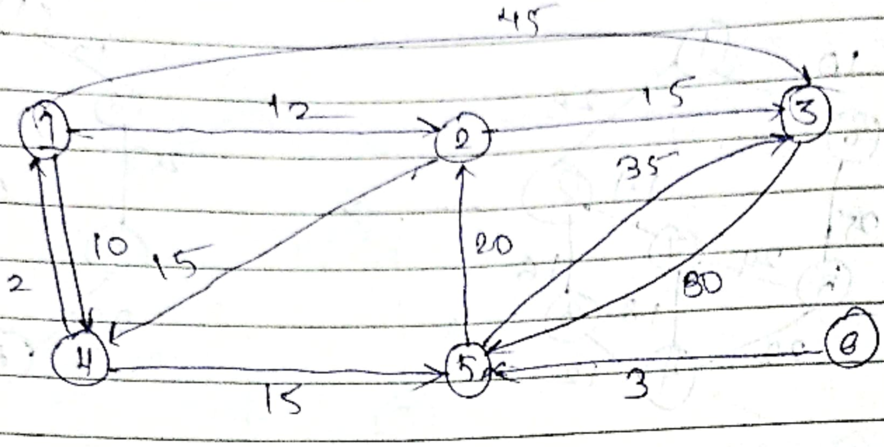
min Heap

|V| = V'

$T(n) = O(n \log n)$

Disadvantage – Not solve negative edge problem

* Dijkstra Algorithm (Single Source Shortest Path Algorithm)



	2	4	1	3	
Selected vertex	2	3	4	5	6
4	12	45	min 10	∞	∞
2	min 12	45	10	25	∞
5	12	27	10	min 25	∞
3	12	27	10	25	∞
	12	27	10	25	∞

$\boxed{1 - 4 - 2 - 3 - 5}$

Analysis (Complexity)

$|E| \approx E'$ & $|V| = V'$

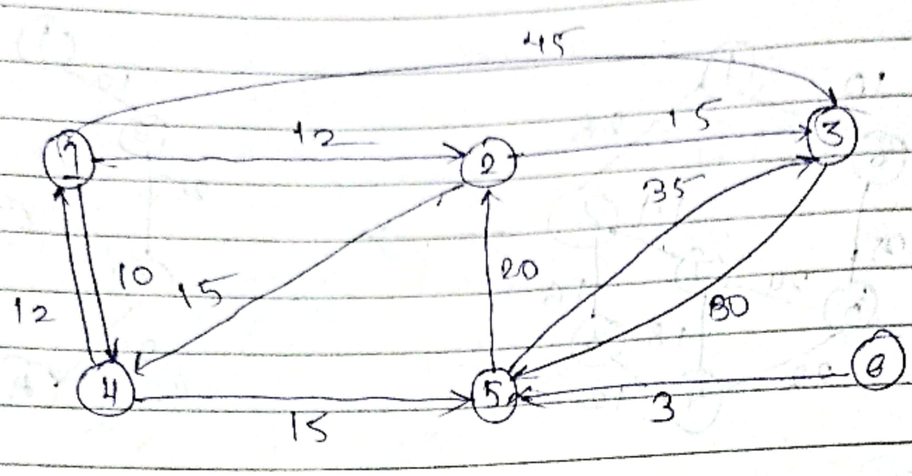
$E = n$ & $|V| = V' = n$

$T(n) = O(n^2)$

Date :

Disadvantage — Not solve negative edge problem

* Dijsktra Algorithm (Single Source Shortest Path Algorithm)



selected vertex	2	3	4	5	6
4	12	45	min 10	∞	∞
2	min 12	45	10	25	∞
5	12	27	10	min 25	∞
3	12	27	10	25	∞
	12	27	10	25	∞

$\boxed{1 - 4 - 2 - 3 - 5}$

Analysis (Complexity)

$|E| \approx E'$ & $|V| = V'$

$E = n$ & $|V| = V' = n$

$T(n) = O(n^2)$

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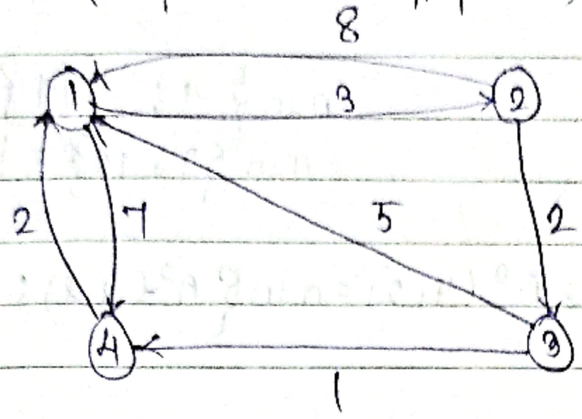
* All pair Shortest Path
(Floyd - warshall Algorithm)



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

$$A^1 = \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix}$$

$A(2,3) = A^0(2,3) = \min\{A^0(2,3);\ A^0(2,1)+A^0(1,3)\}$

$\underline{\min} = \{2;\ 8+\infty\} = 2$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

$A(2,4) = A^0(2,4) = \min\{A^0(2,4);\ A^0(2,1)+A^0(1,4)\}$

$= A^0(2,4) = \min\{\infty;\ 8+7\} = 15$

$A(3,2) = A^0(3,2) = \min\{A^0(3,2);\ A^0(3,1)+A^0(1,2)\}$

$= A^0(3,2) = \min\{\infty;\ 5+3\} = 8$

$$A(3,4) = A^0(3,4) = \min\{A^0(3,4);\ A^0(3,1) + A^0(1,4)\}$$

$$= \min\{1;\ 5+7\}$$
$$= \min\{1;\ 12\} = 1$$

$$A(4,2) = A^0(4,2) = \min\{A^0(4,2);\ A^0(4,1) + A^0(1,2)\}$$

$$= \min\{\infty;\ 2+3\} = 5$$

$$A(4,3) = A^0(4,3) = \min\{A^0(4,3);\ A^0(4,1) + A^0(1,3)\}$$

$$= \min\{\infty;\ 2+\infty\} = \infty$$

$$A^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

$$\downarrow$$

$$A(1,3) = A^2(1,3) = \min\{A^1(1,3);\ A^1(1,2) + A^1(2,3)\}$$

$$= \min\{\infty;\ 3+2\} = 5$$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \cancel{\infty}\,7 & 0 \end{array}$$

$$A(1,4) = A^2(1,4) = \min\{A^1(1,4);\ A^1(1,2) + A^1(2,4)\}$$

$$= \min\{7;\ 3+15\} = 7$$

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$$A(3,4) = A^0(3,4) = \min\{A^0(3,4);\ A^0(3,1) + A^0(1,4)\}$$

$$= \min\{1;\ 5+7\}$$
$$= \min\{1;\ 12\} = 1$$

$$A(4,2) = A^0(4,2) = \min\{A^0(4,2);\ A^0(4,1) + A^0(1,2)\}$$

$$= \min\{\infty;\ 2+3\} = 5$$

$$A(4,3) = A^0(4,3) = \min\{A^0(4,3);\ A^0(4,1) + A^0(1,3)\}$$

$$= \min\{\infty;\ 2+\infty\} = \infty$$

$$A^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

$$A(1,3) = A^2(1,3) = \min\{A^1(1,3);\ A^1(1,2) + A^1(2,3)\}$$

$$= \min\{\infty;\ 3+2\} = 5$$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$$A(1,4) = A^2(1,4) = \min\{A^1(1,4);\ A^1(1,2) + A^1(2,4)\}$$

$$= \min\{7;\ 3+15\} = 7$$

$A(3,1) = A^2(3,1) = \cancel{\min\{A^1(3,1)+A}$
$= \min\{A^1(3,1);\ A^1(3,2) + A^1(2,1)\}$
$= \min\{5;\ 8+8\} = 5$

$A(3,4) = A^2(3,4) = \min\{A^1(3,4);\ A^1(3,2) + A^1(2,4)\}$
$= \min\{1;\ 8+5\} = 1$

$A(4,1) = A^2(4,1) = \min\{A^1(4,1);\ A^1(4,2) + A^1(2,1)\}$

$= \min\{2;\ 5+8\} = 2$

$A(4,3) = A^2(4,3) = \min\{A^1(4,3);\ A^1(4,2) + A^1(2,3)\}$
$= \min\{0;\ 5+2\} = 7$

$$A^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 15 & 8 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$A(1,2) = A^3(1,2) = \min\{A^2(1,2);\ \cancel{A}A^2(1,3) + A^2(3,2)\}$

$= \min\{3;\ \infty+8\} = 3$

$A(1,4) = A^3(1,4) = \min\{A^2(1,4);\ A^2(1,3) + A^2(3,4)\}$
$= \min\{7;\ \cancel{\infty}5+1\} = \cancel{8}6$

Date :

$A(2,1) = A^3(2,1) = \min\{A^2(2,1); A^2(2,3) + A^2(3,2)\}$

$= \min\{8; 2+7\} = 7$ //

$A(2,4) = \min\{A^2(2,4); A^2(2,3) + A^2(3,4)$

$= \min\{15; 2+1\} = 3$ //

$A^3(4,1) = \min\{A^2(2,4); A^2($

$= 2$ //

$A^3(4,2) =$

$= 5$

$$A^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

Formula : $A^k(i,j)$:

$\min(A^{k-1}(i,j), A^{k-1}(i,k) + A^{k-1}(k,j))$

Algo:

```
for ( k=1 ; k<=n; k++)
  {
   for (i=1; i<=n; i++)
   {
     for(j=1; j<=n; j++)
     {
        A^k(i,j) = min{A^k-1(i,j); A^k-1(i,k)
                        + A^k-1(k,j)}
     }
   }
  }
```

Time Complexity
$= O(n^3)$ //

Date :

$A(2,1) = A^3(2,1) = \min\{A^2(2,1);\ A^2(2,3) + A^2(3,2)\}$

$= \min\{8;\ 2+ \} = 7$

$A^3(2,4) = \min\{A^2(2,4);\ A^2(2,3) + A^2(3,4)\}$

$= \min\{15;\ 2+1\} = 3$

$A^3(4,1) = \min\{A^2(2,4);\ A^2($

$= 2$

$A^3(4,2) =$

$= 5$

$$A^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

Formula : $A^k(i,j)$:

$\min\left(A^{k-1}(i,j),\ A^{k-1}(i,k) + A^{k-1}(k,j)\right)$

Algo:

```
for ( k=1 ; k<=n; k++)
{
  for (i=1; i<=n; i++)
  {
    for(j=1; j<=n; j++)
    {
       A^k(i,j) = min{A^k-1(i,j); A^k-1(i,k)
                      + A^k-1(k,j)}
    }
  }
}
```

Time Complexity
$= O(n^3)$

day
Thursday

ADSA

Date :

Dynamic Programming

* 0/1 Knapsack Algorithm

Q.1) $m = 8$, $P = \{\cancel{1, 2, \ldots, 6}\} = \{1, 2, 5, 6\}$

$W = \{2, 3, 4, 5\}$

Soln

$S_0 = \{(0,0)\}$

$S_0^1 = \{(1,2)\}$

$S_1 = \{(0,0), (1,2)\}$

$S_1^1 = \{(2,3), (3,5)\}$

$S_2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_2^1 = \{(5,4), (6,6), (7,7), (8,9)\}$

$S_3 = \{(0,0), (1,2), (2,3), (3,5), (5,4), (6,6), (7,7), \cancel{(8,9)}\}$

$S_3^1 = \{(6,5), (7,7), (8,8), (11,9), (12,11), (13,12)\}$

$S_4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), \cancel{(6,6)}, (7,7), (8,8), \cancel{(11,9)}, \cancel{(12,11)}, \cancel{(13,12)}\}$

$S_4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), (7,7), (8,8)\}$

i) $(8,8) \in S_4$ $\therefore x_4 = 1$

ii) $(8,8) \notin S_3$ $\therefore x_3 = 0$

iii) ~~(8,8)~~ $(8-6, 8-5) = (2,3) \in S_2 = x_2 = 1$

iv) $(2-3)$, ~~(2-2)~~ $3,3) = (0,0)$, $\notin S_1 = x_1 = 0$

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Date :

Dynamic Programming

* 0/1 Knapsack Algorithm

Ex1) $m = 8$, $P = \{\cancel{1, 2, \ldots, 6}\} = \{1, 2, 5, 6\}$

$W = \{2, 3, 4, 5\}$

Sol

$S_0 = \{(0,0)\}$

$S_0' = \{(1,2)\}$

$S_1 = \{(0,0), (1,2)\}$

$S_1' = \{(2,3), (3,5)\}$

$S_2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_2' = \{(5,4), (6,6), (7,7), (8,9)\}$

$S_3 = \{(0,0), (1,2), (2,3), (3,5), (5,4),$
$(6,6), (7,7), \cancel{(8,9)}\}$

$S_3' = \{(6,5), (7,7), (8,8), (11,9), (12,11),$
$(13,12)\}$

$S_4 = \{(0,0), (1,2), (2,3), (5,4), (6,5),$
$\cancel{(6,6)}, (7,7), (8,8), \cancel{(11,9)},$
$\cancel{(12,11)}, \cancel{(13,12)}\}$

$S_4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), (7,7),$
$(8,8)\}$

i) $(8,8) \in S_4 \quad \therefore x_4 = 1$

ii) $(8,8) \notin S_3 \quad \therefore x_3 = 0$

iii) $\cancel{(8,8)}$ $(8-6, 8-5) = (2,3) \in S_2 = x_2 = 1$

iv) $(2-3), \cancel{(2,3)}$ $3, 3) = (0,0) \notin S_1 = x_1 = 0$

$S = \{0, 1, 0, 1\}$

Ex 2.) $M = 9$, $P = \{18, 25, 27, 10\}$

$w = \{2, 4, 5, 3\}$

Sol:

$S_0 = \{(0,0)\}$

$S_0^1 = \{(18,2)\}$

$S_1 = \{(0,0), (18,2)\}$

$S_1^1 = \{(25,4), (43,6)\}$

$S_2 = \{(0,0), (18,2), (25,4), (43,6)\}$

$\cancel{S_2^1 = \{(27,5), (43,9), (50,11), (68,13)\}}$

$\cancel{S_3 = \{(0,0), (18,2), (25,4), (43,6),}$ $\cancel{(25,}$

$S_2^1 = \{(27,5), (45,7), (52,9), (70,11)\}$

$S_3 = \{(0,0), (18,2), (25,4), (43,6), (27,5), (45,7), (52,9), (70,11)\}$

$S_3^1 = \{(10,3), (28,5), (35,7), (37,8), (53,9), (55,10), (62,12)\}$

$S_4 = \{(0,0), (18,2), (25,4), \cancel{(10,3)}, \cancel{(27,5)}, (28,5), \cancel{(43,6)}, (43,6), (45,7), \cancel{(52,9)}, (35,8), \cancel{(37,8)}, (53,9), \cancel{(55,10)}, \cancel{(62,12)}\}$

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$S_4 = \{(0,0),(18,2),(25,4),(28,5),(43,6),$
$(45,7),(53,9)\}$

① $(53,9) \in S_4 \quad x_4 = 1$

② $(53-10, 9-3) = (43,6) \notin S_3 = x_3 = 0$

③ $(43,6) \in S_2 = x_2 = 1$

④ $(18,2)$
$(18-18, 2-2) = (0,0) \in S_1 \quad x_1 = 1$

$g = \{1, 1, 0, 1\}$

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Dynamic Programming

* LCS (Longest Common Subsequence)

Ex) X = BDCB

Y = BACDB

Z = { (BDB, BCB) }

x \ Y	B	A	C	D	B
B	↖1	←1	←1	←1	↖1
D	↑1	←1	←1	↖2	←2
C	↑1	←1	↖2	←2	←2
B	↖1	←1	↑2	←2	↖3

x

B D C B

Y: B A C D B

↖1 ←1 ↑2 ←2 ←2

BCB

BDB

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Mod 4 Dynamic Programming

Ex1)

$S_1 = \{B, C, D, A, A, C, D\}$

$S_2 = \{A, C, D, B, A, C\}$

$S_1 \backslash S_2$	A	C	D	B	A	C
B	←0	←0	←0	↖1	←1	←1
C	←0	↖1	←1	←1	←1	↖2
D	←0	↑1	↖2	2	←2	←2
A	↖1	←1	↑2	←2	↖3	←3
A	↖1	←1	↑2	↑2	↑3	←3
C	↑1	↖2	←2	←2	↑3	↖4
D	↑1	↑2	↖3	←3	←3	↑4

BABCA
ABAC

Eg 2)

Y_1 = ABA CABB

Y_2 = BAB CAB

ABA
BA.B
AB

A

BAB

BABCA

AB

BA

BA.B
AB

Y_1 | Y_2	B	A	B	C	A	B
A	←0	↖1	←1	←1	←1	←1
B	↖1	←1	↖2	←2	←2	↖2
A	↑1	↖2	←2	←2	↖3	←3
C	↑1	↑2	←2	↖3	←3	←3
A	↑1	↑2	←2	↑3	↖4	←4
B	↖1	↑2	↖3	←3	↑4	↖5
B	↑1	↑2	↑3	←3	↑4	↑5

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Monday

Mod 4. Dynamic Programming

Ex1) $S_1 = \{B, C, D, A, A, C, D\}$

$S_2 = \{A, C, D, B, A, C\}$

$S_1 \backslash S_2$	A	C	D	B	A	C
B	←0	←0	←0	↖1	←1	←1
C	←0	↖1	←1	←1	←1	↖2
D	←0	↑1	↖2	←2	←2	←2
A	↖1	←1	↑2	←2	↖3	←3
A	↖1	←1	↑2	↑2	↑3	←3
C	↑1	↖2	←2	←2	↑3	↖4
D	↑1	↑2	↖3	←3	←3	↑4

BABCA
ABAC

Eg 2)

$Y_1 = ABA\ CABB$

$Y_2 = BAB\ CAB$

ABA
BA.B
AB
A
BAB
BABCA
AB
BA
BAB
AB

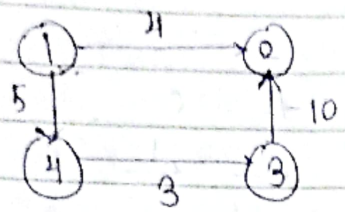
$Y_1 \vert Y_2$	B	A	B	C	A	B
A	←0	↖1	←1	←1	←1	←1
B	↖1	←1	↖2	←2	←2	↖2
A	↑1	↖2	←2	←2	↖3	←3
C	↑1	↑2	←2	↖3	←3	←3
A	↑1	↑2	←2	↑3	↖4	←4
B	↖1	↑2	↖3	←3	↑4	↖5
B	↖1	↑2	↑3	←3	↑4	↑5

mcm (not in mse 2)

Date :

* ~~Bell-man-ford~~ Bellman-Ford Algorithm

↳ (Dynamic Algorithm)



Relaxation Rule

$$\left\{ \begin{array}{l} \text{if } d(u) + c(u,v) < d(v) \\ \text{then} \\ d(v) = d(u) + c(u,v) \end{array} \right\}$$

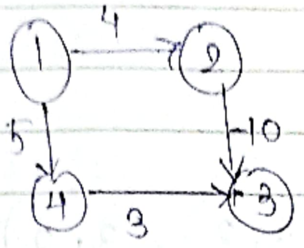
day
Monday

ADSA

Date : 30/09/24

* Bellman Ford Algorithm

→ single source shortest path Algo.
Dij → +ve edges → Greedy Algo.
Bellman-ford Algo → -ve edges



~~no of edges~~
no. of nodes
Relax → $(n-1)^{th}$ times
↳ run to (n-1) times

{ (1,2), (1,4), (3,2), (4,3) }

Edges ←	E	1	1	3	4
distance ←	D	4	5	-10	3

Complexity : No of nodes x no. of edges
$= n \times m = O(n*m)$

if no. of edges = no. of nodes

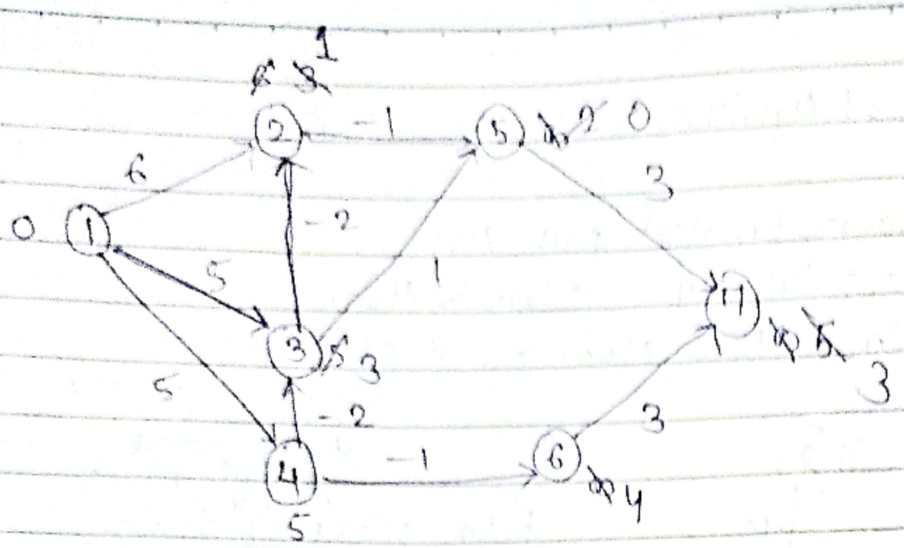
$T.C = n*n = O(n^2)$ → Best case

if graph is complete graph
$T.C = \frac{n(n-1)}{2} * m$ (if n = m)

$T.C. = \frac{n(n-1)}{2} * n = O(n^3)$ → worst case

Date :

Q.2)



(1, 2), (1, 3), (1, 4), (2, 5), (3, 2), (3, 5), (4, 3), (4, 6), (5, 7), (6, 7)

1st Iteration

(1, 2)	(1, 3)	(1, 4)	(2, 5)	(3, 2)	(3, 5)	(4, 3)	(4, 6)	(5, 7)	(6, 7)
6	5	5	-1	-2	1	-2	-1	3	3

(1, 2)	(1, 3)	(1, 4)	(2, 5)	(3, 2)	(3, 5)	(4, 3)
3	3	5	~~2~~	1	1	

Date :

* MCM (Matrix Chain Multiplication)

$A_1 = 30 \times 35$
$A_2 = 35 \times 15$
$A_3 = 15 \times 5$
$A_4 = 5 \times 10$
$A_5 = 10 \times 20$
$A_6 = 20 \times 25$

$$m[i,j] = \begin{cases} 0 & \text{if } i=j \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + P_{i-1} * P_k * P_j\} & \text{if } i<j \end{cases}$$

$$m(1,2) = \min_{i \le k < j} \{ m[1,1] + m(2,2) + P_0 * P_1 * P_2 \}$$
$i=1, j=2$
$\boxed{k=1}$
$$= \min\{0 + 0 + 30 * 35 * 15\} = 15750$$

$$m(2,3) = \min_{i \le k < j} \{ m(2,2) + m(3,3) + P_1 * P_2 * P_3 \}$$
$i=2, j=3$
$i=2,$
$j=3$
$k=2$
$$= \min\{0+0+35*15*5\} = 2625$$

$$m(3,4) = \min_{i \le k < j} \{ m(3,3) + m(4,4) + P_2 * P_3 * P_4 \}$$
$i=3, j=4$
$k=3$
$$= \min\{0+0+ \cancel{35}\ 15 \times 5 \times 10\} = 750$$

Date :

* MCM (Matrix Chain Multiplication)

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$$m[i,j] = \begin{cases} 0 & \text{if } i=j \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + P_{i-1} * P_k * P_j\} & \text{if } i<j \end{cases}$$

$m(1,2) = \min\limits_{i \le k < j} \{ m[1,1] + m(2,2) + P_0 * P_1 * P_2 \}$
$i=1, j=2$
$\boxed{k=1}$

$= \min\{ 0 + 0 + 30 * 35 * 15 \} = 15750$

$m(2,3) = \min\limits_{i \le k < j} \{ m(2,2) + m(3,3) + P_1 * P_2 * P_3 \}$
$i=2, j=3$
$i=2$
$j=3$
$k=2$
$= \min\{ 0 + 0 + 35 * 15 * 5 \} = 2625$

$m(3,4) = \min\limits_{i \le k < j} \{ m(3,3) + m(4,4) + P_2 * P_3 * P_4 \}$
$i=3, j=4$
$k=3$
$= \min\{ 0 + 0 + \cancel{35}\, 15 \times 5 \times 10 \} = 750$

Date :

* MCM (Matrix Chain Multiplication)

$A_1 = 30 \times 35$
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$i=1, j=2$
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$$m(2,3) = \min_{i \le k < j} \{ m(2,2) + m(3,3) + P_1 * P_2 * P_3 \}$$
$i=2, j=3$
$i=2,$
$j=3$
$k=2$
$$= \min\{0+0+35*15*5\} = 2625$$

$$m(3,4) = \min_{i \le k < j} \{ m(3,3) + m(4,4) + P_2 * P_3 * P_4 \}$$
$i=3, j=4$
$k=3$
$$= \min\{0+0+\cancel{35}\, 15 \times 5 \times 10\} = 750$$

Date :

$m(4,5) = \min\limits_{i \le k < j} \{ m(4,4) + m(5,5) + P_3 * P_4 * P_5 \}$
$i=4, j=5$
$k=4$
$= \min\{0 + 0 + 5 \times 10 \times 20\} = 1$

$m(5,6) = \min \{ m(5,5) + m(6,6) + P_4 * P_5 * P_6 \}$
$i=5, j=6 \quad k=5$
$= \min\{0+0+ \quad\quad \} = 5000$

diff = 2

$m(1,3) = \min \begin{cases} m(1,1) + m(2,3) + P_0 * P_1 * P_3 \\ m(1,2) + m(3,3) + P_0 * P_2 * P_3 \end{cases}$
$i=1, j=3 \quad k=2$ or $k=1$
$= \min \begin{cases} 0 + 2625 + 30*35*5 \\ 15750 + 0 + 30*15*5 \end{cases}$
$= \min \begin{cases} 7875 \\ 18000 \end{cases} = \min\{7875\}$
$(k=1)$

$m(2,4) = \min \begin{cases} m(2,2) + m(3,4) + P_1 * P_2 * P_4 \\ m(2,3) + m(4,4) + P_1 * P_3 * P_4 \end{cases}$
$i=2, j=4 \quad k=2,3$
$= \min \begin{cases} 0 + 750 + 35*15*10 \\ 2625 + 0 + 35*5*10 \end{cases}$
$= \min\{4375\}, k=3$

$m(3,5) = \min \begin{cases} m(3,3) + m(4,5) + P_2 * P_3 * P_5 \\ m(3,4) + m(5,5) + P_2 * P_4 * P_5 \end{cases}$
$i=3, j=5 \quad k=3,4$
$= \{ \quad \}$
$= \min(2500) = k=3$

$m(4,6) = 3500, k=5$

Date :

$m(4,5) = \min_{i \le k < j} \{ m(4,4) + m(5,5) + P_3 * P_4 * P_5 \}$
$i=4, j=5$
$k=4$
$= \min\{0+0+5 \times 10 \times 20\} = 1000$

$m(5,6) = \min \{ m(5,5) + m(6,6) + P_4 * P_5 * P_6 \}$
$i=5, j=6 \quad k=5$
$= \min\{0+0+ \} = 5000$

diff = 2

$m(1,3) = \min \begin{cases} m(1,1) + m(2,3) + P_0 * P_1 * P_3 ; \\ m(1,2) + m(3,3) + P_0 * P_2 * P_3 \end{cases}$
$i=1, j=3 \quad k=2$ or $k=1$

$= \min \begin{cases} 0 + 2625 + 30 * 35 * 5 \\ 15750 + 0 + 30 * 15 * 5 \end{cases}$

$= \min \begin{cases} 7875 \\ 18000 \end{cases} = \min\{7875\}$
$(k=1)$

$m(2,4) = \min \begin{cases} m(2,2) + m(3,4) + P_1 * P_2 * P_4 \\ m(2,3) + m(4,4) + P_1 * P_3 * P_4 \end{cases}$
$i=2, j=4 \quad k=2,3$

$= \min \begin{cases} 0 + 750 + 35 * 15 * 10 \\ 2625 + 0 + 35 * 5 * 10 \end{cases}$

$= \min\{4375\}, k=3$

$m(3,5) = \min \begin{cases} m(3,3) + m(4,5) + P_2 * P_3 * P_5 \\ m(3,4) + m(5,5) + P_2 * P_4 * P_5 \end{cases}$
$i=3, j=5 \quad k=3,4$
$= \begin{cases} \\ \end{cases}$

$= \min(2500) = k=3$

$m(4,6) = 3500, \ k=5$

Date :

diff = 3

$m(1,4)$ $= \min(9375)$ $k=3$
$k=1,2,3$

$m(2,5)$ $= \min(7125)$ $k=3$
$k=2,3,4$

$m(3,6)$ $= \min(5375)$ $k=3$
$k=3,4,5$

diff = 4

$m(1,5)$ $= \min(11875) = k=3$
$k=1,2,3,4$

$m(2,6)$ $= \min(10500) = k=3$
$k=2,3,4,5$

diff = 5

$m(1,6)$ $= \min(15125) = k=3$
$k=1,2,3,4,5$