Siddhant Bhardwaj - ISTA 311

Midterm Submission

1. (a) Now, given that $P(A) = 0.5$ & $P(B) = 0.6$
& $P(A \cup B) = 0.9$

Now, we know that,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

So for A & B to be independent,
$P(A \cap B) = 0$.

Now, $0.9 = 0.6 + 0.5 - P(A \cap B)$
$\Rightarrow 0.9 = 1.1 - P(A \cap B)$
$\Rightarrow P(A \cap B) = 0.2$

But $0.2 \neq 0$, $\therefore$ A & B cannot be independent.

(b) Now, we know that, $P(B|A) = \frac{P(B \cap A)}{P(A)}$

So, $P(B|A) = \frac{0.2}{0.5} = \frac{2}{5} = 0.4$

But $P(B) = 0.6$. So, $P(B|A) \not> P(B)$

$\therefore$ We can say that A is not favorable or unfavorable to B.

2. (a) The 1st yes or no question that I would ask is, that would have the maximum possible entropy, would be asking if the card is red or black in color.

(b) By Shannon's information theory, we can say that, $H = \log_2(X)$, where H is the minimum no. of steps needed to arrive at the correct identity of the card & X is the no. of cards in a pack.

So, $H = \log_2(52) = 5.7 \sim 6$

$\therefore$ 6 questions would be needed to arrive at the identity of the card in the most optimal manner.

3. By Shannon's source coding theorem, we can say that the ~~mem~~ minimum possible expected length for a uniquely ~~expected length~~ decodable symbol code is equal to its alphabet entropy.

Now, we also know that,

$$\text{Alphabet Entropy} = \sum_{i=1}^{n} p_i \times \log_2(1/p_i)$$

So, for Alphabet 1, Alphabet Entropy

$$= 0.5 \times \log_2\left(\frac{1}{0.5}\right) + 0.15 \times \log_2\left(\frac{1}{0.15}\right) + 0.2 \times \log_2\left(\frac{1}{0.2}\right)$$

$$+ 0.1 \times \log_2\left(\frac{1}{0.1}\right) + 0.15 \times \log_2\left(\frac{1}{0.15}\right) + 0.15 \times \log_2\left(\frac{1}{0.15}\right)$$

$$+ 0.1 \times \log_2\left(\frac{1}{0.1}\right) = 2.86 \text{ (approx)}.$$

Now, for Alphabet 2,

P.T.O

$$\text{Alphabet Entropy} = 0.05 \times \log_2\left(\frac{1}{0.05}\right) + 0.01 \times \log_2\left(\frac{1}{0.01}\right) + 0.85 \times \log_2\left(\frac{1}{0.85}\right) + 0.03 \times \log_2\left(\frac{1}{0.03}\right) + 0.02 \times \log_2\left(\frac{1}{0.02}\right) + 0.02 \times \log_2\left(\frac{1}{0.02}\right) + 0.02 \times \log_2\left(\frac{1}{0.02}\right)$$

$= 0.9722$ (approx.)

$\therefore$ Since alphabet 2's entropy is less than alphabet 1's entropy, hence, alphabet 2 has shorter average code length compared to alphabet 1.

4. Given, that, the newspaper report states —

"Among children, involved in bicycle ~~accident~~ accidents, the majority are boys". . (i)

& the newspaper headline states —

"Boys more at risk on bicycle". . (ii)

The error arises from the fact that ~~since boys drive bicycles, hence, they~~ since more boys drive bicycles, then the majority of accidents involve ~~the~~ bicycles involves boys. Using the

concept of conditional probabilities
the statement (i) ~~ka~~ makes a statement about the following conditional probability — P(Boys | children accident)

whereas, the statement (ii), makes a statement about the following the conditional probability —

$$P(\text{accident} \mid \text{Boy}).$$

So, the misinterpretation of conditional probabilities causes the error

5. Given, ~~A = Isaac says to~~ that Isaac says a statement and Isabella responds to the query ~~of the~~ of whether it is true with a yes. Also, people on the island lie $2/3^{rd}$ of the time & say the truth $1/3^{rd}$ of the time.

Now, ~~the probab~~ A = Isaac's statement is true, B = Isabella's statement ~~is~~ is yes.

~~So~~ So, P(A|B) is the probability we need to find.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B)$ is the probability that Isaac's statement is true & Isabella's statement of yes is also true. So, $P(A \cap B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

P(B) is the probability that Isabella would say yes. Now, Isabella would lie 2/3 time & say truth 1/3 time

So, If she is lying, then if Isaac's statement is false, her answer would be yes. Also, if is she is saying the truth, she would say yes & Isaac's statement is also true.

$$P(B) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{4}{9} + \frac{1}{9}$$

$$= \frac{5}{9}.$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/9}{5/9} = \frac{1}{5} \text{ (Ans)}$$

6. By Bayes Theorem,

$$P(A_i | B) = \frac{P(B|A_i)\cdot P(A_i)}{\sum_i P(B|A_i)\cdot P(A_i)}$$

~~$= \frac{P(B|A_i) \cdot P(A_i)}{}$~~

Now, A_i = the person has used opioids &
B = the test is positive.

So, $P(A_i|B)$ would be the probability value that is needed to be found in this situation. Hence, the situations that should be discovered are :-

$P(B|A_i) \times P(A_i)$ = P(the test is positive | the person has used opioids) × P(the person has used opioids)

$P(B|\bar{A_i}) \times P(\bar{A_i})$ = P(the test is positive | the person has not used opioids) × P(the person has not used opioids)

So, By Bayes theorem,

P(the person has used opioids | the test is positive)

~~= P(the test is positive | the person has used opioids)~~
P(~~the~~ test is positive | the person has used opioids)
~~+ P(the test is~~ positi

= P(the test is positive | the person has used opioids) × P(the person has used opioids)
/ P(the test is positive | the person has used opioids) × P(the person has used opioids) + P(the test is positive | the person has not

used opioids) x P(the person has not used opioids)

(V_1 m)