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ISTA-311 HW-1 (Answers)

1. All or most cats are animals with fur.

If x is a cat, then x is certainly or likely an animal with fur.

No, the statement is not true when reversed since being an animal with fur does not necessarily imply that the animal is a cat.

2. Since there are 15 people entering a raffle with 3 distinct order-relevant prize positions - 1^{st} prize, 2^{nd} prize & 3^{rd} prize, we can infer that there are 15 choices for the 1^{st} position, 14 for the 2^{nd} (because 1 is already chosen for 1^{st} which excludes him from consideration of 2^{nd}) and 13 for the 3^{rd} (because 2 are already chosen). Thus, the no. of different combinations of winners are $15 \times 14 \times 13 = 2730$.

3. (a) Since all characters are letters, and there is no restriction and the letters are not case-~~sen~~ sensitive, then no. of combinations $= (26)^6$.

(b) Since all the characters are letters & the 1^{st} one is 'q', then it means that except for the 1^{st} position, the remaining

5 positions are ~~all~~ all alphabet characters without any restriction then the no of combinations is $1\times(26)^5$.

(c) Since all characters are different letters, then the combinations are $26\times25\times24\times23\times22\times21$ (because 1st position has 26 characters, 2nd position has one less because only 25 characters remain and so on).
$\therefore$ No. of combinations $= 26\times25\times24\times23\times22\times21$

(d) Since there are 3 letters & 3 numbers, here, since there are 10 possible no. (0-9), ~~& 26 for~~ since it is not case-sensitive, hence, $(26)^3\times(10)^3$ is the no. of combinations possible.

(e) Since there are 6 positions, with 26 possible characters (because it is not case-sensitive) and 10 digits (0-9). Hence, the no. of possibilities are $(26+10)=36$ for 1 position. Thus, $(36)^6$ is the total no of possibilities. Now, if there are only letters, then no of possibilities is $(26)^6$
$\therefore$ No of combinations with atleast one digit $=(36)^6-(26)^6$.

4. Now, by rule, we know that the no. of diagonals in a polygon are given by formula $\frac{n(n-3)}{2}$.

$\therefore$ For $n=8$, No. of diagonals $=\frac{8(8-3)}{2}$
$=4\times5$
$=20$ (Ans)

5. $P(A) = 0.4$ & $P(B) = 0.7$

Now we know, $P(A \cap B) \leq P(A)$, also, $P(A \cup B) \leq 1$.

By, $P(A) + P(B) - 1 \leq P(A \cap B) \leq P(A)$ [By Inclusion-Exclusion principle]

$\Rightarrow 0.4 + 0.7 - 1 \leq P(A \cap B) \leq 0.4$

$\Rightarrow 1.1 - 1 \leq P(A \cap B) \leq 0.4$

$\Rightarrow 0.1 \leq P(A \cap B) \leq 0.4$.

6. (a) We know that for Jane to get an A, she must have ~~3 of 4 answers~~ 3 out of 4 answers as correct. This means that correct ans = 3 or correct ans = 4.

$\therefore$ Probability of Jane getting an A

$= P(\text{Jane gets A}) = P(\text{correct ans} \geq 3)$

$= P(\text{correct ans} = 3) + P(\text{correct ans} = 4)$

$= \frac{4!}{(4-3)!\,3!} \cdot (0.8)^3 (0.2)^{4-3} + (0.8)^4$

$= \frac{4!}{3!} (0.8)^3 (0.2) + (0.8)^4$

$= 4 \times (0.8)^3 \times (0.2) + (0.8)^4$

$= (0.8)^3 [4 \times 0.8 \times 0.2 + 0.8]$

$= (0.8)^3 \times 1.44$ (Ans)

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(b) Since Jane has gotten the problem correct, then it means to get only 2 out of remaining problems correct to get an A.

∴ Probability of Jane getting an A

$= P(\text{Jane gets an A}) = P(\text{correct\_ans} \geq 2)$

$= P(\text{correct\_ans} = 2) + P(\text{correct\_ans} = 3)$

$= \frac{3!}{(3-2)!\,2!} \cdot (0.8)^2 \cdot (0.2) + (0.8)^3$

$= \frac{3!}{2!} (0.8)^2 \cdot (0.2) + (0.8)^3$

$= 3 \times (0.8)^2 \times (0.2) + (0.8)^3$

$= (0.8)^2 \left[3 \times 0.2 + 0.8\right]$

$= (0.8)^2 \left[0.6 + 0.8\right]$

$= (0.8)^2 \times 1.4$ (Ans).

7. Since there are 8 rooks on an 8x8 chessboard, this means that we need to find the probability that none of the rooks can capture each other. Now this is equal to :-

$\frac{\text{Unkillable position}}{\text{All position}}$ So, All position

is equal to $\frac{64!}{(64-8)!} = \frac{64!}{56!}$ because you can't place more than 1 rook on 1 square. Now for 1st piece, we have entire 8x8 chessboard. Since a rook can kill anything in its row & column, the 2nd piece will have 7x7 chessboard, 3rd - 6x6, 4th - 5x5, 5th - 4x4, 6th - 3x3, 7th - 2x2, 8th - 1x1. So,

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we know that potential squares for each ~~of~~ of them are 64, 49, 36, 25, 16, 9, 4, 1
Thus, by counting rule, unkillable positions $= 64 \times 49 \times 36 \times 25 \times 16 \times 9 \times 4 \times 1$

$\therefore$ P(no rook capturing another)

$$= \frac{64 \times 49 \times 36 \times 25 \times 16 \times 9 \times 4 \times 1}{\left(\frac{64!}{56!}\right)}$$