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ISTA 311 - Finals

1. Now, we must look at the def'n of markov chain. A markov chain is a model in which for the order of possible events, the probability of a particular event will be dependent only on the previous state. Hence, since in the given question, the fact that I have chosen a link is dependent on my current page, hence it is a markov chain.

Now, for the state space, it would include all the articles and links that can be visited from each and every page on Wikipedia.

For the transition probability, then if the current page has n different links & since a link is going to be chosen at random, hence the transition probability is going to be (1/n).

2. The rankings are as follows :

(a) 2 (c) 3
(b) 1 (d) 4

3. 0.3 - prior, 0.8 - likelihood,
0.6 - prior, 0.1 - likelihood,
0.77 - posterior

Problems:-

(a) Given, there are 7 chips labelled as 1,2,3 ·7 & 3 are black, 2 are red and 2 are green.

Now, probability of choosing each chip is $1/7$. So, the probability of chip 5 is being selected at 2^{nd} draw is equal to probability that chip 5 was not selected in 1^{st} draw, multiplied by chip selected in 2^{nd} draw.

$\therefore$ P(chip 5 selected in 2^{nd} draw) $= \frac{6}{7} \times \frac{1}{6}$
$= \frac{1}{7}$ (Ans).

(b) Now, given that a green chip is drawn at the 2^{nd} draw, then the probability is equal to the probability of non-green chip at 1^{st} draw times probability of green chip at 2^{nd} draw.

$\therefore$ P(green chip at 2^{nd} draw) $= \frac{5}{7} \times \frac{2}{6}$
$= \frac{5}{21}$

2.(a) If there are N total engines & we have seen engine no. 17, then there must be 17 or more engines in total.

$\therefore$ Two cases $\rightarrow$

(1) If $n = 17$, P(engine no. 17) $= 1/17$

Page No.
Date

(ii) If $n > 17$, then P(engine no. 17) $= \frac{1}{N}$, where $N > 17$.

(b) Now, if the company has no more than 200 engines, then this means that it has a maximum of 200 engines in possession, i.e., N is less than or equal to 200.

$\therefore N \in [1, 200]$.

Hence for values of N in $N = 1, 2, 3 \cdots 200$, then, if $N=1$, then only engine has been produced & so forth. Hence, the uniform prior probabilities would be $(1/N)$.

$\therefore$ Priors $= (1/N)$.

Now, after observing engine no. 17, we can state that the minimum no. of engines is going to be 17.

~~Here, the unnormalized posterior distribution, the can be obtained by likelihood =~~ $\cancel{\frac{1}{x}}$, ~~where~~ $\cancel{x \in [17, 200]}$

Here, in order to obtain unnormalized posterior distribution, we would need to multiply the priors X the likelihood.

$\therefore$ Unnormalized posterior distribution $= \frac{1}{N} \times$ Likelihood(17), where

Likelihood$(x) = \frac{1}{x}$ or 0, ~~depending~~ on if $N < X$.

$\Rightarrow \frac{1}{N} \times \left[\frac{1}{X} \mid 0, \text{ if } N < X\right]$

~~(c) After observing engine 17, the maximum a posteriori estimate of N is 17, is $\frac{1}{17} \times \frac{1}{17}$~~

(c) The value for which the maximum a posteriori estimate of N would be 17. This can be calculated by $\frac{1}{17} \times \frac{1}{17} = 0.034$ (Ans).

(d) Using the python loop, the posterior mean was calculated be around 73.42 (Ans).

3. (a) Given,

	a	b	c	d	e
Freq	0.2	0.1	0.1	0.2	0.4
Length	3	2	4	4	1

$\therefore$ Average code word length $=$
$(0.2 \times 3 + 0.1 \times 2 + 0.1 \times 4 + 0.4 \times 1) / 5$
$= 0.48$

(b) Given, Alphabet 2 -

Symbol	a	b	c	d	e
Freq	0.1	0.2	0.05	0.01	0.64

$$\{a, b, c, d, e\} \xrightarrow{0} \{e\}$$
$$\downarrow 1$$
$$\{a, b, c, d\} \xrightarrow{1} \{b\}$$
$$\downarrow 0$$
$$\{a, c, d\} \xrightarrow{1} \{a\}$$
$$\downarrow 0$$
$$\{c, d\} \xrightarrow{1} \{c\}$$
$$\downarrow 0$$
$$\{d\}$$

$\therefore$ $a = 101$, $b = 11$, $c = 1001$, $d = 1000$ & $e = 0$ } ← Code A.

So, Code A matches with Alphabet 2.

Also, for Alphabet 1,

$$\{a, b, c, d, e\} \rightarrow \{e, d\} \xrightarrow{0} \{e\}, \quad \{e, d\} \xrightarrow{1} \{d\}$$
$$\downarrow 1$$
$$\{a, b, c\} \xrightarrow{1} \{a\}$$
$$\downarrow 0$$
$$\{b, c\} \xrightarrow{0} \{c\}$$
$$\downarrow 1$$
$$\{b\}$$

$\therefore$ Code B matches with Alphabet 1.

4(a) Let theory be T, & application be A & success be S.

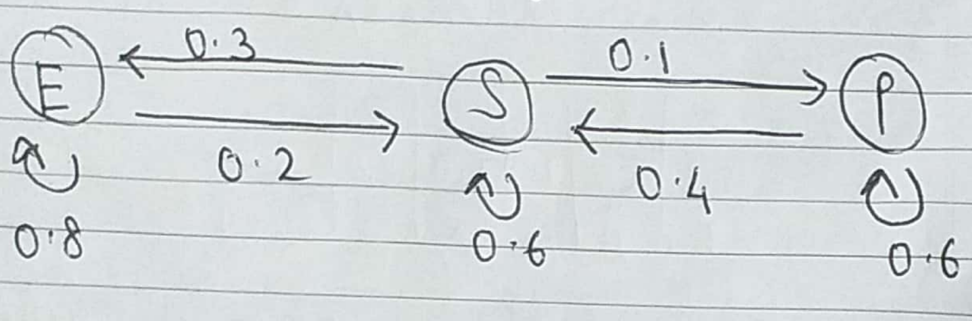
$\therefore P(S|T) = 0.6,\ P(S|A) = 0.8,\ P(T) = 0.7,\ P(A) = 0.3$

$$\therefore P(S) = P(S|T)\cdot P(T) + P(S|A)\cdot P(A)$$
$$= 0.6 \times 0.7 + 0.8 \times 0.3$$
$$= 0.42 + 0.24$$
$$= 0.66 \text{ (Ans)}$$

(b) Now, $P(S|T) = 0.5,\ P(S|A) = 0.9$

$$\therefore P(S) = 0.5 \times 0.7 + 0.9 \times 0.3$$
$$= 0.35 + 0.27$$
$$= 0.62 \text{ (Ans)}$$

5.(a) Let E be excellent, P be poor, & S be satisfactory.



(b)

$$\begin{array}{c|c} & \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \begin{array}{c} E \\ S \\ P \end{array} & \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0.4 & 0.6 \end{bmatrix} \\ & \begin{array}{ccc} E & S & P \end{array} \end{array}$$

(c) File submitted

(d) Approx., the stationary distribution is,

Poor – 11%, Satisfactory – 34.5%,
Excellent – 54.5%

6. Math Joke of the Test :-

Two friends, (Alex & Alice) chatting with each other about their mutual friend, Mike.

Alice – I am a bit worried about Mike's obsession with the difference between sine & cosine.

Alex – Bah! Don't Worry!! It's just a phase!!!