EXERCISE BOOK

ISTA-331

Written HW 3

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1. Given, $P(1) = P(2) = P(3) = P(4) = 1/5$ & $P(5) = P(6) = 1/10$ for loaded die & $P(1 \text{ to } 6) = 1/6$ for fair die

(a) Drawing a Bayes table,

	Prior	P(D|H)	P(D|H) P(H)	Posterior	P(H|D)
H_1	1/2	1/6	1/12	1/12 x 60/8	5/8
H_2	1/2	1/10	1/20	1/20 x 60/8	3/8

where H_1 = choosing a fair die, H_2 = choosing a loaded die

$\therefore P(H_2|D) = P(\text{loaded die} \mid \text{getting a } 6) = 3/8$ (Ans)

(b) Let D = getting a 2 & H_1 = fair die &
H_2 = getting a loaded die

	Prior	Likelihood - P(D|H)	P(D|H) P(H)	Posterior - P(H|D)
H_1	1/2	1/6	1/12	5/11
H_2	1/2	1/5	1/10	6/11

$\therefore P(H_2|D) = P(\text{getting a loaded die} \mid \text{getting a } 2)$

$= 6/11$

2.

2. Now, given that we have 3 cards with properties like :-

(1) Both sides are green. (2) Both sides are red.
(3) One side is red & one side is green.

Let A = event that drawn card shows red on upper side. Let e_1, e_2, e_3 be events of tossing a totally red card, a totally green card & a card with red color on the other side and green color on another side.

$\therefore P(A/e_1) = 1,\ P(A/e_2) = 0,\ P(A/e_3) = 1/2$

[By Bayes Theorem]

$$P(e_1/A) = \frac{P(e_1) \times P(A/e_1)}{P(e_1)\cdot P(A/e_1) + P(e_2)\cdot P(A/e_2) + P(e_3)\cdot P(A/e_3)}$$

$$= \frac{1/3 \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}} = \frac{\frac{1}{3}}{\frac{2+1}{6}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

Hence, P(getting red ~~card~~ color on other side | got a red color) = 2/3 (Ans)

3. Now, we have 2 bowls – Bowl 1 & Bowl 2. Bowl 1 – 1/4 chocolate & 3/4 Vanilla & Bowl 2 – 1/2 chocolate & 1/2 Vanilla.

(a) By treating the 2 cookies as one simultaneous piece of evidence, we get –
H_1 = Bowl 1 , H_2 = Bowl 2 & D = getting 2 chocolate cookies

	Prior	Likelihood	P(D|H)·P(H)	Posterior
H_1	1/2	1/4 × 1/4	1/32	1/5
H_2	1/2	1/2 × 1/2	1/8	4/5

∴ P(Bowl 1 | picking 2 chocolate cookies) = 1/5 &
P(Bowl 2 | picking 2 chocolate cookies) = 4/5.

(b) Assuming the 2 chocolate cookies are picked one after another,
1. posterior probabilities after 1st chocolate cookie

~~H_1 | Prior~~
~~H_2~~

	Prior	Likelihood	P(D|H)·P(H)	Posterior
H_1	1/2	1/4	1/8	1/3
H_2	1/2	1/2	1/4	2/3

∴ Now, using posterior probabilities as priors in 2nd round

	Prior	Likelihood	P(D|H)·P(H)	Posterior
H_1	1/2	1/3	1/6	1/3
H_2	1/2	2/3	1/3	2/3

4. If we draw a chocolate cookie & a vanilla cookie, then the posterior probabilities are calculated as follows:

D_1 = drawing a chocolate cookie, D_2 = drawing a vanilla cookie, H_1 = ~~drawing~~ Bowl 1, H_2 = Bowl 2.

For chocolate cookie

	Prior	Likelihood	$P(D_1 \mid H) \cdot P(H)$	Posterior
H_1	$1/2$	$1/4$	$1/8$	$1/3$
H_2	$1/2$	$1/2$	$1/4$	$2/3$

For Vanilla cookie:

	Prior	Likelihood	$P(D_2 \mid H) \cdot P(H)$	Posterior
H_1	$1/2$	$3/4$	$3/8$	$3/5$
H_2	$1/2$	$1/2$	$1/4$	$2/5$

I do not think that the order in which the cookies are drawn matters. This is because the no. of cookies is so large that picking a single cookie ~~is~~ does not alter the likelihood substantially.