ISTA-311 HW4 Written

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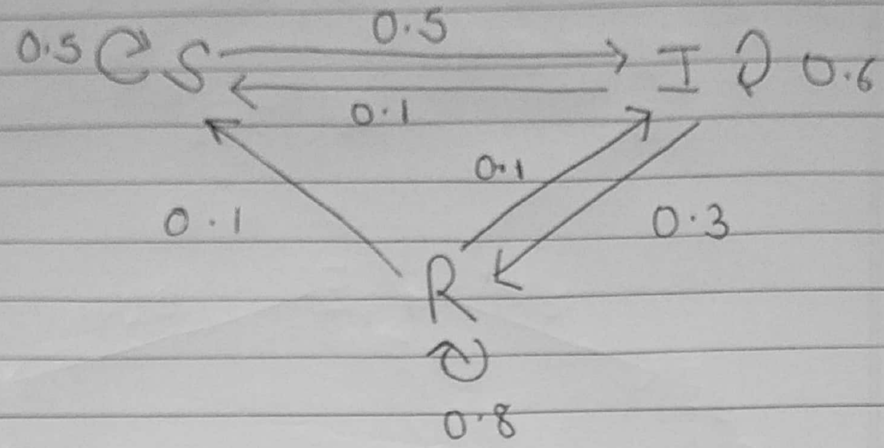
1. (a)



(b) The transition matrix is :-

$$\begin{array}{c|ccc} & S & I & R \\ \hline S & 0.5 & 0.5 & 0 \\ I & 0.1 & 0.6 & 0.3 \\ R & 0.1 & 0.1 & 0.8 \end{array}$$

(c) Now, to get stationary probabilities :-

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow \frac{1}{2}x + \frac{1}{2}y = x \quad \cdots (i)$

$\Rightarrow \frac{1}{10}x + \frac{6}{10}y + \frac{3}{10}z = y \cdot (ii)$

$\Rightarrow \frac{1}{10}x + \frac{1}{10}y + \frac{8}{10}z = z \cdot (iii)$

From (i), $\frac{1}{2}y = \frac{1}{2}x \Rightarrow x = y \cdots (iv)$

Putting (iv) in (iii),

$$\Rightarrow \frac{2}{10}y + \frac{8}{10}z = z \Rightarrow \Rightarrow y = z$$

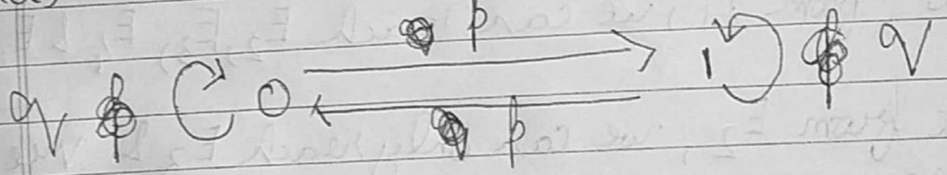
So, $x = y = z$. But since x, y & z are probabilities, so,

$$x + y + z = 1 \Rightarrow 3z = 1 \Rightarrow z = 1/3$$

$$\therefore x = y = z = 1/3$$

(d) Since the long-term behavior of Markov Chain is dependent on the stationary probabilities, and $y = 1/3$, hence, we can say that about $1/3$rd of the population will be infected at any given time.

2. (a)



(b) The transition matrix would be :-

$$\begin{array}{c c} 0 & \\ 1 & \\ & \end{array} \begin{bmatrix} \cancel{p}q & \cancel{q}p \\ \cancel{q}p & \cancel{p}q \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$
$$\quad 0 \qquad 1$$

(C) Now, from state 0 to get to state 0 after 2 steps, will take 2 different routes :-

$0 \rightarrow 1 \rightarrow 0$ .. (i)

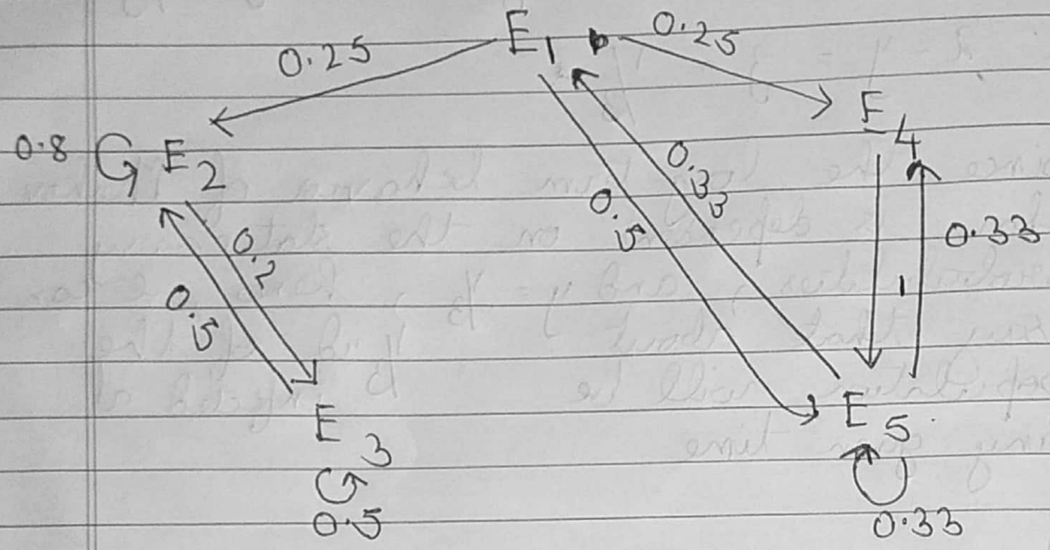
$0 \rightarrow 0 \rightarrow 0$ .. (ii)

For case (i), since the state has changed twice, so, probability = ~~$q \times q = q^2$~~ $p \times p = p^2$

From case (ii), since the state has not changed twice, so, probability = ~~$p \times p = p^2$~~ $q \times q = q^2$

$\therefore$ Ans. $= p^2 + q^2$

3.



Now, from E_1, we can reach E_2, E_3, E_4 & E_5

But from E_2, we can only reach E_3 & vice versa.

$\therefore$ Not all points can reach all other points (Ans)