CL686 Course Project: Generic Description
Closed Loop Simulations using
ARX Model based LQOC and Innovation Bias Approach

1 System Dynamic Simulation

Let the nonlinear system assigned to you be described as follows

$$
\begin{aligned}
\frac{d\mathcal{X}}{dt} &= \mathbf{f}\left(\mathcal{X},\mathcal{U},\mathcal{D}\right) && (1)\\
\mathcal{Y} &= \mathbf{C}\mathcal{X} && (2)\\
\mathcal{X} &\in R^n, \mathcal{U}\in R^m, \mathcal{D}\in R^d, \mathcal{Y}\in R^r
\end{aligned}
$$

and let $(\mathcal{X}_s, \mathcal{U}_s, \mathcal{D}_s)$ represent the specified equilibrium operating point. Dynamics of the system is simulated with specified initial condition $\mathcal{X}(0)$ and with piecewise constant inputs

$$
\mathcal{U}\left(t\right) = \mathcal{U}_s + \mathbf{u}(k) \quad \text{for } kT \leq t < (k+1)T \tag{3}
$$

$$
\mathcal{D}\left(t\right) = \mathcal{D}_s + \mathbf{d}(k) \quad \text{for } kT \leq t < (k+1)T \tag{4}
$$

where $\mathbf{u}(k)$ represents the controller output and $\mathbf{d}(k)$ is a zero mean Gaussian white noise sequence with covariance matrix $\mathbf{Q}_d$. The measurements available from the system at sampling interval T are corrupted with noise

$$
\mathcal{Y}(k) = \mathbf{C}\mathcal{X}(k) + \mathbf{v}(k) \tag{5}
$$

where $\mathbf{v}(k)$ is a zero mean Gaussian white noise sequence with covariance matrix $\mathbf{R}$.

2 LQOC using ARX based State Space Model

Let the state space realization of ARX model identified from data be represented as follows controller design$^{\sim}$

$$
\begin{aligned}
\mathbf{z}(k+1) &= \widetilde{\mathbf{\Phi}}\mathbf{z}(k) + \widetilde{\mathbf{\Gamma}}\mathbf{u}(k) + \mathbf{L}\mathbf{e}(k) && (6)\\
\mathbf{y}(k) &= \widetilde{\mathbf{C}}\mathbf{z}(k) + \mathbf{e}(k) && (7)
\end{aligned}
$$

This model is used as state estimator as follows

$$
\begin{aligned}
\mathbf{y}(k) &= \mathcal{Y}(k) - \mathcal{Y}_s && (8)\\
\mathbf{e}(k) &= \mathbf{y}(k) - \widetilde{\mathbf{C}}\widehat{\mathbf{z}}(k) && (9)\\
\widehat{\mathbf{z}}(k+1) &= \widetilde{\mathbf{\Phi}}\widehat{\mathbf{z}}(k) + \widetilde{\mathbf{\Gamma}}_u\mathbf{u}(k) + \mathbf{L}\mathbf{e}(k) && (10)
\end{aligned}
$$

The linear quadratic optimal controller is implemented as

$$\begin{aligned} \mathbf{u}(k) &= \mathbf{u}_s(k) - \mathbf{G}_\infty \left[\widehat{\mathbf{z}}(k) - \mathbf{z}_s(k)\right] && (11) \\ \mathbf{u}_L &\leq \mathbf{u}(k) \leq \mathbf{u}_H && (12) \end{aligned}$$

where $\widehat{\mathbf{z}}(k)$ is computed using eq. (10) and the controller gain matrix is obtained by solving the algebraic Riccati equation (ARE)

$$\mathbf{G}_\infty = \left(\mathbf{W_u} + \widetilde{\mathbf{\Gamma}}^T \mathbf{S}_\infty \widetilde{\mathbf{\Gamma}}\right)^{-1} \widetilde{\mathbf{\Gamma}}^T \mathbf{S}_\infty \widetilde{\Phi} \quad (13)$$

$$\mathbf{S}_\infty = \left[\widetilde{\Phi} - \Gamma \mathbf{G}_\infty\right]^T \mathbf{S}_\infty \left[\widetilde{\Phi} - \Gamma \mathbf{G}_\infty\right] + \widetilde{\mathbf{C}}^T \mathbf{W}_y \widetilde{\mathbf{C}} + \mathbf{G}_\infty^T \mathbf{W_u} \mathbf{G}_\infty \quad (14)$$

Here, $\mathbf{W}_y$ and $\mathbf{W}_u$ are +ve definite controller tuning matrices of dimension $(r \times r)$ and $(m \times m)$, respectively. Solution of the ARE can be found using Matlab Control System Toolbox function *dlqr* as follows

$$[\mathbf{G}_\infty, \mathbf{S}_\infty, E_v] = dlqr\left(\widetilde{\mathbf{\Phi}},\ \widetilde{\mathbf{\Gamma}}, (\widetilde{\mathbf{C}}^T \mathbf{W}_y \widetilde{\mathbf{C}}),\ \mathbf{W_u}\right)$$

Using the innovation bias approach, the target steady state $\mathbf{z}_s(k)$ and target input $\mathbf{u}_s(k)$ are computed as follows

$$\begin{aligned} \mathbf{u}_s(k) &= \mathbf{K}_u^{-1} \left[\mathbf{r}(k) - \mathbf{K}_e\, \mathbf{e}_f(k)\right] && (15) \\ \mathbf{z}_s(k) &= \left(\mathbf{I} - \widetilde{\mathbf{\Phi}}\right)^{-1} \left[\widetilde{\mathbf{\Gamma}} \mathbf{u}_s(k) + \mathbf{L}\mathbf{e}_f(k)\right] && (16) \end{aligned}$$

$$\mathbf{K}_u = \widetilde{\mathbf{C}} \left(\mathbf{I} - \widetilde{\mathbf{\Phi}}\right)^{-1} \widetilde{\mathbf{\Gamma}} \quad ; \qquad \mathbf{K}_e = \widetilde{\mathbf{C}} \left(\mathbf{I} - \widetilde{\mathbf{\Phi}}\right)^{-1} \mathbf{L} + \mathbf{I} \quad (17)$$

where $\mathbf{r}(k)$ denotes desired setpoint at instant k and

$$\mathbf{e}_f(k) = \mathbf{\Phi}_e\ \mathbf{e}_f(k-1) + [\mathbf{I} - \mathbf{\Phi}_e]\mathbf{e}(k) \quad (18)$$

$$\mathbf{\Phi}_e = \alpha \mathbf{I} \ \text{ where } \ 0 \leq \alpha < 1 \quad (19)$$

represents filtered innovation signal. The innovation signal, $\mathbf{e}(k)$, is computed using eq. (9).