This algebraic restatement of the problem is easy to prove. In fact, it holds for any finite abelian group G, not only for cyclic ones. Let f be a fixed permutation of G and let b be a fixed element of G other than e. This element b exists since the order of G is greater than two. Clearly for all a in G, $a \neq ab$ and so $f(a) \neq f(ab)$. When the element a runs over the elements of G, then the element $f(ab)(f(a))^{-1}$ only runs over the nonidentity elements of G. By the pigeon-hole principle there are distinct elements c and d in G for which

$$f(cb)(f(c))^{-1} = f(db)(f(d))^{-1},$$

or equivalently for which

$$f(cb)f(d) = f(db)f(c).$$

Setting

$$cb = x, \quad d = y, \quad db = u, \quad c = v,$$

we have the conditions in (1) satisfied as desired.

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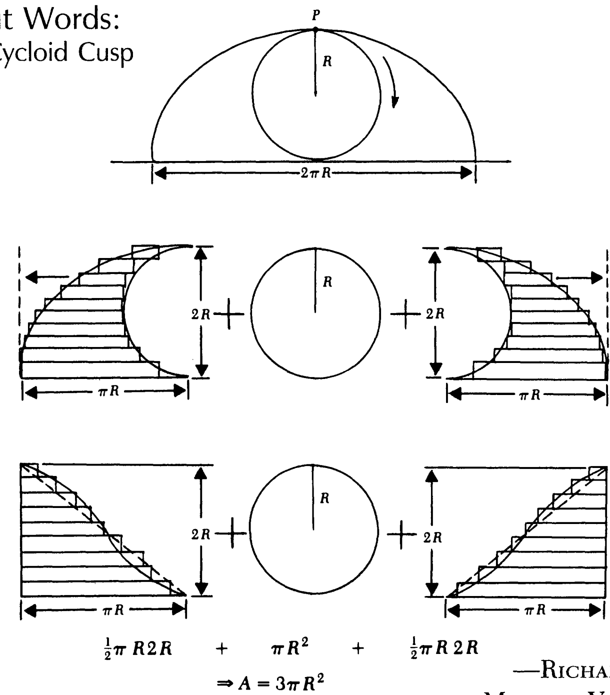
Proof without Words: Area under a Cycloid Cusp



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