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Proof without Words: Area under a Cycloid Cusp

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Source: *Mathematics Magazine*, Vol. 66, No. 1 (Feb., 1993), p. 39

Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America

Stable URL: <https://www.jstor.org/stable/2690472>

Accessed: 18-09-2024 22:44 UTC

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This algebraic restatement of the problem is easy to prove. In fact, it holds for any finite abelian group  $G$ , not only for cyclic ones. Let  $f$  be a fixed permutation of  $G$  and let  $b$  be a fixed element of  $G$  other than  $e$ . This element  $b$  exists since the order of  $G$  is greater than two. Clearly for all  $a$  in  $G$ ,  $a \neq ab$  and so  $f(a) \neq f(ab)$ . When the element  $a$  runs over the elements of  $G$ , then the element  $f(ab)(f(a))^{-1}$  only runs over the nonidentity elements of  $G$ . By the pigeon-hole principle there are distinct elements  $c$  and  $d$  in  $G$  for which

$$f(cb)(f(c))^{-1} = f(db)(f(d))^{-1},$$

or equivalently for which

$$f(cb)f(d) = f(db)f(c).$$

Setting

$$cb = x, \quad d = y, \quad db = u, \quad c = v,$$

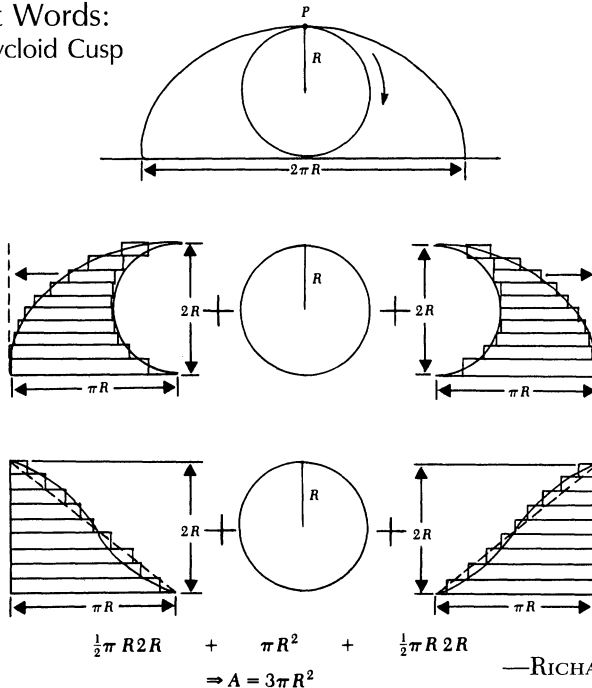
we have the conditions in (1) satisfied as desired.

**Acknowledgments.** I would like to thank the referees for their contributions, significantly improving the quality of exposition.

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**Proof without Words:**  
Area under a Cycloid Cusp



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