Proof Without Words: Varignon's Theorem

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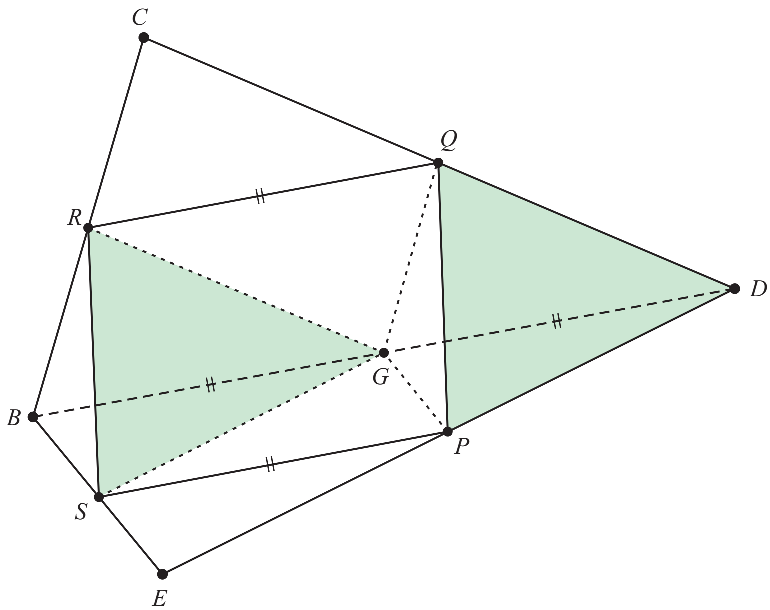
Given a quadrilateral, midpoints of the sides are the vertices of the Varignon parallelogram. The following result may be found, for instance, in [**2**].

Theorem. *The Varignon parallelogram of a convex quadrilateral has area half that of the given quadrilateral. Moreover, the perimeter of the Varignon parallelogram equals the sum of the lengths of the diagonals of the given parallelogram.*

Proof. Let $BG = GD$.



We thank an anonymous reviewer for pointing out that the proof of the perimeter result generalizes to arbitrary quadrilaterals, whereas the proof of the area result does not. We invite the reader to supply appropriate diagrams for nonconvex and crossed quadrilaterals (see [**1**, p. 108]).

Summary. We present a visual proof of Vairgnon's theorem by partitioning the Varignon parallelogram using a midpoint of the quadrilateral diagonal.

References

1. C. Alsina, R. Nelsen, *Charming Proofs: A Journey into Elegant Mathematics*. Mathematical Association of America, Washington, DC, 2010.
2. H. S. M. Coxeter, S. L. Greitzer, *Geometry Revisited*. Mathematical Association of America, Washington, DC, 1967.

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