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Behold! The Pythagorean Theorem via Mean Proportions

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$f(0) = 0$, is it true that

$$\left(\int_0^1 f dx \right)^2 \leq K \int_0^1 |f'|^2 dx$$

for some constant K ?

Beginning with (R), we have $\int_0^1 f dx = \int_0^1 (1-x)f' dx$. Therefore,

$$\begin{aligned} \left(\int_0^1 f dx \right)^2 &= \left(\int_0^1 (1-x)f' dx \right)^2 \\ &\leq \left(\int_0^1 (1-x)|f'| dx \right)^2 \\ &\leq \int_0^1 (1-x)^2 dx \cdot \int_0^1 |f'|^2 dx \\ &= \frac{1}{3} \int_0^1 |f'|^2 dx, \end{aligned}$$

where the second inequality is an application of Schwarz's inequality. Thus, our answer is affirmative, with $K = 1/3$. The interested reader might try to show that $K = 1/3$ is best possible by showing that there exists a continuously differentiable function f on $[0,1]$ with $f(0) = 0$ and $(\int_0^1 f dx)^2 = (1/3) \int_0^1 |f'|^2 dx$.

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