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Proof without Words

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Source: *Mathematics Magazine*, Vol. 65, No. 3 (Jun., 1992), p. 185

Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America

Stable URL: <https://www.jstor.org/stable/2691330>

Accessed: 18-09-2024 23:17 UTC

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the prospective fence buyer: In order to minimize cost of fencing a rectangle with given area, spend half of your money on each kind of fence.

EXAMPLE 3. Let  $X = \{\text{right circular cylinders}\}$  and let  $V$  and  $S$  be the volume and surface functions. The easier of the two problems to solve is to maximize volume when the surface area is given. One finds the answer lies in  $Y = \{\text{square cylinders}\} = \{\text{right circular cylinders with diameter equal to the height}\}$ . Then  $V|_Y = 2\pi r^3$  and  $S|_Y = 6\pi r^2$  for  $r$  the radius and  $V \circ S^{-1}(c) = c^{3/2}/3\sqrt{6\pi}$ . Thus the problem of minimizing the surface area with volume fixed is also solved by a square cylinder. The inequality (3) in this case is

$$V \leq \frac{1}{3\sqrt{6\pi}} S^{3/2}.$$

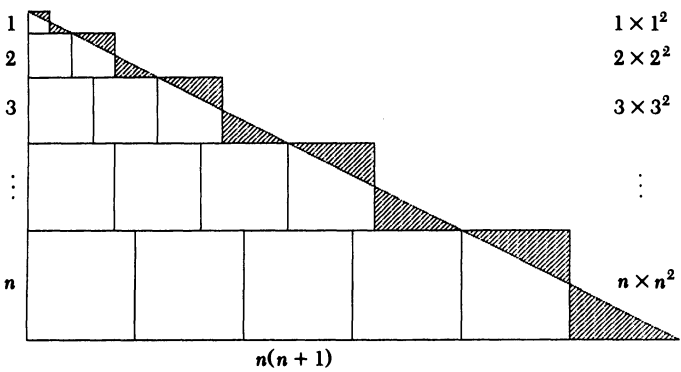
EXAMPLE 4. Let  $X = R_+^n$ ,  $f(x) = [\prod_{i=1}^n x_i]^{1/n}$ , and  $g(x) = (1/n)\sum_{i=1}^n x_i$ . Again the problem of maximizing  $f$  subject to  $g = c$ ,  $x_i \geq 0$  has a solution in  $Y = \{x|x_1 = x_2 = \cdots = x_n\}$  = the “diagonal in  $X$ ”. In this case  $f|_Y = g|_Y = f \circ g^{-1} = \text{identity}$  and (3) is the arithmetic-geometric mean inequality.

REFERENCES

1. I. Niven, *Maxima and Minima without Calculus*, MAA, Washington, DC, 1981.  
2. A. M. Fink, Max-Min without calculus, *The Math Log*, XV 3 (1971).

Proofs without Words

$$1 + 2 + \cdots + n = \frac{1}{2}n(n + 1)$$
$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{1}{2}n(n + 1)\right)^2$$



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