the prospective fence buyer: In order to minimize cost of fencing a rectangle with given area, spend half of your money on each kind of fence.

EXAMPLE 3. Let $X = \{\text{right circular cylinders}\}$ and let V and S be the volume and surface functions. The easier of the two problems to solve is to maximize volume when the surface area is given. One finds the answer lies in $Y = \{\text{square cylinders}\} = \{\text{right circular cylinders with diameter equal to the height}\}$. Then $V|_Y = 2\pi r^3$ and $S|_Y = 6\pi r^2$ for r the radius and $V \circ S^{-1}(c) = c^{3/2}/3\sqrt{6\pi}$. Thus the problem of minimizing the surface area with volume fixed is also solved by a square cylinder. The inequality (3) in this case is

$$V \leqslant \frac{1}{3\sqrt{6\pi}} S^{3/2}.$$

EXAMPLE 4. Let $X = R^n_+$, $f(x) = [\prod_{i=1}^n x_i]^{1/n}$, and $g(x) = (1/n)\sum_{i=1}^n x_i$. Again the problem of maximizing f subject to $g = c$, $x_i \geqslant 0$ has a solution in $Y = \{x \mid x_1 = x_2 = \cdots = x_n\}$ = the "diagonal in X". In this case $f|Y = g|Y = f \circ g^{-1}$ = identity and (3) is the arithmetic-geometric mean inequality.

REFERENCES

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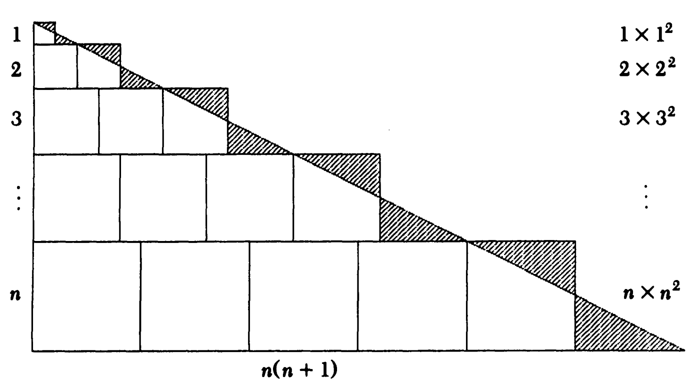
2. A. M. Fink, Max-Min without calculus, *The Math Log*, XV 3 (1971).

Proofs without Words

$$1 + 2 + \cdots + n = \frac{1}{2} n(n+1)$$

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{1}{2} n(n+1)\right)^2$$



—GEORG SCHRAGE
UNIVERSITY OF DORTMUND
4600 DORTMUND, GERMANY