Homework Assignment - 1

CSCI - 677

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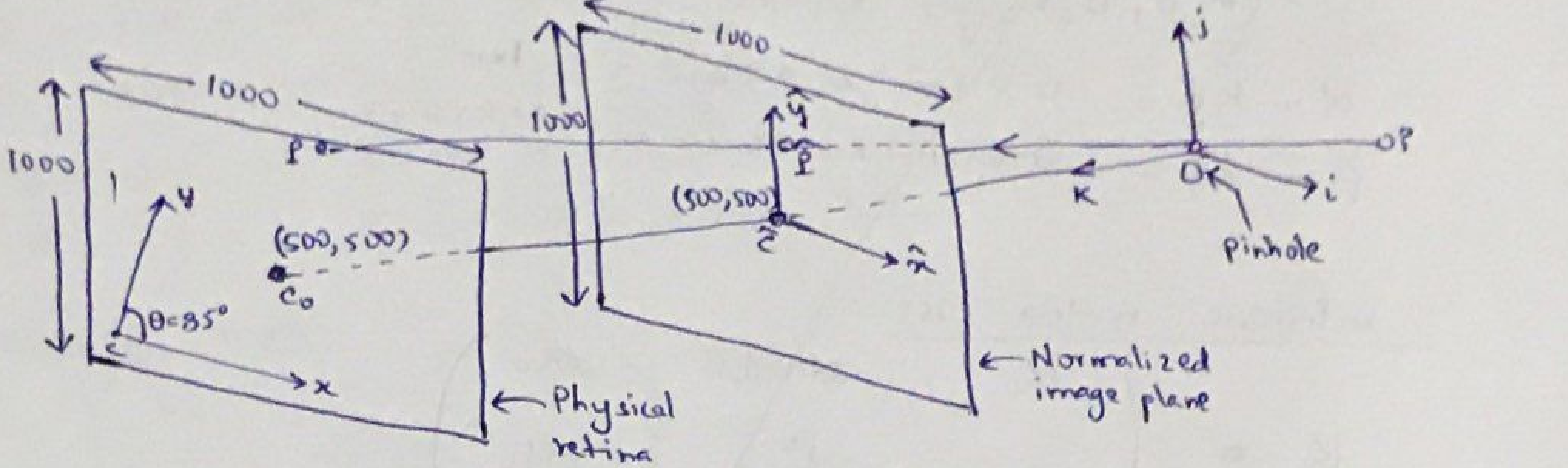
Soln-1: With the data given in the question, the best way to represent it is to reference it to a figure (with co-ordinates and parametric details):



Given: θ = angle b/w x & y axes = $85°$

Pixel spacing along the x-axis = 0.04 mm

Pixel spacing along the y-axis = 0.05 mm

Image dimensions $= (x, y) = (1000, 1000)$

Since, the principal ray intersects the image plane at its center,

$\therefore$ Image center co-ordinates $= C_o = (x_0, y_0) = (500, 500)$.

Focal-length $= f = 25$ mm

To find: Intrinsic matrix, $K = ?$

Soln: The intrinsic matrix is known to be defined by the equations:-

$$x = \alpha \hat{x} - \alpha \cos\theta \, \hat{y} + x_0$$

$$y = \frac{\beta}{\sin\theta} \hat{y} + y_0.$$

which in matrix form is:

$$p = K\hat{p} \quad \text{where } p = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}.$$

and,

$$K = \begin{pmatrix} \alpha & -\alpha \cot\theta & x_0 \\ 0 & \frac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

where; K is called the internal calibration matrix

$(\alpha, \beta, \theta, x_0, y_0)$ are the intrinsic parameters.

$$\text{scaling parameters} \begin{cases} k = \text{pixel} \times mm^{-1} = 1 \times \frac{1}{0.04} mm^{-1} = 25\ mm^{-1} \text{ (per pixel)} \\ L = \text{pixel} \times mm^{-1} = 1 \times \frac{1}{0.05} mm^{-1} = 20\ mm^{-1} \text{ (per pixel)} \end{cases}$$

$\therefore \alpha = kf = 25\ m^{-1} \times 25\ mm = 625$

$\beta = Lf = 20\ mm^{-1} \times 25\ mm = 500$

$\therefore$ intrinsic matrix is:

$$K = \begin{pmatrix} \alpha & -\alpha \cot\theta & x_0 \\ 0 & \frac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow K = \begin{pmatrix} 625 & -625 \times \cot 85^\circ & 500 \\ 0 & \frac{500}{\sin 85^\circ} & 500 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \boxed{K = \begin{pmatrix} 625 & -54.680 & 500 \\ 0 & 501.909 & 500 \\ 0 & 0 & 1 \end{pmatrix}}$$ Ans

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Soln-2: (a). Let homogeneous world co-ordinate be:

$\vec{P}_w^h$

and pixel co-ordinate in homogeneous system $\Rightarrow \vec{p}^h$

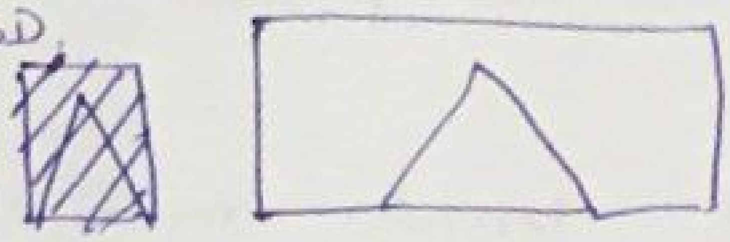
Transformation can be written as:-

$$\vec{p}^h = M_{in}\vec{P}_w^h$$

M_{in} is the intrinsic perspective projection mapping.

Let's consider a set of parallel lines in 3D.

$$\vec{P}_k^s(s) = \begin{pmatrix}\vec{P}_k^0\\ 1\end{pmatrix} + s\begin{pmatrix}\vec{t}\\ 0\end{pmatrix}$$



where; $\vec{P}_0$ is an arbitrary 3D point and the line $\vec{t}$ is a 3D tangent vector.

s is the free parameter for points along the line.

Let's consider a single line.

we know that; $\vec{p}^h = M\vec{P}^h$

$$\Rightarrow \vec{p}^h(s) = M\vec{P}^h(s) = M\begin{pmatrix}\vec{P}_0\\ 1\end{pmatrix} + sM\begin{pmatrix}\vec{t}\\ 0\end{pmatrix}.$$

constant vectors independent of 's'

$$= \vec{p}^h(0) + s\vec{p}_t^h$$

$\Rightarrow$ Homogeneous to pixel coordinates gives-

$$\vec{p}(s) = \frac{\vec{p}^h(s)}{p_3^h(s)} = \frac{\vec{p}^h(0)}{p_3^h(s)} + \frac{s}{p_3^h(s)}\vec{p}_t^h.$$

$$p_3^h(s) = p_3^h(0) + s\,p_{t,3}^h \qquad \text{as } s \to \infty$$

$$\vec{p}(s) = \cancel{\frac{\vec{p}^h(0)}{p_3^h(0) + s\cdot p_{t,3}^h}} \to 0 + \frac{s\vec{p}_t^h}{p_3^h(0) + s p_{t,3}^h}$$

$\Rightarrow \dfrac{\vec{p}_t^h}{p_{t,3}^h} \rightarrow$ This limit point is a constant image point dependent only on the tangent direction $\vec{t}$.

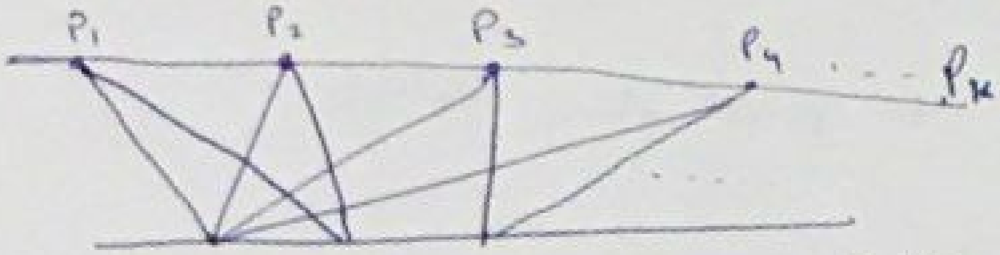
$\therefore$ The perspective projection is:-

$$\Rightarrow \vec{p}_t^h = M \begin{bmatrix} \vec{t} \\ 0 \end{bmatrix}$$

Since at $s \to \infty$ the perspective projection of the points $\vec{p}_t^h(s)$ all converge to the same point $\vec{p}_t^h = M \begin{bmatrix} \vec{t} \\ 0 \end{bmatrix}$

$\Rightarrow$ All parallel lines projects to a <u>vanishing point</u>. <u>Proved</u>

(b). Consider multiple families of parallel lines in a plane.



Consider a family of parallel lines as above a 'kth' family of lines intersect at a point at infinity.

$$\vec{p}_K^h = M \begin{bmatrix} \vec{t}_K \\ 0 \end{bmatrix}.$$

Since the tangent directions are coplanar in 3D, any two points provide a basis.

Let's assume the first two points.

$$\vec{t}_K = a_K \vec{t}_1 + b_K \vec{t}_2 \qquad (a_K, b_K = \text{some constant})$$

$$\vec{p}_K^h = M \begin{bmatrix} a_K \vec{t}_1 + b_K \vec{t}_2 \\ 0 \end{bmatrix} = M a_K \begin{bmatrix} \vec{t}_1 \\ 0 \end{bmatrix} + M b_K \begin{bmatrix} \vec{t}_2 \\ 0 \end{bmatrix}.$$

$$\vec{p}_K^h \Rightarrow a_K \vec{p}_1^h + b_K \vec{p}_2^h$$

Converting homogeneous to image point, we get -

$$\vec{P}_K = \left(\frac{a_K P^{k}_{1,3}}{P^{h}_{K,3}}\right)\vec{P}_1 + \left(\frac{b_K P^{h}_{2,3}}{P^{h}_{K,3}}\right)\vec{P}_2$$

$$\Rightarrow \boxed{\vec{P}_K = \alpha_K \vec{P}_1 + \beta_K \vec{P}_2 .}$$

Hence; the image point $\vec{P}_K$ is an affine combination of the two image points $\vec{P}_1$ and $\vec{P}_2$.

$\Rightarrow$ The horizon must be the line in the image passing through $\vec{P}_1$ and $\vec{P}_2$ which is nothing but a vanishing line.

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