Course: B.Tech (CSBS)   SEM: III

Subject: Computational Statistics   Marks: 20

Date of Exam: 05-10-2020   Duration: 60 minutes

Instructions:

Attempt any 4 out of 5 questions.
Each question carry 5 marks.

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1. Let X_1, X_2, X_3 and X_4 be independent and identically distributed (3x1) random vectors with

$$\mu = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$$

   Consider the linear combinations:

   $$2X_1 + 3X_2 - X_3 + X_4$$
   and
   $$X_1 - 3X_2 + 5X_3 - 2X_4$$

   Find out

   1. Mean Vector.   3. Covariance between them
   2. Covariance Matrix

2. Let X is normally distributed with

$$\mu = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} and \Sigma = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -2 & 0 \\ 2 & 0 & -3 \end{bmatrix}$$

   1) Are X_1 and X_2 independent?   4) What about (X_1, X_2) and X_3?
   2) Are X_1 and X_3 independent?   5) What about X_1 and (X_2, X_3)?
   3) Are X_2 and X_3 independent?   6) Are X1 and Y=2X1-3X2+X3

3. Apply the principal component analysis for the data given below:

Y =

2	1
3	4
5	0
6	7
2	9
3	7

4. Compute the following for Linear Discriminant Analysis:
   a) Compute the global mean (M) using the samples.
   b) Compute the statistics like Mean Vector and Covariance Matrix for samples.
   c) Compute within-class scatter matrix C.

X1 =

4	8
9	4
10	6
6	7
8	9

X2 =

2	2
2	4
4	5
3	7
5	4

5. Factory "ABC" produces very expensive and high quality chip rings that their qualities are measured in terms of curvature and diameter. Results of quality control by experts given in the table below:

Curvature	3.95	3.53	4.57	4.16	3.58	3.16	4.12
Diameter	7.63	8.79	6.65	6.47	5.46	7.22	4.52
Quality	Passed	Passed	Passed	Passed	Not Passed	Not Passed	Not Passed

As a consultant to the factory, you get a task to set up the criteria for automatic quality control. Then, the manager of the factory also wants to test your criteria upon new type of chip rings that even the human experts are argued to each other. The new chip rings have curvature 2.81 and diameter 5.46. Can you solve this problem by employing Discriminant Analysis?