

Name : Siddharth Pandya

Roll No. : 23

Date : 5.10.20

Subject : CS

Ans 5

$$x_1 = \begin{bmatrix} 8.95 & 7.63 \\ 8.53 & 8.79 \\ 4.57 & 6.65 \\ 4.16 & 6.47 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 8.58 & 5.46 \\ 3.16 & 7.22 \\ 4.12 & 4.52 \end{bmatrix}$$

$$\mu_1 = [4.05 \quad 7.385]$$

$$\mu_2 = [2.69 \quad 3.62 \quad 5.73]$$

$$\text{Global Mean} = \mu = [3.235 \quad 6.5575]$$

$$x_1' = \begin{bmatrix} 0.1215 & 1.08 \\ -0.305 & 2.23 \\ 0.735 & 0.1 \\ 0.325 & -0.08 \end{bmatrix}$$

$$x_2' = \begin{bmatrix} -0.255 & -1.09 \\ -0.675 & 0.67 \\ 0.285 & -2.03 \end{bmatrix}$$

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$$C_1 = \frac{1}{N_1} x_1'^T x_1'$$

$$= \frac{1}{4} \begin{bmatrix} 0.115 & -0.305 & 0.735 & 0.325 \\ 1.08 & 2.23 & 0.1 & -0.08 \end{bmatrix} \begin{bmatrix} 0.115 & 1.09 \\ -0.305 & 2.23 \\ 0.735 & 0.1 \\ 0.325 & -0.08 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0.013 + 0.093 + 0.540 + 0.105 & 0.124 + (-0.68) + 0.073 + (-0.26) \\ 0.124 + (-0.68) + 0.073 + (-0.26) & 1.16 + 4.97 + 0.01 + 0.04 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 0.751 & -0.743 \\ -0.743 & 6.78 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.18775 & -0.185 \\ -0.185 & 1.695 \end{bmatrix}$$

$$C_2 = \frac{1}{N_2} x_2'^T x_2'$$

$$= \frac{1}{3} \begin{bmatrix} -0.255 & -0.675 & 0.285 \\ -1.09 & 0.67 & -2.03 \end{bmatrix} \begin{bmatrix} -0.255 & -1.09 \\ -0.675 & 0.67 \\ 0.285 & -2.03 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 0.065 + 0.455 + 0.081 & 0.277 + (-0.452) + (-0.578) \\ 0.277 + (-0.452) + (-0.578) & 1.188 + 0.4489 + 4.120 \end{bmatrix}$$



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$$= \frac{1}{3} \begin{bmatrix} 0.601 & -0.753 \\ -0.753 & 5.7569 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.2 & -0.251 \\ -0.251 & 1.91 \end{bmatrix}$$

$$C = \frac{n_1}{n_1+n_2} C_1 + \frac{n_2}{n_1+n_2} C_2$$

$$= \frac{4}{7} \begin{bmatrix} 0.18775 & -0.185 \\ -0.185 & 1.695 \end{bmatrix} + \frac{3}{7} \begin{bmatrix} 0.2 & -0.251 \\ -0.251 & 1.91 \end{bmatrix}$$

$$= \begin{bmatrix} 0.106 & -0.105 \\ -0.105 & 0.966 \end{bmatrix} + \begin{bmatrix} 0.084 & -0.105 \\ -0.105 & 0.8022 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.19 & -0.21 \\ -0.21 & 1.7682 \end{bmatrix}$$

$$P = \begin{bmatrix} \frac{n_1}{n_1+n_2} \\ \frac{n_2}{n_1+n_2} \end{bmatrix} = \begin{bmatrix} 4/7 \\ 3/7 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 8.841 & 0.7 \\ 0.7 & 0.6 \end{bmatrix}$$

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$$f_1 = \mu_1 C^{-1} x_k - \frac{1}{2} \mu_1 C \mu_1' + \ln P_1$$

$$= (2.81) [4.05 \ 7.385] \begin{bmatrix} 8.841 & 0.7 \\ 0.7 & 0.6 \end{bmatrix} - \frac{1}{2} [4.05 \ 7.385] \begin{bmatrix} 4.05 \\ 7.385 \end{bmatrix}$$

$$\begin{bmatrix} 0.19 & -0.2 \\ -0.2 & 1.768 \end{bmatrix} + \ln \left( \frac{3}{7} \right)$$

$$f_2 = \mu_2 C^{-1} x_k - \frac{1}{2} \mu_2 C \mu_2' + \ln P_2$$

$$= [3.62 \ 5.73] \begin{bmatrix} 8.84 & 0.7 \\ 0.7 & 0.6 \end{bmatrix} (5.81) - \frac{1}{2} [3.62 \ 5.73] \begin{bmatrix} 3.62 \\ 5.73 \end{bmatrix}$$

$$\begin{bmatrix} 0.19 & -0.2 \\ -0.2 & 1.768 \end{bmatrix} + \ln \left( \frac{4}{7} \right)$$

$$\therefore f_2 > f_1$$



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$$i) \quad \mu = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -5 \\ 3 & -5 & 6 \end{bmatrix}$$

$$2x_1 + 3x_2 - x_3 + x_4$$

and

$$x_1 - 3x_2 + 5x_3 - 2x_4$$

$$c_1 x_1 + c_2 x_2 - c_3 x_3 + c_4 x_4$$

$$c_1 = 2, \quad c_2 = 3, \quad c_3 = -1, \quad c_4 = 1$$

$$\begin{aligned} \therefore & (c_1 + c_2 + c_3 + c_4) \mu \\ &= (2 + 3 - 1 + 1) \mu \\ &= 5\mu \end{aligned}$$

$$i) \quad 5\mu = \begin{bmatrix} -15 \\ -5 \\ 5 \end{bmatrix}$$

$$ii) \quad \text{variance matrix: } (c_1^2 + c_2^2 + c_3^2 + c_4^2) \Sigma$$

$$\begin{aligned} &= (2^2 + 3^2 + (-1)^2 + 1^2) \Sigma \\ &= (4 + 9 + 1 + 1) \Sigma \\ &= 15 \Sigma \end{aligned}$$

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$$15\Sigma = \begin{bmatrix} 15 & -30 & 45 \\ -30 & 60 & -75 \\ 45 & -75 & 90 \end{bmatrix}$$

For second case

$$b_1 = 1 \quad b_2 = -3 \quad b_3 = 5 \quad b_4 = -2$$

$$(b_1 + b_2 + b_3 + b_4)\mu =$$

$$(1 + -3 + 5 - 2)\mu$$

$$\mu = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{Covariance} = (b_1^2 + b_2^2 + b_3^2 + b_4^2) \Sigma$$

$$= (1^2 + (-3)^2 + (5)^2 + (-2)^2) \Sigma$$

$$= (1 + 9 + 25 + 4) \Sigma$$

$$= 39 \Sigma$$

$$39\Sigma = \begin{bmatrix} 39 & -78 & 117 \\ -78 & 156 & -195 \\ 117 & -195 & 234 \end{bmatrix}$$



$$\text{iii)} \quad (C_1 b_1 + C_2 b_2 + C_3 b_3 + C_4 b_4) \Sigma$$

$$(2 \times 1 + 3 \times -3 + -1 \times 5 + 1 \times -2) \Sigma$$

$$(2 + (-9) - 5 - 2) \Sigma$$

$$(2 - 9 - 5 - 2) \Sigma$$

$$(-14) \Sigma$$

$$-14 \Sigma = \begin{bmatrix} -14 & 28 & -42 \\ 28 & -56 & 70 \\ -42 & 70 & -84 \end{bmatrix}$$

Ans 3

$$Y = \begin{bmatrix} 1 & 4 & 0 & 7 & 9 & 7 \\ 2 & 3 & 5 & 6 & 2 & 3 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 3 & 5 & 6 & 2 & 3 \\ 1 & 4 & 0 & 7 & 9 & 7 \end{bmatrix}$$

$$\text{Average of row 1} = 3.5$$

$$\text{Average of row 2} = 4.66$$

$$\text{Centered Matrix (A)} = \begin{bmatrix} -1.5 & -0.5 & 1.5 & 2.5 & -1.5 & -0.5 \\ -3.66 & -0.66 & -4.66 & 2.34 & 4.34 & 2.34 \end{bmatrix}$$

$$\text{Covariance matrix} = \frac{AA^T}{n-1}$$

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$$= \frac{1}{5} \begin{bmatrix} -1.5 & -0.5 & 1.5 & 2.5 & -1.5 & -0.5 \\ -3.66 & -0.66 & -4.66 & 2.34 & 4.34 & 2.34 \end{bmatrix} \begin{bmatrix} -1.5 & -3.66 \\ -0.5 & -0.66 \\ 1.5 & -4.66 \\ 2.5 & 2.34 \\ -1.5 & 4.34 \\ -0.5 & 2.34 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2.25 + 0.25 + 2.25 + 6.25 + 2.25 + 0.25 & 5.49 + 0.33 - 6.99 \\ & + 5.85 - 6.51 - 1.17 \\ & 13.39 + 0.43 + 21.71 + \\ & 5.47 + 18.83 + 5.47 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 13.5 & -3 \\ -3 & 65.3 \end{bmatrix}$$

$$= \begin{bmatrix} 2.7 & -0.6 \\ -0.6 & 13.06 \end{bmatrix}$$

$$(A - \lambda I)$$

$$\begin{bmatrix} 2.7 - \lambda & -0.6 \\ -0.6 & 13.06 - \lambda \end{bmatrix}$$



$$(2.7 - \lambda)(13.06 - \lambda) - 0.36 = 0$$

$$2.7 \times 13.06 - 2.7\lambda - 13.06\lambda + \lambda^2 - 0.36 = 0$$

$$35.262 - 15.76\lambda + \lambda^2 - 0.36 = 0$$

$$\lambda^2 - 15.76\lambda + 34.902 = 0$$

$$\lambda = 13.09, 2.66$$