Assignment 8: The Digital Fourier Transform

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1 Abstract

The goal of this assignment is the following.

- To obtain the DFT of various functions.
- To see how the DFT can be used to approximate the CTFT.
- To plot graphs for a better understanding.

2 Assignment

2.1 Importing the standard libraries

```
from pylab import *
```

2.2 Calculating the inverse FFT

To verify that the fft and the ifft are inverses of each other by finding the absolute maximum of the difference between the actual function and the inverse FFT.

```
x=rand(100)
X=fft(x)
y=ifft(X)
c_[x,y]
print(abs(x-y).max())
```

3.332294980281256e-16 is the absolute error that is being printed. The small error is due to the numerical precision of the computer in taking the IFFT. This is due to the finite accuracy of the CPU so that the ifft could not exactly undo what fft did.

2.3 Going through the examples, Example 1

We try to take the FFT of a sinusoid-

```
x=linspace(0,2*pi,128)
y=sin(5*x)
Y=fft(y)
figure(1)
subplot(2,1,1)
plot(abs(Y),lw=2)
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin(5t)$")
grid(True)
subplot(2,1,2)
plot(unwrap(angle(Y)),lw=2)
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
```

We get 2 peaks as expected, but the peaks are in the wrong location and the scale of the x axis is incorrect. We should divide by N to use it as a spectrum. We haven't yet got the frequency axis in place.

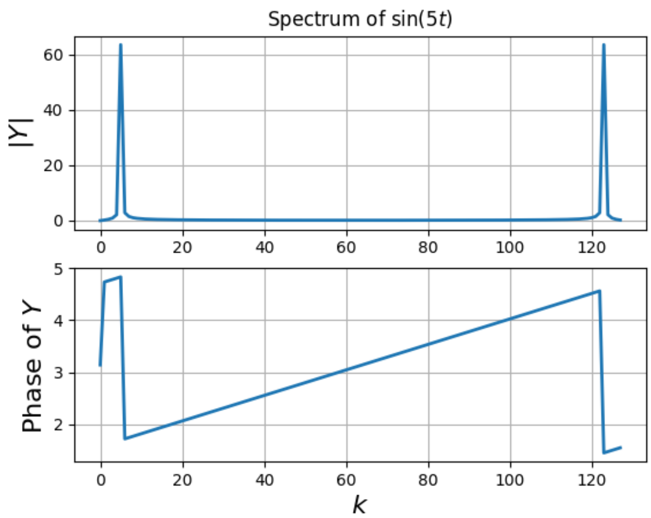

Figure 1

2.4 Going through the examples, Example 2

We now try taking the FFT of a sinusoid using fftshift() to correct the previous error. The error was due to the FFT being from 0 to 2π instead of $-\pi$ to π as we would want. We then divide by the number of samples N. We also have to exclude the last sample to avoid 0 and 2π being treated as the same.

```
x=linspace(0,2*pi,129);x=x[:-1]
y=sin(5*x)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure(2)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin(5t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
```

It now looks like what we expected with 2 peaks at the right frequencies.

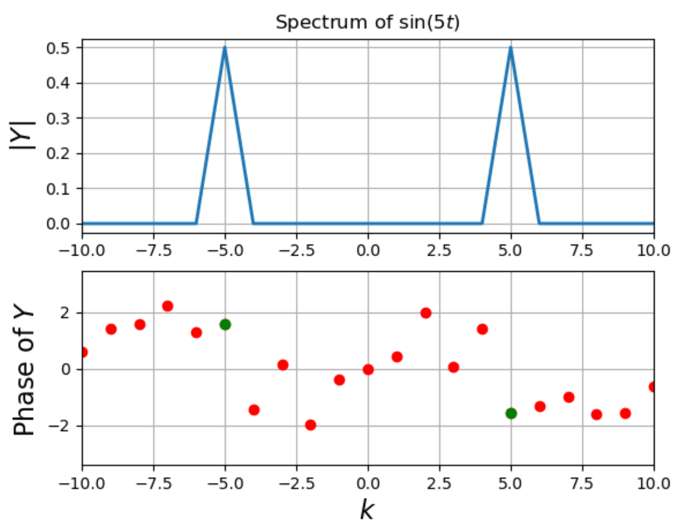

Figure 2

2.5 Going through the examples, Example 3

We take the FFT of an Amplitude Modulated Sinusoid of the form (1 + 0.1 cos (t)) cos (10t)

```
t=linspace(0,2*pi,129);t=t[:-1]
y=(1+0.1*cos(t))*cos(10*t)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure(3)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $(1+0.1cos(t))cos(10t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```

Instead of getting 3 peaks, we only get 1 broad peak. This is because the number of samples are not sufficient to get the necessary information. This is known as aliasing.

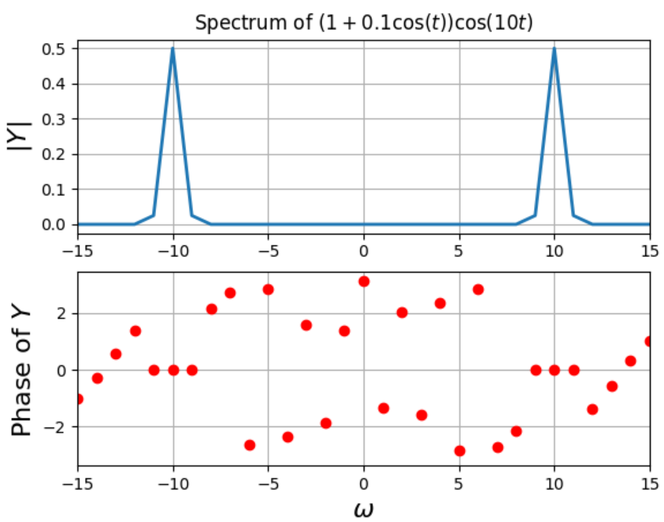

Figure 3

2.6 Going through the examples, Example 4

Increasing the number of samples to 512, we try again.

```
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=(1+0.1*cos(t))*cos(10*t)
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure(4)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\left(1+0.1\cos\left(t\right)\right)\cos\left(10t\right)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```

As expected, we get three peaks at the carrier band and at frequencies above and below the carrier band at frequency difference equal to the modulating frequency. Most of the energy is stored in the carrier band.

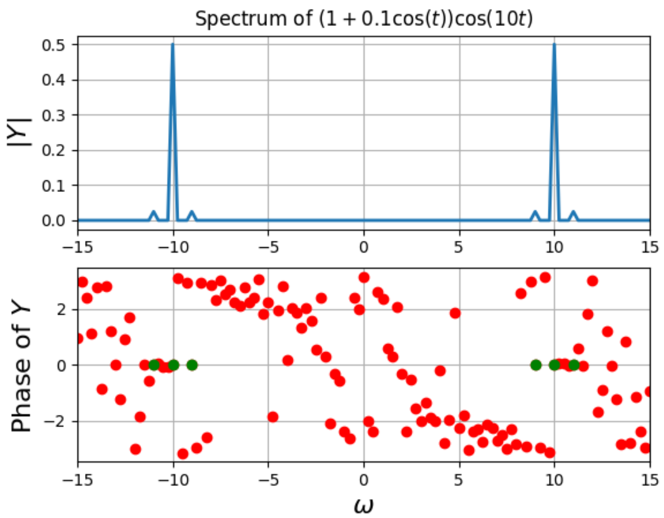

Figure 4

2.7 Going through the examples, Example 3

We take the FFT of an Amplitude Modulated Sinusoid of the form (1 + 0.1 cos (t)) cos (10t)

```
t=linspace(0,2*pi,129);t=t[:-1]
y=(1+0.1*cos(t))*cos(10*t)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure(3)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $(1+0.1cos(t))cos(10t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```

Instead of getting 3 peaks, we only get 1 broad peak. This is because the number of samples are not sufficient to get the necessary information. This is known as aliasing.

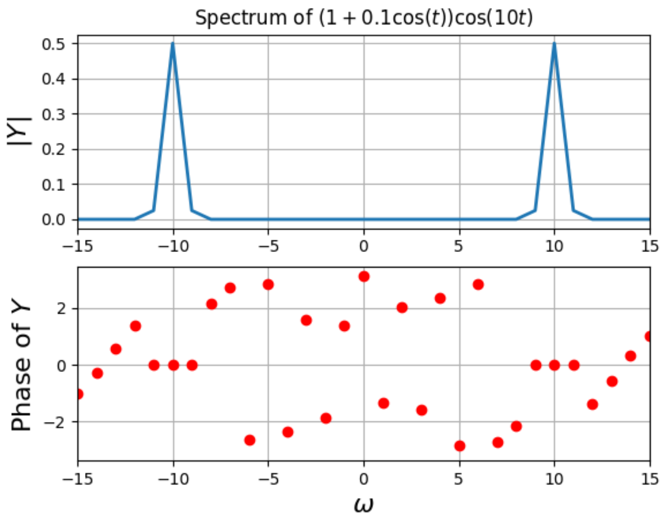

Figure 5

2.8 Question 2: FFT for $cos^3 t$ and $sin^3 t$

Plotting the DFT of the functions $cos^3 t$ and $sin^3 t$ with a sampling frequency of 128, we get

```
#sin^3(t)
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=(sin(t))**3
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure(5)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
xticks(arange(-10, 10, step=2))
title(r"Spectrum of $sin^3 (t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xticks(arange(-10, 10, step=2))
xlim([-10,10])
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)

#cos^3(t)
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=(cos(t))**3
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure(6)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
xticks(arange(-10, 10, step=2))
title(r"Spectrum of $cos^3 (t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'bo',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
```

```
xlim([-10,10])
xticks(arange(-10, 10, step=2))
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```

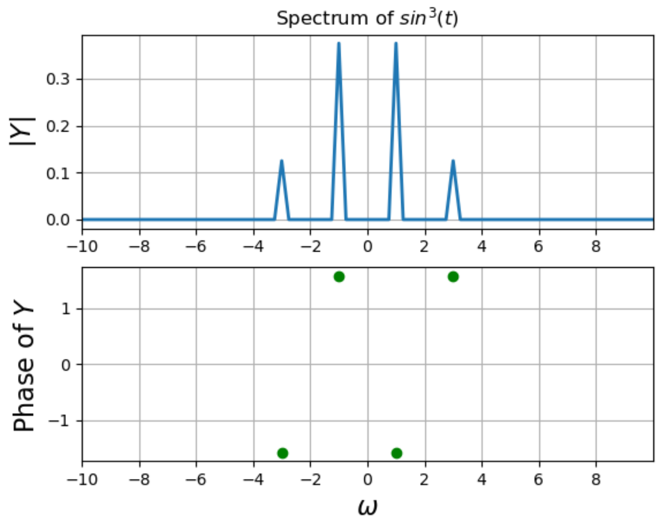

Figure 6

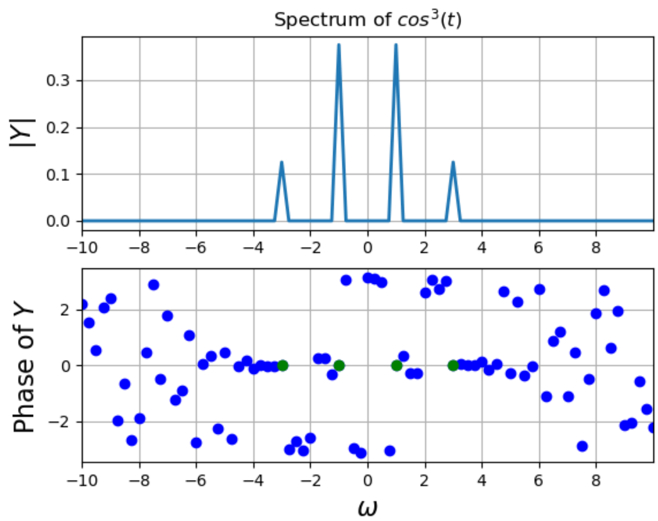

Figure 7

Both the spectra have 4 peaks These correspond to frequencies of ± 1 and ± 3. We get this by splitting them into their identities-

$$cos^3t = \frac{cos3t + 3cost}{4}$$

and

$$sin^3t = \frac{3sint - sin3t}{4}$$

2.9 Question 3: FFT for a frequency modulated sinusoid

We plot the DFT of a frequency modulated sinusoid cos(20t + 5 cos(t))

```
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=cos(20*t+5*cos(t))
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure(7)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-50,50])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of cos(20t+5cos(t))")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-50,50])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```

A lot of frequencies around the carrier band are present. Most of the energy is carried by these frequencies.

2.10 Question 4: FFT of a Gaussian

We use the FFT to estimate the CTFT of the Gaussian distribution function $e^{\frac{-t^2}{2}}$ The CTFT of the above non-periodic function is $\sqrt{2\pi}e^{\frac{-\omega^2}{2}}$ using the appropriate CTFT Formulation.

```
t=linspace(-8*pi,8*pi,1025);t=t[:-1]
y=exp(-(t**2)/2)
Y=fftshift(fft(y))/1024.0
w=linspace(-64,64,1025);w=w[:-1]
figure(8)
subplot(2,1,1)
plot(w,abs(Y),lw=2)
```

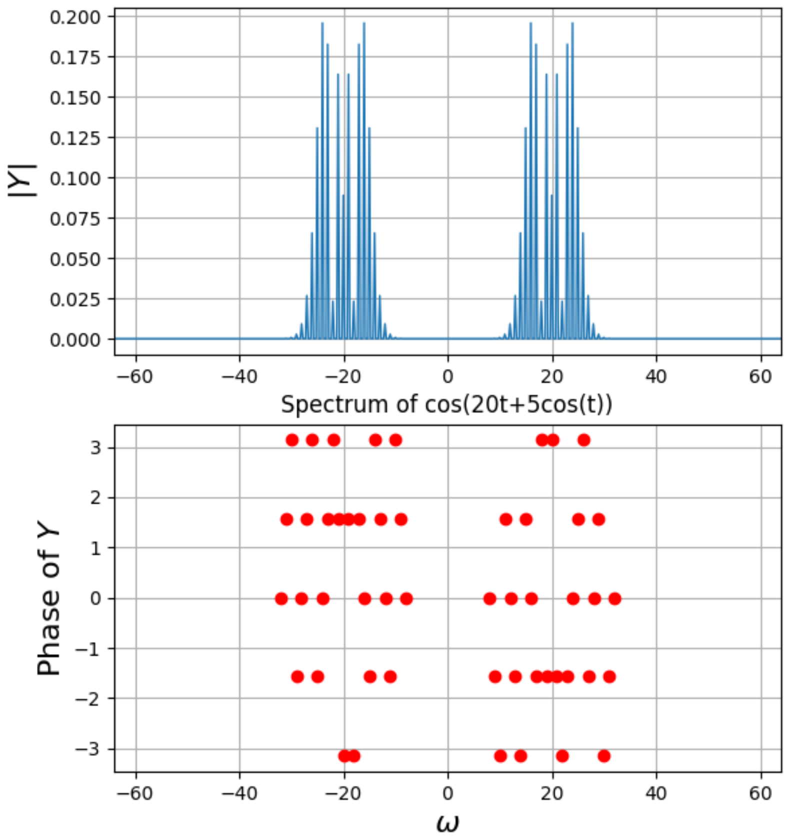

Figure 8

```
xlim([-7.5,7.5])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $exp(-t^2/2)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-7.5,7.5])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
#to calculate error
fast=fft(y)
```

```
inv=ifft(fast)
c_[y,inv]
print (abs(y-inv).max())
```

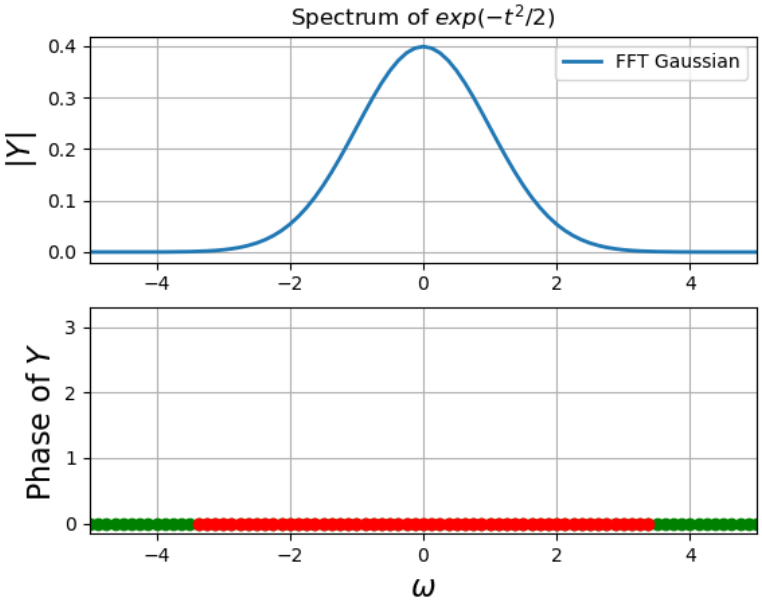

Figure 9

The max error was 1.608160121591368e-16. The error is almost negligible, and can easily be further reduced if necessary. which indicates the IFFTShift was used to remove certain issues with the time domain form of the Gaussian Phase is basically zero (within numerical precision), indicating the CTFT of the Gaussian is real valued and positive for all . In fact, by observation we can say that it is also another Gaussian with same mean and standard deviation. Thus it is clear that the Gaussian is an Eigenfunction of the Fourier Transform.

Only those points whose magnitude is greater than 1e-3 are plotted in red, while the rest are in green.

3 Conclusion

- We analysed the frequency domain spectra of various functions using FFT.
- We saw how the FFT can be used to accurately represent the DFT.