EE5111: Estimation Theory

Heavy-Tailed Distributions

Observations of CVaR for the Cauchy Distribution

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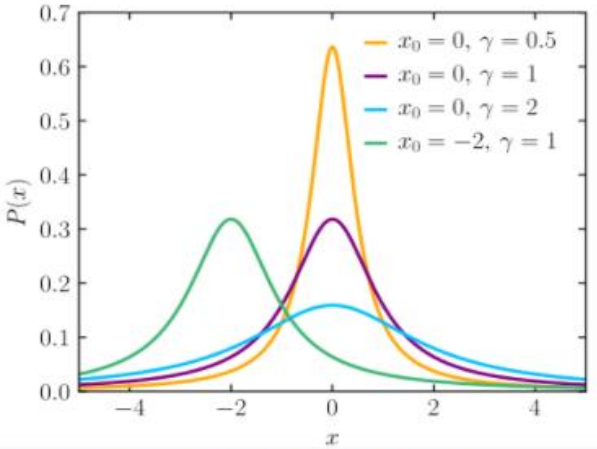
The Cauchy Distribution

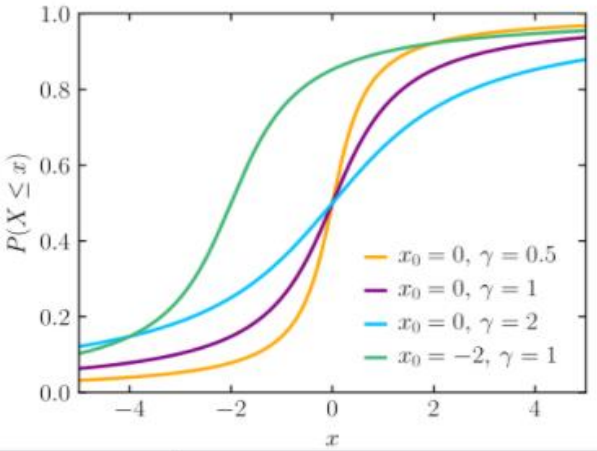
- Distribution of the ratio of **two independent normally** distributed random variables with mean zero.
- Both its **expected value** and its **variance** are **undefined.**
- Does not have finite moments of order ≥ 1; only fractional absolute moments exist.
- Has **no moment generating function**.
- $\mathrm{CVaR} = \int_{VAR}^{\infty} x.\frac{1}{\pi\gamma}\left[\frac{\gamma^2}{(x-x_0)^2+\gamma^2}\right] dx$ where $x_0 = 0$ and $\gamma = 1$

Probability density function

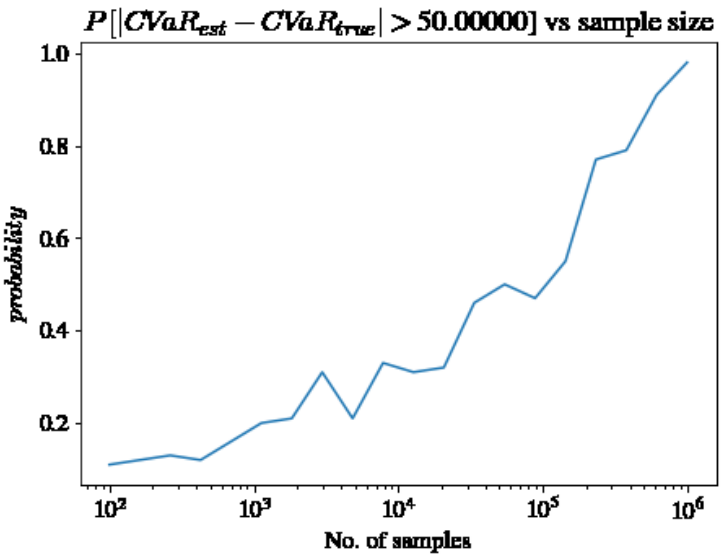


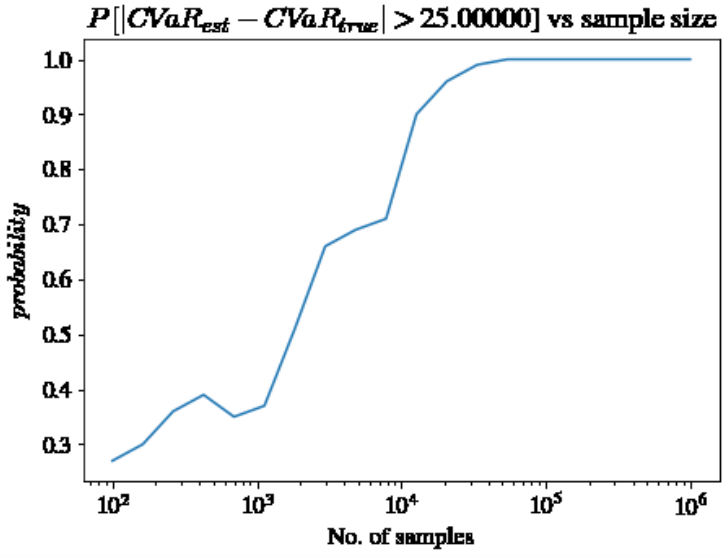
The purple curve is the standard Cauchy distribution

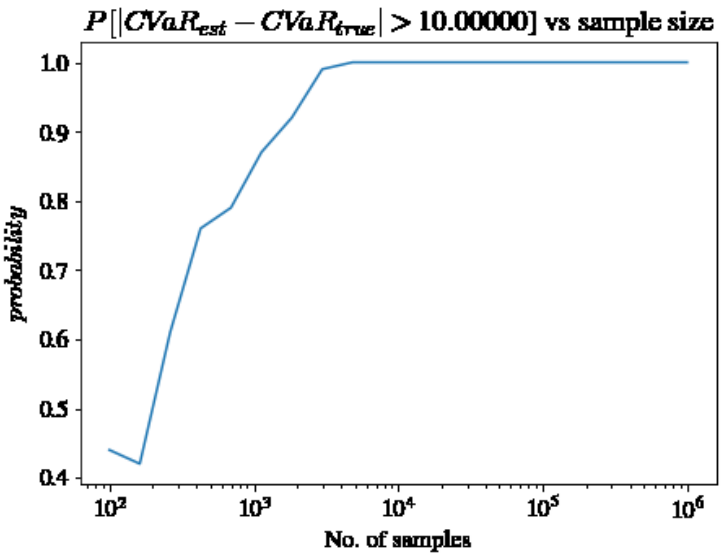
Cumulative distribution function

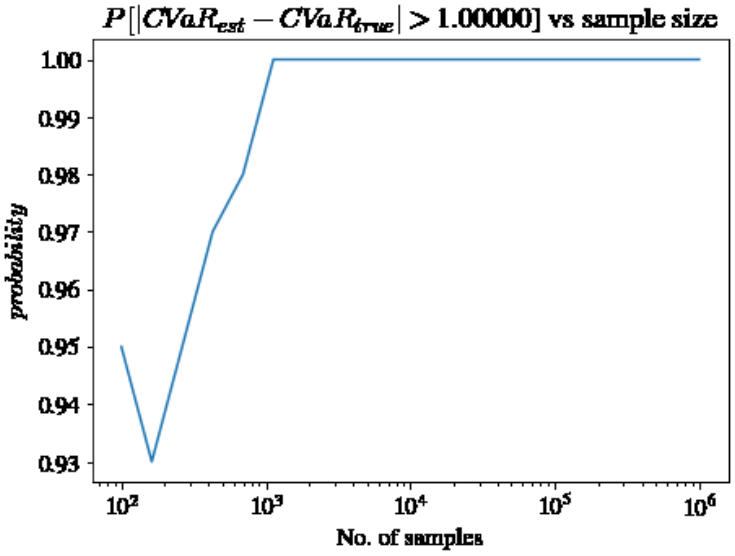


Simulation Results – Cauchy Distribution

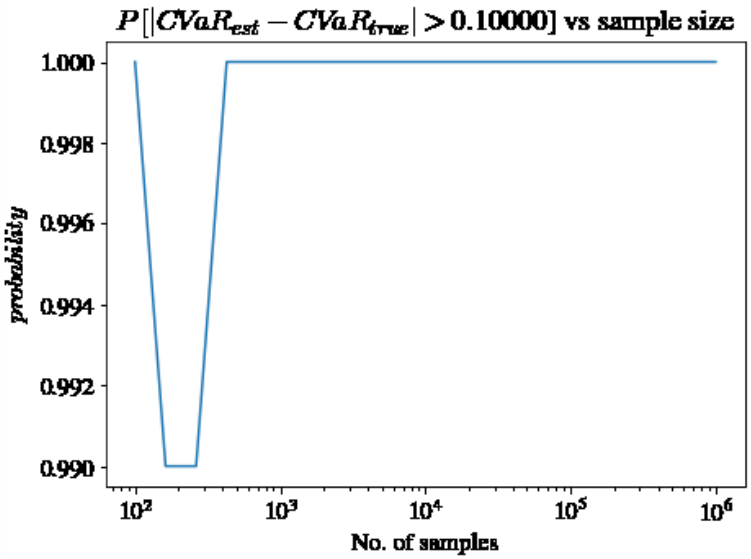


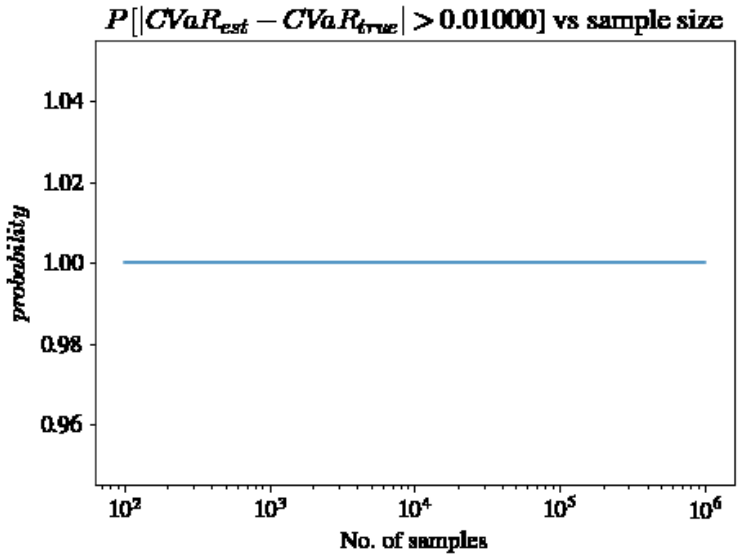






Simulation Results – Cauchy Distribution

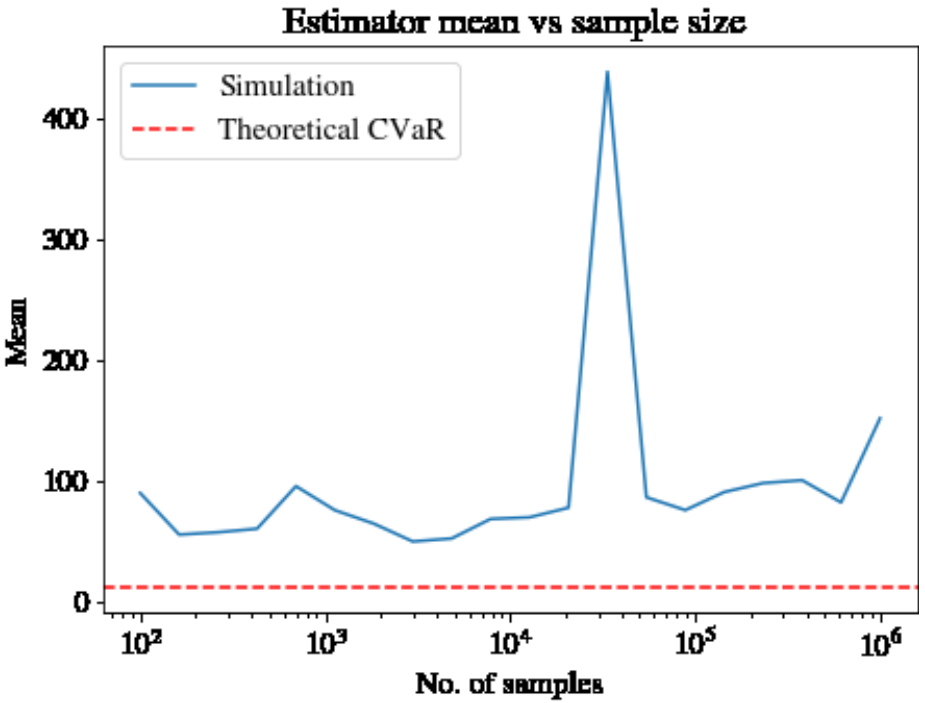


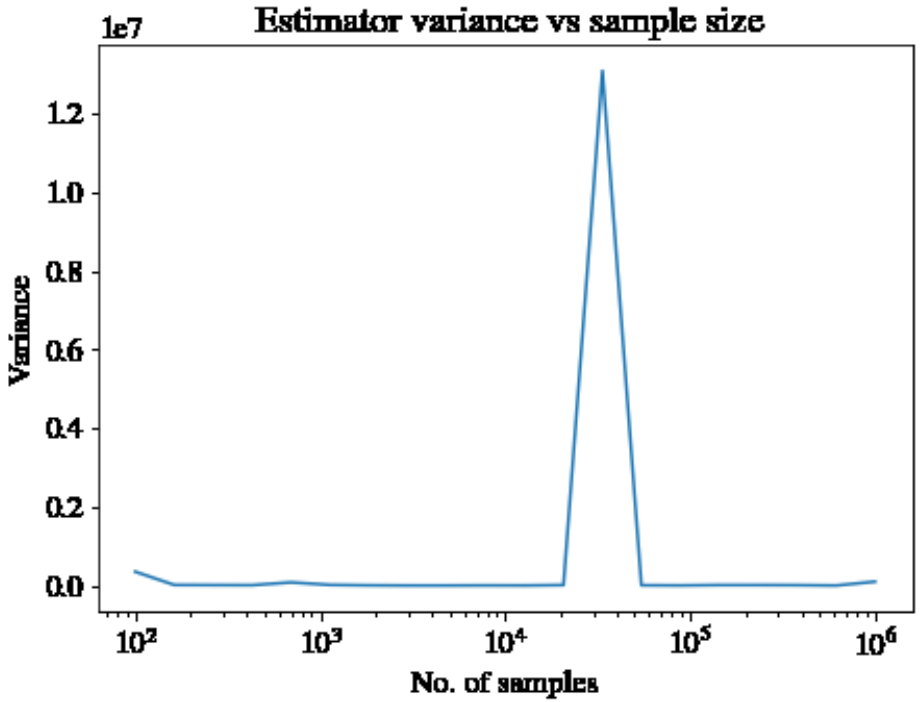


Inference –

- With increasing samples, the estimator worsens (counterintuitive) as the number of extreme samples increases
- Cauchy (I have simulated using standard Cauchy) is a pathological distribution having an undefined mean and variance.
- Extreme values are prone to occur and this totally distorts the graph.

Simulation Results

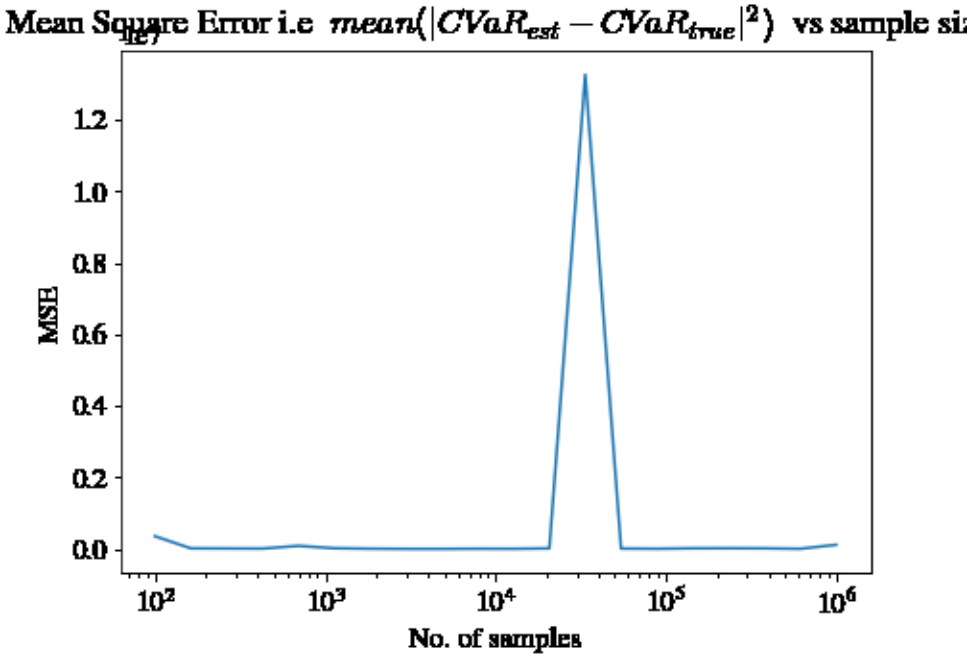


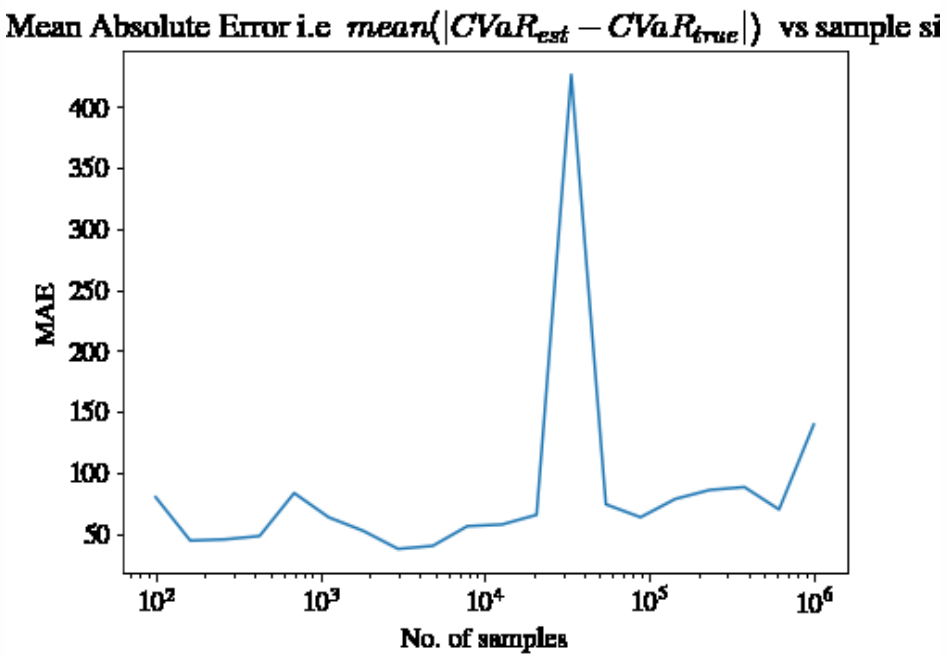


Inference:

- Biased estimator (Mean)
- Not consistent (Variance)

Simulation Results





Inference:

- The Mean Squared Error does not decrease with increasing sample size (with increasing samples, extreme values tend to increase further).
- Infact, there are spikes in between, at around 10^5 samples
- The estimator variance is in the order of 10^7 as seen in the previous slide.
- The mean absolute error is high with random spikes as seen from the graph here.

THANK YOU!