I I I T Bhubaneswar
Imagine, Innovate, Inspire
(A University established by Government of Odisha)

BJT AC Analysis

#LECTURE -

- INTRODUCTION
- IMPORTANT PARAMETERS
- R_E TRANSISTOR MODEL
- HYBRID EQUIVALENT MODEL
- GRAPHICAL DETERMINATION OF H PARAMETERS

INTRODUCTION

- The small-signal ac response of the BJT represents the transistor in the sinusoidal ac domain.
- Concerns in the sinusoidal ac analysis of transistor networks is the magnitude of the input signal.
    - small-signal techniques.
    - large signal techniques.
- There are two models commonly used in the small-signal ac analysis of transistor networks:
    - the r_e model and (for at actual operating condition)
    - the hybrid equivalent model. (for any operating condition) → h

Amplification in the AC Domain

- Transistor can be employed as an amplifying device.
- That is, the output sinusoidal signal is greater than the input sinusoidal signal, or, stated another way, the output ac power is greater than the input ac power.
- The question then arises as to how the ac power output can be greater than the input ac power.
- **Conservation of energy** dictates that over time the total power output of a system cannot be greater than its power input and efficiency can not be greater than 1.
- The factor missing from the discussion above that permits an ac power output greater than the input ac power is the **applied dc power**.
- It is the principal contributor to the total output power even though part of it is dissipated by the device and resistive elements.
- In other words, there is **an "exchange" of dc power to the ac domain that permits establishing a higher output ac power**.
- **DC Analysis and AC Analysis of Transistor are to be done.**

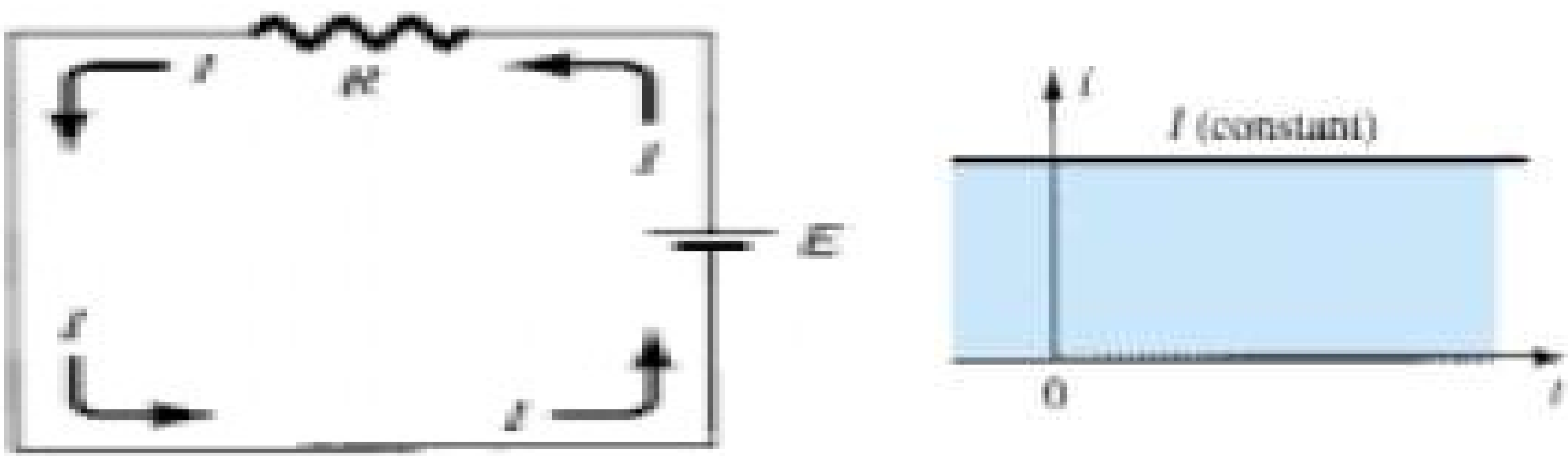
Amplification in the AC Domain

$$\eta = P_{o(\mathrm{ac})}/P_{i(\mathrm{dc})}$$

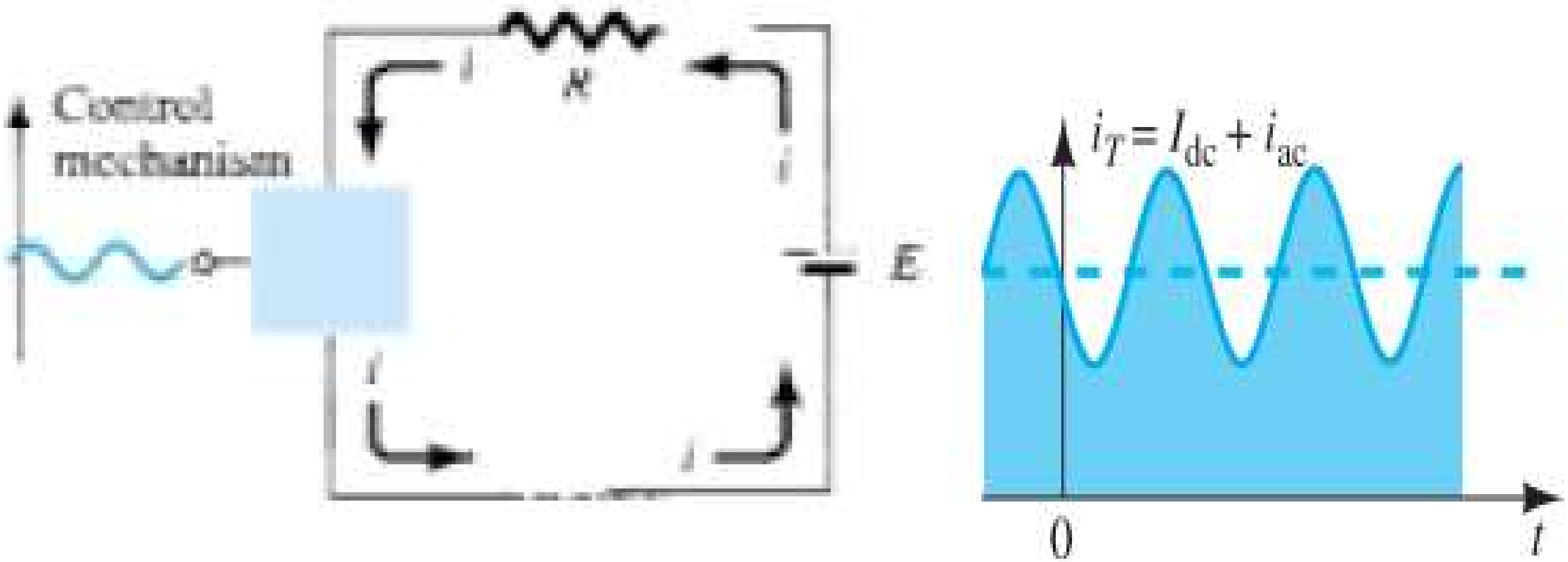
η is conversation efficiency

$P_{o(ac)}$ is the ac power to the load

$P_{i(dc)}$ is the dc power supplied



Steady current established by a dc supply



Effect of a control element on the steady-state flow of the electrical system

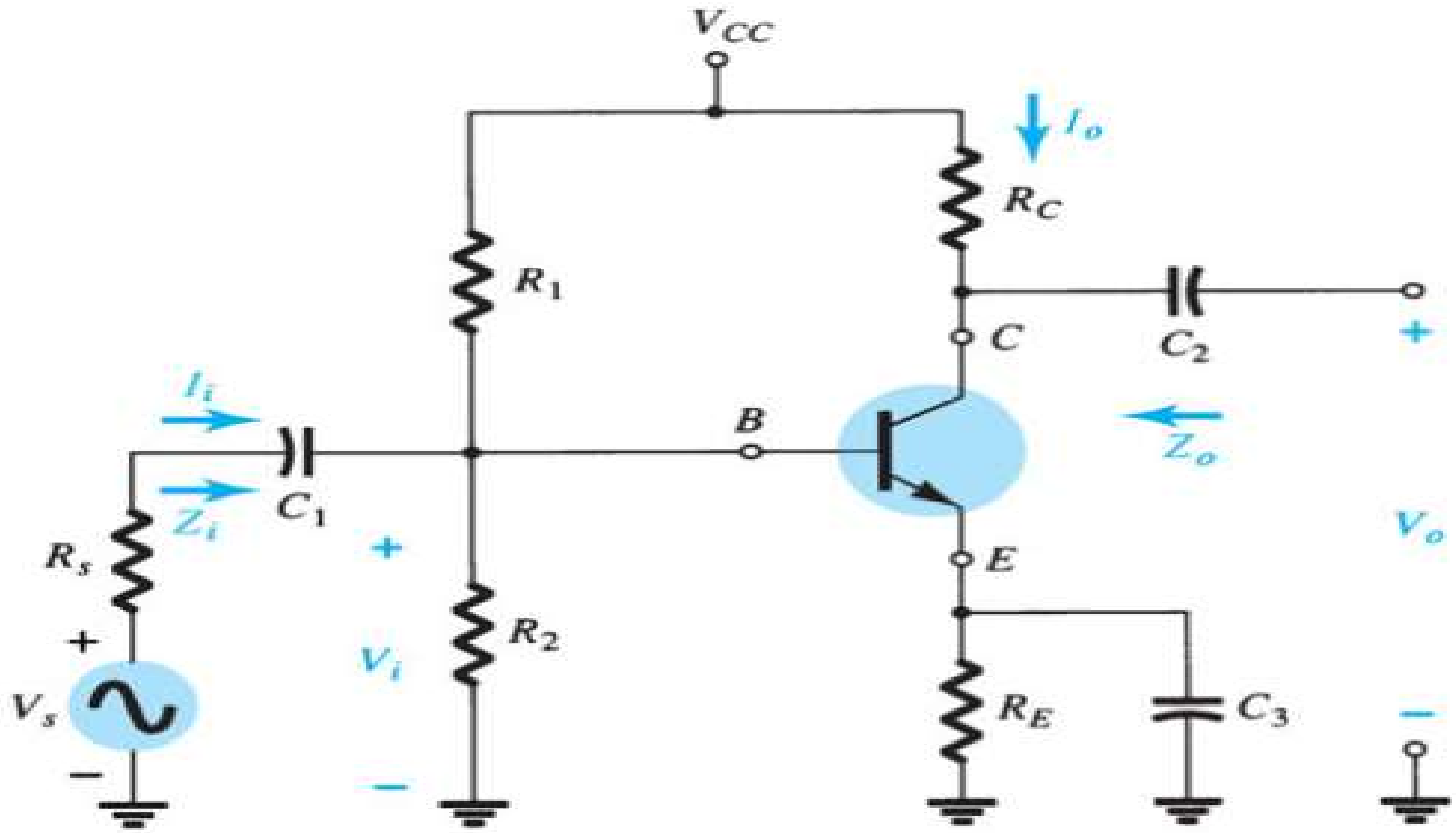
The peak value of the oscillation in the output circuit is controlled by the established dc level.

The superposition theorem is applicable for the analysis and design of the dc and ac components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

- In other words, one can make a complete dc analysis of a system before considering the ac response.
- Once the dc analysis is complete, the ac response can be determined using a completely ac analysis.
- However, that one of the components appearing in the ac analysis of BJT networks will be determined by the dc conditions, so there is still an important link between the two types of analysis.

BJT Transistor Modeling

A model is the combination of circuit elements, properly chosen, that best approximates the actual behaviour of a semiconductor device under specific operating conditions.

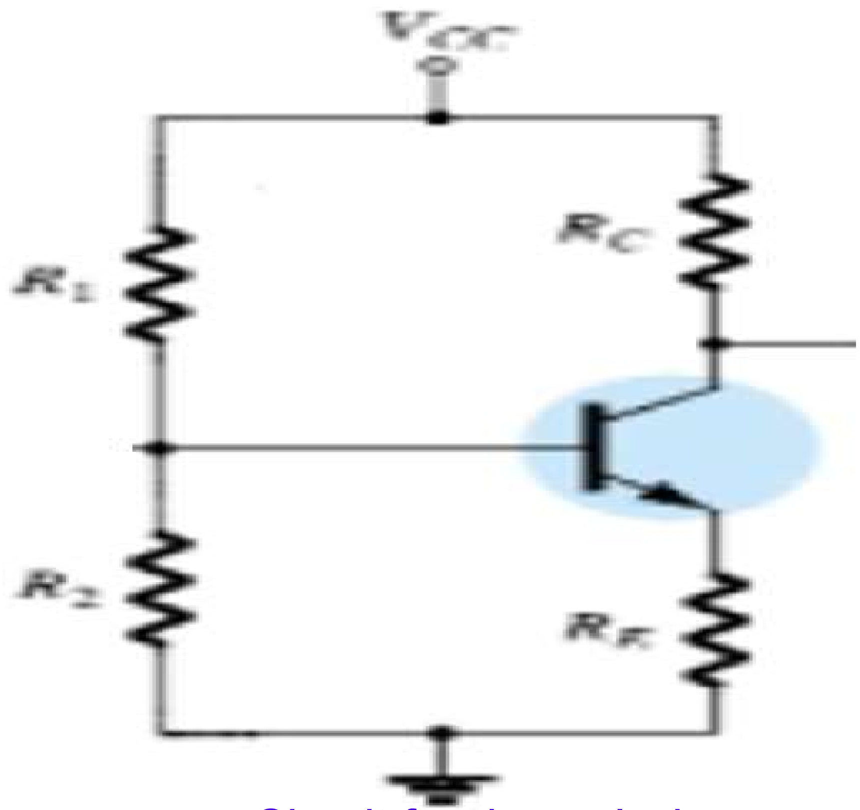


BJT Transistor Modeling

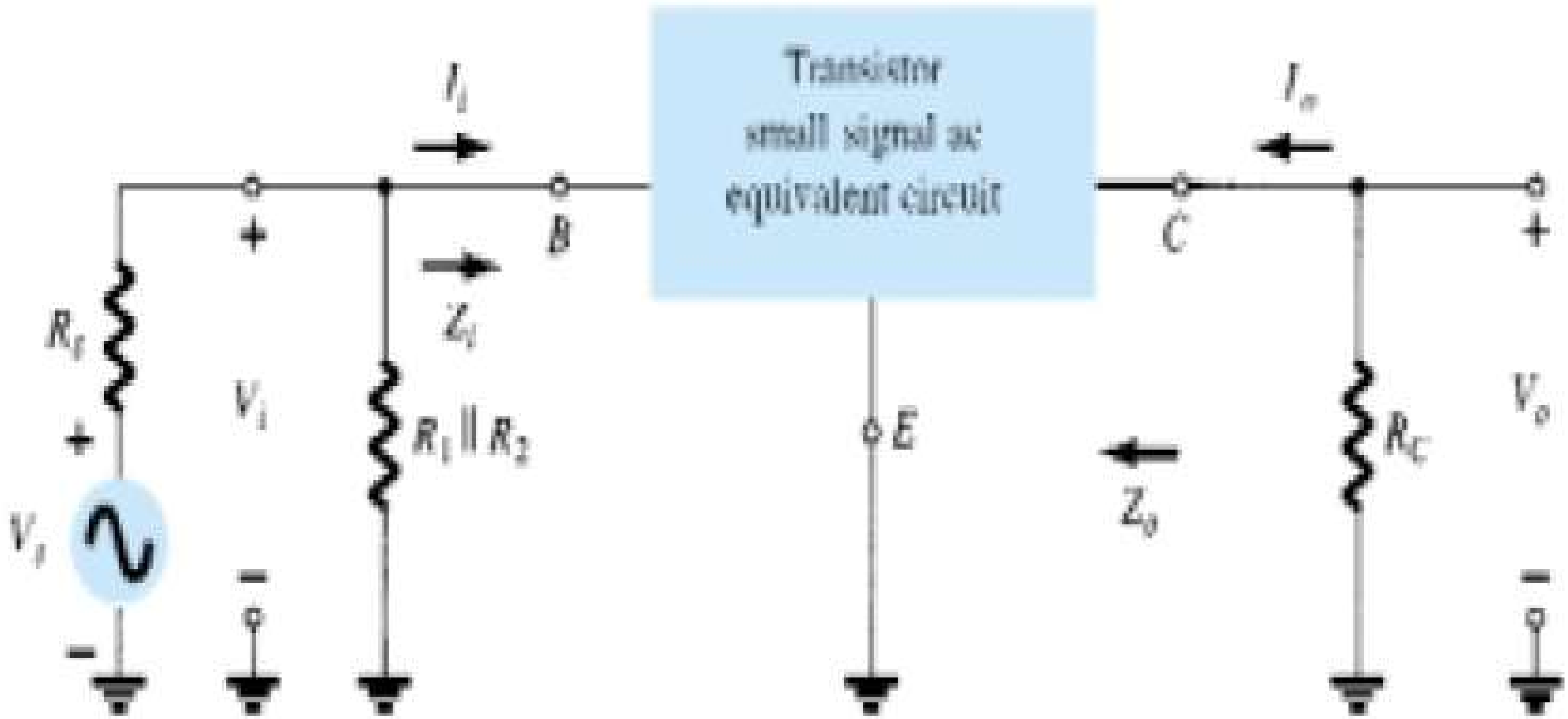
- A model is the combination of circuit elements, properly chosen, that best approximates the actual behaviour of a semiconductor device under specific operating conditions.
- Once the ac equivalent circuit is determined, the schematic symbol for the device can be replaced by this equivalent circuit.
- The basic methods of circuit analysis applied to determine the desired quantities of the network.
- The drawback to using this equivalent circuit, however, is that it is defined for a set of operating conditions that might not match actual operating conditions.
- In most cases, this is not a serious flaw because the actual operating conditions are relatively close to the chosen operating conditions on the data sheets.
- In addition, there is always a variation in actual resistor values and given transistor beta values, so as an approximate approach it was quite reliable.

In summary, therefore, the DC equivalent of a network is obtained by:

1. Setting all AC sources to zero and replacing them by a open-circuit equivalent
2. Replacing all capacitors by a open-circuit equivalent
3. Redrawing the network in a more convenient and logical form



Circuit for dc analysis

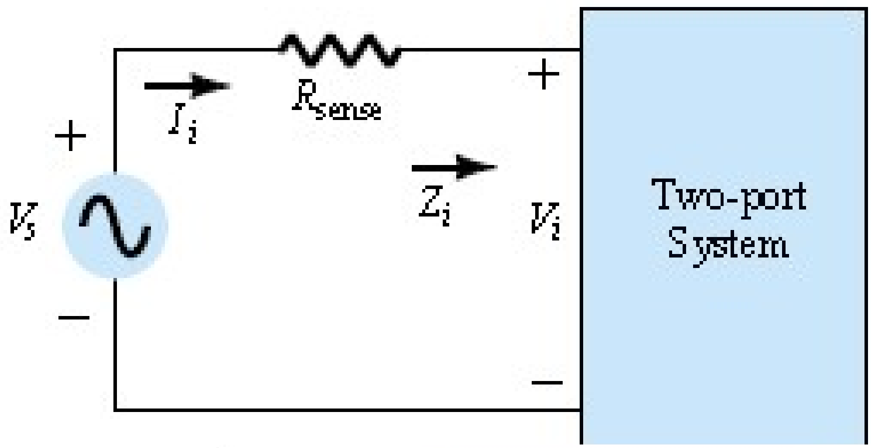


Circuit for small-signal ac analysis

In summary, therefore, the ac equivalent of a network is obtained by:

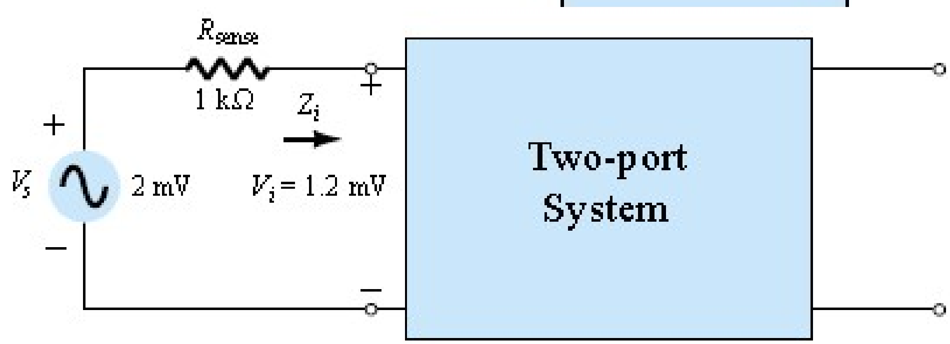
1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
2. Replacing all capacitors by a short-circuit equivalent
3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
4. Redrawing the network in a more convenient and logical form

Important Parameters: Input Impedance (Z_i)



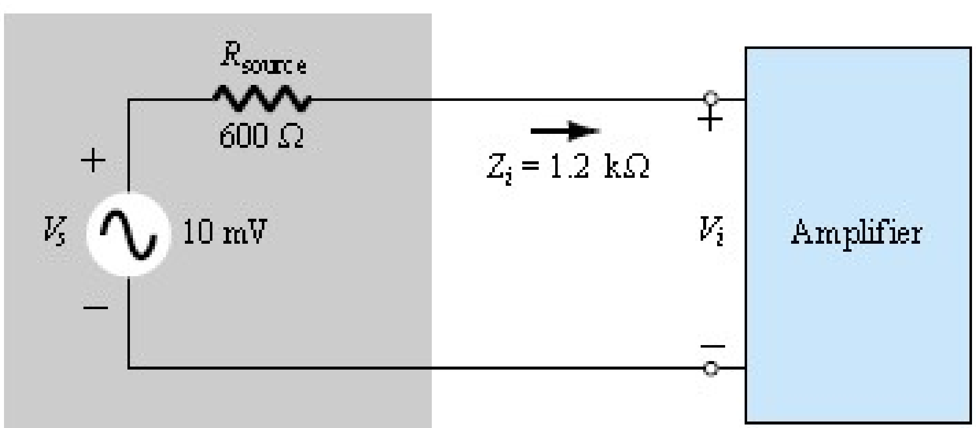
$$I_i = \frac{V_s - V_i}{R_{sense}}$$

$$Z_i = \frac{V_i}{I_i}$$



$$I_i = \frac{V_s - V_i}{R_{sense}} = 0.8\ \mu A$$

$$Z_i = \frac{V_i}{I_i} = 1.5\ k\Omega$$

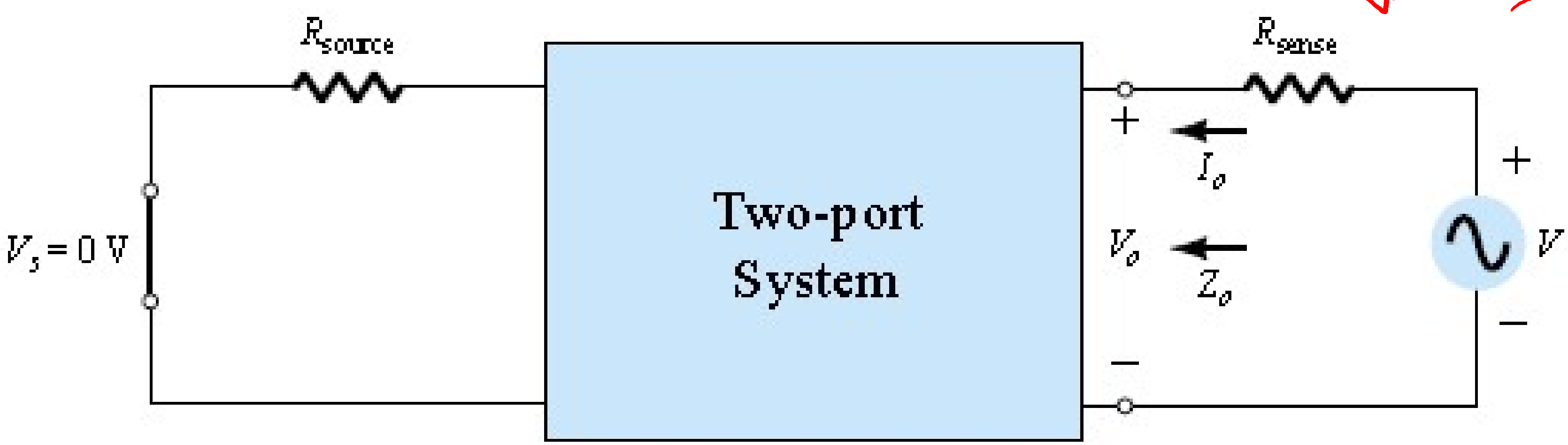


$$V_i = \frac{Z_i V_s}{Z_i + R_{source}} = 6.67\ mV$$

Output Impedance (Z_o)

The output impedance is determined at the output terminals looking back into the system with the applied signal set to zero.

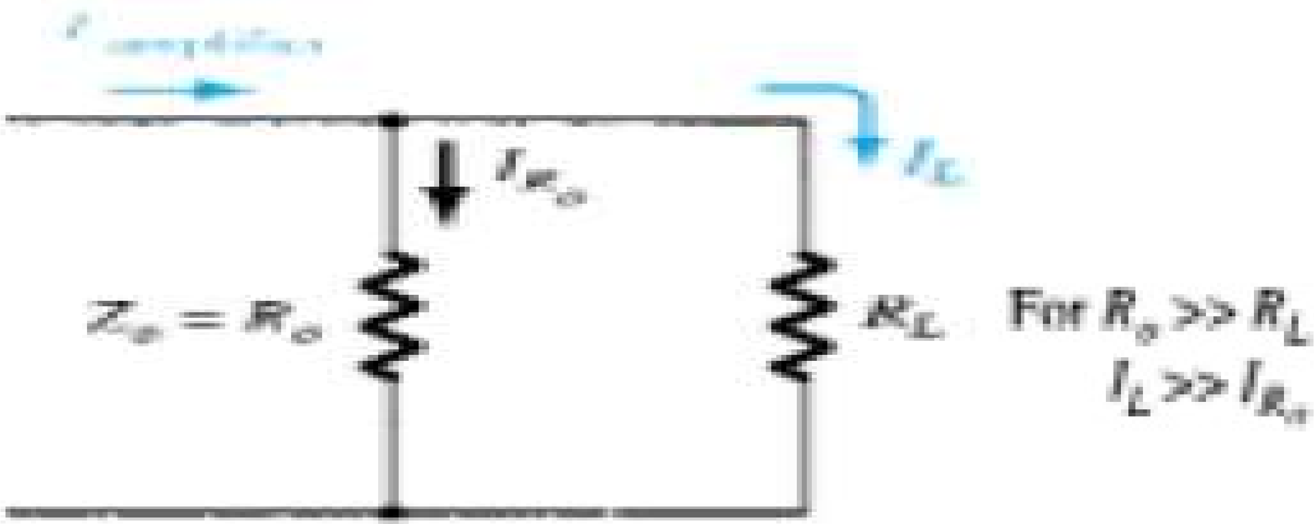
$V - V_o - I_o R_s = 0$

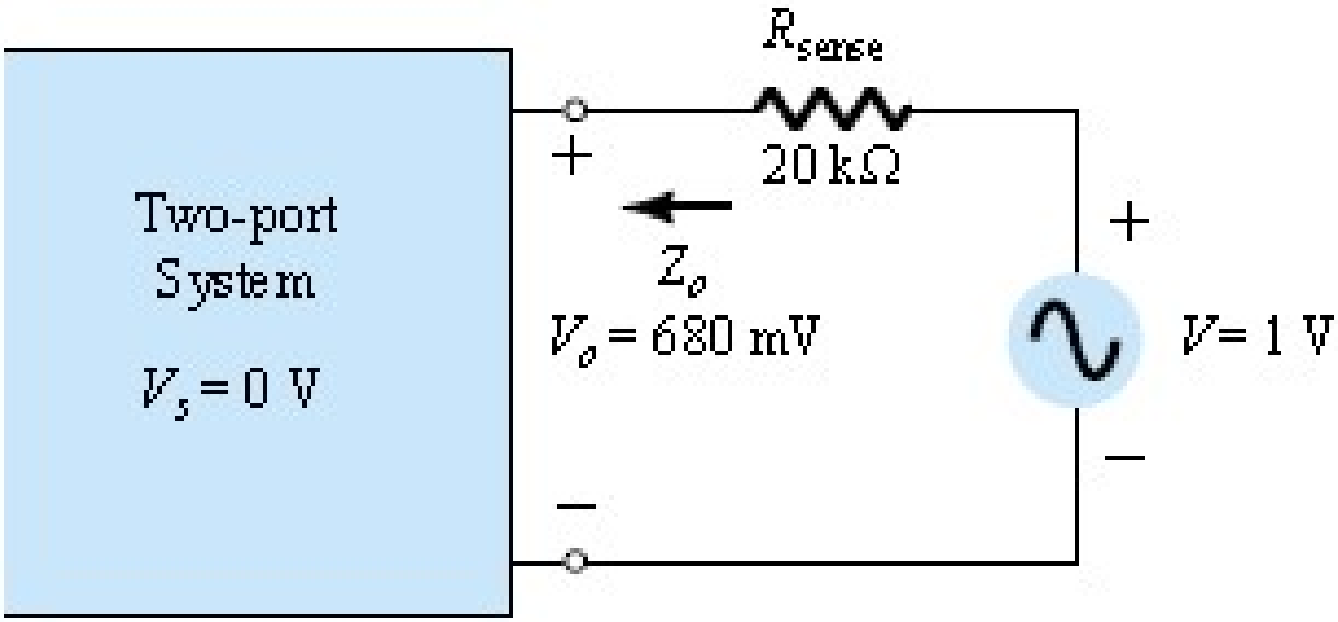


$$I_o = \frac{V - V_o}{R_{sense}}$$

$$Z_o = \frac{V_o}{I_o}$$

- In particular, for frequencies in the low to mid-range (typically 100 kHz):
  - The output impedance of a BJT transistor amplifier is resistive in nature and, depending on the configuration and the placement of the resistive elements,
  - Z_o can vary from a few ohms to a level that can exceed 2 MΩ.
- In addition:
  - An ohmmeter cannot be used to measure the small-signal ac output impedance. since the ohmmeter operates in the dc mode.



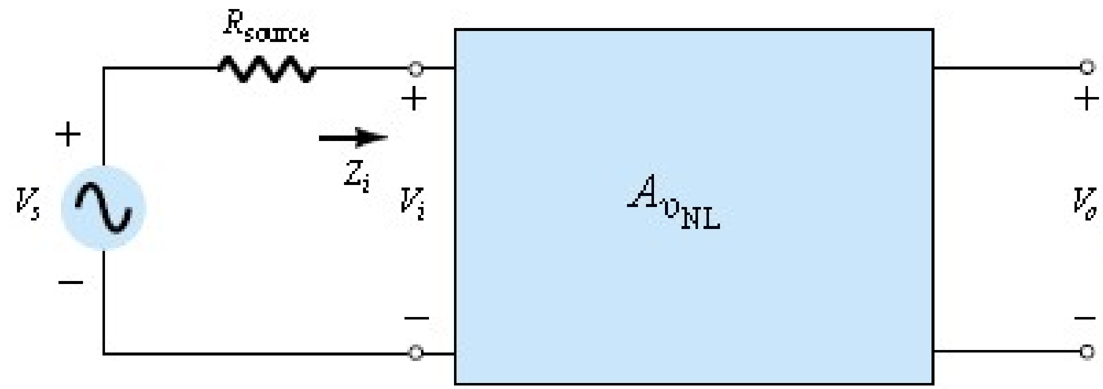


$$I_o = \frac{V - V_o}{R_{sense}} = \frac{1\ \text{V} - 680\ \text{mV}}{20\ \text{k}\Omega} = 16\ \mu\text{A}$$

$$Z_o = \frac{V_o}{I_o} = \frac{680\ \text{mV}}{16\ \mu\text{A}} = \mathbf{42.5\ k\Omega}$$

Voltage Gain (A_V)

$$A_V = \frac{V_o}{V_i}$$



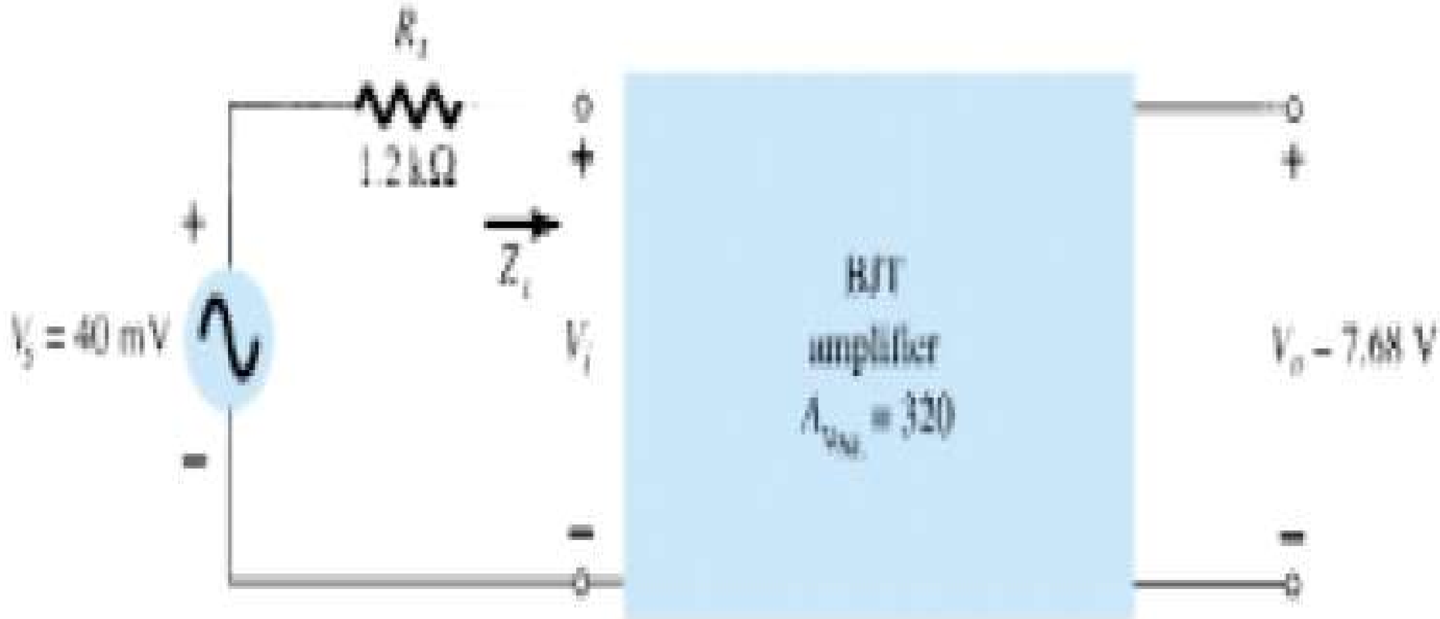
$$A_{v_{NL}} = \left.\frac{V_o}{V_i}\right|_{R_L = \infty\ \Omega \text{ (open circuit)}}$$

Determining the no-load voltage gain.

For transistor amplifiers, the no-load voltage gain is greater than the loaded voltage gain

$$V_i = \frac{Z_i V_s}{Z_i + R_s} \qquad \frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s}$$

$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_i}{V_s} \cdot \frac{V_o}{V_i} = \frac{Z_i}{Z_i + R_s} A_{v_{NL}}$$



(a) V_i.
(b) I_i.
(c) Z_i.
(d) A_{v_s}.

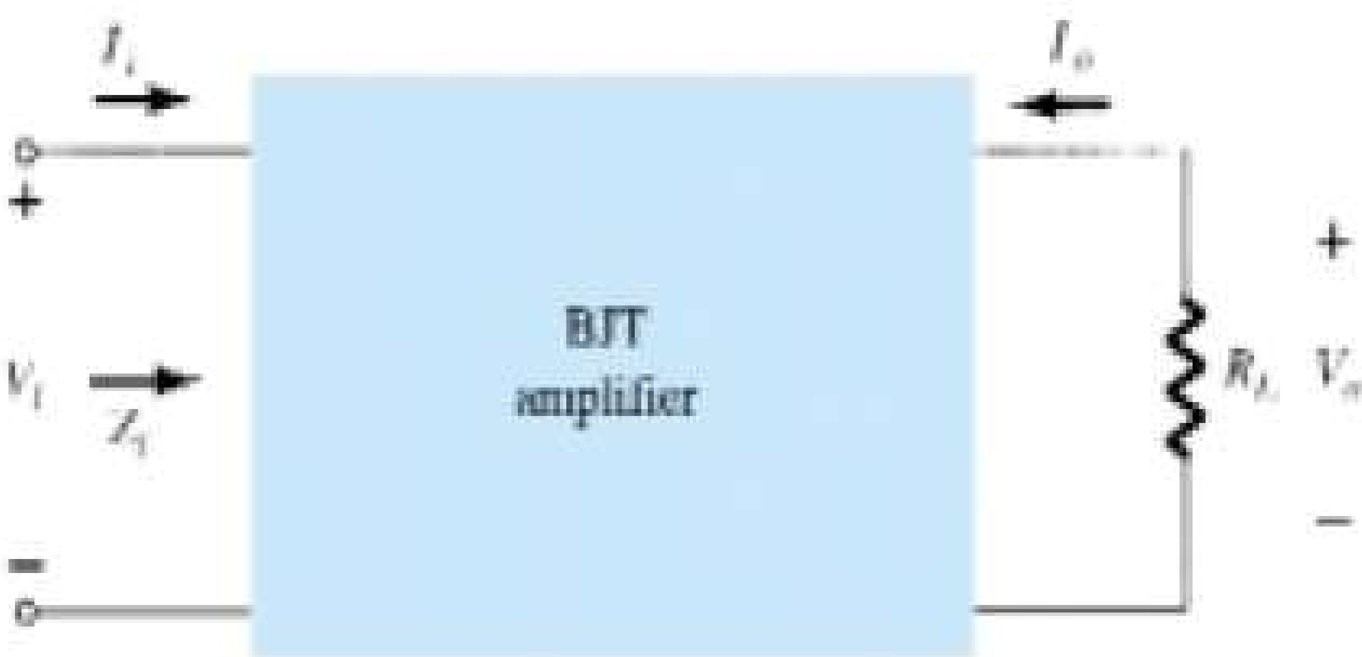
$$A_{v_{NL}} = \frac{V_o}{V_i} \qquad V_i = \frac{V_o}{A_{v_{NL}}} = \frac{7.68\text{ V}}{320} = \mathbf{24\text{ mV}}$$

$$I_i = \frac{V_s - V_i}{R_s} = \mathbf{13.33\ \mu A}$$

$$Z_i = \frac{V_i}{I_i} = \mathbf{1.8\text{ k}\Omega} \qquad A_{v_s} = \frac{Z_i}{Z_i + R_s} A_{v_{NL}} = \frac{1.8\text{ k}\Omega}{1.8\text{ k}\Omega + 1.2\text{ k}\Omega}(320) = \mathbf{192}$$

Current Gain (A_i)

$$A_i = \frac{I_o}{I_i}$$ For BJT amplifiers, $1 < A_i < 100$ [may exceed 100]

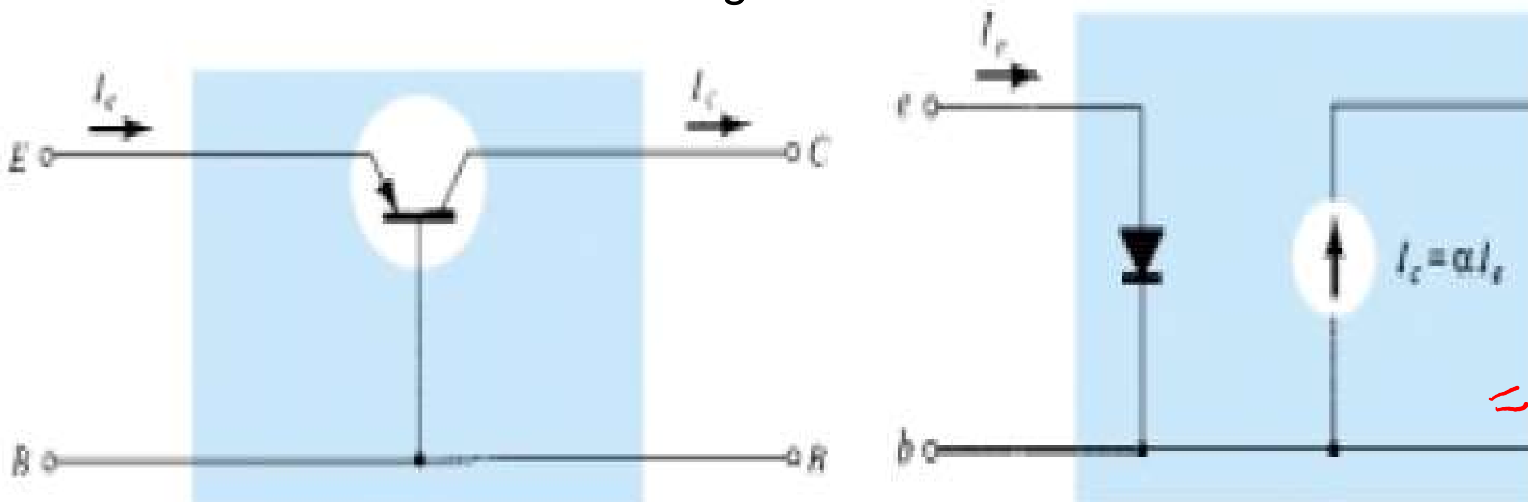


For the loaded situation, $I_i = \frac{V_i}{Z_i}$ $\quad I_o = -\frac{V_o}{R_L}$

$$\Rightarrow A_i = \frac{I_o}{I_i} = -\frac{V_o / R_L}{V_i / Z_i} = -A_V \frac{Z_i}{R_L}$$

r_e Transistor Model: CB Configuration

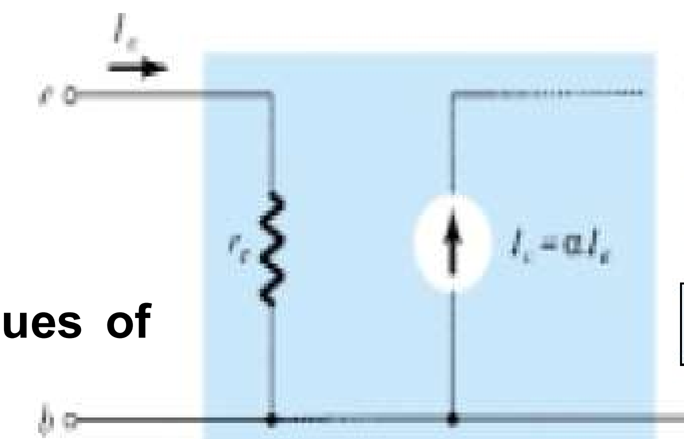
- BJT transistor amplifiers are referred to as current-controlled devices.
- The r_e model employs a diode and controlled current source to duplicate the behavior of a transistor in the region of interest.



$V_o = -I_o R_L$
$= -(-I_c) R_L$

(a) Common-base BJT transistor; (b) r_e model for the configuration

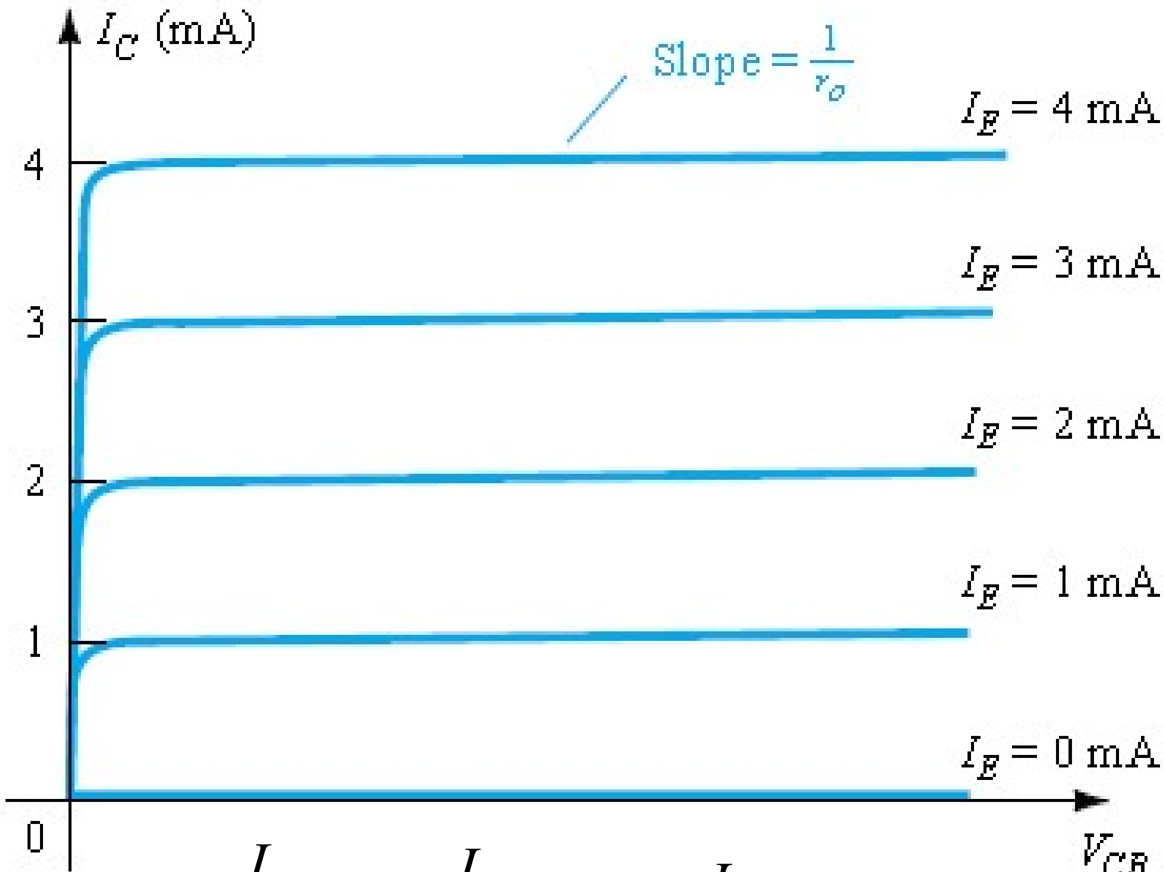
Common-base r_e equivalent circuit



$Z_i \cong r_e$ CB

$Z_o \cong \infty\ \Omega$ CB

For CB configuration, typical values of Zo are in the mega-ohm range.



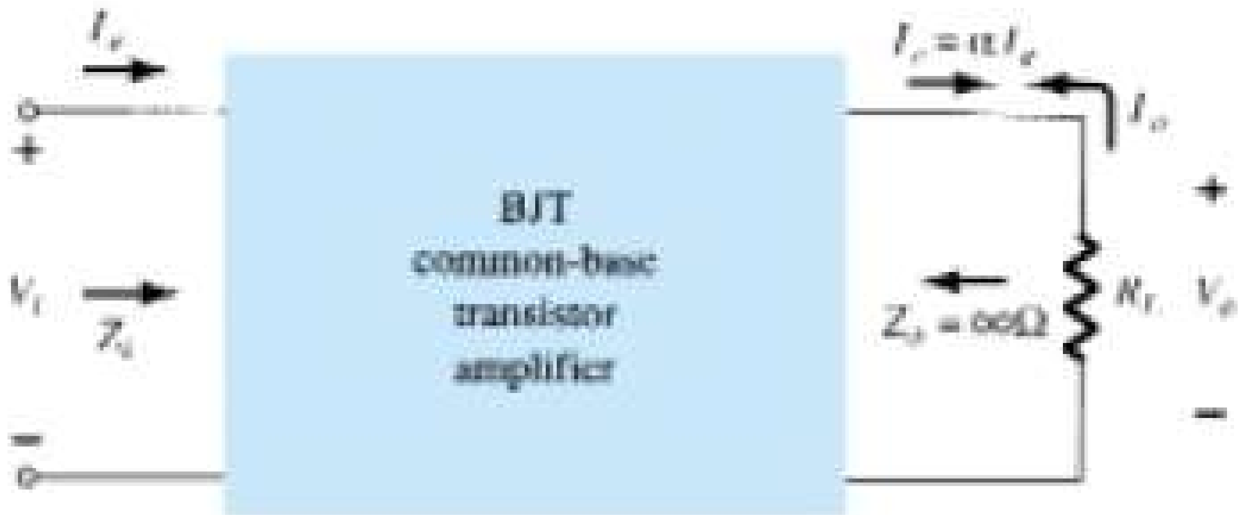
$$V_o = -I_o R_L$$
$$= -(-I_C)R_L$$
$$= \alpha I_e R_L$$

$$V_i = I_e Z_i = I_e r_e$$

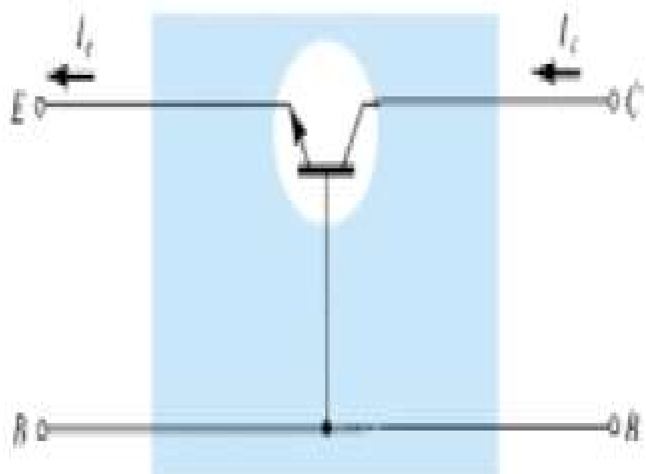
$$(A_V)_{CB} = \frac{V_o}{V_i} = \frac{\alpha I_e R_L}{I_e r_e} \cong \frac{R_L}{r_e}$$

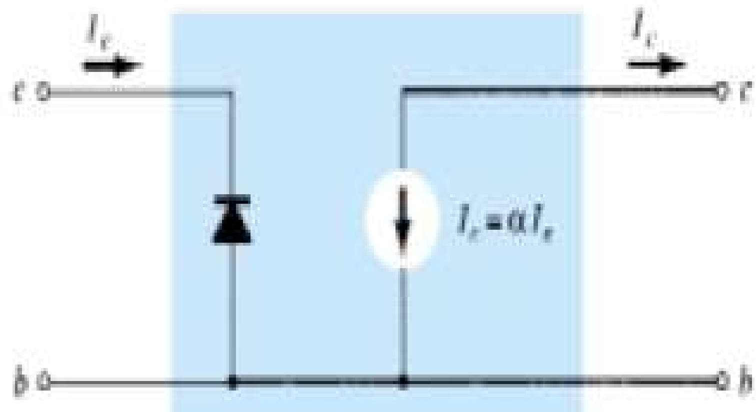
$$A_i = \frac{I_o}{I_i} = -\frac{I_C}{I_e} = -\frac{\alpha I_e}{I_e} \cong -\alpha \cong -1$$

In general, for the CB configuration the input impedance is relatively small and the output impedance quite high.



BJT Transistor Modeling

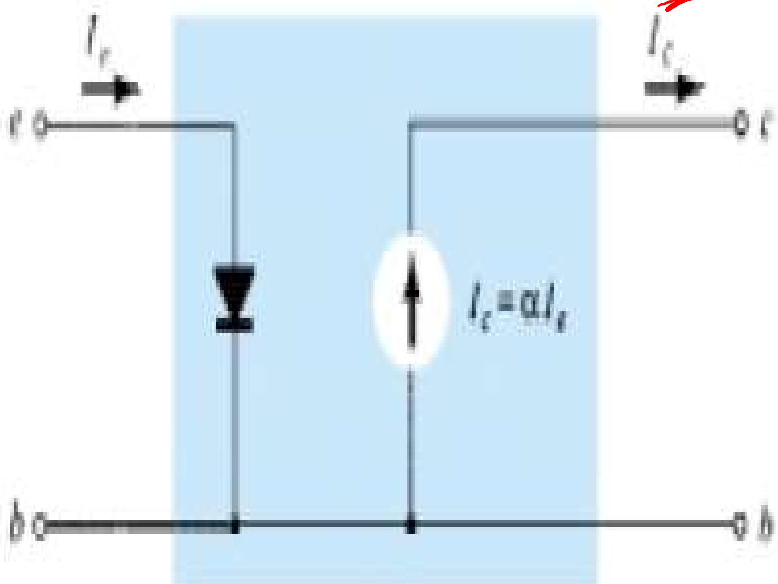




Approximate model for a common-base ***npn transistor*** *configuration*

Question

$A_v \simeq \frac{R_L}{r_e} = \frac{0.56k\Omega}{6.5\Omega} = 86$



For a common-base configuration with I_E = *4 mA*, α = *0.98, and an ac* signal of 2 mV applied between the base and emitter terminals: (a) Determine the input impedance. (b) Calculate the voltage gain if a load of 0.56 kΩ is connected to the output terminals. (c) Find the output impedance and current gain.

Solution

The ac resistance of a diode can be determined by the equation r_{ac} = 26 mV/I_D, where, I_D is the dc current through the diode at the Q (quiescent) point.

$$r_e = \frac{26\,mV}{I_D} = \frac{26\,mV}{4\,mA}$$

$$I_i = I_e = \frac{V_i}{Z_i} = \frac{2\,mV}{6.5\,k\Omega}$$

$$V_o = I_C R_L = \alpha I_e R_L$$

$$A_v = \frac{V_o}{V_i} = \frac{168.86\text{ mV}}{2\text{ mV}} = \mathbf{84.43}$$

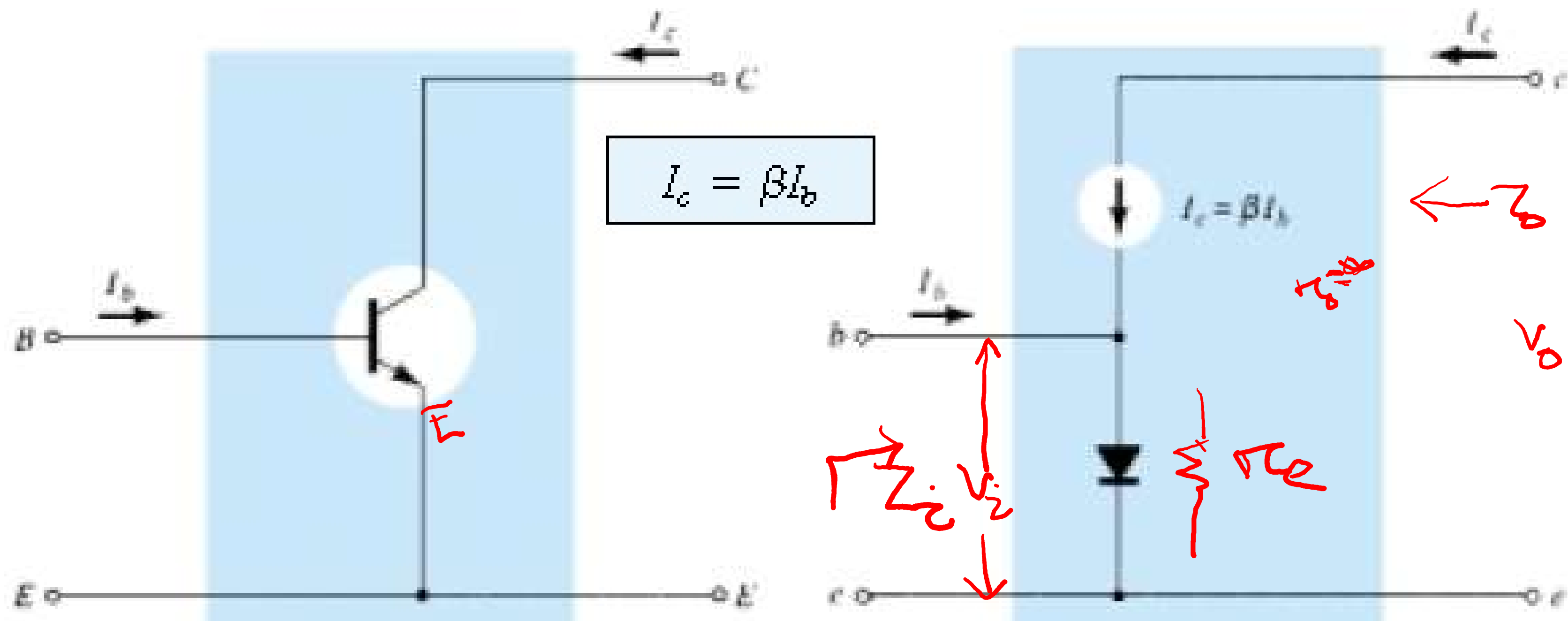
$$A_v = \frac{\alpha R_L}{r_e} = \frac{(0.98)(0.56\ \text{k}\Omega)}{6.5\ \Omega} = \mathbf{84.43}$$

(c) $Z_o \cong \infty\ \Omega$

$$A_i = \frac{I_o}{I_i} = -\alpha = \mathbf{-0.98}$$

r_e Transistor Model: CE Configuration

- BJT transistor amplifiers are referred to as current-controlled devices.



(a) Common-emitter BJT transistor; (b) approximate model for the configuration

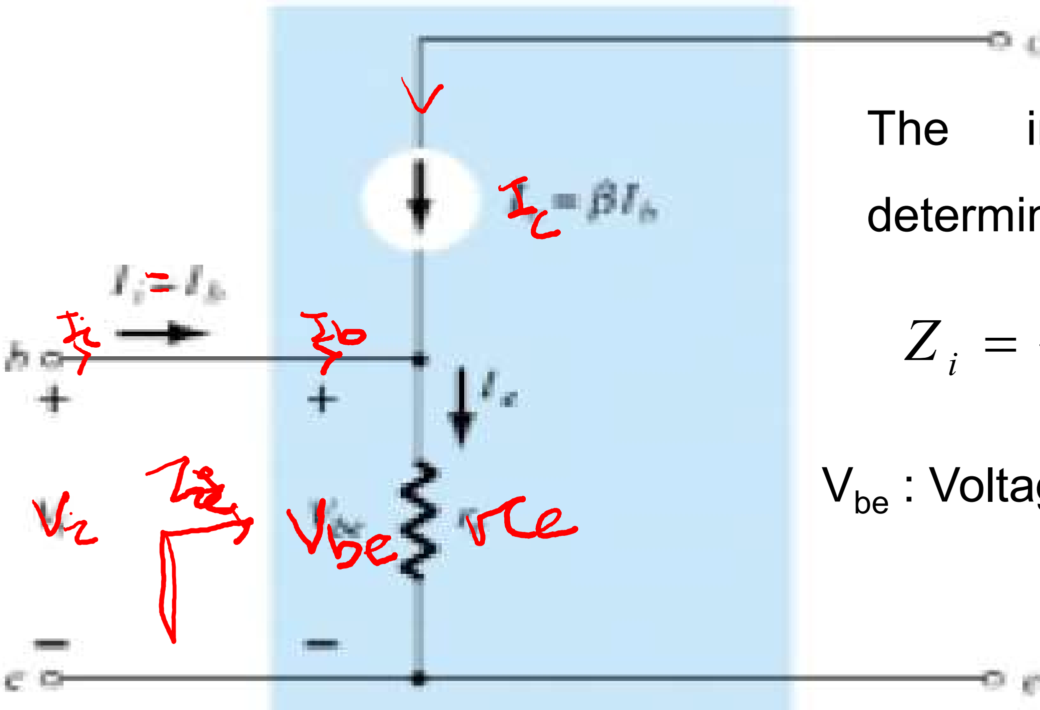
The current through the diode is determined by

$$I_e = I_c + I_b = \beta I_b + I_b$$

$$\boxed{I_e = (\beta + 1)I_b} \qquad \beta_{ac} >> 1 \qquad \boxed{I_e \cong \beta I_b}$$

Input Impedance (Z_i)

$I_e = (\beta+1) I_b$
$\cong \beta I_b$



The input impedance (Z_i) is determined by the following ratio:

$$Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}$$

V_{be} : Voltage is across the diode resistance

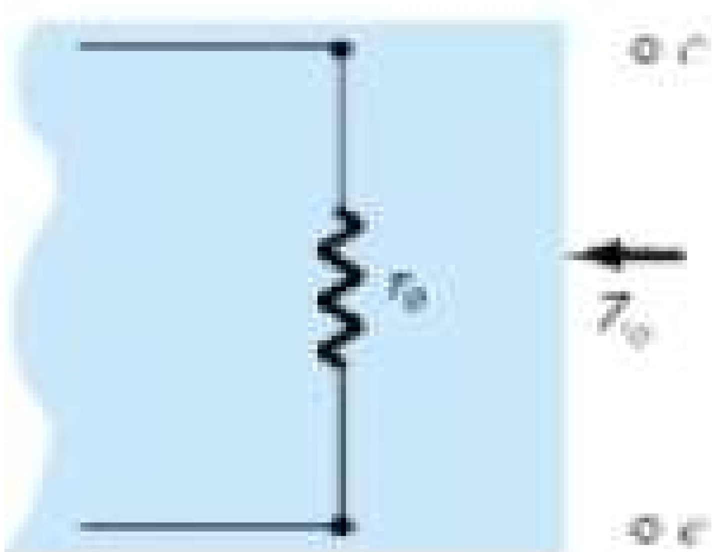
The level of r_e is still determined by the dc current I_E. Using Ohm's law gives

$$V_i = V_{be} = I_e r_e \cong \beta I_b r_e$$

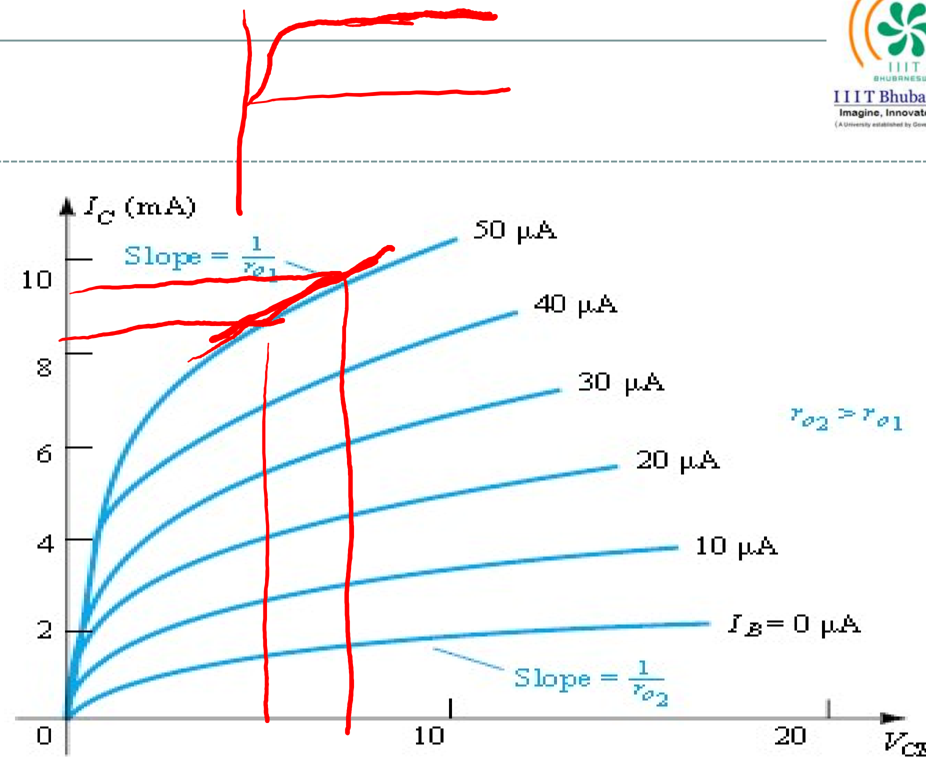
$$Z_i = \frac{V_{be}}{I_b} \cong \frac{\beta I_b r_e}{I_b} \cong \beta r_e$$

Few hundred ohms to the kilo-ohm range, with maximums of about 6–7 kΩ.

Output Impedance (Z_o)



r_o in the transistor equivalent circuit



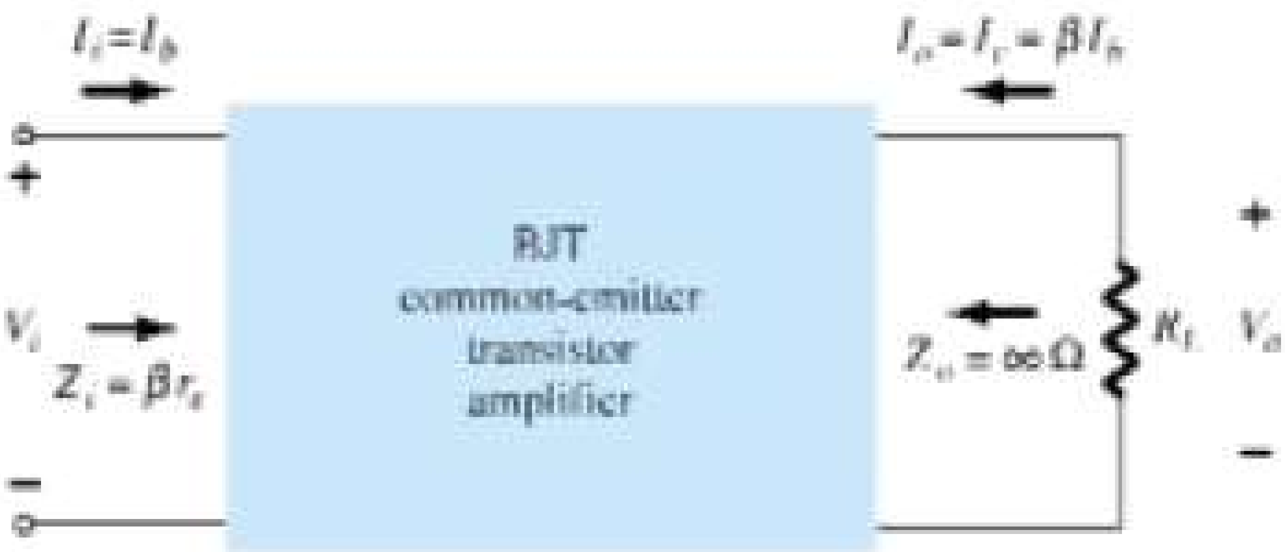
Defining r_o for the CE configuration

For the CE configuration, typical values of Zo are in the range of 40 to 50 kΩ.

if the applied signal is set to zero, the current I_c is 0 A and the output impedance is

$$\boxed{Z_o = r_o}_{CE}$$

if the contribution due to r_o is ignored as in the re model, the output impedance is defined by $Z_o = \infty$



$$V_o = -I_o R_L$$
$$= -I_c R_L$$
$$\Rightarrow V_o = -\beta I_b R_L$$
$$V_i = I_i Z_i$$
$$= \beta I_b r_e$$

Determining Voltage and current gain for the CE amplifier

$$A_V = \frac{V_o}{V_i} = \frac{\beta I_b R_L}{\beta I_b r_e}$$

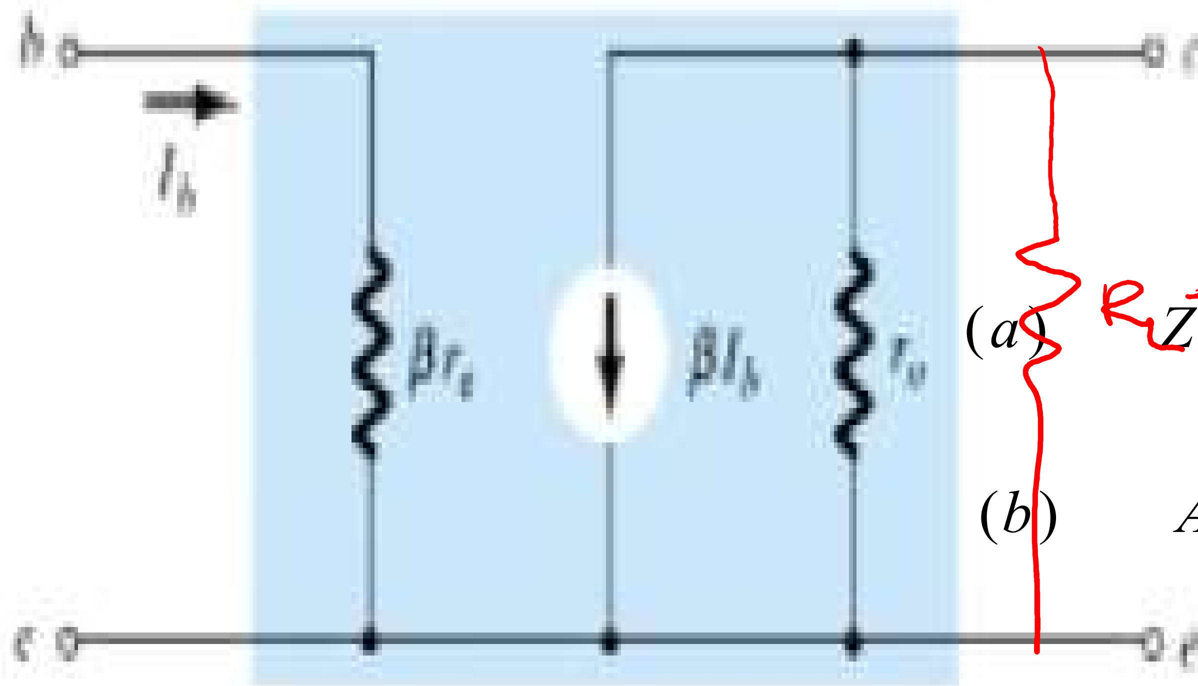
$$\boxed{A_v = -\frac{R_L}{r_e}} \quad CE, r_o = \infty\,\Omega$$

The resulting minus sign for the voltage gain reveals that the output and input voltages are 180° out of phase.

$$A_i = \frac{I_o}{I_i} = \frac{I_c}{I_b} = \frac{\beta I_b}{I_b}$$

$$\boxed{A_i = \beta} \quad CE, r_o = \infty\ \Omega$$

r_e model for the common-emitter transistor configuration.



$$r_e = \frac{26\ \text{mV}}{I_E}$$

$$= \frac{26\ \text{mV}}{3.2\ \text{mA}} = 8.125\ \Omega$$

$R_L = 2k\Omega$

$$(a)\quad Z_i = \beta r_e = (120)(8.125)$$
$$= 975\ \Omega$$

$$(b)\quad A_V = -\frac{R_L}{r_e} = -246.15$$

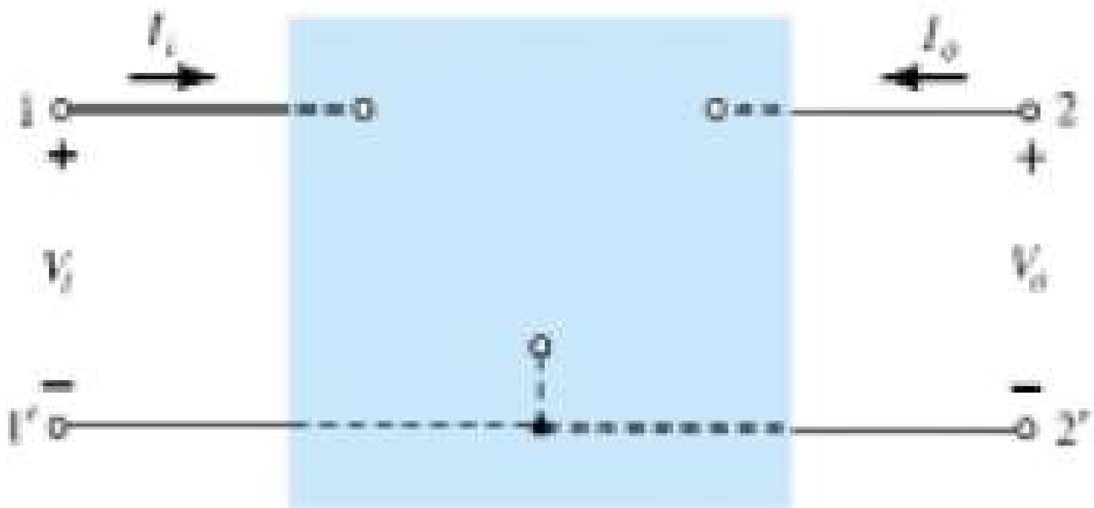
$$(c)\quad A_i = \beta = 120$$

Given β = 120 and I_E = 3.2 mA for a common-emitter configuration with r_o, determine: (a) Zi.

(b) Av if a load of 2 kΩ is applied.

(c) Ai with the 2 kΩ load

The Hybrid Equivalent Model



$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

Hybrid equivalent model: Two-port system

- The term **hybrid** is chosen because the mixture of <u>four variables</u> (V and I) in each equation results in a “hybrid” set of units of measurement for the h-parameters.
- Determine their magnitude can be developed by isolating each and examining the resulting relationship

Short-circuit input-impedance (h_{11})

$$V_i = h_{11} I_i + h_{12} V_o$$

If $V_o = 0$ (short circuit the output terminals) and solve for h_{11}

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o = 0}$$

It is the ratio of the input voltage to the input current with the output terminals shorted, it is called the short-circuit input-impedance parameter.

Open-circuit reverse transfer voltage ratio (h_{12})

$$V_i = h_{11} I_i + h_{12} V_o$$

If I_i is set equal to zero by opening the input leads, then h_{12}

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i = 0}$$

- h_{12} is the ratio of the input voltage to the output voltage with the input current equal to zero.
- It has no units and is called the open-circuit reverse transfer voltage ratio parameter.

Short-circuit forward transfer current ratio (h_{21})

$$I_o = h_{21} I_i + h_{22} V_o$$

If $V_o = 0$ (short circuit the output terminals) and solve for h_{21},

$$h_{21} = \left. \frac{I_o}{I_i} \right|_{V_o=0}$$

It is the ratio of the output current to the input current with the output terminals shorted, it is called the short-circuit forward transfer ratio parameter.

Open-circuit output admittance parameter (h_{22})

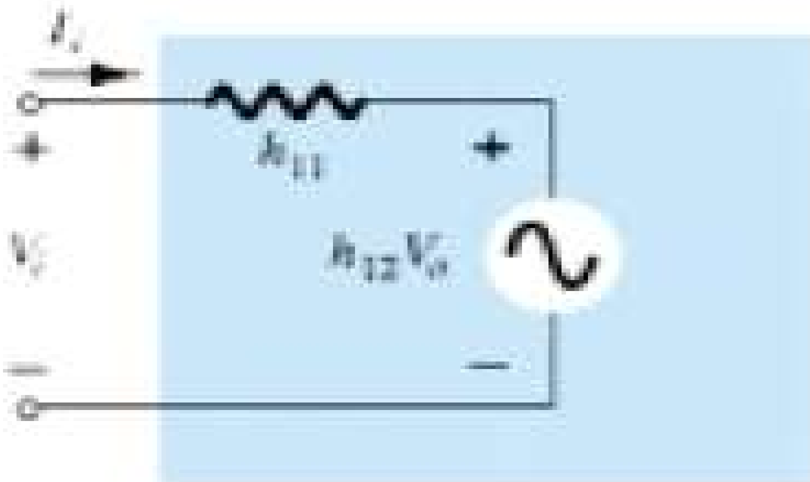
$$I_o = h_{21} I_i + h_{22} V_o$$

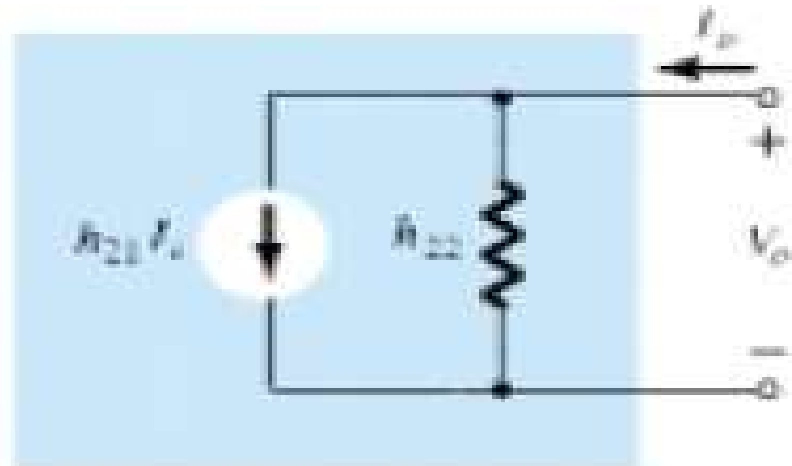
If I_i is set equal to zero by opening the input leads, then h_{22}:

$$h_{22} = \left. \frac{I_o}{V_o} \right|_{I_i=0}$$

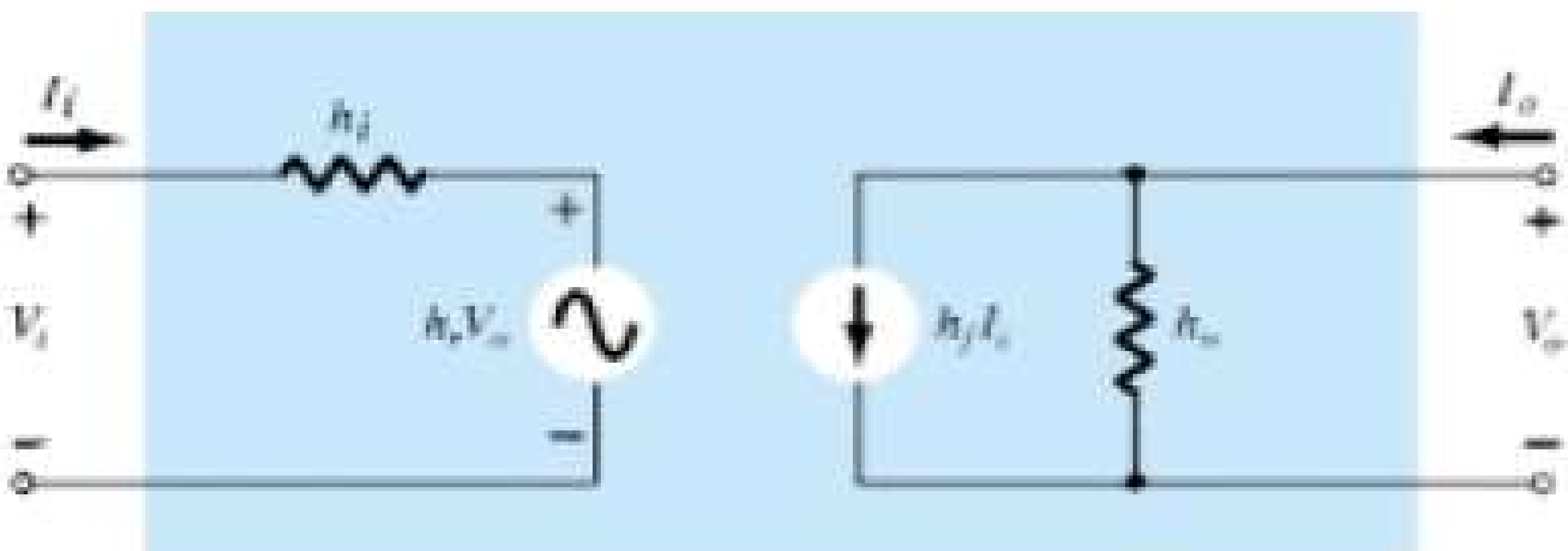
- h_{22} is the ratio of the input current to the output voltage with the input current equal to zero.
- It has no units and is called the open-circuit output admittance parameter.

“ac” equivalent circuit : The Hybrid Equivalent Model





Hybrid input equivalent circuit. Hybrid output equivalent circuit.



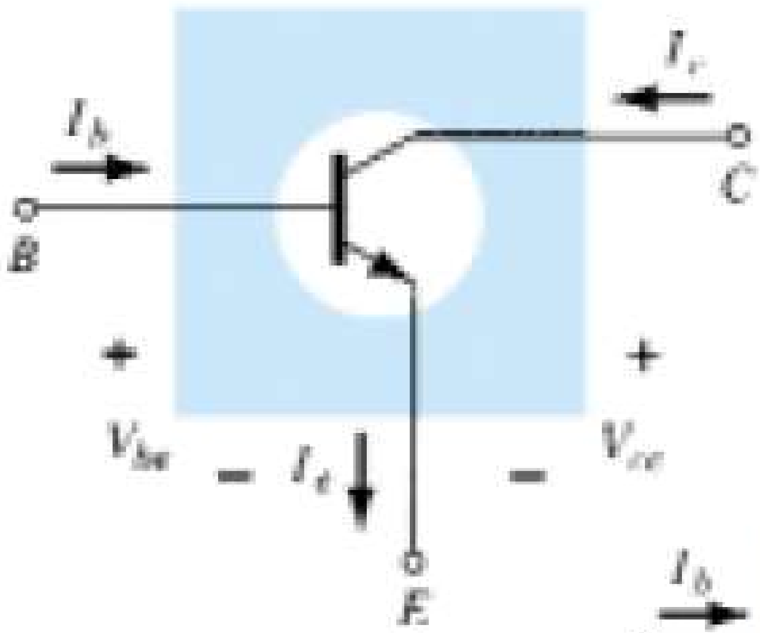
$h_{11} \rightarrow input\ resistance \rightarrow h_i$

$h_{12} \rightarrow reverse\ transfer\ voltage\ ratio \rightarrow h_r$

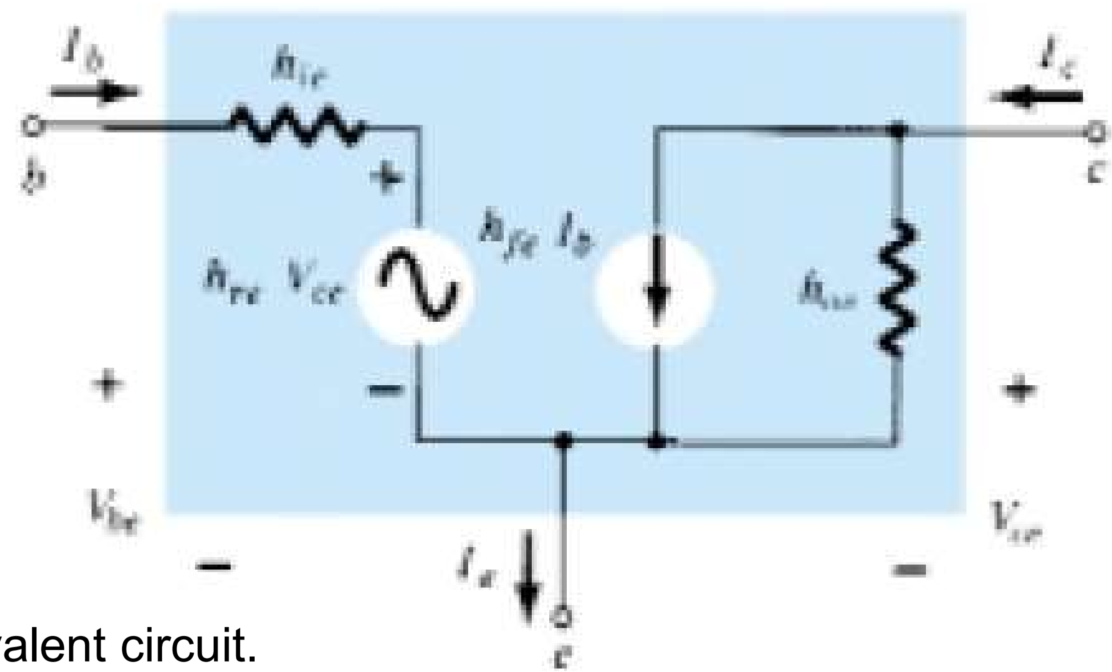
$h_{21} \rightarrow forward\ transfer\ current\ ratio \rightarrow h_f$

$h_{22} \rightarrow output\ conductance \rightarrow h_o$

The Hybrid Equivalent Model: CE Configuration

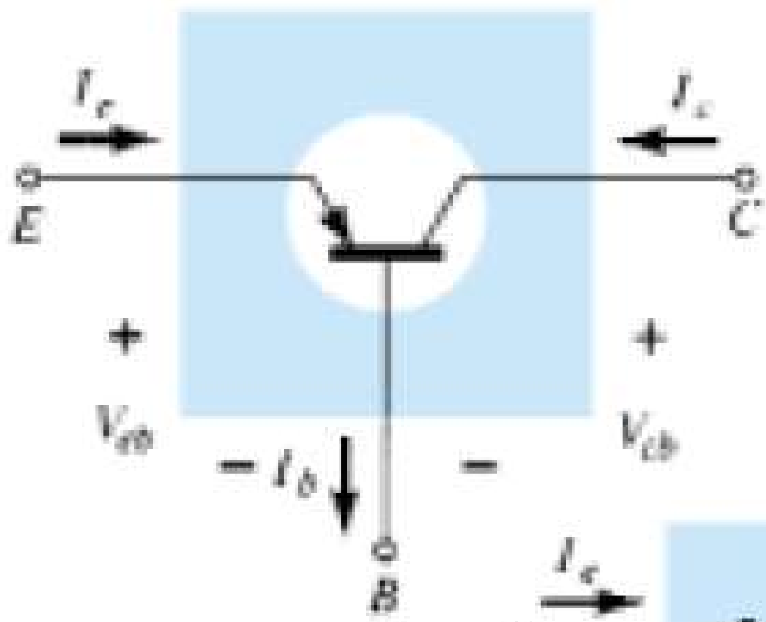


(a) graphical symbol

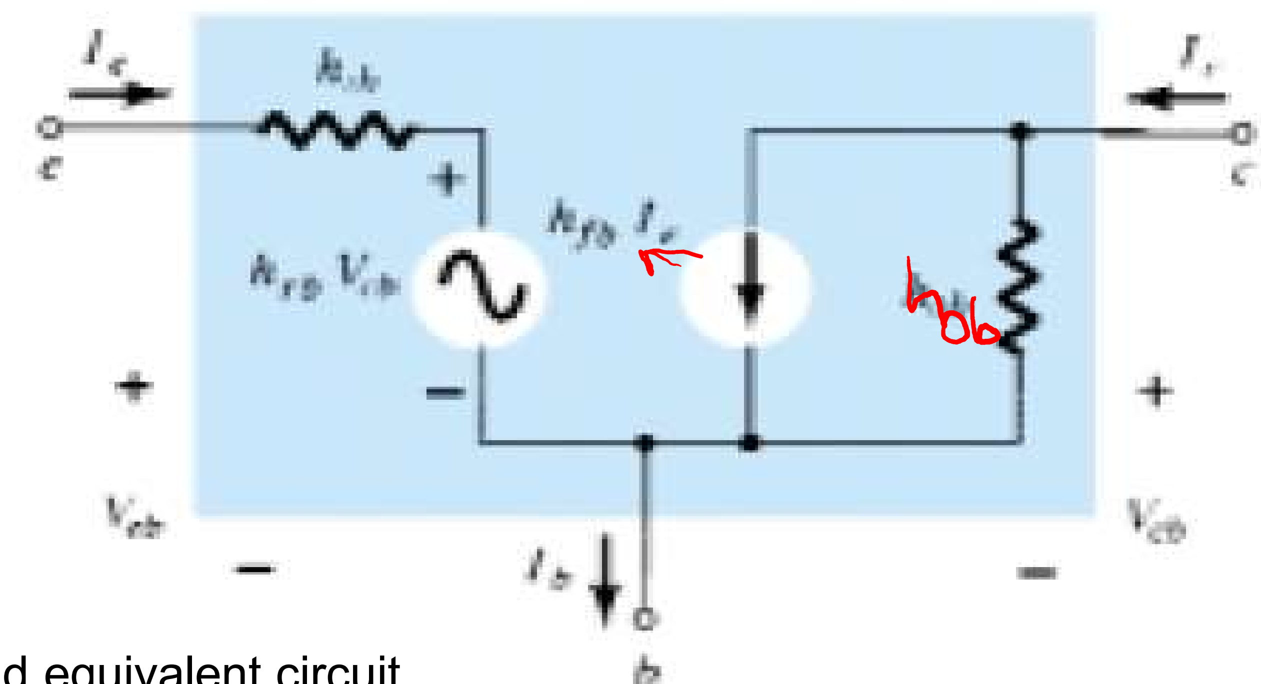


(b) Hybrid equivalent circuit.

The Hybrid Equivalent Model: CB Configuration

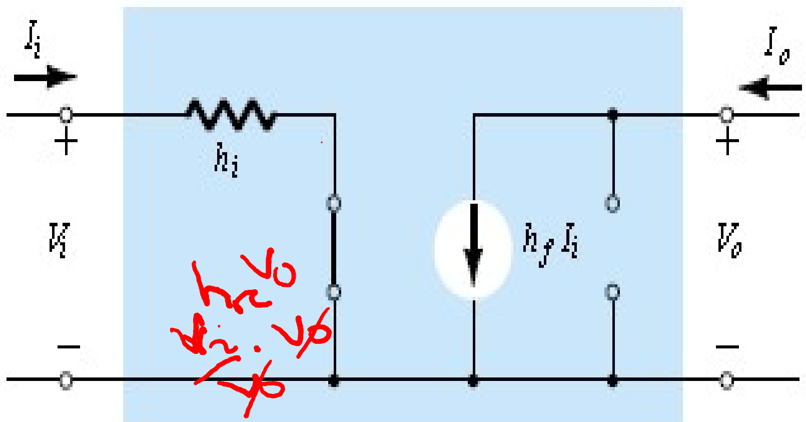


(a) graphical symbol

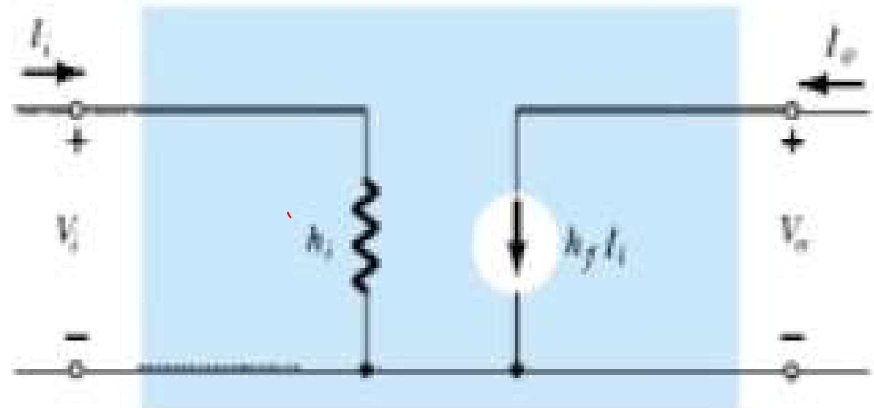


(b) Hybrid equivalent circuit.

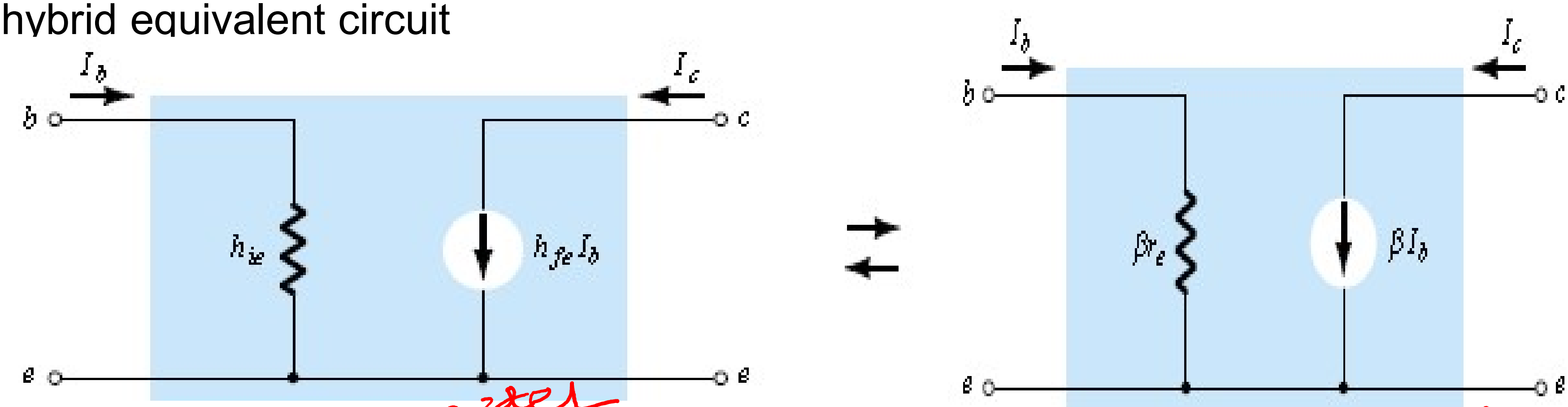
The Hybrid Vs r_e Model



Effect of removing h_{re} and h_{oe} from the hybrid equivalent circuit



Approximate hybrid equi. model



h-model Common-emitter configuration re model

$$\boxed{h_{ie} = \beta r_e}$$

$$\boxed{h_{fe} = \beta_{ac}}$$

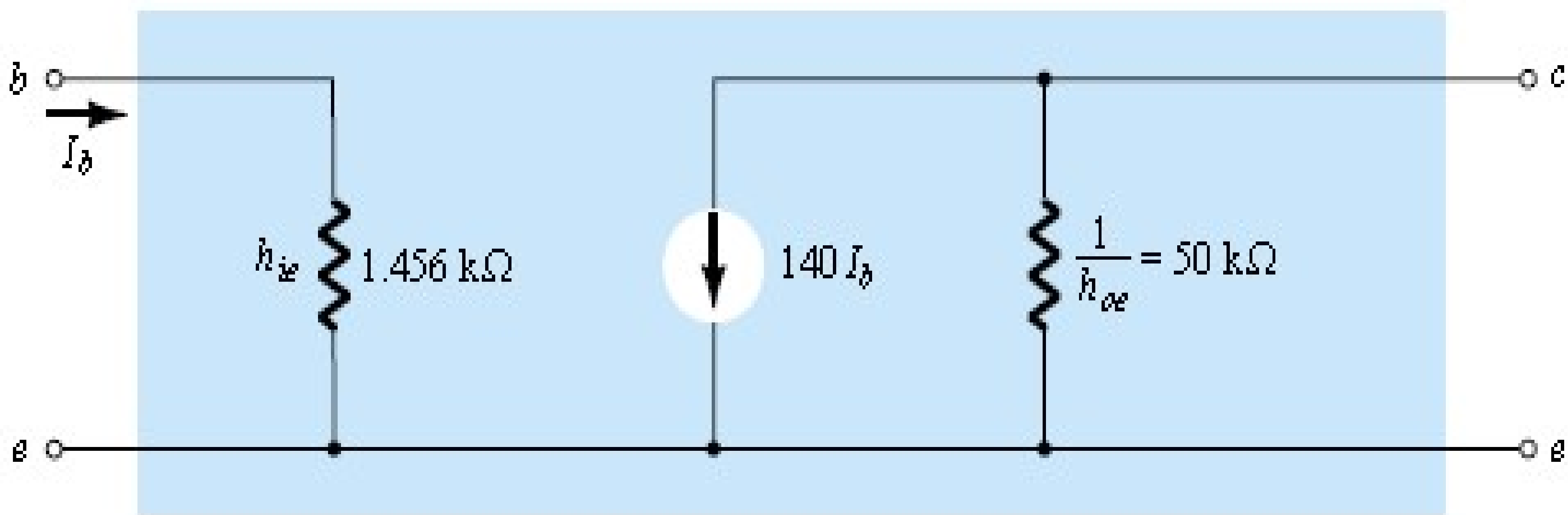
Question:

Given I_E = 2.5 mA, h_{fe} = 140, h_{oe} = 20 μS (μmho), and h_{ob} = 0.5 μS, determine: (a) The common-emitter hybrid equivalent circuit. (b) The common-base r_e model.

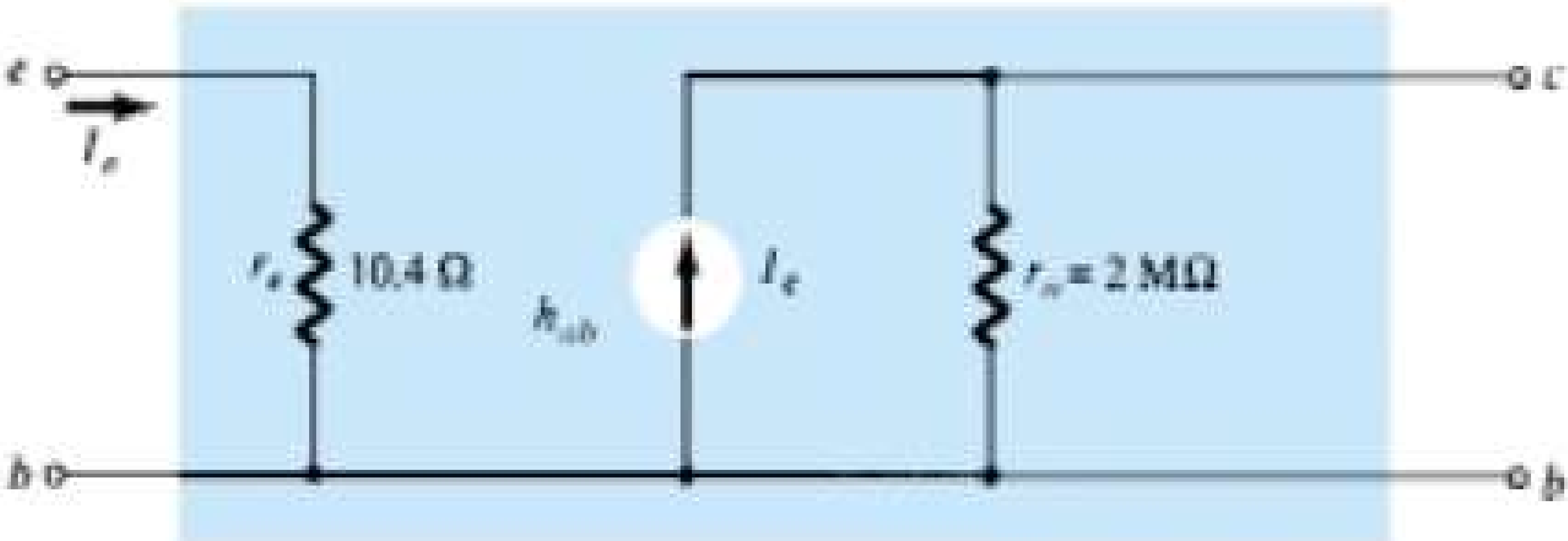
Solution:

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.5\,mA} = 10.4\Omega \qquad h_{ie} = \beta r_e = 140\,(10.4) = 1.456$$

$$r_o = \frac{1}{h_{0e}} = \frac{1}{20\,\mu S} = 50\,k\Omega$$



Common-emitter hybrid equivalent circuit for the parameters



Common-base r_e model for the parameters

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.5\,mA} = 10.4\Omega$$

$$\alpha \cong 1$$

$$r_o = \frac{1}{h_{ob}} = \frac{1}{0.5\,\mu S} = 2\,M\Omega$$

Typical Parameter Values for the CE, CC and CB Transistor Configurations

Parameter	CE	CC	CB
h_i	1 kΩ	1 kΩ	20 Ω
h_r	2.5×10^{-4}	$\cong 1$	3.0×10^{-4}
h_f	50	−50	−0.98
h_o	25 μA/V	25 μA/V	0.5 μA/V
$1/h_o$	40 kΩ	40 kΩ	2 MΩ

Graphical Determination of the h-Parameters

Using partial derivatives (calculus), it can be shown that the magnitude of the *h-parameters* for the small-signal transistor equivalent circuit in the region of operation for the CE configuration can be found using the following equations

$$h_{ie} = \frac{\delta v_i}{\delta i_i} = \frac{\delta v_{be}}{\delta i_b} \cong \left.\frac{\Delta v_{be}}{\Delta i_b}\right|_{V_{CE} = Cons\tan t} \quad (ohms)$$

$$h_{re} = \frac{\delta v_i}{\delta v_o} = \frac{\delta v_{be}}{\delta v_{ce}} \cong \left.\frac{\Delta v_{be}}{\Delta i_{ce}}\right|_{I_B = Cons\tan t} \quad (unitless)$$

$$h_{fe} = \frac{\delta i_o}{\delta i_i} = \frac{\delta i_c}{\delta i_b} \cong \left.\frac{\Delta i_c}{\Delta i_b}\right|_{V_{CE} = Cons\tan t} \quad (unitless)$$

$$h_{oe} = \frac{\delta i_o}{\delta v_o} = \frac{\delta i_c}{\delta v_{ce}} \cong \left.\frac{\Delta i_c}{\Delta v_{ce}}\right|_{I_B = Cons\tan t} \quad (ohms)$$