

Design and Control of a Four-Link Mechanism for High Speed and Dynamic Locomotion^{*}

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Abstract. With limited motor power, a good balance must be made between high speed and high torque to keep a robot run as fast as possible after meeting the torque demand for each joint. A quadruped robot with a novel leg mechanism is designed for high speed and dynamic locomotion. Two design principles are proposed in the design of a leg with low moment of inertia. A four-link transmission unit is proposed to change the motor's fixed-direction rotation into the leg's reciprocating swing. Optimization is carried out to acquire the biggest driving force. Kinematic analysis based on screw theory is made to help find out kinematic characteristics such as operating space and joint angular velocity.

Keywords: dynamic locomotion, low moment of inertia, optimization, screw theory.

1 Introduction

Research in the field of quadruped locomotion started in the fourth century when a four-legged wooden device was built [1]. Developed in the 1960s, walking robots have an advantage over wheeled robots in walking on uneven terrain [2]. Walking robot is divided into monopod robot, biped robot, quadruped robot and multi-legged robot [3].

Two of early representatives are the G.E. Quadruped and the Phoney Poney. Due to the state of development of control systems, they were both controlled with rather simple mechanisms [1]. In 1999 Martin Buehler presented Scout II, a dynamically stable running quadruped robot with a very simple mechanical design [4]. The KOLT robot was developed using electric actuation with mechanical springs for added compliance [5]. The Titan series are walking machines with reptile-like legs [6].

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With good environment adaptability and anti-jamming capability, Bigdog robot is acknowledged as the most robust as well as versatile robot ever since and has a good prospect of applications in the military field [7, 8]. The cheetah robot developed by the Boston Dynamics has reached 18mph in a video posted on its website and becomes the fastest robot ever since. Using made-to-order motors, the MIT cheetah has a good performance in both stability and bio-mimics [9].

This paper introduces the design of a quadruped robot possessing a novel leg mechanism with low moment of inertia. Section 2 presents detailed design of the leg mechanism. In Section 3 the kinematics analysis is done with screw theory. Section 4 describes the set-up of the control system. Conclusions are drawn in Section 5.

2 Detailed Mechanism Design

2.1 Actuator Choosing

Compared with Hydraulic actuators and pneumatic actuators, electric motors are the most widely used actuators in robots. They are inexpensive and available in a big variety of sizes and specifications. Furthermore, they are popular because of their ease and accuracy of control. The battery is the most light power source compared with the oil compressor and the air compressor. That makes it possible to build a light-weight robot with high moving speed. Its biggest disadvantage is limited actuation power and low power density. When the power is set, increasing the motor's rotational speed will decrease the torque, and vice versa.

With comprehensive consideration, high-torque servo motor is chosen as the actuator of the light-weight robot with high moving speed. But a good balance must be made between high speed and high torque. As the overall body mass is limited, a light-weight mechanism is essential.

2.2 Design Principles

In order to solve the problems stated above, two design principles are proposed. One is to reduce the moment of inertia of each leg; the other is to design a proper leg mechanism which is good at fast swing.

To reduce the moment of inertia of the leg, lightweight materials such as aerolite and carbon fiber are selected as the part material and high power density servo motor is utilized. On the other hand, the driving motors should be arranged on the body instead of the leg.

Due to the time cost in each circle of acceleration and deceleration, it's hard to keep swift leg swing at a relatively high speed just by changing the motor's rotating direction. In mechanical design, crank-rocker mechanism and cam mechanism both can realize the swift leg swing at a high speed by changing the motor's fixed-direction rotation into the leg's reciprocating swing.

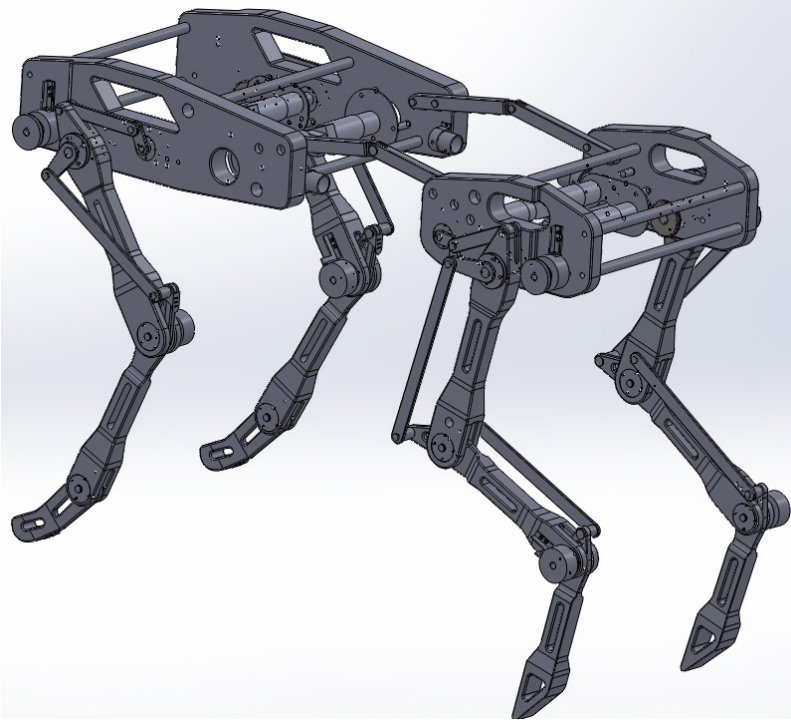


Fig. 1. The overall design of the robot

Based on the two design principles above, a novel mechanism using the crank-rocker mechanism as the transmission unit is developed, as *Fig. 1*. In order to reduce the shank's moment of inertia, the driving motor for the shank is coupled with the rotary shaft of the thigh by a clutch. That is to say, the driving shaft of the shank is also the rotary shaft of the thigh. Since the motor passes right the rotary shaft, zero moment of inertia is put on the thigh.

In order to achieve a better stress distribution, a connecting rod is laid between the thigh and the metatarsal forming a parallelogram to keep the thigh and the metatarsal parallel.

2.3 Setting the Dimension and Angle

Referring to the literature covering the bio-mimics of the cheetah, the dimension and diameter of all the bones is set. The swing angle range of the thigh and the shank is set by measuring the actual angle range of a running cheetah in the video-capture software, *Fig. 2*. Both the dimensions and swing angle ranges is given in *Table 1*.

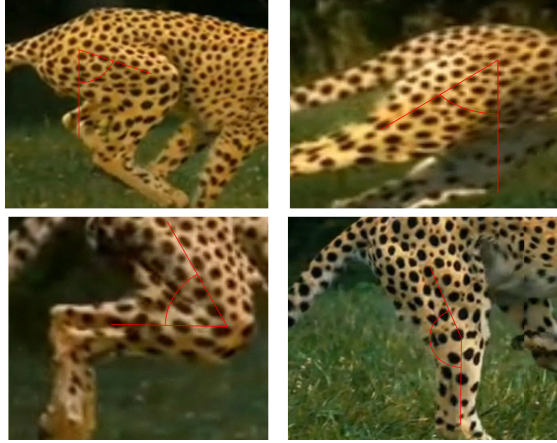


Fig. 2. Angle range of a running cheetah in the video-capture software

Table 1. Dimensions and swing angle ranges of the leg

Specification	Value
Shoulder swing range	130°
Knee swing range	100°
Thigh dimension	250mm
Shank dimension	230mm
Bone diameter	22mm

2.4 Transmission Unit Optimization

The four-link transmission unit can be simplified into the model depicted as *Fig. 3*. The four links are respectively the crank, the connecting rod, the rocker and the fixed link, and their length is the undetermined parameters a , b , c and d . Since the motor's motion is controlled, the angle between the crank and the fixed link θ_1 is known. The next is to find the relationship between θ_1 and the angle between the rocker (which stands for the thigh or the shank) and the fixed link θ_4 . The folln be got, using the cosine theorem.

$$\begin{cases} e = \sqrt{a^2 + b^2 - 2ad \cos(\theta_1)} \\ \alpha = \arccos\left(\frac{d^2 + e^2 - a^2}{2ed}\right) \\ \beta = \arccos\left(\frac{c^2 + e^2 - b^2}{2ec}\right) \\ \theta_3 = \arccos\left(\frac{b^2 + c^2 - e^2}{2bc}\right) \end{cases} \quad (1)$$

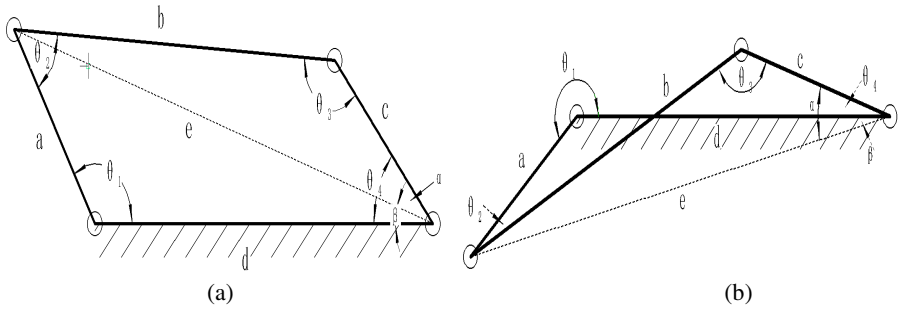


Fig. 3. (a) Configuration 1 of the four-link transmission unit (b) Configuration 2 of the four-link transmission unit

When the configuration is as the Fig. 2. (a),

$$\theta_2 = \arccos\left(\frac{a - d \cos \theta_1}{e}\right) + \arccos\left(\frac{b^2 + e^2 - c^2}{2be}\right) \quad (2)$$

$$\theta_4 = \alpha + \beta = \arccos\left(\frac{d^2 + e^2 - a^2}{2ed}\right) + \arccos\left(\frac{c^2 + e^2 - b^2}{2ec}\right) \quad (3)$$

When the configuration is like Fig. 2. (b),

$$\theta_4 = \beta - \alpha = \arccos\left(\frac{c^2 + e^2 - b^2}{2ec}\right) - \arccos\left(\frac{d^2 + e^2 - a^2}{2ed}\right) \quad (4)$$

The formulas (1)-(4) give the mapping from angle between the rocker (which stands for the thigh or the shank) and the fixed link θ_4 to θ_1 .

According to the swing angle range got in the previous section, the following equations can be got for the two extreme positions.

$$\left\{ \begin{array}{l} e = \sqrt{a^2 + b^2 - 2ad \cos(\theta_1)} \\ \alpha = \arccos\left(\frac{d^2 + e^2 - a^2}{2ed}\right) \\ \beta = \arccos\left(\frac{c^2 + e^2 - b^2}{2ec}\right) \\ \theta_3 = \arccos\left(\frac{b^2 + c^2 - e^2}{2bc}\right) \end{array} \right. \quad (5)$$

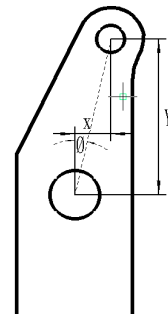


Fig. 4. The point of force application on the connecting rod

where φ is the angle between the center line of the rocker and the line from the center of rotation to the point of force application on the connecting rod. X and Y stand for the coordination of the point of force application on the connecting rod relative to the center of rotation. Using the simultaneous equations above, parameter a and b is set as follows.

$$a = \frac{\left[\sqrt{c^2 + d^2 - 2 \cdot c \cdot d \cdot \cos(150^\circ + \varphi)} - \sqrt{c^2 + d^2 - 2 \cdot c \cdot d \cdot \cos(20^\circ + \varphi)} \right]}{2}$$

$$b = \frac{\left[\sqrt{c^2 + d^2 - 2 \cdot c \cdot d \cdot \cos(150^\circ + \varphi)} + \sqrt{c^2 + d^2 - 2 \cdot c \cdot d \cdot \cos(20^\circ + \varphi)} \right]}{2}$$

At last the relationship between the driving torque M_1 and the equivalent torque M_2 transferred from the motor to the rocker can be calculated as follows. Ignoring the mass of all the rods, the connecting rod can be regarded as a two-force rod. The force applied on the two-force rod by the driving torque M_1 is

$$F = \frac{M_1}{l_1}$$

The equivalent torque M_2 on the rocker by the force F is $M_2 = F \cdot l_2$

Where $l_1 = a \cdot \sin \theta_2$, $l_2 = c \cdot \sin \theta_3$

Thus M_2 can be got, $M_2 = \frac{M_1}{a \cdot \sin \theta_2} \cdot c \cdot \sin \theta_3$

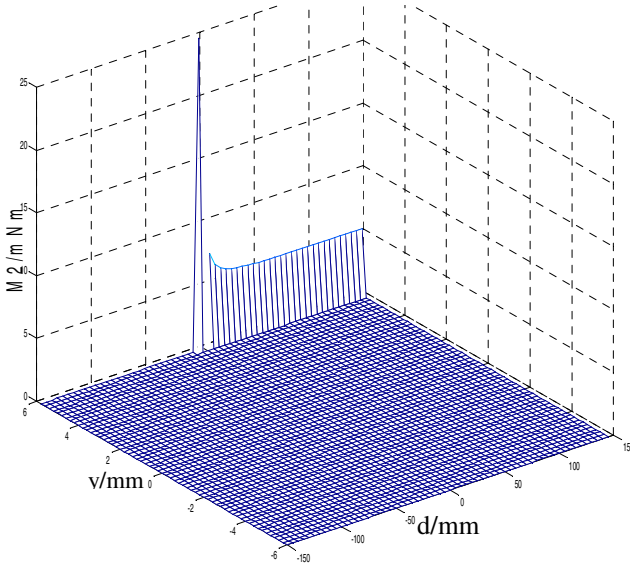


Fig. 5. MATLAB simulation of the equivalent torque M_2

Using the mathematic model proposed above, a curved surface standing for M_2 can be got with three variables x , y , d in the MATLAB as Fig. 5. By selecting the apogee of the curved surface, all the undetermined parameters a , b , c and d can be determined.

The point where most equivalent torque M_2 transferred from the driving torque M_1 is got ($d=-100, y=6, x=10$) in the MATLAB simulation. When $x=10, y=6, d=-100$, we can get $a=29, b=171, c=51$. The whole four-link crank-rocker mechanism is set.

3 Kinematics Analysis

Theories used to do kinematics analysis include the inverse transformation method, the geometric method, the Pieper method and so on. Compared to all the methods above, the screw theory has the following advantages.

(1) The position and orientation of the actuator can be described directly according to the pose of the joint axis. The solution is simplified by just finding out the rotational axis of the kinematic pair in each joint.

(2) Since a spinor represents a group of dual vector, it can be used to describe the position and orientation of a vector, the angular velocity and the linear velocity in kinematics, the force and torque in dynamics. Thus spinor is very simple in the description and clear in geometric concept.

(3) The kinematic model is more precise and efficient by using the screw theory compared to other theories.

The coordination of the leg is as Fig. 6.

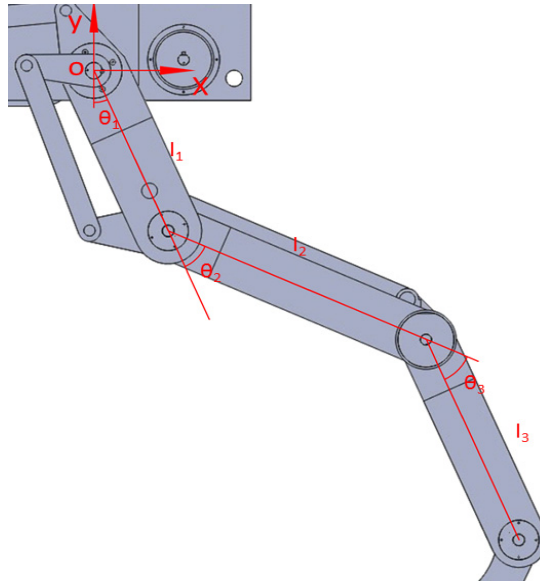


Fig. 6. The coordination of the leg

The initial pose matrix is $g_{ST}(\mathbf{0})$, according to the screw theory,

$$g_{ST}(\mathbf{0}) = \begin{bmatrix} 0 & \\ \mathbf{I}_{3 \times 3} & -(l_1 + l_2) \\ 0 & 1 \end{bmatrix}$$

All the angular velocity and position vector of each joint in the initial state are as follows,

$$\boldsymbol{\omega}_1 = \boldsymbol{\omega}_2 = \boldsymbol{\omega}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{r}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{r}_2 = \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix}, \quad \mathbf{r}_3 = \begin{bmatrix} 0 \\ -l_1 - l_2 \\ 0 \end{bmatrix}$$

Using screw theory, spinors of each joint ξ_1, ξ_2, ξ_3 are as following,

$$\xi_1 = \begin{bmatrix} \boldsymbol{\omega}_1 \\ \mathbf{r}_1 \times \boldsymbol{\omega}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} \boldsymbol{\omega}_2 \\ \mathbf{r}_2 \times \boldsymbol{\omega}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -l_1 \\ 0 \\ 0 \end{bmatrix}, \quad \xi_3 = \begin{bmatrix} \boldsymbol{\omega}_3 \\ \mathbf{r}_3 \times \boldsymbol{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -l_1 - l_2 \\ 0 \\ 0 \end{bmatrix}$$

The kinematic matrix of each joint $e^{\theta_1 \hat{\xi}_1}, e^{\theta_2 \hat{\xi}_2}, e^{\theta_3 \hat{\xi}_3}$ are as follows,

$$e^{\theta_1 \hat{\xi}_1} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad e^{\theta_2 \hat{\xi}_2} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & -l_1 s\theta_2 \\ s\theta_2 & c\theta_2 & 0 & -l_1 (1 - c\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad e^{\theta_3 \hat{\xi}_3} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & -(l_1 + l_2)s\theta_3 \\ s\theta_3 & c\theta_3 & 0 & -(l_1 + l_2)(1 - c\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So finally we get the kinematic matrix of the leg $g_{ST}(\boldsymbol{\theta})$ as following,

$$g_{ST}(\boldsymbol{\theta}) = e^{\theta_1 \hat{\xi}_1} e^{\theta_2 \hat{\xi}_2} e^{\theta_3 \hat{\xi}_3} g_{ST}(\mathbf{0}) \\ = \begin{bmatrix} c\theta_{123} & -s\theta_{123} & 0 & l_1 s\theta_1 + l_2 s\theta_{12} \\ s\theta_{123} & c\theta_{123} & 0 & -l_1 c\theta_1 - l_2 c\theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} x = l_1 s\theta_1 + l_2 s\theta_{12} \\ y = -l_1 c\theta_1 - l_2 c\theta_{12} \\ \varphi = \theta_{123} \end{cases}$$

Fig.7 and Fig. 8 is the operating space of the leg and the angular of the thigh joint calculated in the MATLAB

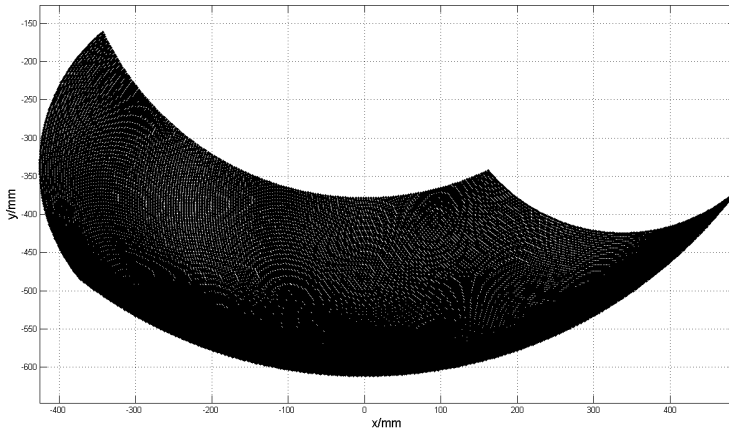


Fig. 7. The operating space of the leg

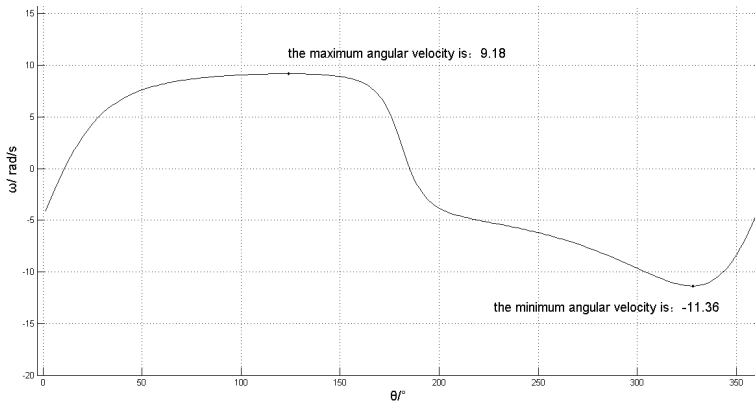


Fig. 8. The angular velocity of the thigh joint

4 Control System

The control system is based on a single centralized host-PC which is responsible for the entire high-level control part. The software system based on a multithread real-time operating system QNX is responsible for pose data acquisition, path planning,

algorithm scheduling as well as handling the internal messaging through PC/104 bus or RS-232.

The angular positions are measured by potentiometers which send analog signals to the A/D converter to get a digital signal. The pose of the robot is measured by the Inertial Measurement Unit(IMU). CANOPEN protocol is developed to transmit data between the motor controller and the host-PC via CAN bus to communicate with the actual robot.

The most outstanding advantage of this control system is the modularization of the system. All the cards composing the hardware system use the pc/104 standard, and is mass-produced by famous companies. In this way, A/D conversion, CAN communication and the host-PC is integrated into a robust and compact system easily. The framework of the control system is as *Fig. 9*.

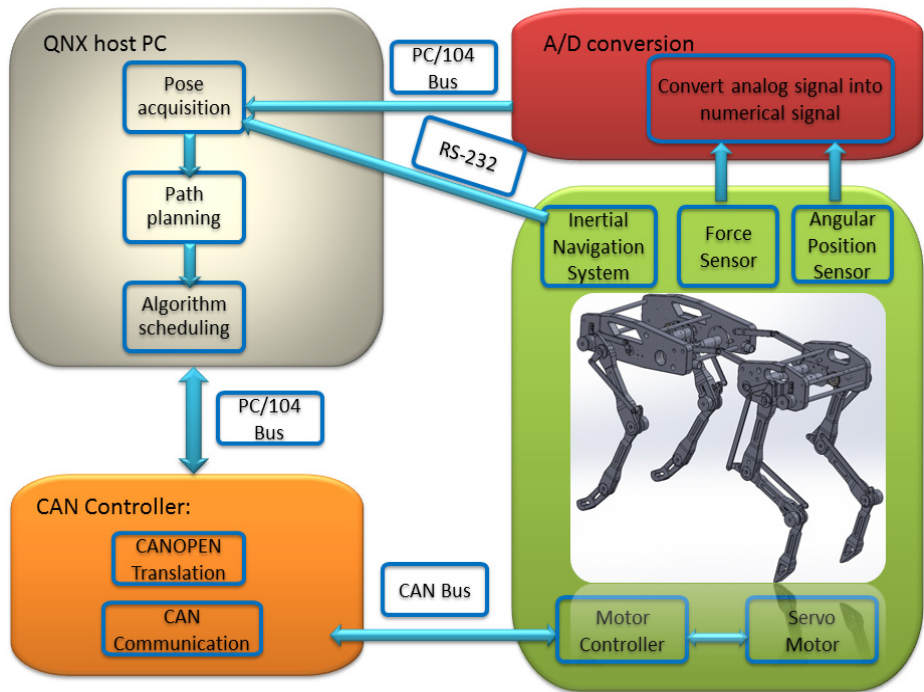


Fig. 9. The framework of the control system

5 Conclusion and Discuss

To achieve high speed and dynamic locomotion, two design principles are suggested to design a leg mechanism with low moment of inertia. A four-link transmission unit is proposed to change the motor's fixed-direction rotation into the leg's reciprocating

swing. Parameters optimization for the transmission unit is carried out to find out parameters to get the most driving force.

Currently, the prototype of the leg mechanism is confined to a planar model. In the following step, the abduction/adduction plane will be taken into account.

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