Indian Institute of Technology Guwahati
Department of Computer Science & Engineering
Design & Analysis of Algorithms (CS 207)
Assignment 1, Date: Jan 9, 2021

Include images where-ever necessary for better explanation

1. What is a greedy algorithm?

2. Does the greedy technique always work?

3. Define bipartite graph.

4. What is perfect matching in a bipartite graph?

5. Define the stable matching problem in a bipartite graph.

6. What is the difference between a perfect matching and a stable matching?

7. What is a blocking pair in stable matching problem?

8. In the stable matching problem with complete preference lists, does there always exist a stable matching?

9. Write pseudo-code of Gale-Shapley algorithm for stable matching problem.

10. Name five applications of Gale-Shapley stable matching algorithm.

11. Let the preference list of 3 men and 3 women be:
$M_1 : \{W_1, W_3, W_2\}$; $M_2 : \{W_3, W_2, W_1\}$; $M_3 : \{W_3, W_1, W_2\}$;
$W_1 : \{M_2, M_1, M_3\}$; $W_2 : \{M_2, M_3, M_1\}$; $W_3 : \{M_1, M_2, M_3\}$.

What can be a stable matching here?

12. Given Students preference order :
    $S_1 = \{T_1, T_2\}$ $S_2 = \{T_1, T_2\}$
    and Teachers Preference :
    $T_1 = \{S_1, S_2\}$ $T_2 = \{S_2, S_1\}$.
    Explain how the GS algorithm would work on this example. Also give the final stable pairs.

13. Is the Gale-Shapley algorithm for the Stable Matching problem a greedy algorithm? Why or why not?

14. How can you implement G-S algorithm efficiently? Explain.

15. What are the data structures used in the implementation of Gale-Shapley Algorithm?

16. Does G-S algorithm guarantee to find a stable matching ? If yes, give proof of correctness.

17. What is the time complexity of Gale - Shapley Algorithm ? Explain the analysis.

18. Given an instance of stable matching problem, do all executions of G-S algorithm yield the same stable matching? Explain your answer with proof.

19. In the Gale-Shapley algorithm, with n men and n women, what is the maximum number of times any woman can be proposed to? Justify.

20. Define *best valid partner* and *worst valid partner* with respect to stable matching problem.

21. (a) In the execution of Gale-Shapley algorithm which follows the policy of *man proposing to woman*, a man's partner becomes worse and worse whereas a woman's partner becomes better and better. Exaplin.

    (b) Consider a stable matching S^* generated by G-S algorithm for an instance of the stable matching problem. Show that each woman is paired with her worst valid partner.

22. Write an algorithm where each woman is paired with her best valid partner.

23. (a) Suppose we are given an instance of the stable matching problem for which there is a man m who is the first choice of all women.
    Prove or give a counterexample: In any stable matching, m must be paired with his first choice.

    (b) If all the men have the same preference list of women, then briefly discuss about the final matches. You can explain with an example.

    (c) Assume that the preference list of all women are same and so is the case for men. Comment on the pairs formed after applying GS algorithm.

24. Suppose S is a stable matching for a given instance I of the Stable Matching problem, not necessarily the one produced by the Gale-Shapley algorithm.

    (a) Does the matching S necessarily become unstable if we reverse the preference list of all the women (and keep the men's preferences intact)?

    (b) What happens when we reverse the preference lists for all the men as well as all the women – does S necessarily become unstable? Justify your answer in each case.
    (Give a proof if the answer is Yes, and a counter example in case the answer is No.)

25. In an execution of the standard GS algorithm, the partners to which a woman gets engaged gets better and better as the algorithm proceeds. How can you change the algorithm to get the reverse thing happening i.e. the partner gets worse and worse for women as the algorithm proceeds?

26. What is rank-maximal matching ? Explain with an example.

27. Can we solve rank-maximal matching by using standard G-S algorithm?

28. Tim has invited m friends over to his birthday party. The birthday cake contains m different types of candies studded on it and no two candies are of same type (which means there are m candies in the cake). He cuts it in such a way that each slice gets exactly one candy. Now each of his friends has his own preference list for candies. Our task is to distribute the m slices among m friends. Is this similar to the stable matching problem? Justify your answer.

Best wishes