

CS506: Quiz #1

Due on Monday, February 11, 2022

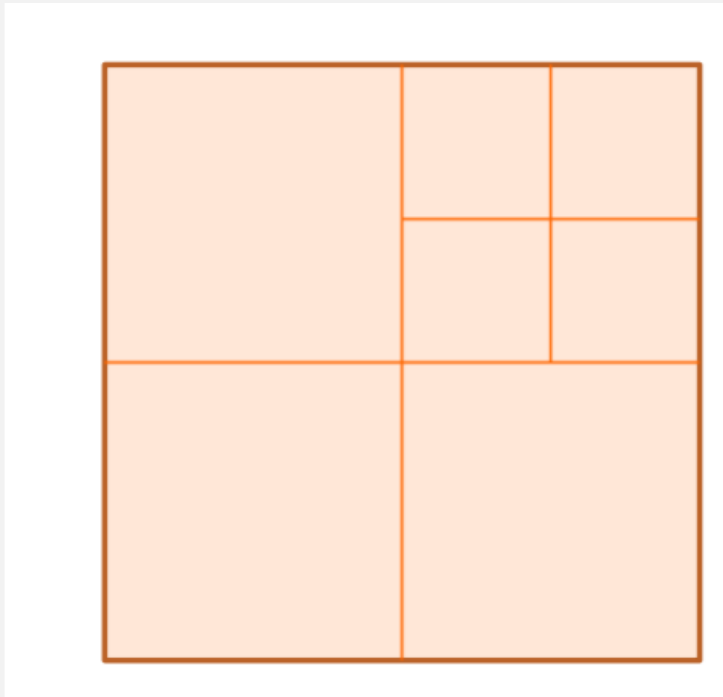
Siddharth Charan 190101085

Question 1 : Give an algorithm for squaring a matrix. Analyse its I/O Complexity. You are required to type your answers and submit a pdf file of your work. \LaTeX generated pdfs are preferred.

We will try to give an algorithm for squaring of a matrix.

Let's say we have a $m \times m$ square matrix A .

To calculate square of it. We can use divide and conquer algorithm. It will be a cache oblivious algorithm. We will break the square matrix in 4 parts. Top left A_1 , top right A_2 , bottom left A_3 and bottom right A_4 of size $m/2 \times m/2$. (Like in below diagram)



Let's say B is squared matrix and corresponding to A_1 sub-matrix, B has B_1 , corresponding to A_2 , we have B_2 and so on...

Then

$B = A \times A$ We can calculate matrix multiplication of those 4 smaller - matrices.

And then, we can get B by calculating, B_1, B_2, B_3, B_4 ...

$$B_1 = A_1 * A_1 + A_2 * A_3$$

$$B_2 = A_1 * A_2 + A_2 * A_4$$

$$B_3 = A_3 * A_1 + A_4 * A_3$$

$$B_4 = A_3 * A_2 + A_4 * A_4$$

We will take 2 sub-matrices at any time (except the cases when we require $A_1 * A_1$ and $A_3 * A_3$) and move them to main memory. We will divide matrix till, we have sub matrix of size smaller or equal to of $M/2$ (Main memory size/2) and then, we will move that sub-matrix in main memory, calculate the multiplication and then, we will follow above steps to get the final square.

Complexity analysis:

Case1

We need to do 8 operations(2 operations for each B1, B2 , B3 and B4) plus some work on half size matrices. If matrix size is greater than block size:

So, we have

$$T(m) = 8 * T(m/2)$$

Case 2

If matrix is smaller, such that both sub-matrices in operation can fit in main memory, then we need to do only $O(m^2/B)$ I/O operation.

$$T(m) = O(m^2/B)$$

It is very similar to calculating transpose of matrix. And we have done calculation for that already.

If we calculate time complexity it comes to $O(n^2/B)$ for nxn matrix.