Gradient Compression For Communication Limited Convex Optimization

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Overview

- Optimization using gradient descent.
  - Iteration complexity.
  - Communication cost of exchanging gradients.
- Compression technique for gradient is used to reduce compression.
- Discuss Gradient compression technique.

Problem formulation

Convex optimization problem,

$$\underset{x \in \mathbb{R}^n}{\text{minimize}}\, f(x). \tag{1}$$

Function f: $\mathbb{R}^n \rightarrow \mathbb{R}$ should satisfify below to conditions:

L-Lipschitz continuous gradient Condition,

$$f(y) \leq f(x) + \langle \nabla f(x), y - x \rangle + \frac{L}{2}\|y - x\|^2. \tag{2}$$

Function is strongly convex,

$$f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle + \frac{\mu}{2}\|y - x\|^2. \tag{3}$$

Cont.

Proposed method,

$$x_{k+1} = x_k - \gamma_k Q(\nabla f(x_k)).$$

Q is quantization, γ is step size.

Instead of using traditional gradient descent,

$$x_{k+1} = x_k - \gamma \nabla f(x_k), \tag{4}$$

Gradient Compression

- Sparsification
  - K-greedy Quantizer Q: $\mathbb{R}^n \to \mathbb{R}^n$

$$[Q_G^K(g)]_i = \begin{cases} [g]_{\pi(i)} & \text{if } i \leq K \\ 0 & \text{otherwise} \end{cases}$$

    where π is a permutation of $\{1, ..., n\}$, and $|g_\pi(k)| \geq |g_\pi(k+1)|$
    g is gradient vector.

- Quantization
  - Ternary Quantizer $Q_T : \mathbb{R}^n \to \mathbb{R}^n$ such that
    $[Q_T(g)]_i = ||g||sgn(g_i)$

Cont.

- Combination of Sparsification and Quatization (Dynamic gradient quatizer)
  - It is defined as $Q_D : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as

$$[Q_D(g)]_i = \begin{cases} \|g\| \ \mathrm{sgn}\ (g_{\mathrm{i}}) & \text{if } i \in I(g) \\ 0 & \text{otherwise} \end{cases}$$

where I(g) is the smallest subset of $\{1, ..., n\}$ such that

$$\sum_{j \in I(g)} |g_i| \geq \|g\|.$$

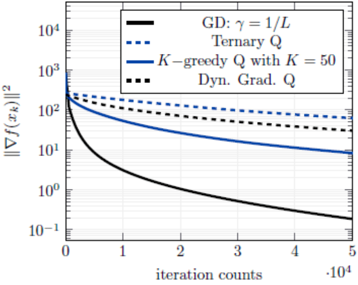
Cont.

- Number of bits required in different Quantizer
  - K-greedy Quantizer: K$(log_2(n) + b)$ bits
  - Ternary Quantizer: $(2n + b)$ bits
  - Dynamic Quantizer: $|I(g)|(log_2(n) + 1) + b$

Experimental results

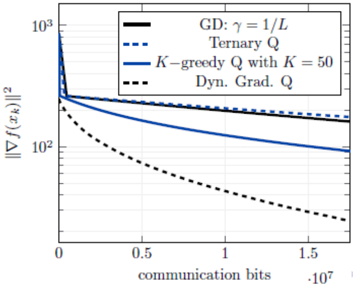
- Let Function $f(x) = 0.5 \times ||Ax - b||$, where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, m = 1000 and n = 800, elements of matrix drawn from uniform distribution [0,1],
  Elements of b is sign of random variable drawn from $\mathcal{N}(0, 1)$.
- Normalized each row of A by its Euclidean norm and computed the Lipschitz constant as $L = \lambda_{max}(A^T A)$.

Cont.



GD: $\gamma = 1/L$
Ternary Q
$K-$greedy Q with $K = 50$
Dyn. Grad. Q
$\|\nabla f(x_k)\|^2$
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
1
2
3
4
5
iteration counts
$\cdot 10^4$

(a)



GD: $\gamma = 1/L$
Ternary Q
$K-$greedy Q with $K = 50$
Dyn. Grad. Q
$\|\nabla f(x_k)\|^2$
10^3
10^2
0
0.5
1
1.5
communication bits
$\cdot 10^7$

References

- https://ieeexplore.ieee.org/document/8619625
- https://en.wikipedia.org/wiki/Gradient_descent