Optimization

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Overview

Problem Statement

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Problem Statement

Express the following problem in matrix form

$$\min_{\textbf{X}} x_{11} + x_{12}$$
 with constrains
$$x_{11} + x_{22} = 1 \text{ and } \textbf{X} \geq 0 \ (\geq \text{means positive definite})$$
 where

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{bmatrix}$$

Solution

We can write

$$x_{11} + x_{12} = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{X}^{\mathsf{T}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$x_{11} + x_{22} = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} & 0 \\ 0 & \mathbf{X} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Solution

Hence, the problem can be reformulated as

$$\min_{\mathbf{X}} \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{X}^{\mathsf{T}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad s.t$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} & 0 \\ 0 & \mathbf{X} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1, \mathbf{X} \ge 0$$

The End