EEC-104: Signals and Systems

Tutorial Sheet – 07

1. A causal discrete-time LTI system is described by

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]$$

where x[n] and y[n] are the input and output of the system, respectively.

(a) Determine the frequency response $H(\Omega)$ of the system.

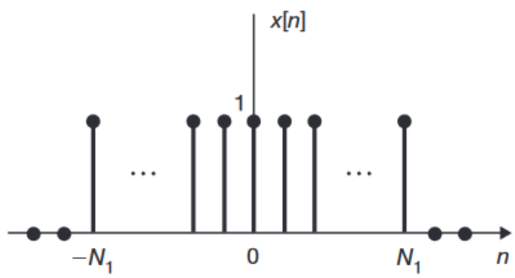
(b) Find the impulse response h[n] of the system

2. Find the Fourier transforms of

a) $x[n] = -a^n u[-n-1] \qquad a \text{ real}$

b) $x[n] = u[n] - u[n-N]$

3. Find the Fourier transform $X(\Omega)$ of the rectangular pulse sequence shown in Fig. Plot $X(\Omega)$ for $N_1 = 4$ and $N_1 = 8$.



4. Sketch $|X(\omega)|$, the amplitude spectrum of a signal $x(t) = 3\cos 6\pi t + \sin 18\pi t + 2\cos(28-\epsilon)\pi t$, where ϵ is a very small number → 0.
   (a) Determine the minimum sampling rate required to be able to reconstruct x(t) from these samples.
   (b) Sketch the amplitude spectrum of the sampled signal when the sampling rate is 25% above the Nyquist rate (show the spectrum over the frequency range ±50 Hz only).
   (c) How would you reconstruct x(t) from these samples?

Laplace Transform

1. Find the Laplace transform X(s) and sketch the pole-zero plot with the ROC for the following signals x(t):

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b) $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

(c) $x(t) = e^{2t}u(t) + e^{-3t}u(-t)$

2. Let $x(t) = e^{-a|t|}$ Find X(s) and sketch the zero-pole plot and the ROC for a > 0 and a < 0.

3. Find the inverse Laplace transform of the following X(s)

$$(a)\quad X(s)=\frac{1}{s+1}, \mathrm{Re}(s) > -1$$

$$(b)\quad X(s)=\frac{1}{s+1}, \mathrm{Re}(s) < -1$$

$$(c)\quad X(s)=\frac{s}{s^2+4}, \mathrm{Re}(s)>0$$

$$(d)\quad X(s)=\frac{s+1}{(s+1)^2+4}, \mathrm{Re}(s) > -1$$

4. Find the inverse Laplace transform of the following X(s).

$$(a)\quad X(s)=\frac{2s+4}{s^2+4s+3}, \mathrm{Re}(s) > -1$$

$$(b)\quad X(s)=\frac{2s+4}{s^2+4s+3}, \mathrm{Re}(s) < -3$$

$$(c)\quad X(s)=\frac{2s+4}{s^2+4s+3}, -3<\mathrm{Re}(s) < -1$$

5. Find the inverse Laplace transform of

$$X(s)=\frac{5s+13}{s(s^2+4s+13)} \qquad \mathrm{Re}(s)>0$$

6. Find the inverse Laplace transform of

$$(a)\quad X(s)=\frac{2s+1}{s+2},\ \mathrm{Re}(s) > -2$$

$$(b)\quad X(s)=\frac{s^2+6s+7}{s^2+3s+2},\ \mathrm{Re}(s) > -1$$

$$(c)\quad X(s)=\frac{s^3+2s^2+6}{s^2+3s},\ \mathrm{Re}(s) > 0$$

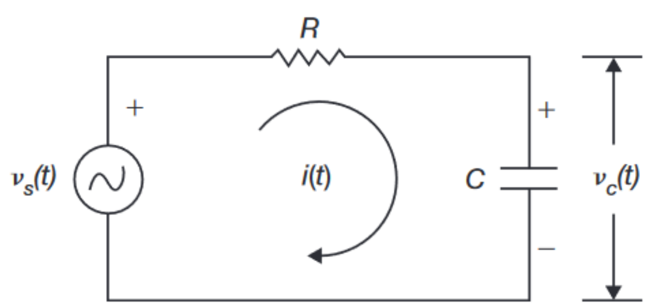
7. Find the inverse Laplace transform of

$$X(s)=\frac{2+2se^{-2s}+4e^{-4s}}{s^2+4s+3} \qquad \mathrm{Re}(s) > -1$$

8. Find the system function H(s) and the impulse response h(t) of the RC circuit in Fig.

   (a) If $x(t) = v_s(t)$ and $y(t) = v_c(t)$.

   (b) If $x(t) = v_s(t)$ and $y(t) = i(t)$.



9. Consider a continuous-time system whose input x(t) and output y(t) are related by

$$\frac{dy(t)}{dt}+ay(t)=x(t)$$

where a is a constant.

(a) Find $y(t)$ with the auxiliary condition $y(0) = y_0$ and

$$x(t) = Ke^{-bt}u(t)$$

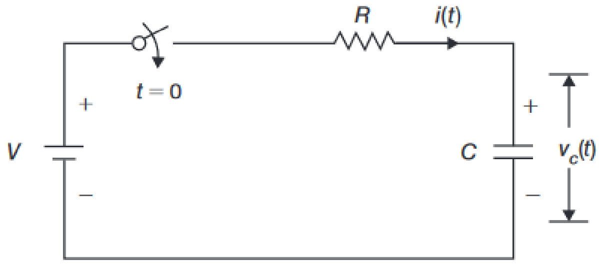
(b) Express $y(t)$ in terms of the zero-input and zero-state responses.

10. Solve the second-order linear differential equation

$$y''(t) + 5y'(t) + 6y(t) = x(t)$$

with the initial conditions $y(0) = 2$, $y'(0) = 1$, and $x(t) = e^{-t}u(t)$.

11. Consider the RC circuit shown in Fig. The switch is closed at t =0. Assume that there is an initial voltage on the capacitor and $v_c(0^-) = v_0$.
(a) Find the current i(t).
(b) Find the voltage across the capacitor $v_c(t)$.



12. In the circuit in Fig. the switch is in the closed position for a long time before it is opened at t > 0. Find the inductor current i(t) for t>0.