ME5205: THEORY OF VIBRATION

PROJECT SUBMISSION

PROBLEM STATEMENT - Develop a suspension system for a four-wheel automotive, that can take extreme road conditions in a semi-urban setting, where there are potholes and bumps. Your goal is to help minimize the excitation transmitted to a rider in the vehicle.

SIDDHARTH DEY

ME18B075

1. INTRODUCTION

The suspension system is mainly responsible for the following three purposes [1]:

- **Road Isolation/ Ride Comfort:** Should provide good 'Ride Comfort' to the passengers by minimizing the effects of perturbations
- **Road Holding:** Minimize the variations in the normal force acting on each wheel as well as the relative displacement of the unsprung mass to improve road-holding characteristics
- **Stability during cornering and acceleration/braking:** To constrain undesired motion from load transfer[1] along certain degrees of freedom such as roll and pitch

The suspension system consists of three main components [2]:

- **Spring:** The elastic element, which provides a force proportional to the input relative displacement between its two ends
- **Damper:** (also called hydraulic shock absorber) The damping element, which produces a dissipative force proportional to the relative velocity between the two ends or the mounting points
- **Mechanical Linkage:** A set of mechanical elements connecting the suspended sprung body to the unsprung mass

2. DIFFERENT TYPES OF SPRINGS, DAMPERS, AND LINKAGES

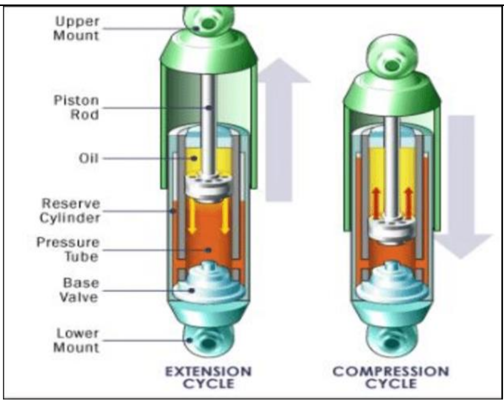
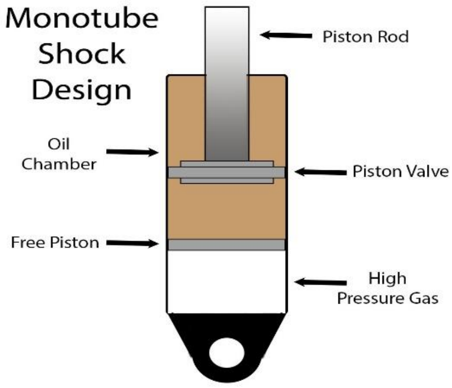
2.1 SPRING

Various types of springs are available depending on the vehicle type and application to provide elastic behavior to the suspended (sprung) mass [1].

LEAF SPRING	COIL SPRING	AIR/PNEUMATIC SPRING
Used in heavy automobiles; consists of 'leaves, which are metal bands fastened through a center bolt and held together with clips	A steel rod or bar is wound into a coil and needs additional support along with the lateral and longitudinal directions	Uses the compressibility of air to absorb vibrations; ECAS – Electronically controlled Air Suspension: controls air pressure to regulate suspension deflection
		

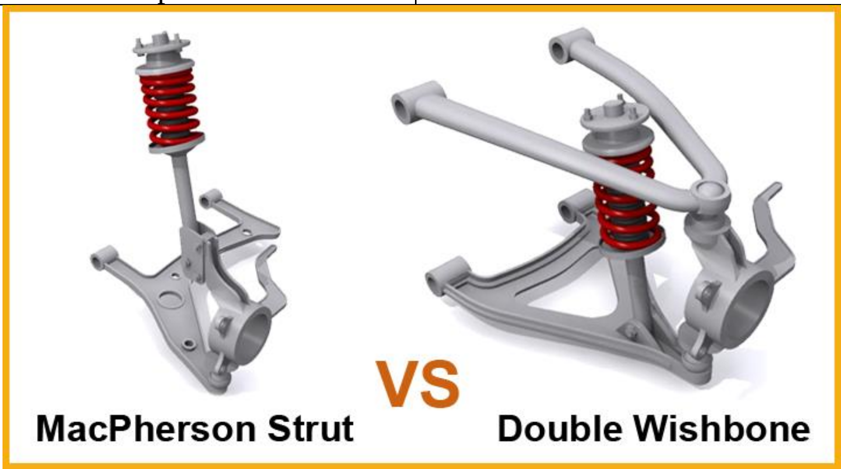
2.2 DAMPER (SHOCK ABSORBER)

It dissipated the kinetic energy of the system in the force form of heat produced by friction. They are mainly of two types [1]:

<table>
<tr><th>Twin-tube shock absorber</th><th>Mono-tube shock absorber</th></tr>
<tr><td>Two tubes in the lower half: Pressure tube and Reserve Tube (reservoir for extra fluid); Base valve through which fluid flows between the two tubes</td><td>Does not have a reserve tube, a floating piston separates the hydraulic dissipative fluid from the pressurized nitrogen gas below the piston</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

2.3 MECHANICAL LINKAGE:

There are both dependent and independent suspension mechanisms but we will be covering only the independent type in this report.

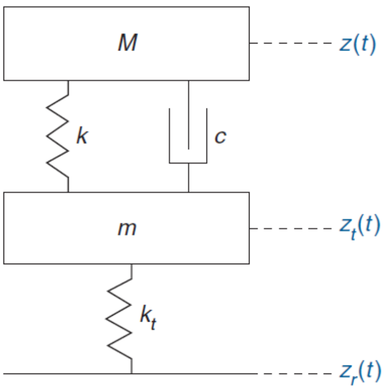
<table>
<tr><th>MacPherson Strut Suspension</th><th>Double Wishbone Suspension</th></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>- Has a strut assembly and a lower control arm
- Advantage: compact and more clearance available for driveshaft in front wheels
- Disadvantage: Wheel camber angle can change due to wheel movement</td><td>- two wishbones (A-arms) serving as the lower and upper suspension links
- Advantage: Better ride and handling characteristics
- Disadvantage: Takes up a lot of space</td></tr>
</table>

3. MATHEMATICAL MODELLING

To capture the dynamics of the vehicle, we use different mathematical models to consider various degrees of freedom and stiffness properties of the sprung and unsprung mass. Broadly speaking, for a four-wheeled passenger vehicle, we usually use one of the following three models [2] – Quarter Car Model, Half Car Model (pitch-oriented model), and Full Car Model. The half-car model considers four degrees of freedom in a single plane, namely the vertical heave of the two unsprung masses and the sprung mass and pitch. This model is used for vehicles moving on a straight road or for two-wheeled vehicles. The full car model contains eight degrees of freedom including the vertical heave of each of the four unsprung mass (tire) and the sprung mass, and the three angular degrees of freedom, yaw, pitch, and roll. Full car models are not usually considered for developing control strategies (which will be the main focus in the later sections) and hence won't be included in this report.

3.1 QUARTER CAR MODEL

This is the most common model used in analysis with only degrees of freedom, the displacements of the sprung and unsprung mass. The unsprung mass considered here is the tire which is assumed to have some stiffness, Kt. Motion only happens along the vertical direction and there is no angular displacement involved. We shall use this model for the control strategies to be developed in the further sections.



Equation 1

$$M\ddot{z} = -k(z - z_t) - c(\dot{z} - \dot{z}_t)$$

Equation 2

$$m\ddot{z}_t = k(z - z_t) + c(\dot{z} - \dot{z}_t) - k_t(z_t - z_r)$$

Figure1: Quarter Car Model

where 'M' and 'm' are the sprung and unsprung mass, 'k' and 'k_t' are the suspensions and tire stiffness coefficients, 'c' is the suspension damping coefficient, 'z' and 'z_t' are the sprung and unsprung mass deflection and 'z_r' is the road disturbance.

4. DESIGN CONSIDERATIONS FOR A PASSIVE QUARTER CAR MODEL

Although there is no hard and fast rule on how to design a suspension mechanism, the following points should be kept in mind while designing:

- **Spring Buckling:** The ratio of the free length of the spring to its mean diameter should be kept less than 4 to prevent buckling of the spring under axial loading.

- **Spring Stiffness:** The stiffness of the spring depends on the mass of the vehicle, the desired pre-compression under static conditions, and several other factors. The spring stiffness formula is given as:
  Eqn (3)

$$K = \frac{G * d^4}{8 * N * D^3}$$

where ‘G’ is the shear modulus of the spring material, ‘d’ is its wire diameter, ‘N’ is the number of active coils (usually the two coils at the end of a compression spring do not play a role) and ‘D’ is its mean diameter. Usually, as a starting point as per the literature review, it is customary to take the natural frequency of the spring to 1-2 Hz for comfort and 3-5 Hz for stiffer springs to get better road-holding. The spring stiffness can thus be obtained by:

Eqn (4)

$$\omega = 2\pi \ sqrt\left(\frac{K}{M}\right)$$

$$K = M * \left(\frac{\omega}{2\pi}\right)^2$$

After getting the initial stiffness value, we can choose the physical parameters as given in the first equation above to get the desired value.

- **Resonance:** This is a challenging obstacle. It should be ensured that the natural frequency of the spring does not match with that of the road disturbance, as it might lead to amplified oscillations due to resonance. Modal analysis of various components of the vehicle should be performed and made sure that the frequency of the different modes of the adjacent components does not come close to each other to prevent the passage of disturbance energy to the inner delicate structures.

- **Vibration Isolators:** We can use soft materials such as rubber washers at the junction of two components for vibration isolation

- **Damping coefficient:** In most applications, underdamped oscillations are desired with faster dissipation. Hence, a damping ratio (ε) close to one is chosen (0.8-0.9). Therefore, once we have fixed the spring stiffness, a good initial guess for an appropriate damping coefficient can be calculated using the following equation:

  Eqn (5)

  $$\varepsilon = \frac{c}{2 * \sqrt{m * k}}$$

  Dampers do not usually show a linear variation of their dissipative force with input relative velocity. The graph resembles a hysteresis plot and hence, we define the equivalent damping coefficient (C_{eq}) which is often mentioned in catalogs.

  Eqn (6)

  $$C_{eq} = \frac{E_d}{\pi * A * \omega^2}$$

  where ‘E_d‘ is the energy dissipated in one cycle (area of the hysteresis loop of the F-v plot), ‘A’ is the amplitude of oscillation and ‘ω’ is the input frequency.

- **Motion Ratio:** When it comes to designing the mechanical linkages, a good motion ratio of close to one should be maintained, where motion ratio is defined as:

  Eqn (7)

$$Motion\ ratio = {}^{Spring\ Displacement}\!\big/_{Wheel\ Displacement}$$

This ensures that disturbance from the road is absorbed properly by the suspension and prevents jerks to the mechanical links.

- **Pole placement:** Relative displacement is related to road-holding and acceleration of the sprung mass to comfort. For the quarter car model, the transfer function of the sprung mass displacement with road disturbance as input is given as [same notations as used in equations (1) & (2)]:
  Eqn (8)

$$F_z = \frac{(c\ k_t)s + (k_t\ k\ )}{(\ M\ m)s^4 + (cm + cM)s^3 + (Mk + mk + Mk_t)s^2 + ck_t s + k_t k}$$

  The poles of this transfer function should lie on the right half of the s-plane for stability. The farther away the poles are from the imaginary axis, the faster stability is reached. The design parameters ‘c’, ‘k_t’, and ‘k’ can also be decided based on the desired pole location. Methods such as the root-locus plot help us in understanding how each of these parameters influences the location of the poles.

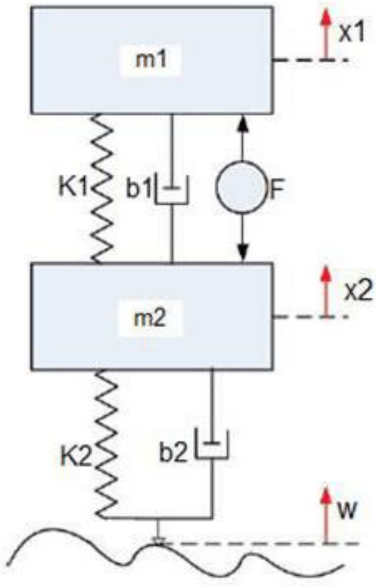
5. CONTROLLABLE SUSPENSION MECHANISMS

So far, we have considered the parameters of the suspension mechanism to be fixed. But there are two systems, namely semi-active suspension and active suspension, which make use of closed-loop control for enhanced performance.

Active Suspension	Semi-Active Suspension
Involves using a separate actuator in addition to the passive spring-damper system that can directly supply energy to the system	Uses a controllable damper whose damping properties can be altered by sending signals from an external ECU
Advantages	
- More freedom as it can apply an external push-pull force	- Effective in terms of space usage - can only dissipate energy, no possibility of making the system unstable
Disadvantages	
- The use of an extra component in each mechanism makes the design complex - Has higher power requirements as it directly supplies energy in the form of an external force	- The force is limited to the first and third quadrants of the Force-velocity graph, i.e, can only supply an opposing force, also making the state space equation non-linear and difficult to control

6. ACTIVE SUSPENSION

As discussed before, the active suspension makes use of a separate actuator that can directly supply a push-pull force to the system. The actuator comes in between the sprung and the unsprung masses and the mathematical model changes as follows:



Equation 9

$$M_1\ddot{X}_1 = -b_1(\dot{X}_1 - \dot{X}_2) - K_1(X_1 - X_2) + U$$

Equation 10

$$M_1\ddot{X}_2 = b_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(\dot{W} - X_2) + K_2(W - X_2) - U$$

(The notation has been explained in the table below)

Figure 2: Active Suspension Quarter Car Model

The system is linear which allows us to use prevalent linear space controllers. We will use a PID controller to achieve a closed-loop response.

6.1 PID CONTROLLER

Let us assume the following parameters to set a plant simulation [3] which represents the quarter-car suspension of a bus.

SYMBOL	SYSTEM PARAMETER	VALUE
M_1	Sprung mass	2500 kg
M_2	Unsprung mass	320 kg
K_1	Spring stiffness of the suspension	80000 N/m
K_2	Spring stiffness of the tire	500000 N/m
b_1	Damping coefficient of suspension	350 Ns / m
b_2	Damping coefficient of tire	15020 N s / m
U	Control force	variable

The equations [3] & [4] can be converted into a linear function by taking the Laplace transform and assuming the initial conditions to be zero. After a few manipulations, we get the transfer function of X1(s) and X2(s) in terms of the actuator input U(s) and road disturbance W(s) as follows:

Eqn (11)

$$\begin{bmatrix} X1(s) \\ X2(s) \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} M_2s^2 + b_2s + K_2 & b_1b_2s^2 + (b_1K_2 + b_2K_1)s + K_1K_2 \\ -M_1s^2 & (M_1b_2s^3 + (M_1K_2 + b_1b_2)s^2 + (b_1K_2 + b_2K_1)s + K_1K_2 \end{bmatrix}\begin{bmatrix} U(s) \\ W(s) \end{bmatrix}$$

where $\Delta = (M_1 s^2 + b_1 s + K_1)\left(M_2 s^2 + (b_1 + b_2)s + (K_1 + K_2)\right) - (b_1 s + K_1)^2$

To get the transfer functions of the relative displacement of the sprung mass to unsprung mass, we set W(s) = 0 to get the control input transfer function and U(s) = 0 to get the road disturbance transfer function.

Eqn (12)

$$G_1(s) = \frac{X_1(s) - X_2(s)}{U(s)} = \frac{(M_1 + M_2)s^2 + b_2 s + K_2}{\Delta}$$

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)} = \frac{-M_1 b_2 s^3 - M_1 b_2 s^2}{\Delta}$$

Now, to improve efficiency, we use a closed-loop PID controller with unit feedback (assuming no sensor noise) and the error between the reference signal and the feedback value of the relative displacement is fed to the controller as input as shown in Figure [3].

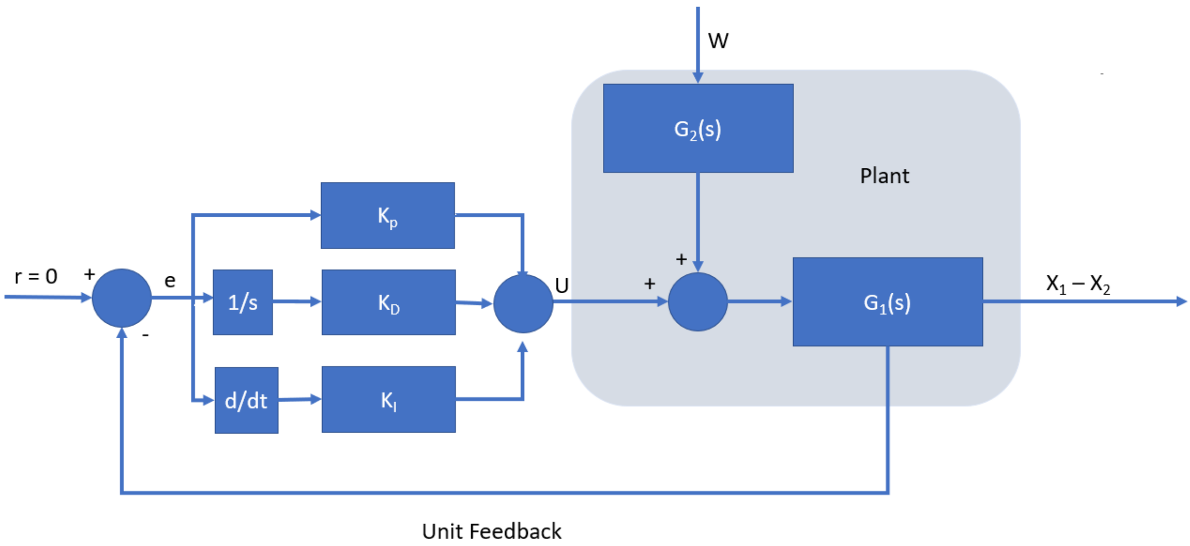
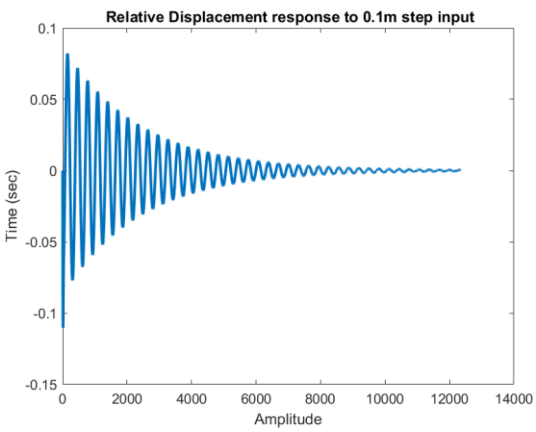
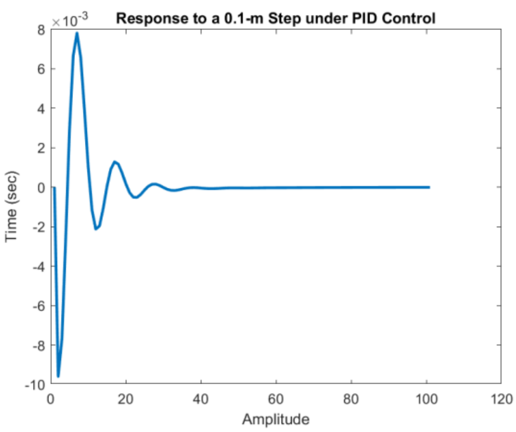


Figure 3: Closed-loop PID block diagram

We take the gain values to be $K_P = 832100$, $K_D = 208025$, and $K_I = 624075$. We give a step input of 0.1m as the road disturbance and observe the relative displacement for both the open-loop and the closed-loop response as shown below.





(a) (b)

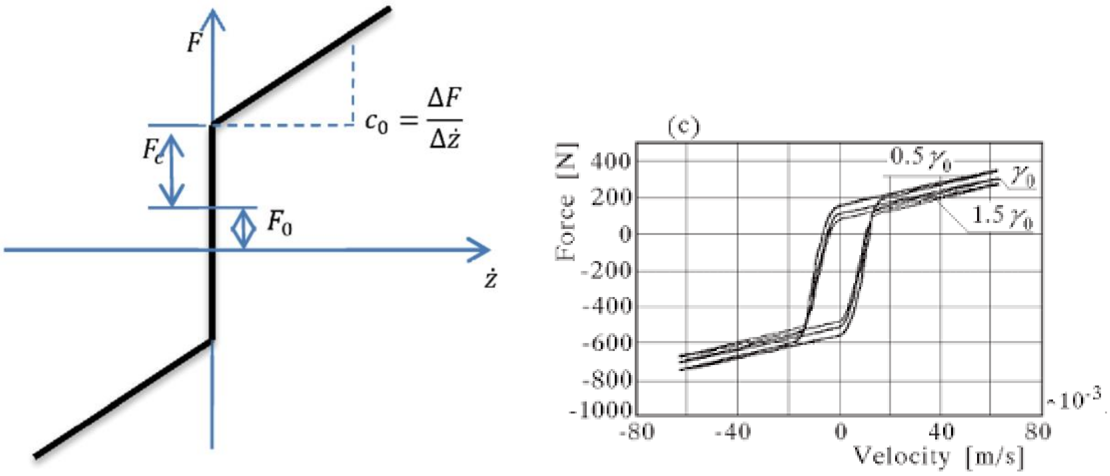
Figure 4: (a) Open-loop (b) Closed-loop PID response of relative displacement between sprung and unsprung mass to 0.1m step input as road disturbance

From Figure 4(a), we can see that the open-loop response has a large overshoot of approximately 8cm and has a very high settling time as well (50ms). Whereas, for the closed-loop response, the overshoot is just 9mm with a settling time of less than 5 secs. Hence, the passengers in the bus will undergo less oscillation and will experience a more comfortable ride.

7. SEMI-ACTIVE SUSPENSION

Before moving on to the discussion of various controllable dampers and devising a control strategy for them, we will first consider a better mathematical model to capture the damper dynamics. Dampers rarely behave linearly. Two factors are contributing to non-linearity: (1) Force offset: In certain dampers, the damper behaves linearly with relative velocity after it overcomes a certain amount of initial force (2) Hysteresis: This is absorbed in almost all cases. The force-velocity graph of the damper forms a closed-loop which results in a different force during expansion and compression strokes. The first problem is dealt with by using the Bingham model and the second one by the Modified Bouc-Wen Model [4].

7.1 NON-LINEAR DAMPER MODELS



(a) (b)

Figure 5: (a) Bingham Model (b) Modified Bouc-Wen Model

Bingham Model:

Eqn (13)

$$F_{damper} = f_c sgn(\dot{x}) + c_0\dot{x} + f_0$$

Where 'sgn' denotes the signum function, $\dot{x}$ is the relative displacement between its two ends and c_0 and f_0 are constants specific to the damper.

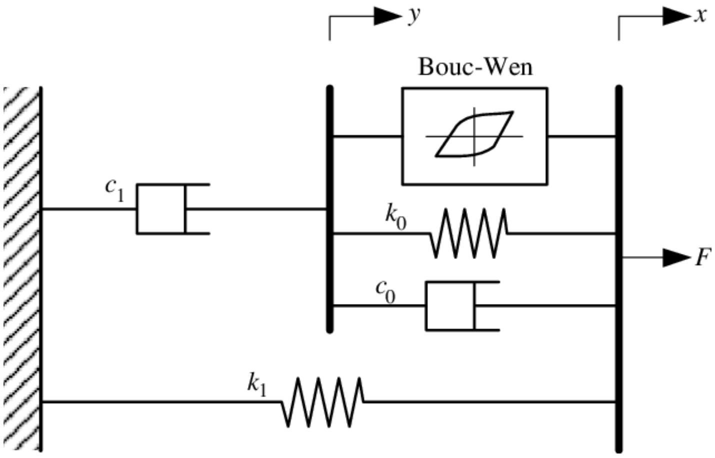


Figure 6: Modified Bouc Wen model characteristic diagram

Modified Bouc-Wen Model:

Eqn (14)

$$F = c_0(\dot{x} - \dot{y}) + k_0(x - y) + k_1 x + \alpha z$$

$$\dot{y} = (c_0 + c_1)^{-1}(c_0\dot{x} + k_0(x - y) + \alpha z)$$

$$\dot{z} = \delta(\dot{x} - \dot{y}) - \beta(\dot{x} - \dot{y})|z|^n - \gamma z|\dot{x} - \dot{y}||z|^{n-1}$$

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u$$

$$c_0 = c_0(u) = c_{0a} + c_{0b} u$$

$$c_1 = c_1(u) = c_{1a} + c_{1b} u$$

$$\dot{u} = -\eta(u - v)$$

Where 'u' is the commanded voltage and 'v' is the applied voltage by the current driver. The modified Bouc-Wen model has many constants specific to the damper associated with it. The non-linear ODEs have been modeled in Simulink (Figure [7]) and the velocity and displacement response are shown below in Figure [8] (The data for the parameters have been taken from ref [5]).

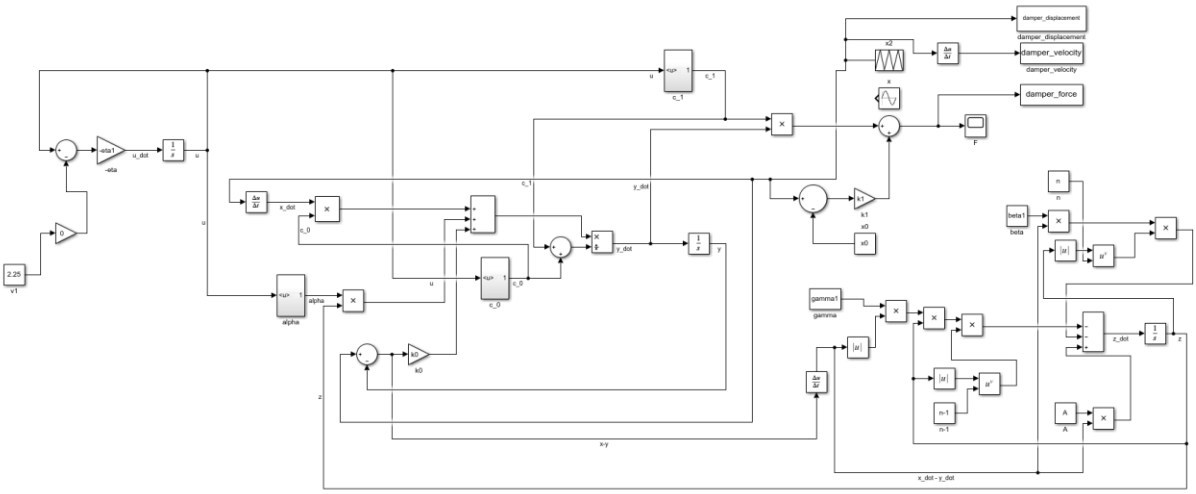
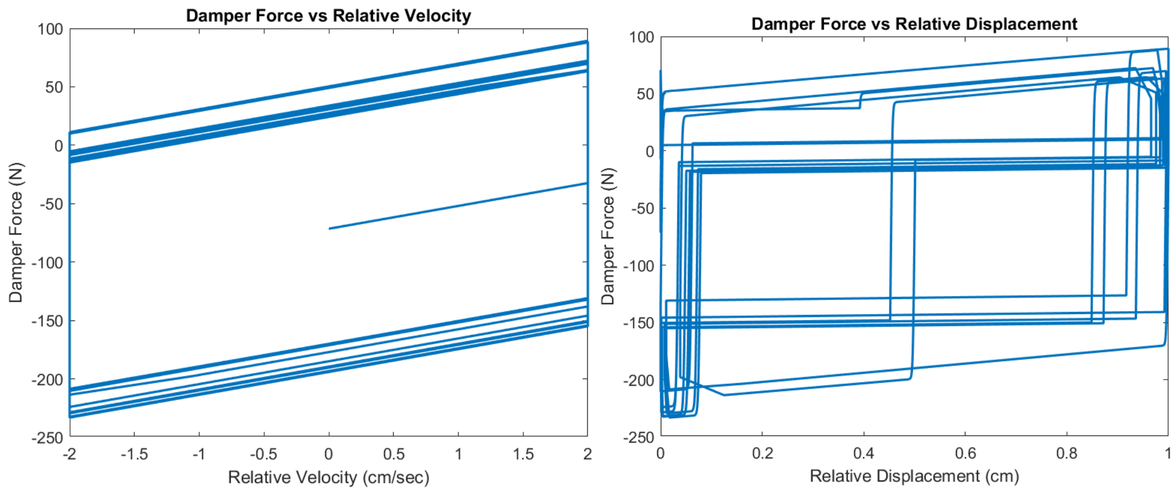


Figure 7: Simulink model of Modified Bouc-Wen Model

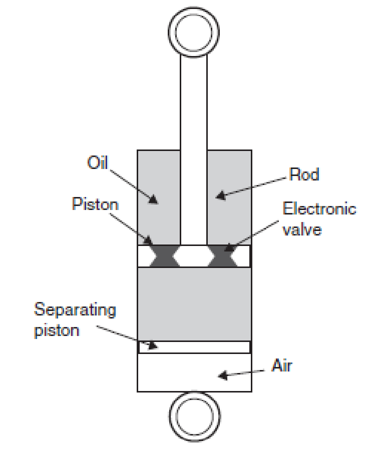
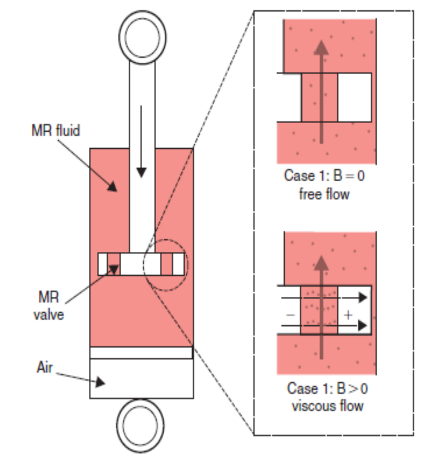
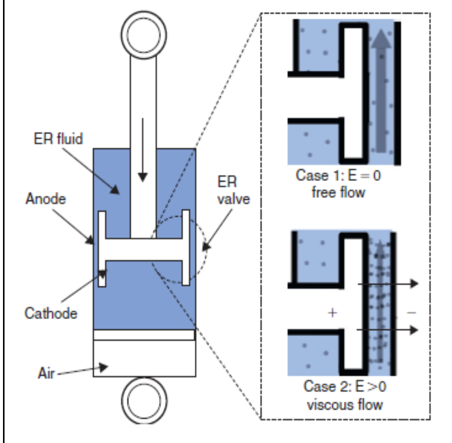


(a) **(b)**

Figure 8: (a) Damper Force vs Relative Velocity (b) Damper Force vs Relative Displacement

7.2 CONTROLLABLE DAMPERS

In semi-active suspension, we need to control the damping characteristics. This can be achieved in many ways, some of which [2] are discussed in the table below:

ELECTROHYDRAULIC (EH) DAMPER	MAGNETORHEOLOGICAL (MR) DAMPER	ELECTRORHEOLOGICAL (ER) DAMPER
		
It consists of electrohydraulic valves which can regulate the diameter of the orifice of the damper piston	It contains MR fluid which is a mixture of oil and micro iron particles whose viscosity can be altered by subjecting to a magnetic field generated by electromagnetic coils	It also contains a similar mixture but is sensitive to an electric field; when no field is applied, the mixture is almost free to flow in the damper orifice

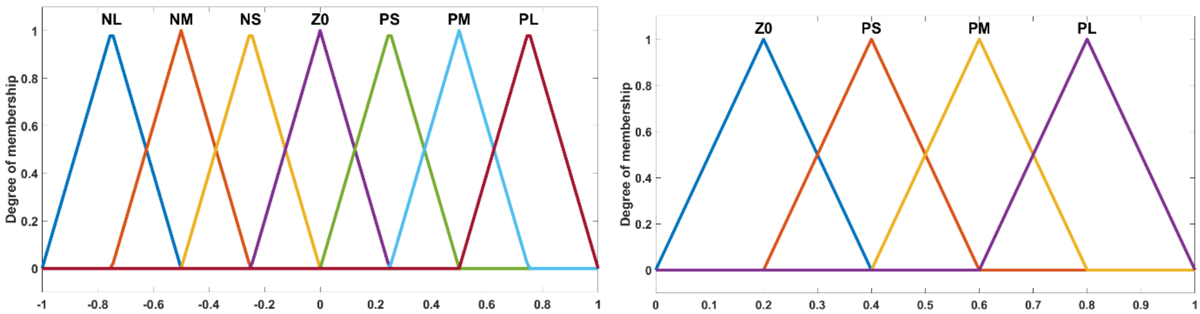
7.3 FUZZY LOGIC CONTROL

We shall use the Modified Bouc-Wen model while devising our semi-active control strategy. But before designing a controller, we need to determine the constants associated with the model for our damper by performing an experiment in an electrodynamic shaker and logging the Force vs Displacement and Force-velocity values, and comparing it with the simulation data to regress the unknown constants of the damper by minimizing an L2 cost function denoting the error between the actual and the simulated data and hence optimize the simulation solution by iteratively updating the constants to reduce error (we can use global optimization algorithms such as the Genetic Algorithm (GA) or the Particle Swarm Optimization (PSO)).

The semi-active suspension is inherently non-linear and hence, we cannot directly use linear controllers. To handle this, we incorporate a nonlinear controller 'Fuzzy logic Control' (ref [5]) via the use of heuristic information. It has three parts; in the first part, we convert the crisp input values, which in our case are the relative displacement and relative velocity, into discrete categories using membership functions by a process called fuzzification. Next, is decision making using an inference table, which is a logic table through which we get a discrete output for the given inputs. Finally, in the defuzzification process, we convert the discrete output into continuous values which are fed to the ECU.

7.3.1. MEMBERSHIP FUNCTIONS AND INFERENCE TABLE

For the fuzzy control, we convert the continuous inputs into linguistic variables or discrete classes through membership functions. The labels used in the membership functions are NL - Negative Large; NM – Negative Medium; NS - Negative Small; Z0 - Zero; PS – Positive Small; PM - Positive Medium; PL - Positive Large. The membership function for both relative displacement and velocity as well as the output is shown in Figure [6]. In the present study, the displacement (D in cm) and velocity (V in cm/s) of the SDOF system are considered as the inputs in the universe of discourse [-0.5 0.5] and [-1 1] respectively



(a) (b)

Figure 9: (a) Membership Function of Relative Displacement and Relative Displacement (Input) (b) Output Membership Function

	$\dot{x}$						
x	NL	NM	NS	Z0	PS	PM	PL
NL	PL	PL	PL	PM	Z0	Z0	Z0
NM	PL	PL	PL	PS	Z0	Z0	PS
NS	PL	PL	PL	Z0	Z0	PS	PM
Z0	PM	PL	PS	Z0	PS	PM	PL
PS	PS	PM	Z0	Z0	PL	PL	PL
PM	Z0	PS	Z0	PS	PL	PL	PL
PL	Z0	Z0	Z0	PM	PL	PL	PL

Figure 10: Inference Table

The inference table is expressed in Figure [8]. This logic is primarily developed from a classic switching based semi-active controller named 'Balance Logic' as shown below:

Eqn (15)

$$V = \begin{cases} V_{max}, & if\ x\dot{x} > 0 \\ 0, & if\ x\dot{x} < 0 \end{cases}$$

where 'V_{max}'is the maximum voltage that can be applied and 'x' is the relative displacement. Let us say 'x' is zero at the equilibrium and positive when in the upper half and vice-versa. $\dot{x}$ is the relative velocity when it moves upward and vice-versa. To understand the balance control logic, let us consider one case where the body is in the upper half and moving upward, i.e, 'x' and '$\dot{x}$' both are positive and hence, x$\dot{x}$ is also positive. Since it is moving away from the equilibrium, we want to provide maximum opposition and thus, we supply maximum voltage to make the dampers as stiff as possible. When 'x' is positive and '$\dot{x}$' is negative, i.e, it is moving downward in the upper half, it is moving towards the equilibrium and we want to provide the least resistance and hence supply zero voltage. The inference table for balance logic is developed on this principle.

7.3.2 RESULTS AND DISCUSSIONS

For brevity, we shall assume a linear model of the equivalent damping coefficient with respect to time of the form:

Eqn (16)

$$f_{MR} = (\alpha I + \beta)\dot{x}$$

Where f_{MR} is the damper force, $\dot{x}$ is the relative velocity and α and β are constants associated with the damper. With the membership functions and inference table defined before, we make the Fuzzy-Logic closed-loop plant of a single degree of freedom sprung mass model (Figure [11]) and design and simulate it in Simulink (Figure[12]).

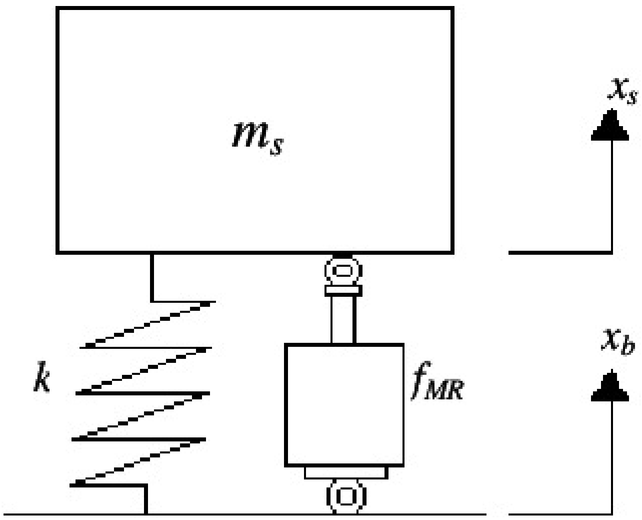


Figure 11: Single Degree of Freedom (sdof) plant

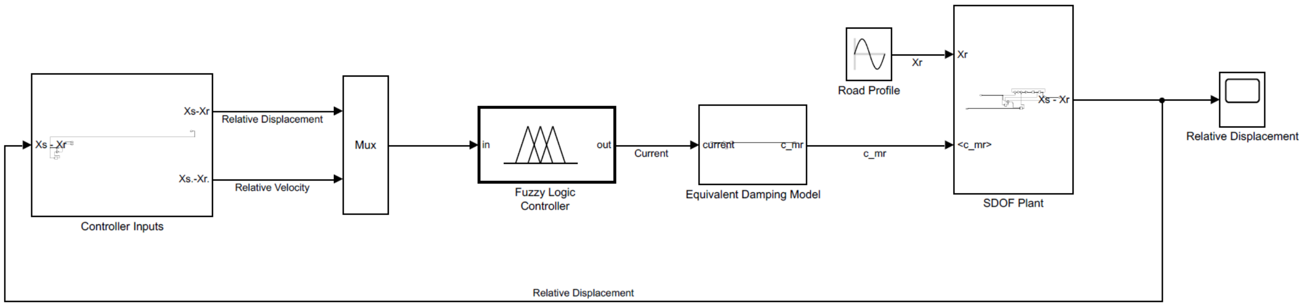
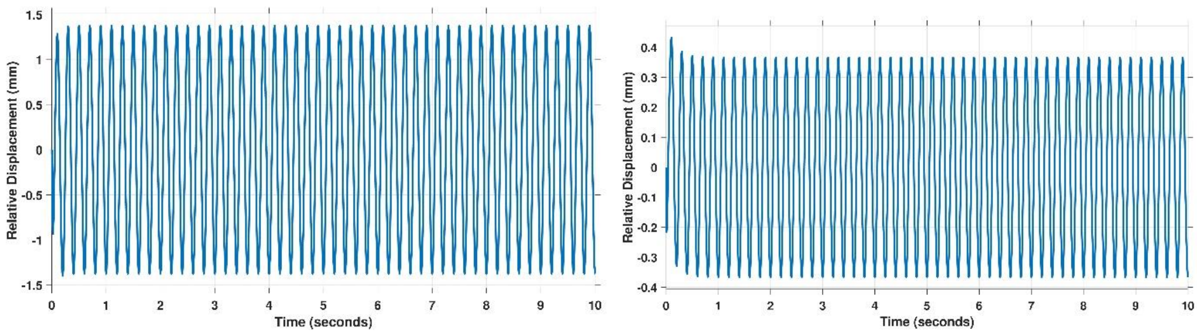


Figure 12: Simulink Model of the Fuzzy-Logic controller for a sdof plant

We compare the relative displacement of the passive and semi-active cases in Figure [13]. We give a sinusoidal road disturbance of 3.5 mm and observe the displacement response as a measure of road-holding. As seen in Figure [13], the peak value in the passive case is 1.39mm and for the semi-active suspension is 0.433mm.



(a) **(b)**

Figure: 13: (a) Relative Displacement response of passive suspension (b) Relative Displacement response of the semi-active suspension

8. CONCLUSION

We studied different mechanisms, springs, and controllable dampers which are used in the industry for suspension. Then we investigated Active and Semi-active suspension to improve road-holding and comfort. In Active Suspension, we assumed an LTI system and used a linear PID controller. For semi-active suspension, we first analyzed various models to reproduce actual damping behavior as observed in experiments and then assuming a Modified Bouc-Wen model, designed a Fuzzy-Logic controller to handle the non-linearity behavior of the plant. We showed in both cases how the performance was improved through simulations.

For future work, we need to consider a full-car model with eight degrees of freedom to better recreate the vehicle response. Many models also include the stiffness of the seats and include additional spring and damping elements to model the same. We also need to model the sensor noise and uncertainty in the actuator and include them as transfer functions in the closed-loop model. For the controller, there is a huge assumption that there is no time delay in the reading made by the sensor and the control force being supplied by the actuator. But in reality, there will be delays due to sensor and actuator response time, time is taken to execute the code among several others, and hence needs to be designed accordingly.

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