

## 1 Asymptotics

(a) Consider the following method:

```
public int beta(int N) {  
    if (N <= 0) {  
        return 0;  
    }  
    Random rand = new Random();  
    int r = rand.nextInt();  
    if (r % 3 == 0) {  
        for (int i = 0; i < N; i += 1) {  
            constant(); // runs in constant time  
        }  
    }  
    return beta(N/2);  
}
```

- (i) What is the best case runtime?
- (ii) What is the worst case runtime?
- (iii) Why can't we say that the best case is when  $N = 0$ ?

(b) Consider the following method:

```
public void delta(int N, boolean bool) {  
    if (N == 0) {  
        System.out.println("hello");  
    }  
    expo(N); // runs in  $2^N$  time  
    if (bool == true) {  
        delta(N-1, bool);  
        delta(N-1, bool);  
    } else {  
        delta(N-1, bool);  
    }  
}
```

- (i) What is the best case runtime?
- (ii) What is the worst case runtime?

## 2 Binary Search Trees

- (a) We implement a binary search tree with the following methods:

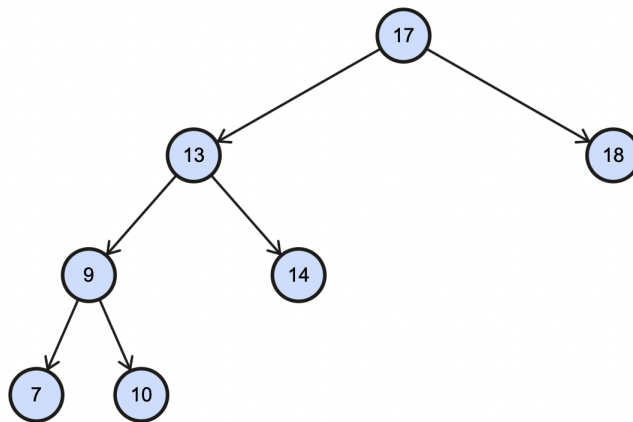
```
// Inserts an item into the binary search tree.
public void insert(T item) {
    // Implementation has been omitted
}

// Deletes an item from the binary search tree.
public void delete(T item) {
    // Implementation has been omitted
}
```

Draw the binary search tree that results from the following operations. Assume we start from an empty tree.

```
insert(5)
insert(7)
insert(10)
insert(6)
insert(3)
insert(1)
insert(4)
delete(10)
delete(7)
```

- (b) Given the following binary search tree:



Suppose we delete the root node. Which node(s) can we replace 17 with as the new root node?

- (c) Suppose we create a BST by inserting the nodes  $V_0, V_1, \dots, V_n$ , where  $V_i$  is strictly smaller than  $V_{i+1}$ , in order. That is, we first insert  $V_0$ , then  $V_1$ , and so on. What is the runtime to find an element in this BST in the worst case, where  $N$  is the number of nodes?