

# Document on Question 28 Exercise(8.1)

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## Abstract

This is a simple document explaining a question about the concept of similar triangles.

Download all python codes from

```
svn co https://github.com/SiddharthPh/Summer2020/trunk/document/codes
```

## QUESTION

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that  $DM = CM$ . Point D is joined to point B. Show that:

- a)  $\triangle AMC \cong \triangle BMD$
- b)  $\triangle DBC$  is a right angle.
- c)  $\triangle DBC \cong \triangle ABC$
- d)  $CM = \frac{1}{2} AB$

## SOLUTION

The python code for the figure is

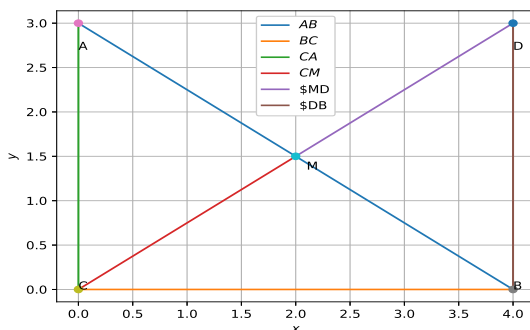
```
./code/triangle.py
```

The latex- tikz code is

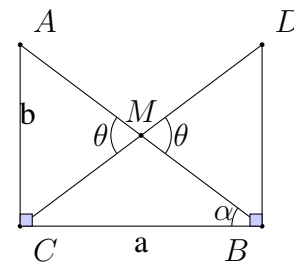
```
./figs/triangle.tex
```

The above latex code can be compiled as standalone document

```
./figs/triangle_fig.tex
```



(a) By Python



(b) By Latex-tikz

Fig. 1: Obtained by executing the above codes.

**Sol.a)**

From the above figure,

$$C = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 0 \\ b \end{pmatrix}, B = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

As, M is the midpoint of AB

$$M = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

Therefore Coordinates of D are

$$D = \begin{pmatrix} a \\ b \end{pmatrix}$$

$\triangle AMC$  and  $\triangle DMB$  are congruent to each other by SAS congruency.

(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]

(ii) Side CM is equal to corresponding side DM [As M is midpoint of DC]

(iii)  $\angle AMC = \angle DMB$  [Vertically Opposite Angles]

**Sol.b)**

In  $\triangle ACB$

$$(AB)^2 = a^2 + b^2 \text{ Since } \angle ACB = 90^\circ \text{ [Pythagorus theorem]}$$

In  $\triangle DBC$  [DB = D-B = b]

$$\cos \angle DBC = [(a^2 + b^2 - (DC)^2)/2ab]$$

By using distance formula i.e  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$  we get that AB=DC from the given coordinates.

$$\cos \angle DBC = [(a^2 + b^2 - (AB)^2)/2ab]$$

$$\cos \angle DBC = 0$$

Therefore,  $\angle DBC$  is right angle

**Sol.c)**

$\triangle ACB$  and  $\triangle DCB$  are congruent to each other in SAS congruency. (i) Both the triangles have a common base, a.

(ii) AC = DB by using distance formula

(iii)  $\angle ACB = \angle DBC = 90^\circ$  [From Solution b]

**Sol.d)**

Since M is the midpoint of CD

$CM = \frac{1}{2} DC$  From Solution b) it is clear that DC=AB

Therefore  $CM = \frac{1}{2} AB$

Hence Proved.