

My Presentation

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Question

Exercise 8.1(Q no.28)

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B. Show that:

a) $\triangle AMC \cong \triangle BMD$

b) $\triangle DBC$ is a right angle.

c) $\triangle DBC \cong \triangle ABC$

d) $CM = \frac{1}{2} AB$

Codes

The python code for the figure is `/code/traingle.py`

The latex- tikz code is `/figs/triangle.tex`

The above latex code can be compiled as standalone document
`/figs/triangle_fig.tex`

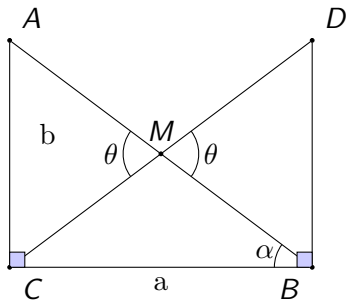


Figure: Right Angled Triangle

Solution a)

From the above figure,

$$C = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 0 \\ b \end{pmatrix}, B = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

As, M is the midpoint of AB

$$M = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

Therefore Coordinates of D are

$$D = \begin{pmatrix} a \\ b \end{pmatrix}$$

$\triangle AMC$ and $\triangle DMB$ are congruent to each other by SAS congruency.

- (i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]
- (ii) Side CM is equal to corresponding side DM [As M is midpoint of DC]
- (iii) $\angle AMC = \angle DMB$ [Vertically Opposite Angles]

Solution b)

In $\triangle ACB$

$$(AB)^2 = a^2 + b^2$$

Since $\angle ACB = 90^\circ$ [Pythagorus theorem]

In $\triangle DBC$

Formula

$\cos \angle DBC =$

$$((a^2 + b^2 - (DC)^2)/2ab)$$

[DB = D-B = b]

By using distance formula i.e $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ we get that $AB=DC$ from the given coordinates.

$\cos \angle DBC =$

$$((a^2 + b^2 - (AB)^2)/2ab)$$

$\cos \angle DBC = 0$

Therefore, $\angle DBC$ is right angle

Solution c)

$\triangle ACB$ and $\triangle DCB$ are congruent to each other in SAS congruency.

(i) Both the triangles have a common base , a.

(ii) $AC = DB$ by using distance formula

(iii) $\angle ACB = \angle DBC = 90^\circ$ [From [◀ Solution b](#)]

Solution d)

Since M is the midpoint of CD

$CM = \frac{1}{2} DC$ From Solution b it is clear that $DC = AB$

Therefore $CM = \frac{1}{2} AB$

Hence Proved.