Presentation Template

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Question

Exercise 8.1(Q no.28)

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:

a) $\triangle AMC \cong \triangle BMD$

b) $\triangle DBC$ is a right angle.

c) $\triangle DBC \cong \triangle ABC$

d) CM = $\frac{1}{2}$ AB

Construction method

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Initial Input Values.	
a	4
b	3
$\angle(ACB)$	90°

Table: To construct $\triangle ACB$

The steps for constructing $\triangle ACB$ are

$$\mathrm{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.1}$$

$$\mathrm{A} = \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.2}$$

$$\mathrm{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.3}$$

Since, M is the midpoint of AB and CD

$$\mathrm{M} = \frac{\mathrm{A}+\mathrm{B}}{2} = \frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix} \mathrm{M} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

$$\mathrm{M} = \frac{\mathrm{C}+\mathrm{D}}{2} \tag{2.4}$$

$$\implies \mathrm{D} = 2\mathrm{M} - \mathrm{C} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.5}$$

Derived Values for *triangleDCB*.	
M	$\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Table: To construct $\triangle DCB$

Codes and Figures

The python code for the figure is

./code/traingle.py

The latex- tikz code is

./figs/triangle.tex

The above latex code can be compiled as standalone document

./figs/triangle_fig.tex

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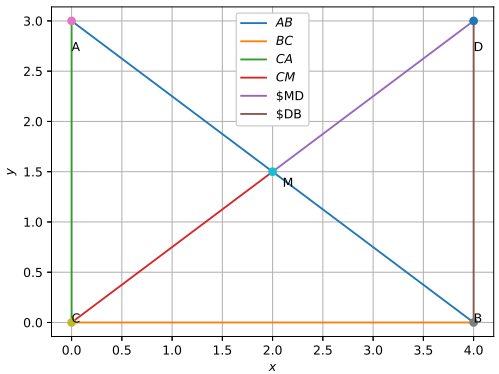
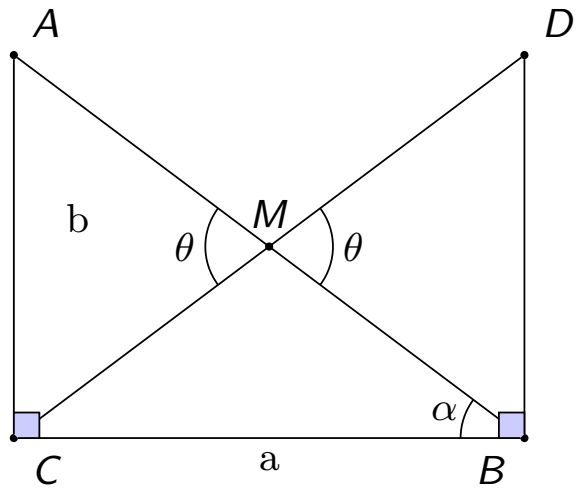


(a) By Python



(b) By Latex-tikz

Solution

Sol a) $\triangle AMC \cong \triangle DMB$ by SAS congruency $\because$
$AM = BM$
$CM = DM$
$\angle AMC = \angle DMB (VerticallyOppositeAngles)$
Sol c)

$$\|\mathrm{A} - \mathrm{B}\| = \left\| \begin{pmatrix} -a \\ b \end{pmatrix} \right\| \tag{3.1}$$

$$\|\mathrm{C} - \mathrm{D}\| = \left\| \begin{pmatrix} -a \\ -b \end{pmatrix} \right\| \tag{3.2}$$

$$\implies \|\mathrm{A} - \mathrm{B}\| = \|\mathrm{C} - \mathrm{D}\| \tag{3.3}$$

$$\text{or, } AB = CD \tag{3.4}$$

From RHS congruence, $\triangle ACB \cong \triangle DCB$

Sol b) From (2.3), (2.1), (2.5)

$$(\mathrm{D} - \mathrm{B})^T (\mathrm{B} - \mathrm{C}) = \begin{pmatrix} 0 & b \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \tag{3.5}$$

$$\implies BD \perp BC \tag{3.6}$$

Sol d) From (3.4), noting that M is the mid point of both AB and CD,

$$\|\mathrm{A} - \mathrm{B}\| = \left\| \begin{pmatrix} -a \\ b \end{pmatrix} \right\| \tag{3.7}$$

$$\|\mathrm{C} - \mathrm{D}\| = \left\| \begin{pmatrix} -a \\ -b \end{pmatrix} \right\| \tag{3.8}$$

$$\implies \|\mathrm{A} - \mathrm{B}\| = \|\mathrm{C} - \mathrm{D}\| \tag{3.9}$$

$$\text{or, } AB = CD \tag{3.10}$$