Math Document Template

Pothukuchi Siddhartha

Download all python codes from

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svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/Probstat/codes
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1 Probability Exercises

1.1 Exercise 1

1.1.1 Problem: Suppose you drop a die at random on the rectangular region shown in Fig.15.6.What is the probability that it will land inside the circle with diameter 1m?

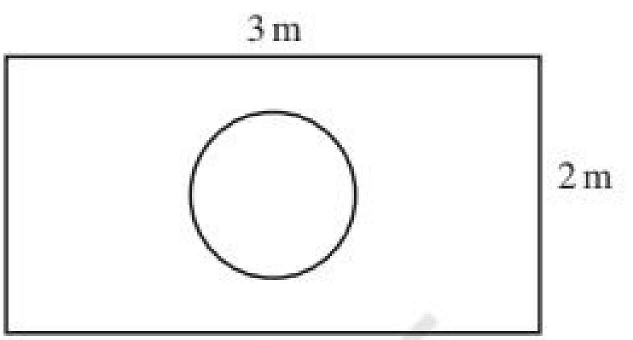

Fig. 15.6

1.1.2 Solution:

1. In the given question,
   The sample size = Total Area of the rectangle=
   $$3x2 = 6m^2 \tag{1.1.2.1.1}$$
   Favourable outcome = Area of Circle=
   $$\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}m^2 \tag{1.1.2.1.2}$$
   Probabilty(P) of the dice landing in the circle=$\frac{\pi}{24}$
   $\therefore$ P = 0.131
   The python code for the figure 1.1.2.1

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prob/codes/prob1.py
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   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given

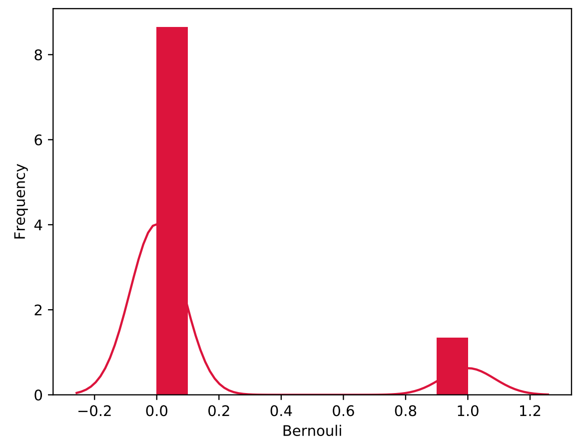

Fig. 1.1.2.1: Bernoulli Distribution.

   below
   Probability mass function(P(X))=$p^x(1-p)^{1-x}$
   $$P(X=0) = 1-p \tag{1.1.2.1.3}$$
   $$P(X=1) = p \tag{1.1.2.1.4}$$
   where p=0.131 given by 1.2.2.1a

1.1.3 Understanding Graph:

1. From the graph (1.1.2.1),
   a) Values on X-axis represent the Bernoulli distribution of data.
   b) Values on Y-axis represent the density of frequency(Histogram estimator) of the data. To calculate the histogram estimator, we have to define the number of bins(Intervals) For the graph in the question,
   $$bins = 10 \tag{1.1.3.1.1}$$
   $$h(binwidth) = \frac{(1-0)}{10} \tag{1.1.3.1.2}$$
   For bin-width h, number of observations n, for bin j, proportion of observations is
   $$p_j = \frac{y_j}{n} \tag{1.1.3.1.3}$$

(Where y_j is the frequency of j$^t h$ bin.)

$$p_0 = \frac{869}{1000} = 0.869 \quad (1.1.3.1.4)$$
$$p_1 = \frac{131}{1000} = 0.131 \quad (1.1.3.1.5)$$

The density estimate is

$$y(x) = \frac{p_j}{h} \quad (1.1.3.1.6)$$
$$y(0) = \frac{0.869}{0.1} = 8.69 \quad (1.1.3.1.7)$$
$$y(0) = \frac{0.131}{0.1} = 1.31 \quad (1.1.3.1.8)$$

To draw the Gaussian Kernel Density curve, Calculate mean and standard deviation for the centre and bandwidth.
See 1.1.3.1 for clear understanding.

$$\mu(Mean) = 0.861 \quad (1.1.3.1.9)$$

$$\sigma^2(\text{Standard Deviation}) = 0.1189 \quad (1.1.3.1.10)$$

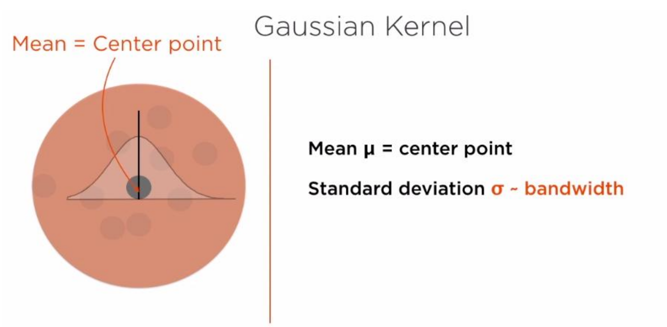

Fig. 1.1.3.1: Gaussian Kernel

1.2 Exercise 2

1.2.1 Problem: A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it ?
(ii) She will not buy it ?

1.2.2 Solution:

1. In the given question,

a) The sample size = Total number of pens(S)=

$$S = 144 \quad (1.2.2.1.1)$$

Favourable outcome = Pens purchased(F1)=

$$F1 = 124 \quad (1.2.2.1.2)$$

Probabilty(P) of the pens purchased by her from the shopkeeper=$\frac{124}{144}$
$\therefore$ P = 0.861
The python code for the figure 1.3.2.1

```
prob/codes/prob2_a.py
```

shows the Bernouli distribution of data.

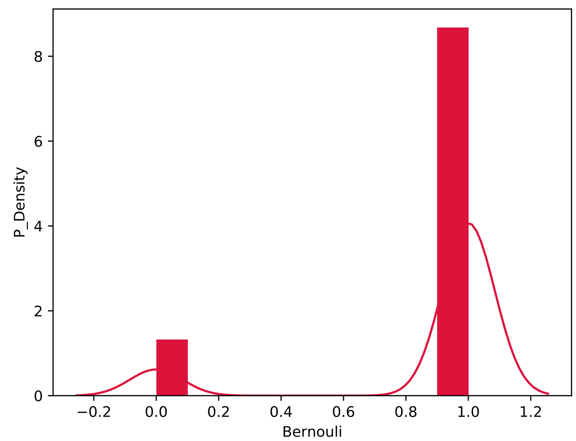

Fig. 1.2.2.1: Bernoulli Distribution.

The Bernoulli Distribution of data is given below
Probability mass function(P(X))=$p^x(1-p)^{1-x}$

$$P(X = 0) = 1 - p \quad (1.2.2.1.3)$$
$$P(X = 1) = p \quad (1.2.2.1.4)$$

where p is the probability of occurence of (X=1)
$\therefore$ p=0.861 given by 1.2.2.1a

2. The sample size = Total number of pens(S)=

$$S = 144 \quad (1.2.2.2.1)$$

Favourable outcome = Pens not purchased(F2)=

$$F2 = 20 \quad (1.2.2.2.2)$$

Probabilty(P) of the pens not purchased by her from the shopkeeper=$\frac{20}{144}$

$\therefore$ P = 0.139
The python code for the figure 1.2.2.2

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prob/codes/prob2_b.py
```

shows the Bernouli distribution of data.

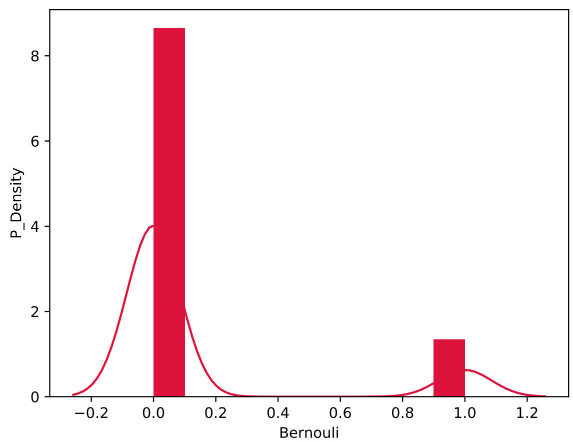

Fig. 1.2.2.2: Bernoulli Distribution.

The Bernoulli Distribution of data is given below
Probability mass function(P(X))=$p^x(1-p)^{1-x}$

$$P(X=0)=1-p \tag{1.2.2.2.3}$$

$$P(X=1)=p \tag{1.2.2.2.4}$$

where p is the probability of occurence of (X=1)
$\therefore$ p=0.139 given by 1.2.2.1a
The understanding of the curves 1.3.2.1 and 1.2.2.2 can be seen in (1.1.3.1b)

1.3 *Exercise 3*

1.3.1 Problem: (i)Complete the following table:

Event:'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	-	-	-	-	-	$\frac{5}{36}$	-	-	-	$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.Therefore,each of them has a probability $\frac{1}{11}$ Do you agree with this argument? Justify your answer.

1.3.2 Solution:

1. In the given question,

   a) The sample size = Total number of possibilities(S)=

$$\begin{pmatrix} \{1\ \ 1\} & \{1\ \ 2\} & \{1\ \ 3\} & \{1\ \ 4\} & \{1\ \ 5\} & \{1\ \ 6\} \\ \{2\ \ 1\} & \{2\ \ 2\} & \{2\ \ 3\} & \{2\ \ 4\} & \{2\ \ 5\} & \{2\ \ 6\} \\ \{3\ \ 1\} & \{3\ \ 2\} & \{3\ \ 3\} & \{3\ \ 4\} & \{3\ \ 5\} & \{3\ \ 6\} \\ \{4\ \ 1\} & \{4\ \ 2\} & \{4\ \ 3\} & \{4\ \ 4\} & \{4\ \ 5\} & \{4\ \ 6\} \\ \{5\ \ 1\} & \{5\ \ 2\} & \{5\ \ 3\} & \{5\ \ 4\} & \{5\ \ 5\} & \{5\ \ 6\} \\ \{6\ \ 1\} & \{6\ \ 2\} & \{6\ \ 3\} & \{6\ \ 4\} & \{6\ \ 5\} & \{6\ \ 6\} \end{pmatrix}$$

S=6x6=36
Favourable outcome for sum=2 (E1)= $\begin{pmatrix} \{1\ \ 1\} \end{pmatrix}$

$$E1=1 \tag{1.3.2.1.1}$$

$$Probabilty(P(E1))=\frac{1}{36}=0.027 \tag{1.3.2.1.2}$$

Favourable outcome for sum=3 (E2)= $\begin{pmatrix} \{1\ \ 2\} & \{2\ \ 1\} \end{pmatrix}$

$$E2=2 \tag{1.3.2.1.3}$$

$$Probabilty(P(E2))=\frac{2}{36}=0.055 \tag{1.3.2.1.4}$$

Favourable outcome for sum=4 (E3)= $\begin{pmatrix} \{1\ \ 3\} & \{2\ \ 2\} & \{3\ \ 1\} \end{pmatrix}$

$$E3=3 \tag{1.3.2.1.5}$$

$$Probabilty(P(E3))=\frac{3}{36}=0.083 \tag{1.3.2.1.6}$$

Favourable outcome for sum=5 (E4)= $\begin{pmatrix} \{1\ \ 4\} & \{2\ \ 3\} & \{3\ \ 2\} & \{4\ \ 1\} \end{pmatrix}$

$$E4=4 \tag{1.3.2.1.7}$$

$$Probabilty(P(E4))=\frac{4}{36}=0.111 \tag{1.3.2.1.8}$$

Favourable outcome for sum=6 (E5)=

$$\left(\begin{Bmatrix}1 & 5\end{Bmatrix} \quad \begin{Bmatrix}2 & 4\end{Bmatrix} \quad \begin{Bmatrix}3 & 3\end{Bmatrix} \quad \begin{Bmatrix}4 & 2\end{Bmatrix} \quad \begin{Bmatrix}5 & 1\end{Bmatrix}\right)$$

$$E5 = 5 \tag{1.3.2.1.9}$$

$$Probabilty(P(E5)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.10}$$

Favourable outcome for sum=7 (E6)= $\left(\begin{Bmatrix}1 & 6\end{Bmatrix} \quad \begin{Bmatrix}2 & 5\end{Bmatrix} \quad \begin{Bmatrix}3 & 4\end{Bmatrix} \quad \begin{Bmatrix}4 & 3\end{Bmatrix} \quad \begin{Bmatrix}5 & 2\end{Bmatrix} \quad \begin{Bmatrix}6 & 1\end{Bmatrix}\right)$

$$E6 = 6 \tag{1.3.2.1.11}$$

$$Probabilty(P(E6)) = \frac{6}{36} = 0.166 \tag{1.3.2.1.12}$$

Favourable outcome for sum=8 (E7)= $\left(\begin{Bmatrix}2 & 6\end{Bmatrix} \quad \begin{Bmatrix}3 & 5\end{Bmatrix} \quad \begin{Bmatrix}4 & 4\end{Bmatrix} \quad \begin{Bmatrix}5 & 3\end{Bmatrix} \quad \begin{Bmatrix}6 & 2\end{Bmatrix}\right)$

$$E7 = 5 \tag{1.3.2.1.13}$$

$$Probabilty(P(E7)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.14}$$

Favourable outcome for sum=9 (E8)= $\left(\begin{Bmatrix}3 & 6\end{Bmatrix} \quad \begin{Bmatrix}4 & 5\end{Bmatrix} \quad \begin{Bmatrix}5 & 4\end{Bmatrix} \quad \begin{Bmatrix}6 & 3\end{Bmatrix}\right)$

$$E8 = 4 \tag{1.3.2.1.15}$$

$$Probabilty(P(E8)) = \frac{4}{36} = 0.111 \tag{1.3.2.1.16}$$

Favourable outcome for sum=10 (E9)= $\left(\begin{Bmatrix}4 & 6\end{Bmatrix} \quad \begin{Bmatrix}5 & 5\end{Bmatrix} \quad \begin{Bmatrix}6 & 4\end{Bmatrix}\right)$

$$E9 = 3 \tag{1.3.2.1.17}$$

$$Probabilty(P(E9)) = \frac{3}{36} = 0.083 \tag{1.3.2.1.18}$$

Favourable outcome for sum=3=11 (E10)= $\left(\begin{Bmatrix}5 & 6\end{Bmatrix} \quad \begin{Bmatrix}6 & 5\end{Bmatrix}\right)$

$$E10 = 2 \tag{1.3.2.1.19}$$

$$Probabilty(P(E10)) = \frac{2}{36} = 0.055 \tag{1.3.2.1.20}$$

Favourable outcome for sum=12 (E11)= $\left(\begin{Bmatrix}6 & 6\end{Bmatrix}\right)$

$$E11 = 1 \tag{1.3.2.1.21}$$

$$Probabilty(P(E11)) = \frac{1}{36} = 0.027 \tag{1.3.2.1.22}$$

Table is completed as follows:

Event:'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

a) The argument mentioned by the student is incorrect.
In the question the event of measurement is sum of possible outcomes of rolling two dice. Here, the probability of occurence of each outcome is not equal.
The different values of probabilities are mentioned in the above solution.
The argument can be supported by the figure 1.3.2.1
The python code for the figure 1.3.2.1

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prob/codes/prob3.py
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shows the random distribution of data. The

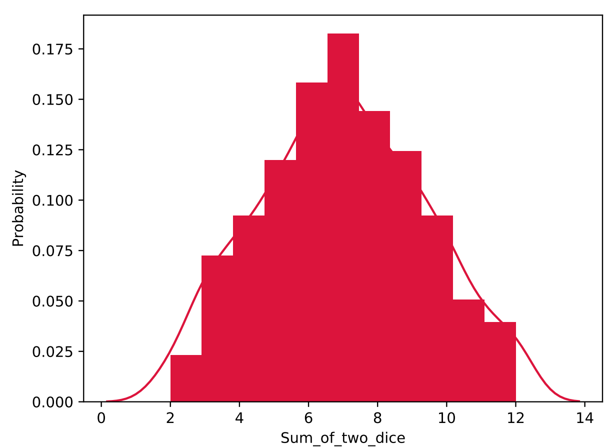

Fig. 1.3.2.1

Distribution of data is given below
Probability mass function(P(X=k))=

$$\begin{Bmatrix} \frac{k-1}{36} & for x < 8 \\ \frac{13-k}{36} & for x => 8 \end{Bmatrix} \tag{1.3.2.1.23}$$

The understanding of the curves 1.3.2.1 can be seen in (1.1.3.1b)