Document on Question 28 Exercise(8.1)

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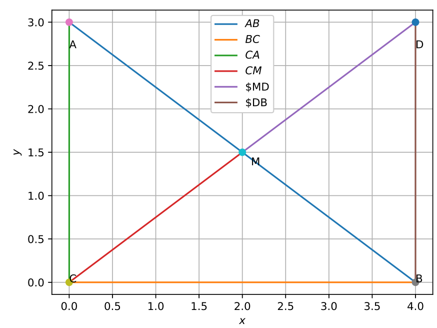
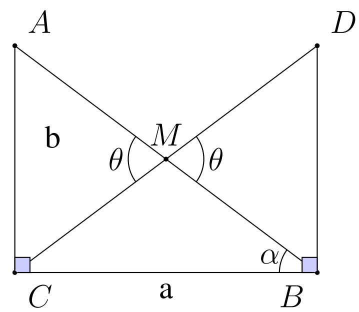
***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/document/codes
```



a) By Python



b) By Latex-tikz

Question

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:
a)$\triangle AMC \cong \triangle BMD$
b)$\triangle DBC$ is a right angle.
c)$\triangle DBC \cong \triangle ABC$
d)CM = $\frac{1}{2}$ AB

Construction

The python code for the figure is

```
./code/traingle.py
```

The latex- tikz code is

```
./figs/triangle.tex
```

The above latex code can be compiled as standalone document

```
./figs/triangle_fig.tex
```

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Initial Input Values.	
a	4
b	3
$\angle(ACB)$	$90°$

TABLE I: To construct $\triangle ACB$

The steps for constructing $\triangle ACB$ are
(i) Let

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(ii)

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

(iii)

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Since, $\mathbf{M}$ is the midpoint of $\mathbf{AB}$ and $\mathbf{CD}$

$$\mathbf{M} = (1/2)(\mathbf{A} + \mathbf{B})$$

$$\mathbf{M} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

$$\mathbf{D} = 2\mathbf{M}$$

$$\mathbf{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Derived Values.	
$\mathbf{M}$	$\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$
$\mathbf{D}$	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

TABLE II: To construct $\triangle DCB$

Solution

From the figure, lets assume $\mathbf{C}$ to be the origin.

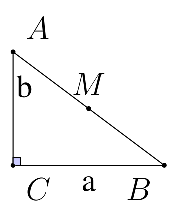


Fig. 1: $\triangle ACB$

$\mathbf{C} = 0$
$\|\mathbf{CA}\|$=b
$\|\mathbf{CB}\|$=a
$\mathbf{M}$ is the position vector of mid-point of $\mathbf{BA}$.
$\mathbf{CM} = \mathbf{CB} + \mathbf{BM}$ [$\mathbf{BM} = (1/2) * \mathbf{BA}$]

$$\mathbf{CM} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -a \\ b/2 \end{pmatrix}$$

Therefore,

$$\mathbf{CM} = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

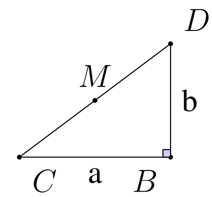


Fig. 2: $\triangle DBC$

From the figure, $\mathbf{CD} = 2(\mathbf{CM})$

$$\mathbf{CD} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Sol.a)

$\triangle AMC$ and $\triangle DMB$ are congruent to each other by SAS congruency.
(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]
(ii)Side CM of is equal to corresponding side DM [As M is midpoint of DC]
(iii)$\angle AMC$ = $\angle DMB$ [Vertically Opposite Angles]
Hence, proved

Sol.b)

In $\triangle ACB$ $(\|\mathbf{BA}\|)^2 = a^2 + b^2$ Since $\angle ACB$ = $90°$[Pythagorus theorem]
In $\triangle DBC$ $cos\angle DBC = [((a^2 + b^2 - (\|\mathbf{CD}\|)^2)/2ab)]$ With the given vector values we get norm of $(\|\mathbf{BA}\|) = (\|\mathbf{CD}\|)$
$cos\angle DBC = [((a^2 + b^2 - (\|\mathbf{CD}\|)^2)/2ab)]$
$cos\angle DBC$=0
Therefore, $\angle DBC$ is right angle

Sol.c)

$\triangle ACB$ and $\triangle DCB$ are congruent to each other in SAS congruency.
(i)Both the triangles have a common base , a.
(ii)AC = DB by using distance formula
(iii)$\angle ACB$ = $\angle DBC$ = $90°$ [From Solution b)]
Hence, proved.

Sol.d)

Since $\mathbf{CM}$ is halfway of $\mathbf{CD}$
$\|\mathbf{CM}\| = \|\mathbf{CD}\|$
From Solution b) it is clear that $\|\mathbf{CD}\| = \|\mathbf{BA}\|$
Therefore $\|\mathbf{CM}\| = \frac{1}{2}\|\mathbf{AB}\|$
Hence, proved.