Document on Question 28 Exercise(8.1)

Pothukuchi Siddhartha

***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/document/codes
```

Question

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:

a)$\triangle AMC \cong \triangle BMD$

b)$\triangle DBC$ is a right angle.

c)$\triangle DBC \cong \triangle ABC$

d)CM = $\frac{1}{2}$ AB

Construction

The python code for the figure is

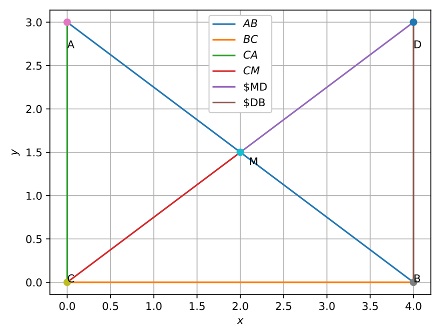
```
./code/traingle.py
```

The latex- tikz code is

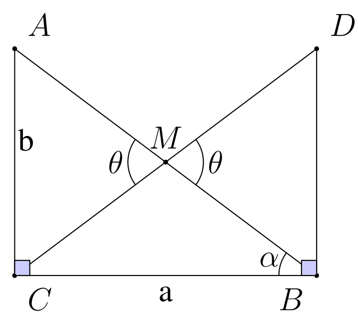
```
./figs/triangle.tex
```

The above latex code can be compiled as standalone document

```
./figs/triangle_fig.tex
```



a) By Python



b) By Latex-tikz

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Initial Input Values.	
$\vec{BC}(a)$	$4\hat{i}$
$\vec{AC}(b)$	$3\hat{j}$
$\angle(ACB)$	$90°$

To construct $\triangle ACB$

Derived Values.	
$\vec{CM}$	$2\hat{i} + 1.5\hat{j}$
$\vec{CD}$	$4\hat{i} + 3\hat{j}$

To construct $\triangle DCB$

Solution

From the figure, lets assume $\vec{C}$ to be the origin.

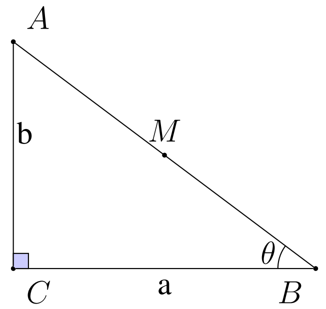


Fig. 1: $\triangle ACB$

$\vec{C} = 0$
$\vec{CA} = b\hat{j}$
$\vec{CB} = a\hat{i}$
$\vec{M}$ is the position vector of mid-point of $\vec{BA}$.
$\vec{CM} = \vec{CB} + \vec{BM}$ $[\vec{BM} = (1/2) * \vec{BA}]$
$\vec{CM} = a\hat{i} + (1/2)(b\hat{j} - a\hat{i})$
Therefore, $\vec{CM} = (1/2)(b\hat{j} + a\hat{i})$

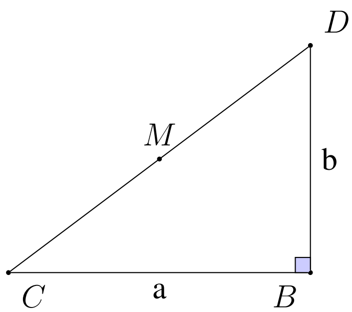


Fig. 2: $\triangle DBC$

From the figure, $\vec{CD} = 2(\vec{CM})$
$\vec{CD} = a\hat{i} + b\hat{j}$

Sol.a)

$\triangle AMC$ and $\triangle DMB$ are congruent to each other by SAS congruency.
(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]
(ii)Side CM of is equal to corresponding side DM [As M is midpoint of DC]
(iii)$\angle AMC$ = $\angle DMB$ [Vertically Opposite Angles]

Sol.b)

In $\triangle ACB$ $(\left\|\vec{BA}\right\|)^2 = a^2 + b^2$ Since $\angle ACB$ = 90°[Pythagorus theorem]
In $\triangle DBC$ $cos\angle DBC = [((a^2 + b^2 - (\left\|\vec{CD}\right\|)^2)/2ab)]$ With the given vector values we get norm of $(\left\|\vec{BA}\right\|) = (\left\|\vec{CD}\right\|)$
$cos\angle DBC = [((a^2 + b^2 - (\left\|\vec{CD}\right\|)^2)/2ab)]$
$cos\angle DBC$=0
Therefore, $\angle DBC$ is right angle

Sol.c)

$\triangle ACB$ and $\triangle DCB$ are congruent to each other in SAS congruency. (i)Both the triangles have a common base , a.
(ii)AC = DB by using distance formula
(iii)$\angle ACB$ = $\angle DBC$ = 90° [From Solution b)]

Sol.d)

Since $\vec{CM}$ is halfway of $\vec{CD}$
$\left\|\vec{CM}\right\| = \left\|\vec{CD}\right\|$
From Solution b) it is clear that $\left\|\vec{CD}\right\| = \left\|\vec{BA}\right\|$
Therefore $\left\|\vec{CM}\right\| = \frac{1}{2}\left\|\vec{AB}\right\|$