Document on Question 28 Exercise(8.1)

Pothukuchi Siddhartha

Abstract

This a simple document explaining a question about the concept of similar triangles.

Download all python codes from

```
svn co https://github.com/SiddharthPh/Summer2020/trunk/document/codes
```

Question

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:
a)$\triangle AMC \cong \triangle BMD$
b)$\triangle DBC$ is a right angle.
c)$\triangle DBC \cong \triangle ABC$
d)CM = $\frac{1}{2}$ AB

Solution

The python code for the figure is

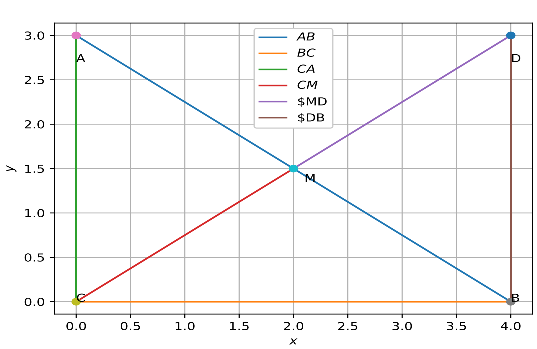
```
./code/traingle.py
```

The latex- tikz code is

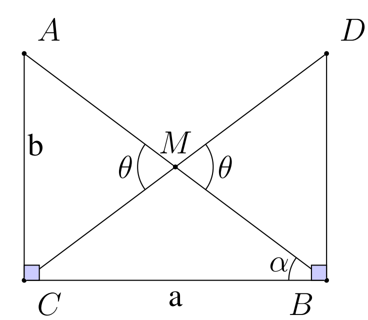
```
./figs/triangle.tex
```

The above latex code can be compiled as standalone document

```
./figs/triangle_fig.tex
```



(a) By Python



(b) By Latex-tikz

Fig. 1: Obtained by executing the above codes.

Sol.a)

From the above figure,

$$C = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 0 \\ b \end{pmatrix}, B = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

As, M is the midpoint of AB

$$M = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

Therefore Coordinates of D are

$$D = \begin{pmatrix} a \\ b \end{pmatrix}$$

$\triangle AMC$ and $\triangle DMB$ are congruent to each other by SAS congruency.
(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]
(ii)Side CM of is equal to corresponding side DM [As M is midpoint of DC]
(iii)$\angle AMC$ = $\angle DMB$ [Vertically Opposite Angles]

Sol.b)

In $\triangle ACB$
$(AB)^2 = a^2 + b^2$ Since $\angle ACB$ = 90°[Pythagorus theorem]

In $\triangle DBC$ [DB = D-B = b]

$cos\angle DBC = [((a^2 + b^2 - (DC)^2)/2ab)]$

By using distance formula i.e $\sqrt{(x1 - x2)^2 + (y1 - y1)^2}$ we get that AB=DC from the given coordinates.
$cos\angle DBC = [((a^2 + b^2 - (AB)^2)/2ab)]$
$cos\angle DBC = 0$
Therefore, $\angle DBC$ is right angle

Sol.c)

$\triangle ACB$ and $\triangle DCB$ are congruent to each other in SAS congruency. (i)Both the triangles have a common base , a.
(ii)AC = DB by using distance formula
(iii)$\angle ACB$ = $\angle DBC$ = 90° [From Solution b]

Sol.d)

Since M is the midpoint of CD
CM=$\frac{1}{2}$ DC From Solution b) it is clear that DC=AB
Therefore CM=$\frac{1}{2}$AB
Hence Proved.