

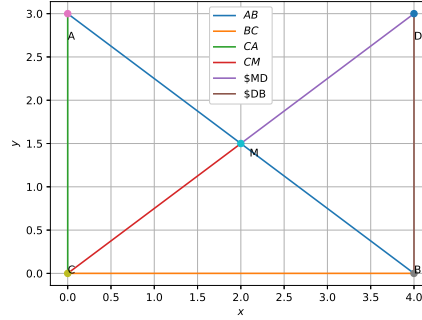
# Document on Question 28 Exercise(8.1)

Pothukuchi Siddhartha

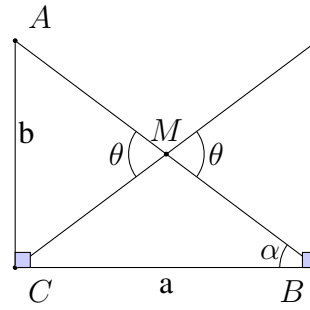
**Abstract**—This a simple document explaining a question about the concept of similar triangles.

Download all python codes from

svn co <https://github.com/SiddharthPh/Summer2020/trunk/document/codes>



a) By Python



b) By Latex-tikz

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:

- $\triangle AMC \cong \triangle BMD$
- $\triangle DBC$  is a right angle.
- $\triangle DBC \cong \triangle ABC$
- $CM = \frac{1}{2} AB$

## CONSTRUCTION

The python code for the figure is

`./code/traingle.py`

The latex- tikz code is

`./figs/triangle.tex`

The above latex code can be compiled as standalone document

`./figs/triangle_fig.tex`

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Initial Input Values.	
a	4
b	3
$\angle(ACB)$	$90^\circ$

TABLE I: To construct  $\triangle ACB$

The steps for constructing  $\triangle ACB$  are  
(i) Let

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(ii)

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

(iii)

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Since, M is the midpoint of AB and CD

$$\mathbf{M} = (1/2)(\mathbf{A} + \mathbf{B})$$

$$\mathbf{M} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

$$\mathbf{D} = 2\mathbf{M}$$

$$\mathbf{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Derived Values.	
M	$\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

TABLE II: To construct  $\triangle DCB$

### SOLUTION

From the figure, lets assume C to be the origin.

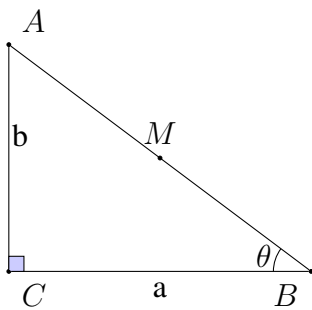


Fig. 1:  $\triangle ACB$

$$\mathbf{C} = 0$$

$$\|\mathbf{CA}\| = b$$

$$\|\mathbf{CB}\| = a$$

M is the position vector of mid-point of BA.

$$\mathbf{CM} = \mathbf{CB} + \mathbf{BM} \quad [\mathbf{BM} = (1/2) * \mathbf{BA}]$$

$$\mathbf{CM} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -a/2 \\ b/2 \end{pmatrix}$$

Therefore,

$$\mathbf{CM} = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

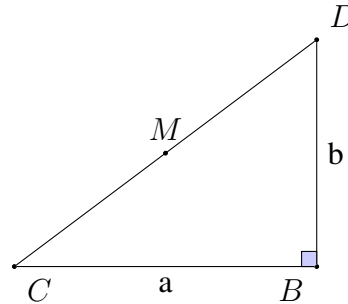


Fig. 2:  $\triangle DCB$

From the figure,  $\mathbf{CD} = 2(\mathbf{CM})$

$$\mathbf{CD} = \begin{pmatrix} a \\ b \end{pmatrix}$$

### Sol.a)

$\triangle AMC$  and  $\triangle DMB$  are congruent to each other by SAS congruency.

(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]

(ii) Side CM is equal to corresponding side DM [As M is midpoint of DC]

(iii)  $\angle AMC = \angle DMB$  [Vertically Opposite Angles]  
Hence, proved

### Sol.b)

In  $\triangle ACB$   $(\|\mathbf{BA}\|)^2 = a^2 + b^2$  Since  $\angle ACB = 90^\circ$  [Pythagorus theorem]

In  $\triangle DBC$   $\cos \angle DBC = [((a^2 + b^2 - (\|\mathbf{CD}\|)^2)/2ab)]$  With the given vector values we get norm of  $(\|\mathbf{BA}\|) = (\|\mathbf{CD}\|)$

$$\cos \angle DBC = [((a^2 + b^2 - (\|\mathbf{CD}\|)^2)/2ab)]$$

$$\cos \angle DBC = 0$$

Therefore,  $\angle DBC$  is right angle

### Sol.c)

$\triangle ACB$  and  $\triangle DCB$  are congruent to each other in SAS congruency.

(i) Both the triangles have a common base, a.

(ii)  $AC = DB$  by using distance formula

(iii)  $\angle ACB = \angle DBC = 90^\circ$  [From Solution b)]

Hence, proved.

***Sol.d)***

Since CM is halfway of CD

$$\|\mathbf{CM}\| = \|\mathbf{CD}\|$$

From Solution b) it is clear that  $\|\mathbf{CD}\| = \|\mathbf{BA}\|$

$$\text{Therefore } \|\mathbf{CM}\| = \frac{1}{2} \|\mathbf{AB}\|$$

Hence, proved.