

## Maths for Programmers

### Tips:

- (i) Stay Calm :- Don't panic, don't stop listening, you are not being graded.
- (ii) Rewind :- you will ~~learn~~ learn much faster
- (iii) Explain :- try to explain the material out loud to yourself, a friend, or a rubber ducky.

### Discrete Maths :-

Deals with <sup>discrete</sup> finite or ~~continuous~~ sets of elements rather than ~~infinite~~ continuous or infinite sets of elements.

eg:- a program running infinite steps of execution is approximated from continuous sets to discrete sets.

eg of infinite set is circle.

sets: collection of distinct objects called elements or members.

$e(\text{symbol})$  :-  $u_2$  an element of

Common sets:

$\phi = \{\}$  null set

$N = \{1, 2, 3, \dots\}$

$N_0 = \{0, 1, 2, 3, \dots\}$

$Z = \{-2, -1, 0, 1, 2, \dots\}$

$(\dots)$   $\Rightarrow$  ellipsis, indicate progression

## Union Operations:

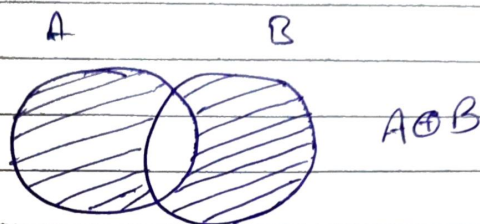
Symmetric differences  $\oplus$

$$A \oplus B = (A \setminus B) \cup (B \setminus A)$$

$$\mathbb{R} \setminus \mathbb{Q} = \mathbb{I}$$

Real  $\hookrightarrow$  rational no.  $\hookrightarrow$  irrational no.

## Venn Diagrams



Proper Subsets: For a set  $B$ , all elements of  $B$  are in set  $A$  and there are some elements of  $A$  not in  $B$ .  
denotation:-  $C \subset \mathcal{U}$

Complement:  $B^c = \{x \in \mathcal{U} : x \notin B\}$   
or  $B^c \hookrightarrow$  universal set

Denotation:- subset:  $C \subseteq \mathcal{U}$

proper subset:  $C \subset \mathcal{U}$

Idempotence:  
 $A \cup A = A$   
 $A \cap A = A$

Identities: performing operations on a set with a given identity element.

$$A \cup \phi = A \quad (A \text{ union null set})$$

$$\mathcal{U} \cup A = \mathcal{U}$$

$$A \cap \mathcal{U} = A$$

$$\phi \cap A = \phi$$

Law of Involution:

$$(A^c)^c = A$$

Cyclical nature of function.

Rational no.: — a no. that can be represented as the quotient of two integers such that  $q \neq 0$

Any rational no. (that is, a fraction in lowest terms) can be written as either a terminating or a repeating decimal.

eg:  $10x = 9.999...$

$$x = 0.999...$$

$$10x - x = 9x = 9$$

$$x = 1$$

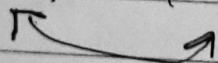
$$10x = 10$$

$$\mathbb{Q} = \{ a/b : a, b \in \mathbb{Z}, b \neq 0 \}$$

Distributivity:

Case 1:  $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Case 2:



De Morgan's Law:

$$(A \cup B)^c \subseteq A^c \cap B^c$$

~~$$(A \cup B)^c \subseteq A^c \cap B^c$$~~

$$(A \cap B)^c = A^c \cup B^c$$

Logic: "Book of Proof" by Hammack covers foundational mathematics.

An excerpt: Logic is a systematic way of thinking that allows us to deduce new info from old info & to parse the meaning of sentences.

logic  $\rightarrow$  math  $\rightarrow$  algorithm  $\rightarrow$  code



Proposition : It is simply a declarative statement with a verifiable truth value. Denoted by lowercase letters.

$P =$  "Rain falls from the sky"

$7 = 5 + 83$

$q = \alpha$

$q =$  "Pune is in USA"

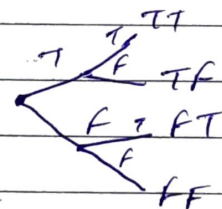
Composite Proposition : They are made up of subproposition.  
Conjunction & Disjunction.

AND,  $P \wedge q$

OR,  $P \vee q$

Truth Tables:

P	q	$P \wedge q$	$P \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F



Idempotence : & Identity

Primitive proposition

P	$P \vee P$	$P \wedge P$	T	F	$P \vee F$	$T \vee P$	$P \wedge T$	$P \wedge F$
T	T	T	T	F	T	F	T	F
F	F	F	T	F	F	T	F	F

$$P \equiv P \vee P \equiv P \wedge P$$

$$P \equiv P \vee F \equiv P \wedge T$$

$$T \equiv P \vee T = F \equiv P \wedge F$$

Complements:

$$P \vee \neg P \equiv T$$

$$T \equiv \neg F$$

$$P \wedge \neg P \equiv F$$

$$F \equiv \neg T$$

negation of read as "not P"

Involution:  $(\neg \neg) P \rightarrow P$  logical equivalent

Tautology :- always true

Fallacy always false

Conditional Statements: a conditional statement contains a hypothesis and a conclusion. These are more formally known as antecedent and a consequent.

conditional:  $P \Rightarrow Q$  ( $P$  implies  $Q$ )

Converse:  $q \Rightarrow p$

inverse :  $\neg p \Rightarrow \neg q$

contrapositive:  $\neg q \Rightarrow \neg p$  (logically equivalent to original conditional)

## Logical Quantifiers:

Propositional function:  $P(x)$

(takes on a value true or false for everything fed to it)

Universal:  $\forall$  (for every)  $(\forall x \in \mathcal{X}) p(x); \forall x p(x)$

Existential:  $\exists$  (there exists)  $(\exists x \in \mathcal{U}) p(x)$   
 $\Downarrow$   
 at least one  $x$

Tautologies:

law of excluded middle:  $P \vee \neg P$

Law of Contradictions :  $\neg (P \wedge \neg P)$

Modus Tollens :  $[(P \Rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$   
 ("Denying the consequent")