Falcon 9 Landing Trajectory Optimization:

Fuel and Performance Considerations

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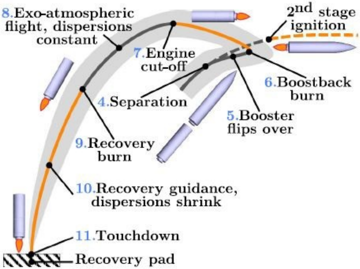
Introduction

- Reusable rocket boosters, such as SpaceX Falcon 9, have revolutionized space transportation by significantly reducing launch costs.
- A key challenge is ensuring precise and safe landings while minimizing fuel consumption for maximum payload capacity.
- Powered descent is the most fuel-critical phase, requiring accurate thrust control for deceleration and trajectory correction.
- Optimization of the thrust magnitude and direction profile can significantly improve landing accuracy and fuel efficiency.
- This study analyzes **fuel and performance optimization strategies** for the final powered descent segment from approximately 2 km altitude down to touchdown (phases 9-11: recovery burn to touchdown).



Problem Statement

Workflow: **Problem Definition** → *Assumptions* → *Implementation* → *Results*

- This case study analyzes **fuel and performance optimization strategies** for Falcon 9 powered descent phase.
- Study isolates the final powered descent from ∼2 km altitude to landing - the most fuel-critical segment.
- Goal: determine optimal thrust profiles using two optimization objectives:
  - **Fuel-Optimal:** Minimizes propellant consumption as primary objective
  - **Performance-Optimal:** Prioritizes landing accuracy (1000×) with secondary time minimization (0.1×)
- Both formulations target identical landing objectives:
  1. Pinpoint landing: position error = 0
  2. Soft landing: final velocities = 0 (with -2 m/s vertical touchdown)
  3. Path constraints: thrust limits, angle limits, altitude > 0
- Initial conditions derived from integrated full recovery trajectories (80–120 km separation through landing approach).
- **Analysis examines trade-offs between fuel efficiency and landing performance strategies.**

Assumptions

Workflow: Problem Definition → **Assumptions** → *Implementation* → *Results*

The following assumptions simplify the powered descent problem and define the scope of the analysis.

1. **2D point-mass dynamics:** (x-horizontal, z-vertical). Rotational dynamics and trim controls (grid fins) used in higher altitudes are omitted [1,2].
2. **Constant gravity field:** $g = 9.81\ m/s^2$ [1].
3. **Atmospheric model:** Exponential density $\rho(z) = \rho_0 e^{-z/H}$, consistent with [1,2].
4. **Quadratic drag:** $D = 0.5 \rho C_D A v^2$. Lift is neglected [1].
5. **Perfect thrust control:** Instantaneous thrust magnitude and gimbal angle control [1].
6. **No disturbances:** Wind and sensor errors ignored [1].
7. **Initial conditions:** Derived from full-phase trajectory studies (80–120 km separation through re-entry to 2 km powered descent start) [1,2].

Note: These assumptions simplify the real system and may affect the fidelity of results.

Initial Conditions (Powered Descent Start at 2 km)

Values are chosen consistently with Falcon 9–style parameters reported in [1][2][3]:

Parameter	Symbol	Value
Horizontal position	x_0	$-100\,m$
Vertical position	z_0	$2000\,m$
Horizontal velocity	v_{x0}	$30\,m/s$
Vertical velocity	v_{z0}	$-236\,m/s$
Initial mass	m_0	$80,000\,kg$
Final conditions	$x_f, z_f, v_{x_f}, v_{z_f}$	0
Minimum final mass	m_f	$\geq 25,000\,kg$

Note: These conditions are consistent with full recovery trajectories from separation to 2 km [1][2]. At powered descent start, horizontal position is generally close to zero and horizontal velocity is nearly zero due to aerodynamic and grid-fin corrections [3][4]. For modeling purposes, however, non-zero initial conditions are used: horizontal position $-100\,m$ and horizontal velocity $30\,m/s$ to represent realistic trajectory dispersions and provide a more challenging optimization scenario.

Model Parameters

The following parameters are used consistently with Falcon 9–style specifications from the literature:

Parameter	Symbol	Value	Description	Ref
Specific impulse	I_{sp}	$350\,s$	Engine efficiency metric	[2]
Standard gravity	g_0	$9.81\,m/s^2$	Reference gravitational accel.	-
Gravitational accel.	g	$9.81\,m/s^2$	Local gravity field	-
Sea-level air density	ρ_0	$1.225\,kg/m^3$	Atmospheric density at $z = 0$	[1]
Atmospheric scale height	H	$8400\,m$	Exponential decay parameter	[1]
Drag coefficient	C_D	1.64	Aerodynamic drag parameter	[2]
Reference area	A	$33.18\,m^2$	Vehicle cross-sectional area	[2]
Minimum thrust	T_{min}	$200\,kN$	Lower thrust bound	[2]
Maximum thrust	T_{max}	$900\,kN$	Upper thrust bound	[2]
Max thrust angle	θ_{max}	15°	Thrust vectoring limit	[5]
Glide slope limit	γ_{max}	30°	Trajectory constraint	[1]
Dry mass	m_{dry}	$25,000\,kg$	Vehicle empty mass	[2]

State and Control Variables

State Vector:

- $\mathbf{x}(t), \mathbf{z}(t)$ – horizontal and vertical positions $[\mathrm{m}]$
- $\mathbf{v_x}(t), \mathbf{v_z}(t)$ – horizontal and vertical velocities $[\mathrm{m/s}]$
- $\mathbf{m}(t)$ – vehicle mass $[\mathrm{kg}]$

$$\mathbf{x}(t) = [x(t), z(t), v_x(t), v_z(t), m(t)]^T$$

Control Vector:

- $\mathbf{T}(t)$ – thrust magnitude $[\mathrm{N}]$
- $\theta(t)$ – thrust angle (from vertical) $[\mathrm{deg}]$

$$\mathbf{u}(t) = [T(t), \theta(t)]^T$$

System Dynamics

Equations of Motion:

$$\dot{x}(t) = v_x(t) \qquad \text{(horizontal position update)}$$

$$\dot{z}(t) = v_z(t) \qquad \text{(vertical position update)}$$

$$\dot{v}_x(t) = \frac{T(t)\sin(\theta(t))}{m(t)} - \frac{D(t)v_x(t)}{m(t)v(t)} \qquad \text{(horizontal acceleration)}$$

$$\dot{v}_z(t) = \frac{T(t)\cos(\theta(t))}{m(t)} - g - \frac{D(t)v_z(t)}{m(t)v(t)} \qquad \text{(vertical acceleration)}$$

$$\dot{m}(t) = -\frac{T(t)}{I_{sp}g_0} \qquad \text{(mass flow rate)}$$

Drag and Atmosphere Models:

$$D(t) = \frac{1}{2}\rho(z(t))C_D A v(t)^2, \quad \rho(z) = \rho_0 e^{-z/H}, \quad v(t) = \sqrt{v_x(t)^2 + v_z(t)^2}$$

Optimal Control Problem

Formulation 1 - Fuel-Optimal:

$$\min_{T(t),\theta(t)} \quad -m(t_f) \quad \text{(maximize final mass)}$$

Formulation 2 - Performance-Optimal:

$$\min_{T(t),\theta(t)} \quad 1000 \cdot \|e_{terminal}\|^2 + 0.1 \cdot t_f$$

where $e_{terminal} = [x(t_f), z(t_f), v_x(t_f), v_z(t_f) + 2]^T$

Boundary Conditions (Target for Both):

$$\begin{aligned}
\mathbf{x}(\mathbf{0}) &= -100\,m, & \mathbf{x}(\mathbf{t_f}) &= 0 \\
\mathbf{z}(\mathbf{0}) &= 2000\,m, & \mathbf{z}(\mathbf{t_f}) &= 0 \\
\mathbf{v_x}(\mathbf{0}) &= 30\,m/s, & \mathbf{v_x}(\mathbf{t_f}) &= 0 \\
\mathbf{v_z}(\mathbf{0}) &= -236\,m/s, & \mathbf{v_z}(\mathbf{t_f}) &= -2\,m/s \\
\mathbf{m}(\mathbf{0}) &= 30,000\,kg, & \mathbf{m}(\mathbf{t_f}) &\geq 25,000\,kg
\end{aligned}$$

Path Constraints:

$$\mathbf{T}_{\min} \leq T(t) \leq \mathbf{T}_{\max}, \quad |\theta(\mathbf{t})| \leq \theta_{\max}$$
$$z(t) \geq 0$$

Decision Variables: $T(t)$ (thrust magnitude), $\theta(t)$ (thrust angle)

Key Difference: Fuel-optimal minimizes propellant consumption; Performance-optimal prioritizes landing accuracy (1000$\times$) with secondary time minimization (0.1$\times$)
Implementation Note: Convergence tolerances adjusted for numerical stability while maintaining identical problem objectives

Implementation Approach

Numerical solution methodology for dual trajectory optimization formulations.

Goal: Robust numerical framework for fuel and performance optimization analysis.

Workflow: Problem Definition → *Assumptions* → **Implementation** → *Results*

Software Framework:

- **Platform:** Python with CasADi for symbolic optimization
- **Solver:** IPOPT with gradient-based scaling and adaptive barrier strategy
- **Method:** Direct collocation (Euler integration, $N = 35$)
- Multi-scale variable normalization for numerical conditioning

Implementation Approach (continued)

Constraint implementation and convergence strategies for reliable optimization results.

Constraint Implementation:

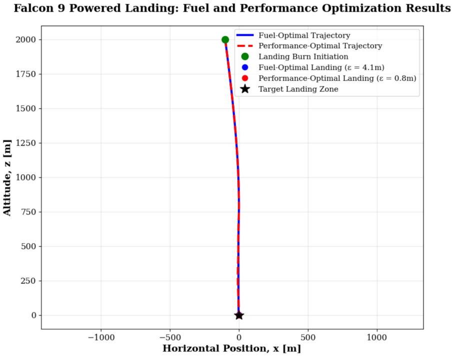
- **Control bounds:** Thrust [200, 900] kN, gimbal angle ± 15
- **Landing targets:** Position $\pm$3m, velocity $\pm$3m/s, soft touchdown (-2 m/s)
- **Mass requirements:**
  - Fuel-optimal: Final mass $\geq 25{,}000$ kg (strict)
  - Performance-optimal: Final mass $\geq 24{,}700$ kg (300kg relaxation)
- **Operational limits:** Flight time 15-45s, altitude ≥ 0

Convergence Strategy:

- **Physics-based initialization:** Three-phase trajectory (correction $\rightarrow$ approach $\rightarrow$ landing)
- **Formulation-specific profiles:** Conservative (0.3-0.8 $T_{\max}$) vs Aggressive (0.9 $T_{\max}$)
- **Constraint tolerance:** Performance-optimal uses $1.8\times$ relaxation factor
- **Solver settings:** 3000 iterations max, $1\text{e-}4$ optimality tolerance

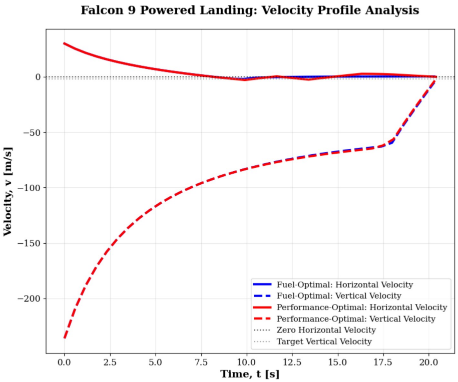
Trajectory Analysis

- Both trajectories initiate from 2 km altitude with identical initial conditions and follow controlled descent profiles.
- **Fuel-optimal approach:** Conservative strategy achieving 4.08 m landing accuracy with minimal propellant usage (1677 kg consumed).
- **Performance-optimal approach:** Precision-focused strategy achieving 0.75 m landing accuracy with 1718 kg fuel consumption.
- Landing precision improvement of 5.4× achieved with 41 kg (2.4%) additional propellant expenditure.
- Flight duration nearly identical: 20.30s (fuel-optimal) vs 20.39s (performance-optimal).
- **Analysis:** Fuel-optimal prioritizes efficiency, while performance-optimal prioritizes precision landing.



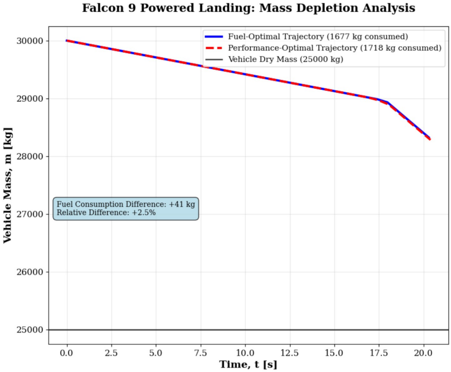
Velocity Profile Analysis

- Both formulations achieve progressive velocity reduction from initial conditions (30 m/s horizontal, -236 m/s vertical) to soft-landing targets.
- **Fuel-optimal approach:** Smooth, gradual velocity changes prioritizing propellant conservation throughout descent phase.
- **Performance-optimal approach:** More aggressive deceleration patterns, particularly in vertical velocity management for enhanced terminal precision.
- Horizontal velocity nullified by both approaches within 7-8 seconds, maintaining near-zero lateral motion thereafter.
- Vertical velocity profiles demonstrate controlled deceleration toward -2 m/s soft-landing target with similar overall patterns.



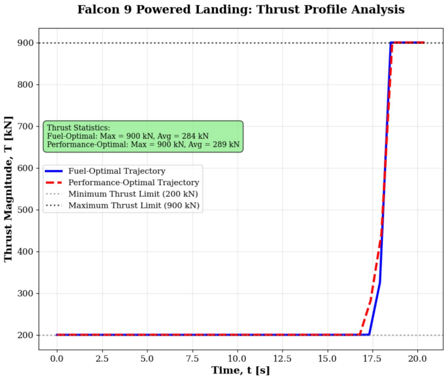
Mass Depletion Profile Analysis

- Both formulations demonstrate steady propellant consumption patterns from initial mass (30,000 kg) through landing sequence.
- **Fuel-optimal approach:** Consumed 1,677 kg propellant over 20.30s flight duration, maintaining substantial safety margin above dry mass.
- **Performance-optimal approach:** Utilized 1,718 kg propellant during 20.39s flight, representing 41 kg (2.4%) additional consumption.
- Mass depletion profiles show nearly overlapping curves, indicating similar overall thrust utilization strategies between formulations.
- Both trajectories maintain operational safety margins with final masses well above 25,000 kg dry mass constraint.



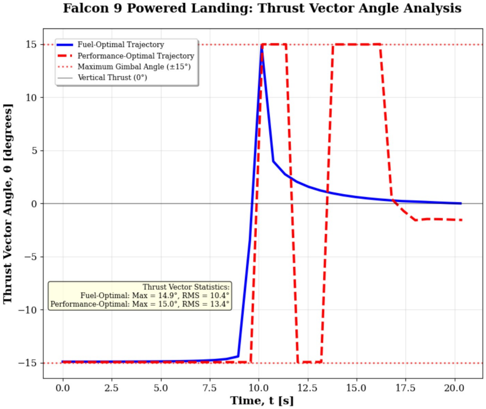
Thrust Magnitude Profile Analysis

- Both formulations operate within engine operational limits $200\text{ kN} \leq T \leq 900\text{ kN}$ throughout landing sequence.
- **Fuel-optimal approach:** Maintains minimum thrust (200 kN) for $\sim$18s, then maximum thrust for final approach. Average thrust ≈ 284 kN.
- **Performance-optimal approach:** Similar two-phase strategy with earlier transition to maximum thrust. Average thrust ≈ 289 kN ($+5$ kN difference).
- Maximum thrust (900 kN) commanded in final $\sim$3s by both formulations to achieve soft landing velocity targets.
- Performance-optimal reaches high thrust marginally sooner with extended maximum thrust duration for enhanced terminal precision.



Thrust Vector Angle Profile Analysis

- Both formulations operate within gimbal angle constraints ($\pm15°$) with 0° representing vertical thrust orientation throughout descent.
- **Fuel-optimal approach:** Conservative thrust vector profile with smooth correction maneuver peaking at 14.9°, RMS angle of 10.4°.
- **Performance-optimal approach:** Aggressive bang-bang control strategy utilizing maximum gimbal authority (15.0°) with higher control activity, RMS angle of 13.4°.
- Fuel-optimal demonstrates continuous, smooth profile minimizing thrust vector deflections while achieving adequate lateral velocity correction.
- Performance-optimal exhibits discrete switching pattern with rapid transitions between constraint limits for enhanced trajectory precision.



Conclusion

This slide presents the key findings from the powered descent optimization analysis. *Workflow: Problem Definition → Assumptions → Implementation →* **Results** In this

case study, we formulated and analyzed two optimization approaches for powered descent of a reusable rocket booster using a 2D point-mass model with quadratic drag and gimbaled thrust. The study examined fuel-optimal and performance-optimal strategies while satisfying state safety constraints and control limits. **Key Findings:**

- **Fuel Consumption Analysis:**
  - Fuel-optimal approach consumed **1,677 kg** propellant over **20.30 s** descent, achieving **28,323 kg** final mass (3,323 kg margin above dry mass).
  - Performance-optimal approach utilized **1,718 kg** (41 kg or 2.4% increase) over **20.39 s**, demonstrating minimal fuel penalty for enhanced precision.
- **Landing Precision Results:**
  - Fuel-optimal approach: **4.08 m** landing accuracy
  - Performance-optimal approach: **0.75 m** landing accuracy (**5.4× improvement**)

- **Trajectory & Velocity Analysis:**
  - Both formulations produce virtually identical descent trajectories and velocity profiles with overlapping deceleration patterns.
  - Enhanced precision achieved through refined final approach control execution rather than fundamentally different strategies.
- **Control Strategy Analysis:**
  - **Thrust magnitude:** Both implement two-phase strategy (200 kN minimum for 18 s, then 900 kN maximum) with performance-optimal showing extended high-thrust duration.
  - **Thrust vector angle:** Fuel-optimal uses smooth deflection (max 14.9°); performance-optimal employs aggressive bang-bang control with higher RMS activity (13.4° vs 10.4°).
- **Operational Insights:**
  - Both approaches demonstrate similar overall flight strategies with differences in terminal precision execution.
  - Performance-optimal achieves superior accuracy through intensive control authority utilization during final approach.

Overall Assessment: Analysis demonstrates that sub-meter terminal precision can be achieved with minimal additional fuel expenditure, providing valuable insights for precision landing mission planning and illustrating the effectiveness of optimal control methodologies for reusable launch vehicle trajectory optimization.

References

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