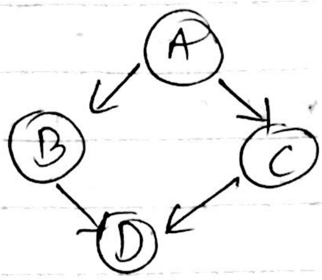
Part II

Q.1

1



	A	¬A
B	0.4	0.9
¬B	0.6	0.1

	A	¬A
C	0.6	0.2
¬C	0.4	0.8

	BC	¬BC	B ¬C	¬B ¬C
D	0.8	0.6	0.5	0.3
¬D	0.2	0.4	0.5	0.7

$$Pr(C \wedge D) = Pr(C=1, D=1)$$

$$= Pr(A=a, B=b, C=1, D=1)$$

$$= \sum_{a,b} Pr(A=a) \cdot P(B=b \mid A=a)\, P(C=1 \mid A=a)\, P(D=1 \mid B=b, C=1)$$

$$= \{ P(A=1) \cdot P(B=1 \mid A=1) \cdot P(C=1 \mid A=1)\, P(D=1 \mid B=1, C=1) \} + \{ P(A=0) \cdot P(B=1 \mid A=0) \cdot P(C=1 \mid A=0) \cdot$$

$+ \{ P(A=1) \cdot P(B=0 | A=1) \cdot P(C=1 | A=1)$
$P(D=1 | B=0, C=1) \}$
$+ \{ P(A=0) \cdot P(B=0 | A=0) \cdot P(C=1 | A=0)$
$P(D=1 | B=0, C=1) \})$

$= \{(0.8 * 0.4 * 0.6 * 0.5) +$
$(0.2 * 0.4 * 0.2 * 0.8) +$
$(0.8 * 0.6 * 0.6 * 0.6) +$
$(0.2 * 0.1 * 0.2 * 0.6)$

$= 0.1536 + 0.1728 + 0.0288 + 0.0024$

$= 0.3576$

Q.2 $P(HIV) = 0.05\% = 0.0005$
$P(+ | HIV) = 0.98$
$P(+ | \neg HIV) = 0.03$

→ Tom has tested positiv
$\therefore P(HIV | +) = ?$

$$P(HIV | +) = \frac{P(+ | HIV) \, P(HIV)}{P(+)} \quad \text{—— (1)}$$

$P(+) = P(+ | HIV) \, P(HIV) + P(+ | \neg HIV)$
$P(\neg HIV)$

$$= 0.98 * 0.0005 + 0.03 * 0.9995$$

$$= 0.00049 + 0.0291$$

$$= 0.0304 \longrightarrow (2).$$

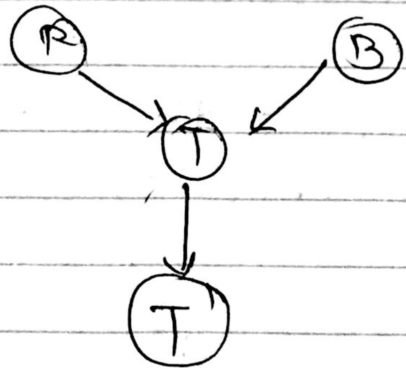
From (1) and (2),

$$P(HIV \mid +) = \frac{0.98 * 0.0005}{0.0304}$$

$$= 0.01067$$

$$= \underline{1.067\%}$$

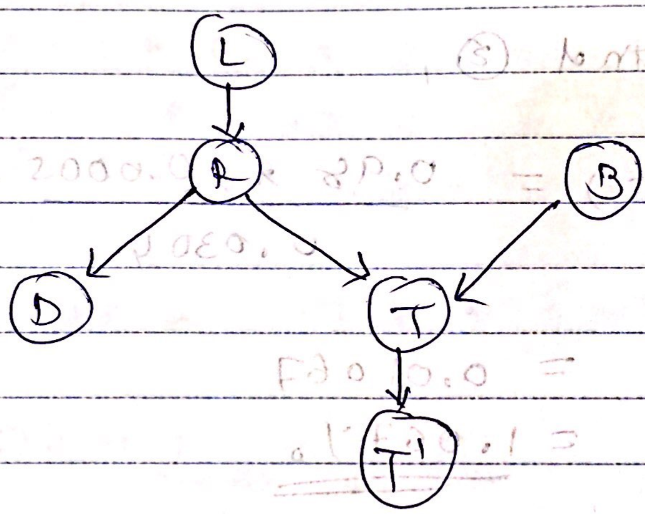
Q.3



* $R \perp\!\!\!\perp B$: conditionally independent as T is blocking node.
* $R \perp\!\!\!\perp B \mid T$: Not conditinally independent
  → There exist a path R – T – B

→ $R \perp\!\!\!\perp B \mid T'$: Not CI.
There is a path from R–T–B as T' is know, (we can say We have evidence of T'')

Q.4



→ $L \perp\!\!\!\perp T' \mid T$: Conditinally Independent

~~Yes~~ For sequencial path L–R–T–T'
T is given. So it is a blocking node.

→ $L \perp\!\!\!\perp B$: Conditinally independent

Here T is blocking node, so L and B are CI.

→ $L \perp\!\!\!\perp B \mid T$: Not conditinally independent
Here T is convegant of R and B
and T is evidence.

∴ ~~L T is~~ there is path L – R – T – B
∴ Not CI.

→ $L \perp\!\!\!\perp B \mid T'$: Not CI

Here, T' is given, so there is
path L – R – T – B.
∴ Not CI.

→ $L \perp\!\!\!\perp B \mid T, R$: Yes, Conditinal Independent

~~Yo~~ Here, L → R → T
∴ R is sequential and has evidence
So it is conditionally independent

* Though R → T → B is not CI.
But over all it is blocked at node R

∴ It is CI.

5(a) Given: H is totally Independent

(a) $n = 20$ $m = 2$

$\therefore$ distribution $= n * m = 20 * 2 = \underline{\underline{40}}$

(b) $n = 20$ $m = 5$

$\therefore$ distribution $= n * m = 5 * 20 = 100$

(c) $n = 500$ $m = 10$

$\therefore$ distribution $= 10 * 500 = 5000$.

5(b)

For 1 ~~parent~~ root node (variable) table size $= m$

For 1 node with single parent,

table size $= m^{1+1} = m^2$

For 2 node with two parents,

table size $= 2 * m^{2+1} = 2 * m^3$

For $(n-4)$ node with 3 parents,

table size $= (n-4) * m^{3+1} = (n-4) m^4$

Total table size $= m + m^2 + 2 * m^3 + (n-4) m^4$

d. $n = 20$ and $m = 2$

table size $= 2 + 2^2 + 2 * 2^3 + 16 * 2^4$
$= 278$

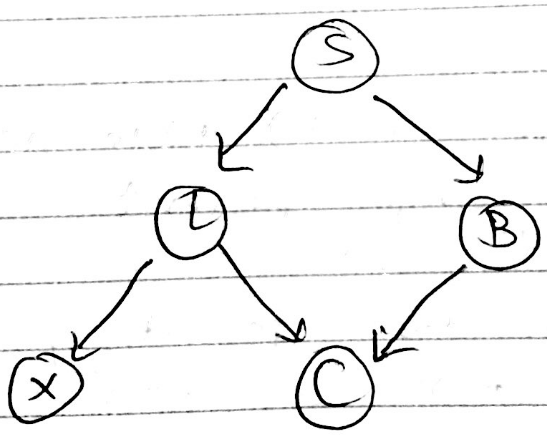
b. $n = 20$ and $m = 5$

table size $= 5 + 25 + 2 * 125 + 16 * 5^4$
$= 10,280$

c. $n = 500$ and $m = 10$

table size $= 10 + 100 + 2 * 1000 + 496 * 10^4$
$= 49,62,110.$

Q.6

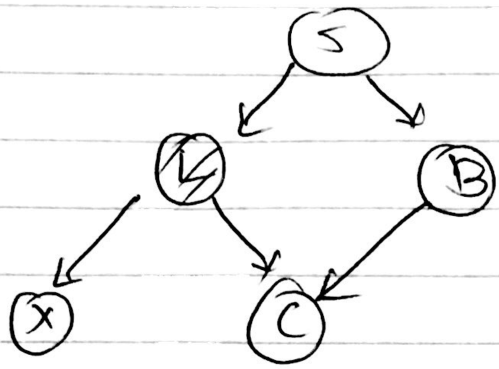


(a) S is known and it is divergent node. Therefore it is conditionally independent.

Here S is blocking.

$\therefore \{X, B\}$ and $\{I, B\}$ are CI

(b) L is known

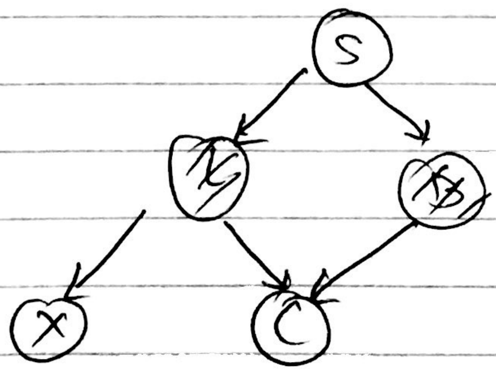


\# L is sequential and 

is divergent

$\therefore$ pairs $\{x, s\}$, $\{x, C\}$ and $\{x, B\}$ are independent (CI).

(c)



* 

Divergent   Sequential

$\therefore$ $\{s, x\}$, $\{x, c\}$ and $\{s, c\}$ are CI

Q.7

(1)

$p(lo \mid io) = 0.5$

$p(lo \mid \sim io) = 0.05$

$p(io = yes) = 0.6$

$p(io = no) = 0.4$

$p(co \mid io) = 0.8$

$p(co \mid \sim io) = 0.1$

(2) $$P(LO \mid CO) = \frac{P(lo, co)}{P(co)}$$

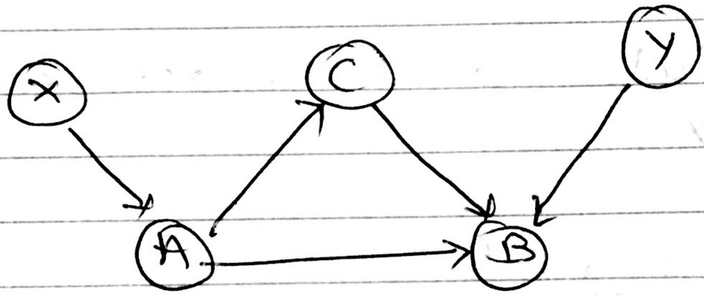
$$= \frac{P(lo, co \mid io)\, P(io) + P(lo, co \mid \sim io)\, P(\sim io)}{P(co)}$$

$$= \frac{P(10|i0)\, P(c0|i0)\, P(i0) + P(10|-i0)\, P(c0|-i0)\, P(-i0)}{P(c0)}$$

$$= \frac{0.5 * 0.8 * 0.6 + 0.05 * 0.1 * 0.4}{0.52}$$

$$= \underline{\underline{0.465}}$$

<u>Q.8</u>



* Set of all nodes that d-separate X and Y.

(a) Checking if A d-separates X & Y

There is a path (1):

X – A – C – B – Y.

path is blocked by B.

* path (2)

X – A – B – Y

path is blocked at A

∴ node A de-separate X & Y.

(ii) <u>Node B</u>

There is a path
X − A − B − Y which is blocked
at A.

path(2) : X − A − C − B − Y is blocked
at B

i.e. Node B does not de-separate
X & Y.

(iii) <u>For node C</u>

path (1) : X − A − C − B − Y
path is blocked at A
path (2) : X − A − B − Y
path is blocked at B

∴ Node C de-separate X and Y.

(iv) check for node {A, B}

There exist a path between X-Y

X − A − C − B − Y is not de-separated.

(v) Check for node A & C
All paths are blocked hence its d-separated.

(vi) check for node B, C
There exists path between X & Y
X - A - B - Y, hence its not d-separated.

(vii) Check for node A, B, C.
All path from X to Y are blocked hence set d-separated.

So, sets are
{A}, {C}, {A, C} and {A, B, C}.