Q1 Rmd

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Question 1

We are given a gamma distribution with density f, where

$$f(x) \propto x^5 e^{-x}, x > 0$$

Question 1.a, 1.b, 1.c, 1.d

We will implement the Metropolis-Hastings algorithm as a function where the proposal distribution can be specified. We will then run the algorithm to generate 10^4 samples each for the Log-Normal $LN(X_t, 1)$, the chi-squared $\chi^2_{\lfloor X_t+1 \rfloor}$, and the exponential Exp(1) distributions. The result of this is reported in Figures 1, 2 and 3 respectively.

In Figure 1, we see that the used Lognormal distribution has poor convergence qualities. It appears to spend many iterations rejecting new samples. In Figure 2 we see a seemingly much better result. The algorithm appears to accept most new samples and the samples fall into a fairly constant range throughout the sampling. When comparing the histograms of Figure 1 and 2, it is very clear that the lognormal proposal distribution leads to a heavily oversampled lower half of the true distribution (roughly the interval $[4, 5]$. This is also evident from the included true density curves. In Figure 3 we see a result that is very similar to that of Figure 2. The samples quickly fall into a fairly constant range of values, and the histogram of samples is very similar to the one generated from using the Chisquared proposal distribution in Figure 2. For all three proposal distributions, we have removed the first 50 generated values as an assumed burn-in period. That period is seemingly not long enough for the Lognormal proposal distribution, and clearly fully sufficient for the Chisquared and exponential proposal distributions.

Question 1.e, 1.f

One can estimate the expected value from each of the samples by averaging over them, for the log normal proposal, this yields 4.09. For the chi-squared proposal, this yields 6.08. For the exponential proposal, this yields 5.97. The Gamma distribution of interest can quickly be identified as having parameters $k = 6$, and $\theta = 1$, and thus expected value 6.

Old code

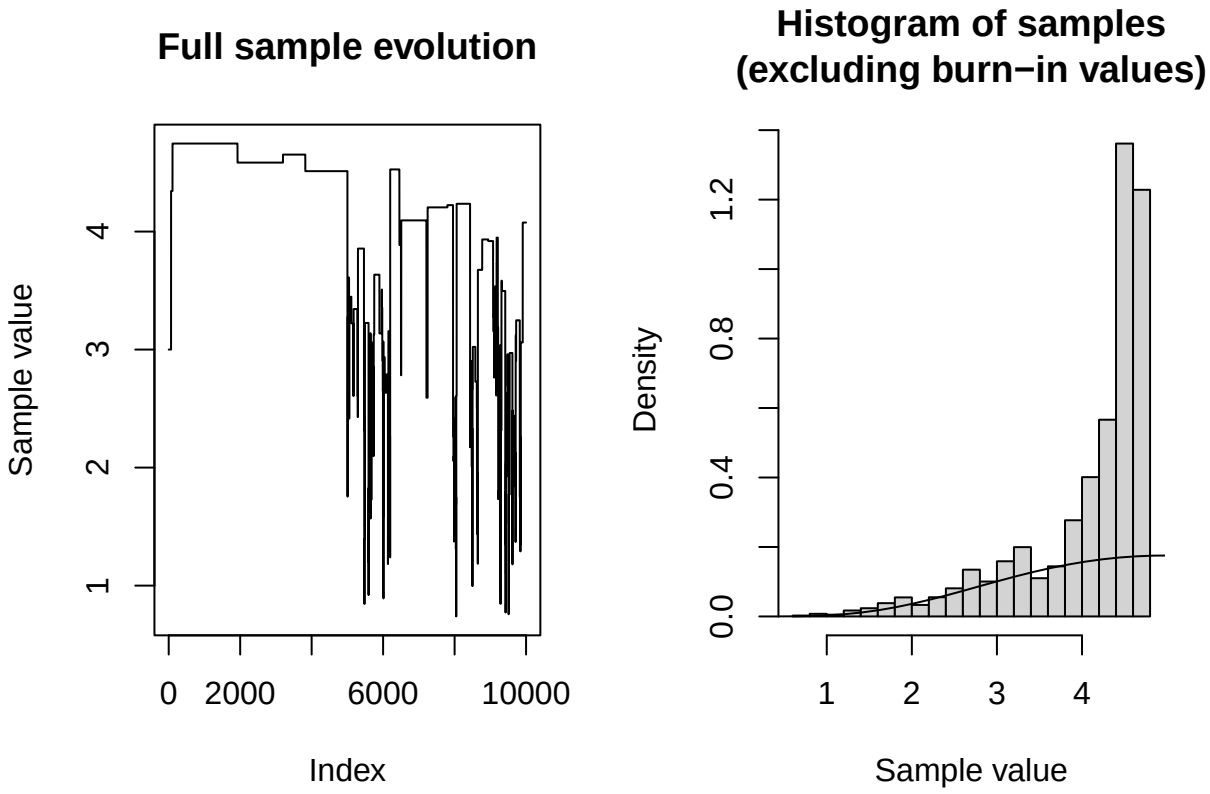

Figure 1: Lognormal porposal distribution

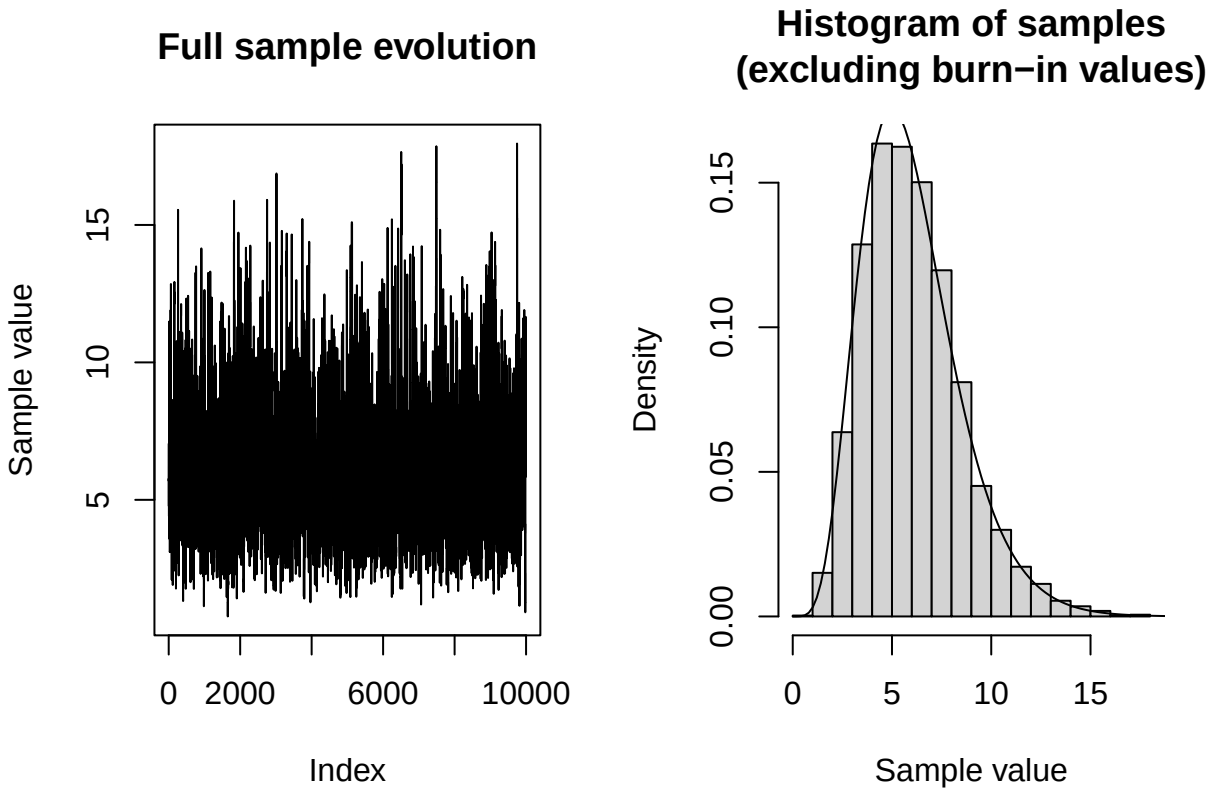

Figure 2: Chisquared proposal distribution

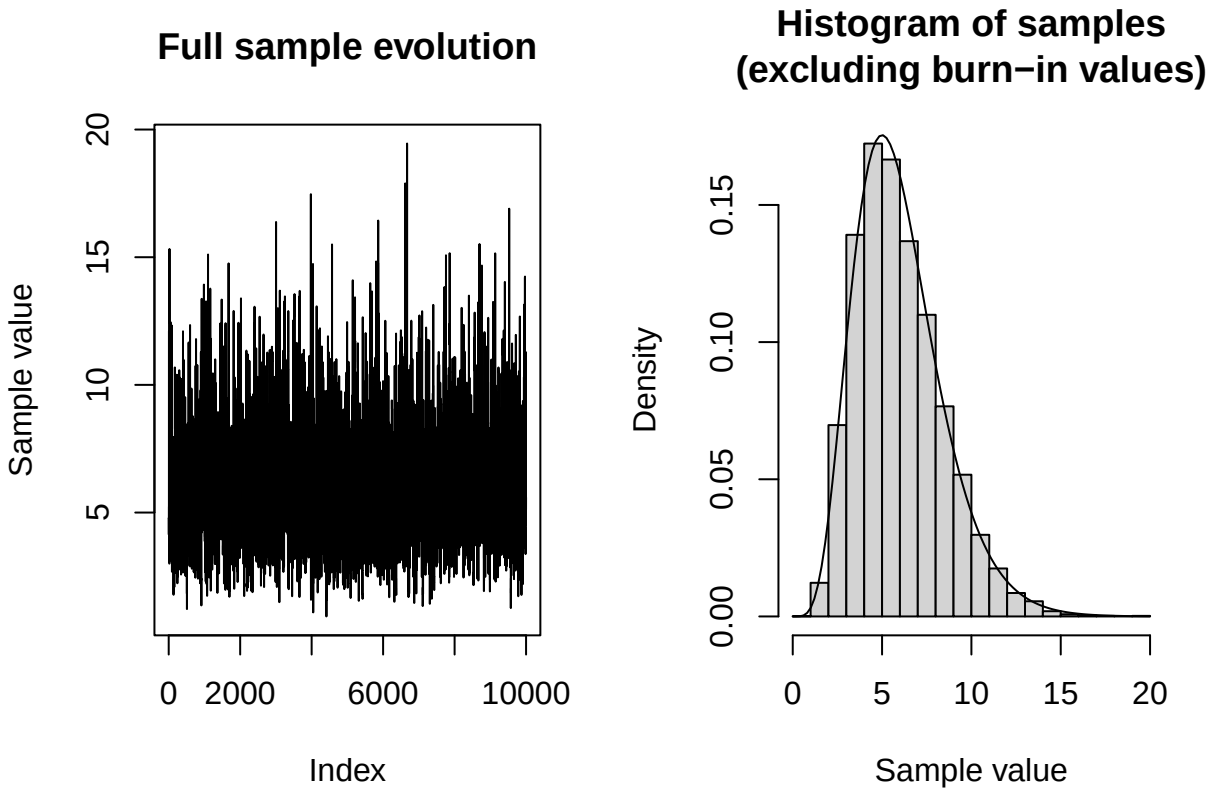

Figure 3: Exponential proposal distribution