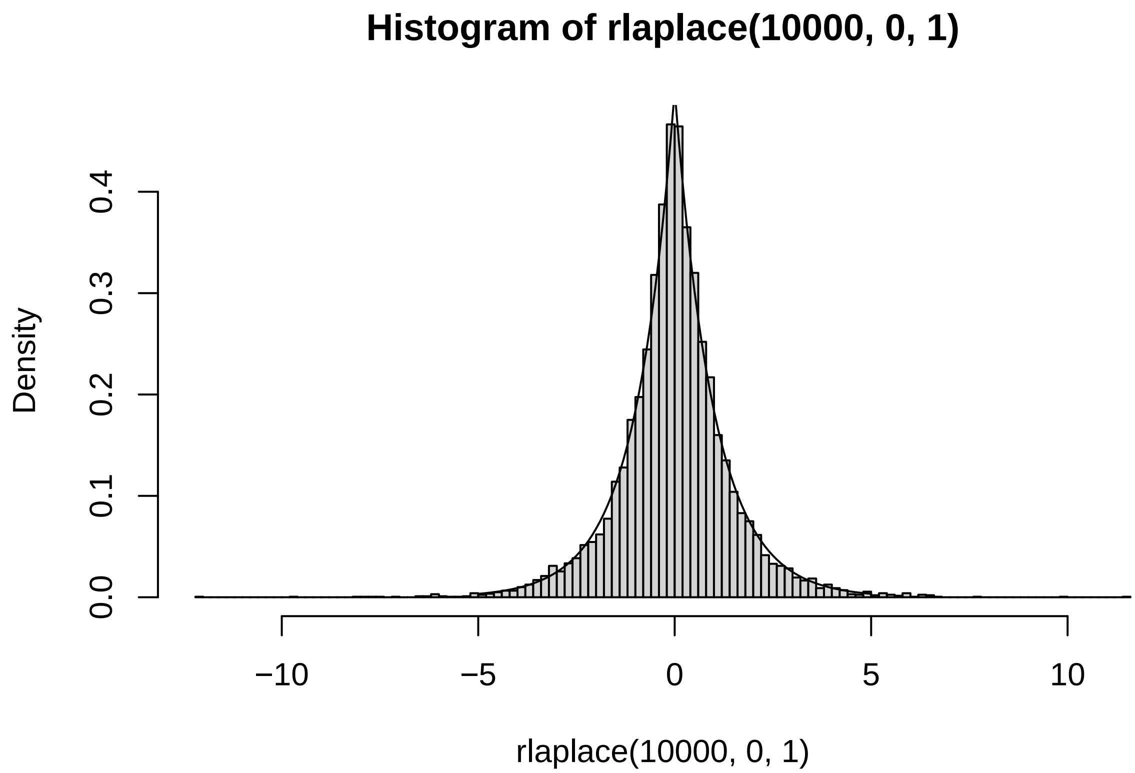
Q 2

Question 2

a

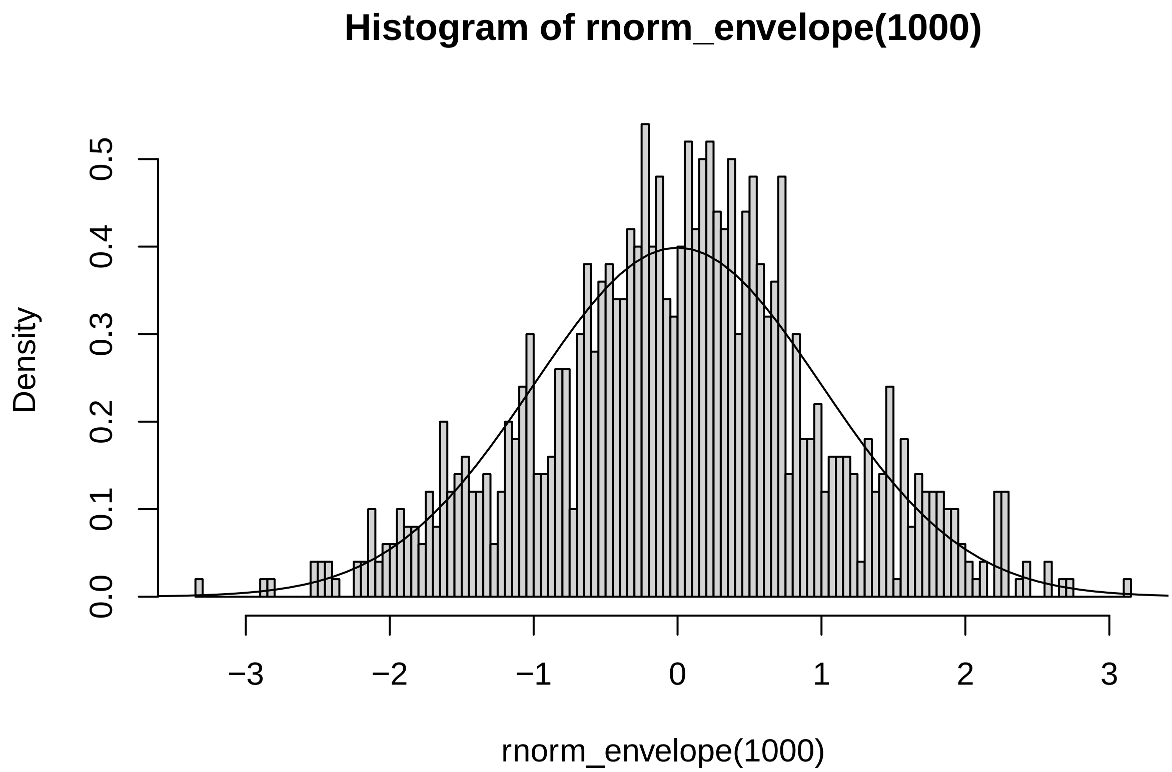
```r
# Inverse CDF formula from
# https://en.wikipedia.org/wiki/Laplace_distribution

# Generate laplace variates using inverse cumulative distribution function
rlaplace <- function(n=1, location=0, scale=1){
  output_vector <- c()
  u <- runif(n)
  for (i in 1:n){
    inverse_F <- location - scale*sign(u[i] - 0.5)*log(1-2*abs(u[i]-0.5))
    output_vector <- c(output_vector, inverse_F)
  }
  return(output_vector)
}

dlaplace <- function(x, location, scale){
  (scale / 2) * exp(-scale*abs(x-location))
}
```

```r
hist(rlaplace(10000, 0, 1), breaks=100, freq = F)
points(x = seq(-5, 5, 0.1), y=dlaplace(seq(-5, 5, 0.1), 0, 1), type="l")
```


Histogram of rlaplace(10000, 0, 1)
Density
0.0
0.1
0.2
0.3
0.4
−10
−5
0
5
10
rlaplace(10000, 0, 1)

Looks good.

b



c

We will now use the rejection sampling method (see reference) to generate standard normal variates. For the envelope we will use the Laplace distribution with location parameter 0, and scale parameter 1, scaled appropriately by a constant a. In order to find the optimal a, we construct the following expression and then derive the optimal (maximal) a that satisfies the inequality.

$$h(x) = \frac{g(x)}{f(x)} = \frac{1/2 \cdot \exp(-x)}{1/\sqrt{2\pi} \cdot \exp(-\frac{x^2}{2})} = \frac{\sqrt{\pi}}{\sqrt{2}} \cdot \exp(\frac{x^2}{2} - x) \geq a$$

$$h'(x) = \frac{\sqrt{\pi}}{\sqrt{2}} \cdot (x-1) \cdot \exp(\frac{x^2}{2} - x)$$

It is clear that $h(x)$ grows very large as x approaches $\pm\infty$, and thus the lone extrema at $x = 1$ is a global minima. This implies that the optimal a is

$$a = h(1) = \frac{\sqrt{\pi}}{\sqrt{2}} \cdot \exp(\frac{1^2}{2} - 1) \approx 0.76 \text{ (rounded down)}$$