Q1 Rmd

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Question 1

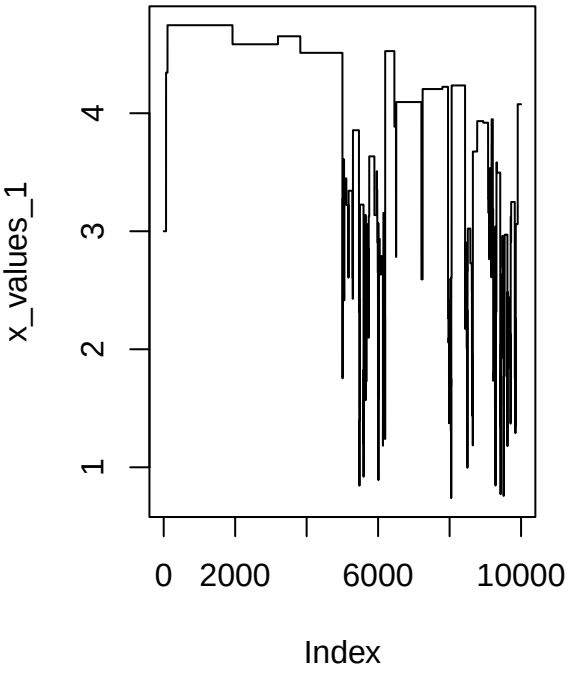
We are given a gamma distribution with density

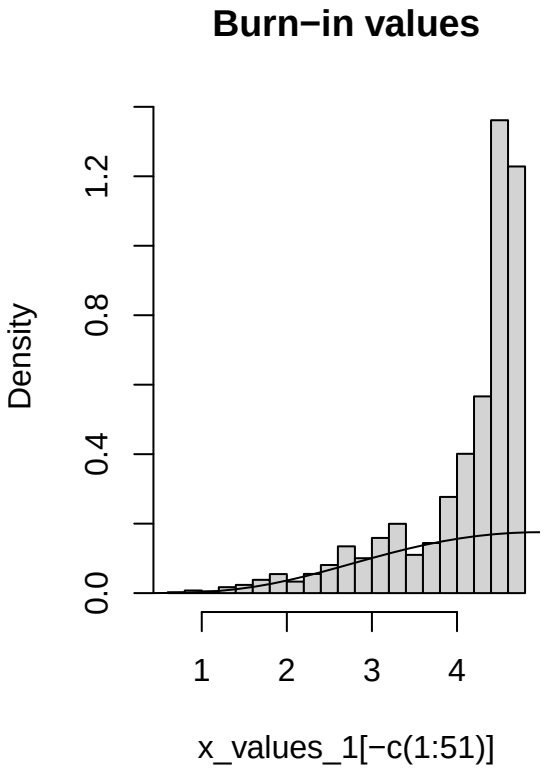
$$f(x) \propto x^5 e^{-x}, x > 0$$

Question 1.a, 1.b, 1.c

We will implement the Metropolis-Hastings algorithm as a function where the proposal distribution can be specified. We will then run the algorithm to generate 10^4 samples each for the Log-Normal $LN(X_t, 1)$, the chi-squared $\chi^2_{\lfloor X_t+1 \rfloor}$, and the THIS distributions. The result of this is reported in Figures 1, 2 and 3 respectively.

Question 1.d


x_values_1
1
2
3
4
0
2000
6000
10000
Index


Burn−in values
Density
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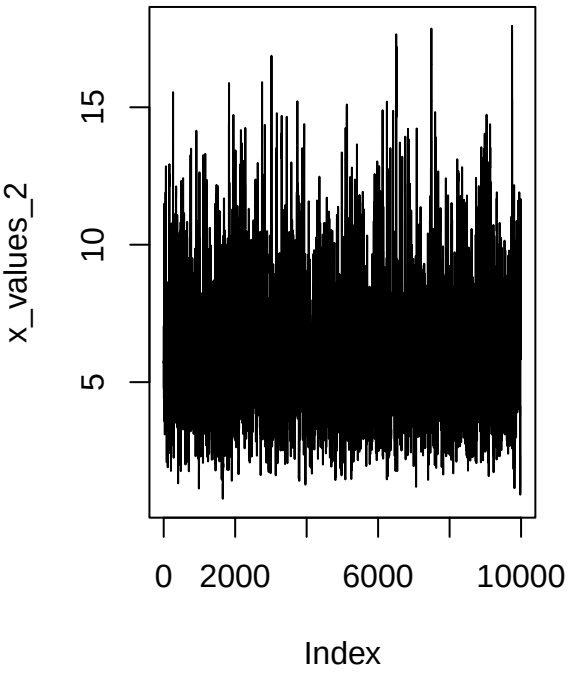
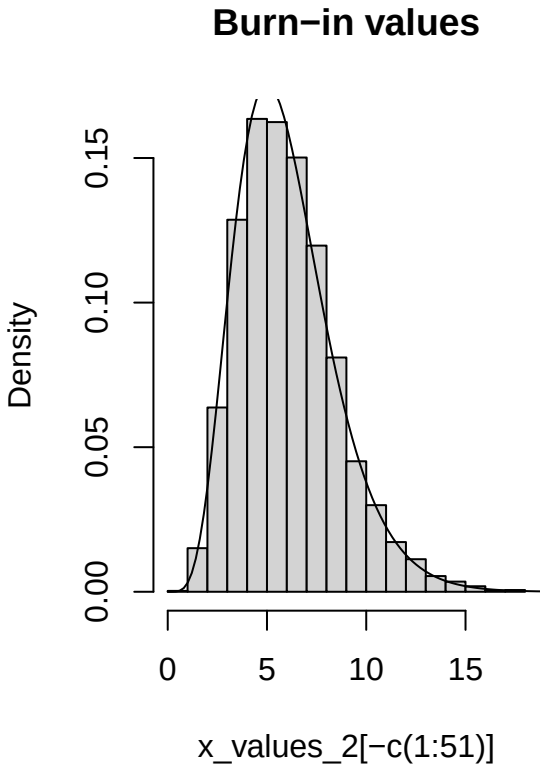
0.4
0.8
1.2
1
2
3
4
x_values_1[−c(1:51)]


x_values_2
5
10
15
0
2000
6000
10000
Index


Burn−in values
Density
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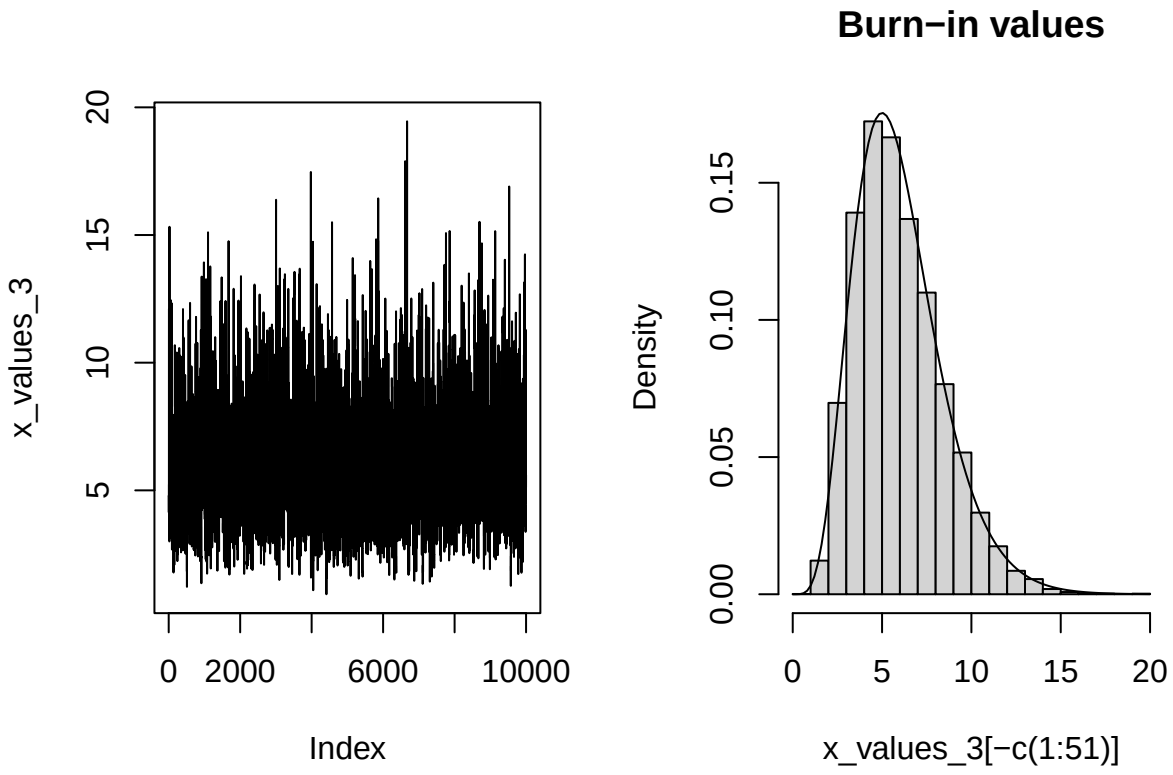
0.10
0.15
0
5
10
15
x_values_2[−c(1:51)]



Question 1.e, 1.f

One can estimate the expected value from each of the samples by averaging over them, for the log normal proposal, this yields 4.09. For the chi-squared proposal, this yields 6.08. For the exponential proposal, this yields 5.97. The Gamma distribution of interest can quickly be identified as having parameters $k = 6$, and $\theta = 1$, and thus expected value 6.

Old code