Lab 1 report

Simon Jorstedt & Siddhesh Sreedar

2023-11-05

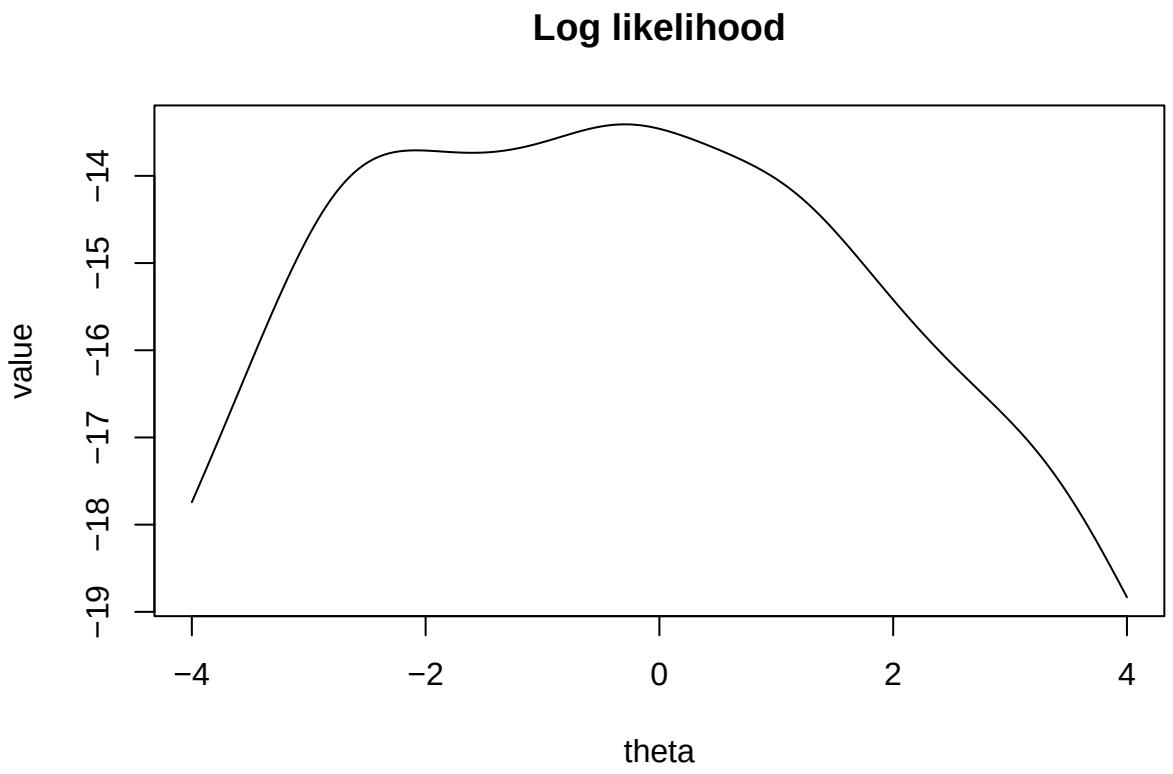
Question 1

```
# Problem setup

## Data parameter values
theta <- seq(-4, 4, 0.01)
x <- c(-2.8, 3.4, 1.2, -.3, -2.6)

# Log-likelihood function
loglike <- function(theta){-5*log(pi)-sum(log(1+(x-theta)^2))}

# Log-likelihood first derivative function
loglike_derivative <- function(theta){sum(2*(x-theta)/(1+(x-theta)^2))}
```

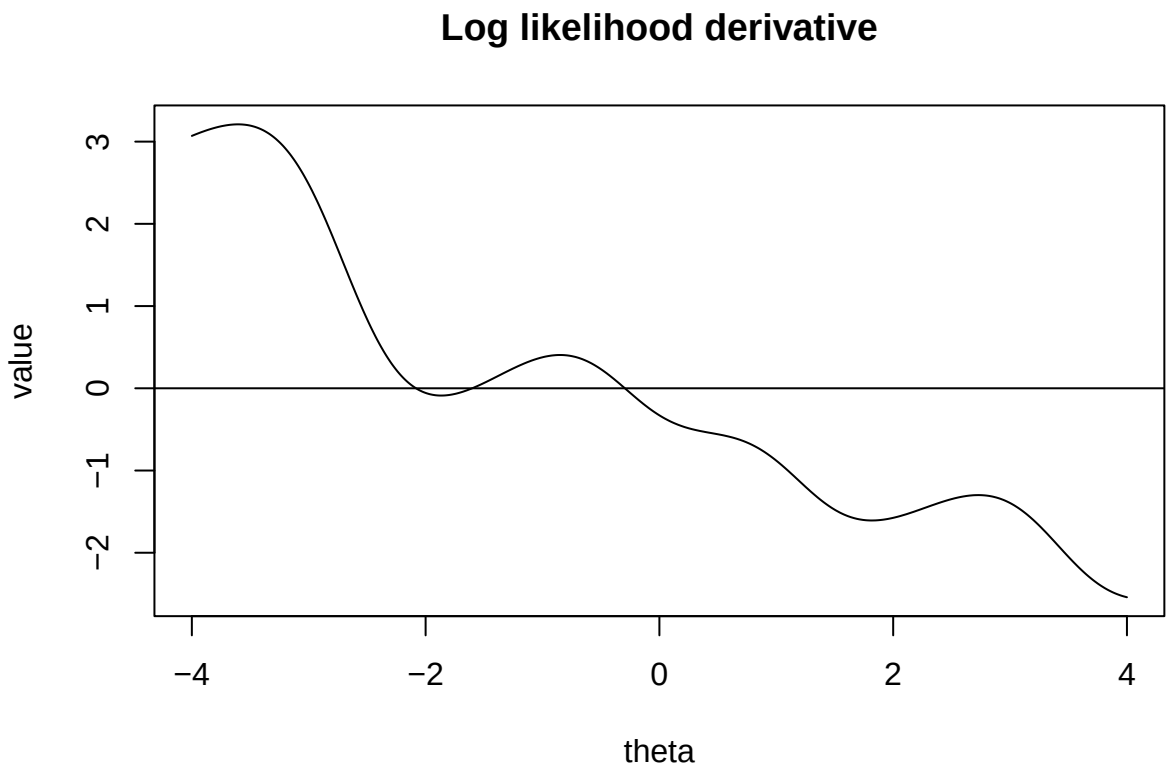
Subquestion a

> Plot the log likelihood function for the given data in the range from -4 to 4. Plot the derivative in the same range and check visually how often the derivative is equal to 0.

```
# Log likelihood plot
y_a1 <- c()
for (theta_i in theta){
  y_a1 <- c(y_a1, loglike(theta_i))
}

plot(theta,
     y_a1,
     type="l",
     main = "Log likelihood",
     xlab = "theta",
     ylab = "value")
```




```r
# Log likelihood derivative plot
y_a2 <- c()
for (theta_i in theta){
  y_a2 <- c(y_a2, loglike_derivative(theta_i))
}

plot(theta,
     y_a2,
     type="l",
     main = "Log likelihood derivative",
     xlab = "theta",
     ylab = "value")
abline(b=0, a=0)
```



The derivative crosses y=0 three times.

Subquestion b

```
# Bisection function

bisect_estimate <- function(a, b, epsilon = 0.000001){
  # a is the left start guess
  # b is the right start guess
  # epsilon is the desired convergence error

  y1 <- loglike_derivative(a)
  y2<- loglike_derivative(b)
  stopifnot("Invalid boundaries `a` and `b`" = y1 * y2 < 0)

  x_t <- c()
  # In the beginning, x_t is either empty or only has one value.
  # When x_t has at least two values, evaluate the last two values with
  # respect to the convergence error.
  while(length(x_t) < 2 || abs(tail(x_t,1)-tail(x_t,2)[1]) > epsilon){
    y1 <- loglike_derivative(a) #sum(2*(x-a)/(1+(x-a)^2))
    y2<- loglike_derivative(b) #sum(2*(x-b)/(1+(x-b)^2))

    # Calculate new boundary
```

```
    x_t <- c(x_t,(a + b)/2)

    if(loglike_derivative(a)*loglike_derivative(tail(x_t,1)) <= 0){
      b <- tail(x_t,1)
    }
    if(loglike_derivative(b)*loglike_derivative(tail(x_t,1)) < 0){
      a <- tail(x_t,1)
    }
  }
  cat("Estimate of x_value: ", tail(x_t, 1))
}
```

Subquestion c

The global maximum is achieved at about x = -0.2952452

Subquestion d

Question 2