Indian Institute of Technology Gandhinagar

MA 201 Project Report

Using Ordinary Differential Equations To Analyse Indian Stock Indices

16th November 2021

Arya | 20110025

Ayush Gupta | 20110031

Aditya Bhujbal | 20110040

Piyush Dhirwani | 20110138

Siddhesh Kanawade | 20110199

Contribution and Work Distribution

Throughout the project, we conducted multiple meetings to divide work distribution and responsibilities. All team members worked on all parts of the project throughout. Though the rough work distribution of project is as follows:

Arya - She worked on the documentation process of Model A and Model D. She also assisted in the derivation of Model A. She also helped in writing future development of the model.

Aditya - He worked on the derivation and execution of Model D. He worked on solving the ODE for Model 4. He further worked on the Kaggle dataset and used MS Excel for analysis and manipulation of the data and Tableau to plot and analyse the findings of Model 4. He also provided technical support to the team.

Ayush - He helped in the derivation of Model C. He further worked on the documentation and editing of the final report. He also helped in writing the future goals and preface of the project. He also provided assistance in the plotting of graphs using Tableau.

Piyush - He coordinated the entire project among all team members. He solved the differential equations and also worked on implementing the Model A and Model C. Along with this, he also proofread the final report and assisted in multiple stages of the project.

Siddhesh - He worked on solving and implementation of Model A and helped in the derivation of equations for Model D. He worked on writing the project report's introduction, preface and conclusion. He also was responsible for proofreading the project report and guided throughout its drafting process.

Preface

The main objective of this project is to study the ordinary differential equations in analysing the Indian Stock Indices and predict the stock indices movement. We will treat the stock indices as a dynamic system that iterates with time. We will also be using the logistic growth model, originally developed by mathematician Verhulst to predict population growth in predicting the indices depending on certain factors. We have used Kaggle's open-source dataset of BSE’s Sensex and NSE’s Nifty 50.

Through the development of the project, we had used Tableau (data analytics and visualisation) software to plot and compare graphs, an online graph plotting software to plot graphs and to solve complex differential equations which are out of our scope. We have tried to implement all techniques taught in course work to solve the differential equations. This project is the stepping stone in our advanced career.

Table of Contents

1. Introduction ……………………………………………………………………………..5
2. Derivation…………………………………………………………..................................6
    a. Model A……………………………………………………………………………..6
    b. Model B…………………………………………………………………………..…6
    c. Model C ………………………………………………………………………….8
    d. Model D……………………………………………………………………………10
3. Implementation ………………………………………………..11
    a. Model A……………………………………………………………………………11
    b. Model B……………………………………………………………………………12
    c. Model C……………………………………………………………………………12
    d. Model D ……………………………………………………………………………13
4. Conclusion……………………………………………………………………………….18
5. Future Development…………………………………………………………………...18
6. Acknowledgment………………………………………………………………………...19
7. References…..……………………………………………………………………………20

Introduction

Modelling stock prices is one of the major topics in the financial field.

In bull market, the stock prices continue to grow over a period of time, whereas in bear market the prices continue to decline. Bear market refers to the financial market that is currently on a downtrend which drops steeply as a result of the selling pressure. Mean reversion, in finance, suggests that various phenomena of interest such as asset prices and volatility of returns eventually revert to their long-term average levels. The mean reversion theory has led to many investment strategies, from stock trading techniques to options pricing models.[1]

In this project, we have studied a series of daily closing prices for the indices of two of India's largest exchanges, namely, SENSEX for the Bombay Stock Exchange and NIFTY 50 of the National Stock Exchange. We have used the open source database available on Kaggle to evaluate these models. We have based our model on the logistical growth model and Newton's law of cooling. The major assumption we have used is that the stock prices are majorly related to time.

We have further solved the differential equations for the coefficient values. We have also used the data analytics and visualization software, Tableau along with MS Excel to manipulate complex datasets and finally plot the results obtained alongside the original dataset to derive conclusions and present a 'proof of concept.'

Derivations

The rise and fall of the stock prices may be described by the logistic growth model and we combine the concept of logistic growth model and dynamic integration to build dynamic models which are illustrated below

Model A (Dynamic Logistic Model)

$$\frac{ds(t)}{dt} = \alpha_1 s^2(t) + \beta_1 s(t)$$

S(t)= s *s(to)=so*

$$\frac{ds}{dt} = \alpha_1 s^2 + \beta_1 s$$

$$\frac{ds}{\alpha_1 s^2+\beta_1 s} = dt$$

+

(Using variable separable method)

$$\frac{ds}{S^2\left(\alpha 1+\frac{\beta 1}{S}\right)}= dt$$

Let $\alpha 1 + \frac{\beta 1}{S} = u$

$$\frac{\beta 1}{S^2} ds = du$$

$$\frac{1}{s^2}\ ds = -\frac{du}{\beta 1}.$$

$$-\frac{du}{\beta 1\, u} = dt$$

$$\frac{du}{u} = -\beta 1\, dt$$

Integrating both sides

$$\int \frac{du}{u} = -\beta 1 \int dt \ln u = -\beta t + c$$

$$\ln\left(\alpha 1 + \frac{\beta 1}{So}\right) = -\beta 1 t + c$$

$$S(to) = So$$

$$\ln\left(\alpha 1 + \frac{\beta 1}{So}\right) = -\beta 1 to + c$$

$$c = \ln\left(\alpha 1 + \frac{\beta 1}{So}\right) + \beta 1 to$$

Put value of c in equation 1,

$$\ln\left(\alpha 1 + \frac{\beta 1}{S}\right) = -\beta 1 t + \ln\left(\alpha 1 + \frac{\beta 1}{So}\right) + \beta 1 t_o$$

$$\ln\left(\frac{\alpha 1 + \frac{\beta 1}{S}}{\alpha 1 + \frac{\beta 1}{So}}\right) = -\beta 1(t - to)$$

$$\frac{\alpha 1 + \frac{\beta 1}{S}}{\alpha 1 + \frac{\beta 1}{So}} = e^{-\beta 1(t-t0)}$$

$$\alpha 1 + \frac{\beta 1}{S} = e^{-\beta 1(t-to)} \left(\alpha 1 + \frac{\beta 1}{So}\right)$$

$$s = \frac{\beta 1}{e^{-\beta 1(t-to)}\left(\alpha 1 + \frac{\beta 1}{So}\right) - \alpha 1}$$

Model B (dynamic transformed logistic model)

$$\frac{d^2 s(t)}{dt^2} = \alpha_2(t)\left(\frac{ds(t)}{dt}\right)^2 + \beta_2(t)\left(\frac{ds(t)}{dt}\right)$$

We assume coefficients $\alpha_2(t)$ and $\beta_2(t)$ are constants for short time and hence we write the parametrized equation as:

$$s(t) = s$$

$$\frac{d^2s}{dt^2} = \alpha_2\left(\frac{ds}{dt}\right)^2 + \beta_2\left(\frac{ds}{dt}\right)$$

$$v(t) = v$$

$$let \ \ \frac{ds}{dt} = v$$

$$\frac{d^2s}{dt^2} = \frac{dv}{dt}$$

$$\frac{dv}{dt} = \alpha_2 v^2 + \beta_2 v$$

$$\frac{dv(t)}{dt} = \alpha_2 v^2(t) + \beta_2 v(t)$$

$$v(t) = v$$

$$v(t_o) = v_o$$

$$\frac{dv}{dt} = \alpha_2 v^2 + \ \beta_2 v$$

$$\frac{ds}{\alpha_2 v^2 + \beta_2 \ v} = dt$$

Using variable separable method

$$\frac{dv}{v^2\left(\alpha_2 + \frac{\beta_2}{v}\right)} = dt$$

$$Let \ \alpha_2 + \frac{\beta_2}{v} = u$$

$$\frac{\beta_2}{v^2} dv \ = du$$

$$\frac{1}{v^2} \ dv = \ -\frac{du}{\beta_2}$$

$$-\frac{du}{\beta_2 u} = dt$$

$$\frac{du}{u} = \ -\beta_2 \ dt$$

Integrating both sides

$$\int \frac{du}{u} = \ -\beta_2 \int dt$$

$$\ln u = -\beta_2 t + c$$

$$\ln\left(\alpha_2 + \frac{\beta_2}{v_o}\right) = \ -\beta_2 t + c$$

$$v(t_o) = v_o$$

$$\ln\left(\alpha_2 + \frac{\beta_2}{v_o}\right) = \ -\beta_2 t_o + c$$

$$c = \ln\left(\alpha_2 + \frac{\beta_2}{v_o}\right) + \beta_2 t_o$$

Putting value of c in equation 1,

$$\ln\left(\alpha_2 + \frac{\beta_2}{v}\right) = \ -\beta_2 t + \ln\left(\alpha_2 + \frac{\beta_2}{v_o}\right) + \beta_2 t_o$$

$$\ln\left(\frac{\alpha_2 + \frac{\beta_2}{v}}{\alpha_2 + \frac{\beta_2}{v_o}}\right) = \ -\beta_2(t - t_o)$$

$$\frac{\alpha_2 + \frac{\beta_2}{v}}{\alpha_2 + \frac{\beta_2}{v_o}} = \ e^{-\beta_2(t-t_0)}$$

$$\alpha_2 + \frac{\beta_2}{v} = e^{-\beta_2(t-t_o)} \ \left(\alpha_2 + \frac{\beta_2}{v_o}\right)$$

$$v = \frac{\beta_2}{e^{-\beta_2(t-t_o)}\left(\alpha_2 + \frac{\beta_2}{v_o}\right) - \alpha_2}$$

$$v = \frac{\beta_2}{e^{-\beta_2(t-t_0)}\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2}$$

$$v = \frac{\beta_2 e^{\beta_2(t-t_0)}}{\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}}$$

$$\frac{ds}{dt} = v$$

$$\frac{ds}{dt} = \frac{\beta_2 e^{\beta_2(t-t_0)}}{\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}}$$

$$As\ \alpha_{2,}\ \beta_2, v_0, t_0\ are\ constants$$

$$Let\ \left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)} = u$$

$$-\alpha_2 \beta_2 e^{\beta_2(t-t_0)} dt = du$$

$$\beta_2 e^{\beta_2(t-t_0)} dt = -\frac{du}{\alpha_2}$$

$$dt = -\frac{du}{\alpha_2 u}$$

$$integrating\ on\ both\ sides,$$

$$\int ds = \int -\frac{du}{\alpha_2 u}$$

$$s = -\frac{1}{\alpha_2} ln\, u + c$$

$$s = -\frac{1}{\alpha_2} ln\left(\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right) + c$$

$$s(t_0) = s_0$$

$$s_0 = -\frac{1}{\alpha_2} ln\left(\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t_0-t_0)}\right) + c$$

$$s_0 + \frac{1}{\alpha_2} ln\left(\frac{\beta_2}{v_0}\right) = c$$

$$s = -\frac{1}{\alpha_2} ln\left(\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right) + s_0 + \frac{1}{\alpha_2} ln\left(\frac{\beta_2}{v_0}\right)$$

$$s = s_0 - \frac{1}{\alpha_2}\left[ln\left(\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right) - ln\left(\frac{\beta_2}{v_0}\right)\right]$$

$$s = s_0 - \frac{1}{\alpha_2}\left[ln\left(\frac{\left\{\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right\}}{\frac{\beta_2}{v_0}}\right)\right]$$

$$\boldsymbol{s(t) = s(t_0) - \frac{1}{\alpha_2}\left[ln\left(\frac{\left\{\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right\}}{\frac{\beta_2}{v_0}}\right)\right]}$$

Model C (dynamic relative growth rate transformed logistic model)

$$\frac{d\delta(t)}{dt} = \alpha_3 \delta(t)^2 + \beta_3 \delta(t)$$

Relative growth rate of the stock price is given by

$$\delta(t) = \frac{\left(\frac{ds(t)}{dt}\right)}{s(t)}$$

we assume coefficients $\alpha_2(t)$ and $\beta_2(t)$ are constants for short time and hence we write the parametrized equation as:

$$\frac{d\delta(t)}{dt} = \alpha_3 \delta^2(t) + \beta_3 \delta(t)$$

$$\delta(t) = \delta$$

$$\delta(t_o) = \delta_o$$

$$\frac{d\delta}{dt} = \alpha_3\delta^2 + \beta_3\delta$$

$$\frac{d\delta}{\alpha_3\delta^2 + \beta_3\delta} = dt$$

(Using variable separable method)

$$\frac{d\delta}{\delta^2\left(\alpha_3+\frac{\beta_3}{\delta}\right)} = dt$$

$$Let\ \alpha_3 + \frac{\beta_3}{\delta} = u$$

$$\frac{\beta_3}{\delta^2}d\delta = du$$

$$\frac{1}{\delta^2}\ d\delta = -\frac{du}{\beta_3}$$

$$-\frac{du}{\beta_3 u} = dt$$

$$\frac{du}{u} = -\beta_3\ dt$$

Integrating both sides

$$\int \frac{du}{u} = -\beta_3\int\ dt$$

$$\ln u = -\beta_3 t + c$$

$$\ln\left(\alpha_3 + \frac{\beta_3}{\delta_o}\right) = -\beta_3 t + c$$

$$\delta(t_o) = \delta_o$$

$$\ln\left(\alpha_1 + \frac{\beta_1}{\delta_o}\right) = -\beta_3 t_o + c$$

$$c = \ln\left(\alpha_3 + \frac{\beta_3}{\delta_o}\right) + \beta_3 t_o$$

Putting value of c in equation 1,

$$\ln\left(\alpha_3 + \frac{\beta_3}{\delta}\right) = -\beta_3 t + \ln\left(\alpha_3 + \frac{\beta_3}{\delta_o}\right) + \beta_3 t_o$$

$$\ln\left(\frac{\alpha_3+\frac{\beta_3}{\delta}}{\alpha_3+\frac{\beta_3}{\delta_o}}\right) = -\beta_3(t - t_o)$$

$$\frac{\alpha_3+\frac{\beta_3}{\delta}}{\alpha_3+\frac{\beta_3}{\delta_o}} = e^{-\beta_3(t-t_0)}$$

$$\alpha_3 + \frac{\beta_3}{\delta} = e^{-\beta_3(t-t_o)}\left(\alpha_3 + \frac{\beta_3}{\delta_o}\right)$$

$$\delta = \frac{\beta_3}{e^{-\beta_3(t-t_o)}\left(\alpha_3 + \frac{\beta_3}{\delta_o}\right) - \alpha_3}$$

Using previously identical method,

$$\delta(t) = \frac{\beta_3}{e^{-\beta_3(t-t_o)}\left(\alpha_3+\frac{\beta_3}{\delta(t_o)}\right)-\alpha_3}$$

$$\delta(t) = \frac{\left(\frac{ds(t)}{dt}\right)}{s(t)}$$

$$\frac{1}{s}\frac{ds}{dt} = \frac{\beta_3}{e^{-\beta_3(t-t_o)}\left(\alpha_3 + \frac{\beta_3}{\delta(t_o)}\right) - \alpha_3}$$

$$\frac{ds}{s} = \frac{\beta_3\ dt}{e^{-\beta_3(t-t_o)}\left(\alpha_3 + \frac{\beta_3}{\delta(t_o)}\right) - \alpha_3}$$

Again, integrating on both sides

$$\int \frac{ds}{s} = \int \frac{\beta_3\ dt}{e^{-\beta_3(t-t_o)}\left(\alpha_3 + \frac{\beta_3}{\delta(t_o)}\right) - \alpha_3}$$

We have integral of R.H.S on previous pages

$$ln\ s = -\frac{1}{\alpha_3} ln\left(\left(\alpha_3 + \frac{\beta_3}{\delta_0}\right) - \alpha_3 e^{\beta_3(t-t_0)}\right) + c$$

$$s = e^{-\frac{1}{\alpha_3}ln\left(\left(\alpha_3+\frac{\beta_3}{\delta_0}\right)-\alpha_3 e^{\beta_3(t-t_0)}\right)}\ e^c$$

$Considering\ e^c\ as\ one\ constant$

$$s = c\,.\,e^{-\frac{1}{\alpha_3}ln\left(\left(\alpha_3+\frac{\beta_3}{\delta_0}\right)-\alpha_3 e^{\beta_3(t-t_0)}\right)}$$

$$s(t_0) = s_0$$

$$s_0 = c\,.\,e^{-\frac{1}{\alpha_3}ln\left(\left(\alpha_3+\frac{\beta_3}{\delta_0}\right)-\alpha_3 e^{\beta_3(t_0-t_0)}\right)}$$

$$c = s_0\ e^{\frac{1}{\alpha_3}ln\left(\frac{\beta_3}{\delta_0}\right)}$$

$$s = s_0\ e^{\frac{1}{\alpha_3}ln\left(\frac{\beta_3}{\delta_0}\right)}\ e^{-\frac{1}{\alpha_3}ln\left(\left(\alpha_3+\frac{\beta_3}{\delta_0}\right)-\alpha_3 e^{\beta_3(t-t_0)}\right)}$$

$$\boldsymbol{s = s_0\ e^{\frac{1}{\alpha_3}ln\left(\frac{\frac{\beta_3}{\delta_0}}{\left(\alpha_3+\frac{\beta_3}{\delta_0}\right)-\alpha_3 e^{\beta_3(t-t_0)}}\right)}}$$

Model D (dynamic general Newton model)

We have used the concept of Newton's Law of Cooling in forming the differential equation for this model

$$\frac{ds(t)}{dt} = \alpha_4(t)[s(t) - A]$$

$s(t_o) = s_o$
$s(t) = s$
$\frac{ds}{dt} = \alpha_4(s - A)$
$\frac{ds}{s-A} = \alpha_4 dt$ variable separable method

Integrating both sides

$$\int \frac{ds}{s - A} = \alpha_4 \int dt$$

Using condition 1 from t_o to t

$$\int_{s_o}^{s} \frac{ds}{s - A} = \alpha_4 \int_{t_o}^{t} dt$$

$$[\ln(s - A)]_{so}^{s} = \alpha_4(t - t_o)$$

$\ln \frac{s-A}{s_o-A} = \alpha_4(t - t_o)$

$\frac{s-A}{s_o-A} = e^{\alpha_4(t-t0)}$

$s - A = (s_o - A)e^{\alpha_4(t-t0)}$

$$\boldsymbol{s = A + (s_o - A)e^{\alpha_4(t-t0)}}$$

Implementation

Model A:

Equation after solving:

$$s = \beta_1 / e^{-\beta_1(t-t_o)}(\alpha_1 + \frac{\beta_1}{s_o}) - \alpha_1 \;\; \dots (1)$$

Assumptions:

1. Stock prices change dynamically in reality, but we assume α_1, β_1 as constant for short interval of time.
2. We assume α_1, β_1 to be constant before 4 trading days from the day to which stock price is to be forecasted.

$$t \; \varepsilon \; [t_o, t_3]$$

Concept:

From the given data set try to plot the graph of the stock prices and find value of $\alpha_1 \; and \; \beta_1$.

Since $|s - s_O| \lll s \; \rightarrow [\alpha_1 = very \; small]$

After performing deep analysis of given data set, we get $\alpha_1 = 0.000000375$

Now, put value of s in equation (1) for each trading day before forecasted day and calculate corresponding β.

Let,

For

$$s = s_1 \; ; \; \beta = \beta_1 \; ; t_1 - t_o = 1$$
$$s = s_2 ; \; \beta = \beta_1 \; ; \; t_2 - t_0 = 2$$
$$s = s_3 ; \; \beta = \beta_2 \; ; \; t_3 - t_o = 3$$

Now, on forecasted day:

Weighted mean is:

$$\beta = \frac{16 * \beta_1 + 4 * \beta_2 + \beta_3}{21} \quad ;$$

$$where$$

$$\alpha = \alpha_1 = 0.000000375$$

coefficient of weighted mean is found graphically

Note: We get two values of β_i , one positive and other negative. But we consider the negative value after performing in depth analysis of given dataset.

Example:

$$t_o = 1st \; Feb \; 2019$$
$$s_o = 36311.74$$

$\boldsymbol{\beta_1}$:

$$s_1 = \frac{\beta_1}{e^{-\beta_1(1)}\left(\frac{\beta_1}{s_o} + \alpha_1\right) - \alpha_1}$$

Here, α = too small (neglect term in subtraction)

$$36456.22 = \frac{\beta}{e^{-\beta}\left(\frac{\beta}{36311.74} + 0.000000375\right)}$$

On solving, using a graph calculator, we get:

$$\beta_1 = -0.11831$$

$\boldsymbol{\beta_2}$:

$$36573.04 = \frac{\beta_2}{e^{-2\beta_2}\,(\frac{\beta_2}{36311.74} + 0.000000375)}$$

On solving, using a graph calculator, we get:

$$\beta_2 = -0.08439$$

$\boldsymbol{\beta_3}$:

$$36714.54 = \frac{\beta_3}{e^{-3\beta_3}\left(\frac{\beta_3}{36311.74} + 0.000000375\right)}$$

On solving, using a graph calculator, we get:

$$\beta_3 = -0.06927$$

Now,

$$\beta = \frac{16 * (-0.11831) + 4 * (-0.08439) + (-0.06927)}{21}$$

$$\beta = -0.109513$$

$$s_5 = -\frac{0.109513}{e^{0.109513}\left(-\frac{0.109513}{363111.74} + 0.000000375\right)}$$

$$s_5 = 37166.435445$$

Observations:

$$s_5\ (theorotically) = 37166.433445$$
$$s_5\ (practically) = 37026.56$$

Thus, the above model gives acceptable results, but can be made better.

Model B:

Dynamic Transformed Logistic Model

Equation after solving,

$$\boldsymbol{s(t) = s(t_0) - \frac{1}{\alpha_2}\left[ln\left(\frac{\left\{\left(\alpha_2 + \frac{\beta_2}{v_0}\right) - \alpha_2 e^{\beta_2(t-t_0)}\right\}}{\frac{\beta_2}{v_0}}\right)\right]}$$

$$\boldsymbol{s(t) = stock\ price\ at\ any\ time\ t}$$

Assumptions:

1. Stock prices change dynamically in reality, but we assume $\boldsymbol{\alpha_2, \beta_2}$ as constant for short interval of time.
2. We assume $\boldsymbol{\alpha_2, \beta_2}$ to be constant before three to four trading days from the day when stock price is to be forecasted $\boldsymbol{t} \in [\boldsymbol{t_0}, \boldsymbol{t_3}]$.

Method for Solving Differential Equation:

$$\boldsymbol{\frac{dv}{\alpha_2 v^2 + \beta_2 v} = dt}$$

Variable separable method is used to solve the equations.

Note: Currently it is not possible to plot the theoretical curves due to limited access to tools and vast dataset. We have manually tried and checked about 30 trading days and then assigned values to the arbitrary constants.

Model C:

Equation after solving,

$$\boldsymbol{s = s_0\, e^{\frac{1}{\alpha_3} ln\left(\frac{\frac{\beta_3}{\delta_0}}{\left(\alpha_3 + \frac{\beta_3}{\delta_0}\right) - \alpha_3 e^{\beta_3(t-t_0)}}\right)}}$$

Method for Solving Differential Equation:

$$\boldsymbol{\frac{d\delta(t)}{\alpha_3 \delta(t)^2 + \beta_3 \delta(t)} = dt}$$

Variable separable method is used to solve the equations.

Observation: Model C describes the relationship between the relative growth rate of the stock prize if $\boldsymbol{\alpha_3 < 0}$ and $\boldsymbol{\beta_3 > 0}$, this implies the return depict closer to growth logistic model.

Model D:

Equation after solving

$$s = e^{\alpha t + c} + A$$

Assumptions:

Stock prices change dynamically in reality but we will assume that α, c and A remain constant for a short interval of time.

Concept: The model has been derived from Newton's law of cooling.

From this, we determine the following differential equation

$$\frac{dx}{dt} = \alpha(x - A)$$

Where α, A are constant for t instant.

$$\frac{dx}{x - A} = A\,dt$$

$$\int \frac{dx}{(x - A)} = \alpha dt$$

$$\mathbf{ln}(\boldsymbol{A - x}) = \boldsymbol{\alpha t + c} \quad \dots (1)$$

Now, we will consider three successive dates to find to find constant for t_4 day.

$$t_1 = 0$$
$$t_2 = 1$$
$$t_3 = 2$$

$t_4 = 1$ … (derived from graphs and reference material)

For t = 0

$$s_o = p$$

$$\ln(|p - A|) = \alpha \qquad \dots(1)$$

For t = 1

$$s_1 = q$$
$$\ln(|q - A|) = c + \alpha \qquad \dots(2)$$

For t = 2

$$s_2 = r$$
$$\ln(|r - A|) = c + 2\alpha \quad \dots(3)$$

p, q, r are taken from excel sheet

from (1), (2) and (3)

$$A = \frac{pq - q^2}{p + r - 2q}$$

$$c = \ln|r - A|$$

$$\alpha = \ln|q - A| - k$$

Using these equations, we analysed the excel data. The same is attached in appendix.

The analysed data can be plotted on a 'index-value vs date' graph. Comparing the two graphs we can observe that the model is working.

Observations:

The model works in predicting the index movement in most circumstances.

The model also succeeds in correctly predicting the amount by which market will move a fair number of times.

The model completely fails for the values with high values of α and we observe a sharp spike in graphs.

Note: Few sharp spikes have been excluded from the graph due to representation scalability issue.

Model succeeds in predicting Nifty 50 movement better than the Sensex 30 index.

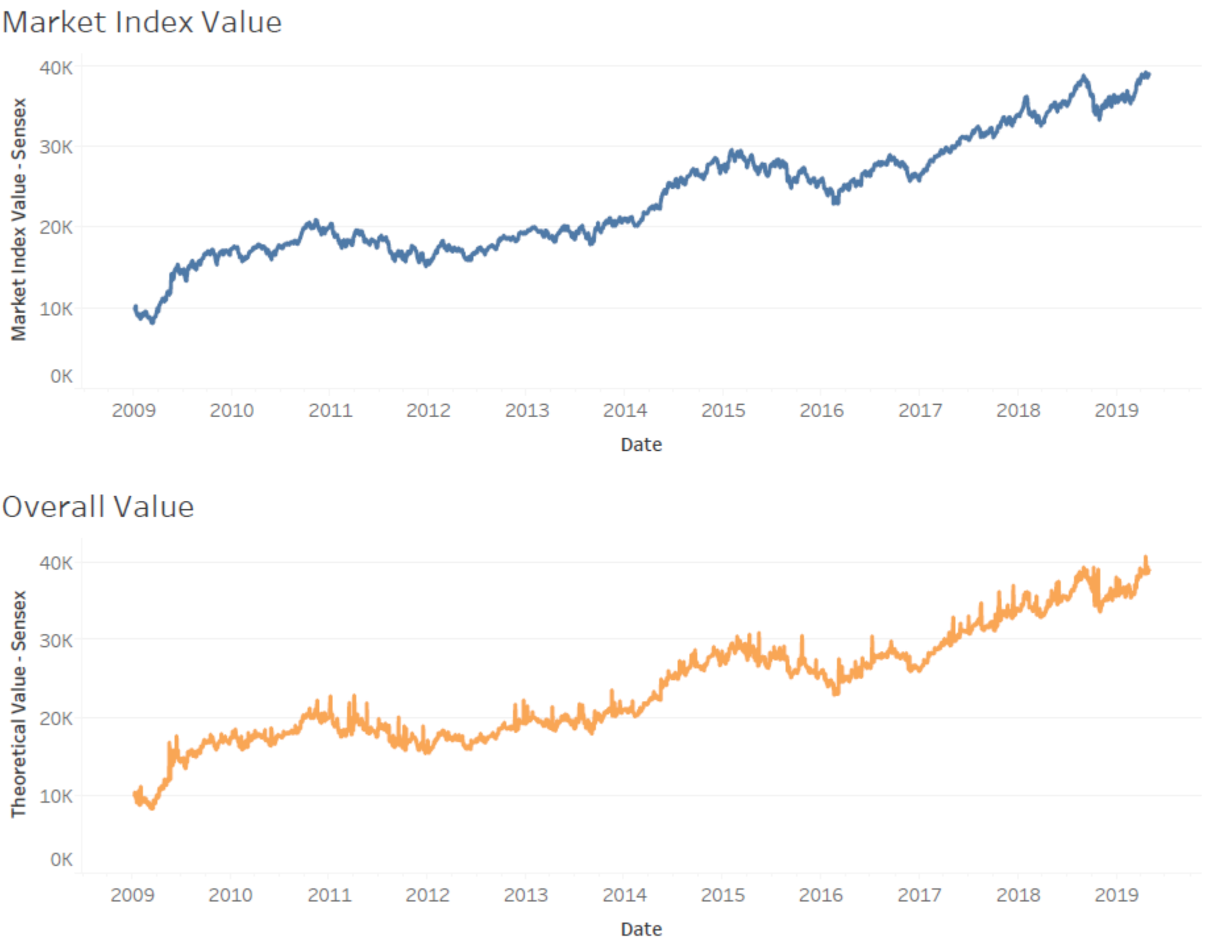

Fig.1 Analysis of Sensex ‘s closing value and theoretical value for duration of 10 years (2009-19)

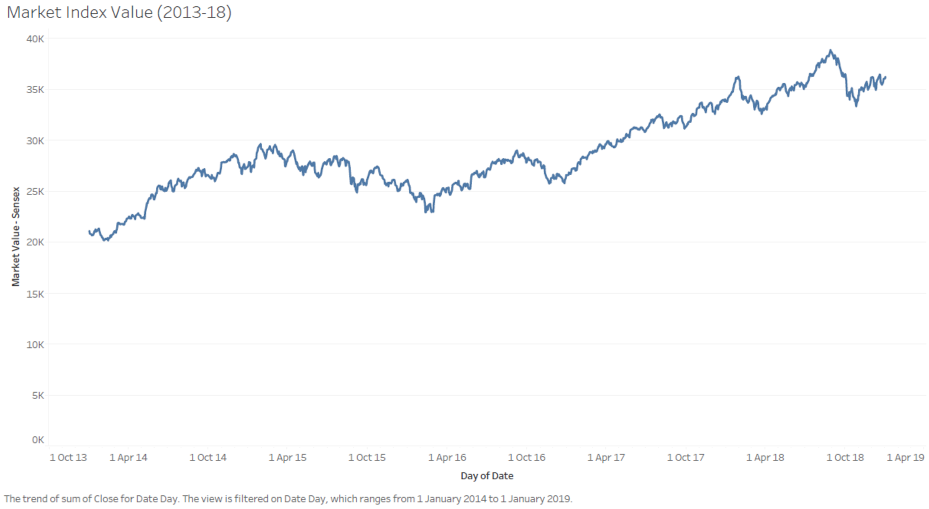

Fig. 2.a – Sensex's closing value for duration of last 5 years

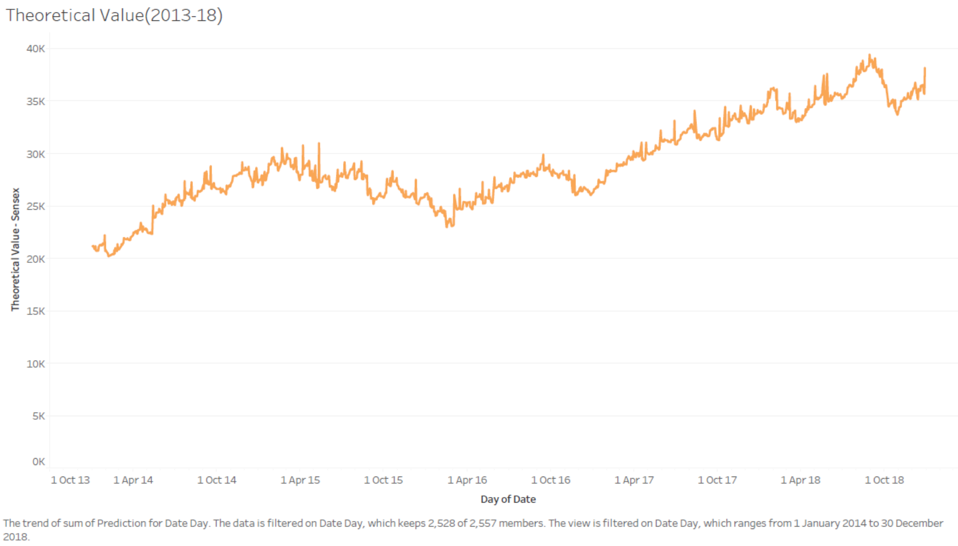

Fig. 2.b – Sensex's derived theoretical value for duration of last 5 years

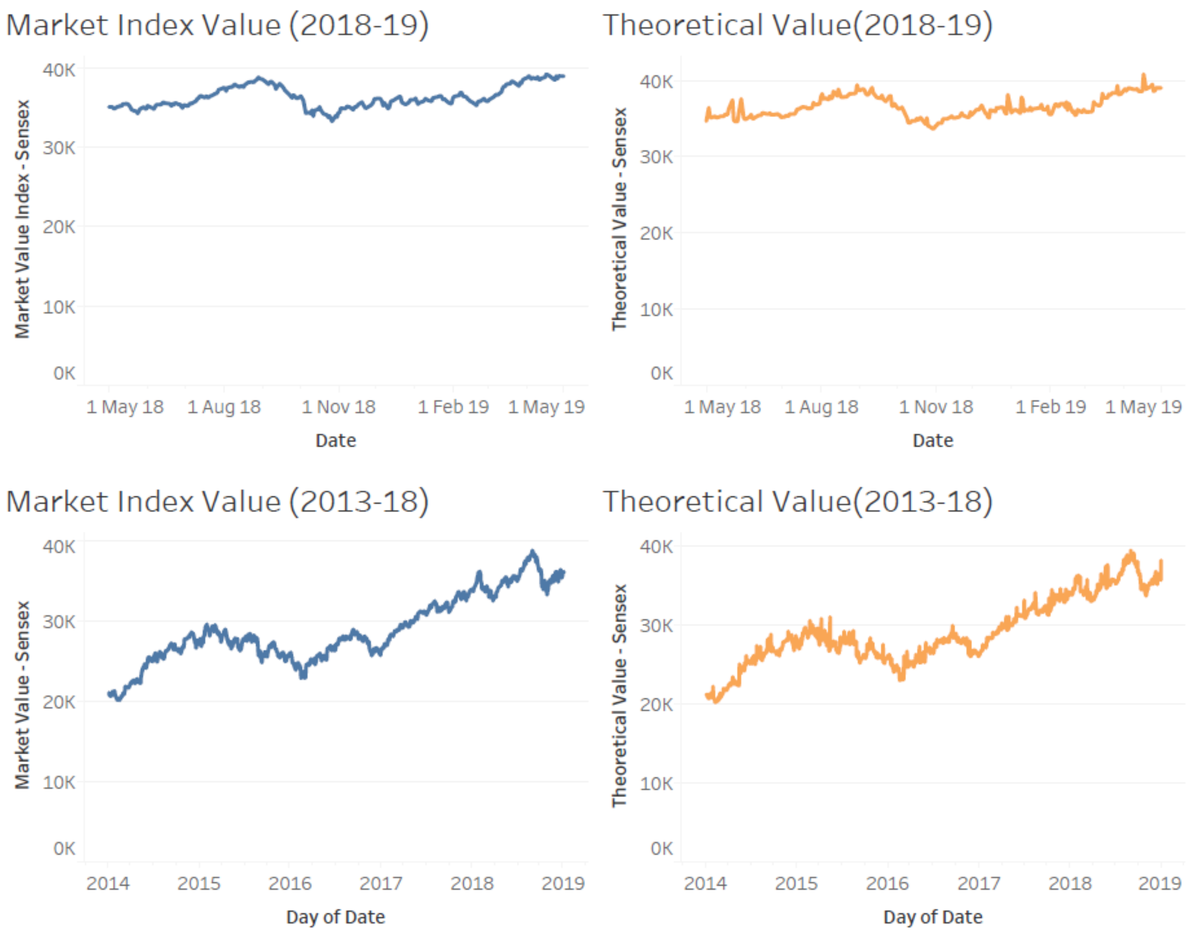

Fig.3 Analysis of Sensex 's closing value and theoretical value

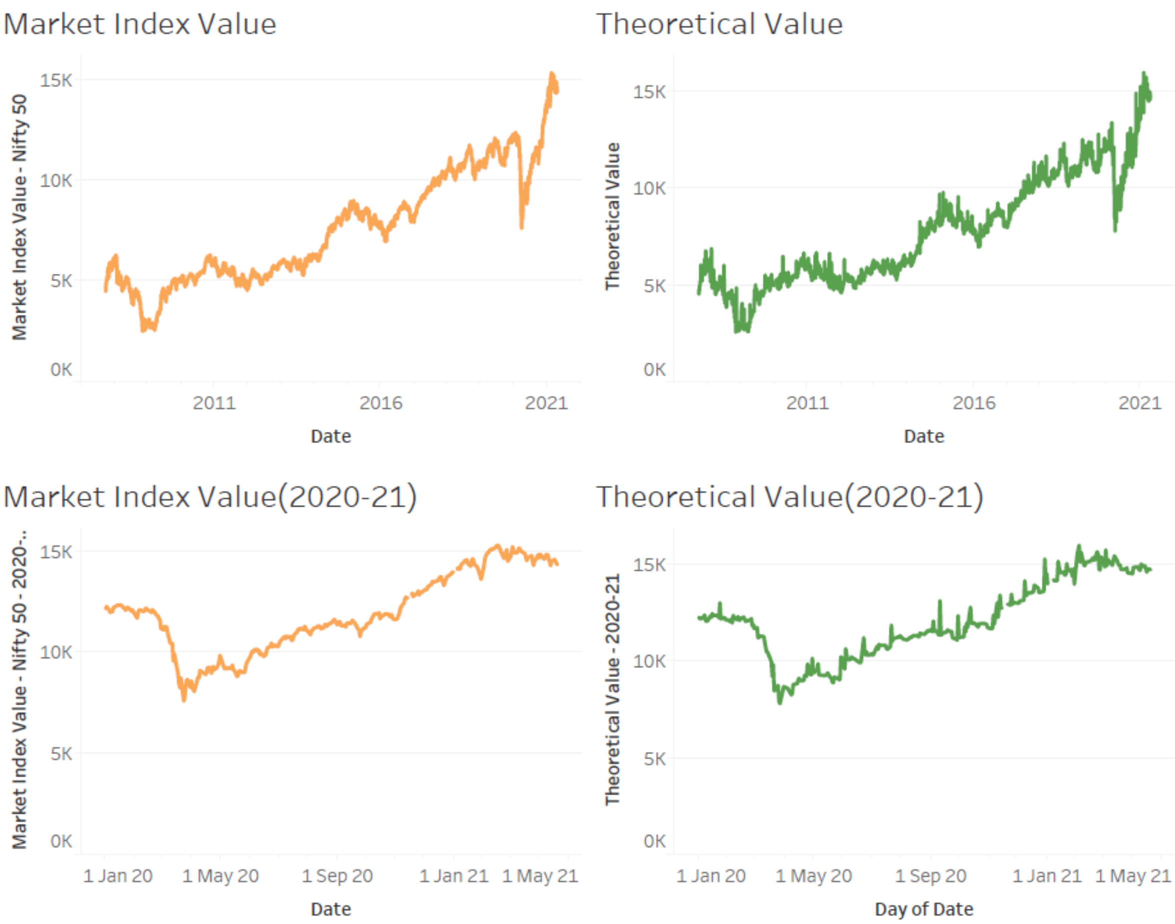

Fig.4 Analysis of Nifty 50 ‘s closing value and theoretical value for duration of 10 years (2011-2021) and same for 1 year (2020-21)

Conclusion

Bull market (upward trend) and bear market (downward trend) is used to describe the velocity and the related growth rate of the stock prices.

Model D is based on law of Newton's cooling is better than other models based on logistic growth model.

Forecasting errors:

MAPE: Mean Absolute Percentage Error

RMSPE: Root Mean Square Percentage Error

MAPE is minimum in Model D and maximum in Model A, hence Model D is has better accuracy than other models.

Similar is true with RMSPE.

Future Development

To develop a way to solve differential equations. The approach may include to learn tech stack necessary to code in any proprietary multi-paradigm programming language and numeric computing environment like MATLAB, Wolfram, etc

To include more variation in type of dataset and improve accuracy of dynamic models
This model can be used as a reference to the machine learning models which can further be used to predict the movement of stock indices and even the individual stock

Acknowledgment

We would like to extend warm gratitude to Prof.Chetan Pahljani, Prof. Satyajit Pramanik and Prof. Baradhwaj Collepa, who instructed us throughout the course. Their lectures proved to be helpful to get a clear understanding of the basics and different aspects of differential equations. We would also like to thank all the tutors and teaching assistants who guided us in the course and for the project. We could apply the concepts of differential equations in financial modelling due to their tutoring and valuable support.

Git Repository:

All the datasets used for modelling of Model D, along with derived Tableau files and manipulated MS Excel sheets have been uploaded in a Git repository. You can access the repository here.

References:

- Kaggle Nifty 50 - https://www.kaggle.com/oneplustricks/nifty-50-20072021
- Kaggle Sensex - https://www.kaggle.com/shailendrayadav/bse-sensex-prediction/data
- [1]https://www.investopedia.com/terms/m/meanreversion.asp#:~:text=Mean%20reversion%2C%20in%20finance%2C%20suggests,techniques%20to%20options%20pricing%20models.
- Financial Modelling by Ordinary and Stochastic Differential Equation by *Abodolsadeh Neisy; Moslem Peymany*
- Application of Parameterized and Nonlinear Ordinary Differential Equations and Dynamic Systems: An Example of the Taiwan Stock Index by *Meng-Rong Li*; *Tsung-Jui Chiang-Lin*; and *Yong-Shiuan Lee*
- https://www.investopedia.com/articles/investing/082614/how-stock-market-works.asp
- https://www.khanacademy.org/math/ap-calculus-bc/bc-differential-equations-new/bc-7-9/v/solving-logistic-differential-equation-part-1
- https://en.wikipedia.org/wiki/BSE_SENSEX