$$\mu_{PD_{BASELINE_{TRAN}}} = -\bar{m}\left[K_p(r - r_{REF}) + K_D(\dot{r} - \dot{r}_{REF}) \cancel{- \ddot{r}_{REF}}\right]$$

$$\dot{\omega}_{CMD} = \dot{J}\left[-K_p(\eta - \eta_{REF}) \cancel{+ \dot{\eta}_{REF}}\right] + J\left[-K_p(\dot{\eta} - \dot{\eta}_{REF}) + \ddot{\eta}_{REF}\right]$$

$$\tau_{BASELINE_{PI}} = -\bar{I}\left[K_p(\omega - \omega_{REF}) + K_D(\dot{\omega} - \dot{\omega}_{REF}) + K_I\int(\omega - \omega_{REF})dt \cancel{- \dot{\omega}_{REF}}\right]$$

$$\frac{d\vec{T}}{ds} = \kappa\vec{N}$$

$$\frac{d\vec{N}}{ds} = -\kappa\vec{T} + \cancel{\tau\vec{B}}$$

$$\frac{d\vec{B}}{ds} = -\cancel{\tau}\vec{N}$$

$$\tau = 0$$

$$S(t) = \int_0^t \|r'(\sigma)\|\, d\sigma$$

$$p(t) = p_1 t^m + p_2 t^{m-1} + \dots + p_m t + p_{m+1}$$

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$$x(t) = x_1 t^m + x_2 t^{m-1} + \dots + x_m t + x_{m+1}$$

$$y(t) = y_1 t^m + y_2 t^{m-1} + \dots + y_m t + y_{m+1}$$

$$z(t) = z_1 t^m + z_2 t^{m-1} + \dots + z_m t + z_{m+1}$$

$$\|r(t)\| = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$$

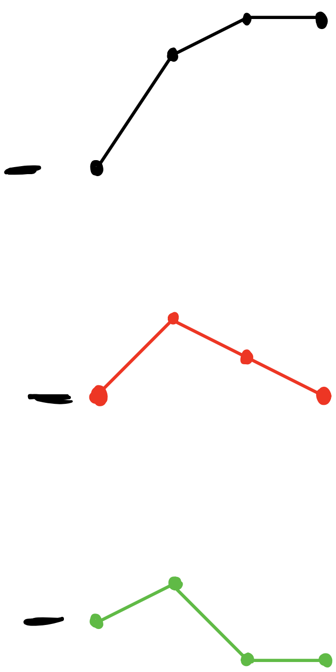
$f(x) = \cos x \Rightarrow \dot{f}(x) = -\sin x \, \dot{x}$

$g(x) = \sin x \Rightarrow \dot{g}(x) = \cos x \, \dot{x}$

IF

$\sin(\psi)\sin(\theta)\dot{\psi}\dot{\theta}$

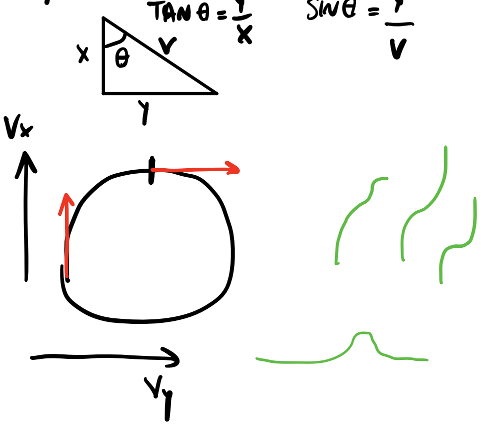
x 1
y 1
z 1
2
2
2
3
3
3



$x(\bar{t})$ $y(\bar{t})$ $z(\bar{t})$

$V = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$ $\sin\theta = \frac{y}{V}$



IF $t_A \leq t_{sim} < t_B$

CHECK POL COEFF

$\psi = \arctan \dfrac{V_y(t)}{V_x(t)}$

$$X(t) = \frac{V_y(t)}{V_x(t)}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\dot{\psi} = \frac{d}{dt}\arctan X(t) = \frac{1}{1+X(t)^2}\,\dot{X}(t)$$

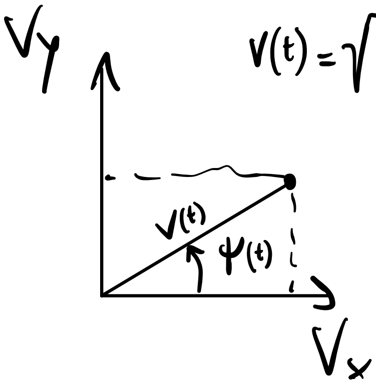
$$\ddot{\psi} = \frac{d^2}{dt^2}\arctan X(t) = \frac{\left(X(t)^2+1\right)\ddot{X}(t) - 2X(t)\,\dot{X}(t)^2}{\left(X(t)^2+1\right)^2}$$

$$\dot{X}(t) = \frac{d}{dt}\left(\frac{V_y(t)}{V_x(t)}\right) = \frac{V_y'(t)\,V_x(t) - V_y(t)\,V_x'(t)}{V_x(t)^2}$$

$$\ddot{X}(t) = \frac{d^2}{dt^2}\left(\frac{V_y(t)}{V_x(t)}\right) = \frac{V_x(t)^2 V_y''(t) - V_x(t)\left(2V_y'(t)V_x'(t) + V_y(t)V_x''(t)\right) + 2V_y(t)\,V_x'(t)^2}{V_x(t)^3}$$

$$\frac{d^2}{dx^2}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)^2 f''(x) - g(x)\left(2f'(x)g'(x) + f(x)g''(x)\right) + 2f(x)g'(x)^2}{g(x)^3}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$



$$V(t)=\sqrt{V_x(t)^2+V_y(t)^2}$$

IMAGINARY NUMBER i

$$\vec{V}(t)=V(t)\left(\cos\psi(t)+i\sin\psi(t)\right)$$

$$\vec{V}(t)=V(t)\,e^{i\psi(t)}$$

$\vec{a}(t)=a_x(t)+i\,a_y(t)$ TRAJECTORY ACCELERATION ON THE X-Y PLANE → KNOWN

$$\vec{a}(t)=a(t)\,e^{i\psi_a(t)} \qquad a(t)=\sqrt{a_x(t)^2+a_y(t)^2} \qquad \psi_a(t)=\operatorname{atan2}(a_y(t),a_x(t))$$

$$\vec{a}(t)=a(t)\,e^{i\psi_a(t)}=\frac{d}{dt}\vec{V}(t)=e^{i\psi(t)}\left(V'(t)+i\,V(t)\,\psi'(t)\right)$$

$$\frac{a(t)\,e^{i\psi_a(t)}}{e^{i\psi(t)}}=a(t)\,e^{i(\psi_a(t)-\psi(t))}=V'(t)+i\,V(t)\,\psi'(t)$$

$$a(t)\,e^{i\overbrace{(\psi_a(t)-\psi(t))}^{\Psi_a}}-V'(t)=i\,V(t)\,\psi'(t)$$

$$\boxed{\frac{a(t)\left(\cos\Psi_a(t)+i\sin\Psi_a(t)\right)-V'(t)}{i\,V(t)}=\psi'(t)}$$

GIVEN THE PROPERTY: $\dfrac{1}{a+ib}=\left(\dfrac{a}{a^2+b^2}\right)+\left(\dfrac{-b}{a^2+b^2}\right)i$

$$\frac{1}{i\,V(t)}=\frac{-V(t)\,i}{V(t)^2}=\frac{-i}{V(t)}$$

$$\left[a(t)\left(\cos\overline{\Psi}_a(t) + i \sin\overline{\Psi}_a(t)\right) - v'(t)\right]\frac{1}{i\,V(t)} = \psi'(t)$$

$$\psi'(t) = \left[a(t)\cos\overline{\Psi}_a(t) + i\,a(t)\sin\overline{\Psi}_a(t) - v'(t)\right]\cdot\frac{-i}{V(t)}$$

$$= \frac{i\left(v'(t) - a(t)\cos\overline{\Psi}_a(t)\right) - i^2 a(t)\sin\overline{\Psi}_a(t)}{V(t)}$$

$$\psi'(t) = \frac{a(t)\sin\overline{\Psi}_a(t)}{V(t)} + i\,\frac{v'(t) - a(t)\cos\overline{\Psi}_a(t)}{V(t)}$$

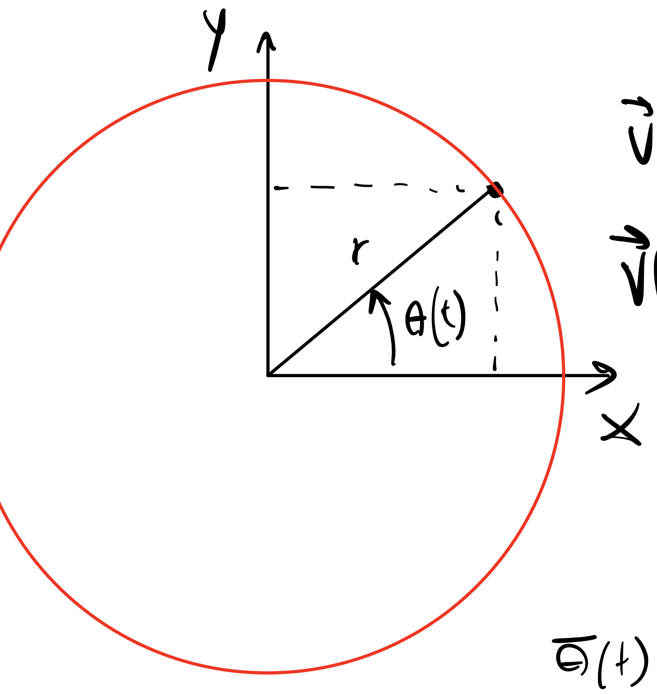
$$\boxed{\psi'(t) = \frac{a(t)\sin\overline{\Psi}_a(t)}{V(t)}}$$

$\psi'(t)$ IS REAL SO WE KEEP ONLY ITS REAL PART

$$\frac{d^2\vec{V}}{dt^2}(t) = e^{i\psi(t)}\left(v''(t) + 2i\,v'(t)\psi'(t) - v(t)\left(\psi'(t)^2 - i\,\psi''(t)\right)\right)$$

FROM HERE I GET $\psi''(t)$

EXAMPLE: UNIFORM CIRCULAR MOTION



$$\vec{z}(t) = r\, e^{i\theta(t)} = x(t) + i\, y(t)$$

$$\vec{v}(t) = x'(t) + i\, y'(t) = v_x(t) + i\, v_y(t) = v(t)\, e^{i\theta_v(t)}$$

$$\vec{v}(t) = \frac{d}{dt}\left(r\, e^{i\theta(t)}\right) = i\, r\, \theta'(t)\, e^{i\theta(t)} =$$

$$= r\theta'(t)\, e^{i\theta(t)}\, e^{i\frac{\pi}{2}} = \underbrace{r\theta'(t)}_{v(t)}\, e^{i\underbrace{(\theta(t)+\frac{\pi}{2})}_{\theta_v(t)}}$$

$$\frac{v(t)\, e^{i\theta_v(t)}}{e^{i\theta(t)}} = i\, r\, \theta'(t)$$

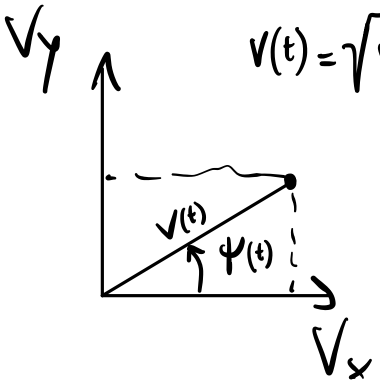
$$\boxed{\begin{aligned} e^{i\theta} &= \cos\theta + i\sin\theta \\ e^{i\frac{\pi}{2}} &= \underbrace{\cos\frac{\pi}{2}}_{=0} + i\underbrace{\sin\frac{\pi}{2}}_{=1} = i \end{aligned}}$$

$$v(t)\, e^{i\overbrace{(\theta_v(t)-\theta(t))}^{\bar{\theta}(t)}} = i\, r\, \theta'(t)$$

$$v(t)\left(\cos\bar{\theta}(t) + i\sin\bar{\theta}(t)\right) = i\, r\, \theta'(t)$$

$$\frac{v(t)}{r}\cos\bar{\theta}(t) + i\frac{v(t)}{r}\sin\bar{\theta}(t) = i\,\theta'(t)$$

$$\theta'(t) = \frac{v(t)}{r}\sin\bar{\theta}(t) = \frac{v(t)}{r}\sin\left(\frac{\pi}{2}\right) = \frac{v(t)}{r}$$



$V(t)=\sqrt{V_x(t)^2+V_y(t)^2}$

IMAGINARY NUMBER i

$$\vec{V}(t)=V(t)\left(\cos\psi(t)+i\sin\psi(t)\right)$$

$$\vec{V}(t)=V(t)\,e^{i\psi(t)}$$

$\vec{a}(t)=a_x(t)+i\,a_y(t)$ TRAJECTORY ACCELERATION ON THE X-Y PLANE → KNOWN

$\vec{a}(t)=a(t)\,e^{i\psi_a(t)}$, $a(t)=\sqrt{a_x(t)^2+a_y(t)^2}$, $\psi_a(t)=\operatorname{atan2}(a_y(t),a_x(t))$

$$\vec{a}(t)=a(t)\,e^{i\psi_a(t)}=\frac{d}{dt}\vec{V}(t)=e^{i\psi(t)}\left(V'(t)+i\,V(t)\,\psi'(t)\right)=$$

$$=V'(t)\,e^{i\psi(t)}+V(t)\,\psi'(t)\,e^{i\frac{\pi}{2}}\,e^{i\psi(t)}=$$

$$=V'(t)\,e^{i\psi(t)}+V(t)\,\psi'(t)\,e^{i\left(\psi(t)+\frac{\pi}{2}\right)}$$

$$a(t)\,e^{i\psi_a(t)}-V'(t)\,e^{i\psi(t)}=V(t)\,\psi'(t)\,e^{i\left(\psi(t)+\frac{\pi}{2}\right)}$$

$$\frac{a(t)}{V(t)}\,e^{i\left(\psi_a(t)-\psi(t)-\frac{\pi}{2}\right)}-\frac{V'(t)}{V(t)}\,e^{i\left(-\frac{\pi}{2}\right)}=\psi'(t)$$

$$e^{i\pi}=\cos\pi+i\sin\pi=-1$$

$$\psi'(t)=\frac{a(t)}{V(t)}\,e^{i\left(\psi_a(t)-\psi(t)-\frac{\pi}{2}\right)}+\frac{V'(t)}{V(t)}\,e^{i\frac{\pi}{2}}$$

$$= \frac{a(t)}{V(t)} \cos\left(\psi_a(t) - \psi(t) - \frac{\pi}{2}\right) + i \frac{a(t)}{V(t)} \sin\left(\psi_a(t) - \psi(t) - \frac{\pi}{2}\right) +$$

$$+ \frac{V'(t)}{V(t)} \left(\overset{=0}{\cancel{\cos\frac{\pi}{2}}} + i\, \overset{=1}{\cancel{\sin\frac{\pi}{2}}} \right) =$$

$$= \frac{a(t)}{V(t)} \cos\left(\psi_a(t) - \psi(t) - \frac{\pi}{2}\right) = \boxed{\frac{a(t)}{V(t)} \sin\left(\psi_a(t) - \psi(t)\right) = \psi'(t)}$$

$$\frac{d^2}{dt^2}\vec{V}(t) = e^{i\psi(t)} \left(V''(t) + 2iV'(t)\psi'(t) - V(t)\left(\psi'(t)^2 - i\psi''(t)\right)\right)$$

$$= \frac{d}{dt}\vec{a}(t) = \vec{J}(t) = J(t)\, e^{i\psi_J(t)}$$

$\vec{J}(t) = J_x(t) + i J_y(t)$ TRAJECTORY JERK ON THE X-Y PLANE ← KNOWN

$$J(t) = \sqrt{J_x(t)^2 + J_y(t)^2} \qquad \psi_J(t) = \operatorname{atan2}\left(J_y(t), J_x(t)\right)$$

$$J(t)e^{i\psi_J(t)} = e^{i\psi(t)} V''(t) + 2V'(t)\psi'(t)\, e^{i\left(\psi(t)+\frac{\pi}{2}\right)} +$$

$$+ V(t)\psi'(t)^2 e^{i\left(\psi(t)+\pi\right)} + V(t)\psi''(t)\, e^{i\left(\psi(t)+\frac{\pi}{2}\right)}$$

$$\psi''(t) = \frac{J(t)e^{i\left(\psi_J(t)-\psi(t)-\frac{\pi}{2}\right)}}{V(t)} - \frac{V''(t)}{V(t)}e^{i\left(-\frac{\pi}{2}\right)} +$$

$$- \frac{2V'(t)\psi'(t)}{V(t)} - \psi'(t)^2 e^{i\frac{\pi}{2}}$$

$$\psi''(t) = \frac{J(t)}{V(t)}e^{i\left(\psi_J(t)-\psi(t)-\frac{\pi}{2}\right)} + \left(\frac{V''(t)}{V(t)} - \psi'(t)^2\right)e^{i\frac{\pi}{2}} +$$

$$- \frac{2V'(t)\psi'(t)}{V(t)}$$

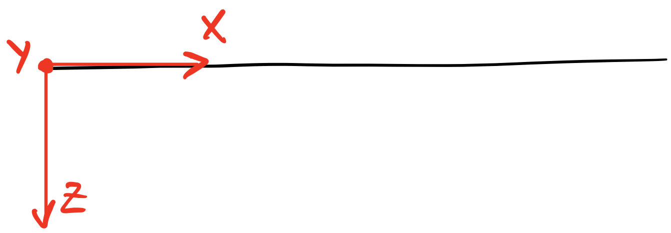
$$\psi''(t) = \frac{J(t)}{V(t)}\cos\left(\psi_J(t)-\psi(t)-\frac{\pi}{2}\right) + i\frac{J(t)}{V(t)}\sin\left(\psi_J(t)-\psi(t)-\frac{\pi}{2}\right) +$$

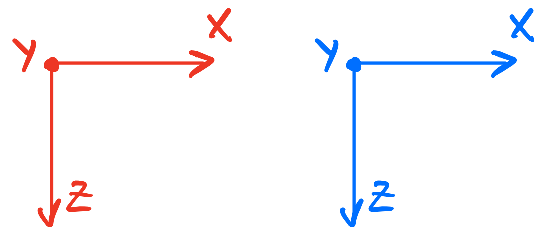
$$+\left(\frac{V''(t)}{V(t)} - \psi'(t)^2\right)\left(\overset{=0}{\cancel{\cos}}\frac{\pi}{2} + i\overset{=1}{\cancel{\sin}}\frac{\pi}{2}\right) - \frac{2V'(t)\psi'(t)}{V(t)} =$$

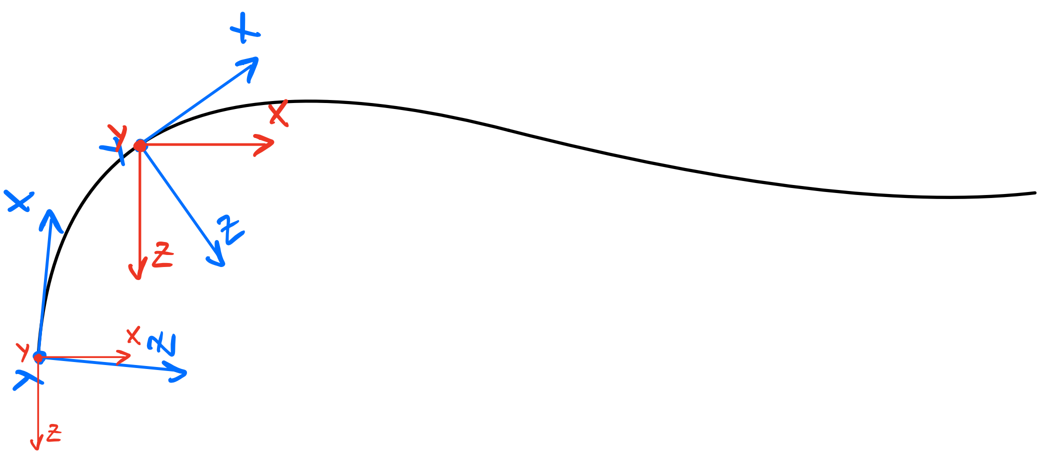
$$\boxed{\psi''(t) = \frac{J(t)}{V(t)}\sin\left(\psi_J(t)-\psi(t)\right) - \frac{2V'(t)\psi'(t)}{V(t)}}$$

$$V(t) = \left(V_x^2(t) + V_y(t)^2 \right)^{\frac{1}{2}}$$

$$V'(t) = \frac{V_x(t)\,V_x'(t) + V_y(t)\,V_y'(t)}{\left(V_x^2(t) + V_y(t)^2 \right)^{\frac{1}{2}}}$$







VALUE = 2D_NORM (POLY_COEFF$_x$, POLY_COEFF$_y$, $\bar{t}$)

$a(t)$: ACCELERATION_NORM

$$a(t) = \left(a_x(t)^2 + a_y(t)^2 \right)^{\frac{1}{2}}$$

$$\sqrt{\left(\text{POLY VAL}(a_x \text{COEFF}, \bar{t})\right)^2 + \left(\text{POLY VAL}(a_x \text{COEFF}, \bar{t})\right)^2}$$

VALUE = 2D_NORM_DERIVATIVE (POLY_COEFF$_x$, POLY_COEFF$_y$, ...

POLY_COEFF$_{x'}$, POLY_COEFF$_{y'}$, $\bar{t}$)

$$v'(t) = \frac{V_x(t) V_x'(t) + V_y(t) V_y'(t)}{\left(V_x^2(t) + V_y(t)^2 \right)^{\frac{1}{2}}}$$