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SECTION - CST SPL-2
ROLL NO. - 43

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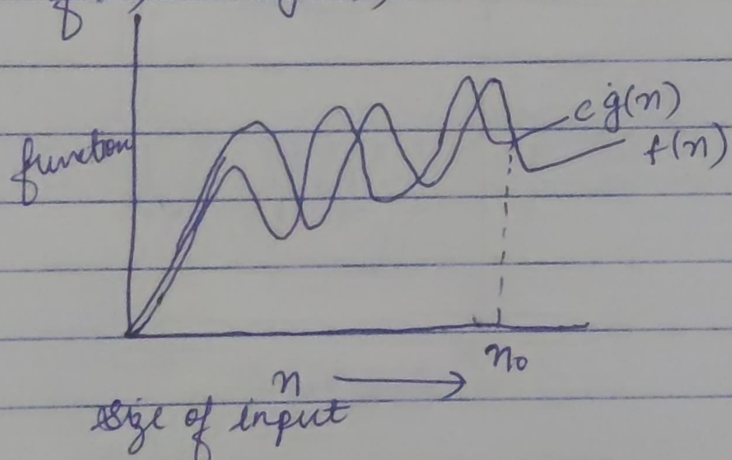
Tutorial-1

1. **Asymptotic notations** - They are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

Different asymptotic notations -

i. Big O(n)

$$f(n) = O(g(n))$$



$$f(n) = O(g(n))$$

iff $f(n) \leq cg(n)$
$\forall n \geq n_0$

for some constant, $c > 0$

g(n) is "tight" upper bound of f(n).

ex. $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq cn^3$

$n^2 + n = O(n^3)$

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ii Big Omega (Ω)

$$f(n) = \Omega(g(n))$$

$g(n)$ is "tight" lower bound of function $f(n)$.

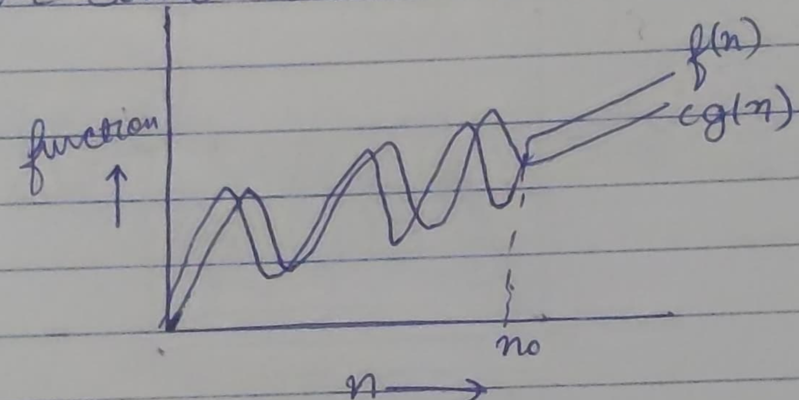
$$f(n) = \Omega(g(n))$$

iff

$$f(n) \geq c\,g(n)$$

$$\forall \quad n \geq n_0$$

for some constant $c > 0$



ex.

$$f(n) = n^3 + 4n^2$$

$$g(n) = n^2$$

$$n^3 + 4n^2 = \Omega(n^2)$$

iii Big Theta (θ)

$$f(n) = \theta(g(n))$$

$g(n)$ is both "tight" upper and "lower" bound of function $f(n)$.

$$f(n) = \theta(g(n))$$

iff

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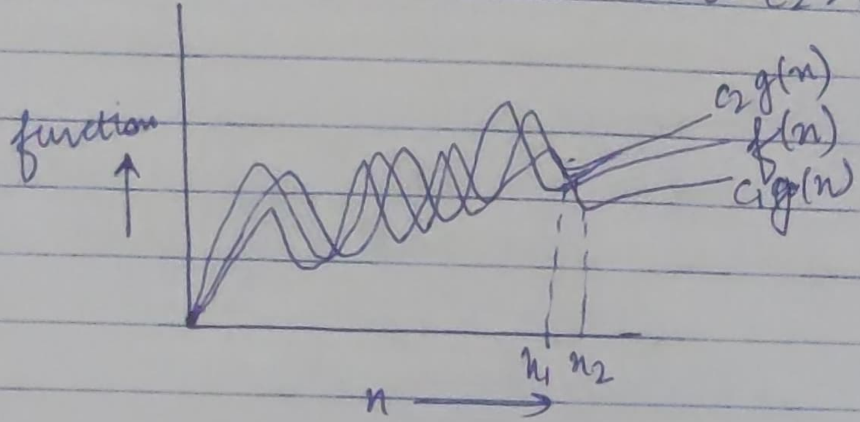
$$c_1 g(n) \le f(n) \le c_2 g(n)$$
$$\forall \quad n \ge \max(n_1, n_2)$$

for some constant $c_1 > 0$ and $c_2 > 0$.



Ex -

$3n + 2 = \Theta(n)$ as $3n + 2 \ge 3n$ and
$3n + 2 \le 4n$ for n, $k_1 = 3$, $k_2 = 4$ & $n_0 = 2$

iv. Small θ (θ) -

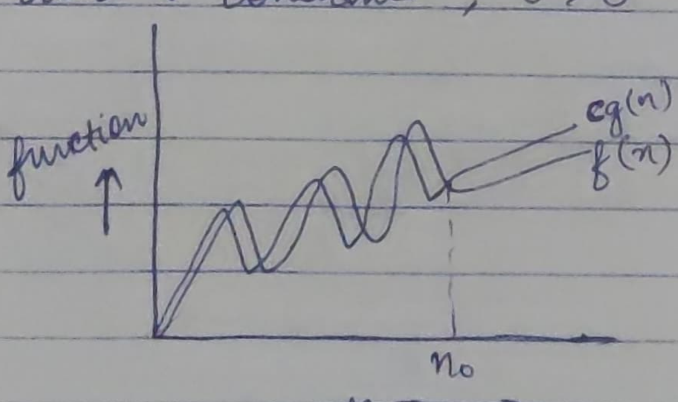
$$f(n) = o(g(n))$$

$g(n)$ is upper bound of function $f(n)$

$$f(n) = o(g(n))$$

when $f(n) < c\,g(n)$
$\forall \ n > n_0$

and $\forall$ constants, $c > 0$



Ex- $f(n) = n^2$

$g(n) = n^3$

$n^2 = O(n^3)$

V. Small omega (ω)

$f(n) \geq \omega(g(n))$

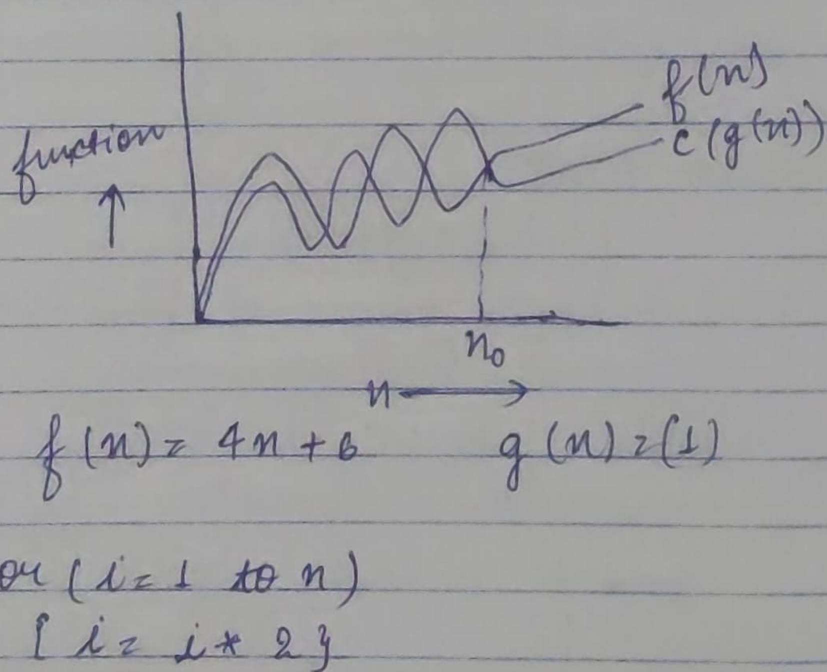
$g(n)$ is lower bound of $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c\,g(n)$

$\forall\ n > n_0$

and $\forall$ constants, $c > 0$



$f(n) = 4n + 6$ $\quad g(n) = (1)$

Ex. for ($i = 1$ to n)

$\{ i = i * 2 \}$

$\rightarrow i = \underbrace{1, 2, 4, 8, 16, \ldots \ldots n}_{k}$ (G.P.)

— $O(k)$

$a = 1,\ r = \frac{2}{1} = 2$

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GP k th value $= t_k = ar^{k-1}$

$n = 1 \times 2^{k-1}$

$n = \frac{2^k}{2}$

$2n = 2^k$

$\log(2n) = k \log 2$

$k = \log_2 2n$

$k = \log_2 2 + \log_2 n$

$k = 1 + \log n$

Time comp $= O(1 + \log_2 n)$

$= O(\log_2 n)$

3. $T(n) = 3T(n-1)$ — (1)

Let $n = n-1$

$\cancel{T(n) =}$ $T(n-1) = 3T(n-2)$ — (2)

Put (2) in (1)

$T(n) = 3 \times 3T(n-2)$ — (3)

Put $n = n-2$

$T(n-2) = 3T(n-3)$ — (4)

Put (4) in (3)

$T(n) = 3 \times 3 \times 3T(n-3)$ — (5)

$T(n) = 3^n T(n-n)$

$= 3^n T(0)$

$= 3^n$

$= O(3^n)$

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4. $T(n) = 2T(n-1) - 1$

$= 2(2T(n-2)-1) - 1$

$= 2^2(T(n-2)) - 2 - 1$

$= 2^3 T(n-3) - 2^2 - 2^1 - 2^0$

$\ldots$

$= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} - \ldots - 2^2 - 2^1 - 2^0$

$= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} - \ldots - 2^2 - 2^1 - 2^0$

$= 2^n - (2^n - 1)$

$T(n) = 1$

5.
```
int i=1, s=1;
while (s<=n){
    i++; s=s+i;
    printf("#");
}
```

$S_i = S_{i-1} + i$

i is incrementing by one step

s in incrementing by value of i

Following will be values after few iterations -

$\Rightarrow i=2, S=3$ 1st iteration

$\Rightarrow i=3, S=6$ 2nd iteration

$\Rightarrow i=4, S=10$ 3rd iteration

Let the value of n be k.

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Values of S ⇒ 1, 3, 6, 10, ...
S represents a series of sum of first n natural numbers for i = k,
$S = \frac{k(k+1)}{2}$ for stopping loop.

$$\frac{k(k+1)}{2} \geq n \Rightarrow \frac{k^2+k}{2} \geq n$$

$$T(n) = O(\sqrt{n})$$

6. void function (int n) {
    int i, count = 0;
    for (i = 1 ; i * i <= n ; i++)
        count ++;
}

$i = 1, 2, 3 \ldots n$
$i^2 = 1, 4, 9, \ldots n$
so $i^2 <= n$ or $i <= \sqrt{n}$

$a_k = a + (k-1)d$
$a = 1 \quad d = 1$
$a_k \leq \sqrt{n}$
$\sqrt{n} = 1 + (k-1).1$
$\sqrt{n} = k$
$T(n) = O(\sqrt{n})$

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7.
```
void function (int n){
    int i, j, k, count = 0;
    for(i = n/2; i <= n; i++){
        for (j = 1; j <= n; j = j*2){
            for (k = 1; k <= n; k = k*2){
                count ++;
            }
        }
    }
}
```

$i = n/2$ $\quad$ $j = \log_2 n$ $\quad$ $k = \log_2 n$

$\vdots$

$\left(\frac{n}{2}+1\right)$ times $\quad$ $\log_2 n$ $\quad$ $\log_2 n$

$$O(i*j*k) = O\left(\left(\frac{n}{2}+1\right) * \log_2 n * \log_2 n\right)$$

$$= O\left(\left(\frac{n}{2}+1\right) \times (\log n)^2\right)$$

$$T(n) = O(n(\log n)^2)$$

8.
```
function (int n){
    if (n == 1) return;
    for (i = 1 to n){
        for (j = 1 to n){
            print ("*");
        }
    }
    function (n-3);
```

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$$T(n) = T(n-3) + n^2 \quad \text{--- (1)}$$

$$T(1) = 1 \quad \text{--- (2)}$$

put $n = n-3$ in (1)

$$T(n-3) = T(n-6) + (n-3)^2 \quad \text{--- (3)}$$

put (3) in (1)

$$T(n) = T(n-6) + (n-3)^2 + n^2 \quad \text{--- (4)}$$

put $n = n-6$ in (1)

$$T(n-6) = T(n-9) + (n-6)^2 \quad \text{--- (5)}$$

put (5) in (4)

$$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n$$

Generalising

$$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2 \; - - - + n^2$$

Let $n - 3k = 1$

$$\frac{n-1}{3} = k$$

$$T(n) = T(1) + \left(n - 3\left(\frac{n-1}{3} - 1\right)\right)^2 + \left(n - 3\left(n - \frac{1}{3}\right)\right)^2 + - - - + n^2$$

$$T(n) = T(1) + [n - ((n-1) - 3]^2 + [n - (n-1-6)]^2 + [n - (n-1-9)]^2 + - - - + n^2$$

$$T(n) = 1 + (3+1)^2 + (6+1)^2 + - - - + n^2$$

$$T(n) = 1^2 + 4^2 + 7^2 \; - - - \; n^2$$

$$T(n) = n^2 + - - - 1$$

$$\boxed{T(n) = O(n^2)}$$

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9.
```
void function(int n){
    for(i=1 to n){
        for(j=1; j<=n; j=j+i){
            printf("*");
        }
    }
}
```

for $i=1$, $j \to n$ times
for $i=2$, $j = 1+3+5 ---+n$

$$a_n = a+(k-1)d$$

$$a = 1 \qquad d = 2$$

$$n = 1+(k-1)\times 2$$

$$\frac{n-1}{2} = k-1$$

$$k = \frac{n-1}{2}+1$$

$$\boxed{k = \frac{n+1}{2}}$$ no. of terms

for $i=2$, $j \to \frac{n+1}{2}$ times

for $i=3$, $j = 1+4+7+ --- n$

$$n = 1+(k-1)\times 3$$

$$\boxed{\frac{n-1}{3}+1 = k}$$

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for $i = 3$, $j = \frac{n+2}{3}$ times

Generalising
for $i = n$, $j = \frac{n+k-1}{k}$ times

Time Complexity is

$$\underbrace{n + \frac{n+1}{2} + \frac{n+2}{3} + \cdots + \frac{n+k-1}{k}}_{n \text{ terms}}$$

General term $= \frac{n+k-1}{k}$

$$\sum_{k=1}^{n} \frac{n+k-1}{k} = \frac{\sum_{1}^{n} n + \sum_{1}^{n} k - \sum 1}{k}$$

$$\Rightarrow \frac{\frac{n(n+1)}{2} + nk - n}{k}$$

$$\Rightarrow \frac{n^2 + \frac{n}{2} + nk - n}{k}$$

$$T(n) = \frac{n^2 + \frac{n}{2} + nk - n}{k}$$

neglecting constant terms

$$\boxed{T(n) = O(n^2)}$$

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10. as given n^k & c^n
relation b/w n^k & c^n is
$$n^k = O(c^n)$$
as $n^k \le d c^n$
$\forall\ n \ge n_0$, d some constant $a > 0$
for $n_0 = 1$
$c = 2$
$\Rightarrow 1^k \le d2$
$n_0 = 1$ & $c = 2$