Date

Time Complexity.

Assignment :- 53

1) What is the time & space complexity of the following code;

```
int a=0, b=0                          → const.
for (i=0; i<N; i++) {                 → N.
} a = a+rand();
for (j=0; j<M; j++) {                 → M.        (N+M.)
    b= b+rand();
}
```

Time complexity; $T(n) = O(n)$

2) What is the time complexity of the following code;

```
int a=0;                              → const.
for(i=0; i<N; i++) {                  → ≈ N.
    for (j=N; j>i; j--) {             → ≈ N/2 times × N.
        a = a+i+j;
    }
}
```

$\frac{N^2}{2} \approx N^2$

Suppose N=5.

i	j
0	5
1	4
2	3
3	2

≈ (N/2 times) चल रहा है 3 times. × (j>i)

Time complexity;

$T(n) : O(n^2)$

3) What is the time complexity of the following code;

```
int i,j,k=0;                          → const
for ( i=n/2, i<=n; i++) {             → ≈ n/2 times + 1
    for (j=2; j<=n; j=j*2) {          → log2N times + 1
        k = k+n/2;
    }
}
```

$T(n) = \frac{n}{2} \times \log_2 n \approx n \log n$.

$\boxed{T(n) = O(n \log n)}$

4) What is the time complexity of the following code:

~~int i, j, k = 0;~~
~~for(i = n/2 ; i < n~~

```
void fun (int n)
{
    for (int i = 0; i < n/2 ; i++)          → ≈ n/2 times.
        for( int j = 1; j + n/2 <= n; j++)  → ≈ n/2 times.
            for (int k = 1 ; k <= n; k = 2*k)  → log_2 n times.
                cout << " Prateek jain";
}
```

rough:

i	j	k
0	1	1
1		2

for n=3

for n=6

i	j	k
0	1	1
1	2	2
2	3	4

$T(n) = \frac{n}{2} \times \frac{n}{2} \times \log_2 n$.

$= \frac{n^2}{4} \log_2 n$.

$\boxed{T(n) = O(n^2 \log n)}$

5) Master theorem.

$T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Here, $a = 3, b = 3, k = 1, p = 0$.

$b^k = 3$

$\therefore (a = b^k)$ ✓

Now. $(p > -1)$ ✓

Then,

$$T(n) = \Theta\, n^{\log_b a} \log^{p+1} n$$

Putting values,

$$= n^{\log_3 3} \log^{0+1} n$$

$$T(n) = O(n \log n)$$

Time complexity: $\boxed{T(n) = \theta(n \log n)}$

6) $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$.

Here, $a = 6, b = 3, k = 2, p = 1$.

$b^k = 3^2 = 9$

$a < b^k$

Now, $p > 0$.

$\therefore T(n) = n^k \log^p n$.

$= n^2 \log^1 n$.

Time complexity: $\therefore \boxed{T(n) = O(n^2 \log n)}$

7) $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Here, $a = 4, b = 2, k = 1, p = -1$

$b^k = 2$

$\therefore a > b^k$

$T(n) = n^{\log_b a}$

$= n^{\log_2 4} = n^2$

Time complexity: $\boxed{T(n) = O(n^2)}$

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8) $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$.

$T(n) = 64T\left(\frac{n}{8}\right) + n^2 \log^{-1} n$.

Here $a = 64$, $b = 8$, $k = 2$, $p = -1$

$b^k = 8^2 = 64$

$\therefore$ $a = b^k$

$T(n) = n^{\log_b a} \log\log n$.

$= n^{\log_8 64} \log\log n$.

$= n^2 \log\log n$.

Time complexity: $\therefore$ $T(n) = O(n^2 \log\log n)$

9) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Here, $a = 7$, $b = 3$, $k = 2$, $p = 0$.

$b^k = 9$

$a < b^k$ ~ $p \geq 0$ ~.

$T(n) = n^k \log^p n$.

$= n^2$

Time complexity: $T(n) = O(n^2)$

10) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$.

Here,

$a = 4$, $b = 2$, $k = 0$, $p = 1$.

$b^k = 1$

So, $a > b^k$ $\quad T(n) = \log n^{\log_b a}$

$= n^{\log_2 4} = n^2$.

Time complexity: $\therefore \boxed{T(n) = O(n^2)}$ //.