Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

1.1. Let the state-space representation of an LTI system be.

$$\dot{x(t)} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix} \tag{1.1.1}$$

$$b1^T = B \tag{1.1.2}$$

and D=0. Find A and C.

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.1.3}$$

Solution:

As we know that

$$Y(s) = H(s) \times U(s) = (\frac{1}{s^3 + 3s^2 + 2s + 1}) \times U(s) \tag{1.1.4}$$

$$H(s) = \frac{Y(s)}{U(s)} = (\frac{x_1(s)}{U(s)}) \times \frac{Y(s)}{x_1(s)} \tag{1.1.5}$$

let

$$x_1(s) = \frac{U(s)}{denominator} \tag{1.1.6}$$

$$Y(s) = x_1(s) \times numerotor \tag{1.1.7}$$

$$s^3 x_1(s) + 3s^2 x_1(s) + 2s x_1(s) + x_1(s) = U(S) \tag{1.1.8}$$

Taking inverse laplace transform we get

$$\dddot{x_1(t)} + \ddot{x_1(t)} + \dot{x_1(t)} + x_1(t) = U(t) \tag{1.1.9}$$

$$\dot{x_1} = x_2 \tag{1.1.10}$$

$$\ddot{x_1} = \dot{x_2} = x_3 \tag{1.1.11}$$

$$\dddot{x_1} = \ddot{x_2} = \dot{x_3} \tag{1.1.12}$$

$$\begin{bmatrix} sx_1(s) \\ s^2x_1(s) \\ s^3x_1(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ sx_1(s) \\ s^2x1(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \tag{1.1.13}$$

taking inverse laplace transform

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \tag{1.1.14}$$

therfore

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \tag{1.1.15}$$

Since

$$Y(s) = x_1(s) \times numerator \tag{1.1.16}$$

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therefore

$$Y(s) = x_1(s) \tag{1.1.17}$$

$$\tag{1.1.18}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ sx_1(s) \\ s^2x_1(s) \end{bmatrix} \tag{1.1.19}$$

taking inverse laplace transform

$$\tag{1.1.20}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{1.1.21}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{1.1.22}$$

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot