CONTROL SYSTEMS

HOMEWORK-1

BHUKYA SIDDHU
EE18BTECH11004

IITH

February 19, 2020

Question 33

Let the state-space representation of an LTI system be.
$\dot{x}(t) = Ax(t) + Bu(t)$
y(t)=Cx(t)+Du(t)
A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$b1^T = B$
and D=0. Find A and C.
$H(s) = \dfrac{1}{s^3 + 3s^2 + 2s + 1}$

Solution

STATE MODEL

Let U1(t) and U2(t) are the inputs of the MIMO system and y1(t),y2(t) are the output of the system and x1(t) and x2(t) are the state variables.

so output equation is,

$$y1(t) = C_{11} \times x1(t) + C_{12} \times x2(t) + d_{11} \times U1(t) + d_{12} \times U2(t) \quad (1)$$

$$y2(t) = C_{21} \times x1(t) + C_{22} \times x2(t) + d_{21} \times U1(t) + d_{22} \times U2(t) \quad (2)$$

$$\begin{bmatrix} y1(t) \\ y2(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{11} & C_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{11} & d_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

therefore Y(t)=C.X(t)+D.U(t)

$$x\dot{1}(t) = a_{11} \times x1(t) + a_{12} \times x2(t) + b_{11} \times U1(t) + b_{12} \times U2(tx \quad (3)$$

$$x\dot{2}(t) = a_{21} \times x1(t) + a_{22} \times x2(t) + b_{21} \times U1(t) + b_{22} \times U2(t) \quad (4)$$

$$\begin{bmatrix} \dot{x1}(t) \\ \dot{x2}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{11} & b_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

$therefore\ \dot{X}(t)$=A.X(t)+B.U(t)

FINDING TRANSFER FUNCTION

$So,\ \dot{X}(t)$=A.X(t)+B.U(t) $\ be \quad equation \quad 1$
and Y(t)=C.X(t)+D.U(t) $\ be \quad equation \quad 2$
by applying laplace transforms on both sides of equation 1
we get
S.X(S)-X(0)=A.X(S)+B.U(S)
S.X(S)-A.X(S)=B.U(S)+X(0)
(SI-A)X(S)=X(0)+B.U(S)
X(S)=X(0)([SI-A])$^{-}1 + B.([SI - A])^{-}1.U(S)$
$\ Laplace \quad transform \quad of \quad equation \quad 2 \quad and \quad sub \ \ X(s)$

Y(S)=C.X(S)+D.U(S)
Y(S)=C.[X(0)([SI-A])$^{-}1 + B.([SI-A])^{-}1.U(S)] + D.U(S)$
If X(0)=0
then Y(S)=C.[B.([SI-A])$^{-}1.U(S)] + D.U(S)$
$\frac{Y(S)}{U(S)} = C.[B.([SI-A])^{-}1] + D = H(S)$

As we know that

$$Y(s) = H(s) \times U(s) = (\frac{1}{s^3 + 3s^2 + 2s + 1}) \times U(s)$$

let X(S)= $\frac{U(S)}{denominator}$

Y(S)=X(S)$\times numerotor$

s$^3X(s) + 3s^2X(s) + 2sX(s) + X(s) = U(S)$

$$\begin{bmatrix} sx(s) \\ s^2x(s) \\ s^3x(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x(s) \\ sx(s) \\ s^2x(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U$$

therfore A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$

Since Y(S)=X(S)$\times numerator$
therefore Y(S)=X(S);

Y= $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x(s) \\ sx(s) \\ s^2x(s) \end{bmatrix}$

C=$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$