EE3025-Assignment 1

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(5.3) The system with h(n) is defined to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{0.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 Solution

Method I :
BIBO Stability Criteria : A system is said to be stable, if the bounded input produces the bounded output.
Let x(n) be a bounded sequence.So,

$$|x(n)| < M_x \tag{1.0.1}$$

Where M_x is a finite value.From the convolution property,

$$y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k) \tag{1.0.2}$$

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{1.0.3}$$

$$|y(n)| \leq M_x \sum_{-\infty}^{\infty} |h(k)| \tag{1.0.4}$$

Because all x(k) less than M_x.
As M_x is finite, for $|y(n)|$ to be finite

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \tag{1.0.5}$$

$$\text{then } |y(n)| \leq M_y < \infty \tag{1.0.6}$$

Therefore we can say that the output is bounded if the impulse response is absolutely summable.

Given difference equation:

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.7}$$

$$y(n) = 0 \text{ for } y < 0 \tag{1.0.8}$$

and the given bounded input is

$$x(n) = \{1, 1, 2, 4, 3, 1\} \tag{1.0.9}$$

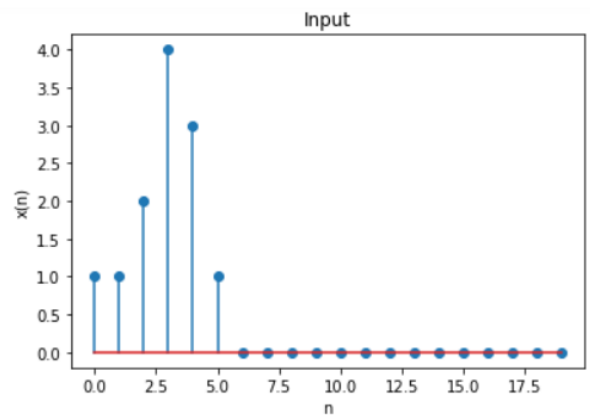

Fig. 0: Given input

and the output we get,

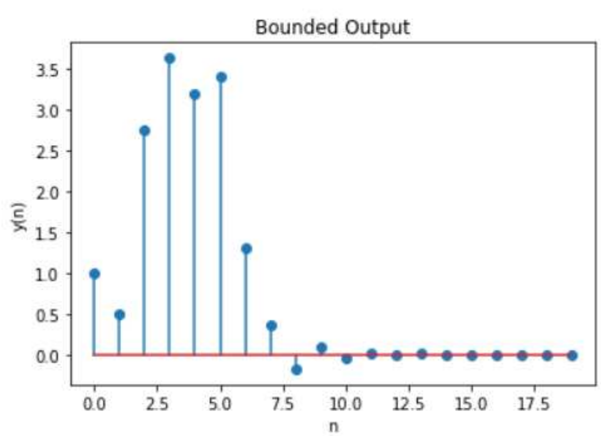

Fig. 0: Bounded output

So we can see that we are getting bounded output for the bounded input.Therefore, the given system is stable.

Method 2 :
Given difference equation,

$$y(n)+\frac{1}{2}y(n-1)=x(n)+x(n-2) \tag{1.0.10}$$

By applying Z-transform we get,

$$Y(z)+\frac{1}{2}z^{-1}Y(z)=X(z)+z^{-2}X(z) \tag{1.0.11}$$

$$Y(z)=\frac{2(z^2+1)}{z(2z+1)}X(z) \tag{1.0.12}$$

Therefore H(z) is

$$H(z)=\frac{2(z^2+1)}{z(2z+1)} \tag{1.0.13}$$

$$H(z)=\frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{1.0.14}$$

$$H(z)=z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}}+\frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{1.0.15}$$

By applying inverse z-transform we get,

$$h(n)=\left[\frac{-1}{2}\right]^{n}u(n)+\left[\frac{-1}{2}\right]^{n-2}u(n-2) \tag{1.0.16}$$

As the condition for the system to be stable is

$$\sum_{n=-\infty}^{\infty}|h(n)|<\infty \tag{1.0.17}$$

By substituting h(n) we get,

$$\sum_{n=-\infty}^{\infty}|h(n)|=\sum_{n=-\infty}^{\infty}\left|\left[\frac{-1}{2}\right]^{n}u(n)+\left[\frac{-1}{2}\right]^{n-2}u(n-2)\right| \tag{1.0.18}$$

$$=\sum_{n=-\infty}^{\infty}\left|(-1)^{n}\left(\left[\frac{-1}{2}\right]^{n}u(n)+\left[\frac{-1}{2}\right]^{n-2}u(n-2)\right)\right| \tag{1.0.19}$$

$$=\sum_{n=-\infty}^{\infty}\left|\left[\frac{1}{2}\right]^{n}u(n)+\left[\frac{1}{2}\right]^{n-2}u(n-2)\right| \tag{1.0.20}$$

$$=\sum_{n=-\infty}^{\infty}\left[\frac{1}{2}\right]^{n}u(n)+\sum_{n=-\infty}^{\infty}\left[\frac{1}{2}\right]^{n-2}u(n-2) \tag{1.0.21}$$

$$=\sum_{n=0}^{\infty}\left[\frac{1}{2}\right]^{n}+\sum_{n=2}^{\infty}\left[\frac{1}{2}\right]^{n-2} \tag{1.0.22}$$

$$=\left[\frac{1}{1-\frac{1}{2}}\right]+\left[\frac{1}{1-\frac{1}{2}}\right] \tag{1.0.23}$$

$$=2+2 \tag{1.0.24}$$

$$=4<\infty \tag{1.0.25}$$

As the given system satisfies the stability condition.Therefore, the system is stable.

Method 3 :
The condition for the system to be stable is,

$$\sum_{n=-\infty}^{\infty}|h(n)|<\infty \tag{1.0.26}$$

The above equation can be written as,

$$\sum_{n=-\infty}^{\infty}|h(n)|\left|z^{-n}\right|_{|z|=1}<\infty \tag{1.0.27}$$

$$\sum_{n=-\infty}^{\infty}\left|h(n)z^{-n}\right|_{|z|=1}<\infty \tag{1.0.28}$$

From triangle inequality,

$$\sum_{n=-\infty}^{\infty}\left|h(n)z^{-n}\right|_{|z|=1}<\left|\sum_{n=-\infty}^{\infty}h(n)z^{-n}\right|_{|z|=1} \tag{1.0.29}$$

$$\therefore |H(z)|_{|z|=1}<\infty \tag{1.0.30}$$

For the system to be stable unit circle should lie in the ROC of the system.

From the equation 1.0.15,

$$H(z)=z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}}+\frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{1.0.31}$$

$$=z^{-1}\left[1-\frac{1}{2z}+\left(\frac{1}{2z}\right)^{2}+....+z^{-2}\left[1-\frac{1}{2z}+\left(\frac{1}{2z}\right)^{2}+....\right]\right] \tag{1.0.32}$$

The above expansion is infinite G.P, so the summation is finite if $z\neq 0$ and

$$\left|\frac{1}{2z}\right| < 1 \implies |z| > \frac{1}{2} \tag{1.0.33}$$

From the equation 1.0.33 ROC of the system consists the unit circle.Thus, the given system is stable.

Below plot is ROC of the given system,

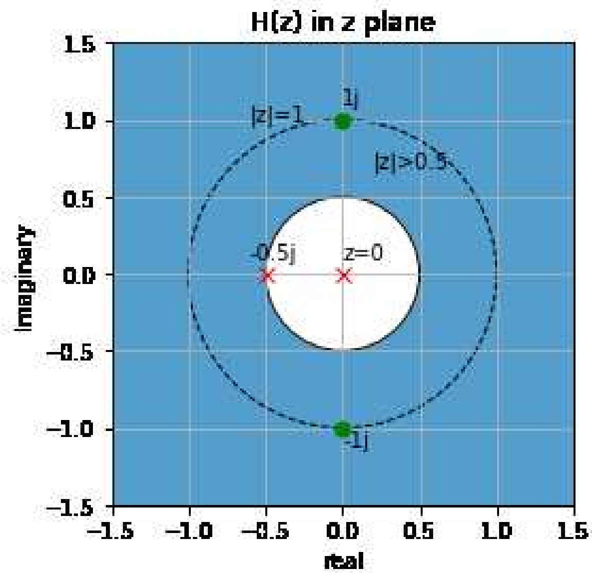

Fig. 0: Plot of H(z) in Z-Plane

From the above plot we can verify that unit circle is lying in the ROC of the system. Therefore, the system is stable.