

Control Systems

G V V Sharma*

CONTENTS

1	Stability	1
1.1	Second order System	1
2	Routh Hurwitz Criterion	2
3	Compensators	2
4	Nyquist Plot	2

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

1 STABILITY

1.1 Second order System

1.1.1. Let the state-space representation of an LTI system be.

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (1.1.1)$$

$$b1^T = B \quad (1.1.2)$$

and D=0. Find A and C.

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \quad (1.1.3)$$

Solution:

FINDING TRANSFER FUNCTION

so

$$\dot{X}(t) = AX(t) + BU(t)$$

$$Y(t) = CX(t) + DU(t)$$

by applying laplace transforms on both sides of equation 1 we get

$$S.X(S) - X(0) = A.X(S) + B.U(S)$$

$$S.X(S) - A.X(S) = B.U(S) + X(0)$$

$$(SI - A)X(S) = X(0) + B.U(S)$$

$$X(S) = X(0)([SI - A])^{-1} + ([SI - A])^{-1}B)U(S)$$

Laplace transform of equation 2 and sub X(s)

$$Y(S) = C.X(S) + D.U(S)$$

$$Y(S) = C.[X(0)([SI - A])^{-1} + ([SI - A])^{-1}B)U(S)] + DU(S)$$

If X(0)=0

$$\text{then } Y(S) = C[([SI - A])^{-1}B)U(S)] + DU(S)$$

$$\frac{Y(S)}{U(S)} = C[([SI - A])^{-1}B) + D = H(S)$$

As we know that

$$Y(s) = H(s) \times U(s) = \left(\frac{1}{s^3 + 3s^2 + 2s + 1} \right) \times U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \left(\frac{\frac{Y(s)}{x_1(s)}}{\frac{U(s)}{x_1(s)}} \right) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

$$\text{let } x_1(s) = \frac{U(s)}{s^3 + 3s^2 + 2s + 1}$$

$$Y(s) = x_1(s) \times 1$$

$$s^3 x_1(s) + 3s^2 x_1(s) + 2s x_1(s) + x_1(s) = U(s) \quad (1.1.4)$$

Taking inverse laplace transform we get

$$\ddot{x}_1(t) + \dot{x}_1(t) + x_1(t) = U(t)$$

$$\dot{x}_1 = x_2$$

$$\ddot{x}_1 = \dot{x}_2 = x_3$$

$$\ddot{x}_1 = \dot{x}_2 = x_3$$

so equation 1.1.4 can be written as

$$\begin{bmatrix} s x_1(s) \\ s^2 x_1(s) \\ s^3 x_1(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ s x_1(s) \\ s^2 x_1(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \quad (1.1.5)$$

taking inverse laplace transform

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \quad (1.1.6)$$

therefore

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \quad (1.1.7)$$

Since $Y(s) = x_1(s) \times \text{numerator}$

therefore $Y(s) = x_1(s)$

$$(1.1.8)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ s x_1(s) \\ s^2 x_1(s) \end{bmatrix} \quad (1.1.9)$$

taking inverse laplace transform

$$(1.1.10)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (1.1.11)$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepalli@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad (1.1.12)$$

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT