Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

1.1. State the general model of a state space system specifying the dimensions of the matrices and vectors.
Solution:

$$\mathbf{x}\dot{(t)} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + Du(t)$$

x(.) is called the ”state vector”
y(.) is called the ”output vector”
u(.) is called the ”input(control) vector”
A(.) is the ”state or system matrix”, dim[A(.)]=nxn
B(.) is the ”input matrix”,dim[B(.)]=nxp
C(.) is the ”output matrix”,dim[C(.)]=qxn
D(.) is the ”feedthrough matrix”,dim[A(.)]=qxp
$x\dot{(t)} = \frac{d}{dt}x(t)$

1.2. Find the transfer function $\mathbf{H}(s)$ for the general system.
Solution:
FINDING TRANSFER FUNCTION

$$X\dot{(t)} = AX(t) + BU(t)$$
$$Y(t) = CX(t) + DU(t)$$

by applying laplace transforms on both sides of equation 1 we get

$$s.X(s) - X(0) = A.X(s) + B.U(s)$$
$$s.X(s) - A.X(s) = B.U(s) + X(0)$$
$$(sI - A)X(s) = X(0) + B.U(s)$$
$$X(s) = X(0)([sI - A])^{-1} + (([sI - A])^{-1}B)U(s)$$

Laplace transform of equation 2 and sub X(s)

$$Y(s) = C.X(s) + D.U(s)$$

$$Y(s) = C.[X(0)([sI - A])^{-1} + (([sI - A])^{-1}B)U(s)] + DU(s)$$

If X(0)=0

$$Y(s) = C[(([sI - A])^{-1}B)U(s)] + DU(s)$$
$$\frac{Y(s)}{U(s)} = C[(([sI - A])^{-1}B)] + D = H(s)$$

1.3. Given

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$
$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$$b1^T = B$$

and D=0, find A and C.
Solution:
In the generic form D is feedforward matrix of dimension qxp
where q is number of outputs and p is number of inputs from given question as D is given 0(scalar), so q=p=1
therefore it is single input and single output system
Hence U is the input and Y is the output which are scalar
Given that B is 3x1 so, n=3
so, A is 3x3,C is 1x3
As we know that

$$Y(s) = H(s) \times U(s) = (\frac{1}{s^3 + 3s^2 + 2s + 1}) \times U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = (\frac{(\frac{Y(s)}{x_1(s)})}{\frac{U(s)}{x_1(s)}}) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

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let

$$x_1(s) = \frac{U(s)}{s^3 + 3s^2 + 2s + 1}$$

$$Y(s) = x_1(s) \times 1$$

$$s^3 x_1(s) + 3s^2 x_1(s) + 2s x_1(s) + x_1(s) = U(s) \qquad (1.3.1)$$

Taking inverse laplace transform we get

$$\dddot{x_1}(t) + \ddot{x_1}(t) + \dot{x_1}(t) + x_1(t) = U(t)$$

$$\begin{aligned} \dot{x_1} &= x_2 \\ \ddot{x_1} = \dot{x_2} &= x_3 \\ \dddot{x_1} = \ddot{x_2} &= \dot{x_3} \end{aligned}$$

so equation 1.3.1 can be written as

$$\begin{bmatrix} s x_1(s) \\ s^2 x_1(s) \\ s^3 x_1(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ s x_1(s) \\ s^2 x1(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U$$

taking inverse laplace transform

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U$$

therfore

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

Since $Y(s) = x_1(s) \times numerator$
therefore $Y(s) = x_1(s)$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ s x_1(s) \\ s^2 x_1(s) \end{bmatrix}$$

taking inverse laplace transform

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot