Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

1.1 Second order System

1.1. Let the state-space representation of an LTI system be.

$$\dot{x(t)} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix} \tag{1.1.1}$$

$$b1^T = B \tag{1.1.2}$$

and D=0. Find A and C.

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.1.3}$$

Solution:
FINDING TRANSFER FUNCTION
so

$$\dot{X(t)} = AX(t) + BU(t)$$
$$Y(t) = CX(t) + DU(t)$$

by applying laplace transforms on both sides of equation 1 we get
S.X(S)-X(0)=A.X(S)+B.U(S)
S.X(S)-A.X(S)=B.U(S)+X(0)
(SI-A)X(S)=X(0)+B.U(S)
$X(S) = X(0)([SI - A])^{-1} + B([SI - A])^{-1}U(S)$
Laplace transform of equation 2 and sub X(s)
Y(S)=C.X(S)+D.U(S)
$Y(S) = C.[X(0)([SI-A])^{-1} + B([SI-A])^{-1}U(S)] + DU(S)$
If X(0)=0
then $Y(S) = C[B([SI - A])^{-1}U(S)] + DU(S)$
$\frac{Y(S)}{U(S)} = C[B([SI - A])^{-1}] + D = H(S)$

As we know that
$Y(s) = H(s) \times U(s) = (\frac{1}{s^3+3s^2+2s+1}) \times U(s)$

$H(s) = \frac{Y(s)}{U(s)} = (\frac{x_1(s)}{U(s)}) \times \frac{Y(s)}{x_1(s)}$

let $x_1(s) = \frac{U(s)}{denominator}$

$Y(s) = x_1(s) \times numerotor$

$$s^3x_1(s) + 3s^2x_1(s) + 2sx_1(s) + x_1(s) = U(S) \tag{1.1.4}$$

Taking inverse laplace transform we get
$\dddot{x_1(t)} + \ddot{x_1(t)} + \dot{x_1(t)} + x_1(t) = U(t)$
$\dot{x_1} = x_2$
$\ddot{x_1} = \dot{x_2} = x_3$
$\dddot{x_1} = \ddot{x_2} = \dot{x_3}$
so equation 1.1.4 can be written as

$$\begin{bmatrix} sx_1(s) \\ s^2x_1(s) \\ s^3x_1(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ sx_1(s) \\ s^2x1(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \tag{1.1.5}$$

taking inverse laplace transform

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U \tag{1.1.6}$$

therfore

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \tag{1.1.7}$$

Since $Y(s) = x_1(s) \times numerator$
therefore $Y(s) = x_1(s)$

$$\tag{1.1.8}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1(s) \\ sx_1(s) \\ s^2x_1(s) \end{bmatrix} \tag{1.1.9}$$

taking inverse laplace transform

$$\tag{1.1.10}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{1.1.11}$$

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$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{1.1.12}$$

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot