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Introduction

In this assignment, we are asked to explore how the hidden layers in a simple single-hidden layer network learn to represent features within the input data. Input data is an array form of 26 capital letters of the English alphabet. Each letter has 9x9 pixels and this is transformed into a vector of length 81 giving one input node for each pixel in the letter. You can see 9x9 pixelated capital letters in Figure 1. Therefore, by default, we have 81 input nodes, 6 hidden nodes, and 8 output nodes. During the analysis, we will see that hidden nodes cluster letters by identifying learned features such as finding mutual patterns such as diagonal, vertical, horizontal, rounded, and straight lines. We will train our model to put our 26 pixelated letters into the correct class assignments. For example, letters 'D', 'O' and 'Q' can be grouped to 'O' like shape because they all seem to share similar pixels.

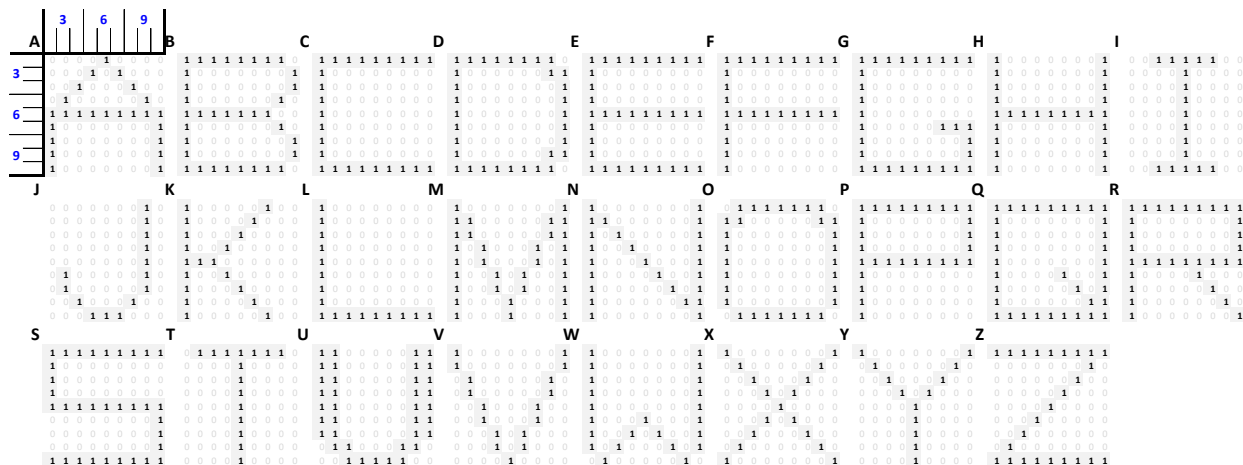


Figure 1

We changed the following parameters during our analysis:

- **numHiddenNodes:** I started with a small number of hidden nodes, and then increased them. The change in the number of nodes had an impact on the activations.
- **numOutputNodes:** During our experiment, I started with 8 classes (default) and I was able to change the class assignments to improve our results.
- **noise:** introduced noise to reduce overfitting and being able to better generalize and faster learning. 10% (0.1) of our pixels were randomly flipped for each character retrieved during the training step. For

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example, according to figure 1, we can see that pixelated O and Q do not have the same walls. So, noise should take care of this problem.

- **eta:** the learning rate helped us to determine how big steps we had to take at each iteration while moving toward a minimum of a loss function.

Analysis

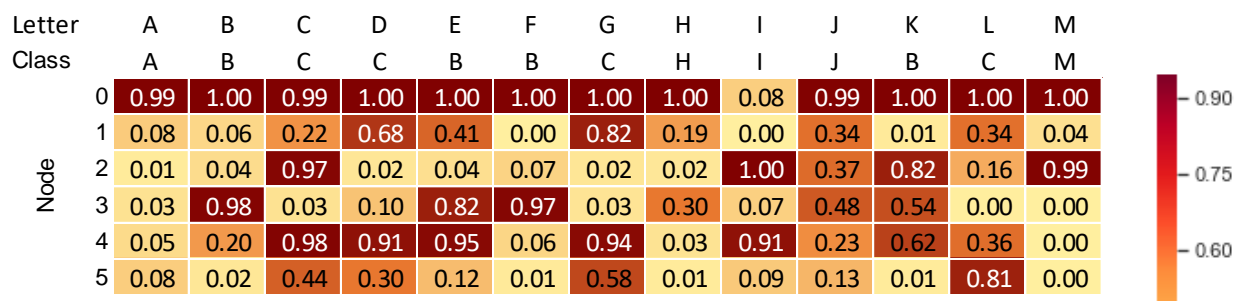
If we do not use backpropagation to minimize the cost function, we will have 19.23% accuracy rate for the “before training.” However, in our analysis, we are interested in backpropagation, “after training” results and interpretations. When I ran the model with default parameters (numHiddenNodes = 6, numOutputNodes = 8, trainingNoise = 0.00, and eta or learning rate = 0.5, SSE threshold epsilon = 0.01), we were able to classify about 85% of the 26 capital letters to the desired 8 classes. It took 2,290 iterations (out of max iteration 5,000 iterations), which has slower convergence. We can achieve lower iterations by modifying the default learning rate (eta = 1.0) in our future analysis. Let’s analyze results; we can see that the following capital letter A, C, H, and J misclassified.

Letter	A	B	E	F	K	P	R	C	D	G	L	O	Q	S	H	I	T	J	M	N	V	W	X	Y	Z	U
Desired	0	1	1	1	1	1	1	2	2	2	2	2	2	2	3	4	4	5	6	6	6	6	6	6	6	7
Selected	6	1	1	1	1	1	1	6	2	2	2	2	2	2	1	4	4	1	6	6	6	6	6	6	6	7

Percent Accuracy 84.62

Figure 2

We can see that the desired class should be rearranged and reclassified. For example, according to figure 1, the pixelated letter A is closer to the pixelated letter H, and H is close to F. The pixelated letter J is closer to the pixelated letter U. So, we can reclassify them next time when we run the model. Now we can focus on interpreting the hidden nodes.



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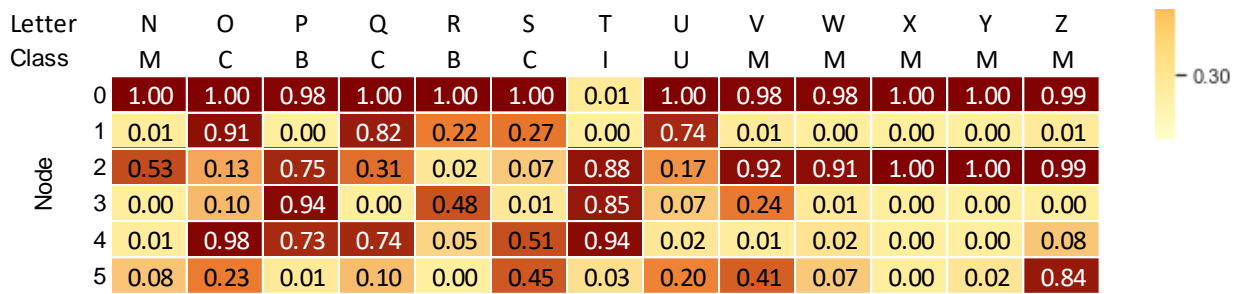
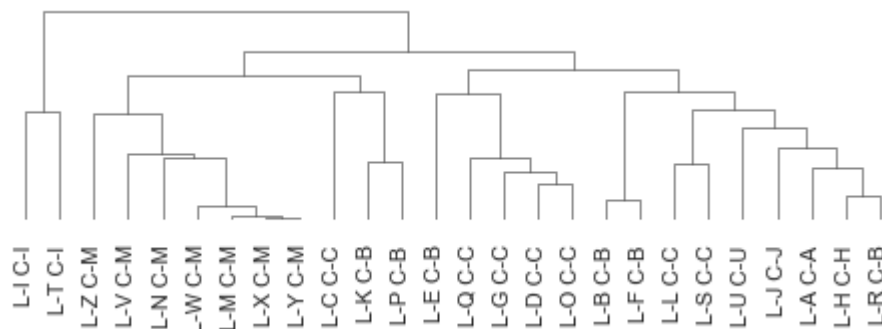


Figure 3

- **H0:** it reacts to all letters except I and T (negative weight) which fall to I class
- **H1:** it reacts to mostly rounded and squared letters which are part of C class such as O, Q, G, U, D
- **H2:** it reacts to white space of Y, I, X, Z, M, C, V, W, T, K, and P – unclear features
- **H3:** it reacts to mostly B class such as B, F, P, E, K, and T
- **H4:** it reacts to square and vertical line of C, O, E, G, T, D, I, Q, and P – somewhat unclear features
- **H5:** it reacts to letters Z, L, G or mostly C class – somewhat unclear features

We can see this visually in the dendrogram.



Letters	A	H	F	E	S	Z	B	P	R	K	Q	O	D	L	C	G	J	U	V	M	N	W	X	Y	T	I
Classes	A	A	B	B	B	B	B	B	B	B	C	C	C	C	C	C	U	U	M	M	M	M	M	M	I	I
Desired	0	0	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	4	4	4	4	4	4	5	5

Figure 4

I reduced output nodes from 8 to 6 and reclassified some 9x9 pixelized capital letters such as 'A' and 'H' as A-class, 'S' and 'Z' as B-class, and 'J' and 'U' as U-class. However, I hold the rest of the parameters at the same default level. After reclassifying the letters, the accuracy of the model went up from 84.62% to 96.15% (or 25/26; 'H' letter misclassified) and the iterations reduced from 2,290 to 1,463.

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When I introduced the 10% noise during training the model, the accuracy score went up to 100%, and iterations slightly increased to 1,616. Therefore, 10% of our pixels were randomly flipped for each character retrieved during the training and the hidden nodes had less tendency to assign weights to the same pixels; now 'H' and 'A' letters classified as A class. Here is the hidden node visual after reclassifying letters and introducing noise.

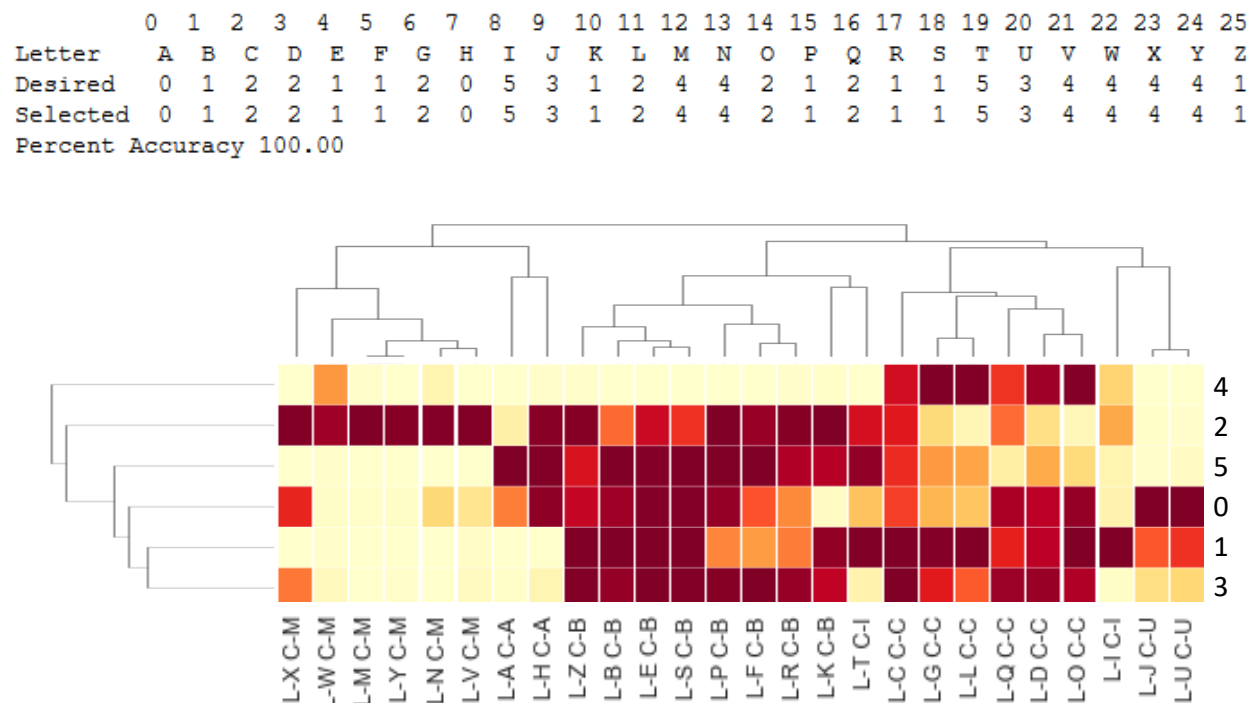


Figure 5

- **H4:** it reacts to C class only
- **H2:** it reacts to mostly M and B classes
- **H5:** it reacts B and A classes primarily. This hidden node groups the letters in a nice way.

Letter	F	S	A	P	B	H	E	T	R	K	Z	C	G
Class_L	B	B	A	B	B	A	B	I	B	B	B	C	C
Hidden node 5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9	0.9	0.8	0.7	0.5

Letter	L	D	O	Q	I	U	J	W	X	V	N	Y	M
Class_L	C	C	C	C	I	U	U	M	M	M	M	M	M
Hidden node 5	0.4	0.4	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

- **H0:** it reacts oval, square, and B class; it does not react M and I classes
- **H1:** it reacts B and C classes which have a bottom or top line
- **H3:** it is somewhat like the previous node, H1

By looking at figure 5, hidden nodes are somewhat duplicating each other. There might be too many hidden nodes.

Conclusion

I started with a small number of hidden nodes, and then increased them and saw what happened with the node activations. I can see that with hidden node 2, my model did not converge (reaches to max iteration 5,000), and accuracy dropped to 53.85%. Thus, not enough hidden nodes were not able to pick up enough features to cluster the letters. We did not have enough hidden nodes. The lowest hidden node with high accuracy was 4 and it was able to cluster 9x9 pixelated capital letters correctly at a learning rate 0.5. However, when I increased the learning rate from 0.5 to 1.0, it took 955 iterations only. So, it took big steps at each iteration while moving toward a minimum of a loss function. As I increased the hidden nodes, my model overfits and training accuracy started dropping. By looking at figure 6, with too few hidden nodes, the model was not able to pick up enough features to distinguish the pixelated capital letters. Too many, and the hidden nodes were tied to specific features, and the model was not able to generalize.

Changes	Learning Rate	Accuracy	Iteration
default parameters (output = 8)	0.5	84.62	2290
reclassifying letters (output = 6)	0.5	96.15	1463
training noise change from 0.0 to 0.1	0.5	100	1616
Hidden nodes = 2	0.5	53.85	5000
Hidden nodes = 3	0.5	80.77	3889
Hidden nodes = 4	0.5	100	2531
Hidden nodes = 4	1.0	100	955
Hidden nodes = 5	0.5	100	2322
Hidden nodes = 6	0.5	100	1616
Hidden nodes = 7	0.5	88.46	1191
Hidden nodes = 8	0.5	100	1100
Hidden nodes = 9	0.5	88.46	1200

Figure 6