UNIT- V

Small Signal Low-Frequency Transistor Amplifier Models BJT

1. Introduction

Amplifiers are the unsung heroes of modern technology. They play a crucial role in strengthening weak electrical signals, enabling them to drive speakers, power antennas, and shape the sounds and images that entertain, inform, and inspire us. Whether you're listening to music, watching television, or communicating through your smartphone, amplifiers are working silently in the background to ensure the signals reach your senses with clarity and impact. In this chapter, we venture into the world of amplifiers, unraveling their inner workings, different types, and applications in a wide array of fields.

2. CE Amplifier

CE amplifier also works in a similar manner. In order to make the amplifier work properly some additional components are added in the circuit as shown in figure 1.

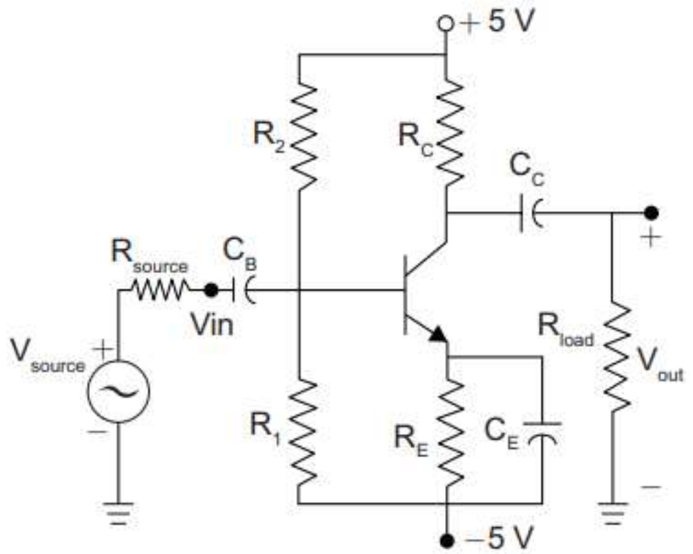


Figure 1: *CE Amplifier*

C_B, C_C -Coupling Capacitor and C_3 – Emitter Bypass Capacitor. C_B, C_C, C_E values are chosen high, so that under ac these provide short circuit. The coupling capacitor (C_C) is used to pass the ac input signal and block the dc voltage from the preceding circuit. This prevents dc in the circuitry on the left of the coupling capacitor from affecting the bias. The coupling capacitor also blocks the bias of the transistor from reaching the input signal source. The emitter bypass capacitor (C_E) is used to bypass the R_E and short circuits the ac signal through C_E since voltage gain decreases because of presence of R_E. R_1 and R_2 forms part of the voltage divider bias circuit to maintain the transistor in the active region.

Transistor Small Signal Model

The behavior of the BJT in the sinusoidal domain is quite different from its dc domain. In dc domain the transistor acts as a switch operating either in saturation or cut-off region. In the ac domain the transistor works in active region and the response depends on capacitance across terminals, input impedance, output conductance, etc. In order to analyze these effects small signal ac equivalent circuit is required. Hence, BJT is modeled as equivalent circuit that represents the AC characteristics of the transistor. The model uses circuit elements that approximate the behavior of the transistor. There are two models commonly used in small signal AC analysis of a transistor:

i. Hybrid equivalent model
ii. r_e model

Hybrid equivalent model is defined based on hybrid parameters, which are given in the next section. The r_e model and large signal models are beyond the scope of this text book.

7.9.1 Hybrid Parameters

Transistor is a three terminal device and can be connected as a two port network by keeping one terminal common to both input and output ports. All the transistor amplifiers are two port networks having two voltages and two currents. The positive directions of voltages and currents are shown in Figure 2. The process of obtaining the h-parameters is summarized in table (1).

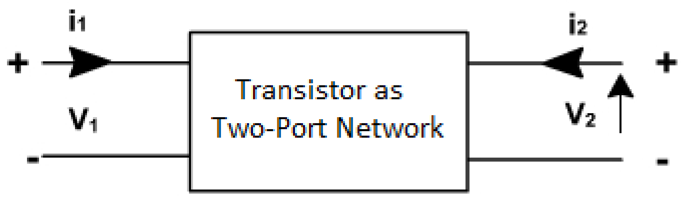


Figure 2: *Transistor as a Two Port Network*

In a two port network, out of four quantities two are independent and two are dependent. For a transistor as two port network, the input current i_1 and output voltage v_2 are taken independent and other two quantities i_2 and v_1 can be expressed in terms of i_1 and v_2.

$$v_1 = f_1(i_1, v_2) \quad \dots (7.73)$$

$$i_2 = f_2(i_1, v_2) \quad \dots (7.74)$$

These functions can be related with hybrid parameters as given below.

$$v_1 = h_{11}i_1 + h_{12}v_2 \quad \dots (7.75)$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \quad \dots (7.76)$$

The parameters h_{11}, h_{12}, h_{21} and h_{22} are called as h-parameters.

The h-parameters are defined as below:

h_{11} is the input impedance obtained with output short circuit (v_2=0).

$$h_{11} = h_i = \left.\frac{v_1}{i_1}\right|_{v_2=0} \quad \dots (7.77)$$

h_{12} is the fraction of output voltage at input with input open circuited or is also called as reverse voltage gain with input open circuited (i_2=0).

$$h_{12} = h_r = \left.\frac{v_1}{v_2}\right|_{i_2=0} \quad \dots (7.78)$$

h_{21} is negative of current gain with output short circuited (v_2=0). The current entering the load is negative of I_2. This is also known as forward short circuit current gain.

$$h_{21} = h_f = \left.\frac{i_2}{i_1}\right|_{v_2=0} \quad \ldots (7.79)$$

h_{22} is output admittance with input open circuited (i_2=0).

$$h_{22} = h_o = \left.\frac{i_2}{v_1}\right|_{i_2=0} \quad \ldots (7.80)$$

Table (1): *Summary of h-parameters*

h-parameter	**Meaning**	**Process to obtain h-parameters**
h_{11}	Input impedance	Output Short circuited
h_{12}	Reverse voltage gain	Input Open Circuited
h_{21}	Forward current gain	Output Short circuited
h_{22}	Output admittance	Input Open Circuited

7.9.2 h - parameter Model

The h-parameters are used to represent the two-port transistor network with its equivalent model. The h-parameter equivalent model is shown in figure (3).

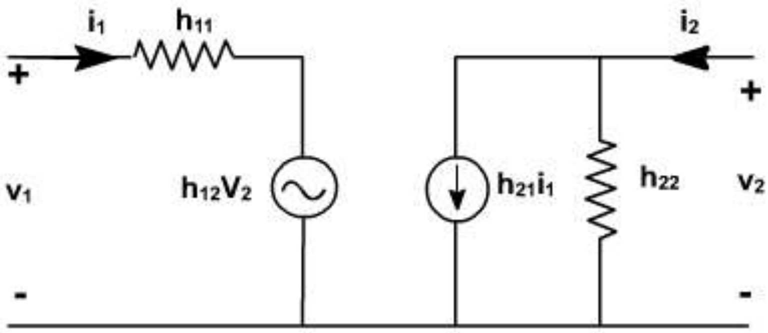


Figure (3): *h-parameter model*

The input circuit appears as a resistance h_{11} in series with a voltage generator h_{12} v_2. This circuit is derived from eqn. (7.75). The input portion is a Thevenin's equivalent, or voltage generator with series resistance. The output circuit involves two components; a current generator h_{21} i_1 and shunt resistance h_{22} and is derived from eqn. (7.76). The output section is Norton equivalent, or current generator with shunt resistance. Thus the circuit is called as hybrid equivalent model. The symbol '*h*' is simply the abbreviation of the word hybrid.

The major usages of equivalent circuit are: First, it isolates the input and output circuits, their interaction being accounted for by the two controlled sources. Secondly, the two sections of the circuit are in a form which makes it simple to take into account source and load circuits.

The h – parameters, however, will change with each configuration. To distinguish which parameter has been used or which is available, a second subscript has been added to the h – parameter notation. For the common – base configuration, the lowercase letter b is added, and for common emitter and common collector configurations, the letters e and c are used respectively and are tabulated in table (7.4).

Table (7.4): *h-parameters for various transistor configurations*

h-parameter	Notation in CB	Notation in CE	Notation in CC
h_{11}	h_{ib}	h_{ie}	h_{ic}
h_{12}	h_{rb}	h_{re}	h_{rc}
h_{21}	h_{fb}	h_{fe}	h_{fc}
h_{22}	h_{ob}	h_{oe}	h_{oc}

7.9.3 Determination of h-parameters from Transistor Characteristics

To determine the four h-parameters of transistor amplifier, input and output characteristic are used. Input characteristic depicts the relationship between input voltage and input current with output voltage as parameter. The output characteristic depicts the relationship between output voltage and output current with input current as parameter. Figure (4) shows the output characteristics of CE amplifier.

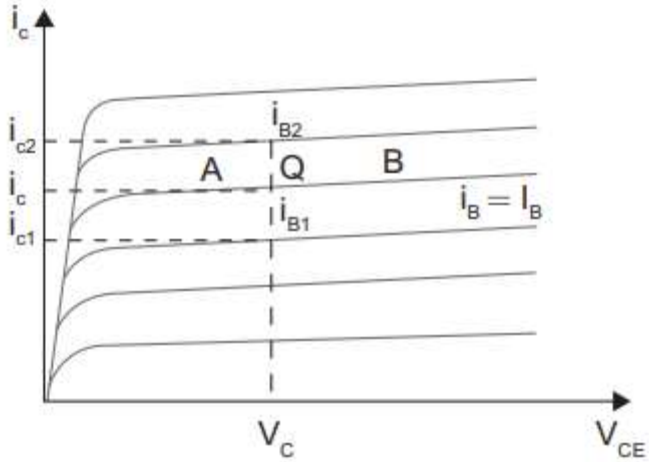


Figure (4): *CE Output characteristics*

The forward short circuit current gain can be obtained as

$$h_{fe} = \left.\frac{\partial I_C}{\partial i_B}\right|_{V_C} = \frac{i_{C2} - i_{C1}}{i_{b2} - i_{b1}} \qquad \text{... (7.81)}$$

The current increments are taken around the quiescent point Q which corresponds to $i_B = I_B$ and to the collector voltage $V_{CE} = V_C$

The value of h_{oe} at the quiescent operating point is given by the slope of the output characteristic at the operating point (i.e. slope of tangent AB).

$$h_{oe} = \left.\frac{\partial i_C}{\partial V_C}\right|_{i_B} \qquad \text{... (7.82)}$$

The parameters h_{ie} and h_{re} can be calculated with the help of CE input characteristic curve, and is shown in figure (5).

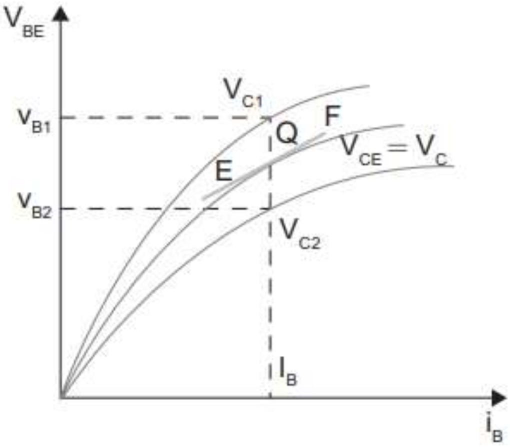


Figure (5): *CE Input Characteristics*

h_{ie} is the slope of the appropriate input on figure (7.30), at the operating point (slope of tangent EF at Q).

$$h_{ie} = \frac{\partial V_B}{\partial i_B} \approx \left.\frac{\Delta V_B}{\Delta i_B}\right|_{V_C} \qquad \text{... (7.83)}$$

A vertical line on the input characteristic represents constant base current. The parameter h_{re} can be obtained from the ratio (V_{B2}– V_{B1}) and (V_{C2}– V_{C1}) for at Q.

$$h_{re} = \frac{\partial V_B}{\partial V_C} = \left.\frac{\Delta V_B}{\Delta V_C}\right|_{I_B} = \frac{V_{B2} - V_{B1}}{V_{C2} - V_{C1}} \qquad \text{... (7.84)}$$

7.9.4 Analysis of Amplifier using h-Parameters

The transistor amplifier can be analyzed with the parameters current gain, voltage gain, input impedance and output impedance. To carry out the analysis the small signal model of the transistor is necessary or h-parameters are to be known. The two port network in the below Fig. represents a transistor in any of the three possible configuration. Transistor amplifier is constructed by connecting an external load and signal source to the two-port network. The two port network represented in figure (6) is replaced with its small signal hybrid model and is shown in figure (7). The circuit thus constructed is valid for any type of load, such as resistive, or impedance or another transistor.

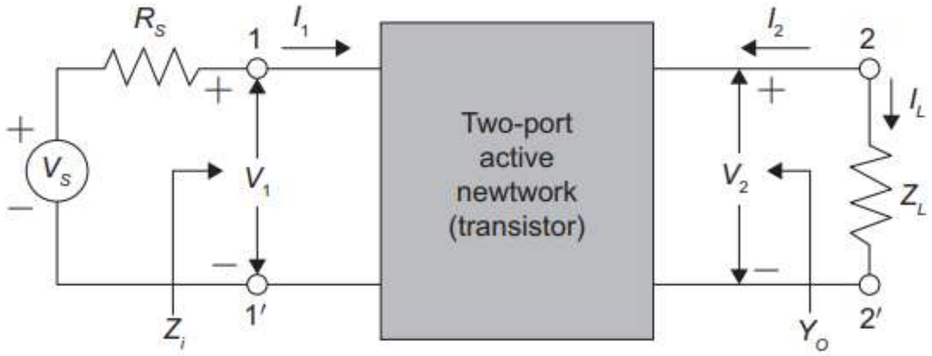


Figure (6): *Basic Amplifier Circuit as Two-Port network*

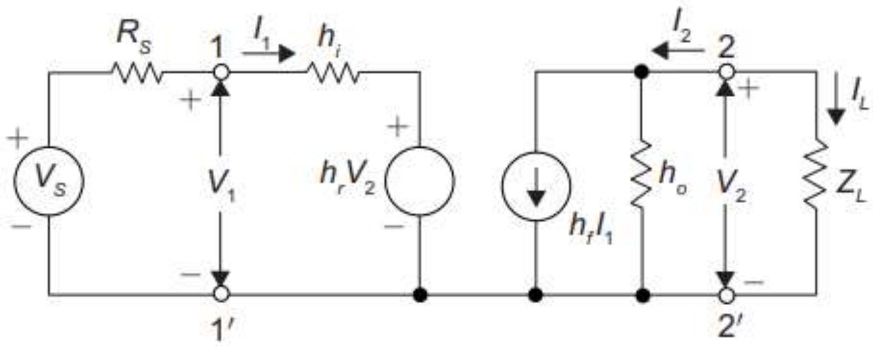


Figure (7): *h-Parameter Model of Transistor Amplifie***r**

(i) Current Gain, A_I: For a transistor amplifier, the current gain is defined as the ratio of output current to input current.

$$A_I = \frac{I_L}{I_1} = -\frac{I_2}{I_1} \qquad \text{... (7.85)}$$

From the circuit given in figure (7), we have

$$I_2 = h_f I_1 + h_o V_2 \qquad \text{... (7.86)}$$

Substituting $V_2 = -I_2 Z_L$ in the above equation, we obtain

$$I_2 = h_f I_1 - h_o I_2 Z_L \qquad \text{... (7.87)}$$

$$h_f I_1 = I_2 + h_o I_2 Z_L \qquad \text{... (7.88)}$$

$$A_I = -\frac{I_2}{I_1} = \frac{-h_f}{1 + h_o Z_L} \qquad \text{... (7.89)}$$

(ii) Input Impedance, Z_i: In the circuit shown in figure (7.32), R_s is the source internal resistance. The impedance we see looking into the amplifier input terminals is the amplifier input impedance and is represented as

$$Z_i = \frac{V_1}{I_1} \quad \text{... (7.90)}$$

From the input circuit of figure (7.32),

$$V_1 = h_i I_1 + h_r V_2 \quad \text{... (7.91)}$$

Hence, $$Z_i = \frac{h_i I_1 + h_r V_2}{I_1} \quad \text{... (7.92)}$$

Substituting $V_2 = -I_2 Z_L = A_I I_1 Z_L$ in the above equation, we obtain

$$Z_i = h_i + h_r A_I Z_L \quad \text{... (7.93)}$$

Substituting for A_I, we get

$$Z_i = h_i - \frac{h_f}{1 + h_o Z_L} h_r Z_L \quad \text{... (7.94)}$$

$$Z_i = h_i - \frac{h_f h_r}{Z_L(\frac{1}{Z_L} + h_o)} Z_L \quad \text{... (7.95)}$$

Defining the load admittance as $Y_L = 1/Z_L$

Then, $$Z_i = h_i - \frac{h_f h_r}{Y_L + h_o} \quad \text{... (7.96)}$$

It is evident that the input impedance is a function of load impedance.

(iii) Voltage Gain, A_V: The voltage gain is defined as the ratio of output voltage to input voltage and is represented mathematically as

$$A_V = \frac{V_2}{V_1} \quad \text{... (7.97)}$$

Substituting $V_2 = -I_2 Z_L = A_I I_1 Z_L$ in the above equation, we obtain

$$A_V = \frac{A_I I_1 Z_L}{V_1} = \frac{A_I Z_L}{Z_i} \quad \text{... (7.98)}$$

(iv) Output Admittance, Y_O: The output impedance $Z_O = 1/Y_O$ is obtained by setting the source voltage V_s to zero and the load impedance Z_L to infinity and by driving the output terminals from a generator V_2. If the current drawn from V_2 is I_2, then $Y_o = \frac{I_2}{V_2}$ with $V_s = 0$ and Z_L equal to infinity.

From the circuit of figure (7.32), $I_2 = h_f I_1 + h_o V_2$ ... (7.99)

Divide the above eqn. (7.99) by V_2, $\frac{I_2}{V_2} = h_f I_1 + h_o V_2$ ... (7.100)

With $V_s = 0$, applying KVL in the circuit, we get

$$R_s I_1 + h_i I_1 + h_r V_2 = 0 \quad \text{... (7.101)}$$

$$I_1(R_s + h_i) + h_r V_2 = 0 \quad \text{... (7.102)}$$

Hence, $$\frac{I_1}{V_2} = \frac{-h_r}{R_s + h_i} \quad \text{... (7.103)}$$

Substituting eqn. (7.103) in the above eqn. (7.100)

$$\frac{I_2}{V_2} = h_f(\frac{-h_r}{R_s + h_i}) + h_o \quad \text{... (7.104)}$$

$$Y_o = h_o - \frac{h_f h_r}{h_i + R_s} \quad \text{... (7.105)}$$

It is evident from the above eqn. (7.105), the output admittance is a function of source resistance.

Solved Problem 6: A CE amplifier has h-parameters as given by h_{ie}=1000 Ω, h_{re}=2x10^{-4}, h_{fe}=50 and h_{oe}=25 μ mho. If both the load and source resistances are 1 KΩ, calculate the i. Current gain and ii. Voltage gain.

Solution: Given data: R_s=1 KΩ and R_L=1 KΩ, h_{ie}=1000 Ω, h_{re}=2x10^{-4}, h_{fe}=50 and h_{oe}=25 μ mho

i. Current Gain,

$$A_I = \frac{-h_f}{1+h_o Z_L}$$

$$A_I = \frac{-50}{1+25\times10^{-6}\times1\times10^{3}} = -48.78$$

ii. Voltage gain,

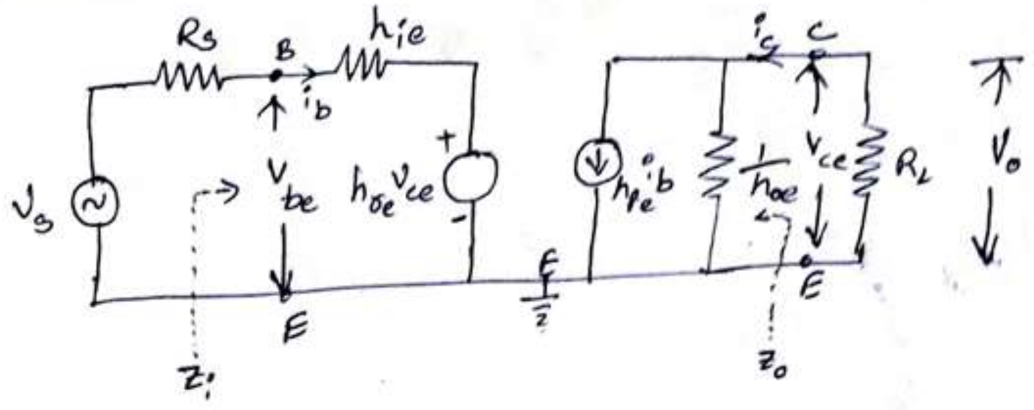
$$A_V = \frac{A_I Z_L}{Z_i} = \frac{-h_{fe}}{(h_{oe}+\frac{1}{R_L})Z_{in}}$$

$$Z_{in} = h_i - \frac{h_f h_r}{Z_L(\frac{1}{Z_L}+h_o)} Z_L$$

$$Z_{in} = 1000 - \frac{50\times2\times10^{-4}}{25\times10^{-6}+1\times10^{-3}} = 990.24\Omega$$

Hence, $A_V = \frac{-h_{fe}}{(h_{oe}+\frac{1}{R_L})Z_{in}} = \frac{-50}{(25\times10^{-6}+1\times10^{-3})\times990.24} = -49.26$

Approximate Analysis of CE amplifier using h-parameter :



Typical values of h-parameters for common Emitter

$h_{ie} = 1k\Omega$.

$h_{re} = 2.5 \times 10^{-4}$

$h_{fe} = 50$

$h_{oe} = 25 \mu S$

$\frac{1}{h_{oe}} = 40 k\Omega$

$\Rightarrow \quad h_{re} = \frac{v_1}{v_2}$

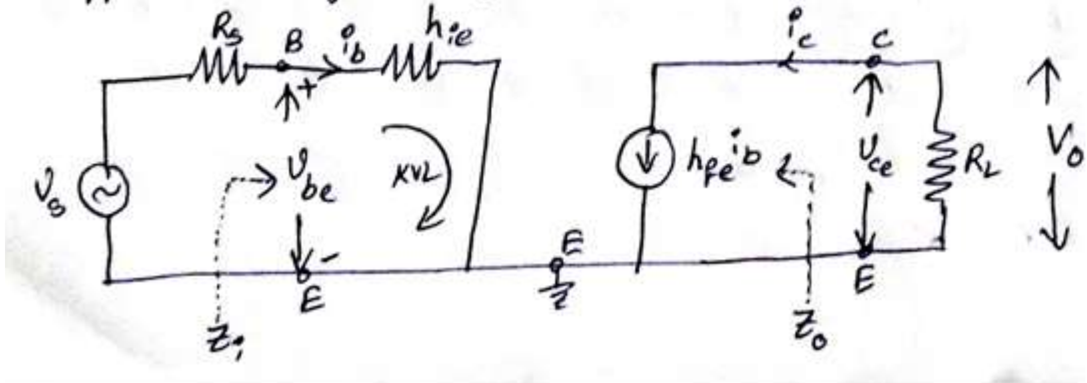
$v_2 >> v_1$ then $h_{re} \simeq 0$ then $h_{re} v_{ce} \Rightarrow$ short circuit

$\downarrow$
0.00025

$\Rightarrow \frac{1}{h_{oe}} = 40 k\Omega$ then very less current will flow through this and negligible

$\therefore \frac{1}{h_{oe}} \doteq$ open circuit

Approximate hybrid equivalent model



i) current gain

$$A_i = \frac{i_c}{i_b} = \frac{h_{fe}\, i_b}{i_b} = h_{fe}$$

$$\boxed{A_i = h_{fe} = \beta}$$

ii) Voltage gain

$$A_v = \frac{\text{o/p voltage}}{\text{i/p voltage}} = \frac{v_{ce}}{v_{be}}$$

$$\Rightarrow v_{ce} = v_o = i_L \cdot R_L = -i_c * R_L$$

$$v_{ce} = -h_{fe}\, i_b R_L$$

Apply KVL to i/p loop.

$$\Rightarrow v_{be} - i_b h_{ie} = 0$$

$$\Rightarrow v_{be} = i_b h_{ie}$$

then $$A_v = \frac{-h_{fe}\, i_b R_L}{i_b h_{ie}} = \frac{-h_{fe} R_L}{h_{ie}}$$

$$\boxed{A_v = \frac{-h_{fe} R_L}{h_{ie}} = \frac{-\beta R_L}{R_i} = \frac{\beta R_L}{R_i} \angle 180^\circ}$$

iii) Input impedance

$$Z_i = \frac{\text{I/p voltage}}{\text{I/p current}}$$

$$= \frac{v_{be}}{i_b} = \frac{\cancel{i_b} h_{ie}}{\cancel{i_b}} = h_{ie}$$

$$\boxed{Z_i = h_{ie}}$$

iv) output impedance: It is the ratio of v_c to i_c with $V_s = 0$

$\Rightarrow$ $v_s = 0$ means there is no voltage source and $i_b = 0$

then $i_c = h_{fe} i_b = 0$

$\rightarrow Z_o = \frac{v_{ce}}{0} = \alpha$

Comparison of CE, CB and CC configurations

The typical h-parameters of the transistor are listed in table (7.5). The various parameters such as Current gain, Voltage gain, Power gain, Input resistance and Output resistance of the three configurations are calculated and presented in table (7.6). The values of R_L and R_S are taken as 3 KΩ.

Table (7.5): *Typical h-parameters for different configurations*

Parameter	CE	CB	CC
h_i	1100 Ω	22 Ω	1100 Ω
h_r	2.5×10^{-4}	3×10^{-4}	1
h_f	50	-0.98	-51
h_o	24 μA/V	0.49 μA/V	25 μA/V

Table (7.6): *Performance comparison of three configurations*

Parameter	CE	CB	CC
A_I	-46.5	0.98	-47.5
A_V	-131	131	0.989
A_P	6091.5	128.38	46.98
R_i	1065 Ω	22.6 Ω	144 KΩ
R_o	45.5 KΩ	1.72 M Ω	80.5 Ω

Comparison of Amplifier Configurations

Characteristic	Common Base	Common Emitter	Common Collector
Power Gain	moderate	highest	moderate
Voltage Gain	highest	moderate	Lowest (less than 1)
Current Gain	lowest (less than1)	moderate	highest
Input Impedance	lowest	moderate	highest
Output Impedance	highest	moderate	lowest
Phase Inversion	none	180° out of phase	none
Application	RF amplifier	universal	isolation