Solutions for SM1001908

Chapter – 1
(Numbers – I)

Concept Review Questions

Solutions for questions 1 to 80:

1. $(2^1)(2^2)(2^3)(2^4)(2^5) = 2^{(1+2+3+4+5)}$
$= 2^{15} = 32768$ Choice (C)

2. **(a)** The sum of an even number of odd numbers is always even. Choice (A)

(b) The product of any number of numbers is odd only if all of them are odd. As the parities of the composite numbers are unknown, we cannot comment on the parity of the product.
Choice (C)

(c) We get different parities for the sum for different sets of composite numbers.
∴ we cannot say. Choice (C)

(d) If one or more of the numbers is/are 2, the product will be even. Otherwise the product will be odd. As the parities of the prime numbers are unknown, we cannot comment on the parity of the product. Choice (C)

(e) The first prime number is even and the remaining prime numbers are odd. The sum of N odd numbers will be even if N is even and will be odd if N is odd. As there are 9 odd numbers, their sum is odd. ∴ The sum will be odd. Choice (B)

3. 19019 = 19 (1001) = (19) (13) (11) (7)
∴ 19019 has 4 prime factors. Ans: (4)

4. **(a)** Let $x = 0.2\overline{55} = 0.2\overline{5}$

$10x = 2.\overline{5}$ -- (1),

$100x = 25.\overline{5}$ --(2)

Subtracting (1) from (2)

$x = \frac{23}{90}$ Choice (A)

(b) Let $x = 0.3\overline{21}$

$10x = 3.\overline{21}$ --(1)

$1000x = 321.\overline{21}$ --(2)

Subtracting (1) from (2), $x = \frac{318}{990} = \frac{53}{165}$
Choice (A)

(c) Let $x = 0.32\overline{1}$

$100x = 32.\overline{1}$ --(1)

$1000x = 321.\overline{1}$ --(2)

Subtracting (1) from (2),

$x = \frac{289}{900}$ Choice (A)

(d) Let $x = 1.1\overline{16}$

$10x = 11.\overline{16}$

$1000x = 1116.\overline{16}$

1000x – 10x = 1105

⇒ x = 1105/990 = 221/198

Choice (B)

5. Choice (A)
$851 = 30^2 - 7^2 = (23)(37)$
∴ Choice (A) is not prime
Choice (B)
$589 = 25^2 - 6^2 = (19)(31)$
∴ Choice (B) is not prime.
Choice (C) is divisible by 3.
Choice (D) is prime. Choice (D)

6. Twin primes are prime numbers, which differ by 2.
In Choice (A), 133 is divisible by 7 and hence it is not a prime
In Choice (B), the numbers are twin primes.
In Choice (C), 159 is divisible by 3 and hence it is not prime.
Choice (D)

7. Choice (A)
Sum of the digits in the odd places= 32
Sum of the digits in the even places = 21
(Sum of the digits in the odd places)
– (Sum of the digits in the even places) is divisible by 11.
∴ Choice (A) is divisible by 11.
Choices (B) and (C) are not divisible by 11. Choice (A)

8. Sum of the digits of the given number i.e. 28 + X must be divisible by 9.
∴ X must be 8. Ans: (8)

9. The given number must have the number formed by its last 5 digits divisible by 32. The number formed by the last 5 digits of the number = 10000 U + 8672.
8672 is divisible by 32. If U is odd, 10000 U is not divisible by 32. If U is even, 10000 U is divisible by 32. ∴ We cannot say.
Choice (C)

10. The number formed by the last k digits of a number must be divisible by 5^k for the number to be divisible by 5^k.
∴ The number formed by the last 4 digits of the number i.e. 9025 must be divisible by 625 for the number to be divisible by 625. As 9025 is not divisible by 625, the number is not divisible by 625. Choice (B)

11. The difference between any number and the sum of its digits is always divisible by 9. Choice (B)

12. As $a^n - a$ is divisible by 10, i.e. the last digit of a^n is a.
∴ a is 0, 1, 5 or 6 Choice (A)

13. The only composite number n, for which (n – 1)! is not divisible by n, is 4. Ans: (4)

14. One more than the product of any 4 consecutive natural must be a perfect square. Choice (A)

15. If the odd natural number is more than or equal to 3 its factorial's parity would be even
1! = 1. ∴ 1! is the only odd number satisfying the given condition. Ans: (1)

16. The product of any N consecutive natural numbers is divisible by N!, any for all values of N.
∴ When N = 7, any such product is divisible by 7! = 5040.
Choice (A)

17. $2^{10} \times (10^2) = (2^{10})(2)^2(5)^2 = 2^{12}(5^2)$
Number of factors of $2^{10} \times (10^2) = (12 + 1)(2 + 1) = 39$
Ans: (39)

18. A number, which has an even number of factors, is not a perfect square. It would be a perfect cube only if the index of each of the prime factors is divisible by 3.
Choice (D)

19. A number of the form $(a^x)(b^y)(c^z)$ ….where a, b, c, ….. are all prime numbers can be written as a product of 2 coprimes in 2^{n-1} ways, where n is the number of distinct prime factors.
As $5^4\,7^6$ has 2 distinct prime factors, it can be written as a product of 2 co primes in 2^{2-1} i.e., in 2 ways. Choice (B)

20. A number of the form (a^m) (b^n) $(c)^p$ ….where a, b, c, … are all prime numbers can be written as a product of 2 distinct numbers in $\frac{(m+1)(n+1)(p+1).....}{2}$ ways if it is not a perfect square and in $\frac{(m+1)(n+1)(p+1)-1}{2}$ ways if it is a perfect square.
The given number is a perfect square.
$\therefore$ It can be written as a product of 2 distinct numbers in $\frac{(4+1)(6+1)-1}{2}$ i.e. 17 ways. Choice (D)

21. The sum of the factors of a number of the form $a^m\ b^n\ c^p$…. where a, b, c, …. are all primes is given by

$$\frac{a^{m+1}-1}{a-1}\cdot\frac{b^{n+1}-1}{b-1}\cdot\frac{c^{p+1}-1}{c-1}$$

The sum of the factors of $2^4 \times (3^3)$= $\frac{2^5-1}{2-1}\cdot\frac{3^4-1}{3-1}$

= $31 \times (40) = 1240$ Ans: (1240)

22. For any perfect number, the sum of its factors is twice the number. Choice (B)

23. There are $\frac{N}{2}$ odd natural numbers less than any even number N.
$\therefore$ $\frac{N}{2}$ numbers are coprime to N.
$\therefore$ $\frac{2^{24}}{2} = 2^{23}$ numbers are less than 2^{24} and coprime to it.
Choice (B)

24. The number of numbers less than N and coprime to it $= N\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = \frac{4}{15}N$ Choice (B)

25. The sum of all the coprimes of N less than N
$= \frac{N}{2}$ (Number of co primes to N less than N)
$72 = 2^3 \times (3^2)$
Sum of all co primes of 72 less than 72
$= \frac{72}{2}\left(72\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\right) = 864$ Ans: (864)

26. **(a)** $b + \frac{1}{b} = 4$

Squaring both sides, $2 + b^2 + \left(\frac{1}{b}\right)^2 = 16$

$b^2 + \frac{1}{b^2} = 14$ Choice (D)

(b) $b - \frac{1}{b} = 4$

Squaring both sides, $-2 + b^2 + \left(\frac{1}{b}\right)^2 = 16$

$b^2 + \frac{1}{b^2} = 18$ Choice (B)

27. $a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$
$= 27000 - 3(176)\,(30) = 11160$ Choice (C)

28. If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$ Choice (C)

29. $a^3 + b^3 + c^3 = 3abc$
$a^3 + b^3 + c^3 - 3abc = 0$
$(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$(a+b+c)\ \frac{\left((a-b)^2+(b-c)^2+(a-c)^2\right)}{2} = 0$

$a + b + c = 0$ or $a - b = b - c = c - a = 0$ i.e. $a = b = c$ or both.
Choice (D)

30. $\frac{10.23^3 - 4.77^3}{10.23^2 + 4.77^2 + (10.23)(4.77)} = 10.23 - 4.77 = 5.46$

$\left(\because \frac{a^3-b^3}{a^2+b^2+ab} = a - b\right)$ Ans: (5.46)

31. Product of the numbers = (LCM) (HCF)
(30) (Other number) = (120) (6)
Other number = 24 Choice (C)

32. If the LCM of two or more numbers equals their product, they must be coprime $\therefore$ HCF (P, R) = 1 Choice (A)

33. L.C.M of any number of mutual co-primes is equal to their product. Choice (A)

34. LCM (150, 180, 270) = LCM (30×5, 30×6, 30×9)
= 30 LCM (5, 6, 9) = 30 (90) = 2700 Ans: (2700)

35. HCF (63, 42, 105) = HCF (7×9, 7×6, 7×21)
= 7 HCF (9, 6, 21) = 7 (3) = 21 Ans: (21)

36. **(a)** LCM of 42,72,90
$2 \times 3 \times 7$, $2^3 \times 3^2$, $2 \times 3^2 \times 5$
LCM = 2520 ($2^3 \times 3^2 \times 7 \times 5$)
HCF = 2×3 = 6. Choice (A)

(b) $810 = 3^4 \times 2 \times 5$
$720 = 2^4 \times 3^2 \times 5$
LCM = $2^4 \times 3^4 \times 5$ = 6480
HCF = $3^2 \times 2 \times 5$ = 90. Choice (D)

(c) $1830 = 61 \times 3 \times 2 \times 5$
$1098 = 2 \times 549 = 2 \times 3^2 \times 61$
LCM = $61 \times 3^2 \times 2 \times 5$ = 5490
HCF = $61 \times 2 \times 3$ = 366. Choice (C)

(d) LCM of numerators = 10
HCF of numerators = 1
LCM of denominators = 24
HCF of denominators = 1
LCM of fractions = LCM of num/HCF of den = 10/1 = 10
HCF of fractions = HCF of num/LCM of den =1/24
Choice (B)

(e) LCM of numerators = 176
HCF of numerators = 1
LCM of denominators = 50
HCF of denominators = 5
LCM of fractions = LCM of num/HCF of den = 176/5
HCF of fractions = HCF of num/LCM of den = 1/50
Choice (A)

37. LCM $\left(\frac{15}{4}, \frac{25}{6}, \frac{45}{8}\right)$

$= \frac{LCM\ (15,\ 25,\ 45)}{HCF\ (4,\ 6,\ 8)}$

$= \frac{LCM\ (5\times 3,\ 5\ \times 5,\ 5\times 9)}{HCF\ (2\times 2,\ 2\times 3,\ 2\times 4)}$

$= \frac{5\ LCM\ (3,\ 5,\ 9)}{2\ HCF\ (2,\ 3,\ 4)} = \frac{5\ (9)\ (5)}{2} = \frac{225}{2}$ Choice (B)

38. Time to toll together again = LCM of 5, 6, 10, 12, 15
= 60 seconds. Ans: (60)

39. Let the side of the smallest square be S cm.
S = 8a = 6b where a and b are natural numbers.
S = LCM (8a, 6b) = 24 LCM (a, b)
Which is minimum when LCM (a, b) = 1
$\therefore$ Required area = 24^2 = 576. Ans: (576)

40. Two numbers whose HCF equals their LCM must be equal. Choice (C)

41. The number formed by the last three digits of a number must be divisible by 8 for the number to be divisible by 8. The least natural number which should be added to the number formed by the last 3 digits of the given number to make it divisible by 8 is 3. Ans: (3)

42. Any prime number greater than 3 must be in the form 6 (A natural number) $\pm$ 1.
$\therefore$ mk must be divisible by 6.
Choice (D)

43. The greatest number which divides the product of any 10 even numbers is 2^{10}. Choice (B)

44. **(a)** To obtain a perfect square, the index of each of the prime factor must be even.
$\therefore$ Least natural number = (3) (5) = 15
Choice (B)

(b) Least natural number = 3. Choice (C)

45. Least natural number = (Least perfect cube greater than 599) – 599 = 729 – 599 = 130. Choice (C)

46. Given 1764 is a perfect cube
i.e. (49) (36) k is a perfect cube $\Rightarrow 7^2\,(6^2)$ k is a perfect cube
$\therefore$ Least value of k is (6) (7) = 42 Ans: (42)

47. N = $(2^4)\,(3^3)\,(7^3)$ K is a perfect square and a perfect cube. Hence the index of each of its prime factors must be divisible by 6
$\therefore$ Least value of k is $2^2\,(3^4)\,(7^3)$
$\therefore$ Total number of factors of k is (3) (5) (4) = 60
Choice (D)

48. Any perfect square having its last 2 digits equal must end with a 4. 144 is an example of such a number.
Choice (B)

49. The least natural number = LCM (7, 8) + 2 = 58
Choice (C)

50. The least natural number = LCM (18, 24) – 7 = 65
Ans: (65)

51. The general form of the numbers leaving remainders of 6 and 8 when divided by 7 and 11 respectively are $7K_1 + 6$ and $11K_2 + 8$ where K_1 and K_2 are natural numbers.
$7K_1 + 6 = 11K_2 + 8$ $K_1 = K_2 + \dfrac{2(2K_2+1)}{7}$
The least value of K_2 satisfying the condition that K_1 is a natural number is 3.
$\therefore$ The least natural number = 11 (3) + 8 = 41
Choice (C)

52. The largest number = HCF (127 – 7, 156 – 6) = 30
Ans: (30)

53. 349247 is odd,
$\therefore$ All the powers of 2 are co-prime to it. There are an infinite number of powers of 2.
An infinite number of positive integers are co-prime to it.
Choice (D)

54. Four-digit numbers divisible by 5, 12 and 18 are divisible by LCM (5, 12, 18) i.e. 180. They are of the form 180 K where K is a natural number.
1000 < 180 K < 10000
$5\frac{5}{9} < K < 55\frac{5}{9}$ i.e., K = 6, 7,………….55
$\therefore$ K has 50 possible values. Ans: (50)

55. **(a)** Least number = [3(6) +2] (8) + 5 =165
Choice (D)

(b) General value of N is (8) (6) (4) k + 165
i.e. 192 k + 165, where k = 0,1,2 -----
Hence the tenth number in the sequence is 192(9) + 165 = 1893 Choice (A)

56. Number of zeros at the end of 150! = Index of the highest power of 5 in 150!
30 numbers have 5 in them, 6 have 5^2 in them and one has 5^3 in it
$\therefore$ Index of the greatest power of 5 in 150! = 30 + 6 + 1 = 37
Ans: (37)

57. 256! has $128(2^1s) + 64(2^2s) + 32(2^3s) + (16\ 2^4s), 8(2^5s) + 4(2^6s) + 2(2^7s) + 1(2^8s)$.
$\therefore$ The greatest power of 2 that divides 256! is 2^{255}.
Note: The index of the greatest power of 2 that divides 2^n! is $2^n – 1$. Choice (B)

58. **(a)** For a number to be divisible by 2, the number must be even. Also the divisibility rule of 9 is sum of the digits of the number should be a multiple of 9. Satisfying both the conditions, the number is 4032
Choice (C)

(b) For a number to be divisible by the numbers 2, 3, 4, 6, 8, 9 it is enough that the number is divisible by 8 and 9. Divisibility rule of 8: last three digits of the given number is a multiple of 8
Divisibility rule of 9: Sum of the digits of the number is a multiple of 9.
Satisfying both the conditions the number is 4608
Choice (B)

(c) For a number to be divisible by 3, 8 and 12, it is enough to check the divisibility of 3 and 8.
Divisibility rule of 3: sum of the digits of the number is a multiple of 3.
Divisibility rule of 8 is known in the earlier problem.
Satisfying the conditions, the number is 4248
Choice (A)

(d) For a number to be divisible by 2, 4, 8 and 11 it is enough to check the divisibility of the number by 8 and 11. Satisfying the conditions, the number is 4752
Choice (C)

(e) For a number to be divisible by 2, 3, 9, 5 and 10 check the divisibility of the number by 9 and 10. Satisfying the conditions, the number is 3780. Choice (B)

(f) $24 = 8 \times 3$
For a number to be divisible by 24, check the divisibility of the numbers by 8 and 3.
The number satisfying the conditions is 3384.
Choice (B)

(g) $22 = 11 \times 2$
$33 = 11 \times 3$
For a number to be divisible by both 22 and 33 check the divisibility of the number by 2, 3 and 11.

Divisibility rule of 11:
Take the sum of the alternate digits starting from units digit i.e., units sum also the sum and of the alternate digits starting from ten's digit i.e. ten's sum. The difference of units sum and tens sum must be 0 or a multiple of 11.
The number divisible by 2, 3 and 11 is 4356.
Choice (B)

(h) $36 = 9 \times 4$; $24 = 8 \times 3$ Check out the divisibility of the number of 8 and 9. The number satisfying the condition is 6336.
Choice (D)

(i) $40 = 5 \times 8$; $72 = 8 \times 9$ its enough to check out the divisibility of the number by 5, 8 and 9
$\therefore$ the number divisible is 7560.
Choice (A)

59. For a number to be a multiple of 9, the sum of the digits must be a multiple of 9. If the sum of the digits is not a multiple of 9, add the required number to make it a multiple of 9

(a) 3 + 2 + 0 + 5 + 2 + 6 = 18
As the number is already a multiple of 9, answer is 0
Choice (C)

(b) 2 + 4 + 3 + 1 + 2 = 12
6 is to be added to make it a multiple of 9
Choice (A)

(c) 2 + 3 + 4 + 7 + 9 + 0 + 4 = 29
7 is to be added to make it a multiple of 9
Choice (B)

(d) 7 + 8 + 9 + 4 + 5 + 7 = 40
5 is to be added Choice (D)

(e) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45
0 is to be added Choice (D)

60. **(a)** 243741
units sum = 12
tens sum = 9
difference = 3
as units sum > tens sum, add (11 –difference) to the number to make it a multiple of 11
i.e., 11 – 3 = 8 Choice (B)

(b) 321423
units sum = 9
tens sum = 6
units sum > tens sum, add(11 –difference)
i.e., 11 – 3 = 8 Choice (D)

(c) 243081
units sum = 5
tens sum = 13
difference = 8
units sum < tens sum, add difference i.e., 8
Choice (A)

(d) 723111
units sum = 4
tens sum = 11
difference = 7
units sum < tens sum, add difference i.e., 7
Choice (C)

(e) 123456789
units sum = 25
tens sum = 20
difference = 5
units sum > tens sum, add (11 – difference)
i.e., 11 – 5 = 6. Choice (B)

61. **(a)** $9000 = 9 \times 1000$
$= 3^2 \times 10^3 = 2^3 \times 3^2 \times 5^3$ Choice (C)

(b) $1936 = 11 \times 176 = 11 \times 11 \times 16$
$= 11^2 \times 2^4$ Choice (C)

(c) $3969 = 9 \times 441$
$= 3^2 \times 3^2 \times 7^2 = 3^4 \times 7^2$ Choice (C)

(d) $14553 = 11 \times 1323$
$= 11 \times 3 \times 21^2 = 11 \times 3^3 \times 7^2$ Choice (C)

62. **(i)** **(a)** $248 \times 555 + 148 \times 445$
$= (100 + 148)\ 555 + 148 \times 445$
$= 100 \times 555 + 148\ (555 + 445)$
$= 203500$ Choice (A)

(b) 4 1/2 + 3 1/5 – 2 1/10 – 4 1/20 = $1\frac{11}{20}$
Choice (B)

(c) $$\frac{(3.37)^3 + 3\times3.37(6.63)^2 + 3\times6.63(3.37)^2 + (6.63)^3}{(3.37)^2 + 2\times3.37\times6.63 + (6.63)^2}$$

$$= \frac{(3.37+6.63)^3}{(3.37+6.63)^2} = 3.37 + 6.63 = 10$$

Choice (C)

(ii) **(a)** $77 \times 335 + 37 \times 665 - 40 \times 335$
$= (77 - 40)\ 335 + 37 \times 665 = 37 \times 335 + 37 \times 665$
$= 37(335 + 665) = 37000$ Choice (D)

(b) $2\frac{7}{12} + 3\frac{1}{4} - 1\frac{1}{2} + 2\frac{1}{6} - 2\frac{1}{3} = 4\frac{2}{12} = 4\frac{1}{6}$
Choice (B)

(c) $(3.13)^2 + (4.25)^2 + (2.62)^2 + 6.26 \times 4.25 + 8.5 \times 2.62 + 5.24 \times 3.13$
$= (3.13)^2 + (4.25)^2 + (2.62)^2 + 2 \times 3.13 \times 4.25 + 2 \times 4.25 \times 2.62 + 2 \times 2.62 \times 3.13 = (3.13 + 4.25 + 2.62)^2$
[i.e., $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$]
$= (10)^2 = 100$ Choice (B)

63. **(a)**

```
  1 | 1 71 61 | 131
    | 1
    | ---------
 23 |   71
    |   69
    |   ---------
261 |     261
    |     261
    |     ---------
    |       0
    |     -------
```

$\therefore \sqrt{17161} = 131$ Choice (C)

(b)

```
  7 | 5929 | 77
    | 49
    | ---------
147 | 1029
    | 1029
    | ---------
    |    0
    | ---------
```

$\sqrt{5929} = 77$ Choice (A)

(c)

```
  1   24964   158
    | 1
    | -----
 25 | 149
    | 125
    | ---------
308 | 2464
    | 2464
    | ---------
    |    0
    | ---------
```

$\sqrt{24964} = 158$ Choice (A)

(d)

```
  5  2809 | 53
          | 25
          | ---------
    103   | 309
          | 309
          | --------
          |   0
          | --------
```

$\sqrt{2809} = 53$ Choice (D)

(e)

```
   1 | 231.04 | 15.2
     | 1
     | -----
  25 | 131
     | 125
     | ---------
30.2 |   6.04
     |   6.04
     |   --------
     |     0
     |   --------
```

$\sqrt{231.04} = 15.2$ Choice (D)

(f)

```
  1   | 1 76 89 | 133
      | 1
      | ------
 23   |  76
      |  69
      | ---------
263   |   789
      |   789
      |  --------
      |     0
      |  --------
```

$\therefore \sqrt{17689} = 133.$ Choice (C)

64. Two numbers having no common factor except one are called as relative primes. Among the options the pairs which are relative primes are 57,61; 396,455; and 6561, 1024 Choice (B)

65. For any two numbers, LCM $\times$ GCD = Product of the two numbers.
$\therefore$ (432) (18) = (54) (other number)
Other number = $\frac{(432)(18)}{54} = 144$

Ans: (144)

66. Least number satisfying the given conditions is 9.
Ans: (9)

67. LCM of 22,33,55 = $2 \times 3 \times 5 \times 11 = 330$
Smallest three digit number = 330 + 5 = 335
Largest three digit number = $330 \times 3 + 5 = 995$
Choice (B)

68. As the difference between the divisors and the remainder is same in both the cases
Smallest number = LCM of divisors – (common difference of divisors and remainders)
= LCM of (8,12) – 5 = 24 – 5 =19.
Choice (D)

69. Two different dividends and two remainders,
$\therefore$ take HCF of difference of dividends and remainders
6850 – 50 = 6800
2575 – 25 = 2550
$6800 = 17 \times 2^4 \times 5^2$
$2550 = 17 \times 150 = 17 \times 2 \times 3 \times 5^2$
HCF = $17 \times 2 \times 5^2 = 850$
$\therefore$HCF = 850 is the greatest number
Choice (B)

70. Three dividends and the same remainder (not mentioned) then take difference of two pairs of dividends and find the HCF
134 – 96= 38
229 – 134 = 95
HCF of 38, 95 = 19
$\therefore$HCF = 19 Ans: (19)

71. Take difference of two pairs of dividends and find their HCF
140 – 68 = 72
248 – 140 = 108
HCF of 72, 108
$36 \times 2 = 72$
$36 \times 3 = 108$
HCF = 36. Choice (A)

72. The largest number = HCF (218 – 146, 434 – 218)
= HCF (72, 216) = 72
Choice (B)

73. Take HCF of difference of dividers and remainders.
HCF of 3300 – 23 = 3277 and 3640 – 24 = 3616
$3616 = 2^5 \times 113$
Checking divisibility of 3277 by 113
$3277 = 29 \times 113$
HCF = 113 Ans: (113)

74. LCM of 38 and 57 = 114
Remainder when 1994 is divided by 114 is 56. Number to be added to 1994 to make it a multiple of 114 is 58. in order to leave a remainder of 28, the number to be added is 58 + 28 = 86 Ans: (86)

75. $p = 2^3 . 3^2. 7^2. 11^6$
$q = 2^2. 3^1. 5^4. 11^2. 13^2$
The common prime factors of p and q are 2, 3 and 11.
$\therefore$ GCD (p, q) must have only these prime factors.
$\therefore$ GCD (p, q) = $2^{\min(3, 2)}. 3^{\min(2, 1)}. 11^{\min(6, 2)}$
$= 2^2. 3^1. 11^2 = (4) (3) (121) = 1452$
Choice (B)

76. Least positive integer divisible by $2^2.3.5, 3.5^2.7$ and $5.7.11^2$ is their L.C.M. = $2^2. 3.5^2. 7. 11^2$.
$\therefore$ Its distinct prime factors are 2, 3, 5, 7 and 11.
Number of distinct prime factors = 5. Ans: (5)

Solutions for questions 77 and 78:

When finding the LCM/HCF of 2 or more numbers, each number must be involved in the LCM/HCF functions at least once.

77. Choice (A)

78. Choice (A)

79. Sum of the first N natural numbers = $\frac{N(N+1)}{2}$
$= \frac{N(N+1)}{2} = x^2 \Rightarrow \frac{N(N+1)}{2}$ is a perfect square.
Let us go by the choices.
Choice (A): When N = 1, $\frac{N(N+1)}{2} = 1$ which is a perfect square

When N = 9, $\frac{N(N+1)}{2} = 45$ which is not a perfect square.
$\therefore$ Choice (A) is ruled out.
Choice (B): When N = 1, $\frac{N(N+1)}{2}$ is a perfect square.
When N = 7, $\frac{N(N+1)}{2} = 28$ which is not a perfect square.
$\therefore$ Choice (B) is ruled out.
Choice (C): When N = 1, $\frac{N(N+1)}{2}$ which is a perfect square.

When N = 8, $\frac{N(N+1)}{2} = \frac{8(9)}{2} = 36$ which is a perfect square
When N = 48, $\frac{N(N+1)}{2} = (24)(49)$ which is not a perfect square
$\therefore$ Choice (C) is ruled out.
Choice (D): When N = 1 or 8, $\frac{N(N+1)}{2}$ is a perfect square. (proved above)
When N = 49, $\frac{N(N+1)}{2} = (49), (25)$ which is a perfect square
Choice (D) follows. Choice (D)

80. If $x = \sqrt{2}$ and $y = \sqrt{3}$,
$x + y – xy = \sqrt{2} + \sqrt{3} – \sqrt{6}$
In this case, x + y – xy is irrational.
If $x = \sqrt{2}$ and $y = –\sqrt{2}$,
$x + y – xy = \sqrt{2} + (–\sqrt{2}) – (\sqrt{2})(–\sqrt{2}) = 2$
In this case, x + y – xy is rational.
$\therefore$ We can only conclude that x + y – xy is real
($\because$ Any real number is one which is either rational or irrational) Choice (A)

Exercise – 1(a)

Solutions for questions 1 to 40:

1. In the number 315642, the 1st, 3rd and 5th digits. (i.e., 3, 5 and 4 respectively) are termed being 3, 5 and 4 and would be termed the odd digits. The 2nd, 4th and 6th digits (i.e., 1, 6 and 2 respectively) are termed the even digits.
Sum of odd digits = 3 + 5 + 4 = 12.
Sum of even digits = 1 + 6 + 2 = 9
Hence the sum of the odd digits – the sum of the even digits = 12 – 9 = 3
Hence if 3 is added to the number, sum of the even digits = 9 + 3 = 12 = sum of the odd digits, thereby the number formed becomes divisible by 11. Ans: (3)

2. The given number is 5668x25y and this is divisible by 48.
$\Rightarrow$ The number is divisible by 8, 16 and 3.
$\Rightarrow$ 25y is divisible by 8; $\Rightarrow$ y = 6.
Because 16 is a factor, x25y is divisible by 16.
But y = 6; i.e., x256 is divisible by 16. Whenever, x has any of the even numbers 2, 4, 6 or 8 as its value, x256 is divisible by 16. $\Rightarrow$ x = 2 or 4 or 6 or 8.
Sum of the digits of the number = (38 + x). For this to be divisible by 3, x shall be 4, (from among the above 4 alternatives). x + y = 4 + 6 = 10. Choice (A)

3. Let the even natural numbers be 2k, 2k + 2, 2k + 4 and 2k + 6.
N = 16 + (2k) (2k + 2) (2k + 4) (2k + 6)
= 16(1 + k(k + 1) (k + 2) (k + 3))
= 16(1 + k(k + 3) (k + 1) (k + 2))
= $16(1 + (k^2 + 3k) (k^2 + 3k + 2)$
= $16(1 + (k^2 + 3k)^2 + 2(k^2 + 3k))$
= $16(k^2 + 3k + 1)^2$
$k^2 + 3k + 1$ is odd for any positive integral value of k.
k $\Rightarrow$ $(k^2 + 3k + 1)^2$ is also odd.
$\therefore$ $16(k^2 + 3k + 1)^2$ is a perfect square divisible by 16.
Hence only (B) and (D) are true. Choice (C)

4. Let the four prime numbers be a, b, c and d.
Given $a \times b \times c = 2431$ and $b \times c \times d = 4199$
$\therefore \frac{a\times b\times c}{b\times c\times d} = \frac{2431}{4199} \Rightarrow \frac{a}{d} = \frac{11}{19}$ $\therefore$ d = 19 Ans: (19)

5. Prime numbers less than 5 are 2 and 3.
2^3 and 3^3 leave respective remainders of 2 and 3 when divided by 6.
Prime numbers greater than or equal to 5 are of the form 6k ± 1 where k is a natural number.
$(6k + 1)^3$ leaves a remainder of 1 when divided by 6.
$(6k – 1)^3$ leaves a remainder of 5 when divided by 6.
$\therefore$ Sum of all the distinct possible remainders is 11.
Choice (B)

6. The required divisor is obtained by considering the HCF of (698 – 9, 450 – 8)
HCF of (689, 442) is 13. Choice (B)

7. Let the number be N.
68488 = N.K_1 + R and
67516 = NK_2 + R where K_1 and K_2 are natural numbers and R is the remainder 68488 – 67515 = N(K_1 – K_2).
972 = N(K_1 – K_2)
N must be a factor of 972.
972 = $1 \times 972 = 2 \times 480$
= 3×324
= $4 \times 243 = 6 \times 162 = 9 \times 108$
$\therefore$ N has 6 possibilities. Ans: (6)

8. Let the smallest three-digit number required be N. N leaves a remainder of 3 when divided by 7 and when divided by 5 leaves a remainder of 1. Hence N is of the form k[LCM (7, 5)] – 4= 35k – 4 where k is a positive integer and 4 is the difference between the divisor and the remainder. When the divisor = 7, remainder = 3 and when the divisor = 5, remainder is 1. The number N leaves a remainder of 5 when divided by 6. Hence N = 6a + 5 where a is the quotient when N is divided by 6
N = 6a + 5 = 35 k – 4 $\Rightarrow$ 6a + 5 + 4 = 35k
6a + 9 = 35k ——— (1)
When we put various integer values of a starting from 1, in (1) and see where we get k as an integer, we see that only when a is at least 16, k assumes an integer value of 3 for the first time.
Hence, N = 35 (3) – 4 = 105 – 4 = 101 Choice (A)

9. LCM of 9 and 11 is 99. When the smallest four-digit number 1000 is divided by 99, we have the remainder as 10. Hence, 1000 – 10 = 990 is divisible by 99
Thus the smallest four-digit number which is divisible by 9 and 11 is 990 + 99 = 1089
The smallest four-digit number which when divided by 9 leaves a remainder 5 and when divided by 11 leaves a remainder 7 = 1089 – C, where C is the common difference between the divisor and the remainder in both the cases.
C = 9 – 5 = 4 or C = 11 – 7 = 4
Hence the smallest four-digit number required
= 1089 – 4 = 1085 Choice (D)

10. LCM of 7, 9 and 11 = 7 x 9 x 11 = 63 x 11 = 693.
Dividing the largest 4-digit number 9999 by 693 we get a remainder of 297. Subtracting 297 form 9999, we have 9999 – 297 = 9702 which is exactly divisible by 693 and hence by 7, 9 and 11. The largest 4-digit number which when divided by 7, 9 and 11 leaves a remainder of 5 in each case = 9702 + 5 = 9707 Choice (C)

11. If the soldiers are arranged in rows of 8 or 15 or 20, one soldier is left to stand alone in the last row. Hence if the total number of soldiers is divided by 8 or 15 or 20, the remainder will be 1. Similarly, if the total number of soldiers is divided by 9 or 13, the remainder will be 4. An option satisfying both these conditions is only.
Alternate method:
Number of soldiers on the field = LCM (8, 15, 20)c + 1
= (120c + 1), where c is a constant
Number of soldiers on the field = LCM (9, 13)k + 4
= 117k + 4 where k is a constant.
Hence 120c + 1 = 117k + 4.
The above equation is satisfied when k = 1 for c = 1
Thus number of soldiers in the field = 1(120) + 1 = 121
Choice (D)

12. Let the number of sweets with Rohan we N.
N = K_1 LCM(12, 16, 18) + 1 = 17 K_2, where K_1 and K_2 are natural numbers.
$144K_1 + 1 = 17K_2$
$8K_1 + \frac{8K_1+1}{17} = K_2$
The least value of K_2 (17) is realized when K_1 = 2.
$\therefore$ The least value of N = 289.
N must be of the form 289 + K LCM (144, 17) = 2448K + 289, where K is a whole number.
$\therefore$ 2448K + 289 < 10000
$\therefore$ K can be 0, 1, 2 or 3 i.e., it has 4 possible values.
Choice (C)

13. If side of each identical square tile is x, then the number of tiles required
$= \frac{\text{Area of the floor}}{\text{Area of each square tile}} = \frac{870\,\text{cm}\times 638\,\text{cm}}{x\times x}$
The number of identical square tiles will be minimum if area of each identical square tile (x^2) is maximum. Also, to completely each of the cover the floor, the side of the tile should be a factor of the dimensions of the room.
$\Rightarrow$ x must be the HCF of 870 and 638
$\therefore$x = 58
Hence minimum number of identical square tiles
$= \frac{870\,\text{cm} \times 638\,\text{cm}}{58\,\text{cm} \times 58\,\text{cm}}$ = 165 Ans: (165)

14. Weight of each piece (in kg)

$= \text{HCF}\left(6\frac{1}{8}, 10\frac{1}{2}, 8\frac{3}{4}, 3\frac{15}{16}\right)$

$= \text{HCF}\left(\frac{49}{8}, \frac{21}{2}, \frac{35}{4}, \frac{63}{16}\right) = \frac{\text{HCF}\,(44, 21, 35, 63)}{\text{LCM}\,(8, 2, 4, 16)} = \frac{7}{16}$

Number of pieces obtained $= \dfrac{\frac{49}{8}+\frac{21}{2}+\frac{35}{4}+\frac{63}{16}}{\frac{7}{16}} = 67$

Ans: (67)

15. The two numbers are of the form 6x and 6y since the HCF of the two numbers is 6, where x and y are co-primes.
(6x) (6y) = 4320 36xy = 4320
xy = 120
Now $120 = 2^3 \times 3^1 \times 5^1$
$\therefore$Number of possible pairs $= 2^{n-1}$ where n is the number of distinct prime factors of 120,
$\therefore$Number of pairs $= 2^{3-1} = 2^2 = 4$ Choice (C)

16. Since the HCF of the two numbers is 7, we have 7x and 7y as the two numbers where x and y are co-primes. 7x – 7y = 7(x – y) = 21, x – y = 21/7 = 3, x = y + 3
The LCM of the two numbers is 7xy = 196
xy = 196/7 = 28 $\Rightarrow$ (y + 3)y = 28 $\Rightarrow$ $y^2 + 3y = 28$
$y^2 + 3y - 28 = 0 \Rightarrow (y + 7)(y - 4) = 0$
Since y can't be negative, y = 4
x = 28/y = 28/4 = 7
Hence the larger of the two numbers is 7x = 7 x 7 = 49

Alternate method:

Going by the options, option (A) says the larger number is 28. The smaller number would then be 28 – 21 = 7. LCM of 28 and 7 is 28. Option (B) says the larger number is 35. Since 196 is not a multiple of 35, option (B) is ruled out. Option (C) says the larger number is 42; smaller number would then be 42 – 21 = 21
LCM of 42 and 21 is 42. Hence not possible.
Option (D) says the larger number is 49. Smaller number would then be 49 – 21 = 28
LCM of 49 and 28 is 196 Choice (D)

17. 11111111 = 11(1010101) = 11(101) (10001)
= 11(101) (11025 –1024) = 11(101) $(105^2 - 32^2)$
= 11(101) (137) (73)

Sum of all the factors $= \frac{11^2-1}{11-1} \cdot \frac{101^2-1}{101-1} \cdot \frac{137^2-1}{137-1} \cdot$

$\frac{73^2-1}{73-1}$ = 12 (102) (138) (74) = 12499488

Ans: (12499488)

18. Expressing 152100 as product of prime factors, we have $152100 = 2^2 \times 3^2 \times 5^2 \times 13^2$
Number of ways in which 152100 can be expressed as a product of two different factors
= 1/2 [(2 + 1)(2 + 1)(2 + 1)(2 + 1) – 1]
= 1/2 [81 – 1] = 1/2 [80] = 40 Choice (D)

19. Total number of factors of N = 45.
If N = a^p, b^q, c^r ….., then, (p + 1) (q + 1) (r + 1) …. = 45
45 can be written as $3 \times 3 \times 5$, if the number of factors of 45 is to be maximum.
$\Rightarrow$ (p + 1) (q + 1) (r + 1) ….. $= 3 \times 3 \times 5$;
$\Rightarrow$ N has a maximum of 3 distinct prime factors with respective powers p, q and r.
Maximum number of distinct prime factors of N = 3.

Ans: (3)

20. N = 10! = $2(3)(2^2)(5)\,2(3)\,7(2^3)\,(3^2)\,2(5) = 2^8 3^4 5^2 7^1$
$\therefore$ 10! has 9(5)(3)(2) or 270 factors
The product of all these factors is $(10!)^{135}$.

Choice (A)

21. N = $6^8\,8^6 = 2^{26}\,3^8$. $\therefore$ N has 243 factors.

The product of all these factors of N is $N^{243/2} = \left(6^8 8^6\right)^{\frac{243}{2}}$
$= (6^4\,8^3)^{243}$. Choice (C)

22. $X^2 - 8X = (Y^2 + 2Y)^2 - 8(Y^2 + 2Y) = (Y^2 + 2Y)(Y^2 + 2Y - 8)$
= Y(Y + 2) (Y + 4) (Y – 2)
Let Y = 2a, where a is a natural number.
$\therefore$ $X^2 - 8X$ = 2a(2a + 2) (2a + 4) (2a – 2)
= 16(a – 2) (a – 1) a(a + 2)
= 16 (Product of 4 consecutive natural numbers).
The product of 4 consecutive natural numbers is always divisible by 24.
$\therefore$ $X^2 - 8X$ is always divisible by 384 but not always divisible by 384(2) or 768. Eg. when a = 1, $X^2–8X$ = 384, which is not divisible by 768.

Choice (C)

23. Number of three-digit numbers = 900
Number of three-digit numbers which are divisible neither by 2 nor by 3 = 900 $\times$ (1 – 1/2) $\times$ (1 – 1/3)
= 900 $\times$ 1/2 $\times$ 2/3 = 300 Ans: (300)

24. Let sum of all co-primes to 2016, which are less than 2016 = S
2016 = 2 x 1008 = 2 x 2 x 504 = 2 x 2 x 2 x 7 x 36
$= 2^3 \text{ x } 7 \text{ x } 3^2 \text{ x } 2^2 = 2^5 \text{ x } 7 \text{ x } 3^2$

Hence S = $\frac{2016}{2}$ x 2016 x (1 – 1/2) x (1 – 1/7) (1 – 1/3)
= 2016/2 x 2016 x 1/2 x 6/7 x 2/3 = 580608 Choice (D)

25. 24 = $2^3 \times 3$. The largest power of 3 contained in 360! can be calculated by the method indicated below.

$$\begin{array}{l|l}
3 & \underline{360} = 120 \\
3 & \underline{120} = 40 \\
3 & \underline{40} = 13 \\
3 & \underline{13} = 4 \\
3 & \underline{4} = 1
\end{array} \qquad \text{Total} = 178$$

Hence the largest power of 3 in 360! is 178.
Similarly we can calculate the largest power of 2 in 360!, by the method indicated below.

$$\begin{array}{l|l}
2 & \underline{360} = 180 \\
2 & \underline{180} = 90 \\
2 & \underline{90} = 45 \\
2 & \underline{45} = 22 \\
2 & \underline{22} = 11 \\
2 & \underline{11} = 5 \\
2 & \underline{5} = 2 \\
2 & \underline{2} = 1
\end{array} \qquad \text{Total} = 356$$

Hence the largest power of 2 in 360! is 356. The largest power of 2^3 in 360! is the quotient of 356/3 = 118.
Hence the largest power of 2^3 x 3 in 360! is 118 which is the largest common power of $(2^3 \times 3)$ contained in 360!

Choice (B)

26. Suppose, a = 3, b = 4, c = 5 and d = 2
Then they satisfy $a^d + b^d = c^d$
In this case, the minimum of a, b and c is at least d.

Choice (A)

27. Given divisors are 8, 6 and 5 and their respective remainders are 1, 1 and 2.
$\therefore$The number is of the form of $(8 \times 6 \times 5)k + (8 \times (2 \times 6 + 1) + 1)$ = 240k + 105
As the HCF of 240 and 105 is 15, the number is divisible by both 3 and 5. Choice (D)

28. Let the number missed be x.
The correct sum = 800 + x.
This must be in the form = $\frac{N(N+1)}{2}$
where N is a natural number.

When N = 39, $\frac{N(N+1)}{2}=780$

When N = 40, $\frac{N(N+1)}{2}=820$

When N = 41, $\frac{N(N+1)}{2}=861$

As 800 + x > 800, N ≥ 40 If N = 40, x = 20
For every increase in N by 1. x will increase by N + 1.
∴ If N ≥ 41, x > N. This is not possible.
∴ N = 40 and x = 20 Ans: (20)

29. $N^7 - N = N(N^6 - 1) = N((N^3)^2 - 1) = N(N^3 - 1)(N^3 + 1)$
$N(N - 1)(N^2 + N + 1)(N + 1)(N^2 - N + 1)$
$= N(N - 1)(N + 1)(N^2 + N + 1)(N^2 - N + 1)$
N – 1, N and N + 1 are consecutive integers. The product of any three consecutive integers is divisible by 6.
∴ $N^7 - N$ is divisible by 6.
∴ the remainder is 0. Choice (A)

30. There are 9 single digits pages. Number of digits required to number single digit pages = 9.
There are 90 two digit pages. Number of digits required to number two digit page s= 180.
There are 900 there digit pages.
Number of digits required to number three digit pages = 2700.
Number of four digit pages = 501.
Number of digits required to number four digits pages = 2004.
∴ Total number of keys to be pressed
= 9 + 180 + 2700 + 2004 = 4893.
Ans: (4893)

31. X is a set of integers whose elements when arranged in ascending order form an arithmetic progression whose first term is 9 and common difference is 6. Let us say it has n elements.
375 = 9 + (n – 1)6 ; 62 = n
In an arithmetic progression with even number of terms (say n), the sum of the k^{th} term, from the start and k^{th} term from the end will be the same.
∴ Maximum number of elements of y will occur when y = {9, 15, 21, ... 189}
or v = {195, 201, 207, . . ., 375}.
In either case, y has 31 elements. Choice (C)

32. Given w + x + y + z = 8 m + 10.
In m = 1, x + y + z = 6m + 10 = 16
When the sum of three natural numbers is constant, the sum of their squares is minimum when the numbers are as close as possible.
So the four numbers must be 2m + 2, 2m + 2, 2m + 3 and 2m + 3.
∴ the minimum value of $w^2 + x^2 + y^2 + z^2 = (2m + 2)^2 + (2m + 2)^2 + (2m + 3)^2 + (2m + 3)^2 = 16m^2 + 40m + 26$
Choice (D)

33. If x is of the form 6k + 1 where k is a natural number, none of the elements of X_x will be divisible by 6. If x is of the form 6k or 6k + 2 or 6k + 3 or 6k + 4 or 6k + 5, one of the elements of X_x will be divisible by 6.
∴ of the sets X_1 to X_{78}, there will be 13 sets which do not contain 6, or its multiple.
∴ 65 sets will contain 6 or its multiple X_{79} will not contain 6 or its multiple. X_{80} will contain 6 or its multiple.
∴ a total of 66 sets contain 6 or its multiple.
Choice (B)

34 Given value is $0.7\overline{54} + 0.69\overline{2}$

Fractional value of $0.7\overline{54} = \frac{754-7}{990} = \frac{747}{990} = \frac{83}{110}$

Fractional value of $0.69\overline{2} = \frac{692-69}{900} = \frac{623}{900}$

∴ $0.7\overline{54} + 0.69\overline{2} = \frac{83}{110} + \frac{623}{900} = \frac{14323}{9900}$

Choice (C)

35. Let $N = a^2 - b^2 = (a - b)(a + b)$
If a – b and a + b are of opposite parity, a and b will not be natural numbers.
∴ For a and b to be natural numbers, a – b and a + b must be positive.
∴ K(N) represents the number of ways of expressing N in the form (a – b) (a + b), where a – b and a + b are positive.
The numbers in the 5 choices (N), their prime factors, and k(N) are tabulated below.

N	Prime factors	K(N)
110	2(5) (11)	0
105	(3)(5)(7)	4
216	$2^3\,3^3$	4
384	$2^7\,3$	6
450	$2^1 3^2 5^2$	0

If there is only one 2 in N, K(N) = 0
If there is no.2, K(N) = number of ways of expressing N as a product of two factors.
If there is more than one 2, (say if these are 2m or 2m+1) 2's), the 2's can be split in m ways. If there are n of some other prime factors, those factors can be split it (n+1) ways.
∴ For N = 216 = $2^3 3^3$, the 2's can be split as 2, 2^2, i.e., in 1 way. The 3 threes can be split as 0.3; 1.2; 2.1 or 3.0, i.e., in 4 ways.
∴ K(N) = 4.
For N = 384 = $2^7(3)$, the 2's can be split as 2, 2^6; 2^2, 2^5 or 2^3, 2^4 . For each of these split, the one 3 can go with either part.
∴ k(N) = 3 (N) = 6. This is the maximum. Choice (D)

36. Let S be the sum of the 62 page numbers. The sum of the first 62 numbers is 31(63) = 1953.(i.e., in this case S = 1953). If first leaf is left intact and instead leaf 32 (comprising pages 63, 64) is torn off, S would be 1953 – (1 + 2) + (63 + 64) or 1953 + 124. If the leaves torn off are 3 to 33, S would be 1953 + 2(124). In general, S would be of the form 1953 + 124n, i.e, 1953, 2077, 2201, ………..Among the choices, only 2201 is of this form.
Choice (B)

37. Consider an n-digit number.
Consider n = 3 and 4
Let P = abc = 100a + 10b + c
The reverse Q = cba = 100c + 10b + a
∴ P – Q = 99(a – c)
Let P = abcd = 1000a + 100b + 10c + d
The reverse Q = dcba = 1000d + 100c + 10b + a
∴ P – Q = 999(a – d) + 90(b – c)
For n = 3, P – Q is divisible by both 9 and 11
For n = 4, P – Q is divisible by 9 but not necessarily by 11.
In general when n is odd, the difference is divisible by 9 and 11. When n is even, the difference is divisible by 9 but need not be divisible by 11. In the given problem, N is 40, an even number. ∴ The difference must be divisible by 9 but not necessarily by 11. Choice (A)

38. The largest power of 2 in 32! is 31
32! + 33! + 34! + ……90!
$= 2^{31}$ (k + 33k + 34.33k + 35.34.33k + 36.35.34.33.k+ …..)
where k is odd
$= 2^{31}(34.k (1 + 33 + 35.33)$ + a multiple of 36)
$= 2^{31}(34k_1$ + a multiple of 36) where k_1 is odd
$= 2^{31}(34k_1 + 36k_2)$ (say)
$= 2^{31}(2(17k_1 + 18k_2)) = 2^{32}$(an odd number)
Largest power of 2 in the sum is 32. Choice (A)

39. $5x + 4y = 9k_1 + 4$ and $4x + 5y = 9k_2 + 5$
∴ $x - y = (5x + 4y) - (4x + 5y) = 9(k_1 - k_2) - 1$
Rem $\frac{x-y}{9}$ = –1. The corresponding positive remainder is –1 + 9 i.e. 8. Ans: (8)

40. N = 21600 = $6^3\ 10^2 = 2^5 3^3 5^2$
N has 6 (4) (3), i.e. 72 factors. Any factor in of the form $2^a 3^b 5^c$
$24 = 2^3\ 3^1$
$72 = 2^3\ 3^2$
We want all the factors of N, which are multiples of 24 but not of 72. Therefore, a can be 3,4 or 5, b has to be 1, and c can be 0, 1 or 2. The number of such factors = 3 (1) (3) = 9.
Ans: (9)

Exercise – 1(b)

Solutions for questions 1 to 60:

1. (a) $\frac{2}{3}$ of $45 \div 5 \times \left(\overline{2^4 - 1} \div 90\right)$
Applying BODMAS Rule, we have
$\frac{90}{3} \div 5 \times (15 \div 90) = 30 \div 5 \times \frac{1}{6} = 1$
Choice (B)

(b) $5 + 6 \times \frac{1}{3}$ of $9 - \left\{4 - \frac{5}{8} + 2\frac{7}{8} + \frac{3}{4}\right\}$
$= 5 + 6 \times \frac{1}{3} \times 9 - \{4 - \frac{5}{8} + 2 + \frac{7}{8} + \frac{6}{8}\}$
$= 5 + 18 - \{4 + 2 + \frac{6+7-5}{5}\}$
$= 5 + 18 - \{6 + 1\} \quad = 5 + 18 - 7 = 16$
Choice (C)

2. $75^3 - 50^3 - 25^3 = 75^3 + (-50)^3 + (-25)^3$
Now, 75 + (–50) + (–25) = 0,
When a + b + c = 0, we have $a^3 + b^3 + c^3 = 3abc$
$\therefore 75^3 - 50^3 - 25^3$
= 3 x 75 x (–50) x (–25) = 281250 Ans: (281250)

3. For number of this type (i.e., 1234 . . . n n – 1 . . . 1), the number of digits in the square root of the number will be equal to the middle digit of the number.
Choice (C)

4. The given expression is of the form
$= \frac{a^3 + b^3 - c^3 + 3abc}{a^2 + b^2 + c^2 - ab + bc + ca}$
where a = 0.68, b = 0.67 and c = 0.5
$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = a + b - c$
$= 0.68 + 0.67 - 0.5 = 0.85$
Choice (D)

5. When 546789 is divided by 7, the remainder is 5. Hence 5 should be subtracted from 546789 so that it becomes a multiple of 7. Choice (A)

6. The number is divisible by 10. ∴ c = 0
The number is divisible by 8 ∴ B = 1, 3, 5, 7 or 9.
The number is divisible by 9. The sum of all the known digits of the number leaves a remainder of 1.
∴ A + B = 8 or 17.
∴ (A, B) can be (7, 1) (5, 3) (3, 5) (7, 1) or (8, 9).
It can take 5 values. Choice (D)

7. Given the expression $X(X^3 + 2X^2 + 3X + 4) + 36$.
In order for the expression to be divisible by X, 36 must be divisible by X.
∴ Number of values that x can assume = Number of factors of 36 = 9. Ans: (9)

8. $N^3 + 6N^2 + 8N = N(N^2 + 6N + 8) = N(N + 2)(N + 4)]$
= 8M (M+1) (M+2), where N = 2M. The product of 3 consecutive numbers is always divisible by 6.
∴ The given expression is always divisible by 48.
Choice (D)

9. The number n, the number of times it occur in N and the number of digits it contributes and the total number of digits are tabulated below.

n	No. of occurrences	No. of digits	Total number of digits
1	1	1	1
2	2	2	3
3	3	3	6
--	--	--	--
9	9	9	45
10	10	20	65
11	11	22	87
12	6	12	99
1 (of 12)	Half	1	100

∴ The last 4 digits of N are 2121
The first 2 comes from the 5th 12
The 12 comes from the 6th 12
The 1 is part of the 7th 12.
∴ Rem $\frac{N}{16}$ = Rem $\frac{2121}{16} = 9$ Ans: (9)

10. If n = 2, $2^5 - 2 = 32 - 2 = 30$
If n = 3, $3^5 - 3 = 243 - 3 = 240$
If n = 4, $4^5 - 4 = 1024 - 4 = 1020$
In all the cases $n^5 - n$ is divisible by both 3 and 5.
Only 5 is there in the options. Choice (A)
General proof
Any value of n where n is a positive integer can be expressed in the form 5k or 5k – 1 or 5k – 2 or 5k – 3 or 5k – 4 where k is an integer.
If n = 5k, n is always divisible by 5
If n = 5k – 1, n + 1 = 5k is always divisible by 5
If n = 5k – 2, $n^2 + 1$ = 5k' is always divisible by 5
If n = 5k – 3, $n^2 + 1$ = 5k' is always divisible by 5
If n = 5k – 4, n – 1 = 5k – 4 – 1 = 5k – 5 = 5(k – 1) and is always divisible by 5.
Hence in general, for any n,
$n(n - 1)(n + 1)(n^2 + 1) = n^5 - n$ is always divisible by 5.

11. $n(2n + 1)(n^2 - 1)(4n^2 + 4n) = n(n^2 - 1)\ 4(n)(n + 1)(2n + 1)$
= n(n – 1) (n + 1) 4 n(n + 1) (2n + 1)
We know that n(n – 1) (n + 1) is a product of three consecutive numbers and hence is divisible by 3! = 6.
n(n + 1)(2n + 1) can be written as n(n + 1)[(n + 2) + (n – 1)]
⇒ n(n + 1)(n + 2) + n(n + 1) (n – 1)
∴n(n + 1)(2n + 1) is divisible by 6.
∴$n(2n + 1)(n^2 - 1)\ 4(n^2 + n)$ is divisible by 6 × 6 × 4 = 144 Choice (D)

12. abcde = 10000a + 1000b + 100c + 10d + e
acdbe = 10000a + 1000c + 100d + 10b + e
The difference of abcde and acdbe
= (10000a + 1000b + 100c + 10d + e) – (10000a + 1000c + 100d + 10b + e) = (990b – 900c – 90d)
18 (55b – 50c – 5d) which is always divisible by 9 and 18
Choice (D)

13. 2^2 and 3^2 when divided by 6 leave remainders of 4 and 3 respectively. The square of all other primes are of the form $(6k \pm 1)^2$ which when divided by 6 leave a remainder of 1. Hence the sum of the distinct possible remainders is 8. Ans: (8)

14. Given, 35 + Dq = N; 1750 + D(50q) = 50N
As 50N leaves a remainder of 11, 1750-11 or 1739 is a multiple of D or D is a factor of 1739 = 1764 – 25 = (42 – 5) (42 + 5). We have to consider those factors of 1739, which are greater than 35 and 11 (∴ 35), because the divisor has to be greater than the remainders. We see that D can be 37, 47 or 1739. Choice (D)

15. HCF of 1/5, 4/15 and 8/25 = 1/75
LCM of 1/5, 4/15 and 8/25 = 8/5
Hence $\frac{\text{(LCM of 1/5, 4/15 and 8/25)}}{\text{HCF of (1/5, 4/15 and 8/25)}}\ \frac{8/5}{1/75} = \frac{8}{5} \times 75 = 120$
Ans: (120)

16. Let the number be x.
$\Rightarrow$ x is the HCF of (971 – 3), (852 – 5)
= H.C.F of (968, 847) is 121
$\therefore$ x = 121 Choice (C)

17. General form of number = K [LCM (5, 6, 7)] – 3
(L.C.M. Model 2)
where K is a natural number.
$\therefore$ $210K_1 - 3 = 47K_2 - 6$.
$210K_1 + 3 = 47K_2$
$$4K_1 + \frac{22K_1 + 3}{47} = K_2$$
Least value of K_2 is realized when $K_1 = 2$.
$\therefore$ Least number = 417 Choice (B)

18. The number is of the form 9k + 6
This number when divided by 7 leaves a remainder 5.
So, 9k + 6 – 5 is divisible by 7
Substituting the smallest value 3 for k the multiple of 7 is obtained.
$\therefore$The smallest number = 9k + 6 = 9 x 3 + 6 = 33
So, the general form of the number satisfying the condition is 33 + 63k
Dividing 999, the largest 3-digit number that is possible, by 63, the quotient is 15. Hence, k = 15 and 63k +33 = 978; which is the largest 3-digit number Choice (D)

19. Each runner will be at the starting point at time intervals which are multiples of 200S, 300S, 360S and 450S. All the four will be together at the starting point at time intervals which are multiples of all the four given time intervals.
$\therefore$ Required time interval in seconds
= LCM (200, 300, 360, 450) = 1800 Ans: (1800)

20. Divisors $\rightarrow$ 7 5
remainders $\rightarrow$ 4 2

$\therefore$Required number = $7 \times 2 + 4 = 18$
Remainder when 18 is divided by 17 is 1 Choice (A)

21. Number of chocolates received by Rajesh must be in the force 3k + 1 where k is the number of chocolates distributed to each of his friends. One of his friends distributed k chocolates equally among four friends.
$\therefore$ k must be divisible by 4.
$\therefore$ k = 4 (a) where a is a natural number.
$\therefore$ k = 4a where a is a natural number.
$\therefore$ 3k + 1 = 12a + 1 Only Choice (C) is of this form.
Choice (C)

22. Divisors : 6 + $\times$ 5 + $\times$ 4

Remainders: 4 3 2
Smallest value of the number [2(5) + 3]6 + 4 = 82
General form of the number = (6) (5) (4) k + 82, where k is a whole number.
When the number has the largest four-digit value
(6) (5) (4)k + 82 < 10000 and k is maximum.
$$\therefore\ k = \left[\frac{9918}{120}\right] = 82$$
$\therefore$ Largest value = 120 (82) + 82 = 9922 Ans: (9922)

23. 88400 – 4(22100) = 4(221) (100) = (2^4) (5^2) (13) (17)
Number of factors of 88400 = (4 + 1) (2 + 1) (1 + 1) (1 + 1) = 60.
Choice (C)

24. 24700 = (247) (100) = (19) (13) (12^2) (5^2)
Number of ways in which a number having n prime factors can be expressed as a product a two co-prime factors = 2^{n-1}
$\therefore$ required number of ways = $2^{4-1} = 8$ Choice (C)

25. Expressing 3780 as product of prime factors we have 3780 = $2^2 \times 3^3 \times 5^1 \times 7^1$.
$\therefore$Number of ways of expressing it as product of two numbers = $\frac{1}{2}$ [(2 + 1) (3 + 1) (1 + 1) (1 + 1)] = 24
Choice (C)

26. The given number is 2304 = 4(576) = $4(2^6)(3^2) = 2^8 3^2$
We need the number of numbers between 1000 and 2000, that are relatively prime 2, 3 or 6.
In any group of 6 consecutive numbers, there are $\phi(6)$ or $\phi(2)\phi(3)$ or 1(2) i.e., 2 numbers that are relatively prime to 6. In the 166 groups, each of 6 consecutive numbers, i.e., 1003-1008 to 1993-1998, there are 2(166) or 332 numbers that are coprime to 6. Besides these, 1001 and 1999 are also coprime to 6. $\therefore$ The number of numbers between 1000 and 2000 and coprime to 6 (or 2304) is 334.
Choice (B)

27. To find the highest power of 3 that will exactly divide 100!, we successively divide 100 with 3

3	100	33
3	33	11
3	11	3
3	3	1

We stop here because 1 is less than 3.
Adding all the quotients,
33 + 11 + 3 + 1 = 48 is the highest power of 3
Choice (C)

28. b = a + 2 and c = a + 4
If a is even, b and c are also even. As a, b and c are prime, this is not possible.
$\therefore$ a, b and c are odd.
Least possible value of a is 3.
If a = 3, b = 5 and c = 7
If a > 3, it must be in the form 6k $\pm$ 1 where k is a natural number. If a is of the form 6k – 1, c is not prime.
(a, b, c) = (3, 5, 7) is the only possibility. Ans: (1)

29. If (R – 1)! is not divisible by R, R is a prime number. Since 1 $\leq$ R $\leq$ 50, the objective is to find the number of prime numbers between 1 and 50. On calculating we find there are 15 primes between 1 and 50. Hence there are 15 values of R such that (R – 1)! is not divisible by R. Also for R = 4, (R – 1)! i.e., 3! is not divisible by 4. Hence, the total number of values is (15 + 1) = 16 Ans: (16)

30.
$$2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{1}{2}}}}} = 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{\frac{5}{2}}}}}$$
$$= 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{2+\frac{4}{5}}}} = 2+\cfrac{1}{2+\cfrac{2}{2+\cfrac{2}{\frac{14}{5}}}}$$
$$= 2+\cfrac{1}{2+\cfrac{2}{2+\frac{10}{14}}} = 2+\cfrac{1}{2+\cfrac{2}{\frac{28}{14}+\frac{10}{14}}} = 2+\cfrac{1}{2+\cfrac{2}{\frac{38}{14}}}$$
$$= 2+\cfrac{1}{2+\frac{28}{38}} = 2+\cfrac{1}{\frac{76}{38}+\frac{28}{38}} = 2+\cfrac{1}{\frac{104}{38}} = 2+\frac{38}{104}$$
$$= \frac{208}{104}+\frac{38}{104} = \frac{246}{104} = 123/52$$
Choice (C)

31. Choice (A)
As q and r are odd, q^2 and r^3 are odd.
$\therefore$ pq^2r^3 is odd
$\therefore$ Choice (A) is always true.
Choice (B)

$(p + q)^2$ is even.
$\therefore$ $(p + q)^2 r^3$ is even.
$\therefore$ Choice (B) is always true.

Choice (C)
q + r is even.
$\therefore$ $(p – q + r)^2 (q + r)$ is even.
$\therefore$ Choice (C) is always true.

Choice (D)
If p = 1, q = 3 and r = 5, pqr leaves a remainder of 3 when divided by 4.

If p = 3, q = 5 and r = 7, pqr leaves a remainder of 1 when divided by 4.
$\therefore$ Choice (D) is not always true. Choice (D)

32. The square of any natural number ends with the same units digit as that of the number if the number ends with 0, 1, 5 or 6.
$\therefore$ $(AB)^2$ = CCB means that B can be 0, 1, 5 or 6. As B is a natural number, B can be 1, 5 or 6. As CCB < 1000, AB $\geq$ 31. As A and B are distinct, AB can be 15, 16,. 21, 25, 26 or 31. Only 15^2 and 21^2 are in the form CCB.
$\therefore$ AB has 2 possibilities. Choice (C)

33. Let the smaller number be n and the larger number be N.
$N^2 + n^3 = 593$
$\Rightarrow$ N < 25 and N + 55 = n², $\Rightarrow$ n² < 55 + 25 $\Rightarrow$ n < 9
for n = 8, $n^3 = 512$ and $N^2 = 593 – 512 = 81 \Rightarrow N = 9$
$\therefore$ N – n = 1 Ans: (1)

34. $N = 216^2 = 6^6 = 2^6 3^6$.
The number of factors of N (say ϕ) is 49.
The product of all the factors of N is $N^{\phi/2}$
The product of all the factors of N is $(216^2)^{49/2} = 216^{49}$.
Choice (A)

35. $N = 1296000 = 6^4\ 10^3 = 2^7\ 3^4\ 5^3$. $\therefore$ N has 8(5) (4) or 160 factors. The product of all these factors is $(1296000)^{80}$.
Choice (A)

36. A minimum of 5 coins are required to pay 69 paise. (1 50 p, 1 10p, 1 5p and 2 2p).
A minimum 3 coins are required to pay ₹1.05 (2 50p and 1 5p)
A minimum of 3 coins are required to pay 85p (1 50 p, 1 25 p and 1 10 p)
$\therefore$ Minimum number of coins required in total = 11
Choice (D)

37. Required divisor = 2 $\times$ 16 – 9 = 23 Ans: (23)

38. Let the number added twice be x.
The correct sum = 860 – x.
This must be in the form = $\frac{N(N+1)}{2}$
where N is a natural number.
When N = 39, $\frac{N(N+1)}{2} = 780$
When N = 40, $\frac{N(N+1)}{2} = 820$
When N = 41, $\frac{N(N+1)}{2} = 861$
860 – x < 860
$\therefore$ N $\leq$ 40.
If N = 40, x = 40.
If N < 40, x > N. This is not possible. Choice (B)

39. Let the L.C.M and H.C.F of a and b be ℓ and h respectively.
Given ℓ – h = 57.
Let k times the h be ℓ
$\therefore$kh – h = 57 $\Rightarrow$ h(k – 1) = 57
Now 57 can be expressed as product of two numbers in the following ways.
(i) (1, 57) (ii) (57, 1) (iii) (3, 19) (iv) (19, 3)
Here, (57, 1) and (19, 3) can be eliminated because, the higher the H.C.F, the higher is the sum and minimum sum is required.
From (i), H.C.F =1 and L.C.M = 58
$\therefore$(a, b) can be (1, 58) or (2, 29)
From (iii), H.C.F = 3 and L.C.M = 60
$\therefore$(a, b) can be (12, 15) or (3, 60)
$\therefore$Minimum possible sum for a and b is 12 + 15 = 27
Choice (B)

40. Going by the options, the first option is 206.
The sum of the first 20 natural numbers is
20(21)/2 = 210
Hence 206 which is just 4 less cannot be the sum of consecutive natural numbers starting from 1.
The sum of the first 29 natural numbers is
(29) (30)/9 = 870/2 = 435
Since 439 is just 4 more than 435, it cannot be the sum of consecutive natural numbers starting from 1.
The sum of the first 40 natural numbers is
(40) (41)/2 = 850.
Since 805 is just 45 less than 850, it cannot represent the sum of consecutive natural numbers starting from 1.
The sum of the first 50 natural numbers is
(50) (51)/2 = 1275

Alternate method:

Sum of the first n natural numbers is n(n + 1)/2.
The given options refer to this value; alternatively n(n + 1) still be equal to double the values given under options.
$\Rightarrow$ we have to find which one among (2 $\times$ 206), (2 $\times$ 439), (2 $\times$ 805) and 2(1275) is equal to n(n + 1).
Among the numbers 412, 878, 1680 and 2550, the only number which can be expressed as the product of two consecutive numbers is 2550 (50 and 51). It represents the sum of first n natural numbers. Choice (B)

41. $\frac{800!}{400!} = 800\ (799)\ \ldots\ldots(401)$
The index of the greatest power (IGP) of 11 that divides $\frac{800!}{400!}$ is obtained as follows.

800	72	6
11	11	

400	36	3
11	11	

$\therefore$ IGP of 11 in 800! is 72 + 6 = 78 while IGP of 11 in 400! is 36 + 3 = 39
IGP of 11 in $\frac{800!}{400!}$ = 78 – 39 = 39 Ans: (39)

42. Let each integer satisfying the given condition be denoted by N.
N^2 exceeds a perfect square by 113.
Let us denote the perfect square by x^2
$N^2 – x^2 = 113$.
(N – x) (N + x) = 113.
113 is prime.
$\therefore$ (N – x, N + x) = (–1, –113) or (1, 113)or (113, 1) or (–113, –1).
$\therefore$ (N, x) = (–57, – 56) or (57, 56) or (57, –56) or (–57, 56)
N has two possible values, –57 or 57. Choice (D)

43. $x^2 – y^2 = 255$
(x – y) (x + y) = 255.
Both x – y and x + y must be positive.
Also x – y < x + y.
$\therefore$ (x – y, x + y) = (1, 255);(3, 85);(5, 51) or (15, 17)
Choice (A)

44. In the first round, employees 2, 4, ….180 made an exit. In the second round, 3, 9, 15 made an exit.
In the third round, 5, 25, 35, 55, 65, 85, 95 made an exit.
The number of round is equal to the number of primes less than 100, which is 25. Choice (A)

45. Let the quotients when N is divided by 7, 8, 9 be q_1, q_2, q_3 respectively.
$N = 7q_1 + 5 = 8q_2 + 6 = 9q_3 + 7$
$N + 2 = 7(q_1 + 1) = 8(q_2 + 1) = 9(q_3 + 1)$
N is the least integer satisfying the given conditions.
$\therefore$ N + 2 must also be the least.
$\therefore$ N + 2 must be the LCM(7, 8, 9) i.e., 504.
$\therefore$ N = 502
Rem $\frac{N}{17} = 9$ Choice (A)

46. Let the original four digit number be abcd
K = abcd – dcba = 1000 a + 100 b + 10 c + d
– (1000 d + 100 c + 10 b + a)
= 999 (a – b) + 90 (b – c) = (37) (27) (a – b) + 90 (b – c)
This is divisible by 74 i.e. by both 37 and 2
K is divisible by 37. $\therefore$ 90 (b – c) must be divisible by 37.
$\therefore$ c – b must be divisible by 37. $\therefore$ Only possibility is b – c is 0 i.e. b = c ------------ (1)
K is divisible by 2 i.e. K is even. $\therefore$ (37) (27) (a – d) must be even. $\therefore$ a – d must be even. The least value of a – d is 2. (If a – d = 0, then 'abcd' = abba and K = abba – abba = 0, which is not positive)
From (1) and (2), the least value of abcd is 2000. This lies between 1900 and 2200. Choice (B)

47. The numbers between 2000 and 2400 (both inclusive), which have only even digits and which are multiplies of 3 are listed below. (The sum of the digits has to be a multiple of 6)

2004	2202	2400
2022	2208	
2028	2220	
2040	2226	
2046	2244	
2064	2262	
2082	2268	
2088	2280	
	2286	

There are 18 such numbers. Ans: (18)

48. Let the numbers be a, b, c
$a^2 + b^2 = ab + c^2$, $a^2 + c^2 = ac + b^2$, $b^2 + c^2 = bc + a^2$
Adding these, we have $a^2 + b^2 + c^2 = ab + bc + ac$
$\therefore 2(a^2 + b^2 + c^2) = 2(ab + bc + ac)$
$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$
This is possible only when a – b = b – c = c – a = 0 i.e, a = b = c Choice (C)

49. Let the three-digit number be abc.
b = a + c
The greatest odd number must have the greatest possible value of a and an odd value of c. (Also b≤9)
$\therefore$a = 8, c = 1 and b = 9
or abc = 891 = $11^1(3^4)$
Number of factors of 891 is (1+1) (4+1) = 10
Note: b = a + c means the number is divisible by 11.
Ans: (10)

50. Consider the number 4620 rather than 4624.
$4620 = 11\ (420) = 11\ (7)(2^2)(3)(5)$
Number of positive integers up to 4620 which are not divisible by any of 2, 7 or 11
$= 4620\left(1-\frac{1}{2}\right)\left(1-\frac{1}{7}\right)\left(1-\frac{1}{11}\right) = 1800$
Of the other four numbers upto 4624, 4621 and 4623 are not divisible by any of 2, 7 or 11.
A total of 1802 positive integers up to 4624 are not divisible by any of 2, 7 and 11. $\therefore$ 2822 positive integers up to 4624 are divisible by at least one of 2, 7 and 11. Choice (A)

51. 16081065 = 5 (3216213) = 5(9) 357357
= 5(9)(357)(1001)
= 5(9)3(7)(17)7(11)(13)
= 7(7)(9)15(11)(13)(17)
This is the only way to express the number as the product of 7 numbers between 5 and 19. The sum of these 7 factors is 79. Ans: (79)

52. The largest power of 2 in 15! is 11
$16! = 16(15!) = 2^4(15!)$. $\therefore$ Largest power of 2 in 16! is 15.
$\therefore$ Each of the factorials 16!, 17!, …..100! have the largest power of 2 greater than or equal to 15.
15! + (16! + 17!+ …… 100!) = 2^{11} (an odd number) + 2^{15} (an integer)
= 2^{11} (an odd number + 16 (an integer))
= 2^{11}(an odd number)
The largest power of 2 in 15! + 16! + 17! + …. 100! Is 11.
Choice (A)

53. The given expression E has the form
$p^3 + q^3 + r^3 - 3pqr$ where p + q + r = 0 (p = 4a + 8b – 12c, q = 8a – 12b + 4c and r = –12a + 4b + 8c)
$E = (p + q + r)\ (p^2 + q^2 + r^2 - pq - qr - rp)$
This is 0 since p + q + r is 0. $\therefore$ E is both non-negative and non-positive. Choice (C)

54. X(p, q, r, s, t) = 32 – 16 (Σp) + 8(Σpq) – 4(Σpqr)
+ 2(Σpqrs) – pqrst = 32 – 16(p + q + r + s + t)
+ 8 (pq+ pr + ps + pt + qr + qs + qt + …+st)
–4 (pqr + pqs + ….+ rst) + 2(pqrs + pqrt + pqst + prst) –
pqrst = (2 – p) (2 – q) (2 – r) (2 – s) (2 – t)
$X\left(\frac{16}{15}, \frac{15}{14}, \frac{14}{13}, \frac{13}{12}, \frac{12}{11}\right) = \left(2-\frac{16}{15}\right)\left(2-\frac{15}{14}\right)$
$\left(2-\frac{14}{13}\right)\left(2-\frac{13}{12}\right)\left(2-\frac{12}{11}\right) = \frac{14}{15}\left(\frac{13}{14}\right)\left(\frac{12}{13}\right)\left(\frac{11}{12}\right)\left(\frac{10}{11}\right)$
$= \frac{2}{3}$ Choice (A)

55. $3^4\ 5^2 = 81\ (25) = 2025$
d_i is either in the form 4K + 1 or 4k – 1
If it is in the first form $(-1)^{\frac{d_i-1}{2}} = (-1)^{2k} = 1$
If θ_i is in the second from, $(-1)^{\frac{\theta_i-1}{2}} = (-1)^{2k-1} = -1$
All the factors of 3^4, 5^2 are odd. Its factors are 1, 3, 9, 27, 81, 25, 15, 75, 45, 225, 135, 675, 405, 2025. Of these, 6 are of the form 4K – 1 and the remaining 9 are of the form 4K + 1
$F(3^4\ 5^2) = 6(-1) + 9(1) = 3$ Choice (B)

56. By remainder theorem, $\text{Rem}\left(\frac{6^{2x}}{7}\right) = 1$ and
$\text{Rem}\left(\frac{6^{2x+1}}{7}\right) = -1$ i.e, 6
Let $6^{2x} = 7a + 1$ $\therefore$ $6^{2x+1} = 42a + 6$
$\therefore \left[\frac{6^{2x}}{7}\right] = \left[\frac{7a+1}{7}\right] = a + 1$ and
$\left[\frac{6^{2x+1}}{7}\right] = \left[\frac{42a+6}{7}\right] = 6a + 1$
Required sum = $(a + 1) + (6a + 1) = (7a + 1) + 1 = 6^{2x} + 1$
Choice (C)

57. $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)\left(1-\frac{1}{25}\right) \ldots\ldots \left(1-\frac{1}{900}\right)$
$= \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{5^2}\right) \ldots\ldots \left(1-\frac{1}{30^2}\right)$
$= \left(1-\frac{1}{2}\right)\left[\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\right]\left[\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\right]\left(1+\frac{1}{4}\right) \ldots\ldots$
$\left(1-\frac{1}{30}\right)\left(1+\frac{1}{30}\right)$
$= \frac{1}{2} \times 1 \times 1 \times \ldots\ldots 1 \times \frac{31}{30} = \frac{31}{60}$. Choice (B)

58. $5400 = 2 \times 10 \times 270 = 2 \times 10 \times 27 \times 10$
$= 2 \times 2 \times 5 \times 3^3 \times 2 \times 5 = 2^3 \times 5^2 \times 3^3$
Hence sum of all the factors of 5400

$$= \frac{2^4-1}{2-1} \times \frac{5^3-1}{5-1} \times \frac{3^4-1}{3-1} = \frac{15}{1} \times \frac{124}{4} \times \frac{80}{2} = 18600$$

Ans: (18600)

59. $(N_1 \oplus 8)\ \#\ (N_2 \oplus 7) = 21$. In other words, the product of the remainders of N_1 divided by 8 and N_2 divided by 7 is 21.
$\therefore$ The only possible remainders when N_1 and N_2 are divided by 8 and 7 are 7 and 3 respectively.
($\because$ Any remainder must be less than the divisor).
N_1 and N_2 are natural numbers not more than 100.
N_1 can be 7, 15, 23, ……. 95
N_2 can be 3, 10, 17, ……. 94
N_1 has 12 possible values and N_2 has 14 possible values.
$\therefore$ (N_1, N_2) has 168 possible values. Ans: (168)

60. Let the numbers be 3x, 4x and 5x.
LCM (3x, 4x, 5x) = x LCM (3, 4, 5) = 60x
Given 60x = 480
x = 8
sum 3x + 4x + 5x =12x = 96 Choice (A)

Solutions for questions 61 to 75:

61. From I, some of the values of x are 101, 116, ………… From II, some of the values of x are 106, 117, ………… From I and II, the difference between divisors and remainders is constant i.e., 4, hence $(15 \times 11) - 4 = 161$ is the only number in the range of (100 and 265).
Choice (C)

62. x < 0.
From statement I, at least one of y and z is < 0
If y and z are negative, then xyz < 0
If only one of y and z is negative, then xyz > 0.
So statement I alone is not sufficient
From statement II, $y + z > 0 \Rightarrow$ at least one of y and z is positive or both are positive.
If both are positive then xyz < 0.
If only one is positive then xyz > 0, so statement II alone is not sufficient.
Combining statements I and II, between y and z, one is negative and other is positive. So xyz > 0. Choice (C)

63. From I, $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow 2(a^2 + b^2 + c^2) = 2(ab + bc + ca)$
$\Rightarrow (a^2 + b^2 - 2ab) + (a^2 + c^2 - 2ca) + (b^2 + c^2 - 2bc) = 0.$
$\Rightarrow (a - b)^2 = (b - c)^2 = (c - a)^2 = 0.$
$\therefore$ a = b = c.
As $abc \neq 0$, $a^3 + b^3 + c^3 \neq 0$.
If a + b + c = 0, $a^3 + b^3 + c^3 = 3abc$. As $abc \neq 0$, $a^3 + b^3 + c^3 \neq 0$
$\therefore$ Either statement is sufficient to answer the question.
Choice (B)

64. From statement I, $x = n^2$.
From statement II, $x = k^3$.
Combining statements I and II
If x = 729 it is a perfect square and a cube.
If x = 64 it is also a perfect square and a cube
$\therefore$x can be even or odd.
Hence, both statements together are also not sufficient.
Choice (D)

65. Let x be the number of soldiers.
From statement I, x is a multiple of the LCM of 3, 5 and 7.
$\therefore x = 105k \Rightarrow$ x can be 105 or 210.
So statement I alone is not sufficient.
From statement II, x is even.
Combining both the statements, we get x = 210.
Choice (C)

66. From statement I, $x + y = dk_1$……….(1)
From statement II, $x - y = dk_2$……….(2)
Using both the statements,
Adding equation (1) and (2) we get
$2x = d(k_1 + k_2)$
Since d is odd, $k_1 + k_2$ is even.
$\frac{x}{d} = \frac{k_1 + k_2}{2}$ = integer.
$\therefore$ x is divisible by d.
Similarly, y is also divisible by d. Choice (C)

67. $\frac{a}{b-c} = 1 \Rightarrow a = b - c$ so we have to find $\frac{a}{b}$
$\therefore$ Statement I alone is sufficient.
From statement II, a and b are co-primes
So $\frac{a}{b}$ may be $\frac{3}{5}$ or $\frac{7}{9}$ or any other such value
So unique value is not possible. Choice (A)

68. $10 < 3^n < 300$ so n = 3, 4, or 5
From statement I, n is the square of an integer.
$\therefore$n = 4.
From statement II, 3^n is the square of an integer.
$\therefore 3^n = 81 \Rightarrow n = 4$.
$\therefore$Either statement alone is sufficient. Choice (B)

69. From statement I,
1 + 2 + 3 + 4 + 6 = 16 (the only possibility)
So, I alone is sufficient.
From statement II, $1 \times 2 \times 3 \times 4 \times 5 = 120$ (the only possibility)
So, II alone is also sufficient. Choice (B)

70. GCD of (2a, 2b) = 10 $\Rightarrow$ Let 2a = 10 k and 2b = 10 m, where k and m are co-primes.
$\Rightarrow$ a = 5k; b = 5m
$\therefore$ GCD of a and b is 5.
From Statement I alone, we can answer.
Statement II does not give any information to solve.
Choice (A)

71. From statement I, x = Nk and y = Nr
Only if k and r are co-primes, then N is the HCF of x and y other wise not. $\therefore$ I alone is not sufficient.
From statement II, $\frac{x}{2} = 2N\,k_1$ and $\frac{y}{4} = 2N\,r_1$
$\therefore$ 4N divides x and y.
$\therefore$ N is not HCF of x and y.
$\therefore$ Statement II alone is sufficient. Choice (A)

72. From statement I,
x = 5k + odd positive integer where k is a non-negative integer.
If k = 1 then x is even.
If k = 2 then x is odd.
Statement I alone is not sufficient
From statement II,
a = 4P + odd positive integer, where P is a non-negative integer.
If P is odd or even x is always odd
So statement II alone is sufficient. Choice (A)

73. From statement I, when x is divided by 8 the remainder is 3. So x = 8k + 3, where k is a whole number, when 8k + 3 is divided by 4 the remainder is 3. So statement I alone is sufficient.
From statement II, when x = 5 the remainder when x is divided by 4 is 1 but when x = 10 the remainder is 2.
So, the question cannot be answered by statement II alone. Choice (A)

74. 2a + 4b + a – b + c = 3 (a + b) + c.
From statement I, we don't know whether c is divisible by 3 or not, so we can't answer the question.
From statement II, c is divisible by 3.
$\therefore$ 3(a + b) + c is divisible by 3.
Statement II alone is sufficient. Choice (A)

75. $p^q = r^q$
From statement I, if q = 3, p = r. If q = 6, p = ±r. I is not sufficient.
From statement II, p = r as q is odd II is sufficient.
Choice (A)

Chapter – 2
(Numbers – II)

Concept Review Questions

Solutions for questions 1 to 25:

1. $a^n + b^n$ is a multiple of a + b if n is odd.
$\therefore 11^{103} + 14^{103}$ is a multiple of 25. Choice (B)

2. $29^{2n} – 11^{2n} = 841^n – 121^n$
$a^n – b^n$ is divisible by a – b, for all positive integral values of n.
$\therefore 841^n – 121^n$ is always divisible by 720. Ans: (720)

3. Let the number be N.
N = 48q + 31 where q is the quotient when N is divided by 48; N = 24 (2q + 1) + 7
∴The remainder when N is divided by 24 is 7.
Choice (B)

4. Let the number be N. N = 18 q + 15 where q is the quotient when N is divided by 18.
If q is of the form 4k + r, where k is a whole number and r is the remainder when q is divided by 4, N = 72k + 18r + 15
r can be 0 or 1 or 2 or 3.
∴ The remainder of N divided by 72 can be 18(0) + 15, 18(1) + 15 or 18(2) + 15 i.e. 15, 33 or 51. Choice (D)

5. $\dfrac{18^{168}}{19} = \dfrac{18^{168}}{18-(-1)}$
By remainder theorem, remainder is $(–1)^{168} = 1$ Ans: (1)

6. The 3rd odd natural number is 5.
∴ The product ends with 5. Ans: (5)

7. The 5th and 10th even natural numbers are 10 and 20 respectively.
The last two digits of the product are both 0. Choice (D)

8. The remainder of any number divided by 9 is the remainder of the sum of its digits divided by 9. The sum of the digits of the given number is 37. ∴ The required remainder is 1.
Ans: (1)

9. The remainder of any number divided by 25 is the remainder of the number formed by its last 2 digits divided by 25. The number formed by the last two digits is 37. The remainder is 12. Choice (B)

10. The required number must be the 4-digit number of the form 19k+7, where k is the greatest natural number satisfying 19 k + 7 < 10000
$k < \dfrac{9993}{19} = 525\dfrac{18}{19}$
∴ k = 525
∴ The required number = (19) (525) + 7 = 9982
Ans: (9982)

11. The tens digit of a perfect square ending in 6 must be odd.
∴ pqr26 cannot be a perfect square. Choice (B)

12. Any perfect square ending with 5 must have a tens digit of 2.
∴ 1a4b75 is not a perfect square. Choice (B)

13. 3036 is not a perfect square, whereas 1936 is a perfect square. ∴ We cannot say Choice (C)

14. For any value of C from 1 to 9, C36 cannot be a perfect square.
Choice (B)

15. (a) Perfect squares of the form P6Q are 169, 361 and 961.
∴ P must be odd. Choice (A)
(b) The only perfect square of the form A5B is 256.
∴ A is even. Choice (B)
(c) 121 as well as 441 are perfect squares ∴ We cannot say. Choice (C)

16. $3^9\, 5^{11}\, 15^{13} = 3^9\, 5^{11}\, (3.5)^{13} = 3^9\, 5^{11}\, 3^{13}\, 5^{13} = 3^{22}\, 5^{24}$
The index of each of the prime factors of $3^9\, 5^{11}\, 15^{13}$ is even.
∴ The number is a perfect square. Choice (A)

17. The product of a x digit number and a y digit number must have either (x + y – 1) digits or (x + y) digits.∴ The product can have either 18 or 19 digits. Choice (C)

18. The product of a x digit number, y digit number and a z digit number must have either (x + y + z – 2), (x + y + z – 1) or (x + y + z) digits.∴ The product can have either 22, 23 or 24 digits. Choice (D)

19. The least 13-digit number is 10^{12}. Its square root is 10^6 i.e. a 7 digit number. The greatest 13 digit number is $10^{13} – 1$. Its square root is less than the square root of $10^{13} = 10(10^6)^2$
$\sqrt{10^{13}} = 10^6$ (3.3) which has 7 digits.
∴ The square root must have 7 digits. Choice (A)

20. 3000 < 3PQR < 4000
$(3000)^4 < (3PQR)^4 < (4000)^4$
$(81)\,(10^{12}) < (3PQR)^4 < (256)\,(10^{12})$
$(81)\,(10^{12})$ has 14 digits while $(256)\,(10^{12})$ has 15 digits.
∴ $(3PQR)^4$ has either 14 or 15 digits. Choice (C)

21. The smallest and largest 25 digit numbers are 10^{24} and $10^{25} – 1$ respectively. Their respective cube roots are 10^8 and $\sqrt[3]{10}\,(10^8)$. In either case, the cube root of a 25-digit number will be a 9-digit number. Choice (A)

22. The factorial of any natural number greater than 4 ends with a 0.∴ The units digit of the sum = units digit of
1! + 2! + 3! + 4! = 3 Ans: (3)

23. (a) Considering only the units digit of the numbers in multiplication, units digit of the product = $\underline{8} \times \underline{4} \times \underline{2} \times \underline{3}$ = 2 Choice (A)
(b) Consider each of the numbers and find their units place.
Units place of 7^{48}
Cycle of 7 = 7,9,3,1 i.e., period is 4
48/4 = 12, no remainder i.e. 12 cycles
∴units place of 7^{48} is 1.
Units place of 3^{56}
Cycle of 3 is 3,9,7,1 i.e., period of 4
56/4 = 14, no remainder i.e. 14 cycles.
∴units place of 3^{56} is 1
Units place of 165^{35}
Units place of 5 raised to any power is 5.
∴Units place of 165^{35} is 5
∴Units place of $7^{48} \times 3^{56} \times 165^{35}$ is $1 \times 1 \times 5 = 5$
Choice (B)
(c) Units digit of 8^{4n}:
4n is a multiple of 4
cycle of 8 is 8, 4, 2, 6 i.e., period is 4
4n/4 = n no remainder i.e., n cycles.
∴Units place of 8^{4n} is 6.
Units digit of 6^n:
Units place of 6 raised to any power is 6.
∴Units place of 6^n is 6
Units digit of 9^{2n}
2n is a multiple of 2
Cycle of 9 is 9, 1 i.e., period is 2.
2n/2 = n i.e., no remainder i.e. n cycles.
∴Units place of 9^{2n} is 1.
Units digit of $8^{4n} \times 6^n \times 9^{2n} = 6 \times 6 \times 1 = 6$
Choice (C)
(d) Units digit of the product 31. 32. 33. - - - - - 39 = units digit of 1. 2. 3. 4. 5. - - - -9 = the units digit of product of 2 and 5 which is 0.
∴ Required units digit = 0. Choice (A)

24. $2^{3n} – 1 = (2^3)^n – 1 = 8^n – 1^n$
$x^n – y^n$ is divisible by (x – y) for all the values of n i.e., even or odd
$\therefore 8^n – 1$ is divisible by 7 for all values of n Choice (C)

25. When 2^1 is divided by 7 the remainder is 2
When 2^2 is divided by 7 the remainder is 4
When 2^3 is divided by 7 the remainder is 1
When 2^4 is divided by 7 the remainder is 2
$\therefore$the cycle is 2,4,1 its period is 3
when 63/3, remainder is 0 i.e., 21 cycles.
$\therefore$The remainder is 1 Ans: (1)

Exercise – 2(a)

Solutions for questions 1 to 25:

1. (a) The units digit of 8 repeats after every four powers. Expressing 173 in terms of 4, we have $8^{173} = 8^{4 \times 43 + 1}$ and hence the last digit of 8^{173} and 8^1 should be the same. Hence 8^{173} has the units digit of 8.
Choice (C)

(b) Last digit of 518^{163} is the same as the last digit of 8^{163}; $8^{163} = 8^{(4 \times 40) + 3}$. Since the last digit of the power of 8 is 3, 8^{163} will have the same units digit as 8^3 whose last digit is 2, 142^{157} will have same units digit as 2^{157} $2^{157} = 2^{(4 \times 39 +1)}$ cycle for the last digit of power of 2 is also 4.
Hence 2^{157} will have the same units digit as 2^1 whose units digit is 2.Hence $518^{163} + 142^{157}$ will have the last digit of 2 + 2 = 4 Choice (B)

(c) 1567^{143} has the same last digit as 7^{143}
$7^{143} = 7^{(4 \times 35) + 3}$
Since the last digit of the power of 7 has a cycle of 4, 7^{143} will have the same last digit as 7^3 i.e., 3.
1239^{197} has the same last digit as 9^{197}.
For 9^{197}, since power of 9 is odd its last digit is 9.
Hence, 1239^{197} has last digit of 9.
2566^{1027} has the same last digit as 6^{1027}, i.e., a 6, since 6 raised to any power will always have a last digit of 6.
Hence, last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$ will be the last digit of 3 x 9 x 6 = 162
i.e., 2 Choice (A)

2. This is of the form $(43)^a – (21)^a$, where a = 5n. This is always divisible by 43 – 21 = 22.
Hence it is also divisible by 11. Choice (A)

3. By observation, factorial of any number greater than 6, is divisible by 7.
$\therefore$The effective remainder of 1! + 2! + 3! + ... + 49! is nothing but the reminder obtained when 1! + 2! + 3! + 4! + 5! + 6! is divided by 7.
1! + 2! + 3! . . . . 6! = 873
The remainder when 873 is divided by 7 is 5. Ans: (5)

4. Given dividend is 3^{147} and divisor is 11.
Looking at the remainders when 3^{147} is divided by 11, they are as follows.
For $3^1 = 3$; $3^2 = 9$; $3^3 = 5$; $3^4 = 4$; $3^5 = 1$
$\therefore$For every 5 powers remainders are repeated.
$\therefore \frac{3^{147}}{11} = \frac{3^{5\times29+2}}{11}$, Remainder is that of $= \frac{3^2}{11}$, i.e., 9
Choice (B)

5. $21^3 + 23^3 + 25^3 + 27^3 = (24–3)^3 + (24+3)^3 + (24–1)^3 + (24+1)^3$
$= 2[24^3 + 3(24)(3^2)] + 2[24^3 + 3(24)(1)^2]$
$= 2[2(24^3) + 3(24)(10)] = 96[24^2 + 15]$
$\therefore (21^3 + 23^3 + 25^3 + 27^3)$ when divided by 96 leaves a remainder of 0. Ans: (0)

6. $N = 10^{51} – 750$
I: The remainder of 10^n, where n is any odd number, when divided by 11 is always 10.
A more general statement is that, if n is odd, the remainder of A^n divided by A + 1 is always A.
Rem $\left(\frac{N}{11}\right)$ = Rem $\left(\frac{(11k + 10) – (11(68) + 2)}{11}\right) = 8$
I is true.
II. $10^{51} = (7 + 3)^{51}$ = (7 + 3) multiplied 51 times.
(7 + 3) (7 + 3) = M(7), where M(7) denotes an unspecified multiple of $7 + 3^2$
$(7 + 3)^3 = (7 + 3)(M(7) + 3)$
= A multiple of $M(7) + 3^3$.
It follows in general that $(7 + 3)^N = M(7) + 3^N$
$10^{51} = M(7) + 3^{51}$.
Rem $\left(\frac{10^{51}}{7}\right)$ = Rem $\left(\frac{3^{51}}{7}\right)$ = Rem $\left(\frac{(3^3)^{17}}{7}\right)$
= Rem $\left(\frac{27^{17}}{7}\right)$ = Rem $\frac{(28-1)^{17}}{7}$
= Rem $\left(\frac{(-1)^{17}}{7}\right) = –1$
$\therefore 27^{17}$ is 1 less than a multiple of 7, (or 6 more than a multiple of 7) while 750 = 7 (107) + 1
$\therefore$ Rem $\left(\frac{N}{7}\right)$ = Rem $\left[\frac{(M(7)+6)-(7(107)+1)}{7}\right]$ = Rem $\left(\frac{5}{7}\right)$ = 5.
II is true. Both I and II are true. Choice (C)

Note: $10^{51} = 2^{51} \times 5^{51}$. Remainders of 2^N divided by 7 have a cycle of 3. $\therefore$ Rem $\left(\frac{2^{51}}{7}\right)$ can be found ----- (1)
Rem $\left(\frac{5^{51}}{7}\right)$ = Rem $\left(\frac{(5^3)^{17}}{7}\right)$ = Rem $\left(\frac{(18(7)-1)^{17}}{7}\right)$
$= (–1)^{17} = –1$ --------------(2)
From (1) and (2), Rem $\left(\frac{N}{7}\right)$ can be found.

7. Let $E = 5^{8n+4} + 4^{4n+2} – 10$
$= \left(5^4\right)^{2n+1} + \left(4^2\right)^{2n+1} – 10$
$= 625^{2n+1} + 16^{2n+1} – 10$
$a^N + b^N$ is always divisible by a + b when N is odd.
$625^{2n+1} + 16^{2n+1}$ is divisible by 641.
Rem $\left(\frac{E}{641}\right)$ = –10. This is equivalent to the positive remainder of 631. Choice (C)

8. P and Q have the respective forms 20k + 1 and 20k + 2
$\therefore$ They have the respective forms $4k_1 + 1$ and $4k_1 + 2$
($\because k_1 = 5k$)
Units digits of the power of 2 and 8 have cycles of 4 each.
$\therefore 2^P$ and 8^Q have respective units digits of those of $2^1$and 8^2 i.e 2 and 4.
I: As P has the form 20k + 1, its units digit is 1. $\therefore$The units digit of 2P is 2.
Both 2^P and 2P have the same units digit.
$\therefore$ I is true.
II. 8Q has units digit of 6.
$(8^Q + 8Q)$ ends with 0 i.e $8^Q + 8Q$ is divisible by 10.
$\therefore$ II is true.
Both I and II are true. Choice (C)

9. N = ((48) (98) + 7) ((48) (98) + 9) ((48) (98) + 11)
$\therefore$ Required remainder = Remainder when (7) (9) (11) divided by 48 = 21 Ans: (21)

10. As $(a+b)^3 – a^3 – b^3 = 3ab(a+b)$, $N = 161^3 – 77^3 – 84^3$ $= 3(77)(84)(161) = 3^2(7)^3(11)(4)(23)$
$\therefore$ N is divisible by 4, 23, 11, 7 but not by 8. Choice (D) is false.
Choice (D)

11. Let N = 7923, 7923,……… 7923 (comprising 400 digits or 100 groups)
= 79,23,79,23,. ………… 79,23 (comprising 200 groups)

$\text{Rem}\frac{N}{101} = \text{Rem}\frac{23(100)-79(100)}{101} = 23\,(-1) - 79(-1)$

= 56. (∵ By remainder theorem) Ans: (56)

12. Let N= 2424………… 24(comprising 300 digits)
= 242, 424, ……242, 424 (comprising 100 groups)

$\text{Rem}\frac{N}{999} = \text{Rem}\frac{424(50)+242(50)}{999}$

$= \text{Rem}\frac{666(50)}{999} = \text{Rem}\frac{33,300}{999} = 33 + 300 = 333.$

Choice (A)

13. Let N = 73^{382} $\text{Rem}\frac{N}{100} = \text{Rem}\frac{73^2}{100}$ (We can leave out all the 20's in the index) = $\text{Rem}\frac{5329}{100} = 29.$ Choice (C)

14. Let N = 787^{777}. So, $\text{Rem}\frac{N}{100} = \text{Rem}\frac{87^{760+17}}{100} = \text{Rem}\frac{87^{17}}{100}$

$87^2 \equiv 69$
$69^2 \equiv 61$
$61^2 \equiv 21$
$21^2 \equiv 41$
$\therefore 87^{17} = 87^{16}\,87 \equiv (41)(87) \equiv 67$

Alternate method:

$\text{Rem}\frac{787^{777}}{100} = \text{Rem}\frac{87^{17}}{100}$

$(87)^{17}(87)^3 \equiv 87^{20} \equiv 1 \Rightarrow 87^{17} \equiv 87^{-3} = (87^{-1})^3$

Where 87^{-1} is not 1/87, but the inverse of 87 or that number (or one number), which when multiplied by 87 produces 1(i.e. produces a number of the form 100k + 1)

This is an LCM model 3 problem. We are looking for a multiple of 87 which leaves a remainder of 1, when divided by 100

i.e. 87x = 100y + 1 -------- (1)
$\Rightarrow 13y = 87y_1 + 86$ ------(2) (See N_1)
$\Rightarrow 9y_1 = 13y_2 + 5$ ------- (3) (See N_2)
$\Rightarrow 4y_2 = 9y_3 + 4$
$y_3 = 0 \Rightarrow y_2 = 1 \Rightarrow y_1 = 2 \Rightarrow y = 20 \Rightarrow x = 23$
i.e. the inverse of 87 is 23 $\therefore 87^{-3} \equiv 23^3 \equiv (29)(23) \equiv 67$

Choice (D)

Note: (N_1) As the second term on the RHS of (1) which leaves a remainder of 1 when divided by 87, the first term i.e. 100y (as equivalently 13y) leaves a remainder of 86.

(N_2) As the second term of the RHS of (2) leaves a remainder of 8 when divided by 13, the first term leaves a remainder of 5.

Similarly, we can keep decreasing the coefficients until they are small enough for us to see the solution.

15. Let N = 948^{728} $\text{Rem}\frac{N}{100} = \text{Rem}\frac{48^8}{100}$

$= \text{Rem}\frac{(2304)^4}{100} = \text{Rem}\frac{4^4}{100} = 56.$ Ans: (56)

16. Let N = 674^{586} $\text{Rem}\frac{N}{100} = \text{Rem}\frac{74^6}{100} = \text{Rem}\frac{26^6}{100}$

$= \text{Rem}\frac{76^3}{100} = 76.$ Choice (B)

17. $\text{Rem}\frac{98^{100}}{99} = \text{Rem}\frac{(99-1)^{100}}{99}$

$= \text{Rem}\frac{99k+(-1)^{100}}{99} = 1$

Similarly, $\text{Rem}\frac{100^{100}}{99} = \text{Rem}\frac{1^{100}}{99} = 1$

$\therefore \text{Rem}\frac{98^{100}+100^{100}}{99} = 2$ Choice (A)

18. $(1 - 3x + x^2)^{55} = 1 + a_1x + a_2x^2 + \ldots\ldots + a_{110}\,x^{100}$
Setting x = 1, $1 + a_1 + a_2 +$ ------- $a_{110} = (-1)^{55} = -1$

Choice (B)

19. We need $\text{Rem}\frac{2^{123}}{61}$. As the index is close to a multiple of the divisor which is prime, we think of Fermat's Little theorem = $\text{Rem}\frac{2^{60}}{61} = 1 \Rightarrow \text{Rem}\frac{2^{120}}{61} = 1$

$\therefore \text{Rem}\frac{2^{123}}{61} = \text{Rem}\frac{2^3}{61} = 8.$ Ans: (8)

20. Let R = $\text{Rem}\frac{(10^{400})}{199} = \text{Rem}\frac{(10^{198})^2 10^4}{199} = \text{Rem}\frac{10,000}{199}$

$= \text{Rem}\frac{50(200)}{200-1} = 50\,(1) = 50$ Choice (D)

21. $\text{Rem}\frac{14^{400}}{1393} = \text{Rem}\frac{14(14^{399})}{7(199)} = 7\,\text{Rem}\frac{2(14^{399})}{199} = M\text{(say)}$

$\text{Rem}\frac{2(14^{399})}{199} = 2\,\text{Rem}\frac{(14^{198})^2 14^3}{199} = 2\,\text{Rem}\frac{(14)(196)}{199}$

$\equiv 2(14)\,(-3) \equiv -84$

$\therefore \text{Rem}\frac{2(14^{399})}{199} = 199 - 84 = 115$

$\therefore$ M = 7(115) = 805. Choice (B)

22. By Wilson's Theorem, 96! = 97k + 96
$\therefore 97! = 97^2k + 96(97)$ and $100! = 98(99)\,(100)\,[97^2k + 96(97)]$

$\therefore \text{Rem}\frac{100!}{97^2} = \text{Rem}\frac{96(97)(98)(99)(100)}{97^2}$

$= 97\ \text{Rem}\frac{96(98)(99)(100)}{97}$

$\equiv 97\,(-1)\,(1)\,(2)\,(3) \equiv -582 \equiv 97^2 - 582$
= 9409 − 582 = 8827. Choice (D)

23. By Wilson's Theorem 46! = 47k + 46
$\Rightarrow$ 45! 46 = 47k + 46
where k is an integer.
It follows from this equation that k must be a multiple of 46.
$\therefore$ Dividing by 46 on both sides, we get 45! = 47 (an int) + 1.

$\therefore \text{Rem}\frac{45!}{47} = 1.$ Ans: (1)

24. $81(64^{25}) = 81(8)^{50} = 81\,(9-1)^{50}$
$= 81[9^{50} - {}^{50}C_1\,(9)^{49} + {}^{50}C_2\,(9)^{48}\ldots\ldots + {}^{50}C_{48}(9^2) - {}^{50}C_1\,(9) + 1]$
= A multiple of $9^4 + 81(1 - 50(9))$
= A multiple of $9^4 - 36369$
[9^4 = (6561) and $9^4(6)$ = 39366 and –36369 = –39366 + 2997].
$\therefore 81(64^{25})$ = A multiple of $9^4 - 9^4(6) + 2997$ = A multiple of $9^4 + 2997$
$\therefore$ Remainder is 2997.

Alternative method:

$\text{Rem}\ \frac{81(64^{25})}{9^4} = 81\ \text{Rem}\ \frac{64^{25}}{81} = 81\ \text{Rem}\ \frac{8^{50}}{81}$

$= 81\ \text{Rem}\ \frac{(9-1)^{50}}{81}$

$(9-1)^{50} =$
$9^{50} - {}^{50}C_1\,9^{49}1 + \ldots. + {}^{50}C_{48}\,9^2(1^{48}) - {}^{50}C_{49}\,9\,(1^{49}) + 1^{50}$

$\therefore \text{Rem}\ \frac{(9-1)^{50}}{81} = -\text{Rem}\frac{50(9)-1}{81} = -\text{Rem}\frac{449}{81} = -44$

which is equivalent to 37.
$\therefore$ The required remainder is 37(81) = 2997 Choice (B)

25. $(7^N + N^3)$ ends with 0. $\therefore$ It is even. 7^N is odd.
$\therefore$ N^3 must also be odd. $\therefore$ N must be odd.
$\therefore$ N has the form 4K + 1 or 4K + 3
If N has the form 4K + 1, 7^N ends with 7.
$\therefore$ N^3 must end with 3 and N must end with 7.
(ie., N = 4k + 1 = 10p + 7)
$\therefore$ N is 17, 37, 57, 77 or 97.
If N has the form 4K + 3, 7^N ends with 3. $\therefore$ N^3 must end with 7. $\therefore$ N must end with 3. (i.e., N = 4k + 3 = 10p + 3)
N is 3, 23, 43, 63, or 83.
N has a total of 10 values. Ans: (10)

Exercise – 2(b)

Solutions for questions 1 to 35:

1. $314^{779} + 149^{138}$ has the same units digit as that of $4^{779} + 9^{138}$ which has the same units digit as that of 4^1+9^2 whose units digit is 4 + 1 = 5. Ans: (5)

2. $2^{69}/5 = 2^{69}/2^2 + 1 = \dfrac{2 \times 2^{68}}{2^2+1} = \dfrac{2 \times \left(2^2\right)^{34}}{2^2+1}$

Since $2^2 + 1$ divides $2\left(2^2\right)^{34}$, replacing 2^2 by (– 1), the remainder is $2(-1)^{34} = 2$ Choice (A)

3. As the remainder is odd, it can be 1 or 3 only.
$\therefore$ The prime numbers must be of the form 5k + 1 or 5k + 3, where k is a whole number.
5k ends with a 0 and 5.
$\therefore$ 5k + 1 ends with a 1 and 6.
5k + 3 ends with a 3 or 8.
As 5k + 1 and 5k + 3 must be prime, they must end with 1 and 3 respectively.
$\therefore$5k + 1 has 5 possibilities (11, 31, 41, 61, 71) and 5k + 3 has 7 possibilities (3, 13, 23, 43, 53, 73, 83).
$\therefore$ 12 such prime numbers exist. Ans: (12)

4. Consider $2^{125}/33$
$= (2^5)^{25}/2^5 - (-1) = -1$
Actual remainder = –1 + 33 = 32, as divisor is 33.
In the case of $2^{125}/11$, we divide 32 by 11 and get 10 as the remainder Choice (B)

5. Let N = 987654, 987654, ………. 987654
(comprising 750 digits or 125 groups)
= 987, 654, 987, 654, ……..987, 654,
(comprising 250 groups)

$\text{Rem}\dfrac{N}{999} = \text{Rem}\dfrac{(987+654)125}{999}$

$= \text{Rem}\dfrac{(1641)(125)}{999} = \text{Rem}\left[\text{Rem}\dfrac{1641}{999}\text{Rem}\dfrac{125}{999}\right]$

$= \text{Rem}\dfrac{(642)(125)}{999} = \text{Rem}\dfrac{(640)(125)+250}{999}$

$= \text{Rem}\dfrac{80{,}250}{999}$ = 80 + 250 = 330. Choice (C)

6. Let N = 445, 445, ………445
(comprising 525 digits or 175 groups)
U = 445 (88) while Th = 445(87)

$\therefore \text{Rem}\dfrac{N}{1001} = \text{Rem}\dfrac{445(88)-445(87)}{1001}$ = 445.

Choice (C)

7. $\dfrac{7^{1000}}{50} = \dfrac{\left(7^2\right)^{500}}{7^2-(-1)}$

By remainder theorem, remainder = $(-1)^{500}$ = 1 Ans: (1)

8. $N = (767)^{1009}$

$\text{Rem}\left(\dfrac{N}{25}\right) = \text{Rem}\ \dfrac{[\text{Rem}\ [N/100]]}{25}$

As powers of 767 will have a cyclicity of 20, we can find $\text{Rem}\left[\dfrac{(767)^9}{100}\right]$

$= \text{Rem}\ \dfrac{[67]^9}{100}$ (only last 2 digits affect the last 2 digits)

67^1 has last 2 digits = 67
67^2 has last 2 digits = 89 (67^2)
67^4 has last 2 digits = 21 (89^2)
67^8 has last 2 digits = 41 (21^2)
67^9 has last 2 digits = 47 (41×67)

$\Rightarrow \text{Rem}\left[\dfrac{N}{25}\right] = \text{Rem}\ \dfrac{\text{Rem}[N/100]}{25} = \dfrac{47}{25} = 22$

Choice (D)

9. All powers of numbers ending in 76 end in 76.
Choice (A)

10. Let $N = 768^{1234}$ $\text{Rem}\dfrac{N}{100} = \text{Rem}\dfrac{68^{14}}{100}$

$= \text{Rem}\dfrac{32^{14}}{100}$ $(\because 68 \equiv -32)$

$= \text{Rem}\dfrac{2^{70}}{100} = \text{Rem}\dfrac{2^{10}}{100}$ = 24. Choice (A)

11. Let $N = 994^{499}$, $\text{Rem}\dfrac{N}{100} = \text{Rem}\dfrac{94^{19}}{100}$

Method 1:
$(94)^{19}(94)$ ends in 76. Any even number raised to the power of any multiple of 20 ends in 76. We can try out the options.
$04(94) \equiv 76$ (We need not actually try out the other options)
$24(94) \equiv 56$
$64(94) \equiv 16$
$84(94) \equiv 96$
$\therefore$ 94^{19} ends in 04.

Method 2:
Any (non-zero) even number raised to 20 ends with 76.
$\therefore$ 94^{20} ends with 76.
94^{19} ends with 4. ($\because$ 4^{19} = 4 raised to an odd number and hence ends with 4).
Let the tens digit of 94^{19} be x. Then 76 = last two digits of $x4 \times 94 = (10x + 4)(90 + 4)$. The tens digit of this product = units digit (7 + y) where y is the units digit of 4x..
$\therefore$ 7 + y = 7. $\therefore$ y = 0.
$\therefore$ x = 0 or 5. But if x = 5, 94^{19} will not be divisible by 4.
$\therefore$ x = 0. $\therefore$ 94^{19} ends with 04. Ans: (4)

12. $(3 + 2x)^{99} = {}^{99}C_0\, 3^{99} + {}^{99}C_1\, 3^{98}(2x) + \ldots + {}^{99}C_{99}(2x)^{99}$
Setting x = 1,
we get ${}^{99}C_0\, 3^{99} + {}^{99}C_1(2) + \ldots + {}^{99}C_{99}(2)^{99} = 5^{99}$.
Choice (A)

13. $N = 624^{739} = (625 - 1)^{739}$

$\therefore \text{Rem}\dfrac{624^{739}}{125} = \text{Rem}\ \dfrac{625k+(-1)^{739}}{125} = -1 \equiv 124$.

Ans: (124)

14. We need $\text{Rem}\dfrac{17^{325}}{109}$. As the index is close to a multiple of the divisor, which is prime, we apply Fermat's theorem

$\text{Rem}\dfrac{17^{108}}{109} = 1 \Rightarrow \text{Rem}\dfrac{17^{324}}{109} = 1$

$\Rightarrow \text{Rem}\dfrac{17^{325}}{109} = 17$.

Choice (B)

15. $\frac{12^{433}}{438} = \frac{12\left(12^{432}\right)}{6(73)} = \frac{2(12^{72})^6}{73}$

$\therefore$ Rem$\frac{12^{433}}{438}$ = 6. Rem$\frac{2(12^{72})^6}{73}$ = 12.

Choice (A)

16. We need R = Rem$\frac{10^{2000}}{19}$. Now 2000 = 18 (111) + 2

$\therefore$ Rem$\frac{(10^{18})^{111}10^2}{19}$ = Rem$\frac{10^2}{19}$ = 5.

Choice (B)

17. By Wilson's Theorem, 18! = 19k + 18
$\therefore$ 19! = 361k + 18(19) = 361k + 342. Ans: (342)

18. By Wilson's Theorem, 27!(28) = 29k + 28
where k is an integer. It follows from this equation that k must be a multiple of 28.
$\therefore$ Dividing by 28 both sides, we get 27! = 29 (an int) + 1.

$\therefore$ Rem$\frac{27!}{29}$ = 1 Choice (C)

19. $7^{900} = (8-1)^{900}$
$(8-1)^2 = (8-1)(8-1) = 8^2 - 2(8) + 1 = 8k_2 + 1$
$(8-1)^3 = (8k_2 + 1)(8-1) = 8k_3 - 1$
$(8-1)^4 = (8k_3 - 1)(8-1) = 8k_4 + 1$
We can see that $(8-1)^N$ where N is any positive integer has the form $8k + (-1)^N$ $\therefore$ $7^{900} = 8k + 1$
Also 908 = 8(113) + 4
$\therefore 7^{900} - 908 = (8k + 1) - (8m + 4) = 8(k - 1 - m) + 5$
$\therefore$ The remainder is 5 Choice (C)

20. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$. Also 25 + 31 = 27 + 29 = 56 and 112 = 56 (2)
$25^3 + 31^3 = 56\,[25^2 + 31^2 - 25 \times (31)]$ = 56 (an odd number)
Similarly $27^3 + 29^3$ = 56 (an odd number)
$25^3 + 31^3 + 27^3 + 29^3$ = 56 (an even number)
= A number divisible by 112. $\therefore$ Remainder is 0.

Choice (D)

21. A zero at the end of any number will result from the product of a 2 and a 5. As A has only 1 multiple of 2(i.e.2 itself) and 1 multiple of 5 (i.e. 5), B will have only 1 zero at its end.

Ans: (1)

22. Units digit of the factorial of any natural number which is 5 or more is 0. Required units digit = units digit of 1! + 2! + 3! + 4! = 1 + 2 + 6 + 4 = 3. Ans: (3)

23. 422 = 2(211)
If p is any prime number, (p – 1)! + 1 is a multiple of p.
(Wilson's Theorem)
211 is prime.
$\therefore$ 210! + 1 is a multiple of 211.
210! + 1 = 211k (say)
210! = 211k – 1
LHS is even and $\therefore$ RHS must be even. $\therefore$ k is odd.
Let $k = 2k_1 + 1$
$210! = 422k_1 + 210$
LHS is divisible by 210.
$\therefore$ RHS must be divisible by 210.
$\therefore$ $422k_1$ must be divisible by 210.
$\therefore$ k_1 must be divisible by 105.
Let $k_1 = 105k_2$
$210! = 422\,(105k_2) + 210$
Dividing both sides by 210, $209! = 211k_2 + 1$
k_2 must be odd. Let $k_2 = 2k_3 + 1$
$209! = 211\,(2k_3 + 1) + 1 = 422\,k_3 + 212$.
Alternately, 210! +1 is a multiple of 211 (Wilson's Thorem). Subtracting 211 from 210! +1 $\Rightarrow$ 210! – 210 is also a multiple of 211. As 210 is co-prime to 211, 209! – 1 is divisible by 211 $\Rightarrow$ 209! leaves 212 as remainder.

Choice (A)

24. $2^{924} = 2^{920} \cdot 2^4 = (2^{10})^{92} \cdot 2^4 = (1024)^{92} \cdot 16$.
Its last two digits are that of $24^{92} \cdot 16$.
$24^{92} = (24^2)^{46} = (576)^{46}$.
$\therefore$ Last two digits of 2^{924} are those of $76^{46} \cdot 16$.
Last two digits of 76^N where N is any natural number are 76.
Last two digits of 76^{46} are 76.
$\therefore$ Last two digits of 2^{924} = those of (76) (16) i.e 16.

Alternate method:

Last two digits of 2^N, where N $\geq$ 2, show a cyclic pattern, with cycle length 20.
$\therefore$ Last two digits of 2^{924}, 2^{924-20}, $2^{224-20(2)}$, …..$2^{924-20(46)}$ i.e 2^4 are all the same.
$\therefore$ Last two digits of 2^{924} are 16. Choice (A)

25. $P^2 + 7^Q = 2^6 \cdot 5^6 = 10^6$.
Q is odd. $\therefore$ It has the form 4k + 1 or 4k + 3.
If it has the form 4k + 1, 7^Q ends with 7. If it has the form 4k + 3, 7^Q ends with 3.
7^Q ends with 7 or 3. Also RHS of the given equation ends with 0.
$\therefore$ P^2 ends with 3 or 7. But P is an integer i.e. P^2 is a perfect square.$\therefore$ P^2 cannot end with 3 or 7.
$\therefore$ P has no possible value. Ans: (0)

26. 105 = 3 (5) (7)

Rem $\left(\frac{2^{168}}{3}\right)$ = Rem $\left(\frac{(3-1)^{168}}{3}\right)$

= Rem $\left(\frac{(-1)^{168}}{3}\right)$ = 1 -----(1)

Rem $\left(\frac{2^{168}}{5}\right)$ = Rem $\left(\frac{4^{84}}{5}\right)$ = Rem $\left(\frac{(5-1)^{84}}{5}\right)$ = 1 – (2)

Remainders of 2^N divided by 7 have a cycle of 3.

$\therefore$ Rem $\left(\frac{2^{168}}{7}\right)$ = Rem $\left(\frac{2^{56\times 3}}{3}\right)$

= Rem $\left(\frac{2^3}{7}\right)$ = 1 ------(3)

From (1), (2), (3), $2^{168} - 1$ is divisible by 3, 5, 7 and hence by their L.C.M i.e., 105.

$\therefore$ Rem $\left(\frac{2^{168}}{105}\right)$ = 1 Choice (D)

27. $x = \frac{\left(40^{37}\right)^2 - \left(39^{37}\right)^2}{\left(40^{36}+39^{36}\right)\left(40^{37}+39^{37}\right)} = \frac{40^{37} - 39^{37}}{40^{36} + 39^{36}}$
$a^N - b^N = (a - b)(a^{N-1} + a^{N-2}b + \ldots ab^{N-2} + b^{N-1})$
$\therefore$ $40^{37} - 39^{37} = (1)\,[40^{36} + 40^{35}(39) + \ldots 40(39)^{35} + 39^{36})$
-------- (1)
RHS is more than $40^{36} + 39^{36}$. $\therefore$ LHS is also more than $40^{36} + 39^{36}$. $\therefore$ x > 1. Choice (D)

28. Each term in the dividend has the form $(x + 1)^2\,x!$
$(x + 1)^2\,x! = (x + 1)(x + 1)! = (x + 2 - 1)(x + 1)!$
= (x + 2)! – (x + 1)!
Required remainder is that of (19! – 18! + 18! – 17! + …. + 3! – 2!) divided by 19 i.e. of (19! – 2) divided by 19. This is equivalent to – 2. (It is equal to 19 – 2 = 17) Ans: (17)

29. $(70^{20})^{118} + (80^{40})^{59} = 70^{2360} + 80^{2360} = 10^{2360}\,(7^{2360} + 8^{2360})$
= $(7^{2360} + 8^{2360})$ followed by 2360 zeros
The rightmost non zero digit comes from $7^{2360} + 8^{2360}$
Units digit of 7^{2360} = that of 7^4 = 1(cycle of 4)
Units digit of 8^{2360} = that of 8^4 = 6(cycle of 4)
Units digit of $7^{2360} + 8^{2360}$ = 7
Rightmost nonzero digit is 7. Choice (C)

30. The least factorial divisible by 18 is 6! i.e., 720.
$\therefore$ All higher factorials are divisible by 18.
$\therefore$ The remainder of the sum divided by 18 is the remainder of (1! + 2! + 3! + 4! + 5!) divided by 18. This equals 9. Choice (A)

31. $\text{Rem}\left(\frac{7777777777}{16}\right) = \text{Rem}\left(\frac{7777}{16}\right)$ (The last 4 digits of the given number) = 1

$\text{Rem}\left(\frac{7^{262}}{16}\right) = \text{Rem}\left(\frac{7^{2(131)}}{16}\right) = \text{Rem}\left(\frac{(7^2)^{131}}{16}\right)$

$= \text{Rem}\left(\frac{(48+1)^{131}}{16}\right) = \text{Rem}\left(\frac{1^{131}}{16}\right) = 1$

[$\because$ $(48 + 1)^{131}$ of the form $48k + 1^{131}$]

Required remainder is $\text{Rem}\ \frac{1+1}{16} = 2$ Choice (D)

32. Given,
$\alpha = 188^3 + 200^3 + 211^3 + 299^3$, which is an even number.
When an even number is divided by an even number, the remainder would always be even. It can be neither 23, nor 37, nor 9. Choice (D)

33. N = 757677 ………99100101…….119120
The remainder of any large number divided by 9 can be found easily by the following procedure. We break up the number into a number of parts and add the numbers in each part. We then find the remainder of the sum divided by 9. This remainder R (say) equals the remainder of the large number divided by 9. (This also means that the value of R is independent of the breaking points).
We see that 75 + 120 = 78 + 119 = ….= 97 + 98 = 195
$\therefore$ We would find it convenient to break after 75, 76 …..120

$\text{Rem}\left(\frac{N}{9}\right) = \text{Rem}\left(\frac{(195)(x)}{9}\right)$ where x is the number of pairs that have been formed.
X = 23.

$\text{Rem}\left(\frac{N}{9}\right) = \text{Rem}\left(\frac{(195)(23)}{9}\right) = \text{Rem}\left(\frac{(189+6)(23)}{9}\right)$

$= \text{Rem}\frac{6(23)}{9} = 3$

Alternative method:
Let N = 757677 …..99100101 ……..120.
Let M = 75 + 76 +……+ 120
We can think of M rather than N. (They have the same 9's remainder) M is the sum of 46 consecutive numbers.
If n is an odd number, the sum of n consecutive integers is divisible by n (Also if n is an even number, say n = 2m, the sum of n consecutive integers leaves a remainder of m) $\therefore$The sum of 9 (and also 45) consecutive integers is divisible by 9. $\therefore$In M, which has 46 consecutive integers, we can leave out the first 45 and think of only 120, or leave out all the numbers from 76 to 120 and think of only 75.

i.e, Rem $\frac{M}{9}$ = Rem $\frac{75}{9}$ = Rem $\frac{120}{9}$ = 3

Ans: (3)

34. Successive powers of 7 leave the following remainders, when divided by 5.
2, 4, 3, 1 ; 2, 4, 3, 1 ; and so on.

As 1000 = 250(4), Rem $\frac{7^{1000}}{5}$ = 1. Choice (A)

35. (a) The square of any prime number greater than 3 when divided by 6 leaves a remainder of 1. 2^2 when divided by 6 leaves a remainder of 4. 3^2 when divided by 6 leaves a remainder of 3.
$\therefore$ The remainder cannot be 5. Choice (D)

(b) The cube of any prime number greater than 3 leaves a remainder of 1 or 5. 2^3 when divided by 6 leaves a remainder of 2. 3^3 when divided by 6 leaves a remainder of 3.
$\therefore$ The remainder cannot be 4. Choice (D)

Chapter – 3
(Number Systems)

Concept Review Questions

Solutions for questions 1 to 20:

1. To express a number in binary, we use the digits 0 and 1. Choice (C)

2. In octal system we have eight digits. Ans: (8)

3. In duodecimal system, B is 11. Ans: (11)

4. In hexadecimal system we use 16 digits. Ans: (16)

5.
$$\begin{array}{r|l} 2 & 12 \\ 2 & 6-0 \\ 2 & 3-0 \\ & 1-1 \end{array}$$

$\therefore$12 = $(1100)_2$ Choice (B)

6.
$$\begin{array}{r|l} 12 & 1221 \\ 12 & 101-9 \\ & 8-5 \end{array}$$

$\therefore$ 1221 = $(859)_{12}$
$\therefore$ n = 859 Ans: (859)

7. In septeunary system we use the digits 0, 1, 2, 3, 4, 5, 6.
$\therefore$ highest digit is 6. Choice (A)

8.
$$\begin{array}{r|l} 16 & 2346 \\ 16 & 146-A \\ & 9-2 \end{array}$$

$\therefore$ $(2346)_{10} = (92A)_{16}$
$\therefore$ x = 92A Choice (B)

9.
$$\begin{array}{r|l} 8 & 13 \\ & 1-5 \end{array}$$

$(13)_{10} = (15)_8$ Choice (A)

10. $(121)_8 = (1 \times 8^2 + 2 \times 8 + 1 \times 8^0)_{10} = (81)_{10}$
$$\begin{array}{r|l} 2 & 81 \\ 2 & 40-1 \\ 2 & 20-0 \\ 2 & 10-0 \\ 2 & 5-0 \\ 2 & 2-1 \\ 2 & 1-0 \end{array}$$

$\therefore$ $(121)_8 = (81)_{10} = (1010001)_2$
Choice (C)

11. $(3AB)_{12} = (3 \times 12^2 + A \times 12 + B \times 12^0)$
$= (3 \times 144 + 10 \times 12 + 11 \times 1)_{10} = (563)_{10}$
$\therefore$ x = 563 Ans: (563)

12. $(ACD)_{16} = (A \times 16^2 + C \times 16 + D \times 16^0)$
$= (10 \times 256 + 12 \times 16 + 13 \times 1)_{10}$
$= (2765)_{10}$ Choice (A)

13. $(121)_{10}$ is a perfect square.
$(171)_8 = (1 \times 8^2 + 7 \times 8 + 1 \times 8^0) = (121)_{10}$ is a perfect square
$(A1)_{12} = (A \times 12 + 1 \times 12^0)_{10}$ = 120 + 1 = 121 is a perfect square Choice (D)

14. The numerical value of $(1.001)_2 = (1 \times 20 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3})_{10} = (1 + 0 + 0 + 1/8)$
$= 1 + 0.125 = (1.125)_{10}$ Ans: (1.125)

15. 11011
101
1101
111111

1010001
-------------- Choice (B)

16. $(34)_7 + (25)_7 = (62)_7$ Choice (C)

17. 34
– 25

5

$\therefore (34)_6 – (25)_6 = (5)_7$ Choice (D)

18. The largest digit in hexadecimal system is F.
$\therefore$ Three digit largest number is $(FFF)_{16}$ Choice (C)

19. Every even number is divisible by 2. The remainder will be zero, which is the end digit of the binary number.
$\therefore$ The binary representation of even numbers always end with zero. Choice (B)

20.

2	247	
2	123	1
2	61	1
2	30	1
2	15	0
2	7	1
2	3	1
	1	1

$\therefore (247)_{10} = (11110111)_2$
$\therefore$ Option (A) is true

8	247	
8	30	7
	3	6

$\therefore (247)_{10} = (367)_8$
Option (B) is true

12	247	
12	20	7
	1	8

$\therefore (247)_{10} = (187)_{12}$
Option (C) is true. Choice (D)

Exercise – 3(a)

Solutions for questions 1 to 25:

1. The given number is $(176)_{10}$

2 | 176
2 | 88 – 0
2 | 44 – 0
2 | 22 – 0
2 | 11 – 0
2 | 5 – 1
2 | 2 – 1
1 – 0

$\therefore (176)_{10} = (10110000)_2$ Choice (C)

2. The given number is $(472)_{10}$

8 | 472
8 | 59 – 0
7 – 3

$\therefore (472)_{10} = (730)_8$ Ans: (730)

3. The given number is $(523)_{10}$

16 | 523
16 | 32 – 11
2 – 0

$\therefore (523)_{10} = (20B)$
since 11 = B in the hexadecimal system Choice (B)

4. The given number is $(2776)_{10}$

12 | 2776
12 | 231 – 4
12 | 19 – 3
1 – 7

$\therefore (2776)_8 = (1734)_{12}$ Choice (B)

5. The given octal number is $(7464)_8$.
We write each digit as a block of 3 binary digits. Accordingly we get $(111100110100)_8$. We now group the digits as blocks of 4 from right to left.
$= (\underline{1111}\ \underline{0011}\ \underline{0100})_{16} = ((1111)_2\ (0011)_2\ (0100)_2)_{16}$
$= (F34)_{16}$. Choice (A)

6. The given number is $(110001110)_2$.
We group the digits in blocks of 3 and find the octal equivalent for each block of 3 digits
$(110\ 001\ 110)_2 = ((110)_2\ (001)_2\ (110)_2)_8$
$= (616)_8$ Choice (D)

7. The given number is $(1100111011011)_2$.
We group the digits in groups of 4 and find the hexa-decimal equivalent for each group of 4 digits.
$(\underline{0001}\ \underline{1001}\ \underline{1101}\ \underline{1011})_2$
$= ((0001)_2\ (1001)_2\ (1101)_2\ (1011)_2)_{16}$
$= (19DB)_{16}$ Choice (D)

8. The given binary number is $(1101.0101)_2$
Integer part
$1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 = 1 + 4 + 8 = 13$
Fraction part
$0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$
$= 0.25 + 0.0625 = 0.3125$
$\therefore$ The decimal number is 13.3125 Ans: (13.3125)

9. The given number is $(BAD)_{16}$
We know that B = 11, A = 10 and D = 13 in the hexa-decimal system. $\therefore$The decimal number is
$= 11 \times 16^2 + 10 \times 16 + 13 \times 1 = 2816 + 160 + 13 = (2989)_{10}$
Choice (B)

10. $(1101)_2 + (46)_8 + (97)_{10}$
Now, $(1101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$
$= 1 + 4 + 8 = (13)_{10}$
$(46)_8 = 4 \times 8^1 + 6 \times 8^0 = 32 + 6 = (38)_{10}$ $\therefore$We get $(13)_{10} + (38)_{10} + (97)_{10} = (148)_{10}$
Choice (A)

11. The given decimal number is 256
The minimum number of bits required to represent 256 in the binary system is the number of digits in the binary representation of 256.

2 | 256
2 | 128 – 0
2 | 64 – 0
2 | 32 – 0
2 | 16 – 0
2 | 8 – 0
2 | 4 – 0
2 | 2 – 0
1 – 0

$\therefore (256)_{10} = (100000000)_2$
$\therefore$ 9 bits are required Ans: (9)

12. $(256)_{16} - (256)_8$

We convert both numbers to a common base i.e., in base 10

$(256)_{16} = 2 \times 16^2 + 5 \times 16 + 6 \times 1$

$= 512 + 80 + 6 = (598)_{10}$

$(256)_8 = 2 \times 8^2 + 5 \times 8 + 6 \text{ x } 1$

$= 128 + 40 + 6 = (174)_{10}$

$\therefore (598)_{10} - (174)_{10} = (424)_{10}$ Choice (C)

13. The given equation is

$(n)_{n+2} + (n-1)_{n+1} + (n-2)_n + \ldots\ldots + (1)_3$

Since the number is smaller than the given radix in each case, the number will not change even if the radix is changed to 10

For example $(4)_7 = (4)_{10}$

Hence each term of the expression can be written as

$(n)_{10} + (n-1)_{10} + (n-2)_{10} + \ldots\ldots(1)_{10}$

$= (n + (n-1) + (n-2) + \ldots\ldots + 1)_{10}$

$= \left(\frac{n(n+1)}{2}\right)_{10}$ Choice (C)

14. The number 378 when expressed as the sum of powers of 2 is 256 + 64 + 32 + 16 + 8 + 2. Thus we require 6 of these weights. Ans: (6)

15. Since there exists only one way of converting the number $(378)_{10}$ in base 2, i.e., $(378)_{10} = (101111010)_2$, only one such combination exists. Ans: (1)

16. $f((25)_8, (25)_{10}, (25)_{16})$

$(25)_8 = 2 \times 8^1 + 5 \times 8^0 = (21)_{10}$

$(25)_{16} = 2 \times 16^1 + 5 \times 16^0 = 32 + 5 = (37)_{10}$

$\therefore f((21)_{10}, (25)_{10}, (37)_{10})$

$= 21 \times 25 + 25 \times 37 + 37 \times 21 = (2227)_{10}$ Choice (B)

17. Let the original binary number be $(a_1\ a_2\ \ldots.a_n)_2$ where $a_1, a_2, \ldots\ldots.a_n = 0$ or 1 depending upon the value of n.

$\therefore (a_1\ a_2 \ldots\ldots.a_n)_2 = a_n \times 2^0 + a_{n-1} \times 2^1 + \ldots..a_1 \times 2^{n-1}$

If we now concatenate 1 to the end we get, $(a_1\ a_2\ \ldots a_n\ 1)_2$

$= 1 \times 2^0 + a_n \times 2^1 + a_{n-1} \times 2^2 + \ldots.a_1 \ .\ 2^n$

$= 1 + 2(a_n \times 2^0 + a_{n-1} \times 2^1 + \ldots..+ a_1 \times 2^{n-1})$

$= 1 + 2((a_1\ a_2 \ldots\ldots.a_n)_2)$

$\therefore$ The new number is 1 more than double the original number. Choice (D)

18. The given number is $(1161)_8$

$(1161)_8 = 1 \times 8^0 + 6 \times 8^1 + 1 \times 8^2 + 1 \times 8^3$

$= 1 + 48 + 64 + 512 = (625)_{10}$

The square root of $(625)_{10}$ is $(25)_{10}$

$(25)_{10} = (31)_8$

$\therefore \sqrt{(1161)_8} = (31)_8$ Choice (D)

19. $(234)_6 =$

$2 \times 6^2 + 3 \times 6 + 4 \times 1 = 72 + 18 + 4 = (94)_{10}$

$((94)_{10})^2 = 94 \times 94 = (8836)_{10}$

$(8836)_{10}$ in base 6 form:

$$\begin{array}{r|l} 6 & 8836 \\ 6 & 1472 - 4 \\ 6 & 245 - 2 \\ 6 & 40 - 5 \\ 6 & 6 - 4 \\ & 1 - 0 \end{array}$$

$\therefore (8836)_{10} = (104524)_6$ Choice (C)

20. (1000111) = 71

$(101)_2 = 5$

The remainder when 71 is divided by 5 is 1.

Hence, the remainder is 1 when 1000111 is divided by 101 Choice (B)

21. $(120)_8 = 1 \times 8^2 + 2 \times 8^1 = 64 + 16 = (80)_{10}$

$(24)_8 = 2 \text{ x } 8^1 + 4 \times 8^0 = (20)_{10}$

L.C.M. of $(80)_{10}$ and $(20)_{10}$ is $(80)_{10} = (120)_8$ Choice (D)

22. The L.C.M. of 2, 3, 4 and 5 is 60. The number 60 – 1 = 59 leaves a remainder of 1, 2, 3, 4 and 5 respectively. Hence, 1, 2, 3, 4 would be the last digits in the bases 2, 3, 4 and 5. Ans: (59)

23. To find 3-digit numbers of this form we look at the numbers of the form (60n – 1) between 100 and 1000. These are 15 in number. The numbers being (119, 179, ….. 959). Ans: (15)

24. If we look at the choices,

choice (A):

$6 \times 8^0 + 1 \times 8^1 + 0 \times 8^2 + 5 \times 8^3 = 2560 + 8 + 6$

= 2574, which is not a perfect cube.

choice (B): $6 \times 9^0 + 1 \times 9^1 + 0 \times 9^2 + 5 \times 9^3 = 3660$, which is not a perfect cube

choice (C): $6 \times 11^0 + 1 \times 11^1 + 0 \times 11^2 + 5 \times 11^3$

= 6672, which is not a perfect cube

choice (D): $6 \times 7^0 + 1 \times 7^1 + 0 \times 7^2 + 5 \times 7^3 = 1728$

as, $1728 = (12)^3$, 5016 is a perfect cube in base 7. Choice (D)

25. A.M of $(12)_6$ and $(33)_7$ is $(10)_n$

$(12)_6 = 1 \times 6^1 \times 2 \times 6^0 = (8)_{10}$

$(33)_7 = 3 \times 7^1 + 3 \times 7^0 = 21 + 3 = (24)_{10}$

A.M of $(8)_{10}$ and $(24)_{10} = (16)_{10}$

$(16)_{10} = (10)_n$

here n = 16 Ans: (16)

Exercise – 3(b)

Solutions for questions 1 to 30:

1.

$$\begin{array}{r|l} 2 & 108 \\ 2 & 54 - 0 \\ 2 & 27 - 0 \\ 2 & 13 - 1 \\ 2 & 6 - 1 \\ 2 & 3 - 0 \\ & 1 - 1 \end{array}$$

$(108)_{10} = (1101100)_2$ Choice (C)

2.

$$\begin{array}{r|l} 8 & 567 \\ 8 & 70 - 7 \\ 8 & 8 - 6 \\ & 1 - 0 \end{array}$$

$(567)_{10} = (1067)_8$ Choice (D)

3.

$$\begin{array}{r|l} 12 & 1896 \\ 12 & 158 - 0 \\ 12 & 13 - 2 \\ & 1 - 1 \end{array}$$

$(1896)_{10} = (1120)_{12}$ Choice (B)

4.

$$\begin{array}{r|l} 16 & 894 \\ 16 & 55 - E \\ & 3 - 7 \end{array}$$

$(894)_{10} = (37E)_{16}$ Choice (A)

5. $(7640)_8 = 7 \times 8^3 + 6 \times 8^2 + 4 \times 8^1 + 0 \times 8^0$

$= 3584 + 384 + 32$

$= (4000)_{10}$

$$\begin{array}{r|l} 16 & 4000 \\ 16 & 250 - 0 \\ 16 & 15 - A \\ & 0 - E \end{array}$$

$\therefore (7640)_8 = (EA0)_{16}$ Choice (D)

6. Given $(10101101011)_2$
$= (010101101011)_2$
$= [(010)_2\ (101)_2\ (101)_2\ (011)_2]$
$= (2553)_8$ Choice (B)

7. $(ABC)_{16}$
$= C \times 16^0 + B \times 16 + A \times 16^2$
$= 12 \times 1 + 11 \times 16 + 10 \times 256$
$= 12 + 176 + 2560 = (2748)_{10}$ Ans: (2748)

8. Given $(6555)_x - (777)_x = (5556)_x$
$\Rightarrow (6555)_x = (5556)_x + (777)_x$
Consider unit digit we know 6 + 7 = 13 but we have 5 in unit digit, k, 13 – 8 = 5, $\therefore$ x should be 8.
$\therefore (5666)_8 + (457)_8 = (6345)_8$ Choice (D)

9. $(423)_9 = 4 \times 9^2 + 2 \times 9 + 3 \times 9^0$
$= 324 + 18 + 3 = 345$
$(423)_6 = 4 \times 6^2 + 2 \times 6^1 + 3 \times 6^0$
$= 144 + 12 + 3 = 159$
$(423)_9 - (423)_6 = 345 - 159$
$= 186 = (136)_{12}$ Choice (C)

10. The smallest three-digit number in the base 12 system is 100. The largest three digit number in the base 12 system is BBB.
$(100)_{12} = 12^2 = 144.$
$(BBB) = 12^3 - 1 = 1727.$
The number of three digit numbers
$= (1727 - 144) + 1 = 1584$ Ans: (1584)

11. The smallest three-digit number in base 5 system is 100.
The largest three-digit number in base 5 system is (444)
$(100)_5 = 5^2 = 25$
$(444)_5 = 5^3 - 1 = 125 = 1 = 124$
The number of three-digit numbers in the base 5 system is 124 – 25 = 99
Out of these 99 numbers, 24 numbers are also three-digit numbers in the base 10 system.
$\therefore$ The number of three-digit numbers which are actually two-digit numbers in the base 10 system is 99 – 24 = 75
Ans: (75)

12. $(111)_2 + (222)_3 + - - - - - + (666)_7$
$= (111)_2 = 2^3 - 1,\ (222)_3 = 3^3 - 1, \ldots\ldots\ldots$
$(666)_7 = 7^3 - 1$
$\therefore$ The given sum $= 2^3 - 1 + 3^3 - 1 + 4^3 - 1 + - - - - + 7^3 - 1$
$1^3 + 2^3 + 3^3 + - - - - 7^3 - 1 - 1 - 1 + - - - - - 1$ for 7 times
$\left(\dfrac{7(7+1)}{2}\right)^2 - 7 = 784 - 7 = 777 = (777)_{10}$ Choice (C)

13. $(13)_5 = 3 + 5 = 8$
$(13)_8 = 3 + 8 = 11$
$(13)_{12} = 3 + 12 = 15$
$f\ ((13)_5, (13)_8, (13)_{12})$
$= f\ (8, 11, 15) = 8^2 + 11^2 + 15^2$
$= 64 + 121 + 225 = 410$ Choice (D)

14. $(310)_{16} = 0 + 1 \times 16 + 3 \times 16^2$
$= 0 + 16 + 768 = 784$
The square root of 784 is 28.

$$\begin{array}{r|l} 16 & 28 \\ \hline & 1-12 \end{array}$$

$28 = (1C)_{16}$
$\therefore$ The square root of $(310)_{16} = (1C)_{16}$ Choice (C)

15. $(43)_8 = 3 + 4 \times 8 = 35$
The square of 35 = 1225

$$\begin{array}{r|l} 8 & 1225 \\ \hline 8 & 153-1 \\ \hline 8 & 19-1 \\ \hline & 2-3 \end{array}$$

$\therefore$ The square of $(43)_8 = (2311)_8$ Choice (A)

16. $(23232)_4$ in decimal system is 750. $(232)_4$ in decimal system is 46.
$\therefore$ The remainder, when 750 is divided with 46, is 14.
$(14)_{10} = (32)_4$ Choice (B)

17. $(210)_6 = 0 + 6 \times 1 + 6^2 \times 2$
$= 6 + 72 = 78$
$(30)_6 = 0 + 3 \times 6 = 18$
$\therefore$ The L. C. M of 18, 78 is 234.
$(234)_{10} = (1030)_6$ Choice (C)

18. $(11)_7 = 1 + 7 = 8$
$(55)_7 = 5 + 35 = 40$
$(404)_7 = 4 + 0 \times 7 + 4 \times 7^2 = 200$
$\therefore$ 8, 40 and 200 are in G. P Choice (B)

19. Given, $(24)_6 = 4 + 2 \times 6 = 16$
$(34)_7 = 4 + 3 \times 7 = 25$
The geometric mean of 16 and 25 is $\sqrt{16 \times 25} = 20$
Given, geometric mean $= (24)_n$
$(20)_{10} = (24)_n$
$20 = 4 + 2n \Rightarrow 2n = 16 \Rightarrow n = 8$ Ans: (8)

20. From options $(2454)_{11} = 3205$; $(2454)_{12} = 4096$; $(2454)_9 = 1831$, $(2454)_6 = 610$ and $(2454)_7 = 921$
$\therefore$ 4096 is a perfect cube Choice (B)

21.

$$\begin{array}{r|l} 2 & 456 \\ \hline 2 & 228-0 \\ \hline 2 & 114-0 \\ \hline 2 & 57-0 \\ \hline 2 & 28-1 \\ \hline 2 & 14-0 \\ \hline 2 & 7-0 \\ \hline 2 & 3-1 \\ \hline & 1-1 \end{array}$$

$(456)_{10} = (11001000)_2$
$\therefore$ 4 weights can be used. Ans: (4)

22. The minimum weight is 8 kg Ans: (8)

23. $512 = 2^9$
$\therefore$ The minimum number of bits required is 10
Choice (A)

24. L.C. M of 2, 3, 4, 5 and 6 is 60
$\therefore$ The required number is
L.C.M (2, 3, 4, 5, 6) – 1 = 60 – 1 = 59. Ans: (59)

25. The smallest three digit number is 60 + 59 = 119.
All these numbers differ by 60.
$t_n = a + (n - 1)\ d$
$119 + (n - 1)\ 60 < 1000$
$60n < 1000 - 59$
$60n < 941$
$n < \dfrac{941}{60}$
$n < 15.6$
$\therefore$ The value of n is 15.
$\therefore$ The number of three digit numbers = 15 Ans: (15)

26. $0.375 \times 2 = 0.750 \rightarrow 0$
$0.75 \times 2 = 1.50 \rightarrow 1$
$0.5 \times 2 = 1 \rightarrow 1$
$\therefore (0.375)_{10} = (0.011)_2$ Choice (D)

27. $(13.34375)_{10}$
Consider the integer part, i.e. 13.

$$\begin{array}{r|l} 2 & 13 \\ \hline 2 & 6-1 \\ \hline 2 & 3-0 \\ \hline & 1-1 \end{array}$$

$(13)_{10} = (1101)_2$

Consider the fractional part
$0.34375 \times 2 = 0.68750 \to 0$
$0.68750 \times 2 = 1.3750 \to 1$
$1.3750 \times 2 = 1.75000 \to 0$
$0.75 \times 2 = 1.5 \to 1$
$0.5 \times 2 = 1.0 \to 1$
$(13.34375)_{10} = (1101.01011)_2$ Choice (A)

28. Let the two digit number be (xy)
$(xy)_7 = y + 7x$
$(yx)_7 = x + 7y$
Given $(xy)_7 = 3\,(yx)_7$
$y + 7x = 3\,(x + 7y)$
$7x - 3x = 21y - y$
$4x = 20y$
$x = 5y$
$\therefore$ y = 1 and x = 5
$\therefore$ The number is $(51)_7$
$= 1 + 35 = (36)_{10}$ Choice (B)

29. $(11)_2 + (11)_3 + (11)_4 + \text{- - - -}\ (11)_n$
$3 + 4 + 5 + \text{- - -} + n + 1$
$= 1 + 2 + 3 + \text{- - -} + n + 1 - 3$
$= \dfrac{(n+1)(n+2)}{2} - 3$
$= \dfrac{n^2+3n+2-6}{2} = \dfrac{n^2+3n-4}{2}$ Choice (D)

30. Let the number be $(ab)_n = b + na$
$\therefore (ab0)_n = 0 + nb + n^2a = n\,(b + na)$
$\therefore$ n times the original number Choice (C)

Chapter – 4
(Geometry)

Concept Review Questions

Solutions for questions 1 to 45:

1. The given triangle is isosceles
The area of an isosceles triangle whose base is b cm and which has each of its equal sides as a cm = $\dfrac{b\sqrt{4a^2-b^2}}{4}$

$\therefore$ Area of the triangle = $\dfrac{12\sqrt{4(10)^2-12^2}}{4}$ = 48 sq cm
Ans: (48)

2. $15^2 + 20^2 = 25^2$
$\therefore$ The triangle is right angled at the point of the intersection of the sides 15 cm and 20 cm.
$\therefore$ required distance = 15 + 0 + 20 = 35 cm. Choice (D)

3. The given triangle is isosceles. In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre all lie on the median to the base.
$\therefore$ The area of the required quadrilateral is 0. Choice (A)

4. In an equilateral triangle, the centroid and the orthocentre coincide and the centroid divides each median in the ratio 2 : 1 or 1 : 2.
As x > y, x : y = 2 : 1 Choice (C)

5. In an equilateral triangle, the incentre, the centroid and the circumcentre coincide.
$\therefore$ the area of the required triangle is 0. Choice (D)

6. **(i)** Inradius of an equilateral triangle is $\dfrac{1}{2\sqrt{3}}$ (side of the triangle)
$\therefore$ inradius = $\dfrac{1}{2\sqrt{3}}(9) = \dfrac{3\sqrt{3}}{2}$ cm. Choice (B)

(ii) Circumradius of an equilateral triangle is $\dfrac{1}{\sqrt{3}}$ (side of the triangle)
$\therefore$ Circumradius = $3\sqrt{3}$. Choice (A)

7. As I is the incentre,
$\angle BIC = 90 + \dfrac{\angle A}{2} = 130^\circ$ Ans: (130)

8. **(i)** In the given triangle, the square of each side is less than the sum of the squares of the other 2 sides. Such a triangle is acute angled.
$\therefore$ its circumcentre lies inside the triangle.
Choice (A)

(ii) In the given triangle, the square of its greatest side exceeds the sum of the squares of the other 2 sides. Such a triangle is obtuse angled.
$\therefore$ its orthocenter lies outside the triangle.
Choice (B)

9. Incentre Choice (B)

10. As XY || QR,
$\dfrac{PX}{XQ} = \dfrac{PY}{YR}$ (Basic proportionality theorem)
$\dfrac{4}{6} = \dfrac{PY}{8}$; $PY = \dfrac{16}{3}$ cm Ans: (16)

11. Since triangle PQR is right angled at Q,
$QS = \sqrt{(PS)(SR)}$ = 8 cm Ans: (8)

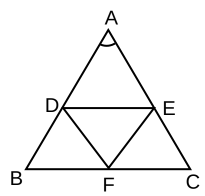
12. The medians in the triangle divide it into 6 triangles of equal area. $\therefore$ Area of Δ BGF = $\dfrac{1}{6}(18) = 3$ sq cm Choice (B)

13. As AD bisects $\angle BAC$
$\dfrac{BD}{DC} = \dfrac{AB}{AC} \Rightarrow \therefore BD = \dfrac{8}{10}(4) = 3.2$ cm Choice (D)

14. $\angle B = 2\angle C$
$\angle C = 180 - 130 = 50^\circ$; $\therefore \angle B = 100^\circ$
$\angle A = 180 - (\angle B + \angle C) = 30^\circ$ Choice (C)

15. AD being the angular bisector,
AB : AC = BD : CD $\Rightarrow \dfrac{AB}{AC} = \dfrac{BD}{CD}$
$\therefore CD = \dfrac{AC.\,BD}{AB} = \dfrac{3 \times 1.5}{2} = 2.25$ cm Ans: (2.25)

16. $\dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{1}{2} = \dfrac{AE}{AC}$
As AC = 12,
$\therefore AE = \dfrac{1}{2}(AC)$ = 6 cm

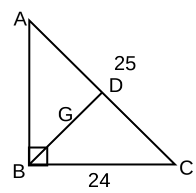


Choice (B)

17. AB is a right angled Triangle
$\therefore$ BD = 12.5 cm

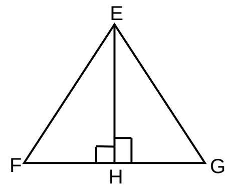
$GD = \dfrac{1}{3}(BD) = \dfrac{12.5}{3}$

= 4.17 cm



Choice (B)

18. Since they have the same base,
$b_1 = b_2$
$h_1 : h_2 = 3 : 5$
$\therefore$ Areas are in the ratio = $\frac{1}{2} b_1 h_1 : \frac{1}{2} b_2 h_2$
$= b_1 h_1 = b_2 h_2 \Rightarrow h_1 : h_2 \; (\because b_1 = b_2)$
$\therefore$ Ratio = $h_1 : h_2 = 3 : 5$ Choice (C)

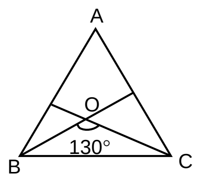
19.


From the figure
$EH^2 = EF^2 - FH^2$
Also $EH^2 = EG^2 - GH^2$
$\therefore EF^2 - FH^2 = EG^2 - GH^2$ Choice (C)

20. $\angle P + \angle Q + \angle R = 180°$
$\angle R = 180° - (\angle P + \angle Q) = 180° - (90° + 30°) = 60°$
In a right angles triangle, the ratio of the sides opposite to the angels 90°, 60° and 30° are in the ratio = $2 : \sqrt{3} : 1$
In Δ PQR, QR, PQ and PR are the sides opposite to the 90°, 60° and 30° angles respectively.
$\therefore PQ : QR : PR = \sqrt{3} : 2 : 1$ Choice (B)

21. As BD and CE are angular bisectors of $\angle B$ & $\angle C$
$\angle OBC = \angle OCB = 22.5°$ ($\angle B = 45°$, $\angle B = \angle C$)
$\angle COD = \angle OBC + \angle OCB = 22.5 + 22.5 = 45°$.
Ans: (45)

22. $\angle BOC = 130°$
But $\angle BOC = 90° + \frac{1}{2}\angle A$,
$\therefore 130° = 90° + \frac{1}{2}\angle A$,
$\angle A = 80°$



Choice (B)

23. The centroid of a triangle is the point of concurrence of its medians. Choice (D)

24. Area of parallelogram = 2 (Area of triangle ABC)
$= 48 \text{ cm}^2$ Ans: (48)

25. $EF = \frac{2}{1+2}(12) + \frac{1}{1+2}(24) = 16$ cm Choice (B)

26. A quadrilateral which is formed by joining the midpoints of another quadrilateral must be a parallelogram whose area is half that of the original quadrilateral.
$\therefore$ the quadrilateral formed must be a parallelogram of area 40 sq cm. Choice (C)

27. Sum of its interior angles = $180(n - 2)$
Sum of its exterior angles = 360^o
$180(n - 2) \le 360$
$n \le 4$; $\therefore$ n = 3 or 4 Ans: (2)

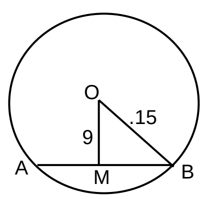
28. Suppose the polygon has n sides
Given, $\frac{n(n-3)}{2} = 3n \Rightarrow n(n - 9) = 0$
As $n \ne 0$, n = 9 Ans: (9)

29. Suppose the polygon has n sides
Each of its interior angles = $\frac{180(n-2)}{n}$
Each of its exterior angles = $\frac{360}{n}$
$\left[\frac{180(n-2)}{n}\right] = 2\left(\frac{360}{n}\right)$
$180\,n(n - 6) = 0$; As $n \ne 0$, n = 6
$\therefore$ The polygon is a hexagon.
Choice (B)

30. Exterior angle of a polygon = $\frac{360°}{n} \Rightarrow E = \frac{180°}{8} = 45°$
Also as I + E = 180° $\Rightarrow$ I = 180 – 45 = 135°.
Choice (C)

31. Let the angle be θ
Given $180° - \theta = 3(90° - \theta°)$
$180° - \theta° = 270° - 3\theta°$
$2\theta° = 90°$
$\theta° = 45°$ Choice (A)

32. Let the number of sides of the polygon be N.
Number of diagonals it has = $\frac{N(N-3)}{2} = 20$
$N^2 - 3N - 40 = 0$
$(N - 8)(N + 5) = 0$
$N > 0$; $\therefore$ N = 8
Each exterior angle = $\left(\frac{360}{8}\right)° = 45°$
Choice (C)

33.


Let AB be the chord with midpoint M.
$AB = 2MB = 2\sqrt{OB^2 - OM^2}$
$= 2\sqrt{15^2 - 9^2} = 24$ cm Ans: (24)

34. $\angle POQ = 2\angle PRQ$
$\therefore \angle PRQ = 50°$ Choice (A)

35. The maximum number of tangents that can be drawn is 3 (2 direct, 1 transverse). Ans: (3)

36. The maximum number of tangents that can drawn is 4(2 direct, 2 transverse). Ans: (4)

37. Only an isosceles trapezium is a cyclic quadrilateral among the first 3 choices. Choice (C)

38. As $\angle BAC$ and $\angle BDC$ are in the same segment,
$\angle BAC = \angle BDC = 30^o$
In Δ BAC
$\angle BAC + \angle ABC + \angle ACB = 180^o$
$\angle ABC = 180^o - (30^o + 60^o) = 90^o$. Choice (C)

39. As AOBC is a quadrilateral,
$\angle AOB + \angle OAC + \angle OBC + \angle ACB = 360^o$
As AC and BC are tangents,
$\angle OAC = \angle OBC = 90^o$
$\therefore \angle AOB = 360^o - (50^o + 2(90^o)) = 130^o$.
Choice (D)

40. As ABCD is a cyclic quadrilateral,
$\angle ABC + \angle ADC = 180^o$
$\angle ADC = 180^o - 85^o = 95^o$ Choice (D)

41. PQR forms a semi circle
$\therefore \angle PRQ = 90^o$
$\therefore PR^2 + RQ^2 = PQ^2$
$PR = \sqrt{30^2 - 18^2} = 24$ cm Ans: (24)

42. As AB and CD are parallel
$\angle HIC = \angle GAH = 130^o$
As GH and JL are parallel,
$\angle HIC = \angle JKL = 130^o$
As CD and EF are parallel,
$\angle JKT = \angle JLE = 130^o$ Choice (B)

43. $l \,//\, m \Rightarrow \angle 6 = \angle 3$
$\therefore \angle 1 + \angle 3 = 120°$
$\angle 1 = \angle 3$
$\therefore \angle 1 = \angle 3 = 60°$
$\angle 4 = 180° - \angle 3 = 120°$ Ans: (120)

44. Let AD be the median drawn from A to BC
By Apollonius theorem
$2(AD^2 + BD^2) = AB^2 + AC^2$
$AD^2 + \left(\frac{BC}{2}\right)^2 = \frac{1}{2}(AB^2 + AC^2)$
$AD = \sqrt{\frac{1}{2}(12^2+16^2) - \left(\frac{8}{2}\right)^2} = 2\sqrt{46}$ cm.
Choice (D)

45. (PQ) (PR) = (PS) (PT)
(4) (18) = 3(3 + ST)
ST = 21 cm Ans: (21)

Exercise – 4(a)

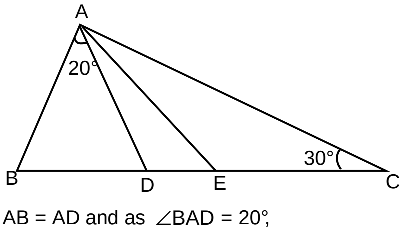
Solutions for questions 1 to 40:

1. Given $\angle 8 = 2\angle 1$
$\angle 8 + \angle 1 = 180°$
$\Rightarrow 2\angle 1 + \angle 1 = 180°$
$\Rightarrow \angle 1 = 60°$ and $\angle 8 = 120°$
$\Rightarrow \angle 8$ and $\angle 4$ are corresponding angles,
$\angle 8 = \angle 4 = 120°$ Ans: (120)

2. As PQ || RS, QAB + ABS = 180°
As AD and $\angle$BD bisect $\angle$QAB and $\angle$SBA,
$2\angle BAD + 2\angle DBA = 180°$
$\angle BAD + \angle DBA = 90°$
Hence the other angle $\angle ADB = 90°$
$AB^2 = AD^2 + BD^2 = 16^2 + 12^2 = 400$
$\Rightarrow$ AB = 20 cm. Ans: (20)

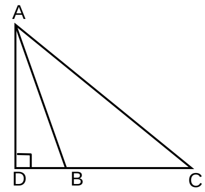
3. $\angle ECD = \angle BCD - 30°$
$\angle BCD = \angle ABC = 80°$ (alternative angles)
Hence $\angle ECD = 80° - 30° = 50°$
As $\angle ECD + \angle CEF = 50° + 130° = 180°$, EF and CD are parallel.
As AB || CD and EF || CD, it follows EF is also parallel to AB.
Hence $\angle BHF + \angle EFH = 180°$
$\angle EFH = 180° - 40° = 140°$.
Note: $\angle BCI = 100°$ is redundant. Choice (A)

4.



As AB = AD and as $\angle BAD = 20°$,
$\angle ADB = \angle ABD = 80°$
As AE = EC, $\angle EAC = \angle ECA = 30°$
$\angle DAE = 180° - [(80° + 20°) + (30° + 30°)] = 20°$
Choice (B)

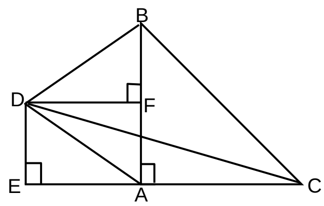
5.



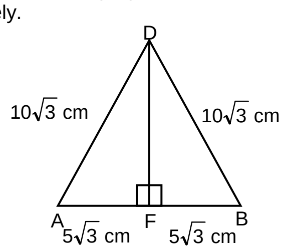
Since D divides BC in the ratio 1 : 3 externally, if DB = x, BC = 2x.
$AB^2 - DB^2 = AD^2$
$= AC^2 - DC^2$ $10^2 - x^2 = 20^2 - (3x)^2$
$100 - x^2 = 400 - 9x^2$
$8x^2 = 300 \Rightarrow x = \sqrt{\frac{300}{8}} = \frac{10\sqrt{3}}{2\sqrt{2}}$
$BC = 2x = \frac{10\sqrt{3}}{\sqrt{2}} = 5\sqrt{6}$ cm.
Choice (D)

6. $a(a + b + c) = d^2$ ——— (1)
$b(a + b + c) = e^2$ ——— (2)
$c(a + b + c) = f^2$ ——— (3)
Adding (1) and (2) and subtracting (3),
$a(a + b + c) + b(a + b + c) - c(a + b + c)$
$= (a + b - c)(a + b + c) = d^2 + e^2 - f^2$
As $a + b > c$, $d^2 + e^2 > f^2$
Similarly $e^2 + f^2 > d^2$ and $d^2 + f^2 > e^2$
$\therefore$ triangle DEF is acute angled.
Choice (A)

7.



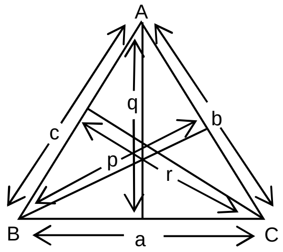
Let DF and DE be perpendicular from D to AC and AB respectively.



Height of $\Delta ABD = DF = \sqrt{AD^2 - AF^2}$
$= \sqrt{(10\sqrt{3})^2 - (5\sqrt{3})^2} = \sqrt{300 - 75} = 15$cm
DF = EA.
$(DC^2 = DE^2 + EC^2)$ and DE = AF
$\Rightarrow DC = \sqrt{(5\sqrt{3})^2 + (15+10)^2}$
$= \sqrt{75 + 625} = 10\sqrt{7}$ cm

Note: The vertex D of the equilateral triangle can be shown on the side of C. But, then, CD becomes minimum. Hence, D is shown as in diagram above.
Choice (A)

8.



If a, b and c are the sides of the triangle and p, q and r are the lengths of the medians as shown, then, by Apollonius theorem,

$2\left(p^2+\left(\frac{b}{2}\right)^2\right) = a^2 + c^2$ ——— (1)

$2\left(q^2+\left(\frac{a}{2}\right)^2\right) = b^2 + {}^2$ ——— (2)

$2\left(r^2+\left(\frac{c}{2}\right)^2\right) = b^2 + a^2$ ——— (3)

(1) + (2) + (3) =

$2(p^2 + q^2 + r^2) + \frac{2b^2}{4}+\frac{2a^2}{4}+\frac{2c^2}{4} = 2a^2 + 2b^2 + 2c^2$

$2(p^2 + q^2 + r^2) = \frac{3}{2}(a^2 + b^2 + c^2)$

Hence $p^2 + q^2 + r^2$

$= \frac{3}{4}(a^2 + b^2 + c^2) = \frac{3}{4}(72) = 54$ sq.cm.

Note: The relation : $3(a^2 + b^2 + c^2) = 4(p^2 + q^2 + r^2)$ can be remembered as a property of triangles.

Ans: (54)

9. $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} = \frac{1}{3}$

Area of triangle ADE = $\frac{1}{9}$ (Δ ABC area)

($\because$ ABC and ADE are similar triangles)

$= \frac{1}{9}(54) = 6$ sq.cm. Ans: (6)

10. $AB^2 = AC^2 - BC^2 = 10^2 - 6^2 = 64$

$\Rightarrow AB = 8$ cm

$FB = \frac{1}{2}AB = 4$ cm

As FB || DE, $\angle BFC = \angle CDE$ (corresponding angles)

$\angle ABC = \angle DEC = 90°$

$\angle DCE = \angle FCB$

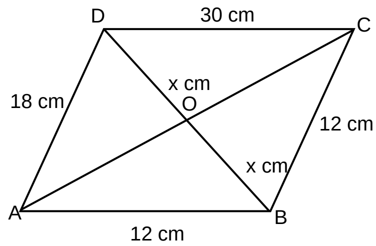
Hence Δ FBC and Δ DEC are similar.

$\frac{BF}{ED} = \frac{BC}{EC}$

$EC = \frac{BC}{BF}\times ED = \frac{6}{4}\times 20 = 30$ units

EB = EC – BC = 30 – 6 = 24 units. Choice (A)

11.



In ΔABC, BO is the median to AC; because the diagonals of a parallelogram bisect.

AO = OC = (1/2)AC = (1/2)(24) = 12 --------- (1)

Applying Apollonius theorem to ΔABC,

$AB^2 + BC^2 = 2(AO^2 + BO^2) \Rightarrow 30^2 + 18^2 = 2(12^2 + BO^2)$

$\Rightarrow 6^2(5^2 + 3^2) = 2 \times 6^2 \times 2^2 + 2BO^2$

$\Rightarrow 6^2(25 + 9 - 8) = 2BO^2 \Rightarrow (18)(26) = BO^2$;

$\Rightarrow BO = 6\sqrt{13}$ $BD = 2BO = 12\sqrt{13}$ Choice (D)

12. As ABCD is a parallelogram, AB || CD. $\Rightarrow$ ABF || CD

Hence $\angle EFB = \angle EDC$

$\angle EBF = \angle DCE$, $\angle BEF = \angle DEC$

(vertically opposite angles)

Hence ΔFEB is similar to ΔDEC

Thus $\frac{EB}{EC} = \frac{FE}{ED} = \frac{FB}{CD}$

As E is the midpoint of BC, EB = EC

Hence $\frac{FB}{CD} = \frac{EC}{EC} = 1 \Rightarrow FB = CD = AB$

AF = AB + BF = 2AB.

Alternate method:

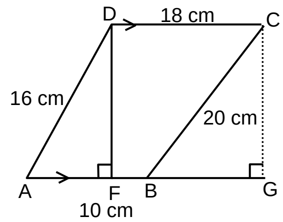
In ΔEFB, DC is parallel to FB.

Hence, $\frac{BE}{EC} = \frac{FE}{ED}$.

As E is the midpoint of BC, BE = EC, hence, FE = ED; i.e., E is the midpoint of FD. In ΔFDA, E is midpoint of FD and EB || DA..

Hence, B is midpoint of AF; FA = 2.AB Choice (B)

13.



For the given set of measurements of the sides, the diagram will be as shown above.

Let DF and CG be perpendiculars to AB, and let AF = x.

In ΔDAF, $DF^2 = 16^2 - x^2 = (256 - x^2)$ ------- (1)

DFGC is a rectangle, hence, CG = DF and FG = DC = 18

In ΔCBG, $CB^2 = CG^2 + GB^2 = DF^2 + (18 - FB)^2$

$\Rightarrow (20)^2 = (256 - x^2) + [18 - (10 - x)]^2$

$\Rightarrow 400 = 256 - x^2 + (8 + x)^2 = 256 - x^2 + 64 + x^2 + 16x$

$\Rightarrow 80 = 16x; \Rightarrow x = 5$ -------- (2)

In ΔACG, $AC^2 = AG^2 + (GC)^2 = (x + 18)^2 + (CG)^2$

$\Rightarrow AC^2 = (23)^2 + (256 - 25) = 529 + 231 = 760$

$AC = \sqrt{760}$ and this is the longer diagonal.

Choice (C)

14. $EF = \frac{2}{3+2}(10) + \frac{3}{3+2}(20) = 16$cm. Choice (C)

15. (AX) (XB) = (CX) (XD)

(8) (14 –8) = (XD+ 8) (XD)

Hence $XD^2 + 8XD - 48 = 0$

(XD + 12)(XD – 4) = 0

XD = 4 cm

CD = CX + XD = XD + 8 + XD = 16 cm. Ans: (16)

16. AB is the common radius of both circles

AC = AD = AB = BC = BD

As AC = BC = AB, $\angle CBA = \angle CAB = 60°$

As AD = BD = AB, $\angle DAB = \angle DBA = 60°$

$\therefore \angle CAD + \angle CBD = 240°$. Choice (C)

17. As AB is the diameter of the circle, $\angle AQB = 90°$
As $\angle PQB$ is 40°, $\angle AQP = \angle AQB - \angle PQB = 50°$
$\angle BXQ$ is the external angle of $\Delta\ \angle AQX$.
Hence, $\angle BXQ = \angle XAQ + \angle XQA = 30° + 50° = 80°$
Choice (A)

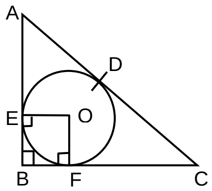
18. Let $\angle XZO$ be x°
As OX = ZO = radii,
$\angle OXZ = \angle OZX = x°$
$\angle XOZ = 180° - (\angle OXZ + \angle OZX) = 180° - 2x°$
$\angle XYZ = 1/2 \angle XOZ$ (Angle subtended at the centre of the circle is twice the angle subtended at the circumference)
Hence in quadrilateral OXYZ,
$20° + 40° + [360° - (180° - 2x°)] + [1/2\ (180° - 2x°)] = 360°$
$60° - 180° + 2x + 90° - x = 0 \Rightarrow x = 30.$

Alternate method:

Join O to Y. In ΔOXY, $\angle OXY = 20°$ (given data) and $\angle OXY = \angle OYX$.
Hence, $\angle XZY = \frac{1}{2} \angle XOY = \frac{1}{2}(140°) = 70°$ ---- (1)
(Angle in the segment is half the angle at the center).
Given that $\angle OZY = 40°$ --------- (2)
Subtracting (2) from (1),
$\angle XZY - \angle OZY = 70° - 40°$
$\Rightarrow \angle XZO = 30°$ Choice (A)

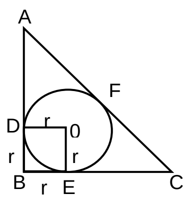
19. $\angle TSR = \angle TPQ = 50°$
(Angles in the same segment are equal)
$\angle PRX = 180° - (30° + 50°) = 100° = \angle STR + \angle TSR$
$\angle STR = 100° - 50° = 50°$
Choice (C)

20. Given that AD : DC = 3 : 2. Let E and F be the points of contact of the incircle with sides. AB and BC respectively.
AD = AE, (tangents to the circle are equal),
EB = BF, FC = CD
As AD : DC : 3 : 2, let AD : DC : BF = 3 : 2 : x



Hence, perimeter of ΔABC
= (3k + 2k) + (2k + xk) + (xk + 3k)
= 10k + 2kx = 2k (5 + x) and this is given equal to 36.
Hence, 2k (5 + x) = 36, $\Rightarrow$ k(5 + x) = 18 -------- (1)
By Pythagoras theorem, $AB^2 + BC^2 = AC^2$
$\Rightarrow (3k + xk)^2 + (xk + 2k)^2 = (3k + 2k)^2$
$\Rightarrow 9 + x^2 + 6x + x^2 + 4 + 4x = 25$
$\Rightarrow 2x^2 + 10x - 12 = 0$; $2(x + 6)(x - 1) = 0$
$\Rightarrow$ x = 1; while x = –6 is not acceptable ------ (2)
Substituting in (1), k (5 + 1) = 18,$\Rightarrow$ k = 3
Hence BF = BE = kx = 3 x 1 = 3 ------- (3)
As can be seen from the diagram, OEBF is a square; and the inradius = BF = BE
$\therefore$ Inradius = 3 cm
Ans: (3)

21. Let its inradius and circumradius be r cm and R cm respectively.
CF = CE and AF = AD (Tangents to the same circles from the enternal points A and C.)
AF + CF = AC = 2R (where R is the circumradius)
AB = r + AD, BC = r + CE

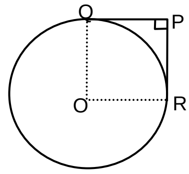


Perimeter of ΔABC = AB + BC + AC
= (r + AD) + (r + CE) + 2R
= 2r + 2R + (AD + CE) = 2r + 2R + (AF + CF)
= 2r + 2R + AC = 2r + 2R + 2R
$\therefore$ Perimeter of the triangle ABC = (2r + 4R)cm
2r + 4R = 24 $\Rightarrow$ r + 2R = 12 cm. Ans: (12)

22. Reflex angle $\angle TOR = 2\ \angle TSR = 240°$
$\angle TOR = 360° - 240° = 120°$
As PT and PR are tangents,
$\angle PTO = \angle PRO = 90°$
$\angle TPR = 360° - (90° + 90° + 120°) = 60°$. Choice (A)

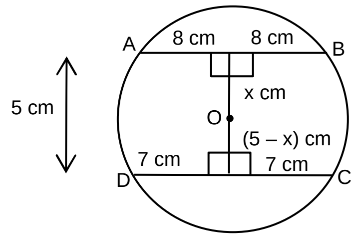
23. (XP) (XQ) = (XR) (XS)
$XS = \frac{(XP)(XQ)}{(XR)} = \frac{(6)(6+4)}{5} = 12$cm.
RS = XS – XR = 7 cm Ans: (7)

24. When the two tangents drawn from a point outside the circle are perpendicular, a square is formed by the outside point, the two points of contact and the center of the circle.
Hence, PQOR is a square.
It is given that PO = 20 cm



$\Rightarrow PQ = 20/\sqrt{2}$ cm.
In a square, the diagonals are equal; $\Rightarrow$ QR = OP = 20 cm
Hence, perimeter of $\Delta PQR = (20/\sqrt{2}) + (20/\sqrt{2}) + 20$
$= 20\,[(2/\sqrt{2}) + 1] = 20(\sqrt{2} + 1) = 20\sqrt{2} + 20$ sq. cm
Choice (D)

25.

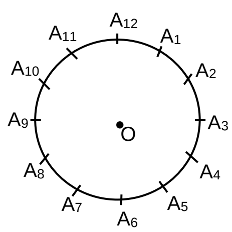


From the above diagram, $OA^2 = x^2 + 8^2$
$OD^2 = (5 - x)^2 + 7^2$.
As $OA^2 = OD^2$, we have $x^2 + 64 = x^2 - 10x + 25 + 49$
$\Rightarrow 10x = 10 \Rightarrow x = 1$
Radius of the circle = $OA = \sqrt{1^2 + 8^2} = \sqrt{65}$ cm.
Choice (D)

26. Length of the transverse common tangent is
$\sqrt{d^2 - (r_1 + r_2)^2} = \sqrt{13^2 - (8+4)^2} = \sqrt{169 - 144} = 5$ cm
Choice (C)

27. $\angle ABE < \angle AEB$ = (As AB > AE)
$\angle BAE = \angle ECD$ (Angles in the same segment are equal)
$\angle ABE = \angle EDC$ (Angles in the same segment are equal)
$\angle ABE = \angle ECD$ (AB || CD)
$\therefore \angle BAE = \angle ABE$
In ΔABE, $\angle BAE + \angle ABE + \angle AEB = 180$
$180° > 3\angle ABE$, $60° > \angle ABE$
$\therefore$ Maximum value of $\angle ABE = 59°$. Ans: (59)

28. In order that the circumcentre lies on one of the sides, the triangle must be right angled.
If A_6A_{12} is one of the sides, there can be 10 other vertices to form 10 distinct right-angled triangles. Similarly, if A_1A_7 is one of the sides, 10 other vertices can be taken to form 10 distinct right-angled triangles.



There are 6 distinct diameters that can be drawn with the given set of twelve points.
Thus the number of right angled triangles formed
$= 6 \times 10 = 60$. Ans: (60)

29. $VZ : ZW = PX : XQ = 3 : 8.$

$\therefore VZ = \frac{3}{11}$ VW and $PX = \frac{3}{11}$ PQ

Also VZ and PX are diameters.

$\therefore VZ = PX = \frac{3}{11}(4.4) = 1.2$

Radius = 2.2. $\therefore$ OZ = OV – VZ = 1. Similarly OX = 1

{$\therefore$ XOZY is a square (XOZY is a rectangle)}

XOZY is a rectangle. $\therefore OZ \perp RS$. $\therefore$ Z is the midpoint of RS. $\therefore RS = 2RZ = 2\sqrt{(2.2)^2 - 1^2} = 1.6\sqrt{6}$

Similarly $TU = 1.6\sqrt{6}$. $\therefore$ Sum of the lengths $= 3.2\sqrt{6}$.

Ans: (3.2)

30. Let the first two sides be 3x and 4x

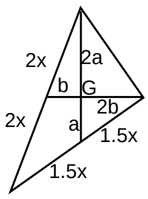
Let G be the point of intersection of the two medians.

In any triangle all the medians are concurrent.

$\therefore$ The third median of the triangle must also pass through G.

$\therefore$ G must be the centroid.

As G is the centroid, it divides each median in the ratio 2 : 1.



$b^2 + (2a)^2 = (2x)^2$ and $a^2 + (2b)^2 = (1.5x)^2$

Adding these $5(a^2 + b^2) = 6.25x^2$

$a^2 + b^2 = 1.25x^2$

Third side $= \sqrt{(2a)^2 + (2b)^2} = x\sqrt{5}$

$= 12\sqrt{5}$ (given) ; $\Rightarrow x = 12$

Smaller of the first two sides = 3x = 36. Choice (A)

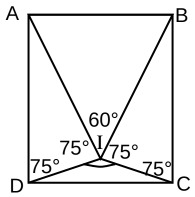
31. Triangle AIB is equilateral

$\therefore$ Each of its angles is 60°.

$\therefore \angle IAD = \angle IBC = 30°$

ABCD is a square and triangle AIB is equilateral.

$\therefore$ AD = AI and BI = BC



$\therefore \angle ADI = \angle AID = \frac{180° - 30°}{2}$

$= 75°$ and $\angle BIC = \angle BCI = 75°$.

$\angle IDC = \angle ICD = 15°$.

$\therefore \angle DIC = 150°$. Choice (C)

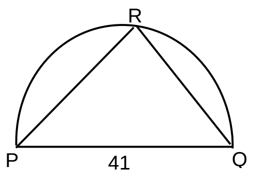
32. $\angle PRQ$ is an angle in a semicircle.

$\therefore$ It is a right angle.

$PR^2 + RQ^2 = PQ^2 = 41^2$

PR and RQ are integers.

$\therefore$ only possible (PR, RQ) values are (9, 40) and (40, 9).

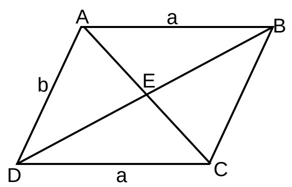


The inradius of any right angled triangle whose sides are a, b, c and c is the hypotenuse is $\frac{a + b - c}{2}$

$\therefore$ The inradius of triangle PQR is $\frac{9 + 40 - 41}{2} = 4$

Choice (A)

33.



Let AB = a and BC = b, let AB > BC

Perimeter of ABCD = 2(AB + BC) = 2(a + b)

2(a + b) = 120

a + b = 60.

Triangles BCE and CDE have CE in common.

Also BE = ED.

$\therefore$ Difference of the perimeters of the triangles is that of a and b. $\therefore$ a – b = 40

a + b = 60 and a – b = 40 $\therefore$ a = 50 and b = 10.

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$

$BD^2 = BC^2 + CD^2 - 2(BC)(CD)\cos\angle BCD$

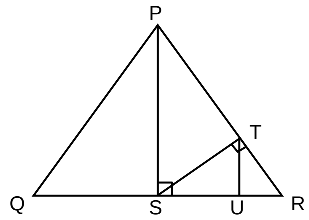
$\angle ABC = 180° - \angle BCD$

$\therefore \cos\angle ABC = -\cos\angle BCD$.

$AC^2 + BD^2 = 2(a^2 + b^2) - 2ab\cos\angle ABC + 2ab\cos\angle ABC$

$= 2(a^2 + b^2) = 2(50^2 + 10^2) = 5200.$ Choice (B)

34.



PQR is equilateral

$\angle R = 60°$, $\angle TSR = 30°$

STU and TUR are 30°– 60°– 90° triangles.

$\therefore$ Ratio of the sides of each is $1 : \sqrt{3} : 2$

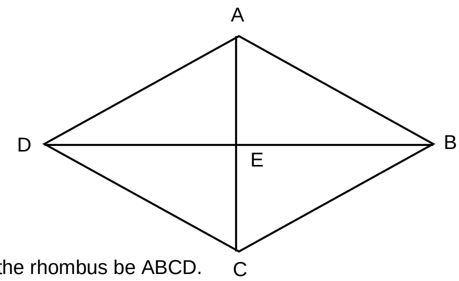
$TU = x$, $SU = \sqrt{3}x$ and $UR = \frac{x}{\sqrt{3}}$. $SR = \frac{4x}{\sqrt{3}}$

S is the midpoint of QR ($\because$ PQR is equilateral)

$QR = 2SR = \frac{8x}{\sqrt{3}}$

Perimeter of PQR = $3QR = \frac{24x}{\sqrt{3}}$ Choice (A)

35.



Let the rhombus be ABCD.

Let $\angle A = 120°$

Let E be the point of intersection of the diagonals

ΔDAE and ΔBAE are congruent (sss)

$\angle DAE = \angle BAE = \frac{\angle A}{2} = 60°$

DAE and BAE are both 30°– 60° – 90° triangles.

$DE = \sqrt{3}$ EA

BD = 2DE, AC = 2EA

$\therefore BD = \sqrt{3}$ AC.

Required ratio = $\sqrt{3} : 1$ Choice (D)

36. Let the sides be 4a, 4b, 4c where a, b, c are positive integers.

Let $4a \geq 4b \geq 4c$.

4a + 4b + 4c = 44

a + b + c = 11. Also $a \geq b \geq c$

If a = 6, a > b + c. This is not possible.

$a \leq 5$.

If a = 5, b + c = 6. (b, c) = (5, 1), (4, 2), (3, 3)

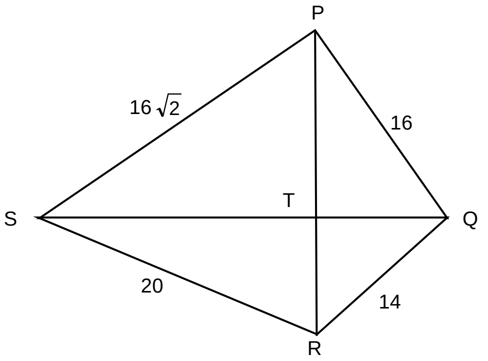
If a = 4, b +c = 7, (b, c) = (4, 3)

a cannot be less than 4 ($\because a + b + c \leq 3a$ i.e. $11 \leq 3a$

i.e. $a \geq 3\frac{2}{3}$)

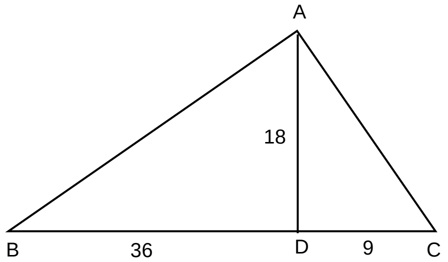
A total of 4 triangles can be formed. Ans: (4)

37.



Let us say the diagonals meet at T.
$PQ^2 = PT^2 + TQ^2$ and $QR^2 = QT^2 + TR^2$
$PQ^2 – QR^2 = PT^2 – QT^2$
$SP^2 – RS^2 = PT^2 – QT^2$
$PQ^2 – QR^2 = SP^2 – RS^2$
$16^2 – 12^2 = SP^2 – 20^2$
$SP = 16\sqrt{2}$. Ans: (16)

38.



$AB^2 = AD^2 + BD^2 = 1620 \Rightarrow AB = 18\sqrt{5}$
$AC^2 = AD^2 + DC^2 = 405 \Rightarrow AC = 9\sqrt{5}$
$AB^2 + AC^2 = BC^2$
$\therefore \angle A = 90°$.
Note: In ΔPQR, if $\angle Q = 90$ and QS is the altitude to PR, then $QS^2 = (PS)(SR)$. Conversely, if QS is the altitude to PR and $QS^2 = (PS)(SR)$, $\angle Q = 90°$ Ans: (90)

39. AB : BC : Altitude to AC = p : q : r.
Let AB = pk, BC = qk, Altitude to AC = rk.

Area of ABC = $\frac{1}{2}$(AB)(BC) = $\frac{1}{2}$ (AC)(Altitude to AC)

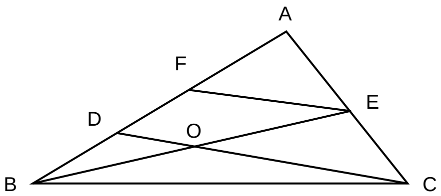
Altitude to AC = $\frac{(AB)(BC)}{AC}$

$$rk = \frac{(pk)(qk)}{\sqrt{(pk)^2 + (qk)^2}} = \frac{kpq}{\sqrt{p^2 + q^2}}$$

$$r = \frac{pq}{\sqrt{p^2 + q^2}} \Rightarrow r^2 = \frac{p^2q^2}{p^2 + q^2}$$

$$\frac{1}{r^2} = \frac{1}{p^2} + \frac{1}{q^2}$$ Choice (C)

40.



BE in the median, ∴ AE = EC
CD bisects BE. ∴ BO = OE.
Let F be the point on AB such that EF II CD.
BD = DF (∵ BO = OE)
DF = FA (∵ CE = EA)
∴ AD : DB = 2 : 1. Ans: (2)

Exercise – 4(b)

Solutions for questions 1 to 55:

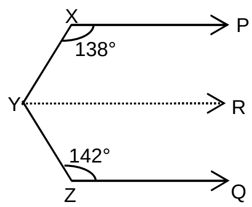
1. $\angle 5 + \angle 7 = 180$ ——— (1)
As per data $\angle 5 = \frac{7}{5}\angle 7$ ——— (2)
Hence from (1) and (2), $\angle 7 = 75$
$\angle 2 = \angle 6$ (corresponding angles)
$\angle 2 + \angle 6 = 2\angle 6 = 2\angle 7$ ($\angle 6$ and $\angle 7$ are vertically opposite angles) = 150°. Ans: (150)

2. $\angle TRS + \angle RTU = 180°$(TU || RS)
$\therefore \angle TRS = 60° \Rightarrow \angle QRS = \angle QRT + \angle TRS = 10° + 360° = 70°$
As the alternate angles $\angle PQR = \angle QRA$, PW || RS
As TU || RS and PW || RS, we have TU || PW
$\therefore \angle PWU + \angle WUT = 180°$
$\therefore \angle WUT = 130°$. Choice (C)

3. $\angle BEF + \angle EFD = 180°$(AB || CD)
$2\angle GEF + 2\angle GFE = 180°$
(EG bisects $\angle BEF$ and FG bisects $\angle EFD$)
$\angle GEF + \angle GFE = 90°$ ——— (1)
In Δ EFG, $\angle EGF + \angle GEF + \angle GFE = 180°$
From (1), $\angle EGF = 90°$
$\therefore FG^2 = EF^2 – EG^2$
$FG = \sqrt{25^2 - 15^2} = 20$ cm. Choice (D)

4. $\angle EAG + \angle ADF = 180°$(As BC ll DF)
$\angle EAG = 180° – 80° = 100°$
$\angle AGB = \angle EAG – \angle ABG = 100° – 30° = 70°$
Choice (A)

5.



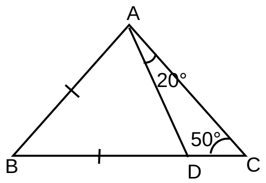
Draw YR parallel to XP and ZQ.
$\angle XYR = 180° – \angle YXP = 180° – 138° = 42°$
$\angle ZYR = 180° – 142° = 38°$
Hence $\angle XYZ = \angle XYR + \angle ZYR = 42° + 38° = 80°$
Ans: (80)

6. Let the angles of triangle be 2x, 3x and 5x.
2x + 3x + 5x = 180°
10x = 180° ⇒ x = 18°
2x = 36°, 3x = 54° and 5x = 90°.
As we have one of the angles is 90°, the triangle i s a right angled triangle, and is also scalene as the other two angles are not equal. Choice (C)

7. Let the perpendicular sides be a and b.
a and b are integers.
$a^2 + b^2 = 41^2$
Only possibilities of (a, b) are (40, 9) and (9, 40). In either case, a + b = 49.
∴ Perimeter = 90 Ans: (90)

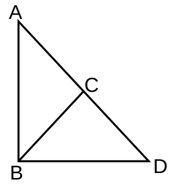
8. Let the angles be 3x°, 4x° and 5x°
3x°+ 4x°+5x°= 180°
x = 15
3x°= 45°, 4x°
= 60° and 5x°= 75°
∴ The triangle is acute angled whose largest angle exceeds 70°. Choice (B)

9.



$\angle BDA = \angle DAC + \angle DCA = 20° + 50° = 70°$
As AB = BD, $\angle BAD = \angle BDA = 70°$
$\angle ABD = 180° - (\angle BAD + \angle BDA)$
$= 180° - (70° + 70°) = 40° = \angle CBA$ Ans: (40)

10. Let $\angle ABC$ be x and $\angle CBD$ be y.
Since AC = BC,
$\angle ABC = \angle CAB = x$.
Since BC = CD
$\angle CBD = \angle CDB = y$
Hence, $\angle ABC + \angle CBD$
$= \angle CAB + \angle CBD$
$\Rightarrow \angle ABD = \angle CAB + \angle CDB$
As the sum of these angles is 180°
$\Rightarrow x + x + y + y = 180°$
$\Rightarrow x + y = 90°$ = the required sum.



Ans: (90)

11. $\angle ABC = \angle AMB + \angle MAB$ and $\angle ACB$
$= \angle ANC + \angle CAN$; as the exterior angle is equal to the sum of the two interior opposite angles.
But $\angle ABC + \angle ACB = p + q$
$= \angle AMB + \angle MAB + \angle ANC + \angle CAN = m° + x° + n° + z°$
$\Rightarrow x° + m° + z° = p° + q° - n°$ Choice (D)

12. $BC^2 = AC^2 - AB^2 = 5^2 - 3^2 = 16 \Rightarrow BC = 4$ units
Let AD be x units.
$AB^2 - AD^2 = BC^2 - DC^2$
$3^2 - x^2 = 4^2 - (5 - x)^2$
Solving the above equation we obtain x = 1.8 units.

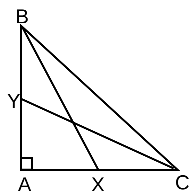
Alternate method:

As per the diagram, $\angle ABC = 90°$ and BD is perpendicular to the hypotenuse. Hence, $AB^2 = AD.\ AC$
$\Rightarrow 3^2 = 5.AD, \Rightarrow AD = 1.8$ Ans: (1.8)

13. Let AB = a cm, BC = b cm and AC = c cm
By Pythagoras theorem

$$\left(\frac{a}{2}\right)^2 + c^2 = CY^2 \quad \text{---------} (1)$$

$$\left(\frac{c}{2}\right)^2 + a^2 = BX^2 \quad \text{---------} (2)$$



On subtraction, $\frac{3a^2}{4} - \frac{3c^2}{4} = BX^2 - CY^2$

$= \frac{3}{4}\left(a^2 - c^2\right) = \frac{3}{4}\left(AB^2 - AC^2\right)$

Hence $\frac{AB^2 - AC^2}{BX^2 - CY^2} = \frac{AB^2 - AC^2}{\frac{3}{4}\left(AB^2 - AC^2\right)} = \frac{4}{3}$

Choice (A)

14. n = 6

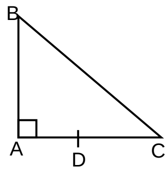
Interior angle = $\left(\frac{2n-4}{n}\right) \times 90 = \frac{8}{6} \times 90 = 120°$

Exterior angle = $\frac{360}{6} = 60°$

Choice (A)

15. Let the lengths of the hypotenuse, the shortest side and the third side be c cm, b cm and a cm
$c + b = 64 \Rightarrow b = 64 - c$
$c = 2(c - b) = 2(2c - 64)$
$c^2 = a^2 + b^2 = [4(c - 32)]^2 + [64 - c]^2$
$16c^2 - 1152c + 0480 = 0$
$c + 2 - 72c + 1280 = 0 \Rightarrow c = 40$ or 32.
But if c = 32, b = 32 i.e., b = c
This is not possible.
$\therefore$ c = 40. Ans: (40)

16.



$BC^2 - BD^2 = AB^2 + (2AD)^2 - (AB^2 + AD^2)$
$= AB^2 + 4AD^2 - AB^2 - AD^2 = 3AD^2$ Choice (C)

17. As triangles ABC and PQR are similar,

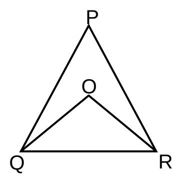
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR}\right)^2$$

$$\Rightarrow \frac{50}{200} = \frac{1}{4} = \left(\frac{40}{x}\right)^2$$

$$= \sqrt{\frac{1}{4}} = \frac{40}{x} \Rightarrow x = \frac{40}{1/2} = 80 \text{ cm.}$$ Choice (D)

18. Perimeter of ACP = AC + CP + PA = AC + a + a
where a = (1/2) of PQ or QR or RP.
AC = (1/2). QR, because A and C are the mid points of PQ and PR.
Hence, the perimeter of ΔACP = a + a + a = 3a
Perimeter of ΔPQR = (2a)3 = 6a.
Ratio = 3a : 6a = 1 : 2
Note: Side of ΔPQR is 8 cm. This information is redundant.
Choice (D)

19. $\angle QPR + \angle PQR + \angle PRQ$
= 180°——— (1)
$\angle QOR + \angle OQR + \angle ORQ$
= 180°



$\angle QOR + \frac{\angle PQR}{2} + \frac{\angle PRQ}{2}$
= 180°——— (2)
$\frac{\angle PQR + \angle PRQ}{2} = 180° - QOR$
$= 180° - 140° = 40°$
$= \left[\frac{1}{2}\left(180^0 - \left(\angle QPR\right)\right)\right]$ (from (1)
$\Rightarrow 180° - \angle QPR = 80° \Rightarrow \angle QPR = 100°$
Choice (A)

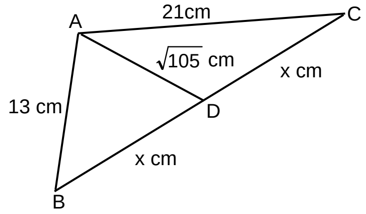
20. $\angle CAB = 80°$
$\therefore \angle ACB + \angle CBA = 100°$
$\therefore \angle DCB + \angle OBC = \frac{\angle ACB}{2} + \frac{\angle ABC}{2} = 50°$.
$\therefore \angle BOC = 180° - (\angle OCB + \angle OBC) = 130°$.
Ans: (130)

21. $\angle EAB = \angle EAD + \angle DAB = 150°$
In ΔEAB, EA = EB
$\therefore \angle AEB = \angle ABE = \frac{180° - \angle EAB}{2} = 15°$
$\angle BED = \angle AED - \angle AEB = 60° - 15° = 45°$
In ΔEFD, $\angle FED + \angle EFD + \angle FDE = 180°$
$45° + \angle EFD + 60° = 180$
$\angle EFD = 75°$ Choice (D)

22. As AD = BC, ABCD is an isosceles trapezium.
Any isosceles trapezium has the property of being cyclic.
$\therefore \angle BCD + \angle DAB = 180°$
$\angle BCD + 100° = 180°$
$\angle BCD = 80°$. Choice (A)

23. $\angle P + \angle Q = 140° + 40° = 180°$
Hence PL is parallel to QN.
$\angle LNM = 180° - \angle PLN = 180° - 130° = 50°$
As LM = LN, $\angle LMN = \angle LNM = 50°$
$\angle MLN = 180° - 2\angle LMN = 180° - (100°) = 80°$
Choice (B)

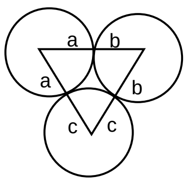
24.



By Appollonius theorem,
$AB^2 + AC^2 = 2(AD^2 + BD^2)$
$13^2 + 21^2 = 2(105 + x^2) \Rightarrow 10\sqrt{2} = x$
$\therefore$ The length of the other diagonal = $2x = 20\sqrt{2}$ cm.
Choice (D)

25. In ΔUTQ and ΔUSP, $\angle U$ is common
$\angle UTQ = \angle USP$ (RQ || SP)
$\therefore$ Triangles UTQ and USP are similar
$\therefore \dfrac{TQ}{PS} = \dfrac{UQ}{UP}$; $\therefore \dfrac{TQ}{RQ} = \dfrac{UQ}{UP} = \dfrac{1}{3}$ (as PS = PQ)
$\therefore UQ = \dfrac{1}{3}UP$ Ans: (3)

26. Let the radii of the three circles be a cm, b cm and c cm.



a + b = 21——— (1)
b + c = 22 ——— (2)
a + c = 23 ——— (3)
Hence 2 (a + b + c) = 66 $\Rightarrow$ a + b + c = 33
Hence c = 12, a = 11 and b = 10.
Thus the radius of the smallest circle is 10 cm.
Choice (A)

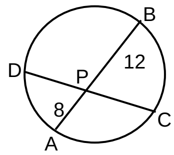
27. As triangle OAB is an equilateral triangle and AB = 4 cm,
OA = OB = (radii of circle) = 4cm
$\therefore$ Circumference of the circle = $2\pi r = 8\pi$ cm Ans: (8)

28. x = 36°, as the angle between the tangent and the chord through the point of contact is equal to the angle in the alternate segment.
$\angle BOP = 2\angle PAB = 2(36°) = 72°$
$y + z + (360° - \angle BOP) + 36° = 360°$
$\Rightarrow y + z = 36°$
Hence $x + y + z = 36° + 36° = 72°$ Choice (C)

29. $\angle BOC = 90 + \dfrac{1}{2}\angle BAC$
(0 is the incentre of triangle ABC)
$\angle BOC = 2\angle BAC$ (0 is the centre of the circle)
$90° + \dfrac{1}{2}\angle BAC = 2\angle BAC$
$60° = \angle BAC$
$\angle BOC = 120°$. Ans: (120)

30. $AD^2 = (AB)(AC) = (4)(4 + AD + 4)$
$AD^2 - 4AD - 32 = 0$
As AD > 0, AD = 8 Choice (A)

31. Given that AP = 8 cm, BP = 12 cm and CD = 22 cm
Let CP = x cm then DP = (22 – x) cm
According to the "chords' segments" theorem, AP x BP = CP x DP



$\Rightarrow (22 - x)x = 8 \times 12$
$\Rightarrow x^2 - 22x + 96 = 0 \Rightarrow x = 6$ or 16
So, the point P divides the chord CD into segments of 6 cm and 16 cm.
$\therefore$ Their difference = 16 – 6 = 10 cm Choice (D)

32. $\angle ABE + \angle AEB + \angle EAB = 180°$
$\angle EAB = \angle EDC$ (Angles in the same segment are equal)
$\angle EDC + 20° + 110° + \angle EDC = 180°$
$\angle EDC = 25°$. Choice (A)

33. $PQ^2 = (QB)(QA)$
$12^2 = (BQ)(QB + BA) = (BQ)(QB + 10)$
$QB^2 + 10QB - 144 = 0$
$(QB + 18)(QB - 8) = 0$ QB = 8 cm.
Choice (A)

34. The distance between the chords exceeds the radius of the circle
$\therefore$ Both chords cannot be on the same side of the centre.
Half of the length of the chord whose length is 144 cm. = 72 cm. Distance of the chord from the centre = $\sqrt{72^2 - 72^2} = 21$ cm
Distance of the other chord from the centre = 81 – 21 = 60 cm
$\dfrac{x}{2} = \sqrt{75^2 - 60^2} = 45$, x = 90 Ans: (90)

35. Exterior angle of the regular polygon = 180° – 162° = 18°
n(number of sides) = $\dfrac{360°}{\text{exterior angle}}$
$= \dfrac{360°}{18°} = 20$ Ans: (20)

36. Sum of the interior angles of the polygon = (2n – 4)90° = [2 (20) – 4] 90° = 3240° Ans: (3240)

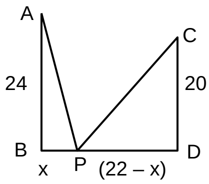
37. Let the angle be x°, then the supplement = (180° – x°)
$x° = \dfrac{2}{3}(180° - x°)$
$3x° = 360° - 2x°$
$5x° = 360° \Rightarrow x = 72°$
Choice (D)

38. Exterior angle of a polygon = $\dfrac{360°}{n}$
Interior angle = $\dfrac{180(n-2)}{2} - 90°$.
$\therefore$ n = 8.
$\therefore$ Exterior angle = 45° and interior angle = 135°
Choice (C)

39. There are two possibilities to be considered
(1) h is the greatest. The eight angles, in degrees, are d – 3, d – 2, d – 1, d, d + 1, d + 2, d + 3, and d + 8 (i.e. h = d + 4)
$\therefore$ 8d + 8 = 360 $\Rightarrow$ d = 44. The angles are 41, 42, 43, 44, 45, 46, 47 and 52

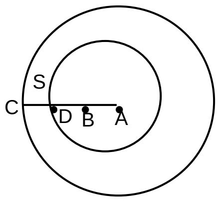
(2) h is the least. The eight angles, in degrees, are d – 3, d – 2, d – 1, d, d + 1, d + 2, d + 3 and d (i.e., h = d – 4)
$\therefore$ 8d = 360 $\Rightarrow$ d = 45.
The angles are 42,43, 44, 45, 46, 47, 48 and 45.
In either case, exactly 3 angles are greater than 45°.
Ans: (3)

40. Let AB and CD be the two walls opposite to each other which are separated by a distance of 22 feet.
Let BP = x feet where P is the foot of the ladder.
Length of the ladder = AP = CP

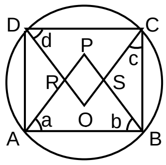


$\Rightarrow AP^2 = CP^2 \Rightarrow 24^2 + x^2 = 20^2 + (22 - x)^2$
$\Rightarrow 44x = 308 \Rightarrow x = 7$
$\therefore AP = \sqrt{24^2 + 7^2} = 25$ Ans: (25)

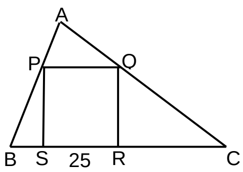
41. The distance between a pair of points, one on each circle, is minimum, when the two points are the points of intersection of the line of centres with the circles. Hence when A and B are the centres of the outer and inner circles respectively, distance between centres = AB = AC – BC
$= r_1 - (s + r_2) = r_1 - r_2 - s$. Choice (B)



42. As ABCD is a quadrilateral,
$2(\angle a + \angle b + \angle c + \angle d) = 360°$.
$a + b + c + d = 180°$
Also as ABP is a triangle, $a + b + P = 180°$……….(1)
Also as DQC is a triangle, $c + d + Q = 180°$………(2)
$(1) + (2) \Rightarrow a + b + c + d + P + Q = 360°$
$\therefore P + Q = 180° (\because a + b + c + d = 180°)$
$\because$ PQRS is a quadrilateral, $\angle PRQ + \angle PSQ = 360° - (\angle P + \angle Q) = 180°$. Choice (C)

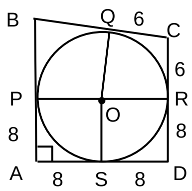


43. Let the triangle be ABC. Let D be the foot of the perpendicular from A to BC (D is not shown in the figure). Let s be the side of the square.
PQ || BC
$\therefore$ Triangles APQ and ABC are similar



$\therefore \frac{PQ}{BC} = \frac{AD - s}{AD} \Rightarrow \frac{s}{25} = \frac{AD - s}{AD}$
The triangle is right-angled at A ($\because 15^2 + 20^2 = 25^2$).
$\therefore$Its area $= \frac{1}{2}(15)(20)$. This is also equal to $\frac{1}{2}(25)(AD)$.
$\therefore AD = 12 \Rightarrow \frac{s}{25} = \frac{12 - s}{12} \Rightarrow s = \frac{300}{37}$
$\therefore$ Perimeter of PQRS $= 4s = \frac{1200}{37}$ Choice (B)

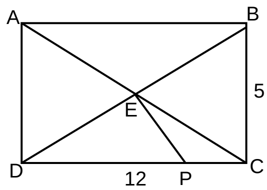
44. Let O be the centre of the circle. The sides of the quadrilateral form tangents to the circle. $\therefore \angle OPA = \angle OSA = 90°$. Also $\angle A = 90°$ and OP = OS = radius
$\therefore$ APOS is a square. . . . . . . (1)
RC = QC = 6 (given)
$\therefore$ RD = CD – RC = 14 – 6 = 8
$\therefore$DS = 8 and
SA = DA – SD = 16 – 8 = 8. $\therefore$ radius is 8
($\because$ From (1)) Ans: (8)



45. Let a, b, c be the sides of each triangle satisfying the given conditions.
The triangles are scalene.
$\therefore$ Let $a < b < c$.
$a + b + c = 24$ where a, b, c are integers.
The longest side of a scalene triangle must be more than one-third of the perimeter and less than half of the perimeter of the triangle.
$8 < c < 12$.
$\therefore$ c is 9, 10 or 11.
If c is 9, a + b = 15. $\therefore$ (a, b) = (7, 8)
If c is 10, a + b = 14. $\therefore$ (a, b) = (5, 9) or (6, 8)
If c is 11, a + b = 13.
$\therefore$ (a, b) = (3, 10), (4, 9), (5, 8) or (6, 7).
(a, b, c) has 7 possible values. Ans: (7)

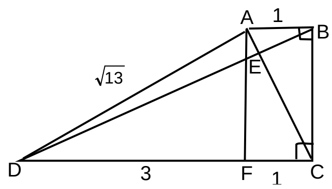
46. I. Let the area of the triangle be A.
The sides of the triangle are $\frac{2A}{4}$, $\frac{2A}{6}$ and $\frac{2A}{9}$.
The sum of any two sides is more than the third side.
$\therefore$ I is correct.
II. Let the area of the triangle be B.
The sides of the triangle are $\frac{2B}{6}$, $\frac{2B}{8}$, $\frac{2B}{15}$.
$\frac{2B}{8} + \frac{2B}{15} > \frac{2B}{6}$.
$\therefore$ II is also correct.
Both I and II are correct. Choice (C)

47. Let the length and the breadth of ABCD be ℓ and b respectively.
$2(\ell + b) = 34$ and $\frac{1}{2}\ell b = 30$
$\ell + b = 17$ and $\ell b = 60$
$\ell > b \therefore (\ell, b) = (12, 5)$. (Let AB = 12, BC = 5)



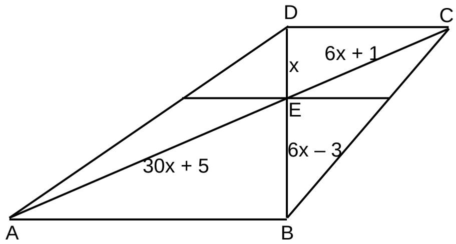
CP : PD = 1 : 3
$\therefore$ CP = 3 and PD = 9
Required distance = EP.
Let EQ be the perpendicular drawn from E to CD.
$EQ = \frac{BC}{2} = 2.5$ and $QP = \frac{DC}{2} - CP = 3$.
$EP = \sqrt{EQ^2 + QP^2} = \frac{\sqrt{61}}{2}$ Choice (D)

48.



AB ||CD
$\therefore \angle BAE = \angle ECD$ and $\angle EBA = \angle EDC$
Triangles AEB and CED are similar. Their corresponding sides are proportional, i.e., $\frac{AE}{CE} = \frac{EB}{ED} = \frac{AB}{CD} = \frac{1}{4}$
Let AE = x and BE = y $\therefore$ EC = 4x and ED = 4y.
Let F be the foot of the perpendicular from A to CD.
$\therefore$ CF = 1, FD = 3
and $AF = \sqrt{AD^2 - DF^2} = \sqrt{13 - 9} = 2$.
$\therefore AC = 5x = \sqrt{AF^2 + FC^2} = \sqrt{4 + 1} = \sqrt{5}$
and $BD = 5y = \sqrt{BC^2 + CD^2} = \sqrt{4 + 16} = \sqrt{20}$
$AE = x = \sqrt{5}/5$ and ED = (4/5) BD $= 4y = 8\sqrt{5}/5$
$\therefore \frac{DE}{AE} = \frac{4y}{x} = \frac{8/\sqrt{5}}{1/\sqrt{5}} = 8$ Ans: (8)

49.



AB || CD and the diagonals meet at E let us draw a line through E which is parallel to AB. This line will also be parallel to CD.
Ratio of the intercepts AE and EC will be the same as that of BE and ED.

$\frac{AE}{EC} = \frac{BE}{ED}$. $\Rightarrow$ Let ED = x $\Rightarrow$ $\frac{30x+5}{6x+1} = \frac{6x-3}{x}$

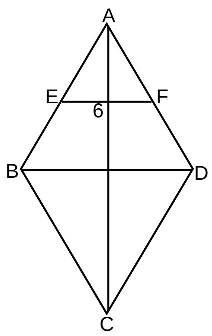
$\Rightarrow 6x^2 - 17x - 3 = 0$

$\Rightarrow (6x + 1)(x - 3) = 0 \Rightarrow x = 3.(\because x > 0)$

i.e., ED = 3

Choice (A)

50. Let ABCD be the rhombus
Let E and F be the midpoints of AB and AD respectively.
The line joining the midpoints of two of the sides of a triangle is parallel to the third side of the triangle. $\therefore$ EF ||BD



$\therefore$ Triangles AEF and ABD are similar
$\therefore$ BD = 2EF = 12.
Let P be the point of intersection of the diagonals BP = PD = $\frac{12}{2}$ = 6 and $BP^2 + PA^2 = BA^2$
Each side of the rhombus is 10.

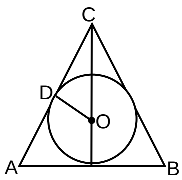
$\therefore$ PA = $\sqrt{10^2 - 6^2}$ = 8.

$\therefore$ The diagonals of the rhombus are 16 and 12.

$\therefore$ Its area = $\frac{1}{2}$ (product of the diagonals) = 96

Choice(C)

51. The smallest triangle that can circumscribe a given circle is equilateral triangle whose side is $2\sqrt{3}$ times the radius of the circle.



Here, a side (AB = 6) is given along with the radius

(OD = $\sqrt{3}$) of the circle.

Here the ratio of the side and the radius

$= \frac{AB}{OD} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

$\therefore$The ΔABC can be equilateral
Hence the perimeter of ΔABC is the least when it is equilateral.
$\therefore$The least perimeter of ΔABC = 3×6 = 18. Ans: (18)

52. $AB^2 + BC^2 = AC^2$. $\therefore \angle B = 90^o$ ($\therefore \angle D = 90^o$, as $\angle B + \angle D = 180^o$)
$\therefore$ Area of the quadrilateral is the total area of two right angled triangles. $\therefore$ Area of the quadrilateral

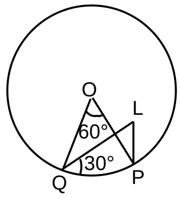
$= \frac{(AB)(B)}{2} + \frac{(AD(DC)}{2}$

$= \frac{(45)(60)}{2} + \frac{(72)\left(\sqrt{75^2 - 72^2}\right)}{2} = 2106$

$\therefore$ Area of the circle = $\pi\left(\frac{AC}{2}\right)^2 = \pi\left(\frac{5625}{4}\right)$

Area of the region inside the circle which is outside the quadrilateral $= \frac{5625\pi}{4} - 2106$ Choice (A)

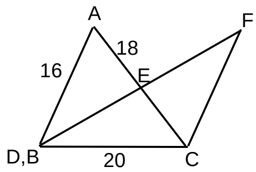
53. Let height of the pole be 'h' and radius of the circle be 'a'
i.e. PL = h and OQ = OP = a PQ is also equal to a
$\therefore$ ΔQPL is a 30°, 90°, 60°triangle
and PL = $\frac{QP}{\sqrt{3}}$.



$\therefore \frac{PL}{OP} = \frac{PL}{QP} = \frac{1}{\sqrt{3}}$

Choice (B)

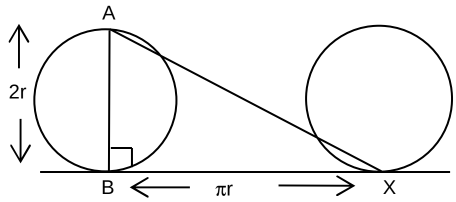
54. In ΔABC, AB = 16, BC = 20 and CA = 18
D and E are points on AB and AC respectively much that
AD + AE = 25 and area of Δ ADE = (1/2) Area of ΔABC



We notice that it we take D = B and E as the midpoint of AC, both the conditions are satisfied. (They would also be satisfied if we take AD = 9 and AE = 16. But even in that case, we would get the same value of DE as the first case.)

$\therefore$ DE = BE = m (say)
Let F be a point in BE extended such that BF = 2 m.
(ΔAEB ≅ ΔCEF. $\therefore$ CF = AB = C (say). Let BC = a)
$\therefore$ In ΔBCF, $(2m)^2 = a^2 + c^2 - 2ac \cos \angle BCF$
$= a^2 + c^2 + 2ac \cos \angle ABC$
($\because$ AB $\|$ FC and $\angle$ABC, $\angle$FCB are supplementary)
In ΔABC, $b^2 = a^2 + c^2 - 2ac \cos\angle ABC$
$\therefore 4m^2 = a^2 + c^2 + (a^2 + c^2 - b^2) = 2a^2 + 2c^2 - b^2$
$= 2(20^2 + 16^2) - 18^2 = 988$ and $DE^2 = BE^2 = M^2 = 247$
Choice (B)

55.



Point A reaches point X, when the wheel makers half a rotation; i.e. BX is equal to half the perimeter of the wheel; i.e. BX = 1/2(2πr) = πr

AX = $\sqrt{4r^2 + \pi^2 r^2} = \sqrt{4(100) + \pi^2(100)} = 10\sqrt{4 + \pi^2}$

Also AB = 2r

$\therefore$ Required ratio $\frac{AX}{AB} = \frac{10\sqrt{4+\pi^2}}{20} = \frac{\sqrt{\pi^2+4}}{2}$

Alternate method:

The problem can be solved, even if the value of the radius is not given. From the figure, it is clear that,
$AX^2 = AB^2 + BX^2 = (2r)^2 + (\pi r)^2 = r^2(\pi^2 + 4)$

Hence, $\frac{AX^2}{AB^2} = \frac{r^2(\pi^2+4)}{4r^2} = \frac{(\pi^2 + 4)}{4}$

$\Rightarrow \frac{AX}{AB} = \frac{1}{2}(\sqrt{\pi^2 + 4})$

Note: Radius = 10 cm is redundant information.
Choice (B)

Solutions for questions 56 to 65:

56. Statement I: The diagonals can be equal even in a non-rectangle. For example, diagonals are equal in an isosceles trapezium.
(If the figure is a rectangle, it must be a square).
I is not sufficient.
Statement II alone is not sufficient as it is not given whether the diagonals are equal and bisect each other.
Using both the statements also we can't answer the question as the four sided figure may be as given below.

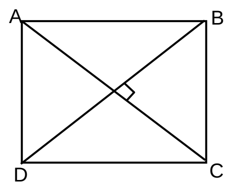


Figure (1)

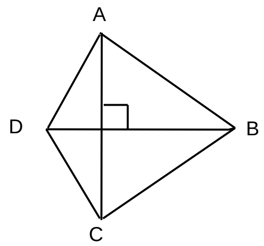
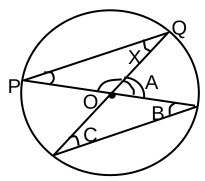


Figure (2)

Figure (1) is a rectangle but figure (2) is not a rectangle. Choice (D)

57. If circumcircle passes through A then we can't say whether $\angle A$ is 90° or not. So statement I alone is not sufficient.
From statement II, $\angle B = 90° \Rightarrow \angle A$ is not equal to 90°.
$\therefore$ Statement II alone is sufficient. Choice (B)

58. From statement I,
$\angle A = 60°$
As OP = OQ = radii
$\angle P = \angle Q = \angle X$ and $\angle POQ$
$= 180° - 60° = 120°$
$\therefore \angle X + \angle X + 120° = 180°$
$\Rightarrow \angle X$ can be determined.
$\therefore$ Statement I alone is sufficient.



From statement II, $\angle B = 30° = \angle C$
$\therefore \angle A = 60°$, $\therefore \angle X = 30°$
$\therefore$ Statement II alone is sufficient. Choice (B)

59. From statement I, AD = 2.
$\Rightarrow$ BD = 2, because $\angle DAB = \angle ABD = 45°$.
$\therefore AB = 2\sqrt{2}$

$BD = 2 \Rightarrow DC = 2\left[\because \tan 30° = \frac{BD}{DC} = \frac{2}{DC} = \frac{1}{\sqrt{3}}\right]$

$= BC = \sqrt{4+12} = 4$
$\therefore$Perimeter $= 2\sqrt{2} + 4 + 2 + 2\sqrt{3}$.
$\therefore$ Statement I alone is sufficient.
But statement II alone is not sufficient as nothing is known about the point E. Choice (A)

60. Statement I is always true for any regular polygon.
From statement II, one of its exterior angles = 60°

$\therefore$ The number of sides $= \frac{360°}{60} = 6$

$\therefore$ Statement II alone is sufficient. Choice (B)

61. From statement I, $AB^2 > BC^2 + AC^2$ so $\angle C$ is the largest, but we can't say which is the smallest angle. So statement I alone is not sufficient.
From statement II, $BC^2 < AB^2 + AC^2$ so we can say that $\angle A$ is not obtuse, but we can't say which is the smallest angle.
Using both statements also we can't say which is the smallest angle in triangle ABC. Choice (D)

62. From statement I, the angles are 30°, 60° & 90° and hence the ratio of the sides can found but not the area. From statement II, the longest side is 5
Using both the statements, in the right angled triangle the hypotenuses is known and the two angles are also known. So we can find the remaining two sides and thereby the area. Choice (C)

63. From statement I,
$\angle BAC + \angle BCA + \angle ABC = 180°$
$50° + \angle BCA + 30° + \angle BCA = 180°$
$\angle BCA = 50°$
So the given triangle is an isosceles triangle.
Statement I alone is sufficient.
From statement II, the altitude bisects the base. Hence it is isosceles. Choice (D)

64. From statement I, AB + AC = 2BC and AC = BC
So AB = AC = BC, so triangle ABC is an equilateral triangle.
In an equilateral triangle the perpendicular line drawn from A bisects BC. So statement I alone is sufficient.
From statement II, AB = AC. So triangle ABC is an isosceles triangle. $\therefore$ The perpendicular line drawn from A bisects BC. So statement II alone is also sufficient.
Choice (B)

65. From statement I,
We don't know whether the diameter of circle B is more or less than the diameter of circle A. So we can't answer the question.
Using both the statements,
If the circle with centre A is bigger than the other circle the two circles will not touch each other. If the circle with centre B is bigger than the other circle they will intersect each other, but the two circles never touch each other externally.
Choice (C)

Chapter – 5
(Mensuration)

Concept Review Questions

Solutions for questions 1 to 50:

1. Area of triangle $= \frac{1}{2}(4)(6)\sin 30° = 6$ sq.cm. Choice (B)

2. Area = rs. Choice (A)

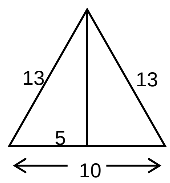
3. Area $= \frac{abc}{4R}$. Choice (D)

4. Area of the equilateral triangle of side a cm $= \frac{\sqrt{3}}{4}a^2$.

$\therefore$ Area of the given equilateral triangle $= \frac{\sqrt{3}}{4}\left(6^2\right)$

$= 9\sqrt{3}$ sq.cm. Choice (D)

5.



$h = \sqrt{13^2 - 5^2} = 12$

Area $= \frac{1}{2} \times 12 \times 10 = 60$ cm². Ans: (60)

6. a = 14cm, b = 48cm, c = 50 cm
This is a right angled triangle
As $a^2 + b^2 = c^2$
$\therefore$ Area = ½ (a) (b) = 336 cm². Choice (B)

7. The triangle formed by joining the midpoints of the sides of another triangle must have an area which is a quarter of the area of the outer triangle.
T_2 is formed by joining the midpoints of the sides of T_1

$\therefore$ T_2's area = $\frac{1}{4}$ (T_1's area)

$\therefore$ Required ratio = 1 : 4 Choice (C)

8. Let the length and the breadth of the rectangular sheet be l m and b m respectively
lb = 1680 . . . (1)
$2(l + b) = 164 \Rightarrow l + b = 82$ . . . . (2)
From (1) and (2), $\ell = 42$ and b = 40. Ans: (42)

9. l = 81m, b = 25m
Area of rect. = $81 \times 25m^2$
Area of square = $a^2 = 81 \times 25 \times 2 \Rightarrow 81 \times 50$
$\Rightarrow 2a^2 = 81 \times 100 = 8100$
$\Rightarrow \sqrt{2a^2} = \sqrt{8100} \Rightarrow a\sqrt{2} = 90$ m Choice (A)

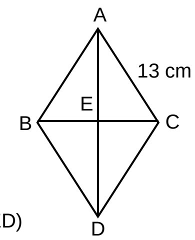
10. Area of triangle = $\sqrt{3/4}\ (2a)^2 = a^2\sqrt{3}$
Area of square = $(2a)^2/2 = 2a^2$
$\therefore$ ratio = $a^2\sqrt{3} : 2a^2 = \sqrt{3} : 2$ Choice (C)

11. Required area = $\frac{1}{2}$ (80) (18) = 720 sq.cm. Ans: (720)

12. Required Area = $\frac{1}{2}$ (4 + 20) (5) = 60 sq.cm. Choice (C)

13. Area of quadrilateral PQRS = Area of PQR + Area of PSR
$= \frac{1}{2}(6)(12) + \frac{1}{2}(4)(12) = 60\,\text{sqcm}$ Choice (A)

14.



Perimeter = 52cm = 4a
$\therefore$ a = 13cm
d = 10 cm = AC
$\therefore$ AE = 5 cm
$\therefore$ ED = $\sqrt{13^2 - 5^2}$ = 12cm
$\therefore$ Area of ABCD = $4 \times$ ½ (AE) (ED)
$= 4 \times \frac{1}{2} \times 5 \times 12 = 120$ sq.cm. Choice (C)

15. Let the length and the breadth of the rectangular sheet be l m and b m respectively
lb = 420
$2(l + b) = 82 \Rightarrow l + b = 41$
Diagonal length (in m)= $\sqrt{l^2 + b^2} = \sqrt{(l + b)^2 - 2lb}$
$= \sqrt{41^2 - 2(420)} = \sqrt{1681 - 840}$
$= \sqrt{841} = 29$ Ans: (29)

16. Let the length and the breadth of the rectangle be 8x and 3x cm respectively.
Length of the wire = Perimeter of the rectangle.
264 = 2(8x + 3x)
264 = 22x
x = 12
Required area (in sq.cm.) = (8x) (3x) = $24x^2$
$= 24(12^2) = (24)(144) = 3456$ Choice (D)

17. Diagonal = $a\sqrt{2} = 6\sqrt{6}$
$\pm$ a = $6\sqrt{3}$ cm
$\therefore$ Area = $\left(6\sqrt{3}\right)^2$ = 108cm² or
Area = $\frac{d^2}{2} = \frac{(6\sqrt{3})^2}{2}$ = 108 cm². Ans: (108)

18. Area = ½ (14 + 8) $\times$ h $\Rightarrow$ 154 = 11 h
h = 14 cm Choice (B)

19. l = 12cm, b = 5 cm
D = $\sqrt{l^2 + b^2}$
length of diagonal = $\sqrt{12^2 + 5^2}$ = 13 cm
Area = l $\times$ b = 12 $\times$ 5 = 60 sq.cm. Choice (A)

20. Field = 25 m $\times$ 15 m
Path width = 3 m
Total Area = 2 $\times$ 25 $\times$ 3
+ 2 $\times$ 15 $\times$ 3 or 31 $\times$ 21 – 25 x 15 = 276 sq.m
+ 4 $\times$ 3 $\times$ 3
= 240 + 36 = 276 sq.m. Choice (A)

21. Area of a trapezium = $\frac{1}{2}$ (Height) (sum of the lengths of the parallel sides)
$\therefore$ Required area (in sq.cm.) = $\frac{1}{2}$ (12) (21 + 3) = 144 cm^2.
Ans: (144)

22. $2\pi r = 21\pi \Rightarrow r = \frac{21}{2}$
Area = $\pi r^2 = \frac{27}{7} \times \frac{27}{2} \times \frac{21}{2}$ = 346.5 cm^2. Choice (D)

23. r = 7cm $\Rightarrow$ 2r = 14 cm = a$\sqrt{2}$
(a is side of square)
$\Rightarrow$ a = 14/$\sqrt{2}$ = 7$\sqrt{2}$
Perimeter of the square = 4a = 4 $\times$ 7$\sqrt{2}$
= 28$\sqrt{2}$ cm. Choice (C)

24. Distance covered by the wheel is one revolution = 2π r
175 $\times$ 2π r = 1100
$\Rightarrow$ 350 $\times$ 22/7 $\times$ r = 1100 $\Rightarrow$ r = 1m
$\therefore$ d = 2r = 2m Ans: (2)

25. Radius of incircle = 1/3 $\times$ altitude = $\frac{1}{3} \times \frac{\sqrt{3}}{2}$ a
Radius of the circumcircle = 2/3 $\times$ altitude = $\frac{2}{3} \times \frac{\sqrt{3}}{2}$a
$\therefore$ $r_1 : r_0$ = 1 : 2
ratio of areas = 1 : 4 Choice (B)

26. Area of the path = π (21²) – π (14)²
$= \frac{22}{7}(21-14)(21+14)$
$= \frac{22}{7} \times 7 \times 35$ = 770 m² Ans: (770)

27. Perimeter of the semi-circle = π r + 2r
$= \frac{22}{7}(14) + 28$ = 72 cm. Ans: (72)

28. Area of the sector = $\frac{72}{360}\ \pi\ (7)^2 = \frac{1}{5}\frac{22}{7}\left(7^2\right)$
= 30.8 sq.cm. Choice (C)

29. Semi Perimeter of the quadrilateral = 15 cm.
Area = $\sqrt{(15-6)(15-7)(15-8)(15-9)} = 12\sqrt{21}$ sq.cm.
Choice (D)

30. Lateral Surface area = (Perimeter of the base) (Height) = 2(6 + 4) (6) = 120 sq.cm. Ans: (120)

31. Total surface area = Lateral surface area + 2(Area of the base) = (4) (6) (10) + 2(6^2) = 312 sq.cm. Choice (B)

32. Volume = (Area of the base) (Height)
$= \frac{\sqrt{3}}{4}(6^2)(20) = 180\sqrt{3}$ sq.cm. Choice (A)

33. Volume = (5) (3) (2) = 30 cubic cm. Ans: (30)

34. Longest diagonal = Body diagonal = $\sqrt{l^2+b^2+h^2}$ cm.
Choice (D)

35. Face diagonal of a cube = $\sqrt{2}$ (side)
$\therefore$ Face diagonal = $6\sqrt{2}$ cm. Ans: (6)

36. Body diagonal of a cube = $\sqrt{3}$ (side)
$\therefore$ Body diagonal = $8\sqrt{3}$ cm. Choice (C)

37. **(i)** Lateral surface area = 2(4) (6 + 5) = 88 sq.cm.
Choice (A)

(ii) Total surface area = Lateral surface area + 2(Area of the base) = 88 + 2(6)(5) = 148 sq.cm. Choice (D)

38. **(i)** Lateral surface area = 4(10^2) = 400 sq.cm.
Choice (A)

(ii) Total Surface area = 6(10^2) = 600 sq.cm.
Choice (B)

39. Total Surface area = 3π(6^2) = 108 π sq.cm. Ans: (108)

40. **(i)** Volume = $\frac{4}{3}\pi$ (12^3) = 2304π cubic cm. Choice (C)

(ii) Surface Area of the sphere = 4π(12^2) = 576π sq.cm.
Choice (A)

41. Curved Surface Area of the hemisphere = 2π(6^2)
= 72π sq.cm. Ans: (72)

42. Volume of the hemisphere = $\frac{2}{3}\pi(6^3)$ = 144 π cubic cm.
Choice (B)

43. If a prism and a pyramid have the same base as well as height, the volume of the prism will be thrice that of the pyramid.
$\therefore$ Required ratio = 3 : 1 Choice (B)

44. Area of parallelogram which has adjacent sides of lengths a cm and b cm with the angle between them being θ = ab sinθ = (8) (10) sin 30°= 40 sq.cm. Ans: (40)

45. **(i)** Lateral Surface area = 1/2 (16) (8) = 64 sq.cm.
Choice (D)

(ii) Total surface area = Lateral Surface area + Area of the base = 64 + 16 = 80 sq.cm. Choice (B)

46. **(i)** The Lateral Surface Area of a frustum whose top radius, bottom radius and slant height are r cm, R cm and l cm is given by πl (R + r).
Lateral Surface Area = π(5) (6 + 8) = 70π sq.cm.
Choice (D)

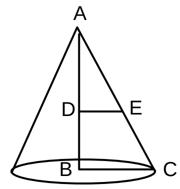
(ii) Total Surface Area = Lateral Surface Area + Base Area + Top Area = 70π + π(R^2 + r^2)
= 70π + π(8^2 + 6^2) = 170π sq.cm. Choice (D)

47. Area of a regular hexagon = $\frac{3\sqrt{3}}{2}$ $(\text{side})^2$

$\therefore$ Required area = $\frac{3\sqrt{3}}{2}(4^2) = 24\sqrt{3}$ sq.cm.

Choice (A)

48. Let DE denote the radius of smaller cone. Let BC denote the radius of the bigger cone.
Triangles ADE and ABC are similar.



$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

$\therefore$ DE = $\frac{1}{2}$BC

$\therefore \frac{\text{Volume of the smaller cone}}{\text{Volume of the bigger cone}} = \frac{\pi\left(\frac{1}{2}BC\right)^2 \frac{1}{2}AD}{\pi BC^2\,AD} = \frac{1}{8}$

$\therefore$ Required ratio = 7 : 8 Choice (B)

49. Required ratio = $\frac{\pi\left(\frac{1}{2}BC\right)AE}{\pi(BC)(AC)}$

As $\frac{AE}{AC} = \frac{1}{2}$, required ratio = 1 : 4 Choice (A)

50. Similar to a method shown in the previous solution it can be shown that the ratio of the sides of the bases of the smaller and larger pyramids is 1 : 2. Let s be the side of the larger pyramid and let h be its height.

Required ratio = $\frac{\frac{1}{3}\left(\frac{s}{2}\right)\left(\frac{s}{2}\right)\frac{h}{2}}{\frac{1}{3}\frac{h}{2}\left(\frac{s}{2}\right)^2 + s^2 + \sqrt{\left(\frac{s}{2}\right)^2 s^2}}$

$= \frac{\frac{1}{3}\left(\frac{s^2h}{8}\right)}{\frac{1}{3}\frac{7}{8}s^2h} = \frac{1}{7}$ Choice (B)

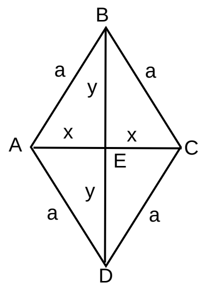
Exercise – 5(a)

Solutions for questions 1 to 35:

1. Let each equal side be a cm. Let the base be b cm.
2a + b = 72 ——— (1)
a = b + 6 ——— (2)
Solving (1) and (2), a = 26 and b = 20
Area of an isosceles triangle whose each equal side is a cm and whose base is b cm is given by
$\frac{b}{4}\sqrt{4a^2 - b^2}$ sq. cm .
a = 26, b = 20
$\therefore$ Area = 240 sq. cm. Choice (A)

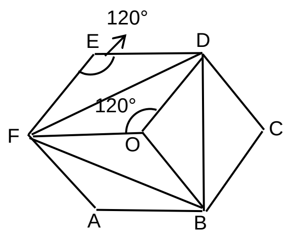
2. Let the sides be a and b. Let the angle between them be θ.
$a^2 + b^2 + \le 4\left(\frac{1}{2}ab\sin\theta\right)$
subtracting 2 ab both sides,
0 ≤ $(a – b)^2$ ≤ 2(ab) (sin θ – 1)
$\therefore$ sin θ ≥ 1
As – 1 ≤ sin θ ≤ 1, sin θ = 1
$\therefore$ θ = 90°
Area of the triangle = $\frac{1}{2}$ ab sinθ = $\frac{1}{2}$ (12) (sin 90°)
= 6 sq.units. Ans: (6)

3. Rhombus ABCD can be divided into two congruent triangles Δ ABC and Δ ACD of perimeter 36 cm each.
2a + 2x = 36 $\Rightarrow$ a + x = 18____ (1)
Rhombus ABCD can be divided into four congruent triangles ΔABE, ΔBCE, ΔADE, and Δ ECD of perimeter 24 cm each.
Hence, a + x + y = 24___ (2)
From equation (1),
a + x = 18. From this information and equation (2),
y = 24 – (a + x) = 24 – 18 = 6 cm.
Triangle ABE is right-angled, $a^2 = x^2 + 6^2$
$a^2 = (18 - a)^2 + 6^2$
$a^2 = 324 - 36a + a^2 + 36$
36a = 360 $\Rightarrow$ a = 10 cm. Choice (A)



4. Let the shorter parallel side of the trapezium be a cm.
Area of the trapezium = $\frac{1}{2}h\,(b_1 + b_2)$, where b_1 = a and h = a; b_2 = 21 $\Rightarrow$ $\frac{1}{2}a\,(a + 21) = 98$
$a^2 + 21a - 196 = 0$
(a + 28) (a – 7) = 0
a = 7 cm.
Hence the height of the trapezium = a = 7 cm. Ans: (7)

5.



The hexagon shown has 6 congruent triangles of which three of than are inside the triangle BFD. (without overlap)
$\therefore$ Area of the triangle = $\frac{3}{6}\times$ (Area of the hexagon)
$\therefore$ K = $\frac{1}{2}$ Choice (D)

6. Area of a parallelogram = (product of two adjacent sides) $\times$ sin (Angle between them) = $20 \times 10 \times \sin 45°$
$= 20 \times 10 \times \frac{1}{\sqrt{2}} = 100\sqrt{2}$ sq. cm. Choice (D)

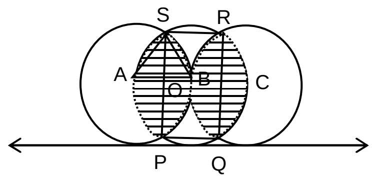
7. Let the radius of the circular wire be r. Let the sides of the triangular wire and the other wire be a and s respectively. Given, the resulting figures enclose the same area.
$\therefore \pi r^2 = \frac{\sqrt{3}}{4}a^2 = s^2$
The lengths of the wires are $2\pi r$, 3a, 4s respectively.
$$\left(\frac{2\pi r}{4s}\right)^2 = \frac{4\pi^2 r^2}{16\,s^2} = \frac{\pi}{4}\left(\frac{\pi r^2}{s^2}\right) = \frac{\pi}{4}(1) < 1$$
$\therefore (2\pi r)^2 < 4s^2$. $\therefore$ $2\pi r < 4s$
$$\left(\frac{4s}{3a}\right)^2 = \frac{16}{9}\left(\frac{s^2}{a^2}\right) = \frac{16}{9}\left(\frac{\sqrt{3}}{4}\right) = \frac{4}{3\sqrt{3}} < 1$$
$\therefore$ 4s < 3a
$2\pi r < 4s < 3a$. Both (I) and (II) follow. Choice (C)

8. Area of the sector = $\frac{1}{2}\ell r$
Perimeter = ℓ + 2r = 108
Radius = 42
Hence, ℓ = 108 – 84 = 24
$\therefore$ Area = $\frac{1}{2}\times 24 \times 42$ = 504 sq.cm.

Alternate method:
Let the central angle of the sector be θ. (inradians)
(42) θ + 42 + 42 = 108
(42) θ = 108 – 84 = 24 ; $\theta = \frac{24}{42}$
Area of the sector = $\left(\frac{\theta}{2}r^2\right) = \frac{(24)}{(42)(2)}$ (42)(42)
= (12) (42) = 504 cm^2. Ans: (504)

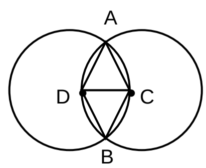
9. The sum of the diameters of the two circles is equal to the side of the square.
If radius of each circle is r, then 2r + 2r = 4 $\Rightarrow$ r = 1
$\therefore$ required ratio is $\frac{4 \times \pi(1)^2}{4^2 - 4\pi(1)^2} = \frac{4\pi}{4[4-\pi]} = \frac{\pi}{4-\pi}$
Choice (D)

10.

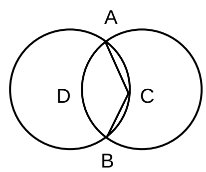


P, Q, R, S are the points of intersection of the circles with the middle circle.
In sector ASBP (with centre A)
AS = AB = AP = r
$\angle$SAP = 120°($\because$ ASB is an equilateral triangle)
Area of the sector ASBP = $\left(\frac{120}{360}\right)\pi r^2 = \frac{\pi r^2}{3}$
Similarly area of the sector BSAP (with centre B) = $\frac{\pi r^2}{3}$
$\therefore$ Area of each shaded region = [$\frac{2\pi r^2}{3}$ – Area of rhombus ASBP]
$$= \left[\frac{2\pi r^2}{3} - (2)\frac{\sqrt{3}r^2}{4}\right] = \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]r^2$$
$\therefore$ Area of the required region = $2r^2\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$
Choice (C)

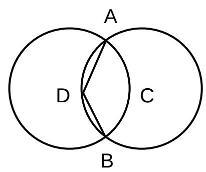
11.



(a)



(b)



(c)

ΔADC and Δ DBC are equilateral triangles.
Required Area = Area of sector DACB (fig (b)) + Area of sector CADB (fig (c)) – Area of the rhombus ACDB (fig (a))

$$= \frac{1}{3} x \pi x 64 + \frac{1}{3} \pi x 64 - 2 x \frac{\sqrt{3}}{4} x 64 = \frac{128\pi}{3} - 32\sqrt{3}$$

Choice (A)

12. Let the central angle of the smallest sector be x°.
The central angles of the 2nd, 3rd, 4th and 5th sectors are 2x°, 4x°, 8x° and 16x° respectively.
Given that
x° + 2x° + 4x° + 8x° + 16x° = 360°

$\Rightarrow$ 31x° = 360° $\Rightarrow$ $x° = \frac{360°}{31}$

$\therefore$ Area of the smallest sector = $\frac{\frac{360°}{31}}{360}\pi(2^2) = \frac{4\pi}{31}$ sq.cm

Choice (D)

13. The semi-perimeter of the cyclic quadrilateral
= $\frac{2+4+6+8}{2}$ = 10 cm.
Area of the cyclic quadrilateral
= $\sqrt{(10-2)(10-4)(10-6)(10-8)}$
= $\sqrt{8\times6\times4\times2}$ = $8\sqrt{6}$ cm^2. Choice (D)

14. Let the side of the equilateral triangle ABC be a cm long and radius of the circle circumscribing it be r cm. Area of the triangle ABC can be expressed as, $\frac{\sqrt{3}}{4}a^2$ as well as

$3\frac{a^3}{4^r}$

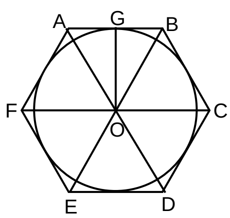
$\therefore \frac{\sqrt{3}}{4}a^2 = \frac{a^3}{4r} \Rightarrow r = \frac{a}{\sqrt{3}}$ . . . . (1)

Let the circumradius of triangle DEF be R cm.
For any equilateral triangle, its circumradius is twice its inradius = R = $\frac{2a}{\sqrt{3}}$ --------- (2)

Ratio of the areas of the outer circle and the inner equilateral triangle = $\pi\left(\frac{2a}{\sqrt{3}}\right)^2 : \frac{\sqrt{3}}{4}a^2 = \frac{4}{3}\pi a^2 : \frac{\sqrt{3}}{4}a^2$

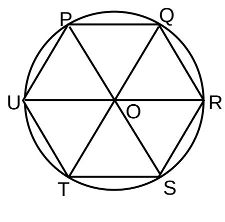
$16\pi : 3\sqrt{3}$. Choice (A)

15. When a regular hexagon is circumscribed around a circle, the hexagon gets divided into six identical equilateral triangles, when each of the vertices is joined to the centre of the circle.
The radius (r) of the circle is equal to the altitude of any of the triangles formed. If a is the side of the hexagon, then



$(\sqrt{3} \cdot a)/2 = r$; a = (2r/ $\sqrt{3}$) and the area of polygon
= (6. $\sqrt{3} \cdot a^2$)/4 = (6$\sqrt{3}$ r²)/3 ---- (1)

When a regular hexagon is inscribed in a circle, radius of the circle is equal to the side of the polygon. Hence r = the side of the polygon; and area of the polygon
= [6. $\sqrt{3}$. (side)²/4], as the polygon gets divided into six identical equilateral triangles. Area of the inscribed polygon



= (6$\sqrt{3}$. r²)/4 ---------- (2)
From (1) and (2) required ratio
= [6$\sqrt{3}$ .r²/4]/[6$\sqrt{3}$ r²/3] = 3 : 4

Alternate method:

Ratio of the areas of two regular hexagons is the ratio of the squares of their sides. If a regular hexagon is inscribed in a circle and another regular hexagon is circumscribed on the same circle, then their sides are in the ratio ($\sqrt{3}$: 2). Hence, ratio of areas is 3 : 4. Choice (C)

16. Area of the remaining plot over which earth dug out is uniformly spread = (70 × 40) – (10 × 5) = 2750 sq. ft.
Volume of the earth dug out = 10 × 5 × 27.5 = 1375 cu.ft

$\therefore$ Rise in the level = $\frac{\text{Volume of the pit}}{\text{Remaining area of the plot}}$

$= \frac{1375}{2750} = \frac{1}{2}$ ft. Ans: (0.5)

17. Let the number of lead shots be N.
(N) (volume of the lead shot) = Increase in the volume of water
$\pi (40)^2 [(8.5 - 6) 10] = N \times \frac{4}{3}\pi (2)^3$
(all measurements are converted into mm)
N = 1600 × 2.5 × 10 × $\frac{3}{4} \times \frac{1}{8}$ = 3750. Choice (B)

18. Let the radius of the base of the cone be r and the slant height of the cone be l.
$\pi r (l + r) = 200\pi$ and $\ell + r = 25$
r = $\frac{200\pi}{\pi(25)}$ = 8
Curved surface area of the cone
= π r l = π (8) (25 – 8) = 136π cm^2. Choice (C)

19. There are three possible cases in which a cone can be cut out of a cuboid. Given l = 56, b = 21, h = 14; where l, b, h are expressed in cms.
Case (i): when the base of the cone is in the plane of lb.
The Radius of the cone = $\frac{21}{2}$ = 10.5 cm.
Height of the cone = 14 cm
V_1 = volume of the cone = $\frac{1}{3}\pi (10.5)^2 (14)$
= $\frac{\pi}{3} \times 7^2\ 1.5^2 \times 7 \times 2$
= $\frac{\pi}{3} (7^2) (31.5)$
Case (ii): when base of the cone is in the plane of bh
The Radius of the cone = $\frac{14}{2}$ = 7 cm
Height of the cone = 56 cm
V_2 = volume of the cone = $\frac{1}{3}\pi (7)^2 (56)$
Case (iii): when the base of the cone is in the plane of h, ℓ
The Radius of the cone = $\frac{14}{2}$ = 7 cm
Height of the cone = 21 cm
V_3 = volume of the cone = $\frac{1}{3}\pi (7)^2 (21)$
Hence, the maximum possible volume occurs in case (ii)
$V_2 = \frac{1}{3}\left(\frac{22}{7}\right)(49)(56)$ = 2874.67 cm^3.

Choice (D)

20. Height of the cylinder = overall height – height of the hemisphere = h.

$h = 21 - \frac{14}{2} = 14$ cm.

Curved surface area of the cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 14 = 616$ cm²

Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2 = 2 \times \frac{22}{7} \times 7^2 = 308.$

The difference between curved surface areas of the cylinder and the hemisphere = 616 – 308 = 308 cm^2.

Ans: (308)

21. If the height is decreased by x cm, decrease in the volume = (1/3) [$\pi r^2 h - \pi r^2 (h - x)$] = $\pi r^2 x$].
It the radius decreases by x cm, decrease in volume
= (1/3) [$\pi r^2 h - \pi (r - x)^2 h$] = (1/3)$\pi$ [$r^2 h - (r^2 - 2xr + x^2)h$]
= (1/3)π [$2xrh - x^2 h$]
Combining the two results, $\pi r^2 x = \pi [2xrh - x^2 h]$.
Cancelling π and x both sides,

$r^2 = 2rh - xh$; $x = \frac{-r^2 + 2rh}{h}$. Choice (A)

22. External radius of the pipe = $\frac{0 \cdot 8}{2} = 0 \cdot 4$ cm = 4 mm.

Internal radius of the pipe
= external radius of the pipe – thickness = 4 – 2 = 2 mm.
Volume of the material of pipe = [π ($4^2 - 2^2$) 280] mm^3

$= \frac{12 \times 280 \times \pi}{1000}$ cm^3

Weight of the pipe = $\frac{15 \times \frac{22}{7} \times 12 \times 280}{1000} = 158 \cdot 4$ gm.

Ans: (158.4)

23. Capacity of the reservoir = $50 \times 30 \times 20 = 30{,}000$ m^3.
Time taken to fill the reservoir

$= \frac{\text{capacity of the reservoir}}{(\text{cross-sectional area of the pipe})(\text{speed of flow through the pipe})}$

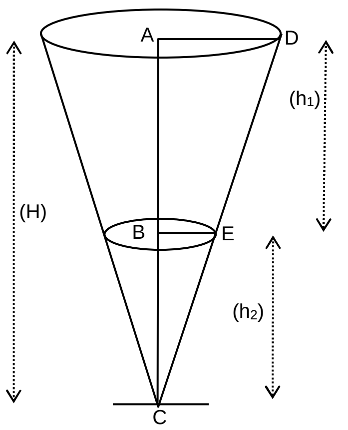
$= \frac{(30{,}000)\text{m}^3}{\left(\frac{25}{100 \times 100}\right)\text{m}^2 \times \frac{10 \times 1000}{1}\frac{\text{m}}{\text{hr}}} = 1200$ hr. Ans: (1200)

24. Capacity of the bucket = $\frac{\pi h}{3}$ [$(21)^2 + (7)^2 + (21) \times (7)$]

= 2548π.

$h = \frac{2548 \times 3}{(441 + 49 + 147)} = \frac{7644}{637} = 12$

Alternate method:



The diagram shows the large cone from which a small cone was cut to form the bucket.
Ratio of the radii of the cones = AD : BE = 21 : 7 = 3 : 1
Because of the similar triangles formed, ratio of the heights of the cone = (H : h_2) is equal to the ratio of the radii; = 3 : 1 (here H = $h_1 + h_2$)
Ratio of the volumes of the two cones is

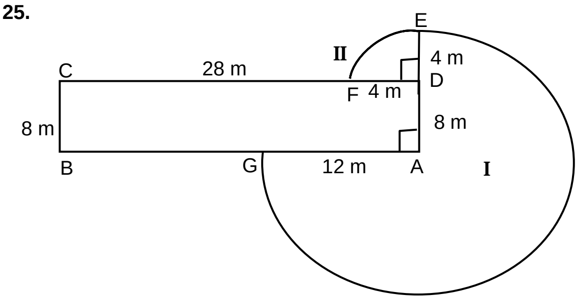
$\frac{1}{3}\pi R^2 H : \frac{1}{3}\pi r^2 h_2 = \frac{R^2}{r^2} \cdot \frac{H}{h_2} = \left(\frac{3}{1}\right)^2 \cdot \left(\frac{3}{1}\right) = 27 : 1$ Hence the volume of the bucket = (27 – 1) parts of the ratio 26 parts of the ratio = 2548π (given data)

$\Rightarrow 26 \times \frac{1}{3} \times \pi \times 7^2 \times h_2 = 2548\pi$

$\Rightarrow h_2 = 6$ and $h_1 = H - h_2 = 3 \times 6 - 6 = 12$ cm.

Choice (B)

25.



From the given information, the area of he field that the cow can graze is represented by the sectors I and II.
(The cow is tied to corner A of the plot)

Area grazed by the cow = $\frac{3^{th}}{4}$ of the area of the circle with radius 12 m + Area of the sector with D as center with radius (12 m – 8 m) i.e. 4 m = $\frac{3}{4}\pi(12)^2 + \frac{90}{360} \times \pi(4)^2$

= $108\pi + 4\pi = 112\pi$ = 352 sq. m Ans: (352)

26. Let the side of each cube be s cm.

Volume of each cube into which C_1 is cut = $\frac{s^3}{N}$.

Side of each of these cubes = $\sqrt[3]{\frac{s^3}{N}}$

Diameter of sphere inscribed in any of these cubes = $\sqrt[3]{\frac{s^3}{N}}$

$\therefore$ Volume of that sphere = $\frac{4}{3}\pi\left(\frac{\sqrt[3]{\frac{s^3}{N}}}{2}\right)^3 = \frac{\pi s^3}{6N}$

Total volume of the spheres inscribed in

$C_1 = V_1 = \left(\frac{\pi s^3}{6N}\right)(N) = \frac{\pi s^3}{6}$

Volume of the sphere inscribed in C_2

$= V_2 = \frac{4}{3}\pi\left(\frac{s}{2}\right)^3 = \frac{\pi s^3}{6}$

Required ratio = 1 : 1. Choice (A)

27. Let the length and the breadth of R, (as well as R_2) be l and b respectively.
Let the side of S be a. A cylinder which is formed by folding along on edge of a rectangle will have that edge as the circumference of its base. The edge adjacent to the edge would be its height. C_1 will have a height of b and a circumference of l.

Its volume = $\pi\left(\frac{l}{2\pi}\right)^2 b = lb\left(\frac{l}{2\pi}\right)$

C_2's volume = $lb\left(\frac{b}{2\pi}\right)$

C_3's volume = $\pi\left(\frac{a}{2\pi}\right)^2 a = a^2\left(\frac{a}{2\pi}\right)$

$a^2 = lb$. As $l > b$, $b < a < l$

$\therefore C_a > C_c > C_b$. Choice (C)

28. The dimensions of the room are 4x, 5x and 7x. Volume of the room = (4x) (5x) (7x)

$140x^3 = 30240$, x = 6 cm.

Difference in the costs for covering the walls with papers of different prices

= (5.50 – 5) 2 [(7x) (5x) + (5x) (4x)]

= $55x^2 = 55(6)^2 = 55\ (6)\ (6)$ = ₹1980. Ans: (1980)

29. Let the radius of the sphere be r cm. The height and the radius of cylinder would be r cm each.

Ratio of the curved surface area of the cylinder and the volume of the sphere

$= \frac{2\pi(r)(h)}{\frac{4}{3}\pi r^3} = \frac{1}{3}$; $\frac{2\pi r^2}{\frac{4}{3}\pi r^3} = \frac{1}{3} \Rightarrow r = \frac{9}{2}$

Volume of the sphere

$= \frac{4}{3}\pi\left(\frac{9}{2}\right)^3 = \frac{4}{3}\pi\times\frac{729}{8} = \frac{243\pi}{2}$ cm^3. Choice (A)

30. Let the side of the square base be a.

Let the height of each tank be h.

Let the radius of the hemispherical tank be r.

The height of the hemispherical tank is also its radius.

$\therefore$ h = r.

$4a = 2\pi r \Rightarrow r = \frac{2a}{\pi}$

Volume of the cuboidal tank = $a^2 h = a^2 r = \frac{2a^3}{\pi}$

Volume of the hemispherical tank = $\frac{2}{3}\pi r^3$

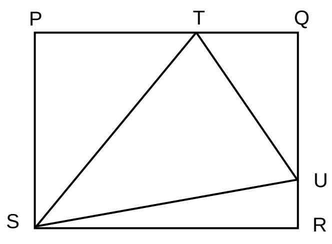
$= \frac{2}{3}\pi\left(\frac{2}{\pi}a\right)^3 = \frac{16}{3\pi^2}a^3$

The ratio of the volumes of the cuboidal tank to the hemispherical tank is $3\pi : 8$.

Difference of the volumes as a fraction of the volume of the cuboidal tank = $\frac{3\pi - 8}{3\pi} \approx \frac{66-56}{66} = \frac{10}{66} \approx 15\%$

Choice (B)

31.



Let TQ = UR = 1.

$\therefore$ PQ = QR = RS = ST = 4 and Ar PQRS = 16

Ar of ΔPTS = (1/2) (4) (3) = 6

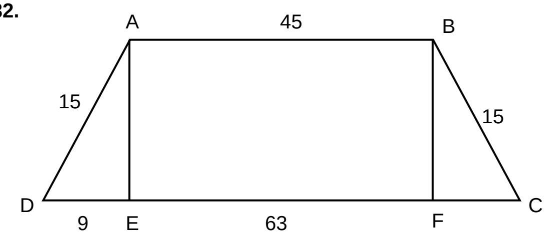
Ar of ΔTQU = (1/2) (1) (3) = 1.5

Ar of ΔURS = (1/2) (4) (1) = 2

Ar of ΔTUS = 16 – 9.5 = 6.5 and $\frac{\text{Ar } \Delta \text{TUS}}{\text{Ar PQRS}} = \frac{13}{32}$

Choice (B)

32.



Let the feet of the perpendiculars drawn from A and B to CD be E and F respectively.

In ΔDAE, DA = 15, DE = 9

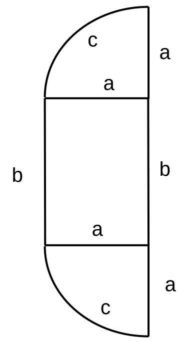
$\therefore$ AE = 12

DE = FC = $\frac{63 - 45}{2} = 9$

Area of the quadrilateral = $\frac{1}{2}$(AE)(AB + CD)

= $\frac{1}{2}$ (12) (45 + 63) = 648. Ans: (648)

33.



The perimeter of the figure = c + b + a + b + a, where c is the arc length of the bottom as well as the top quadrant.

$c = \frac{\pi a}{2}$

Perimeter of the figure = $2\left(\frac{\pi a}{2}\right) + 2b + 2a = \pi a + 2b + 2a$

$\pi a + 2b + 2a = 100$

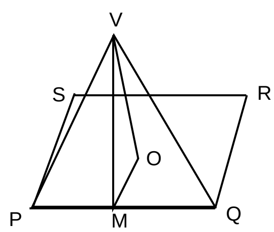
$b = \frac{100 - \pi a - 2a}{2} = 80 - \frac{\pi a}{2} - a$.

Area of the figure (A) = $2\left(\frac{90}{360}\pi a^2\right) + ab$

$= \frac{\pi a^2}{2} + a\left(80 - \frac{\pi a}{2} - a\right) = 80a - a^2$

$\therefore \frac{A + a^2}{a} = 80$ Ans: (80)

34.



Let the base of the pyramid be the square PQRS. Let V be the Vertex, O the centre of the base and M the mid point of PQ. Let OM = 3.

$\therefore$ MQ = MP =3, i.e, PQ = 6.

Semiperimeter of PQRS = 12. Height of pyramid OV = 4.

$\therefore$ VM = 5

Area of $\Delta VPQ = S = \frac{1}{2}(6)(5) = 15.$

If S = 15, A = 36

In general A = $\frac{36}{15}S = 2.4S.$ Ans: (2.4)

35. Let the length, breadth and the height of the rectangular box be ℓ, b and h respectively.
The box is inscribed in a sphere. ∴ The diameter of the sphere is the longest diagonal of the box.

Diameter of the sphere = $\sqrt{\ell^2 + b^2 + h^2}$

$50\sqrt{2} = \sqrt{\ell^2 + b^2 + h^2} \Rightarrow \ell^2 + b^2 + h^2 = 5000$

Total surface area of the box = 9400.
$2(\ell b + bh + \ell h) = 94000$
Sum of the lengths of all the edges of the box
$= 4(\ell + b + h)$
$(\ell + b + h)^2 = \ell^2 + b^2 + h^2 + 2(\ell b + bh + \ell b)$
$= 5000 + 9400 = 14400$
$\Rightarrow \ell + b + h\ \ell = 120° \Rightarrow 4(\ell + b + h) = 480$ Ans: (480)

Exercise – 5(b)

Solutions for questions 1 to 45:

1. Let the hypotenuse be c cm and the other two sides be a cm and b cm. a + b + c = a + b + $\sqrt{a^2+b^2} = 90$

$\frac{1}{2}ab = 270 \Rightarrow ab = 540 \Rightarrow a + b = 90 - c$

(Squaring the above equation)
$(a + b)^2 = (90 - c)^2$
$a^2 + b^2 + 2ab = 8100 - 180c + c^2$
$180c = 8100 + c^2 - 2 \times 540 - c^2$

$c = \frac{8100 - 2\times540}{180} = \frac{7020}{180} = 39$ cm.

Alternate method:

Among the ratios of the sides of a right-angled triangle, 5 : 12 : 13 is one ratio. The sum of the numbers is 5 + 12 + 13 = 30; and 30 is a factor of 90 which is perimeter.
Hence, the sides could be 3 × 5, 3 × 12 and 3 × 13 i.e., 15, 36 and 39. If these are the sides then, area is $\frac{15\times36}{2}$=270, which satisfies the given condition. Hence hypotenuse = 39 cm Choice (A)

2. Semi-perimeter of the triangle
$= \frac{34+50+52}{2} = \frac{136}{2} = 68.$

Area of the triangle = $\sqrt{(68)(68-34)(68-50)(68-52)}$

$= \sqrt{68\times34\times18\times16} = 816$ cm^2. Ans: (816)

3. Let the side of the equilateral triangle be a cm and the height be h cm; then, $\Rightarrow h = \frac{\sqrt{3}}{2}a = 6\sqrt{3}$

Thus, a = 12 cm.

Area of the equilateral triangle = $\left(\sqrt{3}\cdot a^2\right)/4$

$= \frac{\sqrt{3}}{4}(12)^2 = 36\sqrt{3}$ cm^2. Choice (A)

4. Let the inradius of the triangle be r cm.

Semi-perimeter of the triangle s = $\frac{7+4+9}{2}$ = 10 cm.

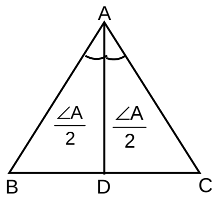
Area of the triangle = $\sqrt{10(10-7)(10-4)(10-9)}$

$= \sqrt{10\times3\times6\times1} = \sqrt{180}$

Area of the triangle = rs, where r is the inradius.

$r = \frac{\sqrt{180}}{10} = \frac{6\sqrt{5}}{10} = \frac{3\sqrt{5}}{5}.$ Choice (C)

5.



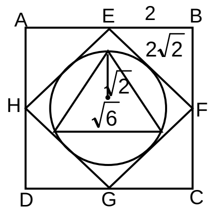
Area of ΔABC = 1/2 × AB × AC × sin 60° = Area of ΔABD + Area of ΔDAC
= 1/2 × AB × AD × sin 30° + 1/2 × AD × AC × sin 30°
⇒ 1/2 .(10)(15)(sin60°)

$= \left(\frac{1}{2}\times10\times AD\times\frac{1}{2}\right)$

$+ \left(\frac{1}{2}\times AD\times15\times\frac{1}{2}\right)$

$150\sqrt{3}$ = AD (10 + 15) ⇒ AD = $6\sqrt{3}$ cm. Choice (D)

6.



Let ABCD be the square and EFGH be the quadrilateral obtained by joining the midpoints of the sides of ABCD.

AE = AH = $\frac{\text{side of ABCD}}{2}$ = 2m.

∴ EH = $2\sqrt{2}$ m.

Similarly EF = FG = GH = $2\sqrt{2}$ m.

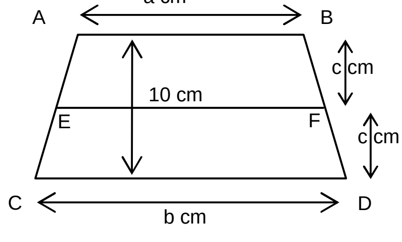
EFGH is also a square and each side is $2\sqrt{2}$ m.
A circle is inscribed in EFGH
∴ Side of EFGH = Diameter of the circle,
∴ Radius of the circle = $\sqrt{2}$ m.
An equilateral triangle was inscribed in the circle.
∴ circumradius of the triangle = $\sqrt{2}$ m.
∴ side of the triangle = $\sqrt{2}\cdot\sqrt{3} = \sqrt{6}$ m

(∵ circumradius of an equilateral triangle of side a = $\frac{a}{\sqrt{3}}$).

Area of the triangle = $\frac{3\sqrt{3}}{2}$ sq.m.

(∵ Area = $\frac{\sqrt{3}}{4}a^2$) Choice (C)

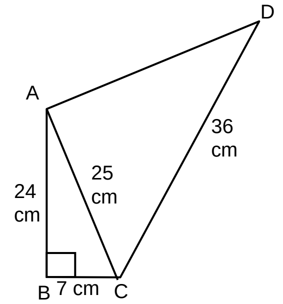
7. Length of the line joining the midpoints of the non-parallel sides of an isosceles trapezium be x



1/2 × 10 × (a + b) = 150; a + b = 30

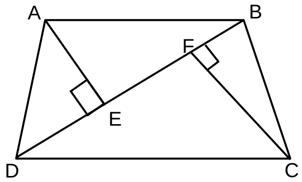
x = length of DE = $\frac{30}{2}$ = 15 cm. Choice (D)

8.



As $AB^2 + BC^2 = 24^2 + 7^2 = 25^2 = AC^2 \Rightarrow \angle ABC = 90°$
Area of the quadrilateral ABCD = $\Delta ABC + \Delta ACD$
$= \frac{1}{2} \times 24 \times 7 + \Delta ACD = 84 +$ Area of the triangle ACD = 309
$\Rightarrow \Delta ACD = 225$ cm². Ans: (225)

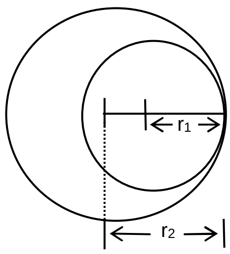
9.



Let the length of the perpendiculars AE and CF be x and (5 + x) respectively.
Area of the trapezium = 1/2 [(x + 5) + x] 30 = 210
Hence the sum of the lengths of the perpendiculars to the diagonal BD = 14 cm.
Note (1): The data that the altitudes differ by 5 cm is redundant.
(2): Even if AD and BC are shown on the parallel sides, the solution does not differ in anyway. Choice (C)

10. Area of the floor
$= \dfrac{\text{Total cost of paving the floor with square tiles}}{\text{Rate of paving the floor with square tiles}}$
$= \dfrac{2240}{7} = 320$ m²
If the breadth of the floor is 'b' and the length of the floor is 'l'
l = 2b
320 = (2b) (b)
$b^2 = \dfrac{320}{2} = 160 \Rightarrow b = \sqrt{160} = 4\sqrt{10}$ m
$l = 2b = 8\sqrt{10}$ m
perimeter of the floor = 2 (l + b)
$= 2\ (8\sqrt{10} + 4\sqrt{10}) = 24\sqrt{10}$ m. Choice (D)

11.



Let the radius of the inner circle be r_1 cm and the radius of outer circle be r_2 cm.
$r_2 = r_1 + 7$
$\pi r_2^2 - \pi r_1^2 = 1078$
$= \pi\ (r_1 + r_2)\ (r_1 - r_2) = 1078$
$\Rightarrow \pi(7)\ (r_1 + r_2) = 1078$
$r_1 + r_2 = 49$ cm Ans: (49)

12. Area of the shaded region = Area of the square ABCD – Total area of 4 sectors $= 14^2 - 4 \times \dfrac{90}{360} \times \pi\ (7)^2$
$= 196 - 4 \times \dfrac{1}{4} \times \dfrac{22}{7} \times 49 = 196 - 154 = 42$ sq. cm.
Choice (A)

13. Semi – perimeter = $\dfrac{3+6+9+12}{2} = 15\text{cm}$
Area = $\sqrt{(15-3)(15-6)(15-9)(15-12)} = 18\sqrt{6}\text{sq.cm}$
Choice (B)

14. Area swept by the hour hand between 11:20 a.m. and 11:55 a.m. = area of sector through which the hour hand swept. Angle of the sector formed for the interval 11:20 a.m. and 11:55 a.m. is the angle of rotation of the hour hand in an interval of 35 minutes = 35 × (3/2) degrees.
Hence required area = $\dfrac{35 \times \frac{1}{2} \times \pi \text{ x } 6^2}{360} = \dfrac{35 \times \frac{1}{2} \times \frac{22}{7} \times 36}{360}$
$= (5 \times 11 \times 36) / 360 = 5.5$ cm² Ans: (5.5)

15. Let the side of the equilateral triangle be a and side of square be s.
$3a = 4s \Rightarrow a = \dfrac{4s}{3}$.
Diagonal of the square = $\sqrt{2}$ s
$\dfrac{\text{Side of the equilateral triangle}}{\text{Diagonal of the square}} = \dfrac{\frac{4s}{3}}{\sqrt{2}s} = \dfrac{4}{3\sqrt{2}} = 4 : 3\sqrt{2}$.
Choice (D)

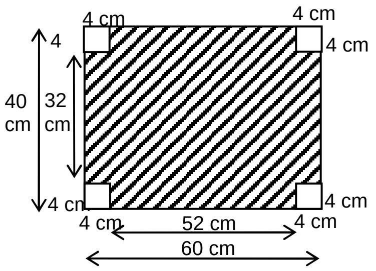
16. Let the radius of the circle be r. Let the sides of the square and the triangle be s and a respectively.
$\pi r^2 = s^2 = \dfrac{\sqrt{3}}{4}a^2 \Rightarrow C = 2\pi r,\ S = 4s,\ T = 3a$
$C^2 = 4\pi^2 r^2 = 4\pi^2 \left(\dfrac{S^2}{\pi}\right) = 4\pi s^2$
$T^2 = 9a^2 = 9\left(\dfrac{S^2}{\frac{\sqrt{3}}{4}}\right) = 12\sqrt{3}s^2$ and $S^2 = 16s^2$
$\Rightarrow C^2 < S^2 < T^2$
$\therefore$ C < S < T. Choice (C)

17. Side of the hexagon = $\dfrac{24}{4} = 6\text{cm}$
Area of the hexagon = $\dfrac{3\sqrt{3}}{2}\left(6^2\right) = 54\sqrt{3}\text{sq.cm}$ .Choice (A)

18. Let the radius of each cylinder be r cm. Let the heights of cylinder A and B be a cm and b cm respectively.
$2\pi rb = \left(1 + \dfrac{300}{100}\right)(2\pi ra) \Rightarrow b = 4a$
$\therefore$ required percentage = $\dfrac{4a - a}{4a} \times 100 = 75\%$ Ans: (75)

19. Let the radius of the sphere be r cm. The height and the radius of cylinder would be r cm each.
Ratio of the curved surface area of the cylinder and the volume of the sphere
$= \dfrac{2\pi(r)(h)}{\frac{4}{3}\pi r^3} = \dfrac{1}{3}\ ;\ \dfrac{2\pi r^2}{\frac{4}{3}\pi r^3} = \dfrac{1}{3} \Rightarrow r = \dfrac{9}{2}$
Volume of the sphere
$= \dfrac{4}{3}\pi\left(\dfrac{9}{2}\right)^3 = \dfrac{4}{3}\pi \times \dfrac{729}{8} = \dfrac{243\pi}{2}$ cm³ Choice (A)

20.



When squares of side 4 cm are cut from a rectangle of the length 60 cm and breadth 40 cm, we get a piece which looks like shaded figure above.
Volume of the cuboid formed when the resulting piece is made into a cuboid = $52 \times 32 \times 4 = 6656$ cm^3

Ans: (6656)

21. Let the length, breadth and height of the cuboid be l cm, b cm and h cm respectively. Given that
$l^2 + b^2 + h^2$
$= \frac{1}{2}[2(lb + lh + bh)] \Rightarrow \frac{1}{2}\left[(l-b)^2 + (b-h)^2 + (l-h)^2\right] = 0$
This is possible only if l = b = h
$\therefore$ The cuboid is a cube its volume = $l^3 = 729 \Rightarrow l = 9$
$\therefore$ Its lateral surface area = $4l^2 = 324$ sq. cm.

Choice (C)

22. Let the original radius of the balloon be r cm. Increase in its surface area = $4\pi(3r)^2 - 4\pi r^2$
$= 4\pi(9r^2 - r^2) = 8(4\pi r^2) = (8)$ (original surface area).

Ans: (8)

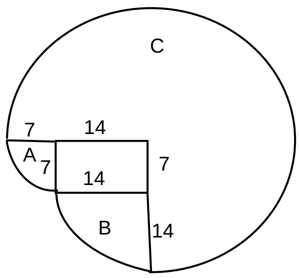
23. Quantity of brick work required
$= \pi\left[\left(\frac{10}{2} + \frac{6}{12}\right)^2 - \left(\frac{10}{2}\right)^2\right] \times 30$
$= \pi[5{\cdot}5^2 - 5^2] \times 30 = \pi[30{\cdot}25 - 25] \times 30$
$= \pi[5{\cdot}25] \times 30 = 157.5\pi$ cubic feet. Choice (B)

24. Area of the field = (Curved surface area of the roller) $\times$ Number of revolutions = $\left(2 \times \frac{22}{7} \times 49 \times 160 \times 600\right)$ cm^2
$= \frac{(44 \times 7 \times 160 \times 600)}{(100)^2}$ m^2 = 2956.8 sq.m Choice (A)

25. Let the radius of the cone be r and height of the cone be h.
$r = \frac{1}{3}h \Rightarrow h = 3r$, Total surface area = $\pi r(r + l)$
As $h \propto r$ and $l^2 = h^2 + r^2$; $r + l \propto r$; $\Rightarrow r(r + l) \propto r^2$.
$\therefore$ Total surface area α r^2.
$\Rightarrow$ Total surface area = kr^2 [k is a constant of proportion]
Percentage error in Surface area
$= \frac{(\text{error in calculating total surface area})}{\text{Actual total surface area}} \times 100$
$= \frac{k(1.01r)^2 - kr^2}{kr^2} \times 100 = 0.0201 \times 100 = 2.01$

Ans: (2.01)

26.



The radii and the central angles of the 3 sectors, A, B and C are tabulated below.

	A	B	C
Radius	7	14	21
Central Angle	90°	90°	270°

Total area = Area (A) + Area (B) + Area (C)
$= \frac{\pi}{4}(7^2) + \frac{\pi}{4}(14)^2 + \frac{3\pi}{4}(21^2) = \frac{\pi}{4}[7^2 + 14^2 + 3(21)^2]$
$= \frac{22}{7} \times \frac{1}{4}[49 + 196 + 1323] = 1232$ Choice (D)

27. Let the radius and the height of the room be r m and h m respectively. The length of the longest rod that can be placed in the room is 29 m.
$\therefore 4r^2 + h^2 = 841$ —— (1)
The curved surface area is 1320 m^2
$\therefore (2)\left(\frac{22}{7}\right)r\,h = 2640 \Rightarrow hr = 210$ —— (2)
(1), (2) $\Rightarrow (2r + h)^2 = 841 + 840 = 1681 = 41^2$ and $(2r - h)^2 = 1$
$\therefore$ 2r + h = 41 or 2r + h = –1
or 2r – h = 1 or 2r – h = –1
$\therefore$ r = 10.5, h = 20 OR r = 10, h = 21
$\therefore$ The height could be 20 m or 21 m. Choice (D)

28. Let the length breadth and height of the cuboid be l cm, b cm and h cm respectively.
Area of the third of the mutually adjacent faces) face
$= \frac{432}{2} - (96 + 48) = 72$sq.cm
lb, lh and bh must be 72, 96 and 48 in any order
(lh) (bh) (lb) = (72) (96) (48)
$\Rightarrow lbh = \sqrt{(72)(96)(48)} = \sqrt{(24)(3)(24)(4)(24)(2)}$
$= 24^2 = 576$ Choice (B)

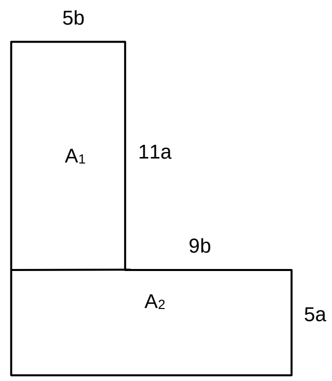
29. As the increase in the volume in both cases must be the same, final volumes must be the same in both cases.
Final volume = $\pi\,\pi(5 + x)^2\ (5) = \pi(5 + 3x)(5^2)$
$\Rightarrow 25 + 10x + x^2 = 25 + 15x \Rightarrow x(x - 5) = 0$
As $x \neq 0$, x = 5. Ans: (5)

30. Given $\ell bh = 140$ cm^3; $\ell b = 28$ cm^2; $bh = 20$ cm^2;
$B = \frac{(\ell b)(bh)}{\ell bh} = \frac{28 \times 20}{140} = 4$ cm; b = 5 cm; ℓ = 7cm.
$\therefore$ Sum of the edges = 4(& + 4 + 5) = 64 cm.

Choice (D)

31. Let the length of the rectangle be l cm. and the breadth of the rectangle be b cm.
lb = 247 and l – 3 = b + 3 $\Rightarrow$ l = b + 6.
(b + 6) b = 247 $b^2 + 6b - 247 = 0$
(b + 19) (b – 13) = 0, b = 13 cm. l = 13 + 6 = 19 cm.
Perimeter of the original rectangle
= 2 (l + b) = 2 (19 + 13) = 64 cm. Ans: (64)

32.



Perimeter = 5b + 11a + 9b + 5a + 14b + 16a = 28b + 32a
28b + 32a = 136 ⇒ 7b + 8a = 34

$b = \frac{34 - 8a}{7}$

Area (A) = $A_1 + A_2$ = 55ab + 70ab = 125ab

$= 125a\left(\frac{34 - 8a}{7}\right)$ ∴ $7A = 4250a - 1000a^2$

∴ p = 4250, q = 1000 and p + q = 5250. Ans: (5250)

33. Let the number of times he would go round before completely mowing half of the lawn be x.
Area of the lawn which would remain after mowing

$= \frac{(40)(30)}{2} = 600$ sq.m ∴ (40 – 2x) (30 – 2x) = 600

⇒ $4x^2 - 140x + 600 = 0$ ⇒ x = 30 or 5
When x = 30, 40 – 2x is negative
∴ x = 5. Ans: (5)

34. Area of the triangle EFG $= \frac{1}{2} \times EG \times AG$ (as height of Δ EFG = DE = AG) $= \frac{1}{2} \times AD \times \frac{1}{3} AB$

$$\frac{\text{Area of the triangle EFG}}{\text{Area of the rectangle ABCD}} = \frac{\frac{1}{2} \times AD \times \frac{1}{3} AB}{AD \times AB} = 1 : 6.$$

Choice (B)

35. $\frac{\Delta ECD}{\Delta ECB} = \frac{3}{2}$ (as heights are equal, areas are in the ratio of the bases). Similarly; $\frac{\Delta ECB}{\Delta EBA} = \frac{4}{3}$

Multiplying the two results, $\frac{\Delta ECD}{\Delta EBA} = \frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$

Choice (A)

36. Let the length of the cuboid be l cm and breadth and height of the cuboid be b cm and h cm.
As l > b
$\ell h = 60$ ____ (1)
$bh = 40$ ____ (2)
$\ell bh = 480$ ____ (3)
Multiplying (1) and (2), $\ell b h^2 = 2400$,

l b h (h) = 2400; $h = \frac{2400}{lbh} = \frac{2400}{480} = 5$

$l = \frac{60}{5} = 12$ cm $b = \frac{40}{5} = 8$ cm

Longest diagonal of the cuboid

$= \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 8^2 + 5^2} = \sqrt{144 + 64 + 25}$

$= \sqrt{208 + 25} = \sqrt{233}$ cm . Choice (B)

37. Let the side of the equilateral triangle be a cm. Let the length and the breadth of the rectangle be l cm and b cm respectively.
2 (l + b) = 3a

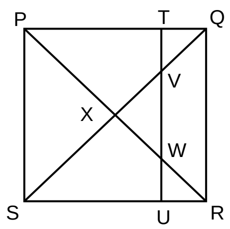
If l = a, $b = \frac{a}{2}$.

Required ratio $= \frac{\sqrt{3}}{4} a^2 : lb = \sqrt{3} : 2$ Choice (B)

38. PQRS is a square.
∴∠PQS = 45°
∴ ΔQTV is isosceles.

∴ $TV = TQ = \frac{QV}{\sqrt{2}} = \frac{b}{\sqrt{2}}$

Also $WU = UR = \frac{b}{\sqrt{2}}$



(∵ WUR is also a 45°, 90°, 45° triangle)

$VW = PS - (TV + WU) = a - 2\frac{b}{\sqrt{2}} = a - \sqrt{2}\, b$

Triangles XVW and XQR are similar (∵ VW||QR and ∠X is common)

Ratio of their corresponding sides $= \frac{VW}{QR} = \frac{a - \sqrt{2}b}{a}$.

∴ Ratio of the areas of XVW and XQR $= \left(\frac{a - \sqrt{2}b}{a}\right)^2$

Choice (A)

39. Volume of the bottle (V) = Volume of the upper cylinder + Volume of the frustum + Volume of the lower cylinder. The height of the frustum is 15 – (8 + 4) or 3 cm.

∴ $V = \pi (4)^2(8) + \frac{\pi}{3} 3[6^2 + 4^2 + (6)(4)] + \pi(6)^2(4)$ cm^3

$= \pi (128 + 76 + 144) = 348\pi$ cm^3. Ans: (348)

40. Let the side of the square be a.
Area of the shaded region = Area of the outer circle – Area of the square
Area of the dotted region = Area of the square – Area of the inner circle
The side of the square is the diameter of the inner circle. The diagonal of the square is the diameter of the outer circle.

Required ratio $= \pi\left(\frac{\sqrt{2}a}{2}\right)^2 - a^2 : a^2 - \pi\left(\frac{a}{2}\right)^2$

$= \frac{\pi}{2} - 1 : 1 - \frac{\pi}{4} = 2(\pi - 2) : 4 - \pi$ Choice (B)

41. Let the side of the square be a.
Height of the pyramid = $3(\sqrt{2}\,a) = 3\sqrt{2}\,a$
Let the length of the perpendicular drawn from the vertex to any of the bases of the triangular regions be h

$h^2 = (3\sqrt{2}a)^2 + (\frac{a}{2})^2$ ⇒ $h = \frac{\sqrt{73}\,a}{2}$

Area of each of the triangular regions $= \frac{1}{2}ah$

$= \frac{\sqrt{73}a^2}{4} = \frac{\sqrt{73}}{4}(\frac{P}{4})^2 = \frac{\sqrt{73}}{64} P^2$ Choice (D)

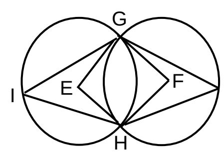
42. Diagonal of the rectangle = Diameter of the circle = 12 units.
For a rectangle whose diagonal is constant, the area as well as its perimeter are maximum when it is a square.
∴ I is true, but II is false Choice (A)

43. Let the radius and the height of the cylinder be r and h respectively.
2πr(r + h) = 440 and r + h = 10
2πr(10) = 440

∴ $r = \frac{22}{22/7} = 7$

∴ h = 3
Volume = $\pi r^2 h = 462$ cm^3. Ans: (462)

44.



The points E and F to be centered
The circles intersect at G and H.
∴ GH is a common chord of the two circles.
∴ ∠GEH = 2 ∠GIH
= 60°.----------(1)

As the circles are congruent, $\angle GFH = 60°$-----(2)

EGFH is a rhombus ($\because$ Each side of EGFH is a radius of one of the circles). Also (1), (2) imply that triangles EGH and FGH are equilateral

Let the radius of each circle be r

Area of EGFH = $2\left(\frac{\sqrt{3}}{4}r^2\right)$

Area of the region common to the two circles

= 2 (Area of the sector EGH – Area of the triangle EGH)

$= 2\left(\frac{60}{360}.\pi r^2 - \frac{\sqrt{3}}{4}r^2\right) = \frac{\pi}{3}r^2 - \frac{\sqrt{3}}{2}r^2$

Ratio = $\frac{\sqrt{3}}{2} : \frac{\pi}{3} - \frac{\sqrt{3}}{2} : 2\pi = 3\sqrt{3} : 2\pi - 3\sqrt{3}$.

Choice (C)

45. Ratio of the densities of the materials is 1 : 2. Also, the pipes have the same weight. $\therefore$ The ratio of their volumes is 2 : 1 (Let the lengths be L_1 and L_2)

Let the outer diameters be 5x and 4x.

Let the inner diameters be a and b

Let the thicknesses be 5y and 4y

a = 5x – 2(5y) and b = 4x – 2(4y) ($\therefore \frac{a}{b} = \frac{5(x-2y)}{4(x-2y)} = \frac{5}{4}$)

Ratio of volumes = $\frac{\frac{\pi((5x)^2 - a^2)}{4}}{\frac{\pi((4x)^2 - b^2)}{4}}\ \frac{L_1}{L_2} = \frac{2}{1}$

$\therefore \frac{L_1}{L_2} = \frac{2}{1}\left(\frac{4}{5}\right)^2 = \frac{32}{25}$ Choice (A)

Solutions for questions 46 to 50:

46. Neither of the statements alone is sufficient as each has only partial information.

Combining statements I and II,

$(1/2)d_1d_2$ = area of rhombus. $1/2 \times 6 \times d_2 = 24 \Rightarrow d_2 = 8$

we have the side of the rhombus = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$ so

s = $\sqrt{3^2 + 4^2} = 5$; $\therefore$ Perimeter = 4s = 20.

Choice (C)

47. From statement I, $6a^2 = 96$, so a can be found and the diagonal $a\sqrt{3}$ can be found.

From statement II, $a^3 = 64$ so a can be found and the diagonal $a\sqrt{3}$ can be found. Choice (B)

48. From statement I, $4\pi r^2 = 120$.

$\Rightarrow r = \sqrt{(120/4\pi)}$; $v = \frac{4}{3}\pi r^3$.

Hence statement I alone is sufficient.

Statement II alone cannot be sufficient since no dimensions of the parallelopiped are known.

Choice (A)

49. From statement I, h = r/2

From statement II, r = $\sqrt{4}$ = 2

Combining statements I and II, r = 2 and h = 1.

Volume = $1/3\pi r^2h$.

So we can find the volume of the cone.

Choice (C)

50. Volume of a cylinder is πr^2h. So either of the statements alone is not sufficient as the relation between radii and heights is given in different statements. Combining statements I and II

$\frac{h_A}{h_B} = \frac{1}{2}; \frac{r_A}{r_B} = \frac{1}{4}$

$v = \pi r^2h$ so

$\frac{v_A}{v_B} = \frac{r^2{}_A h_A}{r^2{}_B h_B} = \frac{1}{32}$

So, both are required to answer the question.

Choice (C)

Chapter – 6
(Coordinate Geometry)

Concept Review Questions

Solutions for questions 1 to 40:

1. (a) The equation of the x - axis is y = 0 Choice (B)

(b) The equation of the y - axis is x = 0 Choice (A)

2. The slope of the line parallel to the x - axis is 0.

Ans: (0)

3. (a) The equation of the line parallel to the x - axis is y = k where k is a real number.

As this line passes through the point (5, 9), 9 = k

$\Rightarrow$ y – 9 = 0 is the required line. Choice (B)

(b) The equation of the line parallel to the y-axis is x = h where h is a real number.

As this line passes through the point (5, 9), 5 = h

$\Rightarrow$ x – 5 = 0 is the required line. Choice (A)

4. We know that the coordinate axes intersect at the origin. So, (0, 0) is their point of intersection. Choice (D)

5. The lines x = 2 and y = 3 intersect at (2, 3). Choice (A)

6. Every point lying on a line satisfies the equation of the line $\Rightarrow$ 2(3) + 3(2) + k = 0 $\Rightarrow$ k = –12 Ans: (–12)

7. In the given point x < 0, y < 0

$\Rightarrow$ it belongs to Q_3. Choice (C)

8. In Q_2, x < 0, y > 0 and the distance of a point from Y-axis is the x-coordinate and the distance from X-axis is the y-coordinate.

$\therefore$ the required point is (–2, 3). Choice (B)

9. The distance of the point from X-axis is its Y-coordinate i.e. 3 units. Ans: (3)

10. The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here, it is given that (x_1, y_1) = (2, –3) and (x_2, y_2) = (–1, 1)

$\therefore$ Required distance = $\sqrt{(2+1)^2 + (-3-1)^2}$

= 5 units Choice (A)

11. The distance from the origin to (x_1, y_1) is $\sqrt{x_1^2 + y_1^2}$

Here (x_1, y_1) = (– 5, – 12)

= $\sqrt{(-5)^2 + (-12)^2}$ = 13 units Choice (C)

12. It is given that PQ is the perpendicular bisector of AB and Q is a point on AB.

$\Rightarrow$ Q is midpoint of AB.

The midpoint is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

i.e. $\left(\frac{-3-7}{2}, \frac{4+2}{2}\right)$ = (–5, 3)

Choice (C)

13. If θ is the angle made by the line with X-axis in the positive direction, then the slope of the line is $\tan\theta$

$= \tan 60° = \sqrt{3}$ Choice (D)

14. (a) Equation of the line with slope m and y-intercept 'C' is given by y = mx + c, here m = – 1, c = 3
i.e., y = – x + 3 or x + y = 3 Choice (B)

(b) Let the equation be y = mx + c

$m = \frac{4}{3}$. When the line cuts the x-axis, y = 0.

$\therefore 0 = \frac{4}{3}(6) + c$

i.e., c = –8.

Required equation is $y = \frac{4}{3}x - 8$

i.e., 3y = 4x – 24 Choice (A)

15. Intercepts made by the line ax + by + c = 0 on coordinate axes are given by

x-intercept $= \frac{-c}{a} = \frac{12}{3} = 4$

y-intercept $= \frac{-c}{b} = \frac{-(-12)}{-4} = -3$ Ans: (4, –3)

16. Slope of the line ax + by + c = 0 is given by $-a/b$

Given line is 3x – 4y + 7 = 0

Slope $= \frac{-3}{-4} = \frac{3}{4}$ Choice (A)

17. The equation of the line, with x-intercept and y-intercept b, is given by $\frac{x}{a} + \frac{y}{b} = 1$

i.e. $\frac{x}{3} + \frac{y}{-2} = 1 \Rightarrow 2x - 3y = 6$ Choice (B)

18. The equation of the line, with slope 'm' and passing through the point (x_1, y_1), is given by $y - y_1 = m(x - x_1)$

Here $m = -2/3$ and $(x_1, y_1) = (-1, 4)$

i.e. $y - 4 = \frac{-2}{3}(x + 1)$

3y – 12 = – 2x – 2
2x + 3y = 10 Choice (C)

19. If two lines with slopes m_1 and m_2 are perpendicular then $m_1 m_2 = -1$

$\Rightarrow m_1(-2) = -1 \Rightarrow m_1 = \frac{1}{2}$ Ans: (0.5)

20. If two lines are parallel their slopes are equal
$\Rightarrow$ Required slope = Slope of the given line

$= \frac{-3}{-4} = \frac{3}{4}$ Choice (B)

21. Slope of the given line is $\frac{-2}{-2} = 1$

Slope in terms of θ is $\tan\theta = 1$
$\Rightarrow \theta = 45° = \pi/4$ Choice (A)

22. Distance from the origin to the line ax + by + c = 0 is given by $\frac{c}{\sqrt{a^2+b^2}}$

$\therefore$ The required distance $= \frac{10}{\sqrt{3^2+4^2}} = 2$ units

Choice (C)

23. Let the required point be P.
P lies on the x-axis.
$\therefore$ P is (a, 0)
Given XP = YP i.e., $XP^2 = YP^2$
$(a-2)^2 - (0-4)^2 = (a-6)^2 + (0-10)^2$
$a^2 - 4a + 4 + 16 = a^2 - 12a + 36 + 100$
a = 29/2 Choice (D)

24. We know that the centroid of a triangle is the same as the centroid of the triangle formed by the mid points of its sides.

$\therefore$ centroid of the triangle $= \left(\frac{-4-2+2}{3}, \frac{0+2+4}{3}\right)$

$= \left(\frac{-4}{3}, 2\right)$ Choice (C)

25. Let P = (–7, 8), Q = (–3, 9), R = (–5, 6) and S = fourth vertex
P, Q, R, S form a parallelogram
$\therefore$ its diagonals bisect each other.
$\therefore$ midpoint of PR = midpoint of QS.
Let S = (a, b)

Midpoint of PR $= \left(\frac{-7+(-5)}{2}, \frac{8+6}{2}\right) = (-6, 7)$.

Midpoint of QS $= \left(\frac{-3+a}{2}, \frac{9+b}{2}\right)$

$(-6, 7) = \left(\frac{-3+a}{2}, \frac{9+b}{2}\right)$

a = –9 and b = 5

Alternate method:

Fourth vertex of a parallelogram is given by
$(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$
= (–7 – 5 – (–3), 8 + 6 – 9) = (–9, 5) Choice (A)

26. $\therefore$ slope of $L_1 \equiv 3x - 4y + 7 = 0$ is 3/4

and slope of $L_2 \equiv ax + 8y - 6 = 0$ is $\frac{-a}{8}$.

Two lines will not intersect only if their slopes are equal.

$\frac{3}{4} = \frac{-a}{8}$, i.e, a = –6 Ans: (–6)

27. Two lines are perpendicular, then product of their slopes is –1. slope of $L_1 = \frac{2}{3}$ and slope of $L_2 = \frac{-3}{b}$.

$\left(\frac{2}{3}\right)\left(\frac{-3}{b}\right) = -1$ i.e., b = 2 Ans: (2)

28. Given equations of the lines are 8x – 3y = 13 and 2x + y = 5
Solving the two equations & we get x = 2; y = 1
$\therefore$ The point intersection (2, 1). Choice (B)

29. We know that of $a_1x + b_1y + 4 = 0$ and $a_2x + b_2y + c_2 = 0$ represent to same line then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

6x + py + 18 = 0 and x + y + q = 0 represent the same line

$\Rightarrow \frac{6}{1} = \frac{p}{1} = \frac{18}{q}$

$\therefore \Rightarrow$ p = 6 and q = 3, p + q = 9 Ans: (9)

30. Let A = (4, 3), B = (0, 7) and C = (–4, 3)

slope of AB $= \frac{7-3}{0-4} = -1$, slope of BC $= \frac{7-3}{0-(-4)} = 1$

(slope of AB) (slope of BC) = –1
$\therefore AB \perp BC$
$\therefore$ triangle ABC is right angled at B.
$\therefore$ orthocentre = (0, 7)
($\because$ for any right angled triangle, orthocentre is the vertex containing right angle). Choice (A)

31. Diagonal of the square = AC = $\sqrt{(7-3)^2+(13-5)^2}$

$= \sqrt{80}$ units.

$\because$ area of a square = $\frac{1}{2}$ (diagonal)^2

Its area = $\frac{1}{2}\left(\sqrt{80}\right)^2$ = 40 sq.units Ans: (40)

32. Since one of the two lines is parallel to Y-axis and other is parallel to X-axis
$\therefore$ The angle between the lines is 90°. Ans: (90)

33. The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\therefore$ required distance = $\frac{|3(2)+4(3)+10|}{\sqrt{3^2+4^2}} = \frac{28}{5}$ units

Choice (B)

34. Let A be (x, y). Given (X, Y) = (2, 4) and (h, k) = (–2, –3)
x = X + h , y = Y + k
$\Rightarrow$ x = 2 –2 =0 and y = 4 – 3 = 1
$\therefore$ (x, y) = (0, 1) Choice (B)

35. The equations relating the coordinates are x = X + h, y = Y + k
Here (x, y) = (4, –2) and (h, k), = (–7, 5)
4 = X –7and–2 = Y + 5
X = 11, Y = –7 and (X, Y) = (11, –7) Choice (D)

36. radius = $\sqrt{g^2+f^2-c} = \sqrt{(-4)^2+(3)^2+11} = 6$

circumference = $2\left(\frac{22}{7}\right)(6) = 12\pi$

Ans: (12)

37. Centre and radius of the circle $(x-a)^2 + (y-b)^2 = r^2$ is (a, b) and r. Here the centre is (3, –2) and r = 6.
Choice (C)

38. The centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is (–g, –f) which is (–4, 3). Choice (A)

39. The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\sqrt{g^2+f^2-c}$. Here, it is $\sqrt{(3)^2+(-2)^2+23} = 6$
Area = $\pi(6)^2 = 36\pi$ Choice (D)

40. Diameter of the circle is $2\sqrt{g^2+f^2-c}$

$= 2\sqrt{(-3)^2+(4)^2+56} = 18$ Ans: (18)

Exercise – 6(a)

Solutions for questions 1 to 19:

1. Centre of the circle $x^2 + y^2 = a^2$ is (0, 0).
Distance = $\sqrt{24^2+7^2} = \sqrt{576+49}$

$= \sqrt{625}$ = 25 units. Ans: (25)

2. Slope = $\frac{y_2-y_1}{x_2-x_1} = \frac{a-(a+b)}{-b-(a-b)}$

$= \frac{a-a-b}{-b-a+b} = \frac{-b}{-a} = \frac{b}{a}$ Choice (A)

3. Equation of a line parallel to y-axis is of form x = k
As, the line passes through the point $\left(\frac{7}{3}, -2\right)$;
$\Rightarrow$ k = 7/3
$\therefore$ Equation of line is x = 7/3 i.e. 3x – 7 = 0. Choice (D)

4. 5x – y + 6 = 0 $\rightarrow$ (1)
4x + 3y + 1 = 0 $\rightarrow$ (2)
Solving (1) and (2), we get x = –1 and y = 1
$\therefore$ (–1, 1) lies in the 2nd quadrant. Choice (B)

5. Centroid = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{2+4+0}{3}, \frac{8-2+6}{3}\right)$ = (2, 4). Choice (D)

6. Let the x-axis divide the line segments in the ratio m : n, at (x, 0).
$\therefore$ (x, 0) = $\left(\frac{m(1)+n(4)}{m+n}, \frac{m(1)+n(7)}{m+n}\right) \Rightarrow \frac{7n+m}{m+n} = 0$

$\Rightarrow$ 7n + m = 0 $\Rightarrow \frac{m}{n} = \frac{-7}{1}$
$\therefore$ the required ratio is 7 : 1 externally. Choice (C)

7. If a set of three points are collinear, then the area of triangle formed by joining these points is equal to zero.
$\Delta = \frac{1}{2}\begin{vmatrix} 1-3 & 1-7 \\ 7-3 & 7-k \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -6 \\ 4 & 7-k \end{vmatrix}$
$\Rightarrow$ (–2) (7 – k) – 4 (–6) = 0
$\Rightarrow$ – 14 + 2k = –24 $\Rightarrow$ 2k = –10 $\Rightarrow$ k = –5. Ans: (–5)

8. Any collinear point will lie on the line formed by the given points. The equation of the line joining (1, 3) and (3, 7) is
$(y-3) = \frac{7-3}{3-1}(x-1)$
$\Rightarrow$ y – 3 = 2x – 2 $\Rightarrow$ 2x – y + 1 = 0
The point (2, 5) satisfies the equation. Choice (D)

9. Area = $\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1-3 & 1-(-1) \\ 1-5 & 1-2 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 2 \\ -4 & -1 \end{vmatrix}$

$= \frac{1}{2}[2-2(-4)] = \frac{1}{2}\times 10$ = 5 sq. units. Ans: (5)

10. Area formed by the line ax + by + c = 0 and co-ordinates axes is given by $\frac{1}{2}\frac{c^2}{|ab|}$
Given line : 4x – 3y = 24, a = 4, b = –3 and c = –24
$\therefore$ Area = $\frac{1}{2}\cdot\frac{(-24)^2}{(4)(-3)} = \frac{1}{2}\cdot\frac{(24)^2}{12}$ = 24 sq. units.
Ans: (24)

11. The distance between (–1, –1) and $\left(\sqrt{3}, -\sqrt{3}\right)$
$= \sqrt{\left(-1-\sqrt{3}\right)^2+\left(-1+\sqrt{3}\right)^2} = \sqrt{2(1+3)} = 2\sqrt{2}$
The distance between ($\sqrt{3}$, $-\sqrt{3}$) and (1, 1)
$= \sqrt{\left(\sqrt{3}-1\right)^2+\left(-\sqrt{3}-1\right)^2} = \sqrt{2(3+1)} = 2\sqrt{2}$
The distance between (–1, –1) and (1, 1)
$\sqrt{(-1-1)^2+(-1-1)^2} = \sqrt{4+4} = 2\sqrt{2}$
Since the distances between the points taken in pairs are equal, they form an equilateral triangle. Choice (D)

12. The vertex containing the right angle is the point of intersection of perpendicular lines
i.e., 3x + y – 4 = 0 and x – 3y + 2 = 0.
Solving the above equations we get x = 1, y = 1.
Choice (B)

13. Roots of the quadratic equation $x^2 - 7x + 6 = 0$ are 6, 1.

$\therefore$ If a = 6, b = 1, then the equation is $\frac{x}{6} + \frac{y}{1} = 1$

$\Rightarrow x + 6y = 6$ and if a = 1, b = 6, then the equation is $\frac{x}{1} + \frac{y}{6} = 1$

$\Rightarrow 6x + y = 6$. Choice (D)

14. Let the fourth vertex be D(x, y)
Since the diagonals bisect each other in a parallelogram, the midpoint of AC coincides with the midpoint of BD.

$\therefore \frac{-1 + x}{2} = \frac{2 + 3}{2}$

$\Rightarrow x = 6$ and $\frac{3 + y}{2} = \frac{4 - 2}{2} \Rightarrow y = -1$

$\therefore D(x, y) \equiv (6, -1)$ Choice (D)

15. The equation of a line making equal intercepts is of the form $\frac{x}{a} + \frac{y}{a} = 1$ (since a = b)

$\Rightarrow x + y = a\ (a \neq 0)$, hence $x + y - 5 = 0$ Choice (A)

16. $x - 2y = 4 \Rightarrow 3x - 6y = 12 \rightarrow (1)$
$-3x + 6y = -2 \Rightarrow 3x - 6y = 2 \rightarrow (2)$
Distance between (1) and (2) is

$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} = \frac{|12 - 2|}{\sqrt{36 + 9}} = \frac{10}{3\sqrt{5}} = \frac{2\sqrt{5}}{3}$ units.

Choice (C)

17. $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|2.1 - 1.4 + 7|}{\sqrt{4 + 1}} = \frac{5}{\sqrt{5}} = \sqrt{5}$ units.

Choice (B)

18. Since the lines are parallel their slopes should be equal.

$\Rightarrow \frac{\sqrt{7}}{\sqrt{3}} = \frac{-\sqrt{3}}{k}; \Rightarrow \sqrt{7}k = -3$

$\Rightarrow k = \frac{-3\sqrt{7}}{7}$ Choice (B)

19. The given parallel lines represent the parallel sides of the squares hence the distance between them gives the length of the side.

$d = \frac{|3 + 3|}{\sqrt{12^2 + 3^2}} = \frac{6}{\sqrt{153}}$

$\therefore$ Area $= d^2 = \frac{36}{153} = \frac{4}{17}$ sq. units. Choice (D)

Solutions for question 20:

20. (i) The given line passes through (0, 5) and (3, 0).

$\therefore y - 5 = \frac{0 - 5}{3 - 0}(x - 0)$

$\Rightarrow 3y - 15 = -5x \Rightarrow 5x + 3y - 15 = 0$ Choice (C)

(ii) The line is passing through origin and (3, 1).

$\therefore$ Hence the equation is $y = \frac{1}{3}x$

$\Rightarrow x - 3y = 0$ Choice (A)

(iii) $m = \tan\theta = \tan 45° = 1$
The line passes through (4, 0) and makes an angle of 45° with x-axis.
$\therefore y = mx + c$
$y = x + c$ (since m = 1)
$0 = 4 + c \Rightarrow c = -4$
$\therefore$ The equation is $x - y = 4$ Choice (B)

(iv) The line makes an intercept of 4 on either axis

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow x + y = 4$. Choice (D)

Solutions for questions 21 to 35:

21. $\cos\theta = \frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}} = \frac{+\sqrt{3} + \sqrt{3}}{\sqrt{1+3}\sqrt{1+3}}$

$= \frac{+2\sqrt{3}}{4} = \frac{+\sqrt{3}}{2}$

$\therefore \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

Alternative method:

Slope of $x + \sqrt{3}y + 6\sqrt{3} = 0$,

$m_1 = \frac{-1}{\sqrt{3}}$ or $\theta_1 = 150°$ and slope of $y + \sqrt{3}x + 2 = 0$,

$m_2 = -\sqrt{3}$ or $\theta_2 = 120°$.

$\therefore$ The angle between the lines = 150 – 120 = 30°

Choice (B)

22. $2x + y - k = 0 \rightarrow (1)$
$4x + y = 13 \rightarrow (2)$
$x - 3y = 13 \rightarrow (3)$
Solving (2) and (3), we get x = 4, y = – 3
Substituting in (1), 8 – 3 – k = 0
$\Rightarrow k = 5$ Ans: (5)

23. Area of quadrilateral required $= \begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$

$\frac{1}{2}\begin{vmatrix} -2-2 & -3-3 \\ -2-2 & 3-(-3) \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -4 & -6 \\ -4 & 6 \end{vmatrix} = \frac{1}{2}((-4)(6) - (-4)(-6))$

= 24 sq.units. Ans: (24)

24. $\therefore$ slope of $L_1 \equiv 2x + 3y - 8 = 0$ is $\frac{-2}{3}$ and slope of $L_2 \equiv kx - 9y + 24 = 0$ is $\frac{k}{9}$

We know that two lines will not intersect only if their slopes are equal.

$\frac{-2}{3} = \frac{k}{9} \Rightarrow k = -6$. Choice (C)

25. slope of first line $= \frac{10 - 7}{3 - 2} = 3$.

slope of second line $= \frac{-1}{3}$

Required equation is $\frac{-1}{3} = \frac{y - 6}{x - 18}$

i.e., $3y + x - 36 = 0$. Choice (D)

26. The slope of $2y + x - 23 = 0$ is $\frac{-1}{2}$

$\therefore$ slope of XY $= \frac{-1}{\frac{-1}{2}} = 2$

The equation XY is $y - 5 = 2(x - 3)$
$y = 2x - 1$
midpoint XY is the point of intersection of $y = 2x - 1$ and $2y + x - 23 = 0$
$\therefore$ midpoint of XY is (5, 9).

Let Y be (a, b), $\left(\frac{3 + a}{2}, \frac{5 + b}{2}\right) = (5, 9)$

(a, b) = (7, 13) Choice (A)

27. The equations relating the coordinates are $x = X + 1$, $y = Y - 1$
Let $f(x, y) = 2x - 3y + 7$ then the transformed equation is given by f(X, Y)
$= 2(X + 1) - 3(Y - 1) + 7 = 0$
$\Rightarrow 2X - 3Y + 12 = 0$ Choice (A)

28. When the axes are rotated through an angle of 45° in anti clockwise direction then the equations relating the coordinates are

x = Xcos45°– Ysin45° and y = Xsin45° + Ycos45°

$x = \frac{X-Y}{\sqrt{2}}$, and $y = \frac{X+Y}{\sqrt{2}}$

∴ The transformed equation of f(x, y) = 0, is $f\left(\frac{x-y}{\sqrt{2}}, \frac{x+y}{\sqrt{2}}\right) = 0$

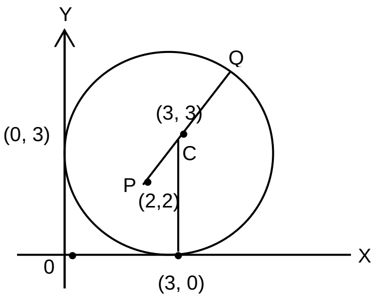
i.e., $\frac{X-Y}{\sqrt{2}} - 2\left(\frac{X+Y}{\sqrt{2}}\right) + 5 = 0$

$\Rightarrow X - Y - 2(X+Y) + 5\sqrt{2} = 0$

$\Rightarrow \quad -X - 3Y + 5\sqrt{2} = 0 \qquad \Rightarrow X + 3Y - 5\sqrt{2} = 0$

Choice (D)

29.



The equation of the circle is $(x-3)^2 + (y-3)^2 = 9$ the center is (3,3) and radius =3

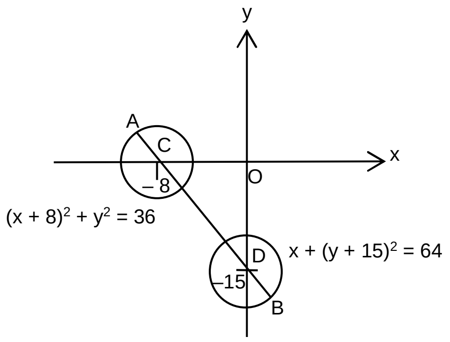
P(2,2) is inside the circle as $(2-3)^2+(2-3)^2<9$

∴The largest possible distance from P to any point on the circle is PQ, where Q is the end of the diameter passing through P

$\therefore PQ = PC + CQ = \sqrt{(3-2)^2 + (3-2)^2} + 3$ (∵CQ = r = 3)

$\therefore PQ = 3 + \sqrt{2}$ Choice (A)

30. The maximum distance between any point on one circle C_1 and any point on another circle C_2 is that between the points which are on the line joining the centers of the circles (between A and B in the figure below).



AB = AC + CD + BD i.e, 6 + CD + 8

$CD = \sqrt{CO^2 + OB^2} = 17$

∴ Maximum distance is 31. Ans: (31)

31. The points we have to consider must satisfy the condition $x^2 + y^2 < 9$.

If $x^2 = 0$, x = 0 and $y^2 < 9$. y = 0, ± 1, ± 2

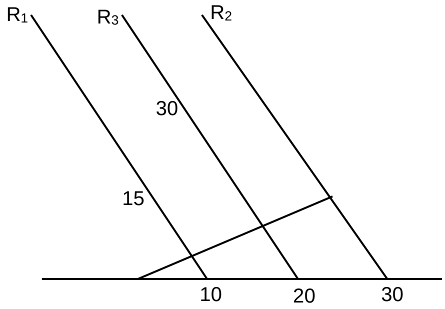
If $x^2 = 1$, x = ± 1 and $y^2 < 8$. y = 0, ± 1, ± 2

If $x^2 = 4$, x = ± 2 and $y^2 < 5$. y = 0, ± 1, ± 2

A total of 25 points satisfy the given condition.

Choice (B)

32.



R_2 is parallel to R_1. Let its equation 3x + 2y = c

R_3 is exactly midway between R_1 and R_2 ∴ Its equation is $3x + 2y = \frac{30+c}{2}$

Given : equation of R_3 is 3x + 2y = 45

$\frac{30+c}{2} = 45$

c = 60

R_1 is closer to the origin than R_2

distance between R_1 and the origin = $\left|\frac{30}{\sqrt{3^2+2^2}}\right| = \frac{30}{\sqrt{13}}$

distance between R_2 and the origin

$= \left|\frac{60}{\sqrt{3^2+2^2}}\right| = \frac{60}{\sqrt{13}}$ Ans: 0.5

33. The point of intersection has integral coordinates. Let this point be (x_0, y_0)

$4x_0 + 5y_0 = 26$ and $y_0 = kx_0 + 2$

$4x_0 + (kx_0 + 2) = 26 \Rightarrow x_0 = \frac{16}{4+5k}$

x_0 is an integer

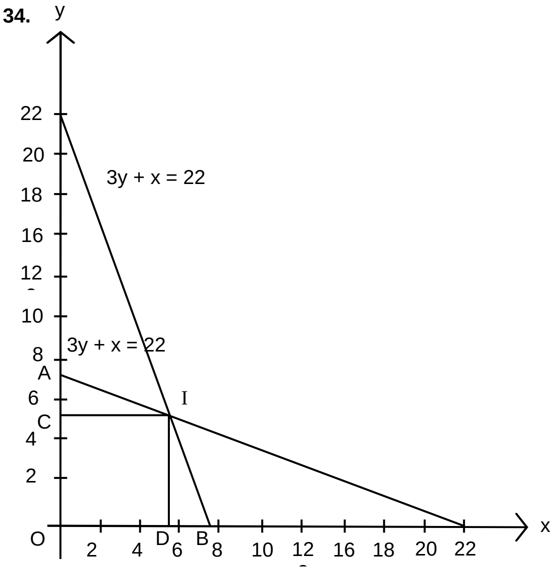
∴ 5 + 5k is a factor (positive or negative) of 16. Also it is odd.

4 + 5k = ±1

$k = \frac{-3}{5}$ or –1

k has only one integer value. It can be verified that when k = –1, y_0 is an integer. Ans: (1)

34.



At I, x + 3y = 22 and 3x + y = 22

Let I be (x_0, y_0)

$x_0 + 3y_0 = 22$ and $3x_0 + y_0 = 22$

Solving these $(x_0, y_0) = \left(\frac{11}{2}, \frac{11}{2}\right)$

The convex quadrilateral formed by the given lines and the coordinate axes is OAIB = S

S = Area of AIO + Area of BIO

$= 2\text{Area of AIO} = (AO)\,(IC) = \frac{22}{3}\left(\frac{11}{2}\right) = \frac{121}{3}$

$\therefore$ 3S = 121 Ans: (121)

35. The distance from the origin to the line 8x – 15y + 140 = 0 is $\left|\frac{8(0)-15\,(0)+140}{\sqrt{8^2+15^2}}\right|$ i.e. $\frac{140}{17}$ i.e. $8\,\frac{4}{17}$

The circle $x^2 + y^2 = 64$ is centered at the origin and has a radius of 8

As the shortest distance from the origin to the line is more than the radius, the line does not meet the circle at even one point. Ans: (0)

Exercise – 6(b)

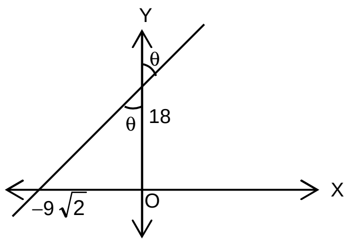
Solutions for questions 1 to 45:

1. Centre of the given circle = (5, 4)

The distance between (8, 8) and (5, 4) = $\sqrt{(8-5)^2+(8-4)^2}$

= 5 units Ans: (5)

2.



Required angle = $\theta = \text{Tan}^{-1}\left(\frac{9\sqrt{2}}{18}\right) = \text{Tan}^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Choice (C)

3. It is given that the parallel lines x + 3y + 7 = 0

$\Rightarrow$ 4x + 6y + 14 = 0 and 4x + 6y + 15 = 0.

Distance between the lines = $\frac{|14-15|}{\sqrt{4^2+6^2}} = \frac{1}{2\sqrt{13}}$

Choice (B)

4. Given, the lines 3x + 4y + 8 = 0 and 12x – 5y + 9 = 0

Angle between the lines

$= \text{Cos}^{-1}\left(\frac{(3)(12)+(4)(-5)}{\sqrt{(3^2+4^2)(12^2+(-5)^2)}}\right) = \text{Cos}^{-1}\left(\frac{16}{65}\right)$

Choice (D)

5. AB = 3CD ($\because$ C and D trisect AB)

AB = $\sqrt{(3-6)^2+(7-13)^2} = 3\sqrt{5}$

CD = $\sqrt{5}$ Choice (A)

6. Let P = (3, –4), Q = (–3, 4) and the third vertex be R.

PQ = $\sqrt{(-3-3)^2+(4-(-4))^2} = 10$

from Choice (A) if R $(4\sqrt{3}\,.3\sqrt{3})$ then

RP = $\sqrt{(3-4\sqrt{3})^2+(-4-3\sqrt{3})^2}$

$= \sqrt{9-24\sqrt{3}+48+16+24\sqrt{3}+27} = 10$

$\therefore$ Choice (A) can be the third vertex, and none of the other choices satisfy the properties of equilateral triangle.

Choice (A)

7. We know that centroid divides each median in the ratio 2 : 1 from vertex.

Given A = (8, 4), D(12, 8)

$\therefore$ Centroid = $\left(\frac{2(12)+8}{3}, \frac{2(8)+4}{3}\right) = \left(\frac{32}{3}, \frac{20}{3}\right)$

Choice (B)

8. Let the third vertex be (x_3, y_3)

Centroid = $\left(\frac{5+7+x_3}{3}, \frac{6+9+y_3}{3}\right)$

$(0, 0) = \left(\frac{12+x_3}{3}, \frac{15+y_3}{3}\right)$

$x_3 = -12$, $y_3 = -15$

$\therefore$ Required vertex is (–12, –15). Choice (D)

9. The slope of the given line 3x + 4y + 11 = 0 is $\frac{-3}{4}$

The slope of the line parallel to it = $\frac{-3}{4}$ ($\because$ parallel lines have equal slopes)

Required equation is y – 4 = $\frac{-3}{4}$ (x – 3)

4y – 16 = –3x + 9

i.e., 4y + 3x – 25 = 0 Choice (C)

10. Equation of the line with equal intercepts is $\frac{x}{a}+\frac{y}{a}=1$

If it is passes through the point (5, 12) then 5 + 12 = a $\Rightarrow$ a = 17

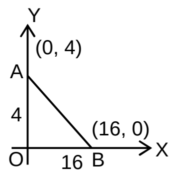
$\therefore$ Equation of the required line is x + y = 17 and intercepts the line are 17, 17 Ans: (17, 17)

11. When x + 4y – 16 = 0 intersects the X-axis, y = 0.

$\therefore$ x + 4(0) – 16 = 0 i.e., x = 16.

When it intersects the Y-axis, x = 0

0 + 4y – 16 = 0 i.e., y = 4



Area of triangle AOB = $\frac{1}{2}$ (OA)(OB) = $\frac{1}{2}$(4)(16)

= 32 sq.units Choice (C)

12. $y^2 - 9y + 18 = 0$

$\Rightarrow y^2 - 6y - 3y + 18 = 0$

i.e., (y – 6) (y – 3) = 0

i.e., y = 6 or 3 by taking one root as slope and the other as Y – intercept

$\therefore$ the required equation of the lines are y = 6x + 3 or y = 3x + 6 Choice (D)

13. The point of intersection of the lines 3x + 4y = 14 and 2x + 3y = 10 is (2, 2)

Given the three lines are concurrent

$\Rightarrow$ point (2, 2) lies on the line 5x + ky = 6.

$\Rightarrow$ 5(2) + k(2) = 16 $\Rightarrow$ k = 3

Ans: (3)

14. The point of intersection of 2x + 3y + 4 = 0, 5x – 7y –19 = 0 is (1, –2)

(1, –2) lies on the line 4x + ky + 6 = 0

4(1) – 2k + 6 = 0

k = 5. Choice (A)

15. Area of triangle = $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$

Let D, E, F be the midpoints = D(1, 5) E(3, 6), F(4, 8)

Area = $\frac{1}{2}\begin{vmatrix} 1-3 & 3-4 \\ 5-6 & 6-8 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} = \frac{1}{2}|4-1|$

= 3/2 sq.units.

$\therefore$ Area of Δle ABC = $4 \times \frac{3}{2}$ = 6 sq.units. Ans: (6)

16. The area of quadrilateral = $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1-5 & 3-3 \\ 3-3 & 1-5 \end{vmatrix} = \frac{1}{2}|16-0| = 8$ sq.units. Choice (C)

17. Given 2x + 3y + 6 + k (x – 4y + 8) = 0

$\Rightarrow$ (2 + k) x + (3 – 4k)y + 6 + 8k = 0 This line is parallel to the Y-axis.

The coefficient of y should be zero.

$3 - 4k = 0 \Rightarrow k = \frac{3}{4}$ Ans: (0.75)

18. Gradient of the line joining the points A (3, 6) and B (4, 9) is

$\frac{9-6}{4-3} = 3$

And slope of the line joining the points A (3, 6) and C (5, k)

$= \frac{k-6}{5-3}$

Given A, B, C are collinear slope of AB = slope of AC

$\Rightarrow \frac{k-6}{2} = 3$

$\therefore$ k = 12 Choice (A)

19. $p^2 - p - 12 = 0$

(p – 4) (p + 3) = 0

p = 4 or –3

If the slope is 4, and x - intercept is –3, then the equation of the line is y = 4x + c where c is the y–intercept. When y = 0, x = –3. $\therefore$ c = 12. So, the line is y = 4x + 12, if the slope is –3, and x – intercept is 4. It can be similarly shown above that the equation of the line can be y = –3x + 12

Choice (C)

20. Gradient of the line joining the points (4, 7) and (6, 11) is

$= \frac{11-7}{6-4} = 2$

Let (x, y) be any point on the line. Then

$2 = \frac{y-7}{x-4} \Rightarrow y = 2x - 1$ Choice (D)

21. Let A=(4, –5), B = (0, 0) and C = (5, 4)

AB = $\sqrt{(0-4)^2 + (0-(-5))^2} = \sqrt{41}$

BC = $\sqrt{(5-0)^2 + (4-0)^2} = \sqrt{41}$

AC = $\sqrt{(5-4)^2 + (4-(-5))^2} = \sqrt{82}$

$\therefore$ AB = BC and $AB^2 + BC^2 = AC^2$

$\therefore$ the triangle is right angled and isosceles. Choice (B)

22. Given the parallel lines (opposite sides of a square) 8x + 4y – 12 = 0 and 2x + y + 4 = 0 $\Rightarrow$ 8x + 4y + 16 = 0.

Distance between opposite sides = $\left|\frac{-12-16}{\sqrt{8^2+4^2}}\right|$

$= \frac{7}{\sqrt{5}}$ perimeter = $4\left(\frac{7}{\sqrt{5}}\right) = \frac{28}{\sqrt{5}}$ Ans: (28)

23. The point of intersection of the lines 2x + 3y – 12 = 0 and 3x + 4y – 17 = 0 is

Given that (3, 2) lies on 4x + ay – 22 = 0.

$\Rightarrow$ 4(3) + a(2) – 22 = 0

$\therefore$ a = 5 Ans: (5)

24. Gradient of AB = tan30°= $\frac{1}{\sqrt{3}}$

equation of AB is y = $\frac{1}{\sqrt{3}}$ x + c where c is the y–Intercept.

It is passing through (3, 4).

$4 = \frac{1}{\sqrt{3}}(3) + c \Rightarrow 4 - \sqrt{3} = c$

$\therefore y = \frac{1}{\sqrt{3}}x + 4 - \sqrt{3}$

At B, x = 0 $\therefore$ y = $4 - \sqrt{3}$

$\therefore$ AB = $\sqrt{(3-0)^2 + \left(4 - \left(4-\sqrt{3}\right)\right)^2} = 2\sqrt{3}$ Choice (B)

25. The ratio in which the X – axis divides the line joining the points (x_1, y_1) and (x_2, y_2) is $-y_1 : y_2$.

Here $y_1 = -1$, $y_2 = 3$

$\therefore$ Required ratio is 1 : 3

$\therefore$ the X – axis divides in a 1 : 3 ratio internally.

Choice (C)

26. Let its x–intercept be a. Its y–intercept =14–a

then the equation of the line is

$\frac{x}{a} + \frac{y}{14-a} = 1$

(3, 4) lies on it. $\Rightarrow \frac{3}{a} + \frac{4}{14-a} = 1$

$42 - 3a + 4a = 14a - a^2$

$a^2 - 13a + 42 = 0$

(a – 6) (a – 7) = 0

If x – intercept is 6, then y – intercept is 8. So, the equation of the line is $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$

When x – intercept is 7 then y – intercept is also 7.

then the equation of the line is x + y = 7. Choice (D)

27. Parallel lines have equal gradients. True.

Perpendicular lines have the product of their gradients as –1. True.

A line parallel to the x – axis has its gradient as 0. So, it is also true.

$\therefore$ None of the three statements is false. Choice (D)

28. Given the lines 4x – 3ky + 4 = 0 $\rightarrow$ (1) and 2x – 5y + 1 = 0 $\rightarrow$ (2) which intersect at a point whose x-coordinate is twice its y - coordinate.

So, let the point of intersection be (2p, p).

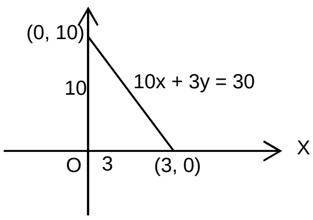
As (2p, p) lies on (2), 4p – 5p + 1 = 0 $\Rightarrow$ p = 1

$\therefore$ The point of intersection is (2, 1).

Now this point (2, 1) also lies on (1).

$\Rightarrow$ 4(2) – 3k (1) + 4 = 0 $\Rightarrow$ k = 4 Ans: (4)

29. At the intersection of the line with the X – axis, y = 0. $\therefore$ 10x = 30



$\Rightarrow$ x = 3

At the intersection of the line with the Y – axis,

x = 0. $\Rightarrow$ 3y = 30

$\therefore$ y = 10.

$\therefore$ Required area = $\frac{1}{2}$ (10)(3) = 15 sq units Choice (C)

30. Required point = $\left(\frac{(2)(2)-(3)(-1)}{2-3}, \frac{(2)(6)-(3)(4)}{2-3}\right)$

= (–7,0) Choice (D)

31. Area of the triangle formed

$= \frac{1}{2}\begin{vmatrix}[(a+1)-(a-1)] & [(a-1)-(a+3)] \\ [(a+2)-(a+1)] & [(a+1)-(a-3)]\end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix}2 & -4 \\ 1 & 4\end{vmatrix} = \frac{1}{2}|2(4)-(-4)(1)|$

$= \frac{1}{2}|8+4| = 6$ sq units Ans: (6)

32. Let the x-intercept be a, then the y – intercept is a + 12.

Then the equation of the line is $\frac{x}{a}+\frac{y}{a+12} = 1$

(2, 12) is a point on it $\Rightarrow \frac{2}{a}+\frac{12}{a+12} = 1$

i.e., $2a + 24 + 12a = 12a + a^2$

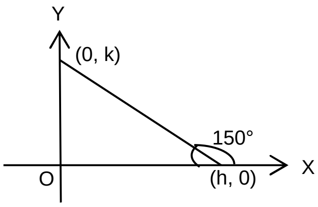
$a^2 - 2a - 24 = 0 \Rightarrow (a-6)(a+4) = 0 \Rightarrow a = 6$ or $a = -4$

When a = 6 then b = 18. Then the slope of the line is $\frac{-b}{a} = \frac{-18}{6} = -3$, which is negative.

When a = –4 then b = 8. Then the slope of the line is $\frac{-b}{a} = \frac{-8}{-4} = 2$

As the slope is positive, the slope of the line is 2. Ans: (2)

33.



Gradient of the line = tan 150° = $-\frac{1}{\sqrt{3}}$

Gradient of the line = $\frac{k-0}{0-h} = -\frac{1}{\sqrt{3}} \Rightarrow k = \frac{1}{\sqrt{3}}h$

Given h + k = 3 $\Rightarrow \frac{h}{\sqrt{3}} + h = 3$

$h = \frac{3\sqrt{3}}{\sqrt{3}+1} = \frac{3\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3\sqrt{3}(\sqrt{3}-1)}{2}$ Choice (A)

34. Gradient of 6x – 3y – 3 = 0 i.e., y = 2x –1 is 2. Required equation will have a gradient of $-\frac{1}{2}$. Its equation is

$-\frac{1}{2} = \frac{y-1}{x-1} \Rightarrow 2y + x = 3.$

Choice (B)

35. The point of intersection of 2x + 3y – 13 = 0 and 3x + 2y – 12 = 0 is (2, 3)

Line 2x + 6y + k = 0 passes through (2, 3)

$\Rightarrow$ 2 (2) + 6 (3) + k = 0

k = –22 Choice (D)

36. The point of intersection of 2x + 3y – 8 = 0 and 2y – 3x – 1 = 0 is (1, 2).

Gradient of the required line = $\frac{4-2}{3-1} = 1$

Required equation is $1 = \frac{y-2}{x-1} \Rightarrow y = x + 1$

Choice (A)

37. Equation of the line passing through (1, 4) and (4, 1) is

$y - 4 = \frac{1-4}{4-1}(x-1)$ (Two point form)

$\Rightarrow y - 4 = -(x-1)$

$\Rightarrow x + y - 5 = 0.$ Choice (A)

38. Given points A (0, 0), B = (–2, 3) and C (6, –9)

slope of (AB) = $\frac{-3}{2}$

slope of (BC) = $\frac{-3}{2}$.

$\therefore$ The points are collinear and will form a straight line.

Choice (D)

39. The equation in the new system is f (X, Y) = aX + bY + C

$\therefore$ The equation in the original system is f(x, y) = 0, where

X = x– h and Y = y – k i.e., X = x +1 and Y = y – 2

$\therefore$ f(x, y) = a(x + 1) + b(y – 2) + c = ax + by + a – 2b + c = 0

Choice (B)

40. $f(X, Y) = X^2 + Y^2 = 2$

X = xcos60° + ysin60°, Y = –xsin 60° + y cos 60°

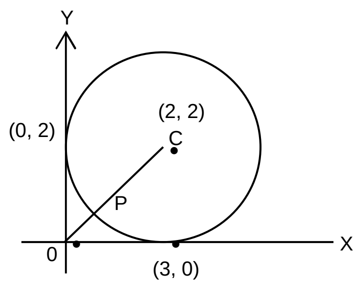
$X = \frac{x+\sqrt{3}y}{2}$, and $Y = \frac{-\sqrt{3}x+y}{2}$

$\therefore f\ \left(\frac{x+\sqrt{3}y}{2}\right)^2 + \left(\frac{-\sqrt{3}x+y}{2}\right)^2 = 2$

$\Rightarrow \frac{4x^2+4y^2}{4} = 2$

$\Rightarrow x^2 + y^2 = 2$ Choice (B)

41.



The given circle is shown in the figure above.

The shortest distance from origin to the circle is OP = OC – PC

Centre of the circle is C (2, 2)

$\therefore OC = \sqrt{2^2+2^2} = 2\sqrt{2}$ and PC is radius = 2

$\therefore$ The required distance OP = $2\sqrt{2} - 2 = 2(\sqrt{2}-1)$

Choice (D)

42. Any secant of a circle must be closer to the circle's centre than any tangent to the circle. In the given problem, the circle is centered at the origin. The origin must be closer to the secant than the tangent.

The distance between the origin and the line

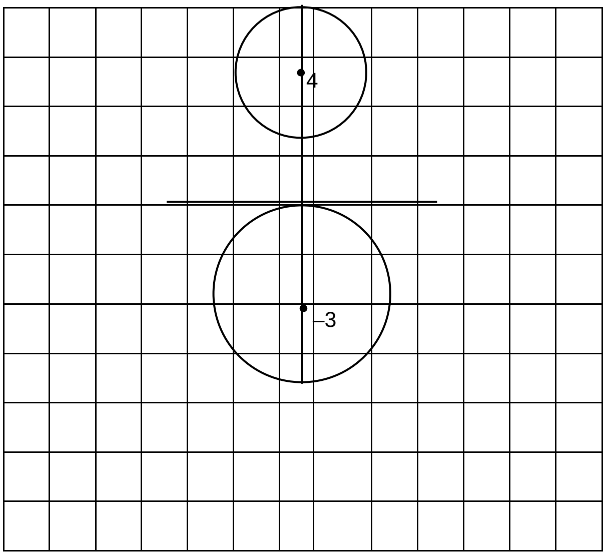
5x – 4y – 20 = 0 is $\frac{|5(0)-4(0)-20|}{\sqrt{5^2+(-4)^2}}$

i.e, $\frac{20}{\sqrt{41}}$. The distance between the origin and the line

5x – 4y + 40 = 0 is $\frac{|5(0)-4(0)+40|}{\sqrt{5^2+(-4)^2}}$ i.e $\frac{40}{\sqrt{41}}$

The line 5x – 4y – 20 = 0 is closer to the origin than the other line. $\therefore$ This must be secant and the other line must be tangent. Radius = Distance from the centre to the line 5x – 4y + 40 = 0. $\therefore$ Radius = $\frac{40}{\sqrt{41}}$ Choice (A)

43. The given circles are $x^2 + y^2 - 8y + 12 = 0$ and $x^2 + y^2 + 6y = 0$



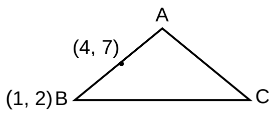
i.e., $x^2 + (y - 4)^2 = 2^2$ and $x^2 + (y + 3)^2 = 3^2$
centre $C_1(0, 4)$ and $r_1 = 2$ and C_2 (0, –3) and $r_2 = 3$
As the centers lie on the y – axis, PQ is the shortest distance between the circles.
$C_1 C_2 = 4 - (-3) = 7$
$PQ = C_1 C_2 - (r_1 + r_2) = 7 - (2 + 3) = 2$ Ans: (2)

44. 3x + 4y = 10 ----- (1) and my – x + 4 = 0 ------- (2) meet at only one point
Solving (1), (2), we have 3(my +4) + 4y = 10
$y = \frac{-2}{3m+4}$, $x = \frac{10m+16}{3m+4}$
y is integer only when m = –1 and m = –2.
For m = –1, y = –2, x = 6
For m = –2, y = 1, x = 2
$\therefore$ Only two integral values of m are possible. Ans: (2)

45. The points we have the consider must satisfy the condition $x^2 + 2y^2 < 24$.
x^2 and $2y^2$ must both be less than 24 i.e
$y^2 < 12$ and $x^2 < 24$
If $y^2 = 0$, $x^2 < 24$ $\therefore$ y = 0 and x = 0, ± 1, ± 2 ± 3, ± 4
If $y^2 = 1$, $x^2 < 22$. $\therefore$ y = ± 1 and x = 0, ± 1, ± 2 ± 3, ± 4
If $y^2 = 4$, $x^2 < 16$. $\therefore$ y = ± 2 and x = 0, ± 1, ± 2 ± 3
If $y^2 = 9$, $x^2 < 6$. $\therefore$ y = ± 3 and x = 0, ± 1, ± 2
A total of 9 + 2(9 + 7 + 5), i.e., 51 points satisfy the given condition. Ans: (51)

Solutions for questions 46 to 50:

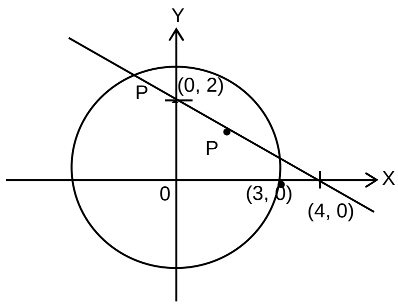
46.



Unless we know the coordinates of C, it is not possible to find the centroid.
$\therefore$ using both the statements also we cannot solve
Choice (D)

47. Using both the statements together we can find the equation of the line using point slope form and then we can check whether (7, 2) lies on L or not.
Hence, both statements together are sufficient.
Choice (C)

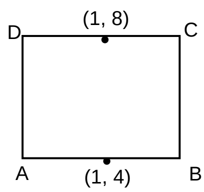
48. Even after using both the statements, P can lie in the 1st or in the 2nd quadrant.



Choice (D)

49. From statement I alone, the area cannot be found as the equation of only one side is known.
From statement II alone, the area cannot be found as the equations of only two sides are known.
From I and II together, the area can be found as all the three equations are known.
Choice (C)

50.



From statement I,
The distance between the midpoints of the sides is length of the side of square the length of the square = 4.
$\therefore$ Area = 16 sq units. Statement I alone is sufficient
From statement II
We can find the length of diagonal
From that we can find area also.
$\therefore$ Statement II alone is also sufficient
Hence, either of the statements is sufficient to answer the question.
Choice (B)

Chapter – 7
(Trigonometry)

Concept Review Questions

Solutions for questions 1 to 35:

1. $\left(\frac{2\pi}{3}\right)^c = \left(\frac{2\pi}{3}\right)^c \times \frac{180°}{\pi^c} = 120°$ Ans: (120)

2. $300° = 300° \times \left(\frac{\pi^c}{180°}\right) = \left(\frac{5\pi}{3}\right)^c$ Choice (B)

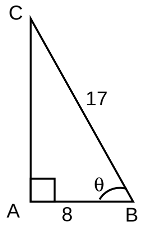
3. 1 revolution = 360° or $2\pi^c$ Choice (C)

4. Minute hand covers an angle of 6° per minute.
In 12 minutes it covers an angle of $12 \times 6 = 72°$
$= 72 \times \frac{\pi}{180} = \frac{2\pi}{5}$ Choice (C)

5. Hour hand covers an angle of $\frac{1}{2}°$ every minute.

In 30 minutes it covers $30 \times \frac{1}{2} = 15° = \frac{\pi}{12}$ Choice (A)

6. Let the right angled triangle be ABC. The given sides are AB = 8 and BC = 17 by Pythagoras theorem.
We have $AC^2 = BC^2 - AB^2 = 17^2 - 8^2$
$AC = \sqrt{225} = 15$
$\sin\theta = \frac{AC}{BC} = \frac{15}{17}$
And $\tan\theta = \frac{15}{8}$

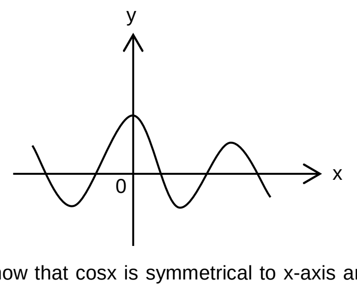


Choice (C)

7. We know that cosθ is positive in Q_1 and Q_4 and tanθ is negative in Q_2 and Q_4. ∴ θ is in Q_4 Choice (D)

8. 500° lies in Q_2 ∴ tan 500° < 0
–200° lies in Q_2 ∴ sin –200° (and cosec –200°) > 0
–500° lies in Q_3 ∴ tan –500° (and cot –500°) > 0
–400° lies in Q_4 ∴ cos –400° (and sec –400°) > 0
∴ The statement in choice (D) is false. Choice (D)

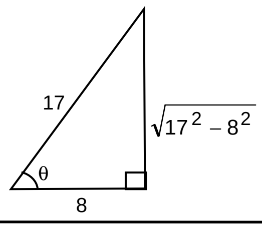
9.



We know that cosx is symmetrical to x-axis and meets the X-axis at infinite number of points. Choice (D)

10. $\frac{\sin\theta \times \cos\theta}{\cos\theta} = \tan\theta \frac{1}{\sec\theta} = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ Choice (C)

11.



Given $\sec\theta = \frac{17}{8}$ and $\theta \notin Q_1$
$\Rightarrow \theta \in Q_4$ (secθ is positive in Q_1 and Q_4)
As a cotθ is negative in Q_4, $\cot\theta = -\frac{8}{15}$ Choice (B)

12. a = cos10° – sin10° and b = cos70° – sin70°
if 0° < θ < 45°, sin θ < cosθ
if θ = 45°, sin θ = cosθ
if 45° < θ < 90°, sin θ > cosθ
∴ a > 0 and b < 0. Choice (C)

13. $(180 + \theta) \in Q_3$ and cot is positive in Q_3.
∴ cot (180 + θ) = cotθ Choice (B)

14. $(1 + \sin\theta)(1 - \sin\theta)\sec^2\theta$
$(1 - \sin^2\theta)\sec^2\theta = (\cos^2\theta)(\sec^2\theta) = 1$ Choice (D)

15. (a) cosec(330)° = cosec(360° – 30°)
= – cosec30° as $(360° - \theta) \in Q_4$ = –2 Ans: (–2)

(b) sec(1020)° = sec(3.360° – 60°) = sec60° = 2
$((360° - \theta) \in Q_4)$ Ans: (2)

16. If the angles of a triangle are in 1 : 2 : 3 ratio then the angles of the triangle are 30°, 60° and 90°.
The ratio of the sides of the triangle is $1 : \sqrt{3} : 2$.
Choice (D)

17. For any value of θ sinθ and cosθ lie between –1 and 1 whereas tanθ and cotθ vary from –∞ to ∞.
Cosecθ and secθ do not lie between –1 and 1
So $\text{cosec}\theta = \frac{1}{2}$ is not possible. Choice (D)

18. We know that $\sec^2\theta - \tan^2\theta = 1$
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
ab = 1 Choice (B)

19. Given $\text{cosec}^4\theta + \cot^4\theta - 2\text{cosec}^2\theta\cot^2\theta$
$= (\text{cosec}^2\theta - \cot^2\theta)^2 = 1^2 = 1$ Ans: (1)

20. Given secθ = –2 and $\cot\theta = -\frac{1}{\sqrt{3}}$
sin θ = cosθ tanθ
$= \frac{1}{\sec\theta.\cot\theta} = \frac{1}{(-2)\left(-\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{2}$ Choice (B)

21. $\text{cosec}\,\frac{3\pi}{4} = \text{cosec}\,135°$
cosec (180 – 45°) = +cosec45° = $\sqrt{2}$ Choice (A)

22. Given $\text{cosec}\theta - \cot\theta = \frac{1}{2} \to (1)$
∴ cosecθ + cotθ = 2 → (2)
$(\because \text{cosec}\theta + \cot\theta = \frac{1}{\text{cosec}\theta - \cot\theta})$
solving (1) and (2) we get $\text{cosec}\theta = \frac{5}{4}$
$\sin\theta = \frac{4}{5}$ Ans: (0.8)

23. Given, $\tan^2\theta + 2\sec^2\theta = \frac{59}{16}$
$\tan^2\theta + 2(1 + \tan^2\theta) = \frac{59}{16}$ $(\because \sec^2\theta = 1 + \tan^2\theta)$
$\tan^2\theta = \frac{9}{16} \Rightarrow \tan\theta = \pm\frac{3}{4}$ Ans: (0.75)

24. We know that $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$
From options, $\sec\theta = 2/5 \Rightarrow \cos\theta = 5/2$ is not possible.
Choice (D)

25. $\sec 46° = \sec(90-44)° = \text{cosec}\,44°$
$\text{cosec}\,46° = \text{cosec}(90-44) = \sec 44°$
$\therefore \sin 44° \sec 46° + \cos 44° \text{cosec}\,46°$
$\sin 44° \text{cosec}\,44° + \cos 44° \sec 44° = 1 + 1 = 2$ Ans: (2)

26. We know that $\tan(90-\theta) = \cot\theta$
$\tan 89° = \tan(90-1°) = \cot 1°$
$\tan 88° = \tan(90-2°) = \cot 2°$
$\tan 46° = \tan(90° - 44°) = \cot 44°$
$\therefore \tan 1° \tan 2° \tan 3° \ldots \tan 46° \tan 47° \ldots \tan 88° \tan 89°$
$= \tan 1° \tan 2° \ldots \tan 44° \cot 44° \cot 43° \ldots \cot 2° \cot 1°$
$= (\tan 1° \cot 1°)(\tan 2° \cot 2°)(\tan 3° \cot 3°) \ldots\ldots\ldots$
$(\cot 44° \tan 44°) = 1$ Ans: (1)

27. $\cos\beta = -\frac{4}{5} \Rightarrow 180° < \beta < 270° \Rightarrow \sin\beta < 0$ – (1)

$\sin^2\beta = 1 - \cos^2\beta = 1 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25}$

$\sin\beta = \frac{-3}{5}$ (From (1))

$\therefore \text{cosec}\,\beta = \frac{-5}{3}$ and $\cot\beta = \frac{\cos\beta}{\sin\beta} = \frac{-4/5}{-3/5} = \frac{4}{3}$

$\text{cosec}\,\beta + \cot\beta = \frac{-5}{3} + \frac{4}{3} = \frac{-1}{3}$ Choice (A)

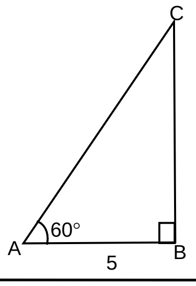
28. The complement of θ is $90-\theta$
The supplement of θ is $180-\theta$
Given, $90° - \theta = \frac{2}{5}(180° - \theta)$
$450° - 5\theta = 360° - 2\theta$
$90° = 3\theta \Rightarrow 30° = \theta$ Ans: (30)

29. Given, $\sin\alpha + \sin\beta + \sin\gamma = 3 \Rightarrow \sin\alpha = \sin\beta = \sin\gamma = 1$
$\alpha = \beta = \gamma = 90°$.
$\frac{\alpha}{2} = \frac{\beta}{2} = \frac{\gamma}{2} = 45°$
$\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot 45° + \cot 45° + \cot 45°$
$= 1 + 1 + 1 = 3$ Ans: (3)

30. We know that area of $\Delta BC = \frac{1}{2}$ absinc
Here $a = 5$, $b = 3\sqrt{2}$ and $\angle C = 45°$
$\Rightarrow \Delta = \frac{1}{2}.(5).(3\sqrt{2})\sin 45°$
$= \frac{1}{2}(.5)(3\sqrt{2})\frac{1}{\sqrt{2}} = 7.5$ sq. untis. Choice (D)

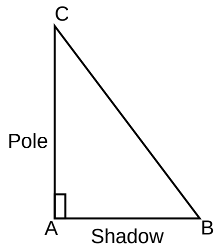
30. Given $a = 5$, $b = 3\sqrt{2}$ and $\angle C = 45°$
By cosine rule we have $c^2 = a^2 + b^2 - 2ab\cos C$
$= 5^2 + (3\sqrt{2})^2 - 2(5)(3\sqrt{2}).\cos 45 = 25 + 18 - 30$
$c^2 = 13 \Rightarrow c = \sqrt{13}$ Choice (B)

31.



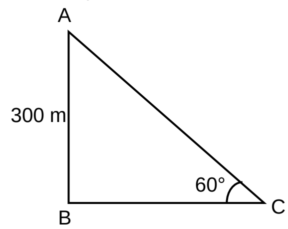
Let BC be the height of the tower.
AB = 50 cm and $\theta = 60°$
$\therefore \tan\theta = \frac{BC}{AB} \Rightarrow \tan 60° = \frac{BC}{50} = 50\sqrt{3}$ Choice (D)

32.



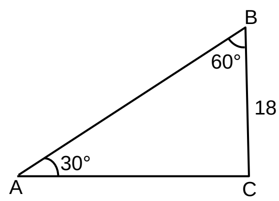
Let AC be the height and AB be the length of the shadow and θ be the angle of elevation of the sun.
If m is the height of the tower then
$AB = \frac{1}{\sqrt{3}}h$
$\tan\theta = \frac{AC}{AB} = \frac{h}{\frac{1}{\sqrt{3}}}$
$\Rightarrow \tan\theta = \sqrt{3}$ and $\theta = 60°$. Ans: (60)

33. Let AB denote the pole with B being its foot.
Let C denote the point.



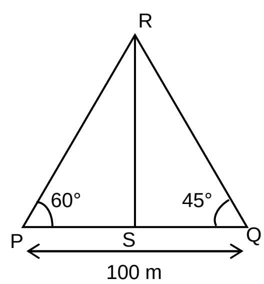
$\frac{AB}{BC} = \tan 60°$
$\frac{300}{BC} = \sqrt{3}$
$BC = \frac{300}{\sqrt{3}} = 100\sqrt{3}$ m
Required distance
$= 100\sqrt{3}$ m
Choice (C)

34.



Let AB be the ladder. Given, $\angle ABC = 60°$
$\therefore \angle BAC = 30°$
In ΔABC $\sin 30° = \frac{18}{AB}$
$AB = \frac{18}{\frac{1}{2}} = 36$ m Choice (A)

35. Let RS be the pole.
Let P and Q be points as shown above.
PS + SQ = 100
In ΔPSR, In ΔRSQ



$\tan 60° = \frac{RS}{PS}$
$\tan 45° = \frac{RS}{SQ}$
$PS = \frac{RS}{\tan 60°}$
$SQ = \frac{RS}{\tan 45°}$

$$\frac{RS}{\tan 60°} + \frac{RS}{\tan 45°} = 100$$

$$RS\left(\frac{1}{\sqrt{3}} + 1\right) = 100$$

$$RS = \frac{100\sqrt{3}}{\sqrt{3}+1} = \frac{100\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = 50\sqrt{3}(\sqrt{3}-1)$$

Choice (C)

Exercise – 7(a)

Solutions for questions 1 to 30:

1. Let the measures of angles of the given triangle be 20°, (20 + d)° and (20 + 2d)°, d being the common difference
$\therefore$ 20° + (20° + d) + (20° + 2d) = 180°
$\Rightarrow$ d = 40°
$\therefore$ The measure of the greatest angle
$= 20° + 2 \times 40° = 100° = \frac{5\pi^c}{9}$

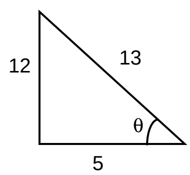
Ans: (5)

2. $\sin\theta + \cos\theta = \frac{-b}{a}$, $\sin\theta \cos\theta = \frac{c}{a}$
Now, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$
$\frac{b^2}{a^2} = 1 + \frac{2c}{a} \Rightarrow \frac{b^2}{a^2} = \frac{a+2c}{a}$
$\therefore b^2 = a^2 + 2ac$. Choice (A)

3. $\tan 22\frac{1}{2}^{\circ} = \sqrt{\frac{1-\cos 45°}{1+\cos 45°}} = \sqrt{\frac{1-(1/\sqrt{2})}{1+(1/\sqrt{2})}}$
$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{\sqrt{2}-1}{1} = \sqrt{2} - 1$. Choice (C)

4. $3\tan^2\theta - 1 = 0, \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$
since 'θ' lies in the III quadrant $\operatorname{cosec}\theta = -2$.
Choice (D)

5. $13\sin\theta - 12 = 0$
$\Rightarrow \sin\theta = \frac{12}{13}$ and 'θ' is acute



$\therefore \frac{\cot\theta - \tan\theta}{\sec\theta - \operatorname{cosec}\theta} = \frac{\frac{5}{12} - \frac{12}{5}}{\frac{13}{5} - \frac{13}{12}} = \frac{25-144}{156-65} = \frac{-17}{13}$.

Choice (B)

6. In the cyclic quadrilateral ABCD, sum of the opposite angles is 180° hence A + C = 180° and B + D = 180°
As A + C = 180°, cosC = (180° – A) = –cosA
As B + D = 180°, cosD = cos (180° – B) = –cosB
$\therefore$ cosA + cosB + cosC + cosD = 0.

Ans: (0)

7. Since sinθ and cosecθ are the roots of the equation $cx^2 + ax + b = 0$; $\sin\theta \cdot \operatorname{cosec}\theta = \frac{b}{c}$ (product of roots)
$\Rightarrow$ b = c, as $\sin\theta \cdot \operatorname{cosec}\theta = 1$.
Choice (A)

8. $\sec\theta + \tan\theta = p$ then $\sec\theta - \tan\theta = \frac{1}{p}$
(As $\sec^2\theta - \tan^2\theta = 1$)
$\Rightarrow 2\sec\theta = p + \frac{1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$
$\therefore \cos\theta = \frac{2p}{p^2+1}$. Choice (C)

9. Since x and $\frac{1}{x}$ are positive,
$x + \frac{1}{x} \geq 2\sqrt{x \times \frac{1}{x}}$ ($\because$ A.M ≥ G.M)
$\Rightarrow x + \frac{1}{x} \geq 2$.
Now $\sin\theta = x + \frac{1}{x}$,
$\Rightarrow \sin\theta \geq 2$, this is not possible as the maximum value that sinθ can take is 1.Hence, no such value of x exists.
Choice (B)

10. Consider an acute angle triangle ABC. Each angle is less than 90° and A + B + C = 180°.
If $\sin\theta > \frac{1}{\sqrt{2}}$, then θ > 45° and if
$\sin\theta < \frac{1}{\sqrt{2}}$, then θ < 45°
Choice (A): If $\sin A < \frac{1}{\sqrt{2}}$, $\sin B < \frac{1}{\sqrt{2}}$ and $\sin C < \frac{1}{\sqrt{2}}$, then A, B, C < 45° and hence A + B + C ≠ 180°
$\therefore$Choice (A) cannot be true.
Choice (B): Let $\sin A < \frac{1}{\sqrt{2}}$, $\sin B < \frac{1}{\sqrt{2}}$, hence A < 45° and B < 45°, or A + B < 90°.
This implies that C > 90°, this is not possible as ΔABC is acute angle triangle.
$\therefore$ Choice (B) is not true.
Choice (C): If $\sin A > \frac{1}{\sqrt{2}}$, $\sin B > \frac{1}{\sqrt{2}}$ and $\sin C > \frac{1}{\sqrt{2}}$, then A, B, C > 45°, this is possible for instance for A = B = C = 60°
$\therefore$ choice (C) can be true.
Choice (D): If $\cos A > \frac{1}{\sqrt{2}}$, $\cos B > \frac{1}{\sqrt{2}}$, then A, B < 45°, hence A + B < 90° Or C > 90°
This is not possible as Δ ABC is acute angle triangle.
$\therefore$ Choice (D) is not true. Choice (C)

11. x = cos50° + cos55° + cos60°
y = sin20° + sin25° + sin30°
Since cos60° = sin30°, we compare the remaining terms. The cosine function is a decreasing function from 0° to 90°, and since cos60° = $\frac{1}{2}$; cos50°, cos55° > $\frac{1}{2}$;
$\Rightarrow$ cos50° + cos55° > 1.
The sine function is a increasing function from 0° to 90°, and since sin30° = $\frac{1}{2}$; , sin20° + sin25° < 1.
$\therefore x > y \Rightarrow \frac{x}{y} > 1$. Choice (A)

12. $\Delta = \frac{1}{2}$; ab sinC = $\frac{1}{2}$; · 9·6 sin45° = $\frac{27}{\sqrt{2}}$ sq. units.
Choice (C)

13. Since the triangle is a right-angled triangle at C, the hypotenuse 'C' is the longest side.
$C = \sqrt{a^2+b^2} \Rightarrow c = \sqrt{65}$ units. Choice (B)

14. (i) The given graph is a reflection of the graph of y = cosx, in x-axis, hence the equation of the given graph can be obtained by changing the sign of y in y = cosx, i.e., y = –cosx. Choice (B)

(ii) The given graph represents the absolute values taken by cos2x (as cos(2π/4) = 1), hence the equation y = $|\cos 2x|$. Choice (B)

(iii) The graph represents a case where in the function y = sinx, the variables are interchanged, hence the equation x = siny. Choice (C)

15. $\dfrac{\cos(90-70)+\sin 50}{\sin 20+\cos(90-40)} = \dfrac{\sin 70+\sin 50}{\sin 20+\sin 40} = \dfrac{2\sin 60\cos 10}{2\sin 30\cos 10}$

$= \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$ Choice (B)

16. $\sin^6x + \cos^6x = (\sin^2x + \cos^2x)^3 - 3\sin^2x\cos^2x(\sin^2x + \cos^2x)$

$= 1 - \dfrac{3}{4}(\sin 2x)^2$

Max value = $1 - \dfrac{3}{4}.0$ ($\therefore$ maximum is obtained when sin 2x is minimum, i.e, sin 2x = 0)

$\therefore$Maximum value = 1

Ans: (1)

17. $h(y) = 3\left[|\sin y| + |\cos y|\right]$

$\sin^2x \le |\sin x|$

$\cos^2x \le |\cos x|$

$\therefore \sin^2x + \cos^2x \le |\sin x| + |\cos x|$

i.e $1 \le |\sin x| + |\cos x| \Rightarrow |\sin x| + |\cos x| \ge 1$

Also sinx + cos x = $\sqrt{2}\left(\dfrac{\sin x}{\sqrt{2}} + \dfrac{\cos x}{\sqrt{2}}\right)$

$= \sqrt{2}$ (sinx cos π/4 + cosx sinπ/4) = $\sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$

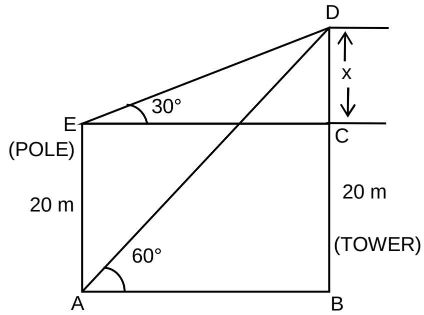
$\therefore$The maximum sum is $\sqrt{2}$; $|\sin x| + |\cos x| \le \sqrt{2}$

Given h(y) = 3 $\left[|\sin y| + |\cos y|\right]$

$\therefore$ minimum value of h(y) is = 3 and maximum value is $3\sqrt{2}$

$\therefore 3 \le h(y) \le 3\sqrt{2}$ Choice (A)

18. Let AE and BD represent the pole and the tower respectively.



In ΔDCE; tan 30° = $\dfrac{x}{AB}$, and

In ΔABD; tan 60°= $\dfrac{x+20}{AB}$ $\dfrac{x}{\tan 30°} = \dfrac{x+20}{\tan 60°}$

$\Rightarrow \sqrt{3}x = \dfrac{x+20}{\sqrt{3}}$

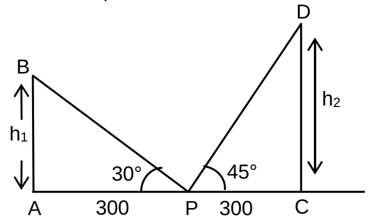
$\Rightarrow$ 2x = 20 or x = 10 m ;

$\Rightarrow$ x = 10.

$\therefore$height of tower is 30m.

Ans: (30)

19. Let AB and CD represent the two towers.



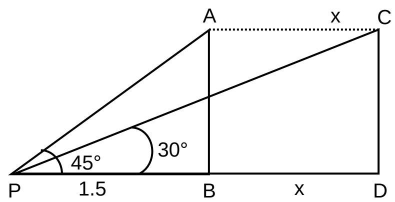
In ΔAPB, tan30°= $\dfrac{h_1}{300}$

$h_1 = \dfrac{300}{\sqrt{3}}$m

In ΔCPD, tan45°= $\dfrac{h_2}{300} \Rightarrow h_2 = 300$ m

$\therefore h_1 : h_2 = \dfrac{300}{\sqrt{3}} : 300 = 1 : \sqrt{3}$ Choice (B)

20.



Let A and C represent the initial and final positions of the aeroplane and P the point of observation.

Distance travelled = AC = x

Given, CD = AB = 1.5 km.

In ΔPCD; tan30°= $\dfrac{1\cdot 5}{1\cdot 5 + x}$

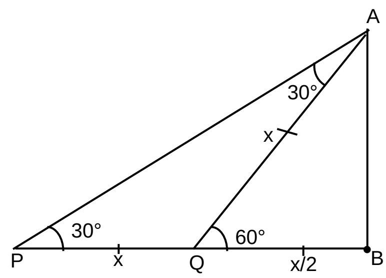
$\Rightarrow 1\cdot 5 + x = (1\cdot 5)\sqrt{3}$

$\therefore x = \sqrt{3}\ (1\cdot 5) - 1\cdot 5 = \dfrac{3}{2}\left(\sqrt{3}-1\right)$

speed = $\dfrac{\text{distance}}{\text{time}} = \dfrac{3}{2}\dfrac{\left(\sqrt{3}-1\right)}{9} = \dfrac{\sqrt{3}-1}{6}$ km/sec.

Choice (A)

21.



Let AB represent the lighthouse, P and Q be the points of observation. In Δ APQ,

∠APQ = ∠PAQ = 30°

$\Rightarrow$ AQ = PQ = x

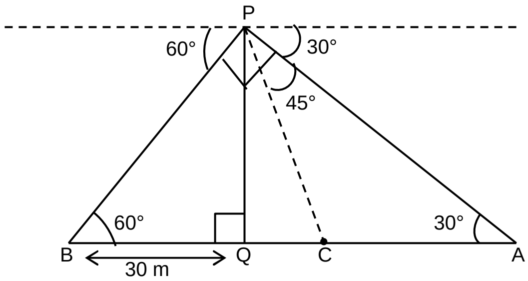
In Δ AQB,

cos60°= $\dfrac{BQ}{x} \Rightarrow BQ = \dfrac{x}{2}$

The steamer takes 10 minutes to travel from P to Q (i.e., a distance of x), then it will take $\dfrac{10}{2}$ i.e., 5 minutes to travel QB (x/2).

$\therefore$ It takes 15 minutes for the steamer to travel from P to B. Hence, at the instant when the steamer is at P, the time is 11:45 a.m. Ans: (11, 45)

22. Let PQ be the tower and A, B be the points of observation. Now, consider the figure below;



Given $\angle APB = 2\ \angle APC$.
$\therefore$ PC is the angle bisector of $\angle APB$.
$\Rightarrow$ AP : BP = AC : BC

In ΔBPQ, $\sin 60° = \dfrac{PQ}{BP}$ and in ΔAPQ, $\sin 30° = \dfrac{PQ}{AP}$

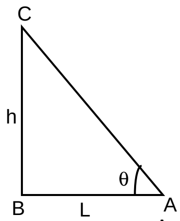
$\therefore$ AP : BP = $\sin 60°$: $\sin 30°$= $\sqrt{3}$: 1

$\Rightarrow$ AC : BC = $\sqrt{3}$: 1

But given, BC = 30 m, hence AC = $30\sqrt{3}$ m .

Choice (A)

23. Given L = $\sqrt{3}$ h



$\tan\theta = \dfrac{h}{L} = \dfrac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta = 30°$.

Ans: (30)

24. Given side AB subtends and angle of 60° at the top of the pole P.
$\therefore$ APB is an equilateral triangle.
$\therefore$ AP = 5m
E is mid point of diagonal

$\Rightarrow$ AE = $\dfrac{1}{2}5\sqrt{2}$ = $\dfrac{5}{\sqrt{2}}$ m

In right triangle APE

$PE^2 = AP^2 - AE^2 = 5^2 - \left(\dfrac{5}{\sqrt{2}}\right)^2$

$\therefore$ PE = $\dfrac{5}{\sqrt{2}}$ m or $2.5\sqrt{2}$ m.

Ans : (2.5)

25. From the given data in Δ PCD $\angle CPD = 30°$, CP = DP
$\angle PCD = \angle PDC = 75°$ and CD = 6m

$\therefore\ \dfrac{CD}{\sin\angle CPD} = \dfrac{PC}{\sin 75^o}$ (sine rule of triangle)

i.e., $\dfrac{6}{\sin 30^0} = \dfrac{PC}{\sin 75^0} \Rightarrow \dfrac{6}{1/2} \times \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ = PC

$\therefore$ PC = $6\left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)$ m

From Δ BCE, $\angle B = 90°$
$\Rightarrow EC^2 = EB^2 = 3^2 + 6^2$

Right EC = $3\sqrt{5}$ m

In Δ EPC, Height of the tower EP

$EP^2 = PC^2 - EC^2 = \left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)^2 - (3\sqrt{5})^2$

= 18 (3 + 1 + 2 $\sqrt{3}$) – 45 = 27 + 36 $\sqrt{3}$

= 9 (3 + 4 $\sqrt{3}$) m.

Choice (D)

26. $\left|\tan^2 \pi x\right| + \left|\cos^2 \pi y\right| = 0 \Rightarrow \left|\tan^2 \pi x\right| = \left|\cos^2 \pi y\right| = 0$

$\Rightarrow \tan\pi x = \cos\pi y = 0$
$\tan\pi x = 0$ implies $\pi x = 0, \pm\pi, \pm 2\pi$ ……and

$\cos\pi y = 0$ implies $\pi y = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}$..

$\therefore$x = 0, ±1, ±2, ……..and y = $\pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{5}{2}$ ……..

The points which satisfy $x^2 + y^2 \le 9$ must be such that $x^2 \le 9$ and $y^2 \le 9$ i.e, $|x| \le 3$ and $|y| \le 3$

The possible values of (x, y) are

$\left(0, \pm\dfrac{1}{2}\right), \left(0, \pm\dfrac{3}{2}\right), \left(0, \pm\dfrac{5}{2}\right), \left(\pm 1, \pm\dfrac{1}{2}\right),$

$\left(\pm 1, \pm\dfrac{3}{2}\right), \left(\pm 1, \pm\dfrac{5}{2}\right), \left(\pm 2, \pm\dfrac{1}{2}\right)\left(\pm 2, \pm\dfrac{3}{2}\right)$

$\therefore$ (x, y) has 26 possible values.

Ans: (26)

27. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\theta}}} = \sqrt{2+\sqrt{2+\sqrt{2(2\cos^2\theta)}}}$

$= \sqrt{2+\sqrt{2.2\cos^2\theta/2}} = \sqrt{2+2\cos\theta/2}$

$= \sqrt{2.2\cos^2\theta/4}$ = 2 cos θ/4

Choice (C)

28. 4 sinA + 6 cosB = 8 and 4 cosA + 6 sinB = 6
$(4 \sin A + 6 \cos B)^2 + (4 \cos A + 6 \sin B)^2 = 8^2 + 6^2$
$16(\sin^2 A + \cos^2 A) + 36 (\cos^2 B + \sin^2 B)$ + 48 sinA cosB + 48 sinB cosA = 100
16(1) + 36(1) + 48 sin(A + B) = 100
Sin(A + B) = 1$\Rightarrow$ A + B = 90° $\Rightarrow$ C = 90°.

Choice (C)

29. $\dfrac{5\sin Q + 4\sin R}{5\sin Q - 4\sin R} = \dfrac{33}{13}$

$\Rightarrow$ 65 sinQ + 52 sinR = 165 sinQ – 132 sinR

$\Rightarrow\ \dfrac{\sin Q}{\sin R} = \dfrac{46}{25}$

From the sin rule, $\dfrac{q}{\sin Q} = \dfrac{r}{\sin R}\ \left(= \dfrac{p}{\sin P}\right)$

$\dfrac{q}{r} = \dfrac{\sin Q}{\sin R} = \dfrac{44}{25}$

p, q, r are integers
Min (PQ + PR) = Min(q + r) = 46 + 25 = 71.

Ans: (71)

30. E = 10 sinx cosx (5 + sinx cosx)

= 5 sin2x (5 + $\dfrac{\sin 2x}{2}$)

$-1 \le \sin 2x \le 1$
Max (E) occurs when sin2x is maximum.

Max (E) = $5\left(\dfrac{11}{2}\right) = \dfrac{55}{2}$

Choice (C)

Exercise – 7(b)

Solutions for questions 1 to 40:

1. Given $\sin\theta + \text{cosec}\theta = \dfrac{5}{2} \Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = \dfrac{5}{2}$

$2\sin^2\theta - 5\sin\theta + 2 = 0$
$2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$
$2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$
$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta - 2 = 0 \Rightarrow \sin\theta = \dfrac{1}{2}$ or 2

As $-1 \le \sin\theta \le 1$, $\sin\theta = \dfrac{1}{2}$

$\sec^2\theta + \cot^2\theta = \dfrac{1}{\cos^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{1}{1-\sin^2\theta} + \dfrac{1-\sin^2\theta}{\sin^2\theta} = \dfrac{1}{1-\left(\frac{1}{2}\right)^2} + \dfrac{1-\left(\frac{1}{2}\right)^2}{\frac{1}{4}}$

$= \dfrac{4}{3} + \dfrac{3}{1} = \dfrac{13}{3}$ Ans: (13)

2. $-1 \le \sin\theta$ or $\cos\theta \le 1$

$\therefore$ From the given options $\sin\theta = \dfrac{3}{2}$ is not possible

Choice (D)

3. $\text{cosec}\,\theta = \dfrac{5}{3}$

$\text{cosec}^2\theta - \cot^2\theta = 1$

$\cot^2\theta = \text{cosec}^2\theta - 1 = \dfrac{16}{9}$

As θ lies in the 1st quadrant

$\cot\theta > 0$

$\therefore \cot\theta = \dfrac{4}{3}$ Choice (A)

4. $\sin\theta + \text{cosec}\theta + \sin^2\theta + \text{cosec}^2\theta = 0$

$\Rightarrow \sin\theta + \sin^2\theta + \dfrac{1}{\sin\theta} + \dfrac{1}{\sin^2\theta} = 0$

$\Rightarrow \sin\theta(\sin\theta + 1) + \dfrac{\sin\theta + 1}{\sin^2\theta} = 0$

$\Rightarrow (\sin\theta + 1)\left(\sin\theta + \dfrac{1}{\sin^2\theta}\right) = 0$

$\Rightarrow \sin\theta + 1 = 0$ or $\sin\theta + \dfrac{1}{\sin^2\theta} = 0$

$\Rightarrow \sin\theta + 1 = 0$ or $\sin^3\theta + 1 = 0 \Rightarrow \sin\theta = -1$

we know that $-\cot^2\theta + \text{cosec}^2\theta = 1 \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$

$\therefore \cot^2\theta = \left(\dfrac{1}{-1}\right)^2 - 1 = 0.$

$\text{Cot}\theta = 0.$ Choice (D)

5. $\sec\theta + \tan\theta = -\dfrac{b}{a}$

$\sec\theta\tan\theta = \dfrac{c}{a}$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\left(\sqrt{(\sec\theta + \tan\theta)^2 - 4\sec\theta\tan\theta}\right)\left(\dfrac{-b}{a}\right) = 1$

$\sqrt{\left(\dfrac{-b}{a}\right)^2 - 4\dfrac{c}{a}} = -\dfrac{a}{b}$

Squaring on both the sides,

$\Rightarrow \dfrac{b^2 - 4ac}{a^2} = \left(-\dfrac{a}{b}\right)^2 = \dfrac{a^2}{b^2}$

$\Rightarrow b^2(b^2 - 4ac) = a^4$

$\Rightarrow b^4 = 4ab^2c + a^4$ Choice (A)

6. $\sin2\theta.\ \text{Sec}3\theta - \text{cosec}2\theta.\ \text{Cot}\theta$

$= \sin120°\sec180° - \text{cosec}120°\cot60°$

$= \left(\dfrac{\sqrt{3}}{2}\right)(-1) - \left(\dfrac{2}{\sqrt{3}}\right)\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{-\sqrt{3}}{2} - \dfrac{2}{3}$ Choice (B)

7. $\cot 22\dfrac{1°}{2} =$

$\sqrt{\dfrac{1+\cos2\left(22\frac{1}{2}^0\right)}{1-\cos2\left(22\frac{1}{2}^0\right)}} = \sqrt{\dfrac{1+\cos45^0}{1-\cos45^0}} = \sqrt{\dfrac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}} = \sqrt{\dfrac{\sqrt{2}+1}{\sqrt{2}-1}}$

$= \sqrt{\dfrac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}} = \sqrt{2} + 1.$ Choice (A)

8. $\angle P + \angle Q + \angle R = 180°$

Triangle PQR is right angled and isosceles.

$\therefore$ one of $\angle P$, $\angle Q$ and $\angle R$ must 90° and each of the other two must be $\dfrac{180° - 90°}{2} = 45°$

$\cos\angle P + \cos\angle Q + \cos\angle R = \cos90° + \cos45° + \cos45°$

$= 0 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$ Choice (A)

9. Given that $\text{cosec}\theta + \cot\theta = x - 1 \quad \rightarrow \quad (1)$

As $\text{cosec}^2\theta - \cot^2\theta = 1$

$\text{cosec}\theta - \cot\theta = \dfrac{1}{\text{cosec}\theta + \cot\theta}.$

$\Rightarrow \text{cosec}\theta - \cot\theta = \dfrac{1}{x-1} \quad \rightarrow \quad (2)$

By [(1) + (2)] ÷ 2, we get

$\text{cosec}\theta = \dfrac{(x-1) + \frac{1}{(x-1)}}{2} = \dfrac{(x-1)^2 + 1}{2(x-1)}$

By [(1) – (2)] ÷ 2, we get

$\cot\theta = \dfrac{(x-1) + \frac{1}{(x-1)}}{2} = \dfrac{(x-1)^2 + 1}{2(x-1)}$

Now, $\cos\theta = \dfrac{\frac{(x-1)^2 - 1}{2(x-1)}}{\frac{(x-1)^2 + 1}{2(x-1)}} = \dfrac{x^2 - 2x}{x^2 - 2x + 2}$ Choice (C)

10. $13\sin\theta - 12 = 0 \Rightarrow \sin\theta = \dfrac{12}{13}$, i.e $\text{cosec}\theta = \dfrac{13}{12}$

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\dfrac{12}{13}\right)^2 = \left(\dfrac{25}{169}\right)$

$\therefore \cos\theta = \dfrac{5}{13}$ (As θ is acute, $\cos\theta > 0$)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{12}{5}$ and $\cot\theta = \dfrac{5}{12}$

$\dfrac{2\cos\theta + 3\tan\theta}{\text{cosec}\theta + \cot\theta} = \dfrac{2\left(\frac{5}{13}\right) + 3\left(\frac{12}{5}\right)}{\frac{13}{12} + \frac{5}{12}} = \dfrac{1036}{195}$

Ans: (1036)

11. Given PQRS is a cyclic quadrilateral $\angle P + \angle R = \angle Q + \angle S = 180°$

$\sin\angle P + \sin\angle Q + \sin(180° - \angle P) + \sin(180° - \angle Q)$

$= \sin\angle P + \sin\angle Q + \sin\angle P + \sin\angle Q$

(As $\sin(180° - \theta) = \sin\theta$) $= 2(\sin\angle P + \sin\angle Q)$

Since the relation between P and Q, is not known the value of the given expression cannot be found

Choice (D)

12. $\cot^6\theta - \mathrm{cosec}^6\theta + 3\mathrm{cosec}^2\theta\cot^2\theta$
$= -[\mathrm{cosec}^6\theta - \cot^6\theta - 3\mathrm{cosec}^2\theta\cot^2\theta]$
$= -[(\mathrm{cosec}^2\theta)^3 - (\cot^2\theta)^3 - 3\mathrm{cosec}^2\theta\cot^2\theta]$
$- [(\mathrm{cosec}^2\theta) - (\cot^2)^3 - 3\,\mathrm{cosec}^2\theta\cot^2\theta\,(\mathrm{cosec}^2\theta - \cot^2\theta)]$
$= -[+1 + 3\,\mathrm{cosec}^2\theta\cot^2\theta - 3\mathrm{cosec}^2\theta\cot^2\theta]$
$= -1.$ Ans: (–1)

13. Given $\tan(\alpha - 45°) + \tan(\alpha + 45°) = 0.$
From options put $\alpha = 0$, $\tan(-45°) + \tan(45) = 0$
Put $\alpha = 90°$, $\tan(90 - 45°) + \tan(90 + 45)$
$= \cot 45° - \cot 45° = 0$
$\therefore$ $\alpha = 0°$ and $90°$ satisfy the given equation. Choice (D)

14. $\sin\alpha = -\dfrac{3}{5}$
$270° < \alpha < 360° \Rightarrow \cos\alpha > 0\ldots$ — (1)
$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(-\dfrac{3}{5}\right)^2 = \dfrac{16}{25}$
$\cos\alpha = \dfrac{4}{5}$
$\therefore$ $\sec\alpha = \dfrac{5}{4}$ and $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-3/5}{4/5} = -\dfrac{3}{4}$
$\sec\alpha + \tan\alpha = \dfrac{1}{2}$ Choice (A)

15. $\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \left(\dfrac{7}{25}\right)^2} = \dfrac{24}{25}$
$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{7}{24}$
$\cot\alpha = \dfrac{24}{7}$, $\mathrm{cosec}\alpha = \dfrac{25}{7}$, $\sec\alpha = \dfrac{25}{24}$
Required value $= \dfrac{\dfrac{24}{7} - \dfrac{25}{7}}{\dfrac{7}{24} - \dfrac{25}{24}} = \dfrac{4}{21}$ Choice (A)

16. Given, $\cos(x + y) = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$ $\sin y = \dfrac{1}{2}$
$\Rightarrow x + y = 75° \Rightarrow y = 30°$
$\therefore$ $x = 45°$
$\therefore$ $x = 45°$, $y = 30°$ Choice (A)

17. The minute hand covers an angle of 6° per minute.
$\therefore$ In 18 minutes the angle covered is 108°. Choice (B)

18. $\log\left[\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{3}}{2}\right] = \log\left[\dfrac{\sqrt{3}}{\sqrt{32}}\right] = \dfrac{1}{2}\log\left(\dfrac{3}{32}\right).$
Choice (D)

19. Let the angles be x, x + 30 and x + 60.
$\therefore$ $x + (x + 30) + (x + 60) = 180 \Rightarrow x = 30$
Since one of the angles is 90°, say angle C, the product cosA cosB cosC = 0, as cos90° = 0.
Choice (A)

20. Given $a = \mathrm{cosec}\,\theta$
$b = \cot\theta$.
Now $\sqrt{\dfrac{a+1}{a-1}} - \sqrt{\dfrac{a-1}{a+1}}$
$= \dfrac{a + 1 - (a - 1)}{\sqrt{a^2 - 1}}$
$= \dfrac{2}{\sqrt{\mathrm{cosec}^2\theta - 1}}$
$= \dfrac{2}{\cot\theta} = \dfrac{2}{b}.$ Choice (A)

21. We know that
$\dfrac{\sin^4 x + \cos^4 x}{2} \ge \sqrt{\sin^4 x.\cos^4 x}$ ($\therefore$ AM ≥ GM)
$\sin^4 x + \cos^4 x \ge 2\,\mathrm{Sin}^2 x\cos^2 x$
$\ge \dfrac{1}{2}\ (\sin 2x)^2$
But max value of sin 2x = 1 $\Rightarrow$ $\sin^4 x + \cos^4 x \ge \dfrac{1}{2}$
$\therefore$ the minimum value of $\sin^4 x + \cos^4 x$ is $\dfrac{1}{2}$ Ans: (0.5)

22. Given $\alpha + \beta = 180°$ and
sum of the roots $\mathrm{cosec}\alpha + \mathrm{cosec}\,\beta = \dfrac{-q}{p}$
$\Rightarrow \mathrm{cosec}\alpha + \mathrm{cosec}\,(180 - \alpha) = \dfrac{-q}{p}$
$\mathrm{cosec}\alpha + \mathrm{cosec}\alpha = \dfrac{-q}{p}$
$2\mathrm{cosec}\,\alpha = \dfrac{-q}{p}$ → (1)
Product of the roots $\mathrm{cosec}\alpha.\mathrm{cosec}\beta = \dfrac{r}{p}$
$\Rightarrow \mathrm{cosec}\,\alpha\ \mathrm{cosec}\,\beta = \mathrm{cosec}\,\alpha\,(\mathrm{cosec}\,\alpha) = \dfrac{r}{p}$
$\Rightarrow \mathrm{cosec}^2\alpha = \dfrac{r}{p}$ → (2)
From (1) and (2), $\left(\dfrac{-q}{2p}\right)^2 = \dfrac{r}{p}$
$\Rightarrow q^2 = 4pr$ Choice (C)

23. PQRS is a cyclic quadrilateral
$\angle P + \angle R = \angle Q + \angle S = 180°$
$\cot\angle P + \cot\angle R = \cot\angle P - \cot\angle P$
(As $\cot(180 - P) = \cot(P) = 0$
similarly $\cot\angle Q + \mathrm{Cot}\,\angle S = 0$
$\therefore$ $\cot\angle P + \cot\angle R - (\cot\angle Q + \cot\angle S) = 0$
Ans: (0)

24. Area of the triangle $= \dfrac{1}{2}$ bc sin A $= \dfrac{1}{2}$ (4) (6) sin 30°
= 6 sq units. Ans: (6)

25. Least angle of the triangle is the angle opposite to the least side. Let this be θ
$\cos\theta = \dfrac{(6\sqrt{3})^2 + 8^2 - 4^2}{2(6\sqrt{3})(8)} = \dfrac{1.625}{\sqrt{3}}$
$\theta = \mathrm{Cos}^{-1}\left(\dfrac{1.625}{\sqrt{3}}\right)$ Ans: (1.625)

26. $r^2 = p^2 + q^2 - 2pq\ \cos\angle R$
$= 8^2 + 10^2 - 2(8)(10)\left(\dfrac{\sqrt{5} + 1}{4}\right)$
$= 164 - 40\ (\sqrt{5} + 1)$
$= 124 - 40\sqrt{5}$
$r = 2\sqrt{31 - 10\sqrt{5}}$ Choice (C)

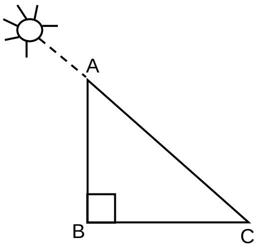
27. The relation which best describes the graph is $y = |\sin x|$

Choice (D)

28. The relation which best describes the graph is $y = |\cos x|$

Choice (D)

29.

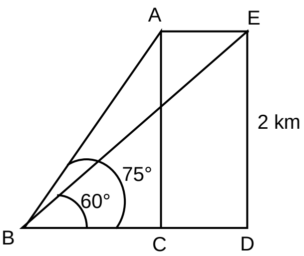


Let AB represent Ajay and BC represent his shadow.

$AB = BC \tan\angle ACB = \dfrac{AB}{BC} = 1$

$\angle ACB = 45°$ Ans: (45)

30.



$\text{Speed} = \dfrac{\text{Distance}}{\text{time}} = \dfrac{AE(\text{in km})}{\dfrac{\frac{1}{2}}{60}(\text{in hrs})}$

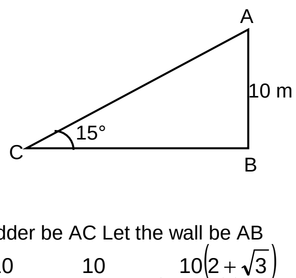
Now $AE = BD - BC = \dfrac{2}{\tan 60^0} - \dfrac{2}{\tan 75^0} = \dfrac{2}{\sqrt{3}} - \dfrac{2}{2+\sqrt{3}}$

$= \dfrac{4}{\sqrt{3}(2+\sqrt{3})}$ km

$\text{Speed} = \dfrac{\dfrac{4}{\sqrt{3}(2+\sqrt{3})}}{\dfrac{1}{120}}$ km/hr $= \dfrac{480}{\sqrt{3}(2+\sqrt{3})}$ km/hr

Ans: (480)

31.



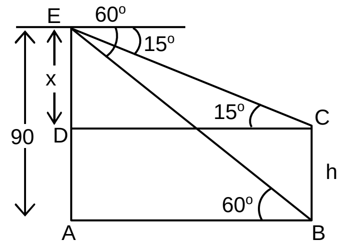
Let the ladder be AC Let the wall be AB

$BC = \dfrac{10}{\tan 15°} = \dfrac{10}{2-\sqrt{3}} = \dfrac{10(2+\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} =$

$10(2 + \sqrt{3})$ m

Choice (C)

32. Let AE be the hill and BC be the tower



Let ED = xm

From ΔEDC, $\tan 15° = \dfrac{ED}{DC} \Rightarrow 2-\sqrt{3} = \dfrac{x}{DC}$

$DC = \dfrac{x}{2-\sqrt{3}} = x(2+\sqrt{3})$ $\rightarrow$ (1)

From ΔAEB, $\tan 60° = \dfrac{AE}{AB} \Rightarrow \sqrt{3} = \dfrac{90}{DC}$ ($\because$ AB = DC)

$DC = \dfrac{90}{\sqrt{3}} = 30\sqrt{3}$ $\rightarrow$ (2)

From (1) and (2) $x(2+\sqrt{3}) = 30\sqrt{3}$

$x = \dfrac{30\sqrt{3}}{2+\sqrt{3}} = 30\sqrt{3}(2-\sqrt{3})$

$\therefore$ height of the tower h = AD = AE − DE

$= 90 - 30\sqrt{3}(2-\sqrt{3})$ m $= 30(3-\sqrt{3}(2-\sqrt{3}))$ m

$= 60(3-\sqrt{3})$m Choice (B)

33. $\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta$

$= (\sec^2\theta)^3 - (\tan^2\theta)^3 - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta)$

$= (\sec^2\theta - \tan^2\theta)^3 = 1^3 = 1$

$\because ((a-b)^3 = a^3 - b^3 - 3ab(a-b)$ Ans: (1)

34. In a triangle ABC, by Sine rule we have

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

Given $a = 3\sqrt{3}$ and $\angle A = 60°$

$\therefore \dfrac{a}{\text{Sin}A} = 2R \Rightarrow \dfrac{3\sqrt{3}}{\sin 60} = 2R$

$R = \dfrac{3\sqrt{3}}{\dfrac{\sqrt{3}}{2}} = 3$ units. Choice (C)

35. Given $a = 4\sqrt{2}$, $b = 4\sqrt{3}$, $\angle A = 45°$

By sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\Rightarrow \dfrac{4\sqrt{2}}{\sin 45} = \dfrac{4\sqrt{3}}{\sin B} = \dfrac{4\sqrt{2}}{\dfrac{1}{\sqrt{2}}} = \dfrac{4\sqrt{3}}{\sin B}$

$\Rightarrow \sin B = \dfrac{\sqrt{3}}{2} \Rightarrow \angle B = 60°$ or $120°$.

Since $\angle C$ is the greatest angle $\angle B = 60°$

$\angle C = 75°$

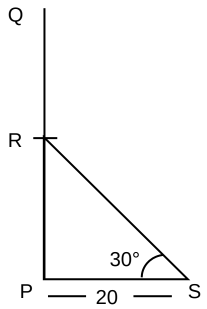
Using sine rule we have

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$, i.e $\dfrac{4\sqrt{2}}{\sin 45^0} = \dfrac{c}{\sin 75^0}$

$= \dfrac{4\sqrt{2}}{1/\sqrt{2}} = \dfrac{c}{\dfrac{\sqrt{3}+1}{2\sqrt{2}}} \Rightarrow c = AB = 2(\sqrt{6} + \sqrt{2})$

Choice (D)

36. Let PQ be the pole and RQ be the upper part of the pole.
From the diagram RQ = RS.



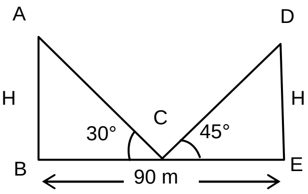
In ΔPRS cos 30°= $\dfrac{PS}{RS}$

$\dfrac{2\sqrt{3}}{2} = \dfrac{20}{RS}$

RS = $\dfrac{40}{\sqrt{3}}$ m

Ans : (40)

37.



Let the height of the buildings be H m.

In ΔABC, $\dfrac{AB}{BC}$ = tan 30°= $\dfrac{1}{\sqrt{3}}$

$\therefore$ $\dfrac{H}{BC} = \dfrac{1}{\sqrt{3}}$ i.e., BC = H $\sqrt{3}$ m

In ΔCDE, $\dfrac{DE}{CE}$ = tan 45°= 1

$\dfrac{H}{CE}$ = 1 i.e., CE = H m

from the diagram, BC + CE = BE = 90 m

$\therefore$ H $\sqrt{3}$ + H = 90

H = $\dfrac{90}{\sqrt{3}+1}$

H = 45 ($\sqrt{3}$ – 1) m

Ans: (45)

38. f(x) = $\dfrac{\sin^5 x - \cos^5 x}{\cos^2 x \sin^2 x}$ and

g(x) = $\dfrac{\cos^5 x - \sin^5 x}{\sin^3 x \cos^3 x}$

$\dfrac{\pi}{4} < x < \dfrac{\pi}{2}$ ----------------(1)

As θ Increases from 0 to $\dfrac{\pi}{2}$, sinθ increases from 0 to 1 and cosθ decreases from 1 to 0. Also

$\sin\dfrac{\pi}{4} = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$.

$\therefore$ sinθ < cosθ when $0 \le \theta \le \dfrac{\pi}{4}$ and sinθ > cosθ when

$\dfrac{\pi}{4} \le \theta \le \dfrac{\pi}{2}$

(1) implies sinx > cosx

$\therefore$ $\sin^5 x > \cos^5 x$

f(x) = $\dfrac{\text{A positive value}}{\text{A positive value}}$ = a positive value

Also g(x) is a negative value.

$\therefore$ f(x) > g(x)

Choice (C)

39. Given 5cosθ + 12sinθ = 13

Dividing both sides by 13, we get

$\dfrac{5}{13}\cos\theta + \dfrac{12}{13}\sin\theta = 1$………(1)

Let $\dfrac{5}{13}$ = cosα and sinα = $\dfrac{12}{13}$

(1) $\Rightarrow$ cosα cosθ + sinα sinθ = 1

$\Rightarrow$ cos(θ – α) = 1 $\Rightarrow$ θ – α = 2πn

$\Rightarrow$ θ = α + 2πn

$\Rightarrow$ tanθ = tanα = $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{12}{5}$

Choice (B)

40. Given α, β are complementary angles, $\alpha + \beta = 90°$

cosα = cos(90°– β) = sinβ

$\cos^2\alpha + \sin^2\beta = \sin^2\beta + \sin^2\beta = 2\sin^2\beta$

$\therefore$ The maximum value of sinβ is 1

$\therefore$ The maximum value of $\cos^2\alpha + \sin^2\beta$ is 2.

Ans: (2)

Solutions for questions 41 to 45:

41. Using statement I alone, cosθ > 0

$\Rightarrow$ θ is in 1st or 4th quadrant.

Hence, it is not in the third quadrant.

Choice (A)

42. Using statement II alone, we can not answer as sinθ < 0

$\Rightarrow$ θ is in

The expression equals 4cosAsinA.

When A = 0°, 4 cosA sinA = 0.

$\therefore$ Statement I alone is sufficient to answer Q_3 or Q_4 statement II we can not answer.

Choice (A)

43. Using statement II alone,

A = B = C = 60°.

tanA + tanB + tanC = $3\sqrt{3}$.

With statement I alone we cannot answer

Choice (A)

44. From statement I, we know that the angle of elevation and the distance. So, we can find the height of the Statue of Liberty.

Similarly from statement II, we can find the height of Eiffel tower.

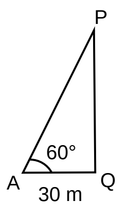
$\therefore$ Using both the statements, we can find which is taller.

Choice (C)

45. From the statement I,

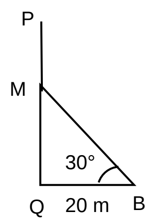
In Δ AQP, BQ = 30m, $\angle$A = 60°

$\therefore$ PQ = 30 $\sqrt{3}$ m



$\therefore$ Statement I is alone sufficient

From the statement II



In ΔBQM, BQ = 20m, $\angle$B = 30°

$\therefore$ QM = 20/ $\sqrt{3}$ and PQ = 40/ $\sqrt{3}$

$\therefore$ Statement II alone is also sufficient.

Hence, either of the statements is sufficient to answer the question.

Choice (B)

Chapter – 8
(Operator Based Questions)

Concept Review Questions

Solutions for questions 1 to 15:

1. Given: $a \alpha b = (ab) - (a + b)$
Here, a = 8, b = 5
$\Rightarrow 8 \alpha 5 = (8 . 5) - (8 + 5) = 27$ Ans: (27)

2. Given: $x - y = \frac{xy}{x+y}$
Here x = 8, y = 6
$\therefore x - y = \frac{8\times6}{8+6} = \frac{48}{14} = \frac{24}{7}$ Choice (B)

3. Given: $a \uparrow b = \frac{a+b}{a-b}$
$4 \uparrow 3 = \frac{4+3}{4-3} = 7$ Ans: (7)

4. Given: f (x, y) = y
$\therefore$ f [3, f (4, f (5, 7))] = f (3, f (4, 7))
= f (3, 7) = 7 Choice (D)

5. Given: $a \% b = (a + b)^2$
$7 \% 3 = (7 + 3)^2 = 100$ Ans: 100

6. Given: $x \leftrightarrow y = (xy)^2$
$3 \leftrightarrow 4 = (3 \times 4)^2 = 144$
also $a \updownarrow b = \left(\frac{a}{b}\right)^2 \Rightarrow (6 \updownarrow 4) = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$
$\frac{(3\leftrightarrow4)}{(6\updownarrow4)} = \frac{144}{9/4} = 64$ Choice (C)

7. Given: f (x, y) = Max of (x, y)
$\Rightarrow$ f (8, 12) = Max (8, 12) = 12 $\rightarrow$ (1)
g (x, y) = L. C. M of (x, y)
g (24, 36) = L. C. M of (24, 36) = 72 $\rightarrow$ (2)
h (x, y, z) = average of (x, y, z)
h (8, 21, 7) = average of (8, 21, 7) = $\frac{8+21+7}{3}$ = 12
$\therefore$ h {f (8, 12). g (24, 36), h (8, 21, 7)
= h (12, 72, 12)
= $\frac{12+72+12}{3}$ = 32 Ans: (32)

8. Given: $a \oplus b = a^2 + b^2$
$\Rightarrow (5 \oplus 3) = 5^2 + 3^2 = 34$
and $(a \odot b) = 2ab$
$(5 \odot 3) = 2.5.3 = 30$
$\therefore (5 \oplus 3) - (5 \odot 3) = 34 - 30 = 4$ Choice (D)

9. Given: $X (x, y) = x^2 - y^2$
$X (32, 20) = 32^2 - 20^2 = 1024 - 400 = 624$
Y (a, b) = a + b
Y (32, 20) = 32 + 20 = 52
$\frac{X(32,20)}{Y(32,20)} = \frac{624}{52} = 12$ Choice (C)

10. Given: P (x, y) = 2x + 3y
P (2, 5) = 2.2 + 3.5 = 19
Q (x, y) = 3x – 2y
Q (1, 1) = 3 – 2 = 1
P (P (2, 5), Q (1, 1)) = P (19, 1)
= 2 × 19 + 3.1
= 38 + 3 = 41 Choice (B)

11. Given: $a{\star}b = \frac{a+b}{a-b}$
$5*3 = \frac{5+3}{5-3} = 4$ and $8*6 = \frac{8+6}{8-6} = \frac{14}{2} = 7$
$\therefore (5*3){\star}(8*6) = (4*7) = \frac{4+7}{4-7} = \frac{-11}{3}$
$x = \frac{-11}{3} \Rightarrow [x] = [-3.66] = -3$ Choice (A)

12. Given: $a \$ b = (a + b)^{a-b}$
$5 \$ 3 = (5 + 3)^{5-3} = 8^2 = 64$ Ans: (64)

13. $a \times b = a^2 - ab + b^2$ and $a \div b = a + b$
$\Rightarrow (a \times b) (a \div b) = (a^2 - ab + b^2) (a + b) = a^3 + b^3$
and $a \alpha \beta = a^2 + ab + b^2$ and $a \beta b = a - b$
$(a \alpha b) (a \beta b) = (a^2 + ab + b^2) (a - b) = a^3 - b^3$
$\frac{(a\times b)(a\div b)-(a\alpha b)(a\beta b)}{b^2} = \frac{a^3+b^3-a^3+b^3}{b^3} = \frac{2b^3}{b^2} = 2b$
Choice (C)

14. Given: $f (x, y) = x^y$ and $g (x, y) = y^x$
$f (3, 4) = 3^4 = 81$ and $g (5, 2) = 2^5 = 32$
$g (2, 7) = 7^2 = 49$
$\therefore \frac{f(3,4)-g(5,2)}{g(2,7)} = \frac{81-32}{49} = 1$ Choice (D)

15. Given [1001%25 + 1002%25 + - - - - + 1025%25]%25
i.e., 1001%25 $\Rightarrow$ R when 1001 is divided by 25 which is 1
1002%25 $\Rightarrow$ R when 1002 is divided by 25 which is 2
“ “ “ “ “ “ “ “ “ “
“ “ “ “ “ “ “ “ “ “
1024%25 $\Rightarrow$ R when 1024 is divided by 25 which is 24
and 1025%25 $\Rightarrow$ R when 1025 is divided by 25 which is 0
$\therefore$ 1001%25 + 1002%25 + - - - - 1025%25
= 1 + 2 + 3 + - - - - + 24 = $\frac{24(24+1)}{2}$ = 300
Required result is 300%25
$\Rightarrow$ Remainder when 300 is divided by 25 which is zero.
Ans: (0)

Exercise – 8(a)

Solutions for questions 1 to 25:

1. $24 \rightarrow 3 = \frac{24}{2\times3} = 4$
$4 \leftarrow 2 = 4 + 2 \times 2 = 8$
$8 \downarrow 4 = 4 \times 8 - 4 = 28$
$28 \uparrow 12 = 3 \times 28 \times 12 = 84 \times 12 = 1008$ Ans: (1008)

2. Consider choice (A):
$21 \downarrow 7 = 4 \times 21 - 7 = 77$
$77 \rightarrow 9 = \frac{77}{2\times9} = \frac{77}{18}$
$77/18 \uparrow 6 = 3 \times 77/18 \times 6 = 77$,
which is a multiple of 11.
Choice (B):
$21 \rightarrow 7 = \frac{21}{2\times7} = \frac{21}{14} = \frac{3}{2}$
$3/2 \uparrow 9 = 3 \times 3/2 \times 9 = 81/2$
$81/2 \downarrow 6 = 4 \times 81/2 - 6 = 156$,
which is not a multiple of 11.
Choice (C) $\Rightarrow 21 \leftarrow 7 = 21 + 2 \times 7 = 35$
$35 \downarrow 9 = 4 \times 35 - 9 = 131$
$131 \rightarrow 6 = 131/12$
which is not a multiple of 11 Choice (A)

3. $14 \downarrow 7 = 4 \times 14 - 7 = 56 - 7 = 49$

$\therefore \sqrt{14 \downarrow 7} = \sqrt{49} = 7$

$9 \uparrow 27 = 3 \times 9 \times 27 = 729$

$\therefore \sqrt[3]{9 \uparrow 27} = \sqrt[3]{729} = 9$

$\therefore \sqrt{14 \downarrow 7} - \sqrt[3]{9 \uparrow 27} = 7 - 9 = -2$ Ans: (–2)

4. In the given numbers 2 is repeated 9 times
$\Rightarrow$ honoured card occurred 9 times. Ans: (9)

5. In the given series of numbers 5 is not followed by 3. Hence, the required number is zero. Ans: (0)

6. If he picks up a honoured card he writes it as 2
In the given number series, 2 occurs consecutively only once.
$\therefore$ The required number is one Ans: (1)

7. For a positive number $\lfloor 2x \rfloor \geq \lfloor x \rfloor + \lfloor x \rfloor$,
for example, $\lfloor 1.2 \rfloor + \lfloor 1.2 \rfloor = \lfloor 2 \times 1.2 \rfloor = \lfloor 2.4 \rfloor = 2$
whereas $\lfloor 1.8 \rfloor + \lfloor 1.8 \rfloor = 2 < \lfloor 2 \times 1.8 \rfloor = \lfloor 3.6 \rfloor = 3$
$\Rightarrow \lfloor 2x \rfloor + \lfloor 2y \rfloor \geq \lfloor x \rfloor + \lfloor x \rfloor + \lfloor y \rfloor + \lfloor y \rfloor$
$\geq \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x \rfloor + \lfloor y \rfloor$
$\geq \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor$ as $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$
Hence $R(x, y) \geq L(x, y)$ Choice (D)

8. For all integral values of x and y; R(x, y) = T(x, y). In the interval (0, 5) there are 6 integers.
So, the number of such pairs $6 \times 6 = 36$.
Choice (C)

9. The order in which operations should be performed: BOSAMD.
$\therefore 13 \times 5 + 35 \div 8 - (2 \times 5)$
$= 13 \times 5 + 35 \div 8 - 10 = 13 \times 5 + 35 \div (-2)$
$= 13 \times 40 \div (-2) = 520 \div (-2) = -260$
Ans: (–260)

10. (15% 6) = L.C.M. of 15, 6 = 30
$(20 \sim 8)$ = G.C.D. of 20, 8 = 4
$\therefore 30 \,\Delta\, 4 = 30^3 - 4^3 = 27000 - 64 = 26936$. Choice (A)

11. L.C.M. of two distinct positive numbers is always greater than their G.C.D.
$\therefore$ choices (A) and (B) are wrong
$(a \$ b) = (a + b)^3 - (a - b)^3$ is always positive, as $a + b > a - b, \Rightarrow (a + b)^3 > (a - b)^3$. Choice (C)

12. Here a = 6, b = 36
Choice (A):
$\Rightarrow (a \% b) \div (a \sim b) + a \,\Delta\, b = 36 \div 6 + (a^3 - b^3)$
$= 6 + (216 - 46856) = 6 - 46640 = -46634$
Choice (B):
$\Rightarrow \sqrt[3]{(a \% b) \times (a \sim b)} = \sqrt[3]{36 \times 6} = 6 = a$
$\therefore$ Choice (B) is true Choice (B)

13. G.C.D. $\times$ L.C.M. = product of two numbers
$(a \sim b) \times (a \% b) = ab$
$\Rightarrow (a \sim b) \times (a \% b)$ is divisible by both a and b.
Choice (D)

14. $p(f(x, x), g(x, -x)) = p(e^{2x}, e^{2x})$
$= \log_e(e^{2x}, e^{2x})$
$= \log_e e^{4x} = 4x$
$= q(e^{6x}, e^{2x})$ Choice (D)

15. $f(p(x, y), q(x, y)) = f(\log xy, \log x/y)$
$= e^{\log xy + \log x/y} = e^{\log x^2} = x^2 = 25$ Ans: (25)

16. $a \$ b = \left(a^2 + b^2\right)^{a^2 - b^2} = (0 + 1)^{0-1} = (1)^{-1} = 1$
$a \,\Delta\, b = 0^{0-1} + 1^{0-1} = 1$
$\therefore a \$ b - a \,\Delta\, b = 1 - 1 = 0$ Ans: (0)

17. Consider choice (A):
$a \$ b = 3 \$ 2 = (9 + 4)^5 = (13)^5 \neq 5^5 . 5$
$\therefore$ Choice (A) is false.
Consider choice (B)
$a \,\Delta\, b = 3^{9-4} + 2^{9-4} = 3^5 \times 2^5$
$= 243 + 32 = 275 = 55(a + b)$
$\therefore$ Choice (B) is true.
Choice (B)

18. Consider choice (A)
$a^{a^2-b^2} = b^{a^2-b^2}$
This is true when $a^2 = b^2$.
$\therefore a \vee b$ can be equal to $a \wedge b$
$\therefore$ Choice (A) is false.
Consider choice (B)
$a \,\Delta\, b = a^{a^2-b^2} + b^{a^2-b^2}$
Which is always greater than 0 as a, b are greater than 0
$\therefore$ Choice (B) is true
Choice (B)

19. Consider choice (A)

$a \$ b = \left(2^2 + 1^2\right)^{2^2 - 1^2} = (5)^3 = 125$

$a \,\Delta\, b = 2^{2^2-1^2} + 1^{2^2-1^2} = 2^3 + 1 = 9$

$\therefore \frac{a\$b}{a\Delta b} = \frac{125}{9}$

$\therefore$Choice (A) is false.
Consider choice (B):
for a = 1 and b = 1, $a \vee b = 1^{1^2-1^2} = 1$
and $a \wedge b = 1^{1^2-1^2} = 1$
$\therefore a \wedge b + a \wedge b = 1 + 1 = 2$
$\therefore$ Choice (B) is false Choice (D)

20. Here a = 3, b = 4, h = 3

$\Delta^2 = \frac{1}{a^2b^2}\left(h^2 + ab\right)$

= 1/144 (9 + 12) = 21/144 > 0

and $\nabla^2 = \frac{1}{a^2b^2}\left(h^2 - ab\right)$

= 1/144 (9 – 12) = –3/144 < 0
Choices (A) and (B) are false.
Also 21/144 > –3/144
$\therefore \Delta^2 > \nabla^2$ Choice (C)

21. With the coefficients of x^2 and y^2 being interchanged the new equation will have a = –7, h = 4, b = 4

$\Delta^2 = \frac{1}{a^2b^2}\left(h^2 + ab\right)$

$= \frac{1}{49 \times 16}(16 - 7 \times 4) = \frac{-12}{49 \times 16}$

$\nabla^2 = \frac{1}{a^2b^2}\left(h^2 - ab\right)$

$= \frac{1}{49 \times 16}(16 - (-7) \times 4) = \frac{44}{49 \times 16}$

$\therefore \Delta^2 < \nabla^2$ Choice (C)

22. $D = \frac{2}{b}\sqrt{h^2 - ab}$

$\Rightarrow D/a = \frac{2}{ab}\sqrt{h^2 - ab}$

$\Rightarrow D/2a = \frac{2}{ab}\sqrt{h^2 - ab} = \nabla$

$\Rightarrow D/2a = \nabla \Rightarrow D = 2a\nabla$
Choice (A)

23. By definition $c^2 = c \otimes c = d$
$c^3 = c^2 \otimes c = d \otimes c = b$
$c^4 = c^3 \otimes c = b \otimes c = a$
$\therefore c^4 = a$
the least value of n = 4.
Choice (B)

24. From table $a^5 = a$, $b^4 = a$, 4c = a and 9d = d
$\therefore (a^5 \otimes b^4) \oplus (4c \otimes 9d)$
$= (a \otimes a) \oplus (a \otimes d)$
$= a \oplus d = d = c \otimes c$
Choice (C)

25. $((b \otimes c) \otimes a) \oplus ((a \oplus b) \oplus d)$
$= (a \otimes a) \oplus (b \oplus b)$
$= a \oplus a = a$ Choice (D)

Exercise – 8(b)

Solutions for questions 1 to 30:

1. $6 \oplus 8 = HCF(6^3, 8^3) = [HCF(6, 8)]^3 = 8$
$(6 \oplus 8)^{1/3} = 8^{\frac{1}{3}} = 2$
$(2 \ominus 4)^{1/3} = [LCM\ (2^3, 4^3)]^{1/3} = LCM(2,4) = 4$
$4 \otimes 1 = 4^2 + 1^2 - 4^2\, 1^2 = 1$
$1 \ominus 1 = LCM\ (1^3, 1)^3 = 1$ Ans: (1)

2. $a \ominus b = LCM\ (4^3, 6^3) = [LCM\ (4,6)]^3 = 12^3 = 1728$
$a \oplus b = HCF\ (4^3, 6^3) = [HCF(4, 6)]^3 = 2^3 = 8$
$\dfrac{a \ominus b}{a \oplus b} = \dfrac{1728}{8} = 216$ which exceeds 200
$\therefore$ Choice (A) is false.
Choice (B)
$(a + 1) \otimes (b - 2) = (4 + 1) \otimes (6 - 2) = 5 \otimes 4$
$= 5^2 + 4^2 - (5)^2\,(4)^2 = -359$
$\therefore$ Choice (B) is false.
Choice (C)
$(a - 1) \oplus (b + 2) = 3 \oplus 8 = 3^2 + 8^2 + (3)^2\,(8)^2 = 649$
$\therefore$ Choice (C) is false. Choice (D)

3. Choice (A)
Any two natural numbers have their HCF less than or equal to their LCM
$\therefore$ Choice (A) is always true.
Choice (B)
$p^3 = q^3 \Rightarrow p = q$
$p \oplus q = 2q^2 + (q^2)^2 = 99$
$(q^2)^2 + 2q^2 - 99 = 0$
$(q^2 + 11)\,(q^2 - 9) = 0$
$q^2 > 0$
$\therefore q^2 = 9$
$\therefore q = \pm 3$
$\therefore$ Choice (B) is true.
Choice (C)
$p^3 = q^3 \Rightarrow p = q$
$p \otimes q = 2q^2 - (q^2)^2 = -8$
$(q^2)^2 - 2q^2 - 8 = 0$
$(q^2 - 4)\,(q^2 + 2) = 0 \Rightarrow q^2 > 0$
$\therefore q^2 = 4 \quad \therefore q = \pm 2$
$\therefore$ Choice (C) is true.
Choice (D)

4. $9 \uparrow 12 = \dfrac{2}{3}(9)\,(12) = 72$
$72 \downarrow 2 = \dfrac{(3)\,(72)}{2} = 108$
$108 \rightarrow 3 = 3(108) + 4(3) = 336$
$336 \leftarrow 1 = 4(336) - 5(1) = 1339.$ Ans: (1339)

5. Choice (A)
$2 \uparrow 3 = \dfrac{2}{3}(2)\,(3) = 4$
$4 \downarrow 5 = 3\left(\dfrac{4}{5}\right) = \dfrac{12}{5}$
$\dfrac{12}{5} \rightarrow 6 = 3\left(\dfrac{12}{5}\right) + 4(6) = 31.2$
$31.2 \leftarrow 7 = 4(31.2) - 5(7)$ which is not an integer

Choice (B)
$2 \rightarrow 3 = 3(2) + 4(3) = 18$
$18 \leftarrow 5 = 4(18) - 5(5) = 47$
$47 \downarrow 6 = 3\left(\dfrac{47}{6}\right) = 23.5$
$23.5 \uparrow 7 = \dfrac{(23.5)\ (7), 2}{3}$
is not an integer.

Choice (C)
$2 \uparrow 3 = \dfrac{2}{3}(2)\,(3) = 4$
$4 \rightarrow 5 = 3(4) + 4(5) = 32$
$32 \downarrow 6 = 3\left(\dfrac{32}{6}\right) = 16$
$16 \leftarrow 7 = 4(16) - 5\,(7) = 29$ is an integer
Choice (D) is not an integer. Choice (C)

6. Choice (A)
$6 \uparrow 1 = \dfrac{2}{3}(6)\,(1) = 4$
$4 \rightarrow 4 = 3(4) + 4(4) = 28$
$28 \downarrow 1 = 3\left(\dfrac{28}{1}\right) = 84$
$84 \leftarrow 7 = 4(84) - 5(7) = 336 - 35 = 301$
$\therefore$Choice (A) is not a perfect square.
Choice (B)
$6 \uparrow 4 = \dfrac{2}{3}(6)\,(4) = 16$
$16 \rightarrow 1 = 3(16) + 4(1) = 52$
$52 \downarrow 1 = 3\left(\dfrac{52}{1}\right) = 156$
$156 \uparrow 7 = \dfrac{2}{3}(156)\,(7) = 728$
$\therefore$ Choice (B) is not a perfect square.
Choice (C)
$6 \uparrow 7 = \dfrac{2}{3}(6)\,(7) = 28$
$28 \rightarrow 1 = 3(28) + 4(1) = 88$
$88 \downarrow 1 = 3\left(\dfrac{88}{1}\right) = 264$
$264 \leftarrow 4 = 4(264) - 5(4) = 1036$
$\therefore$ Choice (C) is not a perfect square.
Choice (D) follows.
Choice (D)

7. $a(x, x) = \dfrac{p^x + p^x}{2} = p^x$
$b(x, x) = \dfrac{p^x - p^x}{2} = 0$
$c(a\,(x, x), b(x, x) = \log_p \dfrac{p^x}{p^x} = 0$
Choice (B)

8. $c(x, y) = \log_p \dfrac{x+y}{x-y}$

$c(x, -y) = \log_p \dfrac{x-y}{x+y}$

Numerator = a $\left(\log\dfrac{x+y}{x-y}, \log\dfrac{x-y}{x+y}\right)$

$= \dfrac{x+y}{x-y} + \dfrac{x-y}{x+y}$

$= 2(x^2 + y^2)/x^2 - y^2$

Similarly, the denominator = $\dfrac{x+y}{x-y} - \dfrac{x-y}{x+y} = \dfrac{4xy}{x^2-y^2}$

$\therefore$ The given expression is $\dfrac{x^2+y^2}{2xy}$ Choice (A)

9. $a(8, 6) = \dfrac{p^8+p^6}{2}$

$b(8, 6) = \dfrac{p^8-p^6}{2}$

$c(a(8, 6), b(8, 6)) =$

$$\log_p \frac{\frac{p^8+p^6}{2} + \frac{p^8-p^6}{2}}{\frac{p^8+p^6}{2} - \left(\frac{p^8-p^6}{2}\right)}$$

$= \log_p \dfrac{p^8}{p^6} = 2$

$c(c(a(8, 6), b(8, 6), 1) = \log_p \dfrac{3}{1} = \log_p 3$ Choice (A)

10. $a(2, 1) = 2^2 + 1^3 = 5$
$c(2, 1) = 2^3 - 1^2 = 7$
$b(2, 1) = 2^3 + 1^2 = 9$
$d(2, 1) = 2^2 - 1^3 = 3$

The required value = $\dfrac{5+7}{9-3} = 2$ Ans: (2)

11. Choice (A)
$a(p, q) = p^2 + q^3$
$d(p, q) = p^2 - q^3$
Given $p^2 + q^3 - (p^2 - q^3) = 128$
$\Rightarrow q^3 = 64 \Rightarrow q = 4$
$\therefore$ Choice (A) is false

Choice (B)

$$\frac{p^3+(p^2)^2}{p^3-(p^2)^2} = \frac{p^3(1+p)}{p^3(1-p)} = 4 \Rightarrow 1 + p = 4 - 4p$$

$\Rightarrow p = 3/5$
$\therefore$ Choice (B) is false.
Choice (C)

$$\frac{a(p^2,p)}{d(p^2,p)} = \frac{p^4+p^3}{p^4-p^3} = \frac{p+1}{p-1} = 3$$

$\Rightarrow$ Using, componendo/dividendo $p = \dfrac{4}{2} = 2$

$\therefore$ Choice (C) is false.

Choice (D)

$$\frac{p^2+q^3}{p^2-q^3} = \frac{p^3+q^2}{p^3-q^2}$$

$p^5 + p^3q^3 - p^2q^2 - q^5 = p^5 - p^3q^3 + p^2q^2 - q^5$
$2p^2q^2 (pq - 1) = 0$
$\Rightarrow p = 0, q = 0$ or $pq = 1$
$\therefore pq = 0$ or 1.
Choice (D) is true. Choice (D)

12. $c(p, p) = 448$
$p^3 - p^2 = 448 = 8(56)$
$\Rightarrow p^2(p - 1) = 8^2(8 - 1)$
Comparing the two sides, 8 is a possible value for p.
Choice (D)

13. $45 + 90 \div 45 \times 5 - 6$
$= 45 + 2 \times 5 - 6$
$= 45 + 10 - 6$
$= 55 - 6 = 49$
(performing the order of operations is BODMAS).
Ans: (49)

Solutions for questions 14 to 16:

Let i be an integer, f be a proper fraction (i.e., $0 \le f < 1$) and r be any real number. We have the following basic results, following from the definition of [x] and (x).

(I) [i] = i
(II) [i + f] = i
(III) [i + r] = i + [r]
(IV) (i) = i + 1
(V) (i + f)] = i +1
(VI) [i + r] = i + (r)
(VII) [r] + 1 = (r)

14. Let A = [A] +a
B = [B] + b
C = [C] + c
D = [D] + d
$0 \le a + b + c + d = s < 4$ . . . (1) and
A + B + C + D = [A] + [B] +[C] + [D] +s
$\therefore$ [A] + [B] + [C] + k
Where k = 0, 1, 2 or 3 (from (1))
$\therefore$ 4 is not a possible value for k. Choice (D)

15. Let A = [A] +a
B = [B] + b
C = [C] + c
$\therefore 0 \le a + b + c = s < 3 \quad \rightarrow \quad (1)$
(A + B + C) = ([A] + [B] +[C] + s)
= [A] + [B] + [C] + (s)
= [A] + [B] + [C] + k $\rightarrow$ (2)
Where k = 1, 2 or 3 (from (1))
Also (A) + (B) + (C)
= [A] + [B] + [C] + 3 $\rightarrow$ (3)
Comparing (2) and (3), I can be 0, –1 or –2. Among the options, only –1 is there. Choice (D)

16. From the basic result (VII), m = 1.
Choice (D)

17. Since all the numbers provided are even and LCM, HCF or the Arithmetic mean of two even numbers is even, the given question reduces to π(even, 2) = 0
Ans: (0)

18. GOS (a, b, c) = $\sqrt[3]{a.b.c}$

GOSS (a, b, c) = $\sqrt[3]{a^2.b^2.c^2}$

GOC (a, b, c) = $\sqrt[3]{a^3.b^3.c^3}$

For a, b, c > 1; $a^3b^3c^3 \ge a^2b^2c^2 \ge abc$

$\Rightarrow \sqrt[3]{a^3b^3c^3} \ge \sqrt[3]{a^2b^2c^2} \ge \sqrt[3]{abc}$

i.e., GOC (a, b, c) $\ge$ GOSS (a, b, c) $\ge$ GOS (a, b, c).
Choices (A), (B) and (C) are clearly true.

While choice (D) i.e., $\sqrt[3]{a^2.b^2.c^2} \ge abc$ is not true.
Choice (D)

19. Consider choice (A):
If one of a, b is negative $a \vee b$ is negative.
Since $a \times b$ is negative while $e^a \times e^b$ is not.
Hence choice (A) is false.
Consider choice (B):
$a \wedge b = e^a e^b$ can be less than 1 when both a, b are negative.
Hence choice (B) is false.
Consider choice (C)
$a \vee b \neq a \wedge b$ for a = b
Choice (C) is true only when a = b = 1 and not for other values.
∴Choice (C) is false
Now as e^a and e^b are always positive, $a \wedge b = e^a e^b$ is always positive. Choice (D)

Solutions for questions 20 to 23:

P > Q > R
a(P, Q, R) = min(P, Q, P) = Q
b(P, Q, R) = max(Q, Q, P) = P
c(P, Q, R) = max(Q, R, R) = Q
d(P, Q, R) = min(Q, R, P) = R
e(P, Q, R) = min(P, Q, R) = R
f(P, Q, R) = min(Q, R, R) = R

20. Choice (A)
$\frac{a(P, Q, R)+c(P, Q, R)}{d(P, Q, R)+e(P, Q, R)} = \frac{2Q}{2R} = \frac{Q}{R}$ which exceeds 1.
∴ Choice (A) exceeds 1.
Choice (B)
$\frac{a(P, Q, R)-e(P, Q, R)}{b(P, Q, R)-f(P, Q, R)} = \frac{Q-R}{P-R} =< 1 \left(\because \frac{Q}{P}, <1\right)$
Choice (C)
e(P, Q, R) + f(P, Q, R) = 2R
a(P, Q, R) + c(P, Q, R) = 2Q
$\frac{a(P, Q, R)+c(P, Q, R)}{e(P, Q, R)+f(P, Q, R)} = \frac{Q}{R}$, which exceeds 1.
∴ Choice (C) exceeds 1
Choice (D) follows. Choice (D)

21. The expression which is undefined must have a denominator of 0.

In choice (D),
d(P, Q, R) – e(P, Q, R) = 0
∴ Denominator = 0
∴ Choice (D) is undefined. Choice (D)

22. Each of the first three choices is equal to 1.
Choice (D)

23. Choice (A)
Given expression is $\frac{P-Q}{Q-R}$, which is

$\frac{\text{positive}}{\text{positive}}$ = positive.

Choice (B)
Given expression is $\frac{Q-P}{R-Q} = \frac{P-Q}{Q-R}$, which is positive.

Choice (C)
Given expression is $\frac{Q-P}{Q-R} = \frac{(P-Q)}{Q-R}$, which is negative.

Choice (D)
Given expression is $\frac{Q-R}{Q-R}$ = 1 which is positive.
Choice (C)

24. $a(-x, x) = 3^{-x+x} = 1$
$b(-x, -x) = 3^{-x-(-x)} = 1$
$d(1, 1) = \log_3 \frac{1}{1} = 0$ Ans: (0)

25. $a(2, 1) = 3^{2+1} = 3^3$
$b(3, 2) = 3^{3-2} = 3^1$
$c(3^3, 3^1) = \log_3 3^3.3^1 = \log_3 3^4 = 4$ Ans: (4)

26. $a(3, 4) = 3^{3+4} = 3^7$ and $b(5, 2) = 3^{5-2} = 3^3$
$\frac{c[a(3, 4), b(5, 2)]}{d[a(3, 4), b(5, 2)]} = \frac{c(3^7, 3^3)}{d(3^7, 3^3)}$
$= \frac{\log_3 (3^7)(3^3)}{\log_3 \frac{3^7}{3^3}} = \frac{10}{4} = \frac{5}{2}$ Ans: (2.5)

27. $8(X \# Y) = 8\left(\frac{XY}{4}\right) = 2XY$
Given: $8(X \# Y) = x \oplus y$
∴ 2XY = X + Y + XY
XY – X – Y = 0
XY – X – Y + 1 = 1
X(Y – 1) – 1(Y – 1) = 1
(X – 1) (Y – 1) = 1
X and Y are integers
∴ X – 1 and Y – 1 are factors of 1.
∴ X – 1 = Y – 1 = ±1
∴ X = Y = 0 or 2
∴ (X, Y) = (0, 0) or (2, 2)
The number of values of (X, Y) is 2. Ans: (2)

28. $1 \oplus 2 = 1 + 2 + 1(2) = 5$
$5 \# 4 = \frac{(5)(4)}{4} = 5$
$5 \oplus 3 = 5 + 3 + (5)(3) = 23$ Ans: (23)

29. p = 1, q = 1, r = 2, s = 3 and t = 4
$\oplus = \sqrt{2^2 - (1)(4)} = 0$
$\ominus = \sqrt{3^2 - (1)(4)} = \sqrt{5}$
Choice (A)
$\oplus^2 > \ominus^2$ Choice (A) is false.
Choice (B)
$\oplus \ominus > 0$
∴ Choice (B) is false.

Choice (C)
$\oplus < \ominus$ ∴ Choice (C) is true. Choice (C)

30. $\oplus = \sqrt{a^2 - ac}$ and $\ominus = \sqrt{b^2 - bc}$
Choice (A) $\oplus = \ominus$
Taking the square for both the expressions, we get
$a^2 - ac = b^2 - bc$
$\Rightarrow a^2 - b^2 = c(a - b)$; $\Rightarrow (a - b)[c - (a + b)] = 0$
$\Rightarrow a - b = 0$ or $c - (a + b) = 0$; a = b or c = a + b,
which is > 0 since a, b > 0
∴ Choice (A) is not necessarily true.

Choice (B)
$\oplus^2 = a^2 - ac > 0$
a (a – c) > 0
∴ a – c > 0, i.e. a > 0
∴ Choice (B) is true.

Choice (C)
$\ominus^2 = b^2 - bc < 0$

b(b – c) < 0
As b > 0, ∴ b – c < 0
∴ Choice (C) is not true. Choice (B)

Chapter – 9
(Statistics)

Concept Review

Solutions for questions 1 to 30:

1. Individual or raw data Choice (C)

2. Grouped data Choice (B)

3. Size of the class = difference between the lower (or upper) limits of two successive classes,
i.e. 19 – 9 = 10 Choice (A)

4. Mid value = Average of the limits of a class.
$= \frac{14.5+25.5}{2} = 20$ Ans: (20)

5. Arithmetic mean = $\frac{\text{Sum of scores}}{\text{number of scores}}$
$= \frac{12+17+15+21+36+40}{6}$
$= \frac{141}{6} = 23.5$ Choice (D)

6. Arithmetic mean of an arithmetic progression
$= \frac{\text{first term}+\text{last term}}{2}$
∴ Arithmetic mean of 8, 15, 22, 29, 36, 43, 50, is
$= \frac{8+50}{2} = 29$ Choice (B)

7. Given arithmetic mean of x_1, x_2, ... x_n is 50. Then the arithmetic mean of $x_1 – 10$, $x_2 – 10$, ... $x_n – 10$ is 50 – 10 = 40 Ans: (40)

8. The arithmetic mean of x_1, x_2 . . . . . x_{50} is k.
Than the arithmetic mean of $cx_1 + 8$, $cx_2 + 8$, ... , $cx_{50} + 8$ is ck + 8 Choice (C)

9. The sum of deviations about mean is always equal to zero Choice (D)

10. The middle observation of the first 49 natural numbers is 25.
∴ Median = 25 Ans: (25)

11. When a, b, c are in arithmetic progression, then b is the arithmetic mean of a and c
∴ Arithmetic mean of x_1, x_3 is x_2 Choice (A)

12. In the given data, the most frequently occurring value = 3 irrespective of the value of x.
∴ Mode = 3 Choice (B)

13. The number of observations is seven (odd)
∴ when arranged in order we have 6, 8, 10, 12, 14, 16
(i) if 10 < x < 12,the median is x
(ii) if x < 10, the median is 10
(iii) if x > 12, the median is 12
∴ y ∈ [10, 12]
or 10 ≤ y ≤ 12 Choice (B)

14. In the given data, 3 and 5 occur the greatest number of times ∴ This data has two modes, 3 and 5.
Such data is called bimodal data. Choice (D)

15. Bimodal data Choice (B)

16. The geometric mean of a and b is $\sqrt{ab}$.
Here a = 12 and b = 3. = $\sqrt{(12)\ (3)} = 6$
Ans: (6)

17. The harmonic mean of x_1, x_2, ... x_n is $\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_n}}$
here n = 4
∴ The harmonic mean of $\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8}$ is
$\frac{4}{2+4+6+8} = \frac{4}{20} = 0.2$
Ans: (0.2)

18. The harmonic mean of 3 and 5 is
Harmonic mean of a, b is $\frac{2ab}{a+b}$
$\frac{2}{1/3+1/5} = \frac{2.5.3}{5+3} = 3.75$ Choice (B)

19. The relation between arithmetic mean (A, M), geometric mean (GM) and harmonic mean (H. M) is
A.M ≥ G.M ≥ H.M (Standard result)
Choice (A)

20. We know that
$G^2 = AH$
$(6)^2 = 12H$ or H = 3 Choice (C)

21. The empirical relation between arithmetic mean, median and mode is
Mode = 3 Median – 2 Mean
24 = 3 Median – 2 (26)
Median = $\frac{24+52}{3} = \frac{76}{3}$
Choice (D)

22. For a symmetric distribution,
Mean = Median = Mode Choice (D)

23. For a moderately symmetric distribution we have
Mode = 3 Median – 2 Mean
Mode – Median = 2 (Median – Mean)
24 = 2 (Median – Mean)
∴ Median – Mean = $\frac{24}{2} = 12$
Ans: (12)

24. Range = Maximum – Minimum
Here the maximum = 83 and the minimum = 19
∴ Range = 83 – 19 = 64
Choice (B)

25. If a constant is added to every observation, then the range is unaltered.
∴ Range = 28
Ans: (28)

26. The given data, arranged in order is 6, 8, 12, 14, 17, 21, 26.
If there are 4n – 1 observations in the data, the $\left(\frac{n+1}{4}\right)^{th}$ observation is Q_1
i.e., $\left(\frac{7+1}{4}\right)^{th}$ = 2^{nd} item i.e., 8
∴ $Q_1 = 8$
Choice (B)

27. The given data arranged in ascending order is 14, 21, 23, 26, 29, 30, 38, 42, 47, 56, 72. If there are 4n – 1 observations, the 3 $\left(\frac{n+1}{4}\right)^{th}$ observation of the data is Q_3, the 3rd quartile is the 3 $\left(\frac{11+1}{4}\right)$ or the 9th observation.

$\therefore Q_3 = 47$ Ans: (47)

28. Mean deviation of two numbers a, b is $\frac{|a-b|}{2}$

$\therefore$ Mean deviation of 24 and 36 is $\frac{|36-24|}{2} = 6$

Ans: (6)

29. If σ is the standard deviation (S.D) of $x_1, x_2, \ldots\ldots x_n$ then S.D of $cx_1 + p, cx_2 + p, \ldots\ldots, cx_n + p$ (where c, p are real numbers) is given by c σ
$\therefore$ Required S. D is 3 σ Choice (D)

30. The given observations are arranged in ascending order, 4, 8, 12, 16, 20, 24, 28, which is an arithmetic progression

$\therefore$ Standard deviation = c.d $\sqrt{\frac{n^2-1}{12}}$,

where c.d is common difference = 4
and, n is number of observation p = 7

$\therefore$ S. D = 4 $\sqrt{\frac{7^2-1}{12}}$ = 8. Choice (C)

Exercise – 9(a)

Solutions for questions 1 to 25:

1. The sum of the squares of first 'n' natural numbers is n(n + 1) (2n + 1)/6
If n = 9, we get the sum as
9(9 + 1) (2(9) + 1)/6 = 285
We deduct $1^2 + 2^2 + 3^2$ from 285 to get 271
We are given 6 numbers. The mean of the required numbers is 271/6 = 45.16. Choice (D)

2. The numbers are in geometric progression. The sum of these numbers is

$4 \cdot \frac{4^6-1}{4-1} = 4 \cdot \frac{4095}{3} = 5460$

The arithmetic mean is 5460/6 = 910 Ans: (910)

3. The numbers form an arithmetic progression. Hence the arithmetic mean is (9 + 105)/2 = 57.
Note: AM (A.P) = AM (the first and the last term)
Choice (C)

4. $\overline{x} = \Sigma x/n$
So $\Sigma x = n\,\overline{x}$
The sum of the numbers is $50 \times 42 = 2100$
On subtracting 75 and 105 from 2100,
we get 2100 – (105 + 75) = 1920.
So, the sum of the remaining set of 48 observations is 1920.
Hence the mean of the remaining set is 1920/48 = 40.
Choice (A)

5. Given $n_1 = 7$, $\overline{x_1} = 36$; $n_2 = 13$, $\overline{x_2} = 46$.

Combined mean $\overline{x_c} = \frac{n_1\,\overline{x_1} + n_2\,\overline{x_2}}{n_1+n_2} = \frac{7\times36+13\times46}{7+13}$

$= \frac{850}{20} = 42.5$

Choice (B)

6. HM $(x_1, x_2, x_3, \ldots\ldots, x_n) = \dfrac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}}$

HM $(1, {}^1/_4, {}^1/_7, {}^1/_8, {}^1/_{10}) = \frac{5}{1+4+7+8+10} = \frac{5}{30} = \frac{1}{6}$.

Choice (C)

7. G.M. (G.P.) = $\sqrt{\text{first term} \times \text{last term}}$
G.M. $(3^1, 3^2, 3^3, \ldots\ldots 3^{99})$
$= \sqrt{3 \times 3^{99}} = 3^{50} = 9^{25}$. Choice (C)

8. Arranging the numbers in ascending order we have, 15.2413, 15.3125, 15.3142, 15.3143, 15.3215, 15.4132, 15.5213. Since there are 7 terms, the median is $\left(\frac{7+1}{2}\right)^{th}$ term i.e., 4th term.
$\therefore$ median = 15.3143. Ans: (15.3143)

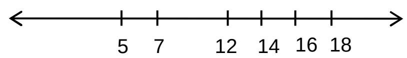
9. Arranging the given elements in order, we have 4, 6, 8, 14, 15, 16, 20, 22
Since the number of elements is 8, the median is the average of $\left(\frac{8}{2}\right)^{th}$ and $\left(\frac{8}{2}+1\right)^{th}$ elements

$\therefore$ Median = $\frac{14+15}{2}$ = 14.5 Choice (C)

10. We need to know the $\left(\frac{17+1}{2}\right)^{th}$ prime or the 9th prime i.e., 23. In fact, we need not list all the 17 primes.
Choice (D)

11. As the two numbers included into the series flank the median (M), the position of the median is not affected. Hence the median remains the same. Choice (B)

12. We first plot the known numbers on the number line as follows.



Now if $x \le 12$ then 12 would be the median.
If $x \ge 14$ then 14 would be the median.
If $12 \le x \le 14$ then x itself would be the median. So $12 \le$ median ≤ 14. Choice (B)

13. 10 has occurred for a maximum number of times (4 times). Inspite of x being an unknown, 10 is still the most found number. Choice (B)

14. Mode = 3 Median – 2 Mean empirical formula:
Mode = 3(4.5) – 2(1.25) = 13.5 – 2.5 = 11 Ans: (11)

15. On adding or subtracting a constant, range does not change. Hence range is r. Choice (D)

16. Here n = 7. Arranging the data in ascending order, we have 5, 9, 10, 13, 15, 16, 20.

Now, $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term = 2nd term = 9

and $Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ term = 6th term = 16

$\therefore$ Q.D. $= \frac{Q_3-Q_1}{2} = \frac{16-9}{2}$ = 3.5. Ans: (3.5)

17. The data given 1, 2, 5, 7, 11, 13, 16, 17

Arithmetic Mean = $\frac{1+2+5+7+11+13+16+17}{8} = 9$

Mean deviation = $\frac{\left|x_i - \bar{x}\right|}{8} = \frac{|1-9|+|2-9|+.....+|17-9|}{8}$

$= \frac{8+7+4+2+2+4+7+8}{8} = \frac{42}{8} = 5.25$

Choice (D)

18. The arithmetic mean of a and b is $\frac{a+b}{2}$

S.D = $\sqrt{\frac{(a-(a+b)/2)^2+(b-(a+b)/2)^2}{2}}$

S.D = $\sqrt{(a-b)^2/4} = \left|\frac{a-b}{2}\right|$. As a > b, $|a-b| = a-b$

Thus SD = (a – b)/2 Choice (D)

19. Let the numbers be x, x + 1, x + 2, x + 3, x + 4, x + 5, x + 6. Here n = 7

This being an arithmetic progression with common difference 1. S.D = $|d|\sqrt{\frac{n^2-1}{12}} = 1\sqrt{\frac{7^2-1}{12}} = 2$

$\Rightarrow$ Variance = $(S.D.)^2 = 2^2 = 4$ Choice (A)

20. We subtract 18 from each number. The series reduces to 0, 0, 0, 0, 0,0, 0, 0, 0, 5

S.D = $\left(\frac{\sum x^2}{n} - (\bar{x})^2\right)^{1/2} = (25/10 - (5/10)^2)^{1/2}$

$= (2.5 - 0.25)^{1/2} = 1.5$ Choice (D)

21. Since the measures of deviation are not affected by origin change, S.D. $(ax_i + b) = a \times SD(x_i)$

$\Rightarrow SD(3x_i + 2) = 3SD(x_i) = 15$ Ans: (15)

22. $\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2 = 3025$ (given)

$\Rightarrow \frac{n(n+1)}{2} = 55$; $\Rightarrow$ n(n + 1) = 110, so, n = 10

Now $\Sigma n^3/n = 3025/10 = 302.5$ Choice (D)

23. $\Sigma(x - 7) = 40$

$\Rightarrow \Sigma x - \Sigma 7 = 40$

Dividing throughout by 10, we get $\Sigma x/10 - \Sigma 7/10 = 40/10$

$\bar{x} - 7 = 4$; $\bar{x} = 11$ Ans: (11)

24. For two positive integers x and y

G = $\sqrt{xy}$

H = (2xy) / (x + y) ; A = (x + y)/2

Now, $G^2 = xy$.

and AH = $\frac{x+y}{2} \times \frac{2xy}{x+y} = xy$.

Hence $G^2 = AH$. Choice (A)

25. AM (a, b) = (a + b)/2

Case (i) $x \geq 0$

a = max (x, – x) = x

b = min (x, – x) = –x

So, $\frac{a+b}{2} = \frac{(x)+(-x)}{2} = 0$

Case (ii) x < 0

a = max (x, – x) = –x $\Rightarrow$ b = min (x, – x) = x

So, $\frac{a+b}{2} = \frac{(-x)+x}{2} = 0$

Thus A.M (a, b) = 0 Ans: (0)

Exercise – 9(b)

Solutions for questions 1 to 25:

1. The sum of the first 80 even natural numbers = 2 + 4 + 6 + 8 + ... + 160 = 2(1+ 2 + 3 + 4 +. . . . + 80) = $\frac{(2)(80)(81)}{2}$.

The arithmetic mean of the first 80 even natural numbers $= \frac{(2)(80)(81)}{2 \times 80} = 81$ Ans: (81)

2. The sum of the first x odd natural numbers = xy. The pth even natural number is 1 more than the pth odd natural number.

$\therefore$ The sum of the first x even natural numbers = xy + x(1) = x(y + 1)

$\therefore$ The arithmetic mean of the first x even natural numbers $= \frac{x(y+1)}{x} = y+1$ Choice (B)

3. The harmonic mean of 1, 3, 6, 8, 16, and 32

$= \frac{6}{\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}} = \frac{192}{55}$ Choice (C)

4. Geometric mean = $\left[(1)(6)(6^2)....(6^{51})\right]^{\frac{1}{52}}$

$\left[6^{1+2+3......+51}\right]^{\frac{1}{52}}$

$\left[6^{\frac{(51)(52)}{2}}\right]^{\frac{1}{52}} = 6^{\frac{51}{2}}$ Choice (B)

5. Mean of the given numbers

$= \frac{16+4+10+18+30+14+2+24+26}{9} = 16$

Mean deviation

$= \frac{\left[\begin{array}{l}|16-16|+|4-16|+|10-16|+|18-16|+\\|30-16|+|14-16|+|2-16|+|24-16|+|26-16|\end{array}\right]}{9}$

$= \frac{68}{9}$ Choice (A)

6. Wrong total of the numbers = (10) (20) = 200. Correct total of the numbers = 200 + 14 + 12 + 16–(6 + 8 + 10) = 218.

Actual mean = $\frac{218}{10} = 21.8$ Ans: (21.8)

7. The average wage of all the employees is

$\frac{(60)(4000)+(20)(5000)}{80} = ₹4250$ Choice (D)

8. Arithmetic mean = $\frac{6+6+6+6+10+10+8+8+12}{9} = 8$

Standard deviation (S.D) = $\sqrt{\frac{\Sigma(xi - x)^2}{n}}$

$= \sqrt{\frac{4(-8+6)^2+2(-8+10)^2+2(-8+8)^2+(-8+12)^2}{9}} = \frac{\sqrt{40}}{3}$

Choice (D)

9. The first 15 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Their median = Middle prime number = 8^{th} prime number 19.

Ans: (19)

10. AM ($ax_1 + b_1, ax_2 + b_2 . . . ax_n + b$) = a(AM($x_1, x_2 . . x_n$)+ b.
Given AM($y_1, y_2, y_3, . . . y_n$) = A

$\therefore$ AM $\left(\frac{3y_1+2}{4}, \frac{3y_2+2}{4} ... \frac{3y_n+2}{4}\right) = \frac{3A+2}{4}$

Choice (A)

11. Sum of all the observations = (25) (24) = 600.
Arithmetic mean of the remaining numbers

$= \frac{600 - 35 - 36}{23} = 23$ Choice (B)

12. Range = Maximum observation – Minimum observation = 60
3 Maximum observation – 3 minimum observation
= 3(60) = 180. Ans: 180

13. We know that the median of ($y_1, y_2 + k, y_3 + k, . . . y_n + k$)
= the median ($y_1, y_2, . . y_n$) + k
$\therefore$ Median of the new set = 60 – 3 = 57

Choice (A)

14. There are 11 terms arranged in an ascending order

$Q_1 = \left(\frac{11+1}{4}\right)^{th}$ term = 3[rd] term = 7.

$Q_3 = 3\left(\frac{11+1}{4}\right)^{th}$ term = 9[th] term = 29.

The Quartile Deviation (Q.D) = $\frac{Q_3 - Q_1}{2} = \frac{29-7}{2} = 11$

Ans: (11)

15. Arithmetic mean = $\frac{2(5)+3(4)+4(6)}{2+3+4} = \frac{46}{9}$

Choice (D)

16. 8 occurs the maximum number of times in the data.
$\therefore$ mode = 8. Ans: (8)

17. The least multiple of 8 greater than 100 = 104 = (13) (8)
The greatest multiple of 8 less than 200 = 192 = (24) (8).
There are 12 multiples of 8 between 100 and 200.

$\therefore$ their sum = $\frac{12}{2}$ [104 + 192]

$\therefore$ their arithmetic mean = $\frac{\frac{12}{2}[104+192]}{12} = 148$

Choice (C)

18. If each observation of a series is divided by k their standard deviation will also be divided by k. $\therefore$ the standard deviation would be divided by 3.

$\therefore$ Hence the standard deviation of the new series is $\frac{\sigma}{3}$.

Choice (B)

19. Let the side of ABCD be x pm.

Time taken by Raju to travel AB = $\frac{x}{S_1}$ hours

Time taken by Raju to travel BC = $\frac{x}{S_2}$ hours.

Average speed = $\frac{2x}{\frac{x}{S_1}+\frac{x}{S_2}} = \frac{2S_1S_2}{S_1+S_2}$.

which is the harmonic mean of S_1 and S_2

Choice (C)

20. $Z_1 = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$

$Z_2 = \frac{Y_2 + Y_3 + Y_4 + Y_5}{4}$

$Z_3 = \frac{Y_3 + Y_4 + Y_5 + Y_6}{4}$ . . . . .

$Z_{n-2} = \frac{Y_{n-2} + Y_{n-1} + Y_n + Y_1}{4}$

$Z_{n-1} = \frac{Y_{n-1} + Y_n + Y_1 + Y_2}{4}$

$Z_n = \frac{Y_n + Y_1 + Y_2 + Y_3}{4}$

It would be seen that each Y_i where $1 \le i \le n$ occurs exactly in four of the Z_i s.
$\therefore$ arithmetic mean of $Z_1, Z_2, . . . Z_n$

$= \frac{4(Y_1 + Y_2 + ... + . Y_n)}{4n} = A$ Ans: 1

21. The sum of the cubes of the first n even natural numbers

$= 8\left(\frac{n(n+1)}{2}\right)^2$

Their arithmetic mean = $8\frac{\left(\frac{n(n+1)}{2}\right)^2}{n} = 2n(n+1)^2$

Choice (B)

22. $\frac{Y_1 + Y_2 + ... + Y_n}{n} = M$

$Y_1 + Y_2 + . . . Y_{n-2} + Y_{n-1} \quad Y_n = Mn$
$Y_1 + Y_2 + . . . Y_{n-2} + Y_n = Mn - Y_{n-1}$
$Y_1 + Y_2 + ... Y_{n-2} + Y_r + Y_n = Mn - Y_{n-1} + Y_r$
$\therefore$ the arithmetic mean of the new series will be

$\frac{Mn - Y_{n-1} + Y_r}{n}$ Choice (D)

23. $\frac{x_1 + x_2 + ... x_n}{n} = x$

$\Rightarrow x_1 + x_2 + . . . + x_n = nx$ . . . (1)

$\frac{y_1 + y_2 + ... + y_n}{n} = y$

$\Rightarrow y_1 + y_2 + ... + y_n = n y$ . . . . (2)
Subtracting (2) from (1),
$(x_1 - y_1) + (x_2 - y_2) + ... + (x_n - y_n) = n(x - y)$

Required mean = $\frac{x_1 - y_1 + x_2 - y_2 + ... + x_n - y_n}{n}$

$= \frac{n(x-y)}{n} = x - y$ Choice (A)

24. Sum of all the observations = (150) (30)
Sum of 100 of them = (100) (30)
Sum of the remaining = (150 – 100) (30)

Arithmetic mean of the remaining = $\frac{(150-100)(30)}{50} = 30$

Ans: 30

25. Given, n = 11; $\sum(xi - x)^2 = 110$

Standard deviation = $\sqrt{\frac{\sum(xi - \bar{x})^2}{n}} = \sqrt{\frac{110}{11}} = \sqrt{10}$

Choice (B)