Chapter – 4
GEOMETRY

In most competitive exams, the problems relating to Geometry usually cover triangles, quadrilaterals and circles. Even though polygons with more than four sides are also covered, the emphasis on such polygons is not as much as it is on triangles and circles. In this chapter, we will look at some properties as well as theorems and riders on parallel lines, angles, triangles (including congruence and similarity of triangles), quadrilaterals, circles and polygons.

ANGLES AND LINES

An angle of 90° is a right angle; an angle less than 90° is acute angle; an angle between 90° and 180° is an obtuse angle; and an angle between 180° and 360° is a reflex angle.

The sum of all angles made on one side of a straight line AB at a point O by any number of lines joining the line AB at O is 180°. In Fig. 4.01 below, the sum of the angles u, v, x, y and z is equal to 180°.

When any number of straight lines meet at a point, the sum of all the angles around that point is 360°. In Fig. 4.02 below, the sum of the angles u, v, w, x, y and z is equal to 360°.

Fig. 4.01

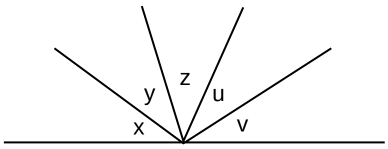
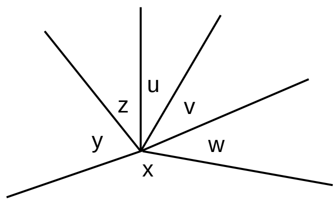
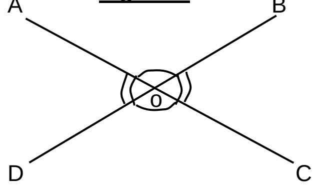


Fig. 4.02



Two angles whose sum is 90° are said to be complementary angles and two angles whose sum is 180° are said to be supplementary angles.
When two straight lines intersect, vertically opposite angles are equal. In Fig. 4.03 given below, $\angle AOB$ and $\angle COD$ are vertically opposite angles and $\angle BOC$ and $\angle AOD$ are vertically opposite angles. So, we have $\angle AOB = \angle COD$ and $\angle BOC = \angle AOD$.

Fig. 4.03



Two lines which make an angle of 90° with each other are said to be PERPENDICULAR to each other.

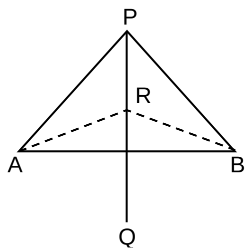
If a line/line segment l_1 passes through the mid-point of another line segment l_2, the line/line segment l_1 is said to be the BISECTOR of the line segment l_2, i.e., the line segment l_2 is divided into two equal parts.

If a line l_1 is drawn at the vertex of an angle dividing the angle into two equal parts, the line l_1 is said to be the ANGLE BISECTOR of the angle. Any point on the angle bisector of an angle is EQUIDISTANT from the two arms of the angle.

If a line l_1 is perpendicular to line segment l_2 as well as passes through the mid-point of the line segment l_2, the line l_1 is said to be the PERPENDICULAR BISECTOR of the line segment l_2.

Any point on the perpendicular bisector of a line segment is EQUIDISTANT from both ends of the line segment.

Fig. 4.04

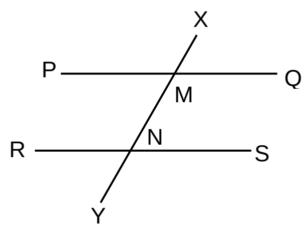


In Fig 4.04, line PQ is the perpendicular bisector of line segment AB. A point P on the perpendicular bisector of AB will be equidistant from A and B, i.e., PA = PB. Similarly, for any point R on the perpendicular bisector PQ, RA = RB.

PARALLEL LINES

When a straight line cuts two or more lines in the same plane, the line is called the TRANSVERSAL. When a transversal XY cuts two parallel lines PQ and RS [as shown in Fig. 4.05], the following are the relationships between various angles that are formed. [M and N are the points of intersection of XY with PQ and RS respectively].

Fig. 4.05



(a) Alternate angles are equal, i.e.
$\angle PMN = \angle MNS$ and $\angle QMN = \angle MNR$

(b) Corresponding angles are equal, i.e.
$\angle XMQ = \angle MNS$; $\angle QMN = \angle SNY$;
$\angle XMP = \angle MNR$; $\angle PMN = \angle RNY$

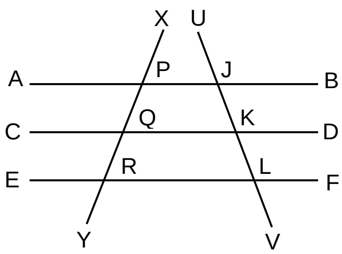
(c) Sum of interior angles on the same side of the transversal is equal to 180°, i.e.
$\angle QMN + \angle MNS = 180°$ and $\angle PMN + \angle MNR = 180°$

(d) Sum of exterior angles on the same side of the transversal is equal to 180°, i.e.
$\angle XMQ + \angle SNY = 180°$, and $\angle XMP + \angle RNY = 180°$

RATIO OF INTERCEPTS

If three or more parallel lines make intercepts on a transversal in a certain proportion, then they make intercepts in the same proportion on any other transversal as well. In Fig. 4.06, the lines AB, CD and EF are parallel and the transversal XY cuts them at the points P, Q and R. If we now take a second transversal, UV, cutting the three parallel lines at the points J, K and L, then we have PQ/QR = JK/KL.

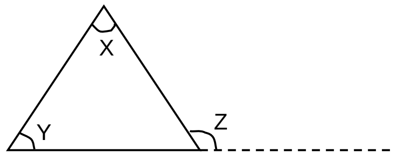
Fig.4.06



Note: If three or more parallel lines make equal intercepts on one transversal, they make equal intercepts on any other transversal as well.

TRIANGLES

Fig. 4.07



Sum of the three angles of a triangle is 180°

The exterior angle of a triangle (at each vertex) is equal to the sum of the two opposite interior angles. (Exterior angle is the angle formed at any vertex, by one side and the extended portion of the second side at that vertex).

A line perpendicular to a side and passing through the midpoint of the side is said to be the **perpendicular bisector** of the side. It is not necessary that the perpendicular bisector of a side should pass through the opposite vertex in a triangle in general.

The perpendicular drawn to a side from the opposite vertex is called the **altitude** to that side.

The line joining the midpoint of a side with the opposite vertex is called the **median** drawn to that side. A median divides the triangle into two equal halves as far as the area is concerned.

An equilateral triangle is one in which all the sides are equal (and hence, all angles are equal, i.e., each of the angles is equal to 60°). An isosceles triangle is one in which two sides are equal (and hence, the angles opposite to them are equal). A scalene triangle is one in which no two sides are equal.

In an isosceles triangle, the unequal side is called the BASE. The angle where the two equal sides meet is called the VERTICAL ANGLE. In an isosceles triangle, the perpendicular drawn to the base from the vertex opposite the base (i.e., the altitude drawn to the base) bisects the base as well as the vertical angle. That is, the altitude drawn to the base will also be the perpendicular bisector of the base as well as the angle bisector of the vertical angle. It will also be the median drawn to the base.

In an equilateral triangle, the perpendicular bisector, the median and the altitude drawn to a particular side coincide and that will also be the angle bisector of the opposite vertex. If a is the side of an equilateral triangle, then its altitude is equal to $\sqrt{3}.a / 2$

Sum of any two sides of a triangle is greater than the third side; difference of any two sides of a triangle is less than the third side.

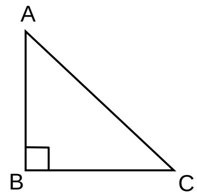
If the sides are arranged in the ascending order of their measurement, the angles opposite the sides (in the same order) will also be in ascending order (i.e., greater angle has greater side opposite to it); if the sides are arranged in descending order of their measurement, the angles opposite the sides in the same order will also be in descending order (i.e., smaller angle has smaller side opposite to it).

There can be only one right angle or only one obtuse angle in any triangle. There can also not be one right angle and an obtuse angle both present at the same time in a triangle.

Hypotenuse is the side opposite the right angle in a right-angled triangle. In a right-angled triangle, the hypotenuse is the longest side. In an obtuse angled triangle, the side opposite the obtuse angle is the longest side.

Fig. 4.08

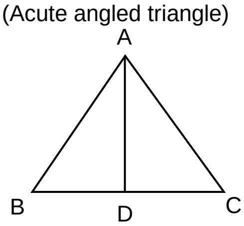
(Right-angled triangle)



In a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. In Fig. 4.08, $AC^2 = AB^2 + BC^2$

Fig. 4.09

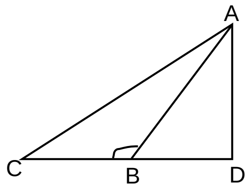
(Acute angled triangle)



In an acute angled triangle, the square of the side opposite the acute angle is less than the sum of the squares of the other two sides by a quantity equal to twice the product of one of these two sides and the projection of the second side on the first side.
In Fig. 4.09, $AC^2 = AB^2 + BC^2 - 2\ BC.BD$

Fig. 4.10

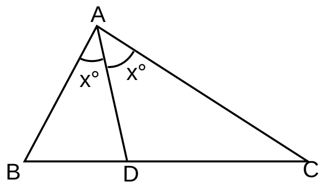
(Obtuse angled triangle)



In an obtuse angled triangle, the square of the side opposite the obtuse angle is greater than the sum of the squares of the other two sides by a quantity equal to twice the product of one of the sides containing the obtuse angle and the projection of the second side on the first side.

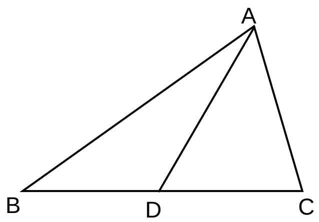
In Fig. 4.10, $AC^2 = AB^2 + BC^2 + 2BC.BD$

Fig. 4.11



In a triangle, the internal bisector of an angle bisects the opposite side in the ratio of the other two sides. In triangle ABC, if AD is the angle bisector of angle A, then BD/DC = AB/AC. This is called the **Angle Bisector Theorem** (refer to Fig. 4.11).

Fig. 4.12

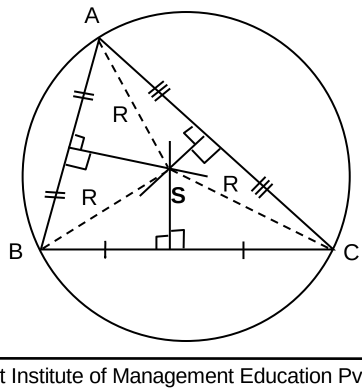


In ΔABC, if AD is the median from A to side BC (meeting BC at its mid point D), then $2(AD^2 + BD^2) = AB^2 + AC^2$. This is called the **Apollonius Theorem**. This will be helpful in calculating the lengths of the three medians given the lengths of the three sides of the triangle (refer to Fig. 4.12).

GEOMETRIC CENTRES OF A TRIANGLE

CIRCUMCENTRE

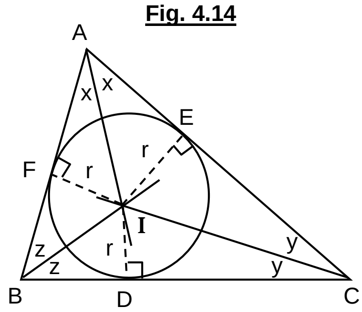
Fig 4.13



The three perpendicular bisectors of a triangle meet at a point called **Circumcentre** of the triangle and it is represented by S. The circumcentre of a triangle is equidistant from its vertices and the distance of circumcentre from each of the three vertices is called circumradius (represented by R) of the triangle. The circle drawn with the circumcentre as centre and circumradius as radius is called the **Cirumcircle** of the triangle and passes through all three vertices of the triangle. (refer to Fig. 4.13)

INCENTRE AND EXCENTRES

Fig. 4.14



The internal bisectors of the three angles of a triangle meet at a point called **Incentre** of the triangle and it is represented by I. Incentre is equidistant from the three sides of the triangle i.e., the perpendiculars drawn from the incentre to the three sides are equal in length and this length is called the inradius (represented by r) of the triangle. The circle drawn with the incentre as centre and the inradius as radius is called the incircle of the triangle and it touches all three sides on the inside.

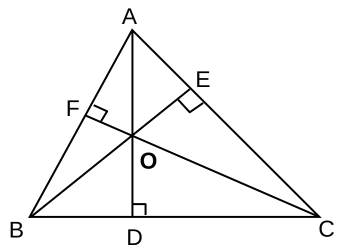
In Fig. 4.14, $\angle BIC = 90° + ½ A$ where I is the incentre.

$\angle CIA = 90° + ½ B$; and $\angle AIB = 90° + ½ C$.

If the internal bisector of one angle and the external bisectors of the other two angles are drawn, they meet at a point called **Excentre.** There will be totally three excentres for the triangle - one corresponding to the internal bisector of each angle.

ORTHOCENTRE

Fig 4.15



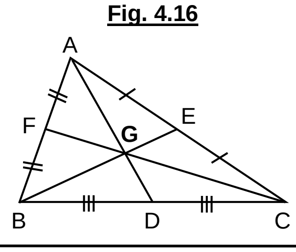
The three altitudes meet at a point called Orthocentre and it is represented by O (refer to Fig. 4.15).

$\angle BOC = 180° - \angle A$; $\angle COA = 180° - \angle B$;

$\angle AOB = 180° - \angle C$.

CENTROID

Fig. 4.16



The three medians of a triangle meet at a point called the **Centroid** and it is represented by G (refer to Fig. 4.16).

Important points about geometric centres of a triangle

In an acute angled triangle, the circumcentre lies inside the triangle. In a right-angled triangle, the circumcentre lies on the hypotenuse of the triangle (it is the midpoint of the hypotenuse). In an obtuse angled triangle, the circumcentre lies outside the triangle.

In an acute angled triangle, the orthocentre lies inside the triangle. In a right-angled triangle, the vertex where the right angle is formed (i.e., the vertex opposite the hypotenuse) is the orthocentre. In an obtuse angled triangle, the orthocentre lies outside the triangle.

In a right-angled triangle the length of the median drawn to the hypotenuse is equal to half the hypotenuse. This median is also the circumradius of the right-angled triangle.

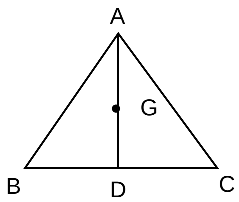
Centroid divides each of the medians in the ratio 2 : 1, the part of the median towards the vertex being twice in length to the part towards the side.

Inradius is less than half of any of the three altitudes of the triangle.

In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre, all lie on the median to the base.

In an equilateral triangle, the centroid, the orthocentre, the circumcentre and the incentre, all coincide.

Fig 4.17



Hence, in equilateral triangle ABC shown in Fig. 4.17, AD is the median, altitude, angle bisector and perpendicular bisector. G is the centroid which divides the median in the ratio 2 : 1. Hence, AG = 2/3 AD and GD = 1/3 AD.

But since AD is also the perpendicular bisector and angle bisector and since G is the circumcentre as well as the incentre, AG will be the circumradius and GD will be the inradius of the equilateral triangle ABC. Since AD is also the altitude, its length is equal to $\sqrt{3}a/2$ where a is the side of the equilateral triangle. Hence, the circumradius of the equilateral triangle

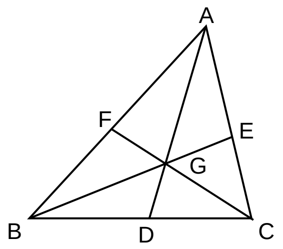
$= \frac{2}{3} \times \frac{\sqrt{3}}{2} \,.\, a = a/\sqrt{3}$ and the inradius $= \frac{1}{3} \times \frac{\sqrt{3}}{2} \,.\, a$

$= a/2\sqrt{3}$

Since the radii of the circumcircle and the incircle of an equilateral triangle are in the ratio 2 : 1, the areas of the circumcircle and the incircle of an equilateral triangle will be in the ratio 4 : 1.

When the three medians of a triangle (i.e., the medians to the three sides of a triangle from the corresponding opposite vertices) are drawn, the resulting six triangles are equal in area and the area of each of these triangles in turn is equal to one-sixth of the area of the original triangle.

Fig 4.18

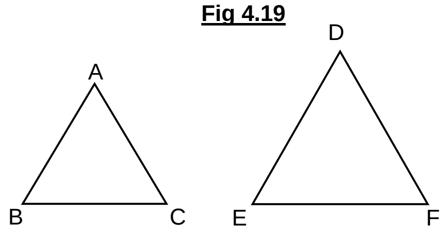


In Fig 4.18, AD, BE and CF are the medians drawn to the three sides. The three medians meet at the centroid G. The six resulting triangles AGF, BGF, BGD, CGD, CGE and AGE are equal in area and each of them is equal to $1/6^{th}$ of the area of triangle ABC.

SIMILARITY OF TRIANGLES

Two triangles are said to be similar if the three angles of one triangle are equal to the three angles of the second triangle. Similar triangles are alike in shape only. The corresponding angles of two similar triangles are equal but the corresponding sides are only proportional and not equal.

Fig 4.19



For example, in Fig 4.19, if ΔABC is similar to ΔDEF where $\angle$A = $\angle$D, $\angle$B = $\angle$E and $\angle$C = $\angle$F, we have the ratios of the corresponding sides equal, as given below.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

By "corresponding sides", we mean that if we take a side opposite to a particular angle in one triangle, we should consider the side opposite to the equal angle in the second triangle. In this case, since AB is the side opposite to $\angle$C in $\angle$ABC, and since $\angle$C = $\angle$F, we have taken DE which is the side opposite to $\angle$F in ΔDEF.

Two triangles are similar if,

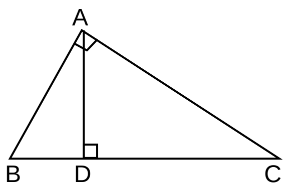
- the three angles of one triangle are respectively equal to the three angles of the second triangle, or
- two sides of one triangle are proportional to two sides of the other and the included angles are equal, or
- if the three sides of one triangle are proportional to the three sides of another triangle.

In two similar triangles,

(a) Ratio of corresponding sides = Ratio of heights (altitudes) = Ratio of the lengths of the medians = Ratio of the lengths of the angular bisectors = Ratio of inradii = Ratio of circumradii = Ratio of perimeters.

(b) Ratio of areas = Ratio of squares of corresponding sides

Fig 4.20



In a right-angled triangle, the altitude drawn to the hypotenuse divides the given triangle into two similar triangles, each of which is in turn similar to the original triangle. In triangle ABC in Fig.4.20, ABC is a right-angled triangle where $\angle A$ is a right angle. AD is the perpendicular drawn to the hypotenuse BC. The triangles ABD, CAD and CBA are similar because of the equal angles given below.

In triangle ABC, $\angle A = 90°$. If $\angle B = \theta$, then $\angle C = 90° - \theta$.

In triangle ABD, $\angle ADB = 90°$. We already know that $\angle B = \theta$; hence $\angle BAD = 90° - \theta$.

In triangle ADC, $\angle ADC = 90°$. We already know that $\angle C = 90° - \theta$; hence $\angle CAD = \theta$.

We can write down the relationship between the sides in these three triangles. The important relationships that emerge out of this exercise are :

1. $AD^2 = BD \cdot DC$;
2. $AB^2 = BC \cdot BD$;
3. $AC^2 = CB \cdot CD$.

CONGRUENCE OF TRIANGLES

Two triangles will be congruent if at least one of the following conditions is satisfied:

Three sides of one triangle are respectively equal to the three sides of the second triangle (normally referred to as the S-S-S rule, i.e., the side-side-side congruence).

Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the second triangle (normally referred to as the S-A-S rule, i.e., side-angle-side congruence).

Two angles and one side of a triangle are respectively equal to two angles and the corresponding side of the second triangle (normally referred to as the A-S-A rule, i.e., angle-side-angle congruence).

Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to hypotenuse and one side of the second right-angled triangle.

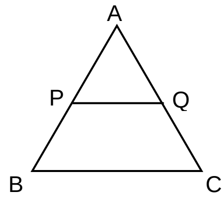
In two congruent triangles,

- the corresponding sides (i.e., sides opposite to corresponding angles) are equal.
- the corresponding angles (angles opposite to corresponding sides) are equal.
- the areas of the two triangles will be equal.

Some more useful points about triangles

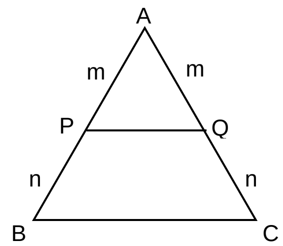
BASIC PROPORTIONALITY THEOREM

Fig. 4.21



A line drawn parallel to one side of a triangle divides the other two sides in the same proportion. For example, in Fig. 4.21, PQ is drawn parallel to BC in ΔABC. This will divide the other two sides AB and AC in the same ratio, i.e., AP/PB = AQ/QC.

Fig. 4.22



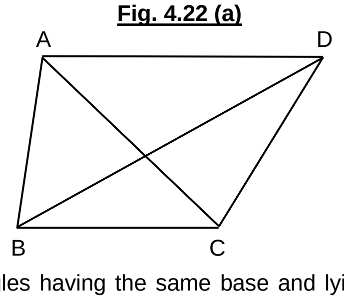
Conversely, a line joining two points (each) dividing two sides of a triangle in the same ratio is parallel to the third side. In Fig. 4.22, P divides AB in the ratio m : n and Q divides AC in the ratio m : n. Now, the line joining P and Q will be parallel to the third side BC and the length of PQ will be equal to $\frac{m}{m+n}$ times the length of BC.

We can say that a line drawn through a point on a side of the triangle parallel to a second side will cut the third side in the same ratio as the first side is divided.

MID-POINT THEOREM

The line joining the midpoints of two sides of a triangle is parallel to the third side and it is half the third side.

Fig. 4.22 (a)



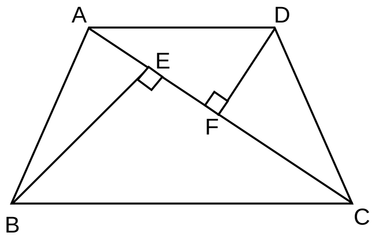
Two triangles having the same base and lying between the same pair of parallel lines have their areas equal (fig.4.22(a)).

AD is parallel to BC. Hence, $\Delta ABC = \Delta DBC$

QUADRILATERALS

Any four-sided closed figure is called a quadrilateral. By imposing certain conditions on the sides and/or angles of a quadrilateral, we can get the figures trapezium, parallelogram, rhombus, rectangle, square.

Fig. 4.23

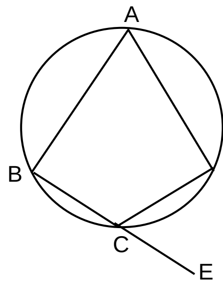


The sum of four angles of a quadrilateral is equal to 360°.

The perpendiculars drawn to a diagonal (in a quadrilateral) from the opposite vertices are called "offsets". In Fig. 4.23, BE and DF are the offsets drawn to the diagonal AC.

If the four vertices of a quadrilateral lie on the circumference of a circle (i.e., if the quadrilateral can be inscribed in a circle) it is called a cyclic quadrilateral (refer to Fig. 4.24). In a cyclic quadrilateral, sum of opposite angles = 180° i.e., in Fig. 4.24, A + C = 180° and B + D = 180°.

Fig. 4.24

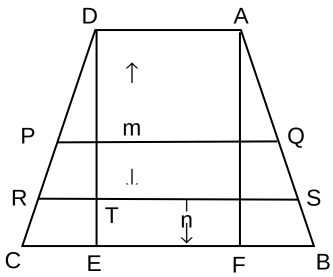


Also, in a cyclic quadrilateral, exterior angle is equal to the interior opposite angle, i.e., in Fig.4.24, $\angle DCE$ is equal to $\angle BAD$.

Now, we will look at different quadrilaterals and their properties.

TRAPEZIUM

Fig. 4.25



If one side of a quadrilateral is parallel to its opposite side, then it is called a trapezium. The two sides other than the parallel sides in a trapezium are called the oblique sides.

In Fig. 4.25, ABCD is a trapezium where AD is parallel to BC.

If the midpoints of the two oblique sides are joined, it is equal in length to the average of the two parallel sides, i.e., in Fig. 4.25, PQ = 1/2 [AD + BC]

In general, if a line is drawn in between the two parallel sides of the trapezium such that it is parallel to the parallel sides and also divides the distance between the two parallel sides in the ratio m : n (where the portion closer to the shorter of the two parallel sides is m), the length of the line is given by :

$\frac{m}{m+n} \times$ Longer side + $\frac{n}{m+n} \times$ Shorter side where shorter side and longer side refer to the shorter and longer of the two parallel sides of the trapezium.

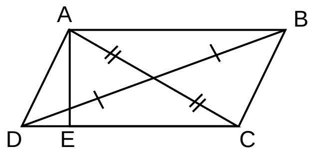
In Fig. 4.25, RS is the line parallel to AD and BC and the ratio of the distances DT and TE is m : n.

The length of RS is given by

$\frac{m}{m+n}$ x BC + $\frac{n}{m+n}$ x AD

PARALLELOGRAM

Fig. 4.26



A quadrilateral in which opposite sides are parallel is called a parallelogram.

In a parallelogram

- Opposite sides are equal
- Opposite angles are equal
- Sum of any two adjacent angles is 180°.
- Each diagonal divides the parallelogram into two congruent triangles.
- The diagonals bisect each other.

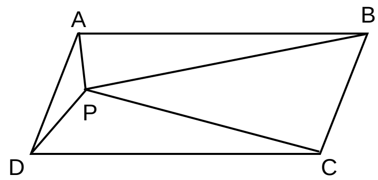
  Conversely, if in a quadrilateral
- the opposite sides are equal or
- the opposite angles are equal or
- the diagonals bisect each other or
- a pair of opposite sides are parallel and equal

  such a quadrilateral is a parallelogram.

If two adjacent angles of a parallelogram are equal, then all four angles will be equal and each in turn will be equal to 90°. Then the figure will be a rectangle.

If any two adjacent sides of a parallelogram are equal, then all four sides will be equal to each other and the figure will be a rhombus.

Fig. 4.27



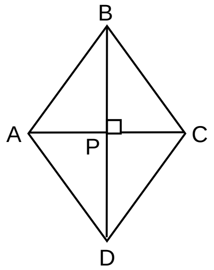
If any point inside a parallelogram is taken and is joined to all the four vertices the four resulting triangles will be such that the sum of the areas of opposite triangles is equal. In Fig. 4.27, P is a point inside the parallelogram ABCD and it is joined to the four vertices of the parallelogram by the lines PA, PB, PC and PD respectively. Then Area of triangle PAB + Area of triangle PCD = Area of triangle PBC + Area of triangle PAD = Half the area of parallelogram ABCD.

If there is a parallelogram and a triangle with the same base and between the same parallel lines, then the area of the triangle will be half that of the parallelogram.

If there is a parallelogram and a rectangle with the same base and between the same parallel lines, then the areas of the parallelogram and the rectangle will be the same. The figure formed by joining the midpoints of the sides of any quadrilateral taken in order, is a parallelogram.

RHOMBUS

Fig. 4.28



A rhombus is a parallelogram in which a pair of adjacent sides are equal (all four sides of a rhombus are equal). Since a rhombus is a parallelogram, all the properties of a parallelogram apply to a rhombus. Further, in a rhombus, the diagonals bisect each other **perpendicularly**.

Conversely, any quadrilateral where the two diagonals bisect each other at right angles will be a rhombus. The four triangles that are formed by the two bisecting diagonals with the four sides of the rhombus will all be congruent. In Fig.5.28, the four triangles PAB, PBC, PCD and PAD are congruent.
Side of a rhombus
$= \sqrt{1/4 \text{ x Sum of squares of the diagonals}}.$

RECTANGLE

A rectangle also is a special type of parallelogram and hence all properties of a parallelogram apply to rectangles also. A rectangle is a parallelogram in which two adjacent angles are equal or each of the angles is equal to 90°.

The diagonals of a rectangle are **equal** (and, of course, bisect each other).

When a rectangle is inscribed in a circle, the diagonals become the diameters of the circle.
If a and b are the two adjacent sides of a rectangle, then the diagonal is given by $\sqrt{a^2+b^2}$.

If a rectangle and a triangle are on the same base and between the same parallels, then the area of the triangle will be equal to half the area of the rectangle.

SQUARE

A square is a rectangle in which all four sides are equal (or a rhombus in which all four angles are equal, i.e., all are right angles) Hence, the diagonals are equal and they bisect at right angles. So, all the properties of a rectangle as well as those of a rhombus hold good for a square.

Diagonal = $\sqrt{2} \times$ Side
When a square is inscribed in a circle, the diagonals become the diameters of the circle.

When a circle is inscribed in a square, the side of the square is equal to the diameter of the circle.

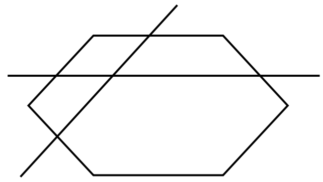
The largest rectangle that can be inscribed in a given circle will be a square.

POLYGON

Any closed figure with three or more sides is called a polygon.

A convex polygon is one in which each of the interior angles is less than 180°. It can be noticed that any straight line drawn cutting a convex polygon passes only two sides of the polygon, as shown in the figure below.

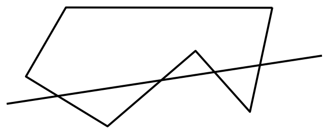
Fig. 4.28 (a)



Convex Polygon

In a concave polygon, it is possible to draw lines passing through more than two sides, as shown in the figure below.

Fig. 4.28 (b)



Concave Polygon

A regular polygon is a convex polygon in which all sides are equal and all angles are equal. A regular polygon can be inscribed in a circle. The centre of the circumscribing circle (the circle in which the polygon is inscribed) of a regular polygon is called the centre of the polygon.

The names of polygons with three, four, five, six, seven, eight, nine and ten sides are respectively triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon.

The sum of interior angles of a convex polygon is equal to (2n – 4) right angles where n is the number of the sides of the polygon.

If each of the sides of a convex polygon is extended, the sum of the external angles thus formed is equal to 4 Right Angles (i.e., 360°).

In a regular polygon of n sides, if each of the interior angles is d°, then $d = \frac{2n-4}{n} \times 90°$ and each exterior angle = $\frac{360°}{n}$.

It will be helpful to remember the interior angles of the following regular polygons:

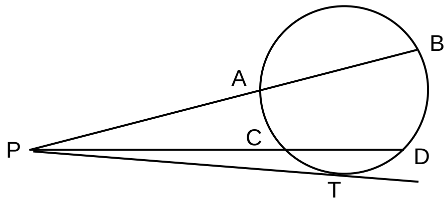
Regular pentagon	:	108°
Regular hexagon	:	120°
Regular octagon	:	135°

If the centre of a regular polygon (with n sides) is joined with each of the vertices, we get n identical triangles inside the polygon. In general, all these triangles are isosceles triangles. Only in case of a regular hexagon, all these triangles are equilateral triangles, i.e., in a regular hexagon, the radius of the circumscribing circle is equal to the side of the hexagon.

A line joining any two non-adjacent vertices of a polygon is called a diagonal. A polygon with n sides will have $\frac{n(n-3)}{2}$ diagonals.

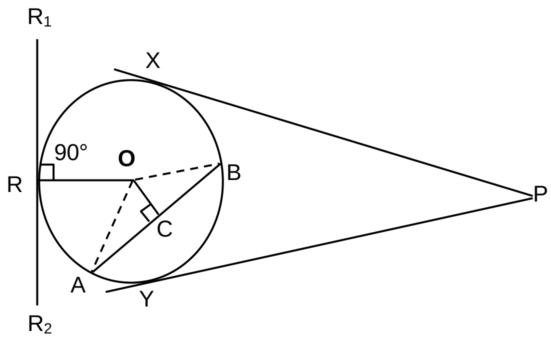
CIRCLES

Fig 4.29



A circle is a closed curve drawn such that any point on the curve is equidistant from a fixed point. The fixed point is called the centre of the circle and the distance from the centre to any point on the circle is called the radius of the circle.

Fig. 4.30



Diameter is a straight line passing through the centre of the circle and joining two points on the circle. A circle is symmetric about any diameter.

A chord is a point joining two points on the circumference of a circle (AB in Fig. 4.30). Diameter is the largest chord in a circle.

A secant is a line intersecting a circle in two distinct points and extending outside the circle also.

A line that touches the circle at only one point is a tangent to the circle (R_1R_2 is a tangent touching the circle at the point R in Fig. 5.30).

If PAB and PCD are two secants (in Fig. 4.29), then PA.PB = PC.PD

If PAB and PCD are secants and PT is a tangent to the circle at T (in Fig. 4.29), then PA.PB = PC.PD = PT^2.

Two tangents can be drawn to the circle from any point outside the circle and these two tangents are equal in length. In Fig. 4.30, P is the external point and the two tangents PX and PY are equal.

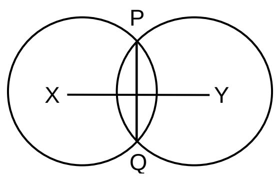
A tangent is perpendicular to the radius drawn at the point of tangency (In Fig. 4.30, $R_1R_2 \perp OR$). Conversely, if a perpendicular is drawn to the tangent at the point of tangency, it passes through the centre of the circle.

A perpendicular drawn from the centre of the circle to a chord bisects the chord (In Fig. 4.30, OC, the perpendicular from O to the chord AB bisects AB) and conversely, the perpendicular bisector of a chord passes through the centre of the circle.

Two chords that are equal in length will be equidistant from the centre, and conversely two chords which are equidistant from the centre of the circle will be of the same length.

One and only one circle passes through any three given non-collinear points.

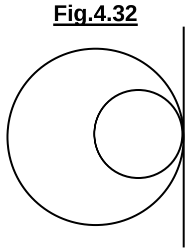
Fig. 4.31



When there are two intersecting circles, the line joining the centres of the two circles will perpendicularly bisect the line joining the points of intersection. In Fig. 4.31, the two circles with centres X and Y respectively intersect at the two points P and Q. The line XY (the line joining the centres) bisects PQ (the line joining the two points of intersection).

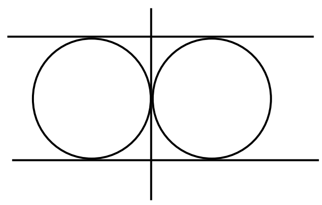
Two circles are said to touch each other if a common tangent can be drawn touching both the circles at the same point. This is called the point of contact of the two circles. The two circles may touch each other internally (as in Fig. 4.32) or externally (As in Fig. 4.33). When two circles touch each other, then the point of contact and the centres of the two circles are collinear, i.e., the point of contact lies on the line joining the centres of the two circles.

Fig.4.32



If two circles touch internally, the distance between the centres of the two circles is equal to the difference in the radii of the two circles. When two circles touch each other externally, then the distance between the centres of the two circles is equal to the sum of the radii of the two circles.

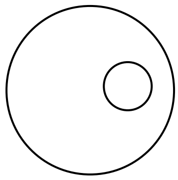
Fig. 4.33



A tangent drawn common to two circles is called a common tangent. In general, for two circles, there can be anywhere from zero to four common tangents drawn depending on the position of one circle in relation to the other.

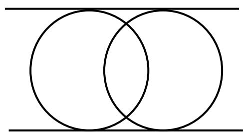
If the common tangent is either parallel to the line of centres or cuts the line joining the centres not between the two circles but on one side of the circles, such a common tangent is called a direct common tangent. A common tangent that cuts the line joining the centres in between the two circles is called transverse common tangent.

Fig. 4.34



If two circles are such that one lies completely inside the other (without touching each other), then there will not be any common tangent to these circles (refer to Fig. 4.34).
Two circles touching each other internally (i.e., still one circle lies inside the other), then there is only one common tangent possible and it is drawn at the point of contact of the two circles (refer to Fig. 4.32).

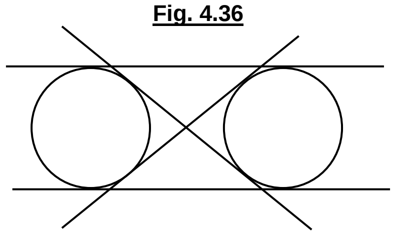
Fig. 4.35



Two intersecting circles have two common tangents. Both these are direct common tangents and the two intersecting circles do not have a transverse common tangent (refer to Fig. 4.35).

Two circles touching each other externally have three common tangents. Out of these, two are direct common tangents and one is a transverse common tangent. The transverse common tangent is at the point of contact (Refer to Fig. 4.33).

Fig. 4.36



Two circles which are non-intersecting and non-enclosing (i.e. one does not lie inside the other) have four common tangents - two direct and two transverse common tangents (Refer to Fig. 4.36).

If r_1 and r_2 are the radii of the two non-intersecting non-enclosing circles,

Length of the direct common tangent

$$= \sqrt{(\text{Distance between centre})^2 - (r_1 - r_2)^2}$$

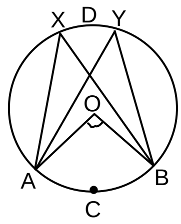
Length of transverse common tangent

$$= \sqrt{(\text{Distance between centre})^2 - (r_1 + r_2)^2}$$

Two circles are said to be concentric if they have the same centre. As is obvious, here the circle with smaller radius lies completely within the circle with bigger radius.

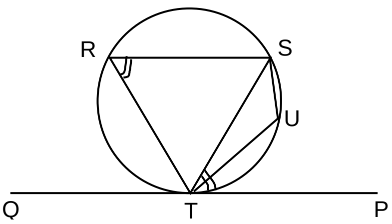
Arcs and Sectors

Fig. 4.37



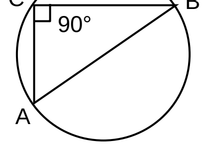
An arc is a segment of a circle. In Fig. 4.37, ACB is called minor arc and ADB is called major arc. In general, if we talk of an arc AB, we refer to the minor arc. AOB is called the angle formed by the arc AB (at the centre of the circle).
The angle subtended by an arc at the centre is double the angle subtended by the arc in the remaining part of the circle.
In Fig. 4.37, $\angle AOB = 2 \,.\, \angle AXB. = 2.\angle AYB$
Angles in the same segment are equal. In Fig. 4.37,
$\angle AXB = \angle AYB$.

Fig. 4.38



The angle between a tangent and a chord through the point of contact of the tangent is equal to the angle made by the chord in the alternate segment (i.e., segment of the circle on the side other than the side of location of the angle between the tangent and the chord). This is normally referred to as the "alternate segment theorem."
In Fig. 4.38, PQ is a tangent to the circle at the point T and TS is a chord drawn at the point of contact. Considering $\angle PTS$ which is the angle between the tangent and the chord, the angle TRS is the angle in the "alternate segment". So, $\angle PTS = \angle TRS$.
Similarly, $\angle QTS = \angle TUS$.

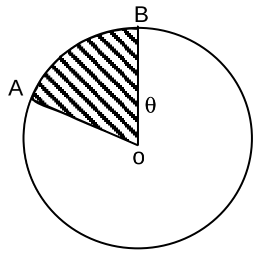
Fig. 4.39



We have already seen in quadrilaterals, the opposite angles of a cyclic quadrilateral are supplementary and that the external angle of a cyclic quadrilateral is equal to the interior opposite angle.

The angle in a semicircle (or the angle the diameter subtends in a semicircle) is a right angle. The converse of the above is also true and is very useful in a number of cases - in a right angled triangle, a semi-circle with the hypotenuse as the diameter can be drawn passing through the third vertex (Refer to Fig. 4.39).

Fig. 4.40



The area formed by an arc and the two radii at the two end points of the arc is called sector.
In Fig. 4.40, the shaded figure AOB is called the minor sector.

AREAS OF PLANE FIGURES

Mensuration is the branch of geometry that deals with the measurement of length, area and volume. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of "solids." Solids are objects, which have three dimensions (plane figures have only two dimensions).

Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

TRIANGLES

The area of a triangle is represented by the symbol Δ. For any triangle, the three sides are represented by a, b and c and the angles opposite these sides represented by A, B and C respectively.

(i) For any triangle in general,

(a) When the measurements of three sides a, b, c are given,

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where

s = $\frac{a+b+c}{2}$

This is called Hero's formula.

(b) When base (b) and altitude (height) to that base are given,
Area = ½ x base x altitude = ½ b.h

(c) **Area = ½ ab . sinC = ½ bc.sinA = ½ ca.sinB**

(d) **Area =** $\frac{abc}{4R}$ where R is the circumradius of the triangle.

(e) **Area = r.s** where r is the inradius of the triangle and s, the semi-perimeter.

Out of these five formulae, the first and the second are the most commonly used and are also more important from the examination point of view.

(ii) For a right angled triangle,

Area = ½ X Product of the sides containing the right angle

(iii) For an equilateral triangle

Area = $\frac{\sqrt{3}\cdot a^2}{4}$ where "a" is the side of the triangle

The height of an equilateral triangle $=\frac{\sqrt{3}\cdot a}{2}$

(iv) For an isosceles triangle

Area = $\frac{b}{4}\sqrt{4a^2-b^2}$ where "a" is length of each of the two equal sides and b is the third side

QUADRILATERALS

(i) For any quadrilateral

Area of the quadrilateral = ½ x One diagonal x Sum of the offsets drawn to that diagonal
Hence, for the quadrilateral ABCD shown in Fig. 4.23, area of quadrilateral ABCD = 1/2 x AC × (BE + DF)

(ii) For a cyclic quadrilateral where the four sides measure a, b, c and d respectively,

Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi-perimeter, i.e., s = (a + b + c + d)/2

(iii) For a trapezium
Area of a trapezium = ½ x Sum of parallel sides x Distance between them
= ½ x (AD + BC) x AE (refer to Fig. 4.25)

(iv) For a parallelogram
(a) **Area = Base x Height**
(b) **Area = Product of two sides x Sine of included angle**

(v) For a rhombus
Area = ½ x Product of the diagonals
Perimeter = 4 x Side of the rhombus

(vi) For a rectangle
Area = Length x Breadth
Perimeter = 2(l + b), where l and b are the length and the breadth of the rectangle respectively

(vii) For a square
(a) **Area = Side^2**
(b) **Area = ½ x Diagonal^2**
[where the diagonal = $\sqrt{2}$ × side]
Perimeter = 4 × Side

(viii) For a regular hexagon
(a) Area = $\frac{3\sqrt{3}}{2}$ (side)^2
(b) Perimeter = 6(side)

(ix) For a polygon
(a) **Area of a regular polygon = ½ x Perimeter x Perpendicular distance from the centre of the polygon to any side**
(Please note that the centre of a regular polygon is equidistant from all its sides)

(b) For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon.

CIRCLE

(i) **Area of the circle** = πr^2 where r is the radius of the circle
Circumference = $2\pi r$

(ii) Sector of a circle
Length of arc = $\frac{\theta}{360°} \times 2\pi r$

Area = $\frac{\theta}{360°} \times \pi r^2$ where θ is the angle of the sector in degrees and r is the radius of the circle.
Area = (1/2)lr; l is length of arc and r is radius.

(iii) Ring : Ring is the space enclosed by two concentric circles.

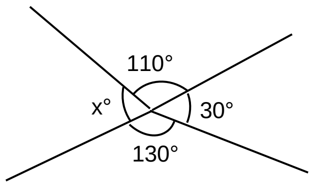
Area = $\pi R^2 - \pi r^2 = \pi(R + r)(R - r)$ where R is the radius of the outer circle and r is the radius of the inner circle.

ELLIPSE

Area = πab where "a" is semi-major axis and "b" is semi-minor axis.
Perimeter = π (a + b)

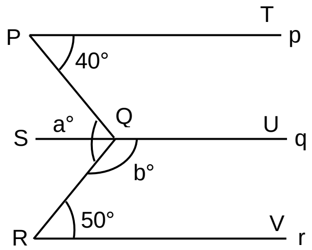
Examples

4.01. Find the value of x in the figure given below.



Sol: The sum of the angles at a point is 360°
$\Rightarrow$ x° + 110° + 30° + 130° = 360°
x = 90

4.02. Find the measures of a° and b° in the figure below, given that the lines p, q and r are parallel to each other.



Sol: $\angle PQR = \angle PQS + \angle SQR$
$\angle PQS = \angle QPT$ (Alternate angles are equal)
$\angle SQR = \angle QRV$ (Alternate angles are equal)
$\therefore \angle PQS + \angle SQR = \angle QPT + \angle QRV = 90°$
$\therefore$ a° = 90°
b + 50° = 180° (co-interior angles are supplementary)
b° = 130°

4.03. An angle equals two-thirds of its supplement. Find it.

Sol: Let the angle be x°. Its supplement is (180° – x°).
Given that $x° = \frac{2}{3}(180° - x°)$
x° = 72°

4.04. An angle equals two-third of its complement. Find it.

Sol: Let the angle be x°. Its complement is (90° – x°).
Given that $x° = \frac{2}{3}(90° - x°) \Rightarrow x° = 36°$

4.05. In a right angled triangle, one of the sides containing the right angle is 30 cm and hypotenuse is 50 cm. Find its area.

Sol: Let the other perpendicular side of the triangle be h cm.
$\sqrt{h^2 + 30^2}$ = 50 (Pythogoras theorem)
$\Rightarrow$ h = 40
$\therefore$ Area = $\frac{1}{2}$ (40) (30) i.e. 600 sq.cm

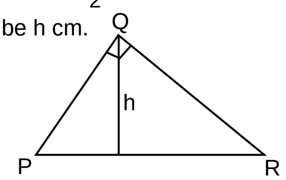
4.06. Find the area of an equilateral triangle of side 6 cm.

Sol: Area of an equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(6^2) = 9\sqrt{3}$ sq.cm

4.07. Triangle PQR is right angled at Q. Find the length of the altitude drawn from Q to PR if PQ = 8 cm and QR = 15 cm.

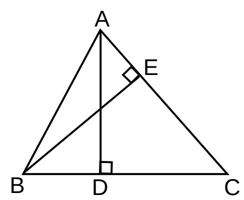
Sol: Area of triangle PQR = $\frac{1}{2}(8)(15) = 60$ sq.cm
Let the altitude be h cm.



$\frac{1}{2}(PR)(h) = 60$

$\frac{1}{2}\left(\sqrt{8^2 + 15^2}\right)$ h = 60 $\Rightarrow$ h = $\frac{120}{17}$

4.08.

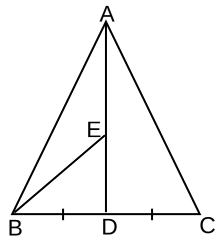


In the figure above,
AD = 4·2 cm, BC = 10·8 cm and BE = 7·2 cm. Find AC.

Sol: Area of ΔABC
$= \frac{1}{2}(AD)(BC) = \frac{1}{2}(BE)(AC)$

$AC = \frac{(AD)(BC)}{BE} = \frac{(4 \cdot 2)(10 \cdot 8)}{7 \cdot 2}$ = 6.3 cm

4.09.



In the figure given above, AD is the median on BC. BE is the median on AD. Find the ratio of the areas of the triangles AEB, BED and ABC.

Sol: Since BE is the median, Area of ΔAEB = Area of ΔBED.
Since AD is the median, Area of ΔABD = Area of ΔADC $\Rightarrow$ Area of Δ AEB

$$\frac{1}{2}\left[\frac{1}{2}(\text{Area of } \Delta\text{ ABC})\right]$$

$\therefore$ Areas of ΔAEB, ΔBED and ΔABC will be in the ratio 1 : 1 : 4.

4.10. In triangles XYZ and PQR, $\angle$X = $\angle$P and $\angle$Z = $\angle$R.
XY = 4·2 cm, YZ = 6·4 cm, PR = 2·8 cm,
QR = 3·2 cm. Find the lengths of PQ and XZ.

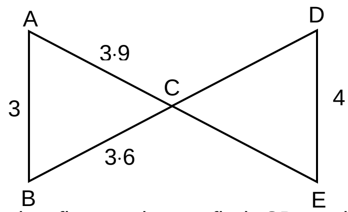
Sol: As $\angle$X = $\angle$P and $\angle$Z = $\angle$R, triangles XYZ and PQR are similar.

$$\therefore \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR}$$

$$\frac{4\cdot 2}{PQ} = \frac{6\cdot 4}{3\cdot 2} = \frac{XZ}{2\cdot 8}$$

PQ = 2·1 cm and XZ = 5·6 cm

4.11.



In the figure above, find CD and CE given $\angle$BAC = $\angle$CDE.

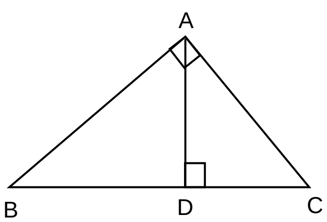
Sol: In triangles ABC and CED, $\angle$ACB = $\angle$DCE (vertically opposite angles are equal)
$\angle$BAC = $\angle$CDE (given)
$\therefore$The Triangles ABC and DEC are similar

$$\frac{BC}{CE} = \frac{AB}{DE} = \frac{AC}{CD}$$

$$\frac{3\cdot 6}{CE} = \frac{3}{4} = \frac{3\cdot 9}{CD}$$

$\therefore$ CE = 4·8 cm and CD = 5·2 cm

4.12.



In the figure above,
AC = $4\sqrt{3}$ cm and BD = 8 cm. Find DC.

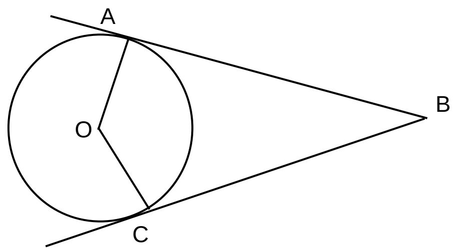
Sol: Let DC = x cm
In triangles ADC and BAC, $\angle$ADC = $\angle$BAC = 90°(given)
$\angle$ACD = $\angle$ACB ($\angle$C is common)
$\therefore$ The triangles BAC and ADC are similar

$$\therefore \frac{AC}{BC} = \frac{AD}{AB} = \frac{DC}{AC}$$

AC2 = (BC) (DC)

$\left(4\sqrt{3}\right)^2 = (x+8)x \Rightarrow (12)\ (4) = (x + 8)\ x$
$\Rightarrow x^2 + 8x - 48 = 0 \Rightarrow x = 4$ or -8
As x > 0, x = 4.

4.13.



In the figure above, BA and BC are tangents to the circle with centre O. AB = 28 cm and OA = 21 cm. Find OB + BC.

Sol: $\angle$OAB = $\angle$OCB = 90°(AB and BC are tangents)
$\therefore$ $OA^2 + AB^2 = OB^2 = OC^2 + CB^2$

$$OB = \sqrt{21^2 + 28^2} = 35 \text{ cm}$$

AB = BC (tangents to the circle from an external point)
$\therefore$ BC = 28 cm
Hence OB + BC = 63 cm

4.14. The angle subtended by an arc at the centre of a circle is 40°. If the area of the sector formed by the arc and the radii of the circle is $68\frac{4}{9}$ sq.cms, find the radius of the circle. $\left(\text{take } \pi = \frac{22}{7}\right)$

Sol: Let the radius of the circle be r cm.

$$\frac{40°}{360°}\left(\frac{22}{7}r^2\right) = \frac{616}{9} \Rightarrow r = 14$$

4.15. Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of the direct common tangent drawn to them.

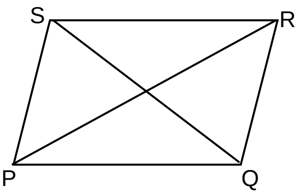
Sol: Length of the direct common tangent

$$= \sqrt{\begin{matrix}(\text{distance between centres})^2 - \\ (\text{difference between radii})^2\end{matrix}}$$

$$= \sqrt{(9+4)^2 - (9-4)^2} = 12 \text{ cm}$$

4.16. PQRS is a parallelogram, Are the areas of triangles PQR and RQS equal?

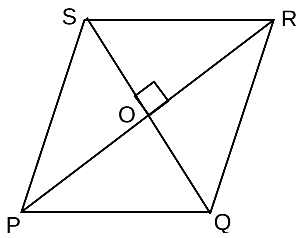
Sol:



In parallelogram PQRS (any),
Δ PQR $\equiv$ Δ RSP and Δ SPQ $\equiv$ Δ QRS
$\Rightarrow$ Area [Δ PQR] = Area [ΔRSP] = $\frac{1}{2}$ Area [PQRS]
and Area [Δ SPQ] = Area [ΔQRS] = $\frac{1}{2}$ Area [PQRS]
$\therefore$ Areas of ΔPQR and ΔRQS are equal.

4.17. PQRS is a rhombus such that PR = 32 cm and QS = 24 cm. Find the perimeter of PQRS.

Sol:



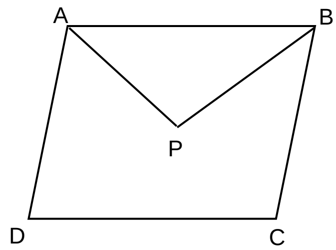
In rhombus, the diagonals bisect each other at right angles.
So, SO = OQ = 12 cm. And
PO = OR = 16 cm.
And ΔSOR is right angled at O.

$\Rightarrow$ SR = $\sqrt{\left(\frac{32}{2}\right)^2 + \left(\frac{24}{2}\right)^2}$

$\therefore$ SR = 20 cm
$\therefore$ Perimeter = 80 cm

4.18. In parallelogram ABCD, the angle bisectors of $\angle$A and $\angle$B intersect at P. Find $\angle$APB.

Sol:



In parallelgram ABCD,
$\angle$A +$\angle$B = 180°
$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} = 90°$
In ΔABP, $\angle$PAB + $\angle$PBA + $\angle$APB = 180°
$\frac{\angle A}{2} + \frac{\angle B}{2} + \angle APB = 180°$
$\angle$APB = 180° − 90°= 90°

4.19. In a polygon, the sum of the interior angles is 1980°. Find the number of sides in it.

Sol: Let the number of sides be n.
Then, the sum of its interior angles
= 180°(n − 2) = 1980°
$\therefore$ n = 13

4.20. In a hexagon, one of the interior angles is 100°. If all the other angles are equal, find each of these angles.

Sol: In a hexagon the sum of the interior angles
= 180°(6 – 2) i.e., 720°
Each of the remaining angles
= $\frac{720° - 100°}{5} = 124°$

Concept Review Questions

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the area of the triangle whose sides are 10 cm, 10 cm and 12 cm (in sq cm).

2. The sides of a triangle are 15 cm, 20 cm and 25 cm. Find the sum of the distances of its orthocentre from the vertices (in cm).
(A) 0 (B) 15 (C) 20 (D) 35

3. In a triangle whose sides are 5 cm, 5 cm and 7 cm, a quadrilateral is formed by taking the centroid, the orthocentre, the circumcentre and the incentre of the triangle as its vertices. If the area of the quadrilateral thus formed is x cm^2, which of the following is true?
(A) $0 \le x < 6.25$ (B) $6.25 \le x < 12.5$
(C) $12.5 \le x < 18.5$ (D) $18.5 \le x$.

4. In an equilateral triangle, the orthocentre divides each median in the ratio x : y. If x > y, then x : y =
(A) 3 : 2 (B) 3 : 1
(C) 2 : 1 (D) None of these

5. The area of a triangle, (in cm^2) formed by the incentre, the centroid and the cirumcentre of an equilateral triangle whose side is 6 cm is
(A) $4.5\sqrt{3}$ (B) $3\sqrt{3}$
(C) $6\sqrt{3}$ (D) None of these

6. **(i)** What is the inradius (in cm) of an equilateral triangle whose side is 9 cm?
(A) $3\sqrt{3}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{3\sqrt{3}}{4}$

(ii) What is the circumradius (in cm) of an equilateral triangle whose side is 9 cm?
(A) $3\sqrt{3}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{3\sqrt{3}}{4}$

7. In Δ ABC, I is the incentre and ∠A = 80º. Find ∠BIC. (in degrees)

8. **(i)** A triangle has its sides as 6 cm, 7 cm and 8 cm. Its circumcentre lies
(A) inside the triangle (B) outside the triangle
(C) on the triangle (D) any of the above

(ii) A triangle has its sides as 4 cm, 6 cm and 8 cm. Its orthocentre lies
(A) inside the triangle (B) outside the triangle
(C) on the triangle (D) Any of the above

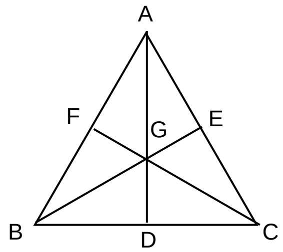
9. The internal bisectors of the angles of a triangle meet at a point, called _____ of the triangle.
(A) Orthocentre (B) Incentre
(C) Centroid (D) Circumcentre

10. In triangle PQR, X and Y are points on PQ and PR respectively such that XY || QR. PX = 4 cm, XQ = 6 cm and YR = 8 cm. Find PY (in cm).

3

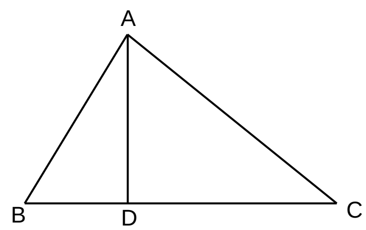
11. In a right angled triangle PQR, QS is an altitude to PR. ∠PQR = 90º, PS = 16 cm and SR = 4 cm. Find QS (in cm).

12.



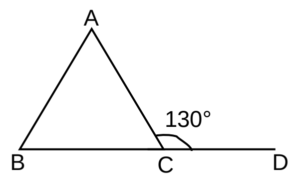
In the figure above, AD, BE and CF are the medians and G is the centroid. If the area of Δ ABC is 18 sq cm, find the area of ΔBGF (in sq cm).
(A) 2 (B) 3
(C) 6 (D) None of these

13.



In the figure above (not to scale), AB = 8 cm and AC = 10 cm. AD bisects ∠BAC, DC = 4 cm. Find BD.
(A) 3.6 (B) 3.7 (C) 3.8 (D) 3.2

14.



In the given figure, ∠ABC = 2∠ACB and ∠ACD = 130°. Find ∠BAC.
(A) 50° (B) 65° (C) 30° (D) 60°

15. In triangle ABC, AD is the angle bisector of ∠BAC. If the length of AB is 2 cm, AC = 3 cm and BD = 1.5 cm, then find the length of CD. (in cm)

16. Points D and E are lying on the sides AB and AC of triangle ABC, such that AB = 2AD, and BC = 2DE. Find AE, given AC = 12 cm.
(A) 9 cm (B) 6 cm
(C) 7.5 cm (D) 4 cm

17. G is the centroid of a triangle ABC. AB = 7 cm, BC = 24 cm and AC = 25 cm. If D is the mid-point of AC, then find the length of GD.
(A) 8.33 cm (B) 4.17 cm
(C) 12.5 cm (D) 6.25 cm

18. Two triangles ABC and ABD have the same base. Find the ratio of their areas, given their heights are in the ratio of 3 : 5.
(A) 5 : 3 (B) 9 : 25
(C) 3 : 5 (D) 25 : 9

19. In triangle EFG, EH is the altitude to FG. Which of the following can be concluded?
(A) $EF^2 - GH^2 = GH^2 - EG^2$
(B) $EF^2 - GH^2 = EG^2 - FH^2$
(C) $EF^2 - FH^2 = EG^2 - GH^2$
(D) None of these

20. In a right angled triangle PQR, $\angle P = 90°$ and $\angle Q = 30°$. Find PQ : QR : PR.
(A) $\sqrt{3} : 1 : 2$ (B) $\sqrt{3} : 2 : 1$
(C) $1 : \sqrt{3} : 4$ (D) $1 : 2 : \sqrt{3}$

21. In the given figure, ABC is an isosceles triangle, where AB = AC, $\angle ABC = 45°$. BD is an angle bisector of $\angle ABC$ and EC is an angle bisector of $\angle ACB$. Find $\angle COD$. (in degrees)

22. Bisectors of angle B and angle C of triangle ABC meet at O. If $\angle BOC = 130°$, find angle A.
(A) 50° (B) 80°
(C) 40° (D) 65°

23. The centroid of a triangle is the
(A) point of concurrence of its perpendicular bisectors.
(B) point of concurrence of its altitudes.
(C) point of concurrence of its angle bisectors.
(D) point of concurrence of its medians.

24. A rectangle, a triangle and a parallelogram lie on the same base and between the same parallel lines. If the area of triangle is 24 sq.cm then, find the area of the parallelogram. (in sq cm)

25. ABCD is a trapezium in which AB || CD. AB = 12 cm and CD = 24 cm. EF is parallel to AB and CD and divides the distance between them in the ratio 1 : 2. Find EF (in cm.)
(A) 14 (B) 16 (C) 18 (D) 20

26. A quadrilateral has an area of 80 sq cm. Its midpoints are joined. Find the type of the quadrilateral thereby formed and its area.
(A) Rectangle, 40 sq cm
(B) Square, 40 sq cm
(C) Parallelogram, 40 sq cm
(D) None of the above

27. A polygon of n sides has the sum of its interior angles at most equal to the sum of its exterior angles. How many possibilities exist for n?

28. The number of diagonals in a polygon is thrice the number of sides in it. Find the number of sides in it.

29. A regular polygon has each of its interior angles as twice of each of its exterior angles. It must be a
(A) Pentagon (B) hexagon
(C) heptagon (D) nonagon

30. Find the interior and exterior angles of a regular polygon of eight sides.
(A) 150°, 30° (B) 140°, 40°
(C) 135°, 45° (D) 108°, 72°

31. An angle is such that its supplement is thrice its complement. Find the angle.
(A) 45° (B) 60°
(C) 75° (D) 30°

32. A regular polygon has 20 diagonals. Find the measure of each exterior angle of the polygon.
(A) 30° (B) 40°
(C) 45° (D) 20°

33. A circle has a radius of 15 cm. A chord is at a distance of 9 cm from its centre. Find the length of the chord (in cm).

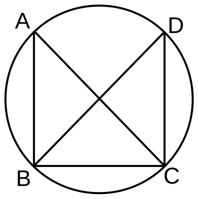
34. P and Q are the points on the circumference of a circle with centre O. $\angle POQ = 100°$. If R is a point on the major arc, find $\angle PRQ$.
(A) 50° (B) 130°
(C) 60° (D) 120°

35. Find the number of common tangents which can be drawn to two circles which touch each other externally.

36. Find the number of common tangents which can be drawn to two non-intersecting and non-enclosing circles.

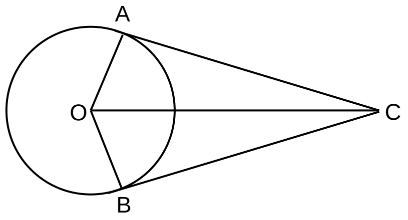
37. Which of the following is/are necessarily cyclic quadrilateral(s)?
(A) Parallelogram
(B) Non isosceles trapezium
(C) Isosceles trapezium
(D) All the above

38.



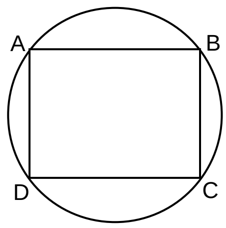
In the figure above, $\angle BDC = 30^o$, $\angle BCA = 60^o$. Find $\angle ABC$.
(A) 100° (B) 95° (C) 90° (D) 85°

39.



In the figure above, O is the centre of the circle. $\angle ACB = 50^o$. AC and BC are tangents to the circles at A and B respectively. Find $\angle AOB$.
(A) 100° (B) 110° (C) 120° (D) 130°

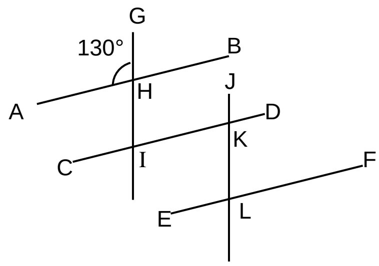
40.



In the figure above, $\angle ABC = 85^o$. Find $\angle ADC$.
(A) 170º (B) 190º (C) 85º (D) 95º

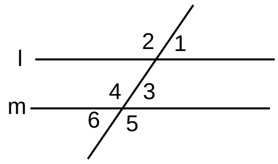
41. The diameter PQ of a circle is 30 cm and R is a point on its circumference such that QR = 18 cm. Find PR (in cm).

42.



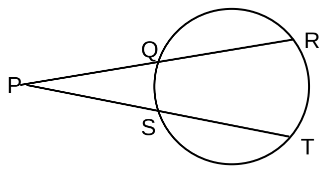
In the figure above, AB, CD and EF are parallel. GH and JL are parallel. Find $\angle JLE$.
(A) 50º (B) 130º (C) 140º (D) 40º

43. In the given figure, lines l and m are parallel. If $\angle 1 + \angle 6 = 120°$, then find $\angle 4$. (in degrees)



44. In Δ ABC, AB = 12 cm, AC = 16 cm and BC = 8 cm. Find the length of the median drawn from A to BC (in cm.).
(A) 7 (B) $\sqrt{46}$ (C) 14 (D) $2\sqrt{46}$

45.



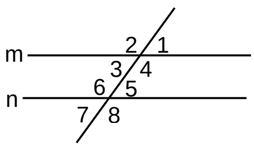
In the figure above, PQ = 4 cm, QR = 14 cm and PS = 3 cm. Find ST.

cm

Exercise – 4(a)

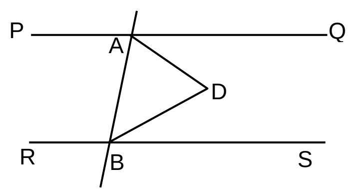
Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. In the given figure, lines m and n are parallel and angle 8 is twice angle 1. Find the measure of $\angle 4$.

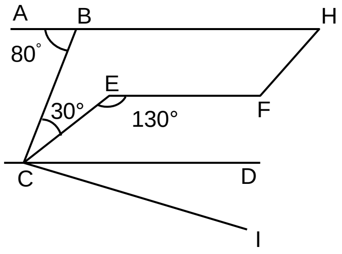


2. In the figure below, PQ is parallel to RS and AB cuts them at A and B respectively. AD and BD are the bisectors of angles QAB and SBA. Find AB if AD = 16 cm and BD = 12 cm.

cm

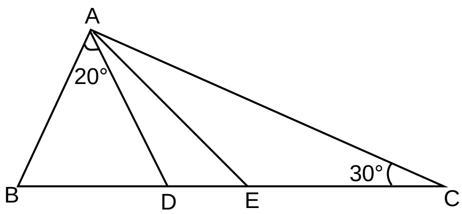


3. In the figure given below, AB || CD. Find the measure of $\angle EFH$ given that, $\angle BHF = 40°$ and $\angle BCI = 100°$.



(A) 140° (B) 40° (C) 100° (D) 160°

4. In the given figure ABC and ADE are triangles such that AB = AD, AE = EC. Find the measure of angle DAE.



(A) 10° (B) 20° (C) 25° (D) 15°

5. In triangle ABC, AB = 10 cm and AC = 20 cm. D is a point such that it divides BC externally in the ratio. 1 : 3. If AD $\perp$ BC, then find the length of BC.

(A) 20 cm (B) $10\sqrt{6}$ cm

(C) 10 cm (D) $5\sqrt{6}$ cm

6. The sides of a triangle ABC are a cm, b cm and c cm. The sides of a triangle DEF are d cm, e cm and f cm, $a(a + b + c) = d^2$, $b(a + b + c) = e^2$ and $c(a + b + c) = f^2$

If the measure of the greatest angle in triangle DEF is x°, which of the following is true?

(A) $60 < x < 90$

(B) $x = 90$

(C) $90 < x \leq 105$

(D) $105 < x \leq 120$

7. In a right angled triangle ABC, AB = $10\sqrt{3}$ cm and BC = 20 cm, $\angle A = 90°$. An equilateral triangle ABD is constructed with base AB and with vertex D, at a maximum possible distance from C. Find the length of CD.

(A) $10\sqrt{7}$ cm

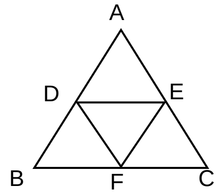
(B) $10\sqrt{11}$ cm

(C) $10\sqrt{12}$ cm

(D) $10\sqrt{14}$ cm

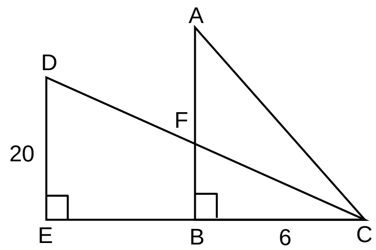
8. What is the sum of the squares of the medians (in sq cm) of a triangle ABC if the sum of the squares of its sides is 72 sq.cm?

9.



In the given figure, $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} = \frac{1}{3}$. If area of triangle ABC is 54 sq.cm, then find the area of triangle ADE. (in sq cm)

10.



In the above figure, F is the midpoint of AB and AC = 10 units. Find the length of EB.

(A) 24 units (B) 20 units

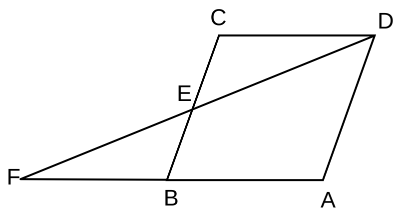
(C) 28 units (D) 26 units

11. If the adjacent sides of a parallelogram are 30 cm and 18 cm and one of the diagonals of the parallelogram is 24 cm long, find the other.

(A) 15 cm (B) 18 cm

(C) 20 cm (D) None of these

12. ABCD is a parallelogram and E is the midpoint of BC as shown in the figure. If DE and AB when produced meet at F, then AF is equal to ______.



(A) $\frac{3}{2}$ AB (B) 2 AB (C) 3 AB (D) $\frac{4}{3}$ AB

13. If the sides AB, BC, CD and DA of a trapezium ABCD measure 10 cm, 20 cm, 18 cm and 16 cm respectively, find the length of the longer diagonal, given that AB is parallel to CD.

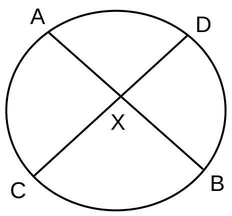
(A) $\sqrt{655}$ cm (B) $\sqrt{256}$ cm

(C) $\sqrt{760}$ cm (D) $\sqrt{840}$ cm

14. ABCD is a trapezium with AB||CD. AB = 10 cm, CD = 20 cm. E and F are points on AD and BC respectively such that AE : ED = BF : FC = 3 : 2. Find EF (in cm).

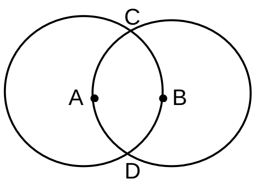
(A) 14 (B) 12 (C) 16 (D) 10

15.



Two chords AB and CD intersect at point X as shown in the figure. If AX = 8 cm, AB = 14 cm and CX – XD = 8 cm, find CD. (in cm)

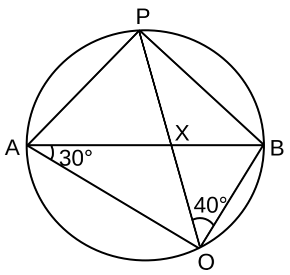
16.



In the above figure, A and B are the centres of two circles. C and D are the points of intersection of the circles. Find $\angle CAD + \angle CBD$.

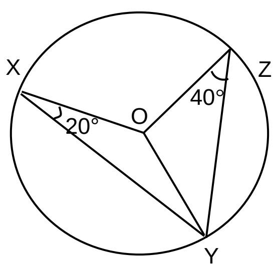
(A) 210° (B) 270° (C) 240° (D) 300°

17. In the figure drawn below, AB represents the diameter of the circle. Find the value of $\angle BXQ$.



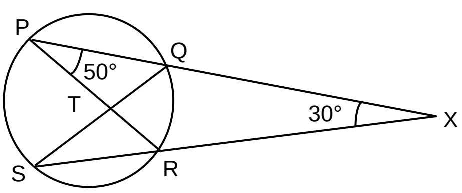
(A) 80° (B) 70° (C) 60° (D) 75°

18. Find $\angle XZO$ from the given figure, given O is the centre of the circle.



(A) 30° (B) 40° (C) 50° (D) 45°

19.

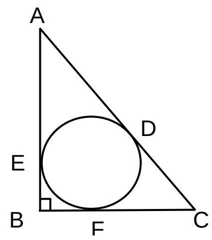


In the above figure, if PQRS is a cyclic quadrilateral then find the value of $\angle STR$.

(A) 30° (B) 40° (C) 50° (D) 60°

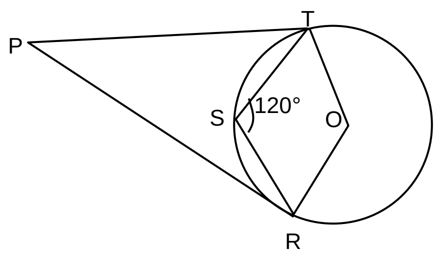
20. A right-angled triangle ABC is right-angled at B. The point of contact of the incircle with the hypotenuse, divides it, in the ratio 3 : 2. If the perimeter of the triangle ABC is 36 cm, find its inradius. (in cm)

.



21. The perimeter of a right angled triangle is 24 cm. Find the sum of its inradius and twice its circumradius. (in cm).

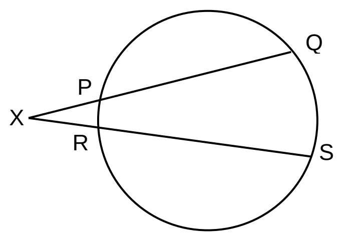
22. In the given figure, PT and PR are two tangents. If O is the centre of the circle and $\angle TSR$ = 120°, find $\angle TPR$.



(A) 60° (B) 80° (C) 85° (D) 75°

23. In the given figure (not drawn to scale), XP = 6 cm; PQ = 4 cm and XR = 5 cm. Find RS.

cm



24. From a point P, which is 20 cm away from the center of a circle, two tangents PQ and PR are drawn to the circle to touch it at Q and R. Given that the tangents are perpendicular, find the perimeter of ΔPQR.

(A) $(20 + 10\sqrt{2})$ cm

(B) $(5 + 20\sqrt{2})$ cm

(C) $(10 + 20\sqrt{2})$ cm

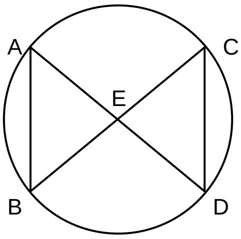
(D) None of these

25. Two parallel chords AB and CD are drawn on opposite sides of a diameter of a circle. AB is 16 cm long and is at a distance of 5 cm from CD. If CD is 14 cm long, then find the radius of the circle.

(A) $\sqrt{60}$ cm (B) $\sqrt{55}$ cm
(C) $\sqrt{85}$ cm (D) None of these

26. Find the length of the transverse common tangent drawn to two circles which have their centres separated by 13 cm and with radii of 8 cm and 4 cm respectively.
(A) 8 cm (B) 6 cm (C) 5 cm (D) 7 cm

27.



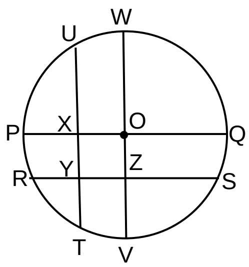
In the figure above, AB is parallel to CD. AD and BC intersect at E. AB > AE. Find the maximum possible value of ∠ABE (in degrees) if its measure is an integer.

$\boxed{}$

28. A_1, A_2, A_3, …… A_{11}, A_{12} are 12 distinct points equally spaced and arranged in the same order on the circumference of a circle. Find the distinct number of triangles which can be formed using these points as vertices such that their circumcentre lies on one of the sides of the triangle.

$\boxed{}$

29.



In the figure shown, O is the centre of the circle. XOZY is a rectangle. VZ : ZW = PX : XQ = 3 : 8. If the diameter of the circle is 4.4, find the sum of the lengths of RS and TU.

$\boxed{}\sqrt{6}$

30. Two of the sides of a triangle are in the ratio 3 : 4. The medians to these sides are perpendicular to each other. If the third side of the triangle is $12\sqrt{5}$, find the smaller of the first two sides of the triangle.
(A) 36 (B) 24 (C) 48 (D) 60

31. ABCD is a square. I is a point inside the square such that triangle AIB is equilateral. ∠CID = ______.
(A) 120° (B) 135° (C) 150° (D) 165°

32. A triangle PQR is drawn in a semicircle such that P and Q are the ends of the diameter of the semicircle and R lies on the arc of the semicircle. The diameter of the semicircle is 41. If the lengths of PR and RQ are integers find the inradius of the triangle PQR.
(A) 4 (B) 6 (C) 5 (D) 7

33. ABCD is a parallelogram whose diagonals meet at E. The perimeter of ABCD is 120. The perimeters of the triangles BCE and CDE differ by 40. Find sum of the squares of the diagonals of ABCD.
(A) 3900
(B) 5200
(C) 2600
(D) Cannot be determined

34. PQR is an equilateral triangle. S is the foot of the perpendicular from P to QR. T is the foot of the perpendicular from S to PR. U is the foot of the perpendicular from T to QR. If TU = x, the perimeter of PQR is ______.
(A) $\dfrac{24x}{\sqrt{3}}$ (B) $\dfrac{12x}{\sqrt{3}}$ (C) $\dfrac{36x}{\sqrt{3}}$ (D) $\dfrac{18x}{\sqrt{3}}$

35. One of the angles of a rhombus is 120°. The ratio of the longer and the shorter diagonals of the rhombus is ______.
(A) $3\sqrt{3}$: 2 (B) 2 : $\sqrt{3}$
(C) $3\sqrt{3}$: 4 (D) $\sqrt{3}$: 1

36. How many distinct triangles having a perimeter of 44 can be formed where each side is a multiple of 4?

$\boxed{}$

37. PQRS is a quadrilateral. The diagonals of PQRS are perpendicular to each other. PQ = 16, QR = 12, RS = 20. If SP = k $\sqrt{2}$, find the value of k.

$\boxed{}$

38. In triangle ABC, D is a point on BC such that BD = 36 and DC = 9. AD = 18. AD ⊥ BC. ∠A = $\boxed{}$ degrees

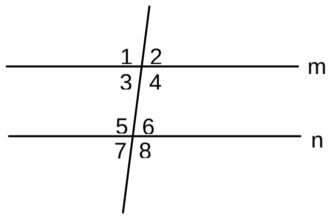
39. In ΔABC, ∠B = 90°. The ratio of lengths of AB, BC and thee altitude to AC is p : q : r. Which of the following can be concluded?
(A) $\dfrac{1}{r^2} = \dfrac{1}{p^2} - \dfrac{1}{q^2}$ (B) $\dfrac{1}{r^2} = \dfrac{1}{q^2} - \dfrac{1}{p^2}$
(C) $\dfrac{1}{r^2} = \dfrac{1}{p^2} + \dfrac{1}{q^2}$ (D) None of these

40. ABC is a triangle. BE is the median drawn to AC. D is a point on AB such that CD bisects BE. $\dfrac{AD}{DB} = \boxed{}$

Exercise – 4(b)

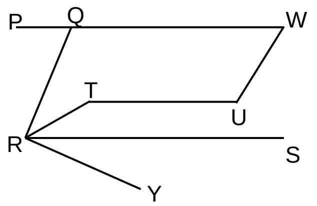
Directions for questions 1 to 55: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1.



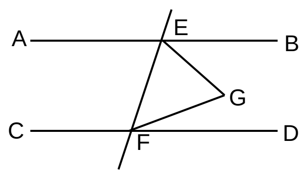
In the above figure, m is parallel to n and five times of $\angle 5$ = seven times of $\angle 7$. Find $\angle 2 + \angle 6$. (in degrees)

2.



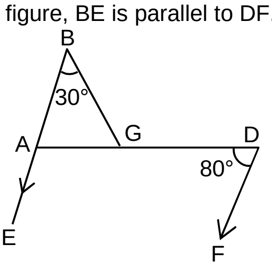
In the figure above (not drawn to scale), TU||RS. Find $\angle$WUT, given $\angle$PWU = 50°, $\angle$PQR = 70°, $\angle$QRT = 10° and $\angle$UTR = 120°.
(A) 110° (B) 120° (C) 130° (D) 140°

3.



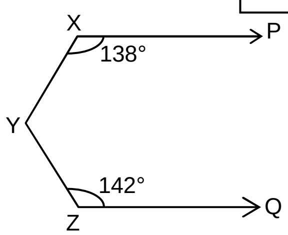
In the figure above, AB||CD. EF cuts AB and CD at E and F respectively EG bisects $\angle$BEF. FG bisects $\angle$EFD. EG = 15 cm. EF = 25 cm. Find FG (in cm).
(A) 12 (B) 16 (C) 8 (D) 20

4. In the given figure, BE is parallel to DF. Find $\angle$AGB.



(A) 70° (B) 75° (C) 80° (D) 90°

5. Find $\angle$XYZ (in degrees) in the following figure, where XP is parallel to ZQ.

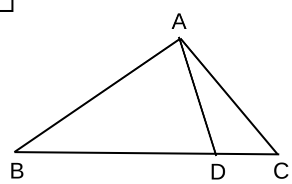


6. The angles of a triangle are in the ratio 2 : 3 : 5. Identify the type of triangle.
(A) Scalene and acute angled
(B) Isosceles and obtuse angled
(C) Scalene and right angled
(D) Isosceles and acute angled

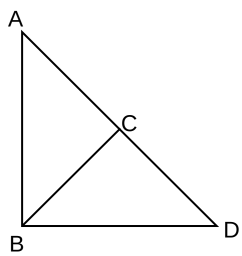
7. A right angled triangle has a hypotenuse of 41. The lengths of its perpendicular sides are integers. The perimeter of the triangle is

8. The angles of a triangle are in the ratio 3 : 4 : 5. If the greatest angle is C°, which of the following is tru e?
(A) C < 75 (B) 75 ≤ C < 90
(C) C = 90 (D) 90 < C < 120

9. In ΔABC, $\angle$C = 50°. D is a point on segment BC such that BD = AB. If $\angle$CAD = 20°, find $\angle$CBA . (in degrees)

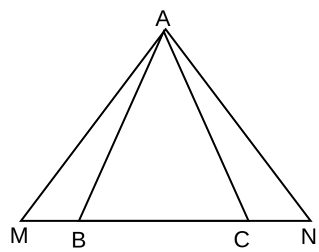


10.



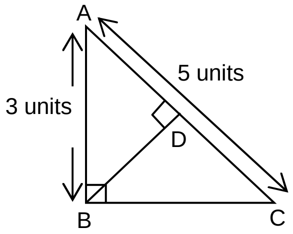
In the above figure, AC = BC and BC = CD. Find the value of $\angle BAC + \angle BDC$. (in degrees)

11. In the figure below, $\angle$AMB = m°, $\angle$MAB = x°, $\angle$BAC = y°, $\angle$CAN = z°, $\angle$ABC = p°, $\angle$ACB = q° and $\angle$ANC = n°



Which of the following is true?
(A) x° + m° + z° = p° + q° + n°
(B) x° + m° + p° = n° + q° + z°
(C) x° + m° + q° = n° + p° – z°
(D) x° + z° + m° + n° = p° + q°

12. Find the length of side AD in the figure given below



☐ units

13. In triangle ABC, $\angle A = 90°$, BX and CY are medians. Find the value of $\dfrac{AB^2 - AC^2}{BX^2 - CY^2}$.

(A) 4/3 (B) 5/3 (C) 7/3 (D) 2

14. Find the interior and exterior angles of a regular polygon of six sides.
(A) 120°, 60° (B) 115°, 65°
(C) 108°, 72° (D) 90°, 90°

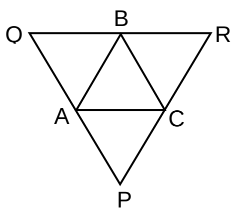
15. In a right angled triangle, the sum of the lengths of the hypotenuse and the shortest side is 64 cm. The length of the other side is twice the difference of the lengths of these sides. Find the length of the hypotenuse (in cm).

☐

16. In triangle ABC, $\angle A = 90°$ and D is the midpoint of AC. The value of $BC^2 - BD^2$ is equal to _______.
(A) $2AD^2$ (B) AD^2 (C) $3AD^2$ (D) $4AD^2$

17. The perimeters of similar triangles ABC and PQR are 40 cm and x cm respectively. Find x if the areas of triangles ABC and PQR are 50 sq.cm and 200 sq.cm respectively.
(A) 20 cm (B) 30 cm (C) 40 cm (D) 80 cm

18. In the given figure triangle PQR is equilateral and its side is 8 cm. A, B and C are the midpoints of PQ, QR and PR respectively. Find the ratio of the perimeter of ΔACP to the perimeter of ΔPQR.

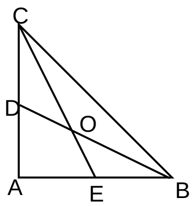


(A) 1 : 4 (B) 1 : 3 (C) 1 : $\sqrt{2}$ (D) 1 : 2

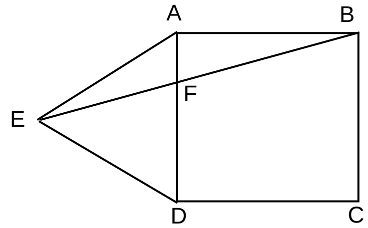
19. In triangle PQR, QO and RO are the bisectors of $\angle PQR$ and $\angle PRQ$ respectively. Find the measurement of $\angle QPR$, if $\angle QOR = 140°$.
(A) 100 ° (B) 70° (C) 80° (D) 110°

20. In the given figure, $\angle CAB = 80°$. BD and CE are angle bisectors of $\angle ABC$ and $\angle ACB$ respectively. Find $\angle BOC$. (in degrees)

☐

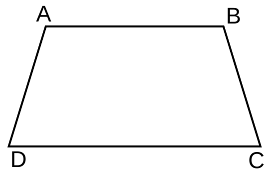


21.



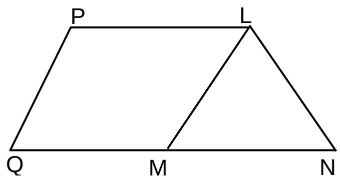
In the above figure, ABCD is a square and triangle ADE is equilateral. Find $\angle EFD$.
(A) 65° (B) 70° (C) 80° (D) 75°

22.



In the figure above, AB is parallel to CD. AD = BC. If $\angle DAB = 100°$, find $\angle BCD$
(A) 80° (B) 60°
(C) 70° (D) 50°

23. In the figure below, $\angle P = 140°$, $\angle Q = 40°$, LM = LN and $\angle PLN = 130°$. Find $\angle MLN$.

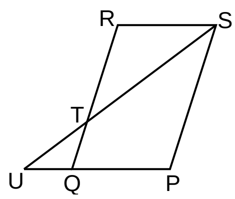


(A) 70° (B) 80°
(C) 85° (D) 90°

24. The adjacent sides of a parallelogram are 21 cm and 13 cm. If one of the diagonals is $2\sqrt{105}$ cm long, find the length of the other diagonal (in cm).
(A) 16 (B) 14
(C) $18\sqrt{2}$ (D) $20\sqrt{2}$

25. PQRS is a parallelogram. T is a point on QR satisfying QT = $\dfrac{QR}{3}$. If PQ and ST are produced to meet at U, as shown in the figure, then PU =

☐ UQ

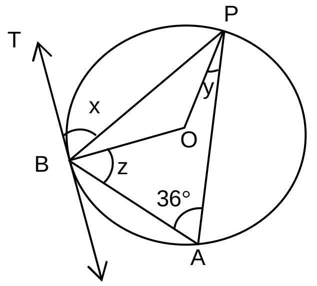


26. The centres of three circles which touch each other externally form a triangle of sides 21 cm, 22 cm and 23 cm. Find the radius of the smallest circle.
(A) 10 cm (B) 11 cm
(C) 12 cm (D) 13 cm

27. An equilateral triangle OAB is drawn such that A and B are the points on the circle with O as the centre of that circle. If AB = 4 cm, find the circumference of the circle. (in cm)

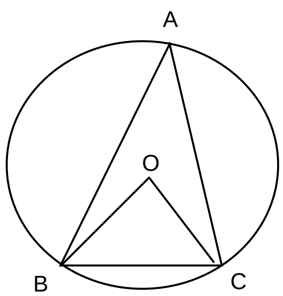
$\boxed{}\ \pi$

28. In the given figure (not drawn to scale) AB is a chord of the circle with centre O. BT is a tangent to the circle. Find the value of x + y + z.



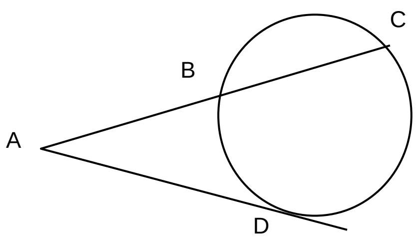
(A) 90°
(B) 80°
(C) 72°
(D) 60°

29.



In the above figure, O is the centre of the circle. It is also the incentre of triangle ABC. Find ∠BOC (in degrees).

30.



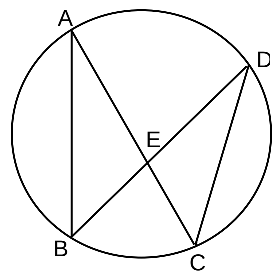
In the above figure, ABC is a secant. AD is a tangent to the circle at D. If AB = 4 cm and BC = AD + AB, find AD. (in cm)

(A) 8
(B) 6
(C) 12
(D) 16

31. Two chords AB and CD of a circle intersect at a point P. The segments AP, CD and PB measure, respectively, 8 cm, 22 cm and 12 cm. What is the difference in the lengths of CP and PD?

(A) 20 cm
(B) 6 cm
(C) 12 cm
(D) 10 cm

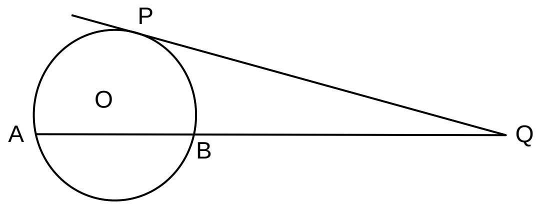
32.



In the above figure, ∠ABE = ∠EDC + 20° and ∠AEB = 110°. Find ∠EDC.

(A) 25°
(B) 120°
(C) 35°
(D) 30°

33. In the given figure, O is the centre of the circle. If tangent PQ = 12 cm and AB = 10 cm, then QB is equal to ______.



(A) 8 cm
(B) 9 cm
(C) 10 cm
(D) 6 cm

34. In a circle of radius 75 cm, two parallel chords have lengths of 144 cm and x cm. They are separated by 81 cm. Find x.

35. If each interior angle of a regular polygon of n sides is 162°, find the value of n.

36. Find the sum of the interior angles of the polygon mentioned in the preceding question.

37. If an angle is equal to two-third of its supplement, find the angle

(A) 18°
(B) 54°
(C) 64°
(D) None of these

38. The exterior angle of a regular polygon is 90° less than its interior angle. Find its interior and exterior angles.

(A) 150°, 30°
(B) 140°, 40°
(C) 135°, 45°
(D) 108°, 72°

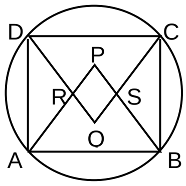
39. There are eight rays in a plane with a common end point. The measures of the eight non overlapping angles, in degrees, are a, b, c, d, e, f, g and h + 4. The numbers a, b, c, d, e, f, g, h are consecutive integers. How many of these angles exceed 45°?

40. While painting the inside of an auditorium, a painter places his ladder against a wall such that the top of the ladder touches the wall at a height of 24 feet. After the work on that wall was over, keeping the foot of the ladder fixed at the same point, it leans against the wall on the opposite side and the top of the ladder touches the wall at a height of 20 feet. If the distance between the walls is 22 feet, what is the length of the ladder? (in feet)

41. Two non-intersecting circles, one lying inside the other, are of radii r_1 and r_2 and $r_1 > r_2$. If the minimum distance between any two points on their circumferences is s, then the distance between their centres is

(A) $r_1 + r_2 - s$ (B) $r_1 - r_2 - s$
(C) $r_1 - r_2 + s$ (D) $r_1 - r_2$

42. In the given figure, AP, BP, CQ, DQ are the bisectors of angles A, B, C and D respectively. Sum of angles PRQ and PSQ is ______.



(A) 90° (B) 120°
(C) 180° (D) Data insufficient

43. The sides of a triangle are 15, 20, 25 A square PQRS is placed in the triangle such that P and Q lie on the sides of lengths 15 and 20 respectively, and RS lies on the side of length 25. Find the perimeter of PQRS.

(A) $\frac{900}{31}$ (B) $\frac{1200}{37}$ (C) $\frac{1800}{43}$ (D) $\frac{2400}{47}$

44. ABCD is a quadrilateral circumscribing a circle. P, Q, R, S are the points of contact of the circle with the sides AB, BC, CD, DA respectively. $\angle A = 90°$, QC = 6, CD = 14 and DA = 16. Find the radius of the circle.

45. How many scalene triangles are there for which the lengths of all the sides (in cm) are integers and the perimeter is 24 cm?

46. Consider the following statements.
I. The altitudes of a triangle can be 4, 6, 9.
II. The altitudes of a triangle can be 6, 8, 15.

Which of the following can be concluded?
(A) Only I is correct.
(B) Only II is correct.
(C) Both I and II are correct.
(D) Neither I nor II is correct.

47. ABCD is a rectangle AB > BC. The perimeter of ABCD is 34 and the area of the triangle ABC is 30. P is a point on CD such that CP : PD = 1 : 3. Find the distance from P to the point of the intersection of the diagonals of ABCD.

(A) $\frac{\sqrt{601}}{4}$ (B) 6 (C) $\frac{\sqrt{79}}{2}$ (D) $\frac{\sqrt{61}}{2}$

48. In the quadrilateral ABCD, $\angle ABC = \angle BCD = 90°$. AD = $\sqrt{13}$, AB = 1 and CD = 4. The diagonals AC and BD intersect at E. Find DE/AE.

49. ABCD is a trapezium, in which AB ||CD. The diagonals of ABCD meet at E. AE = 30ED + 5, BE = 6ED – 3, CE = 6ED + 1. Find ED.

(A) 3 (B) 2 (C) 4 (D) 5

50. A rhombus has a perimeter of 40 cm. The line joining the midpoints of two adjacent sides is 6 cm long. Find the area of the rhombus.

(A) 48 sq.cm. (B) 84 sq.cm.
(C) 96 sq.cm. (D) 372 sq.cm.

51. The radius of a circle is $\sqrt{3}$. Segment AB of length 6 is tangent to this circle at some intermediate point. The other tangents drawn to the circle from A and B intersect at C. Find the minimum perimeter of all such triangles ABC.

52. ABCD is a quadrilateral inscribed in a circle. AB = 45, BC = 60, AC = 75, AD = 72. Find the area of the region inside the circle which is outside the quadrilateral.

(A) $\frac{5625}{4}\pi - 2106$ (B) $\frac{7225}{4}\pi - 2302$

(C) $\frac{5625}{4}\pi - 2502$ (D) $\frac{7225}{4}\pi - 2706$

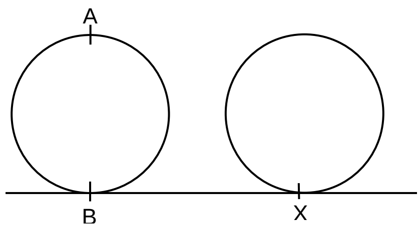
53. A pole, at a point P on the boundary of a circular park subtends an angle of 30° at another point Q on the boundary. If the chord PQ subtends an angle of 60° at the center of the park, then find the ratio of the height of pole to the radius of the park.

(A) 1 : 3 (B) 1 : $\sqrt{3}$ (C) $\sqrt{3}$: 2 (D) 2 : 3

54. In ΔABC, AB = 16, BC = 20, AC = 18. D and E are the points on lines AB and AC respectively such that AD + AE = 25 and the area of ΔADE is half the area of ΔABC. Find DE^2.

(A) 225 (B) 247 (C) 186 (D) 121

55. A circular wheel makes half a rotation such that its topmost point A of the wheel touches the ground at X. If the radius of the wheel is 10 cm, then find (AX)/(AB).



(A) $\sqrt{4\pi^2 + 1}$ (B) $\frac{1}{2}\left(\sqrt{\pi^2 + 4}\right)$

(C) $\sqrt{2\pi^2 + 1}$ (D) $\sqrt{16\pi^2 + 1}$

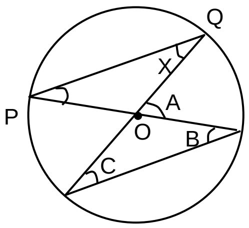
Directions for questions 56 to 65: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using I and II together.

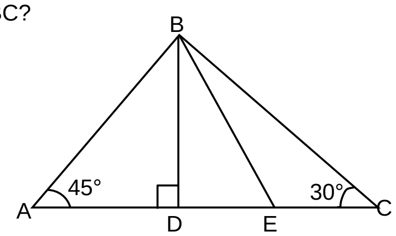
56. Is the four-sided figure a rectangle?
I. The diagonals are equal.
II. The diagonals are perpendicular to each other.

57. In triangle ABC, is angle A a right angle?
I. The circumcircle of triangle ABC passes through A.
II. The circle with AC as diameter passes through B.

58. What is the measure of angle X in the given circle with centre O?
I. $\angle A = 60°$
II. $\angle B = 30°$



59. In the figure below, what is the perimeter of triangle ABC?



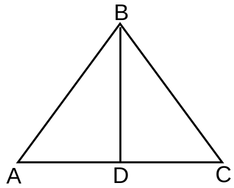
I. The length of AD is 2 cm.
II. The length of EC is $\sqrt{3}$ cm.

60. What is the number of sides in a regular polygon?
I. Sum of the interior angles of the polygon is (2n – 4) x 90° where n is the number of sides of the polygon.
II. One of the exterior angles of the polygon is 60°.

61. Which is the smallest angle of triangle ABC?
I. $AB^2 > BC^2 + AC^2$.
II. $BC^2 < AB^2 + AC^2$.

62. What is the area of triangle ABC?
I. The angles of the triangle are in the ratio 1 : 2 : 3.
II. The longest side of the triangle is 5 cm.

63. Is ABC an isosceles triangle?



I. $\angle BAC = 50°$ and $\angle ABC - \angle BCA = 30°$
II. AD = DC and BD is perpendicular to AC.

64. Does the perpendicular bisector of the side BC of triangle ABC passes through A?
I. AB + AC = 2BC and AC = BC.
II. AB = AC.

65. Do the two circles with centres A and B touch each other externally?
I. The distance between A and B is equal to the diameter of the circle with centre A.
II. One circle is bigger than the other.

Key

Concept Review Questions

1. 48	8. (i) A	15. 2.25	23. D	31. A	39. D
2. D	(ii) B	16. B	24. 48	32. C	40. D
3. A	9. B	17. B	25. B	33. 24	41. 24
4. C	10. 16	18. C	26. C	34. A	42. B
5. D	11. 8	19. C	27. 2	35. 3	43. 120
6. (i) B	12. B	20. B	28. 9	36. 4	44. D
(ii) A	13. D	21. 45	29. B	37. C	45. 21
7. 130	14. C	22. B	30. C	38. C	

Exercise – 4(a)

1. 120	9. 6	17. A	25. D	33. B
2. 20	10. A	18. A	26. C	34. A
3. A	11. D	19. C	27. 59	35. D
4. B	12. B	20. 3	28. 60	36. 4
5. D	13. C	21. 12	29. 3.2	37. 16
6. A	14. C	22. A	30. A	38. 90
7. A	15. 16	23. 7	31. C	39. C
8. 54	16. C	24. D	32. A	40. 2

Exercise – 4(b)

1. 150	12. 1.8	23. B	34. 90	45. 7	56. D
2. C	13. A	24. D	35. 20	46. C	57. B
3. D	14. A	25. 3	36. 3240	47. D	58. B
4. A	15. 40	26. A	37. D	48. 8	59. A
5. 80	16. C	27. 8	38. C	49. A	60. B
6. C	17. D	28. C	39. 3	50. C	61. D
7. 90	18. D	29. 120	40. 25	51. 18	62. C
8. B	19. A	30. A	41. B	52. A	63. D
9. 40	20. 130	31. D	42. C	53. B	64. B
10. 90	21. D	32. A	43. B	54. B	65. C
11. D	22. A	33. A	44. 8	55. B	