Quant Mastery A

Answers and Explanations

ANSWERS AND EXPLANATIONS

1. D
2. D
3. C
4. D
5. B
6. E
7. B
8. B
9. B
10. C
11. D
12. C
13. D
14. E
15. D
16. B
17. E
18. E
19. A
20. B
21. C
22. C
23. E
24. A

1. **(D)**

Nancy works 3 days per week at a rate of x dollars per day. If Nancy works more than 3 days per week, any extra days are paid at double her normal rate. Nancy worked 4 weeks with no overtime, and then worked 3 more weeks, each with y days of overtime. Which of the following represents Nancy's total earnings over this period?

- ○ $3y(7x + 2xy)$
- ○ $3x + 12y$
- ○ $3xy(7x + 2)$
- ○ $3x(2y + 7)$
- ○ $4x(y + 8)$

Step 1: Analyze the Question

We've got a wordy question stem and variables in the answer choices, so we can make this question more concrete and manageable through Picking Numbers.

Step 2: State the Task

We need to determine Nancy's total earnings over a seven-week period, including four regular three-day weeks and three weeks during which she worked overtime days.

Step 3: Approach Strategically

We need two values, so let's set $x = 5$ and $y = 2$. (Note that it's a good idea to avoid 3 and 4 when Picking Numbers for this question, since those numbers appear in the question itself and may cause confusion later when we substitute the values into the answer choices.)

There are two distinct parts of the seven-week period we're asked to consider: the first four weeks, during which she worked three days a week with no overtime, and the final three weeks, during which she worked three days a week plus overtime.

Let's start with the first four weeks. Each week she worked three days at a rate of \$5 a day (her daily rate is x, which we've set equal to 5), for a total of \$60 earned: $4 \times 3 \times 5 = 60$.

In the final three weeks, she worked three days at a rate of \$5 a day, as well as two additional days (the number of overtime days she worked is y, which we've set equal to 2) at a rate of \$10 a day (her overtime days are paid at twice her regular rate, or $2x$), for a total of \$105 earned: $(3 \times 3 \times 5) + (3 \times 2 \times 10) = 105$. Therefore, her total earnings over this period were \$165: $60 + 105 = 165$.

Now let's plug our values for x and y into the answer choices to see which one equals 165:

(A) $3(2)[7(5) + 2(5)(2)] = 6(55) = 330$. Eliminate.

(B) $3(5) + 12(2) = 15 + 24 = 39$. Eliminate.

(C) $3(5)(2)[7(5) + 2] = 30(37) = 1{,}110$. Eliminate.

(D) $3(5)[2(2) + 7] = 15(11) = 165$. Possibly correct.

(E) $4(5)[2 + 8] = 20(10) = 200$. Eliminate.

Only **(D)** gives us 165, so we know we've found the right answer. Note that we can tell that **(A)** and **(C)** will be far too large even before we do the final multiplication. Thinking logically about the answer choices will help to avoid unnecessary calculations.

Step 4: Confirm Your Answer

Reread the question stem, making sure you didn't miss anything about the problem. In particular, make sure that you've substituted your values for x and y consistently and haven't switched them anywhere.

2. (D)

If $5^{8y + 14} = 125^{4y - 14}$, then what is the value of y?

- ○ −14
- ○ −8
- ○ 8
- ○ 14
- ○ 32

Step 1: Analyze the Question

We're asked to solve for a variable located in some rather complicated exponents. The key is to recognize that the base on one side of the equation is a root of that on the other.

Step 2: State the Task

This question asks us to isolate y within the equation.

Step 3: Approach Strategically

Remember this exponent rule: $(x^a)^b = x^{ab}$. With the help of this rule, we can make the bases identical ($125 = 5^3$), yielding an equation that looks like this: $5^{8y + 14} = 5^{3(4y - 14)}$. Since the bases are now equal, we can cancel them and get a single-variable equation:

$$8y + 14 = 3(4y - 14)$$

$$8y + 14 = 12y - 42$$

$$56 = 4y$$

$$14 = y$$

Answer choice **(D)** is correct.

Step 4: Confirm Your Answer

Double-check that you did not make any errors as you applied the exponent rules and simplified the equation. You could plug 14 in for y in the stem equation to check your work, but the numbers get large so quickly that doing so will likely not be very efficient.

3. (C)

If b is an integer, is $a|a| < 2^b$?

(1) $b < 0$

(2) $a = b$

Step 1: Analyze the Question Stem

This is a Yes/No question. For sufficiency, we need to be able to determine whether this inequality is true. Let's analyze the stem by considering the various possible values for a and b and what effect these values would have on the inequality.

First, let's analyze the variable b. Because we are told that b is an integer but not told whether it is positive, negative, or zero, we should first determine whether that matters. If b is positive, 2 to the b power will be an integer equal to the value of 2 or greater. If b is negative, 2 to the b power will be a fraction between 0 and $\frac{1}{2}$. If b is zero, 2 to the b power will equal 1.

Next, let's analyze a: If a is negative, $a|a|$ is also negative, and the answer is "always yes," regardless of the value of b, since 2^b will always be positive. Likewise, if a is zero, then $a|a|$ is also zero, and the answer would still be "always yes." Finally, if a is positive, $a|a|$ is also positive. In this case, the answer could be "yes" or "no," depending on the exact values of a and b.

Step 2: Evaluate the Statements Using 12TEN

Statement (1): The statement tells us that b is negative. According to our analysis above, that tells us that 2^b is between 0 and $\frac{1}{2}$, but it does not tell us anything about a, which could be positive, negative, or zero. The statement is insufficient, thus eliminating **(A)** and **(D)**.

Statement (2): Let's examine some examples to see whether we have sufficiency. If a and b are both equal to 1, then $1 < 2$, and the answer to the question in the stem is "yes." But if a and b are both equal to 2, then $4 = 4$, and the answer to the question in the stem is "no." "Sometimes yes, sometimes no" is an insufficient answer. Eliminate **(B)**.

Because each statement alone is insufficient, we combine the statements to see whether they are sufficient together. Together with the information in the question stem, the two statements tell us that a and b are the same negative integer. Based on number properties, we already found in our analysis of the question stem that a negative integer for b means that the term on the right side of the inequality must be a positive fraction less than or equal to $\frac{1}{2}$. If a is a negative integer, we end up with a negative number on the left side of the inequality, which will always be less than the positive fraction on the right side of the inequality. The answer to the question in the stem is "always yes," so we have sufficiency when we combine the two statements. Choice **(C)** is correct.

4. (D)

If $(x^2 + 8)yz < 0$, $wz > 0$, and $xyz < 0$, then which of the following must be true?

I. $x < 0$
II. $wy < 0$
III. $yz < 0$

- ○ II only
- ○ III only
- ○ I and III only
- ○ II and III only
- ○ I, II, and III

Step 1: Analyze the Question

Let's begin with the inequality $(x^2 + 8)yz < 0$. We know that $x^2 + 8 > 0$, because the square of any real number is 0 or positive, and 8 is positive. When the positive quantity $x^2 + 8$ is multiplied by the quantity yz, the result is negative, so $yz < 0$. Since $yz < 0$, y and z must have opposite signs, and neither can equal zero.

For the second inequality, $wz > 0$, meaning that w and z must have the same sign, and neither can equal zero.

The third given inequality states that $xyz < 0$. Because we already know that $yz < 0$, this tells us that x must be positive.

Step 2: State the Task

We are given three inequality statements and are asked which one or ones must be true.

Step 3: Approach Strategically

Roman numeral III appears most frequently among the answer choices, so we will start there. It states that $yz < 0$. From our analysis of the stem, we know this must be true because y and z must have opposite signs. We can eliminate **(A)**.

Roman numeral II states that $wy < 0$. We know from Step 1 that y and z must have opposite signs, and also that w and z must have the same sign. Therefore, w and y must have opposite signs, and the statement must be true. We can eliminate **(B)** and **(C)** because neither one contains Roman numeral II.

Roman numeral I: As we saw above, x must be positive. Therefore, Roman numeral I is false. Only Roman numerals II and III must be true. Choice **(D)** is correct.

Step 4: Confirm Your Answer

If your scratchwork is well organized, you can glance over it quickly to verify that you correctly confirmed or disconfirmed each statement.

5. (B)

For a bake sale, Simon baked $2n$ more pies than Theresa. Theresa baked half as many pies as Roger, who baked $\frac{1}{3}n$ pies. No other pies were baked for the sale. What fraction of the total pies for sale did Roger bake?

- ○ $\frac{1}{16}$
- ○ $\frac{1}{8}$
- ○ $\frac{3}{16}$
- ○ $\frac{3}{8}$
- ○ $\frac{13}{16}$

Step 1: Analyze the Question

There are variables in the question stem and fractions in the question stem and answer choices, so Picking Numbers is a logical approach. When Picking Numbers for questions involving fractions, the most manageable number to choose is the least common denominator (in this case, the product of the denominators) of the fractions.

Step 2: State the Task

Our task is to find the fraction of the total pies that Roger baked, so we need the total number of pies as well as the number Roger baked.

Step 3: Approach Strategically

The two fractions from the question stem are $\frac{1}{2}$ and $\frac{1}{3}$, so 6, their common denominator, is a good number to pick for n. If $n = 6$, then Roger baked 2 pies $\left(\frac{1}{3}n\right)$, and Theresa baked 1 ($\frac{1}{2}$ of the number that Roger baked). Simon baked 12 (or $2n$) more pies than Theresa did, so Simon baked 13. Adding those numbers up, we get 16 as the total number of pies baked by all three people. Roger baked 2 of them, so the final fraction is $\frac{2}{16}$ or $\frac{1}{8}$. Choice **(B)** is correct.

Step 4: Confirm Your Answer

Reread the question stem, making sure you didn't miss anything about the problem. For instance, make sure that you've used the number of pies that Roger baked, not Simon or Theresa, in your final fraction.

6. (E)

If p is an even integer, and q and r are odd integers, then which of the following CANNOT be an odd integer?

- ○ $\frac{p + q}{r}$
- ○ $\frac{q - r}{p}$
- ○ $\frac{q}{p + r}$
- ○ $\frac{q}{r} + p$
- ○ $\frac{pq}{r} + pr$

Step 1: Analyze the Question

The question asks about odds and evens, so this is clearly a number properties question. We also have variables, so Picking Numbers can help make this question more concrete.

Step 2: State the Task

Find the answer choice that can never produce an odd integer. If an answer choice produces an odd integer, it can be eliminated.

Step 3: Approach Strategically

We know that p must be even, so we'll pick $p = 2$; q and r must be odd, so we'll pick $q = 3$ and $r = 1$. (Note that it's a good idea to pick 1 for r, since r appears quite often in the denominators of the answer choices. Our goal is to eliminate answer choices by making them come out to be odd integers, so choosing a smaller denominator saves us time by increasing the likelihood that we will end up with integer values for the answer choices.) Let's plug these values into the answer choices:

(A) $\frac{2 + 3}{1} = 5$

(B) $\frac{3 - 1}{2} = 1$

(C) $\frac{3}{2 + 1} = 1$

(D) $\frac{3}{1} + 2 = 5$

(E) $\frac{(2)(3)}{1} + 2(1) = 6 + 2 = 8$

Since the correct answer can *never* be odd, we can eliminate **(A)**, **(B)**, **(C)**, and **(D)**, so **(E)** is our correct answer.

Step 4: Confirm Your Answer

Reread the question stem, making sure you didn't miss anything about the question. On this type of question, it's easy to get confused about what you're looking for in the answer—you might, for instance, think that you're looking for an odd answer—so double-check that you've eliminated correctly. Applying the even/odd rules confirms that **(E)** can never be an odd integer as well:

$$\frac{(\text{even})(\text{odd})}{\text{odd}} + (\text{even})(\text{odd})$$
$$= \frac{(\text{even})}{\text{odd}} + \text{even}$$
$$= (\text{non-integer or even}) + \text{even}$$
$$= \text{non-integer or even}$$

7. (B)

Yogurt containers come in 4 different sizes. A grocery store currently has 20 yogurt containers, which have an average size of 7.4 ounces. If there are 7 containers of 4 ounces each and 3 containers of 10 ounces each, and half of the remaining containers contain x ounces of yogurt each while the other half contain $2x$ ounces of yogurt each, what is the value of x?

- ○ 5
- ○ 6
- ○ 8
- ○ 12
- ○ 15

Step 1: Analyze the Question

We've got a lot of numbers in this problem—the average volume of the containers, the number of containers, and the capacity of some of them. We've got integers in the answer choices, so Backsolving is a good approach here.

Step 2: State the Task

We need to determine the capacity (x) of 5 yogurt containers in a group of 20 whose average capacity is 7.4 ounces.

Step 3: Approach Strategically

Using the average formula $\left(\text{Average} = \dfrac{\text{Sum of terms}}{\text{Number of terms}}\right)$, we can determine the total amount of yogurt at the grocery store: $20 \times 7.4 = 148$ ounces. Next, we can subtract the volumes we're given in the question stem to determine how much yogurt is in the remaining containers. Seven containers hold 4 ounces of yogurt, for a total of 28 ounces, and 3 containers hold 10 ounces of yogurt, for a total of 30 ounces. Since $148 - (28 + 30) = 90$, we know that 90 ounces is in the remaining 10 containers.

Now that we've figured out that the 10 remaining containers hold a total of 90 ounces, we can Backsolve. Let's start with **(B)**. Five containers would hold 6 ounces of yogurt, for a total of 30 ounces. The other 5 containers would hold 12 ounces of yogurt ($2x$), for a total of 60 ounces. Since $30 + 60 = 90$, we know we've found the right answer.

Step 4: Confirm Your Answer

Reread the question stem, making sure you didn't miss anything about the problem. Also, make sure that you calculated the correct value for the amount of yogurt in the last 10 containers.

8. (B)

What is the value of x?

(1) $3x^2 - 8x - 35 = 0$

(2) $x^2 - 3x = 7x - 25$

Step 1: Analyze the Question Stem

We are being asked a Value question. The stem here gives us no additional information.

Step 2: Evaluate the Statements Using 12TEN

Statement (1): Let's factor $3x^2 - 8x - 35$ using reverse FOIL. The expression $3x^2 - 8x - 35$ can be factored into $(x + a)(3x + b)$, where a and b are constants. If we test the different factor pairs of 35 for a and b, we find that $3x^2 - 8x - 35 = (x - 5)(3x + 7)$. So the equation $3x^2 - 8x - 35 = 0$ can be written as $(x - 5)(3x + 7) = 0$. To solve for x, we can solve the following equations: $x - 5 = 0$ and $3x + 7 = 0$, giving us $x = 5$ or $x = -\frac{7}{3}$. Since there is more than one possible answer to the stem question, Statement (1) is insufficient. We can eliminate **(A)** and **(D)**.

Statement (2): Let's solve the equation $x^2 - 3x = 7x - 25$. Adding $-7x + 25$ to both sides, we have $x^2 - 10x + 25 = 0$. We can factor $x^2 - 10x + 25$ using reverse FOIL. We need two numbers that sum to –10 and multiply to 25. Since their product is positive, we know that both numbers have the same sign. Since they have a negative sum, we know that the sign the two numbers share must be negative. We are looking for –5 and –5. Thus, $x^2 - 10x + 25 = (x - 5)^2$. (We could also have factored this equation by recalling the Classic Quadratic $(a - b)^2 = a^2 - 2ab + b^2$.) So $(x - 5)^2 = 0$. This means that $x - 5 = 0$. If $x - 5 = 0$, then $x = 5$. Since there is only one possible value for x, Statement (2) is sufficient to answer the question. There is no need to combine statements. Choice **(B)** is correct.

9. (B)

m and n are both integers, and $m \neq n$. Does $n = 0$?

(1) $m = 7$

(2) $mn = n^2$

Step 1: Analyze the Question Stem

To have sufficient information to answer this Yes/No question, we could be given information that would allow us to calculate the value of n, to establish that n cannot be zero, or to determine that $m = 0$. Any of these would allow us to answer the Yes/No question unequivocally.

Step 2: Evaluate the Statements Using 12TEN

Statement (1): If $m = 7$, n may or may not equal zero. This statement is insufficient. Eliminate **(A)** and **(D)**. Evaluating this statement by itself is quite straightforward, but with a clearly insufficient statement like this, keep in mind that the testmaker may be trying to plant a seed in your mind that this statement may be critical, in conjunction with Statement (2), to attaining sufficiency. It's very important that you ignore Statement (1) *completely* when you evaluate (2).

Statement (2) tells us that n must equal zero. If n were not to equal zero, then $mn = n^2$ would require that $m = n$. But we are told in the question stem that $m \neq n$. So n must equal zero. This statement is sufficient. Choice **(B)** is correct.

There is no need to combine statements. Be careful whenever you see an obviously insufficient Statement (1) that you don't fall into the trap of assuming that Statement (1) will be critical to gaining sufficiency in combination with Statement (2). By following the Kaplan Method and evaluating the statements separately first, we determined that Statement (2) is sufficient by itself, and we avoided this trap.

10. (C)

When a computer is rented from store X, there is an initial fee of \$80 and an additional rate of \$20 per day. When a computer is rented from store Y, there is an initial fee of \$180 and a daily computer rental rate of \$17. If the cost of renting a computer for *n* days from store Y is 125% of the cost of renting from store X, what is the value of *n*?

- ○ 5
- ○ 8
- ○ 10
- ○ 13
- ○ 25

Step 1: Analyze the Question

We're asked to compare the cost of renting a computer from two different stores. We've got specific values in the question stem and integers in the answer choices, so Backsolving is a possible approach here.

Step 2: State the Task

We need to determine the length of a rental, in days, such that the cost of renting a computer from store Y is 125% the cost of renting the computer from store X.

Step 3: Approach Strategically

Remember when Backsolving, we should start with **(B)** or **(D)** to minimize the number of answer choices we have to test. Starting with **(B)**, we get a cost of \$80 + 8(\$20) = \$240 to rent a computer at store X. The cost of renting from Y is \$180 + 8(\$17) = \$316. We're looking for the cost of Y to be 125% the cost of X, which we can calculate by dividing the cost of Y by the cost of X. Plugging in our values, we get $\frac{316}{240} \times 100\% \approx 132\%$.

This percentage is higher than 125%, so we need the price at store Y to be somewhat lower relative to the price at store X. To offset the much higher initial cost of renting from store Y, we need to increase the number of days. So we can eliminate **(A)** and **(B)** and test choice **(D).** If the computer is rented for 13 days, we get a cost of \$80 + 13(\$20) = \$340 to rent a computer from store X and a cost of \$180 + 13(\$17) = \$401 to rent from store Y. Since $\frac{401}{340} \times 100\% \approx 118\%$, we know this value is too low, and we need the rental period to be shorter. The only answer between 8 days and 13 days is **(C)**, 10 days.

Step 4: Confirm Your Answer

If time permits, a clear way to confirm your answer would be to plug the value from **(C)**, 10 days, into the question. Doing so gives you a cost of \$80 + 10(\$20) = \$280 to rent a computer from store X and a cost of \$180 + 10(\$17) = \$350 to rent from store Y. Calculating the percentage, you get $\frac{350}{280} \times 100\% = 125\%$, so **(C)** is the correct answer.

But even if you don't have time to confirm your answer in this way, it's always a good idea to reread the question stem, making sure you didn't miss anything about the problem. Also, make sure that you haven't reversed your numbers in calculating the percentages (i.e., divided the cost of X by the cost of Y).

11. (D)

For all real numbers a and b, the operation Ω is defined by $a \, \Omega \, b = a^2 + ab$. If $y = x \, \Omega \, 8$, and $3 \, \Omega \, y = -12$, then what is the sum of the squares of all possible values of x?

- ○ 25
- ○ 29
- ○ 36.5
- ○ 50
- ○ 73

Step 1: Analyze the Question

The testmakers have really thrown everything into the mix to make this question appear intimidating. We have symbolism (but, remember, that's just a matter of substitution), quadratic equations (we can see this coming because there will be multiple possible values of x), and the extra step of finding the sum of the squares in order to get the correct answer at the end.

Step 2: State the Task

We need to substitute the information we are given into the symbol-based equations, determine the possible solutions for x, square those values, and finally add the squares.

Step 3: Approach Strategically

Let's work with the equation $y = x \, \Omega \, 8$ first. We can use the defining equation $a \, \Omega \, b = a^2 + ab$ to write an expression for $x \, \Omega \, 8$. Replacing a with x and b with 8, we get $x \, \Omega \, 8 = x^2 + 8x$. Thus, we have $y = x^2 + 8x$. Now let's replace y with $x^2 + 8$ in the equation $3 \, \Omega \, y = -12$. Then $3 \, \Omega \, (x^2 + 8x) = -12$. Let's now use the defining equation to write an expression for $3 \, \Omega \, (x^2 + 8x)$:

$$\begin{aligned} 3 \, \Omega \, (x^2 + 8x) &= 3^2 + (3)(x^2 + 8x) \\ &= 9 + 3(x^2 + 8x) \\ &= 9 + 3x^2 + 24x \\ &= 3x^2 + 24x + 9 \end{aligned}$$

Since $3 \, \Omega \, (x^2 + 8x) = -12$, we know that $3x^2 + 24x + 9 = -12$. Let's solve this equation for x:

$$\begin{aligned} 3x^2 + 24x + 9 &= -12 \\ 3x^2 + 24x + 21 &= 0 \\ x^2 + 8x + 7 &= 0 \\ (x + 1)(x + 7) &= 0 \end{aligned}$$

So $x + 1 = 0$ or $x + 7 = 0$. If $x + 1 = 0$, then $x = -1$. If $x + 7 = 0$, then $x = -7$. The possible values of x are -1 and -7. The sum of the squares of all possible values of x is $(-1)^2 + (-7)^2 = 1 + 49 = 50$. Choice **(D)** is correct.

Step 4: Confirm Your Answer

There is no way around working through the math for this multi-step problem. The key thing to determine at the end is simply that you answered the question that was asked and that you did not make any avoidable mistakes in your calculations.

12. (C)

Is $x > 3$?

(1) $5^y > 25^8$ and $y = x^2$.

(2) $2^{15x} > (8^{4x})(8)$

Step 1: Analyze the Question Stem

This is a Yes/No question. For sufficiency, a statement will have to tell us that x is definitely larger than 3 (for a "yes") or definitely less than or equal to 3 (for a "no").

Step 2: Evaluate the Statements Using 12TEN

Statement (1): Let's rewrite the inequality $5^y > 25^8$ so that each of the powers has the same base:

$$5^y > 25^8$$

$$5^y > (5^2)^8$$

$$5^y > 5^{16}$$

Therefore, $y > 16$. Because the statement says that $y = x^2$, we know that $x^2 > 16$, and therefore $x > 4$ or $x < -4$. This is insufficient to determine whether x is greater than 3. We can eliminate **(A)** and **(D)**.

Statement (2): Let's rewrite the inequality $2^{15x} > (8^{4x})(8)$ so that each of the powers has the same base. Using our exponent rules, we get the following:

$$2^{15x} > (2^3)^{4x}(2^3)$$

$$2^{15x} > (2^{12x})(2^3)$$

$$2^{15x} > 2^{12x + 3}$$

Because the base 2 in both of the powers 2^{15x} and $2^{12x + 3}$ is greater than 1, we can conclude from the inequality that $15x > 12x + 3$. Let's solve this inequality for x:

$$15x > 12x + 3$$

$$3x > 3$$

$$x > 1$$

If $x > 1$, it is possible for x to be greater than, equal to, or less than 3. For instance, if $x = 4$, then $x > 3$, so the answer to the question is "yes." If $x = 2$, then we do not have that $x > 3$, so the answer to the question is "no." Since more than one answer to the stem question is possible, Statement (2) is insufficient. We can eliminate **(B)**.

Let's combine the statements. From Statement (1), we must have that $x > 4$ or $x < -4$. From Statement (2), we must have that $x > 1$. The requirement that $x > 1$ is incompatible with the possibility that $x < -4$. Therefore, the requirement that $x > 4$ or $x < -4$ together with the requirement that $x > 1$ means that $x > 4$. Since the statements combined tell us that $x > 4$, then we must have $x > 3$, since $4 > 3$. The statements taken together are sufficient to answer the question with a definite "yes." Choice **(C)** is correct.

13. (D)

x and y are positive integers. Is $\sqrt{x}\sqrt{y}$ an integer?

(1) $\frac{x}{y} = \frac{1}{n^2}$, where n is a positive integer.

(2) $\sqrt[3]{x}\,\sqrt[3]{y}$ is the square of an integer.

Step 1: Analyze the Question Stem

This is a Yes/No question. We are told that x and y are positive integers and are asked whether the square root of x times the square root of y is an integer. For sufficiency, a statement will have to demonstrate that the square root of xy is always an integer ("yes") or never an integer ("no").

Step 2: Evaluate the Statements Using 12TEN

Statement (1): Let's cross-multiply the equation $\frac{x}{y} = \frac{1}{n^2}$ which gives us $xn^2 = y$. Now let's substitute $xn^2 = y$ for y in the expression from the question stem: $\sqrt{x}\sqrt{y} = \sqrt{x}\sqrt{xn^2} = \sqrt{x}\sqrt{x}\sqrt{n^2} = xn$.

Since x and n are both integers, the product of x and n, or xn, is also an integer. Thus, $\sqrt{x}\sqrt{y}$ is an integer. Statement (1) is sufficient to answer the question. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): Because x and y are both positive, $\sqrt[3]{x}$ and $\sqrt[3]{y}$ are positive, and therefore $\sqrt[3]{x}\,\sqrt[3]{y}$ is also positive. Since $\sqrt[3]{x}\,\sqrt[3]{y}$ is the square of an integer and is positive, we can say that $\sqrt[3]{x}\,\sqrt[3]{y} = m^2$ where m is a nonzero integer. To remove the cube roots on the left side, cube both sides of the equation. We now have $(\sqrt[3]{x}\,\sqrt[3]{y})^3 = (m^2)^3$, which simplifies to $xy = m^6$. We can now take the square root of both sides of the equation $xy = m^6$, leaving us with $\sqrt{xy} = \sqrt{x}\sqrt{y} = \pm m^3$.

Since m is an integer, m^3 and $-m^3$ are both integers. Thus, $\sqrt{x}\sqrt{y}$ is an integer. Statement (2) is also sufficient to answer the question with a "yes." Choice **(D)** is correct.

14. (E)

Which of the following represents all the possible values of x that are solutions to the equation $5x = |x^2 - 6|$?

- ○ -3, -2, and 0
- ○ -6, -1, 1, and 6
- ○ -6 and 1
- ○ 2 and 3
- ○ 1 and 6

Step 1: Analyze the Question

This is an absolute value question.

Step 2: State the Task

We are asked to solve for all the possible values of x that are consistent with the given equation.

Step 3: Approach Strategically

In the question stem, x is shown both inside and outside the absolute value sign. Remember, absolute value is always a positive number—it indicates how far the information inside the absolute value signs is from zero on the number line. Thus, $5x$ must be positive, so x must be positive. We can eliminate **(A)**, **(B)**, and **(C)**.

The numbers in **(D)** and **(E)** are all different, so we can simply Backsolve, trying out any of the numbers in the remaining answer choices. Let's start with the 1 in this case, in choice **(E)**, since it is easy to work with.

$$5(1) = |1^2 - 6|$$

$$5 = |-5|$$

Since the resulting equation turns out to be true, **(E)** is the correct answer.

Step 4: Confirm Your Answer

Because you Backsolved, you have confirmed your answer. You may be tempted to test some of the other numbers in **(D)** and **(E)** to be sure, but doing so would be a waste of time. Having eliminated **(A)**, **(B)**, and **(C)** because they contain negative values, you need only test one number from **(D)** or **(E)**, since there is no overlap between the numbers listed by these two answer choices. If the number works, then that must be the correct answer, since the question asks for *all* possible solutions for x. Likewise, if the number you test does not work, you know to pick the other answer choice.

15. (D)

Is $y > -4$?

(1) $\left(\frac{1}{7}\right)^{4y} > \left(\frac{1}{7}\right)^{8y + 14}$

(2) $4y^2 + 12y < 0$

Step 1: Analyze the Question Stem

This is a Yes/No question. For sufficiency, we must determine whether y is definitely greater than -4 ("yes") or definitely less than or equal to -4 ("no").

Step 2: Evaluate the Statements Using 12TEN

Statement (1): Since $\frac{1}{7}$ is a positive fraction less than 1, its value will become smaller when raised to a larger exponent. As $\frac{1}{7}$ is the common base for both terms in this inequality, we can simplify the inequality as $4y < 8y + 14$, remembering to switch the direction of the inequality sign to account for the effect of raising a fraction less than 1 to an exponent. Next, we can solve for y:

$$4y < 8y + 14$$

$$-14 < 4y$$

$$-\frac{14}{4} < y$$

Now $-\frac{14}{4} = -\frac{7}{2} = -3.5$. Thus, $y > -3.5$, which is greater than -4. The answer is "always yes." Therefore, Statement (1) is sufficient. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): Let's factor the inequality $4y^2 + 12y < 0$. Factoring out a 4 from the left side, we have $4(y^2 + 3y) < 0$. Factoring out a y from the left side of the inequality, we have $4y(y + 3) < 0$.

When the product of a group of terms is negative, there must be an odd number of negative terms. Here, the product $4y(y + 3)$, which is the product of the three factors 4, y, and $y + 3$, is negative. So there must be either one or three negative factors among these terms. Since 4 is positive, there cannot be three negative factors. Thus, one of the factors y and $y + 3$ must be positive, and the other must be negative. If y were positive, then $y + 3$ would also be positive, of course. Therefore, we can deduce that $y < 0$ and $y + 3 > 0$. Combined, these inequalities give us a range of values for y: $-3 < y < 0$. So y must always be greater than -4. Thus, Statement (2) is sufficient to answer the question. Choice **(D)** is correct.

16. (B)

Betty has 4 times the number of stamps that Cathy does, and Anne has 6 stamps more than Cathy does. If Betty, Cathy, and Anne each increase their number of stamps by 5, which of the following must be true after each person increases her number of stamps?

I. Betty has more stamps than Anne.
II. Anne has 3 more stamps than Cathy.
III. The sum of the numbers of stamps that Betty and Cathy have is a multiple of 5.

- ○ None
- ○ III only
- ○ I and III only
- ○ II and III only
- ○ I, II, and III

Step 1: Analyze the Question

The most efficient way to work with a "must be true" question is to try to come up with a scenario in which each Roman numeral statement is false so that you can eliminate it. Here, the given information involves proportions but includes no actual values. The question stem tells us that, initially, $B = 4C$ and $A = C + 6$. We'll use Picking Numbers to evaluate the Roman numerals.

Step 2: State the Task

We need to determine the relative number of stamps among three people after each increases her collection by five.

Step 3: Approach Strategically

Start with the Roman numeral that shows up most often. That's III, which translates to $(B + 5) + (C + 5) =$ multiple of 5. No matter what numbers we pick, the answer will come out to be an integer. For example, if $C = 1$, then $B = 4$: $(4 + 5) + (1 + 5) = 15$. If we try $C = 7$, then $B = 28$. $(28 + 5) + (7 + 5) = 45$. So, III must be true. We can eliminate **(A)**.

Let's try II next. Suppose $C = 1$ and $A = 7$. Then $C + 5 = 6$ and $A + 5 = 12$; Alice still has 6 more stamps than Cathy. Statement II need not be true. Eliminate **(D)** and **(E)**.

Last, we need to evaluate I. Again, we can Pick Numbers: $C = 1$, $B = 4$, $A = 7$; $B + 5 = 9$ and $A + 5 = 12$. B is less than A in this case, so we can eliminate I. The answer is III only, choice **(B)**.

Step 4: Confirm Your Answer

One of the great advantages of Picking Numbers in Roman numeral problems is that the numbers provide tangible proof that something does (or does not) have to be true. In this case, provided that you made good translations of the information in the question stem (for example, that you did not mistake $A = C + 6$ for $A + 6 = C$ or $A = 6C$), you should be certain of your answer.

17. (E)

Is x^3 an integer?

(1) x^2 is an integer.

(2) The product of $x^3 + \sqrt{7}$ and $x^3 - \sqrt{7}$ is an integer.

Step 1: Analyze the Question Stem

This is a Yes/No question. For a statement to be sufficient, it will cause x^3 to be an integer in every case, or a non-integer in every case. This question requires that we think about different types of numbers, such as fractions and irrational numbers.

Step 2: Evaluate the Statements Using 12TEN

Statement (1) tells us that x^2 is an integer. Does this statement prove that x is an integer? Consider the following scenarios:

If $x = 1$, then $x^2 = 1$ and Statement (1) is true. Then $x^3 = 1$, which is an integer. This scenario answers the stem question with a "yes."

If $x = \sqrt{3}$, then $x^2 = (\sqrt{3})^2 = (\sqrt{3})(\sqrt{3}) = 3$. Because 3 is an integer, Statement (1) is true. In this case, $x^3 = (\sqrt{3})^3 = (\sqrt{3})(\sqrt{3})(\sqrt{3}) = 3\sqrt{3}$. Because $3\sqrt{3}$ is not an integer, we have found an example that makes the statement true but gives us a "no" in response to the question stem.

Because different answers to the question are possible, Statement (1) is insufficient. We can eliminate **(A)** and **(D)**.

Statement (2) tells us that $(x^3 + \sqrt{7})(x^3 - \sqrt{7})$ is an integer. This is the Classic Quadratic $(a + b)(a - b) = a^2 - b^2$. Therefore: $(x^3 + \sqrt{7})(x^3 - \sqrt{7}) = (x^3)^2 - (\sqrt{7})^2 = x^6 - 7$. So $x^6 - 7$ is an integer. Since 7 is an integer, x^6 must itself be an integer. Statement (2) is equivalent to saying that x^6 is an integer.

Now if $x = 1$, then $x^6 = 1^6 = (1)(1)(1)(1)(1)(1) = 1$, which is an integer, so Statement (2) is true. In this case, $x^3 = 1^3 = (1)(1)(1) = 1$, which is an integer, so the answer to the question is "yes."

If $x = \sqrt{3}$, then $x^6 = (\sqrt{3})^6 = (\sqrt{3})(\sqrt{3})(\sqrt{3})(\sqrt{3})(\sqrt{3})(\sqrt{3}) = (3)(3)(3) = 27$, which is an integer, so Statement (2) is true. In this case, $x^3 = (\sqrt{3})^3 = (\sqrt{3})(\sqrt{3})(\sqrt{3}) = 3\sqrt{3}$, which is not an integer, so the answer to the question is "no." Since different answers to the question are possible, Statement (2) is insufficient. We can eliminate choice **(B)**.

The statements taken together continue to be insufficient. The numbers we used to show that each statement by itself is insufficient can also be used to show that the statements taken together are insufficient. If $x = 1$, then both statements are true, and the cube of x is an integer. If $x = \sqrt{3}$, both statements are again true, and the cube of x is *not* an integer. Since different answers to the question are possible, the statements taken together are insufficient. Choice **(E)** is correct.

18. (E)

If a, b, and c are integers such that $0 < a < b < c$, and a is even, b is prime, and c is odd, which of the following is a possible value for abc?

- ○ 5
- ○ 12
- ○ 15
- ○ 33
- ○ 54

Step 1: Analyze the Question

We are told that a, b, and c are integers such that $0 < a < b < c$ and that a is even, b is prime, and c is odd.

Step 2: State the Task

We are given five answer choices and are asked to determine which of them is a possible value for abc.

Step 3: Approach Strategically

Because a is an even number, abc must also be even. This eliminates **(A)**, **(C)**, and **(D)**.

Next, let pick the smallest numbers possible that also meet the criteria. We'll try $a = 2$, $b = 3$, and $c = 5$. In this case, $abc = 30$. Therefore, **(B)** cannot be correct because it is too small, leaving **(E)** as the only answer choice that can be correct. We do not have to determine which values for a, b, and c would result in that answer choice.

Step 4: Confirm Your Answer

Double-checking the given information shows that only choice **(E)** is a possible value for abc. If time permits on Test Day, you can confirm this by determining that the permissible values $a = 2$, $b = 3$, and $c = 9$ result in a product of 54.

19. (A)

The integers x and y are positive, $x > y + 8$, and $y > 8$. If the remainder when $x + y$ is divided by 8 is 7, and the remainder when $x - y$ is divided by 8 is 5, then what is the remainder when $x^2 - y^2$ is divided by 8?

- ○ 3
- ○ 4
- ○ 5
- ○ 6
- ○ 7

Step 1: Analyze the Question

We are given several pieces of information:

x and y are positive integers.

$x > y + 8$

$y > 8$

We can conclude from that information that $y \geq 9$, making $x \geq 18$.

We are also told that when $x + y$ is divided by 8, its remainder is 7, and that when $x - y$ is divided by 8, its remainder is 5.

Step 2: State the Task

The task is to determine the remainder when $x^2 - y^2$ is divided by 8.

Step 3: Approach Strategically

Remember the Classic Quadratic $a^2 - b^2 = (a + b)(a - b)$. So $x^2 - y^2 = (x + y)(x - y)$.

Since the remainder when $x + y$ is divided by 8 is 7, we can say that $x + y = 8N_1 + 7$, where N_1 is a positive integer.

Since the remainder when $x - y$ is divided by 8 is 5, we can say that $x - y = 8N_2 + 5$, where N_2 is a positive integer.

$$\begin{aligned} \text{Now } x^2 - y^2 &= (x + y)\,(x - y) \\ &= (8N_1 + 7)(8N_2 + 5) \\ &= 64N_1N_2 + 40N_1 + 56N_2 + 35 \end{aligned}$$

Because the coefficients 64, 40, and 56 are all multiples of 8 and N_1 and N_2 are integers, it follows that $64N_1N_2$, $40N_1$, and $56N_2$ are also all multiples of 8. The sum of three multiples of 8 is itself a multiple of 8, and therefore the quantity $64N_1N_2 + 40N_1 + 56N_2$ is a multiple of 8. In other words, when $64N_1N_2 + 40N_1 + 56N_2$ is divided by 8, the remainder is 0. This leaves only 35, which when divided by 8 has a remainder of 3.

Therefore, when $x^2 - y^2$ is divided by 8, the remainder is $0 + 3 = 3$. Choice **(A)** is correct.

Step 4: Confirm Your Answer

Rereading the question stem, you can see that you answered the question posed. Double-check that you did not make any errors in substitution or calculation.

20. (B)

On a number line, point A represents 12, point B represents x, and point C represents 24, where $12 < x < 24$. Is the distance from point A to point B less than half the distance from point B to point C?

(1) $x < 17$

(2) $7x + 4 < 3x + 60$

Step 1: Analyze the Question Stem

Let's express algebraically that the distance from point A to point B is less than half the distance from point B to point C. Since $12 < x$, the distance from point A to point B is $x - 12$. Since $x < 24$, the distance from point B to point C is $24 - x$. So we have the inequality $x - 12 < \frac{1}{2}(24 - x)$.

Let's solve this inequality for x:

$$x - 12 < \frac{1}{2}(24 - x)$$

$$2(x - 12) < 24 - x$$

$$2x - 24 < 24 - x$$

$$3x < 48$$

$$x < 16$$

We can consider the question to be, "Is $x < 16$?" Now let's look at the statements.

Step 2: Evaluate the Statements Using 12TEN

Statement (1) says that $x < 17$. So we must have $12 < x < 17$. If $x = 15$, then Statement (1) is true, and we have $x < 16$, so the answer to the question is "yes." If $x = 16.8$, then Statement (1) is true, and we do not have $x < 16$, so the answer to the question is "no."

Since different answers to the question are possible, Statement (1) is insufficient. We can eliminate **(A)** and **(D)**.

Statement (2) says that $7x + 4 < 3x + 60$. Let's solve this inequality for the range of possible values of x.

$$7x + 4 < 3x + 60$$

$$4x < 56$$

$$x < 14$$

Statement (2) says that $x < 14$. So we must have $12 < x < 14$. Since 14 is less than 16, we must have $x < 16$. Statement (2) is sufficient to answer the question with a "yes." Choice **(B)** is correct.

21. (C)

The integers a, b, and c are prime numbers, and $a < b < c$. If $y = a^2bc$, then how many non-prime positive integer factors greater than 1 does y have?

- ○ 4
- ○ 7
- ○ 8
- ○ 10
- ○ 12

Step 1: Analyze the Question

We are told that a, b, and c are prime numbers, and $a < b < c$. We are also told that $y = a^2bc$.

Step 2: State the Task

We are asked how many non-prime factors of y are greater than 1.

Step 3: Approach Strategically

Picking Numbers is a great strategy for this question. Since a, b, and c are prime numbers, where $a < b < c$, let's choose $a = 2$, $b = 3$, and $c = 5$. Then $y = a^2bc = 2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 12 \times 5 = 60$. Thus, $y = 60$.

There are 12 positive integer factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. Of this list, 8 positive integer factors of 60 are greater than 1 and are not prime numbers: 4, 6, 10, 12, 15, 20, 30, and 60. Choice **(C)** is correct.

Step 4: Confirm Your Answer

You can confirm why this would be true regardless of the exact values of a, b, and c by using the given variables and listing out all the ways they can be multiplied together to produce the non-prime factors of a^2bc. You can list out the distinct products that can be formed from these four prime factors as follows: aa, ab, ac, bc, aab, abc, aac, $aabc$. Since these eight are the only possible combinations, **(C)** is the correct answer.

22. (C)

If $abc \neq 0$, is $a > 0$?

(1) $\frac{3a}{b} > 0$

(2) $\frac{b}{c^2} < 0$

Step 1: Analyze the Question Stem

In this Yes/No question, we are told that $abc \neq 0$, which means that none of the three variables is equal to zero. To determine whether $a > 0$, we must be able to find the values of the other two variables or be given an equation defining the relationship between the variables.

Step 2: Evaluate the Statements Using 12TEN

Statement (1): If $a = 1$ and $b = 1$, then Statement (1) is true and the answer to the question is "yes." If $a = -1$ and $b = -1$, then Statement (1) is true and the answer to the question is "no." Statement (1) is insufficient. Eliminate **(A)** and **(D)**.

Statement (2): Since this statement gives us no information about a, this statement is insufficient. We can eliminate **(B)**.

Let's consider the statements together. In Statement (2), we can deduce that the denominator c^2 is positive because the square of any nonzero number is positive. Since $\frac{b}{c^2}$ is negative and c^2 is positive, b must be negative, because a negative divided by a positive is negative.

In Statement (1), since $\frac{3a}{b}$ is positive and b is negative, $3a$ must be negative because a negative divided by a negative is positive. Since $3a$ is negative, a must be negative. In other words, $a < 0$. The statements taken together are sufficient to answer the question with a definitive "no." Choice **(C)** is correct.

23. (E)

Is the integer y a multiple of 4?

(1) $3y^2$ is a multiple of 18.

(2) $y = \frac{p}{q}$, where p is a multiple of 12 and q is a multiple of 3.

Step 1: Analyze the Question Stem

Here, we have a Yes/No question that tests our knowledge of multiples of 4. Recall that, when deciding whether a number is a multiple of 4, we can focus only on the tens and ones digits. In other words: if 12 is a multiple of 4, then 312 must also be a multiple of 4, and 937,528,364,712 must be a multiple of 4 as well.

Step 2: Evaluate the Statements Using 12TEN

Let's consider Statement (1). Since $3y^2$ is a multiple of 18, y^2 must be a multiple of 6. y^2 could be 36, in which case $y = 6$ and y is not a multiple of 4. Those numbers give us a "no." If we double y so that $y = 12$, y^2 is still a multiple of 6, so that's a permissible number. But now since 12 is a multiple of 4, we get a "yes." So we have a "yes" and a "no." Statement (1) is insufficient. Eliminate **(A)** and **(D)**.

Picking Numbers allows us to assess Statement (2) quickly. Choosing $p = 12$ and $q = 3$ yields $y = 4$, which is, of course, a multiple of 4. These numbers give us a "yes." Choosing $p = 36$ and $q = 6$, on the other hand, yields $y = 6$, which is not a multiple of 4. Here, the answer would be "no." "Sometimes yes, sometimes no" means Statement (2) is insufficient. Eliminate **(B)**.

Now let's combine the statements. We can Pick Numbers again, and this time we'll need to be sure to Pick Numbers that satisfy both Statements (1) and (2). We already chose $p = 36$, $q = 6$ in Statement (2). That makes $y = 6$, in which case $3y^2 = 108$, a multiple of 18. The conditions in both statements are satisfied, but y is not a multiple of 4. That would produce a "no" answer to the question stem. The set of numbers $p = 72$, $q = 6$, on the other hand, gives us $y = 12$ when plugged into Statement (2). This means that $3y^2 = 432$, which is a multiple of 18. Here, we've satisfied the conditions of both statements with a value for y that is a multiple of 4. This would give us a "yes" answer to the question stem.

"Sometimes yes, sometimes no" means that the information from the combined statements is insufficient. Choice **(E)** is correct.

24. (A)

What is the value of $7x + 3y$?

(1) $56x + 24y = 520$

(2) $8x + 5y = 79$ and $40x + 25y = 395$

Step 1: Analyze the Question Stem

To determine the value of $7x + 3y$, we would typically need two distinct linear equations that allow us to solve for the values of x and y. On the GMAT, however, we must always be aware that when the testmakers ask us for the value of an expression, they may give us a way to establish that value without knowing the individual values of the two variables.

Step 2: Evaluate the Statements Using 12TEN

Statement (1): We see that the coefficients 56 of x and 24 of y are each eight times the coefficients of the expression in the question stem. If we divide both sides of the equation $56x + 24y = 520$ by 8, on the left side of the resulting equation we will have $7x + 3y$, and on the right side of the resulting equation we will have a number. Thus, we can solve for the value of $7x + 3y$, so this statement is sufficient. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): The two equations given in Statement (2) are not distinct; if you multiply the first equation ($8x + 5y = 79$) by 5, you will have $40x + 25y = 395$. Thus, the two equations $8x + 5y = 79$ and $40x + 25y = 395$ in Statement (2) are equivalent, so they only tell us what the one equation $8x + 5y = 79$ tells us. The equation $8x + 5y = 79$ is a first-degree equation with two variables. Without another distinct linear equation, we cannot determine the values of x and y, nor is there any way in this statement to solve for the value of the expression, $7x + 3y$, in the question stem. Statement (2) is insufficient. There is no need to combine the statements. Choice **(A)** is correct.