

Chapter – 8

OPERATOR BASED QUESTIONS

Questions based on operators or definition based questions have been given in various entrance exams in the past.

The basic idea deals with giving a set of operators/definitions followed by questions which require the use of these operators / definitions.

Examples

Directions for questions 8.01 to 8.03: Consider the following operations:

$$p \alpha q = p^2 + q^2$$

$$p \beta q = p^2 - q^2$$

$$p \cup q = \frac{p^3 + q^3}{p^3 - q^3}$$

$$p \cap q = \frac{p^3 - q^3}{p^3 + q^3}$$

$$p/q = pq$$

8.01. Find the value of $(3 \alpha 2) \cup (3 \beta 2)$.

$$\begin{aligned} \text{Sol: } 3 \alpha 2 &= 3^2 + 2^2 = 13 \\ 3 \beta 2 &= 3^2 - 2^2 = 5 \\ (3 \alpha 2) \cup (3 \beta 2) &= \frac{13^3 + 5^3}{13^3 - 5^3} = \frac{2322}{2072} = \frac{1161}{1036} \end{aligned}$$

8.02. Find the value of $(5 \cup 4) / (5 \cap 4)$

$$\begin{aligned} \text{Sol: } 5 \cup 4 &= \frac{5^3 + 4^3}{5^3 - 4^3} = \frac{189}{61} \\ 5 \cap 4 &= \frac{5^3 - 4^3}{5^3 + 4^3} = \frac{61}{189} \\ \therefore (5 \cup 4) / (5 \cap 4) &= \frac{189}{61} \left(\frac{61}{189} \right) = 1 \end{aligned}$$

8.03. Simplify

$$\left(p^{\frac{3}{2}} \alpha q^{\frac{3}{2}} \right) \alpha \left(p^{\frac{3}{2}} \beta q^{\frac{3}{2}} \right) \bigg/ \left(p^{\frac{3}{2}} \alpha q^{\frac{3}{2}} \right) \beta \left(p^{\frac{3}{2}} \beta q^{\frac{3}{2}} \right)$$

$$\text{Sol: Let } A = p^{\frac{3}{2}} \alpha q^{\frac{3}{2}} \text{ and } B = p^{\frac{3}{2}} \beta q^{\frac{3}{2}}$$

$$\therefore A = p^3 + q^3 \text{ and } B = p^3 - q^3$$

$$\begin{aligned} \text{The required expression is } (A \alpha B) / (A \beta B) \\ = [(p^3 + q^3)^2 + (p^3 - q^3)^2] / [(p^3 + q^3)^2 - (p^3 - q^3)^2] \\ = 2(p^6 + q^6) / (4 p^3 q^3) = 8 p^3 q^3 (p^6 + q^6) \end{aligned}$$

Directions for questions 8.04 to 8.07: Consider the following definitions.

$$m(x, y) = | (x^3 + y^2) - (x^2 + y^3) |$$

$$n(x, y) = x^2 - y^3 + y^2 - x^3$$

$$o(x, y) = \text{LCM}(x, y)$$

$$p(x, y) = \text{HCF}(m(x, y), n(x, y))$$

8.04. Find the value of $m(6, 4)$.

$$\text{Sol: } m(6, 4) = | (6^3 + 4^2) - (6^2 + 4^3) | = 132$$

8.05. Find the value of $n(6, 4)$.

$$\text{Sol: } n(6, 4) = 6^2 - 4^3 + 4^2 - 6^3 = -228$$

8.06. Find the value of $o(m(6, 4), -n(6, 4))$

$$\begin{aligned} \text{Sol: } o(m(6, 4), -n(6, 4)) &= o(132, 228) \\ &= \text{LCM}[12(11), 12(19)] = 12(11)(19) = 2508 \end{aligned}$$

8.07. Find the value of $p(m(6, 4), -n(6, 4))$.

$$\begin{aligned} \text{Sol: The given expression} &= p(132, 228) \\ &= \text{HCF}(132, 228) = 12. \end{aligned}$$

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If $a \alpha b = (ab) - (a + b)$, then the value of $8 \alpha 5$ is
2. If $x - y = \frac{xy}{x+y}$, then the value of $8 - 6$ is .
(A) 2 (B) $\frac{24}{7}$ (C) 17 (D) $\frac{14}{7}$
3. If $a \uparrow b = \frac{a+b}{a-b}$, then the value of $4 \uparrow 3$ is .
4. If $f(x, y) = y$, then find the value of $f[3, f[4, f(5, 7)]]$ is .
(A) 3 (B) 4 (C) 5 (D) 7
5. If $a \% b = (a + b)^2$, then $7 \% 3$ is
6. If $x \leftrightarrow y = (xy)^2$ and $a \downarrow b = \left(\frac{a}{b}\right)^2$, then find the value of $\frac{(3 \leftrightarrow 4)}{(6 \downarrow 4)}$.
(A) 16 (B) 36 (C) 64 (D) 144
7. If $f(x, y) = \text{Max}(x, y)$, $g(x, y) = \text{LCM}(x, y)$ and $h(x, y, z) = \text{average}(x, y, z)$ then find the value of $h\{f(8, 12), g(24, 36), h(8, 21, 7)\}$.
8. Given: $a \oplus b = a^2 + b^2$ and $a \odot b = 2ab$. The value of $(5 \oplus 3) - (5 \odot 3)$ is .
(A) 6 (B) 8 (C) 10 (D) 4
9. Given: $X(x, y) = x^2 - y^2$ and $Y(a, b) = a + b$ then find $\frac{X(32, 20)}{Y(32, 20)}$
(A) 10 (B) 16 (C) 12 (D) 18
10. Given: $P(x, y) = 2x + 3y$, $Q(x, y) = 3x - 2y$, then find $P(P(2, 5), Q(1, 1))$.
(A) 42 (B) 41 (C) 36 (D) 51
11. Given: $a * b = \frac{a+b}{a-b}$, $P = (5 * 3) * (8 * 6)$, and $[x]$ represents the least integer $\geq x$. Find $[P]$.
(A) -3 (B) -5 (C) -4 (D) -6
12. Given: $a \$ b = (a + b)^{a-b}$, $5 \$ 3 =$
13. If $a \times b = a^2 - ab + b^2$, $a \div b = a + b$ and $a \alpha b = a^2 + ab + b^2$ and $a \beta b = a - b$ then find $\frac{(a \times b)(a \div b) - (a \alpha b)(a \beta b)}{b^2}$
(A) a (B) b (C) 2b (D) 2a
14. If $(x, y) = x^y$ and $g(x, y) = y^x$, then find the value of $\frac{f(3, 4) - g(5, 2)}{g(2, 7)}$.
(A) 2 (B) 7 (C) 7^2 (D) 1
15. If $a \% b$ is defined as the remainder(R) when a is divided by b then the value of $[1001 \% 25 + 1002 \% 25 + \dots + 1025 \% 25] \% 25$ is

Exercise – 8(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 to 3: These questions are based on the following information.

$$a \uparrow b = 3a \times b$$

$$a \rightarrow b = \frac{a}{2b}$$

$$a \downarrow b = 4a - b$$

$$a \leftarrow b = a + 2b$$

1. The expression $((24 \rightarrow 3) \leftarrow 2) \downarrow 4) \uparrow 12 =$

2. Which of the following is/are multiples of 11?

- (A) $((21 \downarrow 7) \rightarrow 9) \uparrow 6$ (B) $((21 \rightarrow 7) \uparrow 9) \downarrow 6$
(C) $((21 \leftarrow 7) \downarrow 9) \rightarrow 6$ (D) Both (B) and (C)

3. $\sqrt{14 \downarrow 7} - \sqrt[3]{9 \uparrow 27} =$

Directions for questions 4 to 6: These questions are based on the following information.

Sastry took a pack of cards and started drawing a card from it and replacing it before drawing another card. If he draws a heart numbered card he marks on a paper as number 1. If he picks up a honoured card, he marks it as number 2 and if he picks a spade numbered card, diamond numbered card and a club numbered card he marks them as 3, 4 and 5 respectively. He picks some cards and marks them in the following manner:

1324155234133245541342123345213345522113123

4. How many times did Sastry pick up a honoured card?

5. How many times did he pick a club numbered card immediately after having picked a spade numbered card?

6. How many times did Sastry pick two honoured cards in succession?

Directions for questions 7 and 8: These questions are based on the situation given below.

For real numbers x and y ,
 $L(x, y) = \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor$;
 $R(x, y) = \lfloor 2x \rfloor + \lfloor 2y \rfloor$;
 $T(x, y) = \lceil 2x \rceil + \lceil 2y \rceil$.

Where, $\lfloor x \rfloor$ denotes the greatest integer lesser or equal to x ; and $\lceil x \rceil$ denotes the least integer greater or equal to x .

7. Which of the following is never true for positive real numbers x and y ?

- (A) $L(x, y) \neq R(x, y)$ (B) $L(x, y) = R(x, y)$
(C) $L(x, y) < R(x, y)$ (D) $L(x, y) > R(x, y)$

8. For $x, y \in [0, 5]$, the number of pairs for which $R(x, y) = T(x, y)$, is _____.

- (A) 25 (B) 12 (C) 36 (D) 20

9. The natives of Bannanna island, use the 'BOSAMD' Rule, which gives the order in which the mathematical operations Brackets (B), Of (O), Subtraction (S), Addition (A), Multiplication (M) and Division (D), should be applied.

Find the value of $13 \times 5 + 35 \div 8 - (2 \times 5)$, using the 'BOSAMD' rule.

Directions for questions 10 to 13: These questions are based on the following definitions.

$a \% b$ is LCM of a and b

$a \sim b$ is HCF of a and b

$a \$ b$ is $(a + b)^3 - (a - b)^3$

$a \alpha b$ is $(a + b)^3 + (a - b)^3$

$a \Delta b$ is $(a^3 - b^3)$

10. The value of $(15\% 6) \Delta (20 \sim 8)$ is _____.

- (A) 26936 (B) 24784 (C) 25964 (D) 28419

11. Which of the following statement(s) is/are true for two distinct positive numbers a and b ?

- (A) $(a \sim b)$ is always greater than $(a \% b)$.
(B) $(a \sim b)$ is sometimes equal to $(a \% b)$.
(C) $(a \$ b)$ is always positive.
(D) Only (A) and (B)

12. Which of the following statements is/are true if $a = 6$ and $b = 36$?

- (A) $(a \% b) \div (a \sim b) + (a \Delta b) = 4350$.
(B) $\sqrt[3]{(a \% b) \times (a \sim b)} = a$.
(C) $(a \$ b) < (a \alpha b)$.
(D) More than one of the above.

13. Which of the following statements is true?

- (A) $(a \sim b) (a \% b)$ is divisible by a but not b .
(B) $(a \sim b) (a \% b)$ is divisible by b but not a .
(C) $(a \sim b) (a \% b)$ is divisible by neither a nor b .
(D) $(a \sim b) (a \% b)$ is divisible by both a and b .

Directions for questions 14 and 15: These questions are based on definitions given below.

$f(x, y) = e^{x+y}$; $g(x, y) = e^{x-y}$; $p(x, y) = \log_e(xy)$ and $q(x, y) = \log_e(x/y)$.

14. Find the value of $p(f(x, x), g(x, -x))$.

- (A) $f(2x, 2x)$ (B) $2f(x, x)$
(C) $2g(x, x)$ (D) $q(e^{6x}, e^{2x})$

15. The value of $f(p(x, y), q(x, y))$; for $x = 5$ and $y = 13$ is

Directions for questions 16 to 19: These questions are based on the following data.

$$a \$ b = (a^2 + b^2)^{a^2 - b^2}; a \Delta b = a^{a^2 - b^2} + b^{a^2 - b^2}$$

$$a \vee b = a^{a^2 - b^2}; a \wedge b = b^{a^2 - b^2}$$

16. Find the value of $(a \$ b) - (a \Delta b)$, for $a = 0$ and $b = 1$.

17. Which of the following is true for $a = 3$ and $b = 2$?

- (A) $a \$ b = 5^5 (a + b)$ (B) $a \Delta b = 55(a + b)$
(C) $a \Delta b = 5^5 (a \$ b)$ (D) None of these

18. Which of the following is/are always true for $a, b > 0$?

- (A) $a \vee b$ can never be equal to $a \wedge b$.
(B) $a \Delta b$ is greater than 0.
(C) $a \$ b$ is greater than 1.
(D) More than one of the above

19. Which of the following statements is/are false?

- (A) For $a = 2, b = 1$; $\frac{a \$ b}{a \Delta b} = \frac{25}{81}$.
(B) For $a = 1, b = 1$; $a \vee b + a \wedge b = 1$.
(C) $(a \Delta b) - (a \vee b) - (a \wedge b) = 0$
(D) Both (A) and (B)

Directions for questions 20 to 22: These questions are based on the following data.

For the equation $ax^2 + 2hxy + by^2 = 0$, the operators Δ and ∇ are defined as follows

$$\Delta = \frac{1}{ab} \sqrt{h^2 + ab};$$

$$\nabla = \frac{1}{ab} \sqrt{h^2 - ab}$$

$$\text{and } \nabla^2 = \nabla \times \nabla; \Delta^2 = \Delta \times \Delta.$$

20. Which of the following statements is true for the equation $3x^2 + 6xy + 4y^2 = 0$?

- (A) ∇^2 is positive. (B) Δ^2 is negative.
(C) $\Delta^2 > \nabla^2$ (D) $\Delta^2 < \nabla^2$

21. If a new equation is formed by interchanging the coefficients of x^2 and y^2 in the equation $4x^2 + 8xy - 7y^2 = 0$, then which of the following is/are true for the new equation?

- (A) $\Delta^2 = \nabla^2$ (B) $\Delta^2 > \nabla^2$
(C) $\Delta^2 < \nabla^2$ (D) Both (B) and (C)

22. If $D = \frac{2}{b} \sqrt{h^2 - ab}$, then which of the following is true?

- (A) $D = 2a\nabla$ (B) $\nabla = 2aD$
(C) $2\nabla = aD$ (D) $D = a\nabla$

Directions for questions 23 to 25: Two operators are defined as follows

| \otimes | a | b | c | d |
|-----------|---|---|---|---|
| a | a | b | c | d |
| b | b | d | a | c |
| c | c | a | d | b |
| d | d | c | b | a |

| \oplus | A | b | c | d |
|----------|---|---|---|---|
| a | A | b | c | d |
| b | B | c | d | a |
| c | C | d | a | b |
| d | D | a | b | c |

$$\text{and } a^2 = a \otimes a, a^3 = a^2 \otimes a, 2a = a \oplus a, 3a = 2a \oplus a$$

23. If $c^n = a$, then the least value of n is

- (A) 5 (B) 4 (C) 6 (D) 3

24. $(a^5 \otimes b^4) \oplus (4c \otimes 9d) =$

- (A) $b \oplus d$ (B) $b \otimes c$ (C) $c \otimes c$ (D) $c \oplus c$

25. $[(b \otimes c) \otimes a] \oplus [(a \oplus b) \oplus d] =$

- (A) $c \otimes c$ (B) $b \oplus b$ (C) $b \otimes b$ (D) $a \oplus a$

Exercise – 8(b)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 to 3: These questions are based on the following data.

$$a \oplus b \text{ is HCF of } a^3 \text{ and } b^3$$

$$a \ominus b \text{ is LCM of } a^3 \text{ and } b^3$$

$$a \otimes b \text{ is } a^2 + b^2 - a^2b^2$$

$$a \ominus b \text{ is } a^2 + b^2 + a^2b^2$$

1. Find the value of $((((6 \oplus 8)^{\frac{1}{3}} \ominus 4)^{\frac{1}{3}} \otimes 1) \ominus 1)$

2. If $a = 4$ and $b = 6$, which of the following is true?

- (A) $a \ominus b < 200 (a \oplus b)$
(B) $(a + 1) \otimes (b - 2) = 19$
(C) $(a - 1) \ominus (b + 2) = 87$
(D) None of these

3. Which of the following never holds true?

- (A) $a \oplus b \leq a - b$
(B) If $p^3 = q^3$ and $p \div q = 99$, then $q = \pm 3$
(C) If $p^3 = q^3$ and $p \otimes q = -8$, then $q = \pm 2$
(D) None of these

Directions for questions 4 to 6: These questions are based on the following data.

$$p \uparrow q = \frac{2}{3}pq$$

$$p \downarrow q = 3\frac{p}{q}$$

$$p \rightarrow q = 3p + 4q$$

$$p \leftarrow q = 4p - 5q$$

4. Find the value of $((9 \uparrow 12) \downarrow 2) \rightarrow 3) \leftarrow 1$

5. Which of the following is an integer?

- (A) $((2 \uparrow 3) \downarrow 5) \rightarrow 6 \leftarrow 7$
 (B) $((((2 \rightarrow 3) \leftarrow 5) \downarrow 6) \uparrow 7$
 (C) $((((2 \uparrow 3) \rightarrow 5) \downarrow 6) \leftarrow 7$
 (D) $((((2 \uparrow 3) \rightarrow 5) \leftarrow 7) \downarrow 6$

6. Which of the following is a perfect square?

- (A) $((6 \uparrow 1) \rightarrow 4) \downarrow 1) \leftarrow 7)$
 (B) $((6 \uparrow 4) \rightarrow 1) \downarrow 1) \uparrow 7)$
 (C) $((6 \uparrow 7) \rightarrow 1) \downarrow 1) \leftarrow 4)$
 (D) None of these

Directions for questions 7 to 9: These questions are based on the following data.

$$a(x, y) = \frac{p^x + p^y}{2}; b(x, y) = \frac{p^x - p^y}{2}; c(x, y) = \log_p \frac{x+y}{x-y}$$

7. Find the value of $c[a(x, x), b(x, x)]$.

- (A) 1 (B) 0 (C) $2x$ (D) $-2x$

8. Find the value of $\frac{a[c(x, y), c(x, -y)]}{b[c(x, y), c(x, -y)]}$

- (A) $\frac{x^2 + y^2}{2xy}$ (B) $\frac{x^2 + y^2}{xy}$
 (C) $\frac{2xy}{x^2 + y^2}$ (D) $\frac{xy}{x^2 + y^2}$

9. Find the value of $c(c(a(8, 6), b(8, 6)), 1)$.

- (A) \log_3 (B) \log_4 (C) \log_6 (D) \log_8

Directions for questions 10 to 12: These questions are based on the following data.

For real numbers p and q ,

$$a(p, q) = p^2 + q^3; b(p, q) = p^3 + q^2$$

$$c(p, q) = p^3 - q^2; d(p, q) = p^2 - q^3$$

10. Find the value of $\frac{a(2, 1) + c(2, 1)}{b(2, 1) - d(2, 1)}$

11. Which of the following is true?

- (A) If $a(p, q) - d(p, q) = 128$, then $q = 2$
 (B) If $\frac{b(p, p^2)}{c(p, p^2)} = 4$, then $p = \frac{2}{5}$

(C) If $\frac{a(p^2, p)}{d(p^2, p)} = 3$, then $p = 3$

(D) If $\frac{a(p, q)}{d(p, q)} = \frac{b(p, q)}{c(p, q)}$, then $pq = 0$ or 1

12. If $c(p, p) = 448$, which of the following is a possible value of p ?

- (A) 5 (B) 6 (C) 7 (D) 8

13. To simplify an algebraic expression, the operations are performed in the order BODMAS (B stands for brackets, O for of, D for division, M for multiplication, A for addition and S for subtraction.)

Find the value of $(9 \times 5) + 90 \div 45 \times 5 - 6$.

Directions for questions 14 to 16: These questions are based on the following data.

Let $[x]$ be the greatest integer less than or equal to x and $\{x\}$ be the least integer greater than x .

14. Let $[A + B + C + D] = [A] + [B] + [C] + [D] + k$. Which of the following is not a possible value of k ?

- (A) 0 (B) 1 (C) 2 (D) 4

15. Let $(A + B + C) = (A) + (B) + (C) + i$. Which of the following is a possible value of i ?

- (A) 1 (B) 2 (C) 3 (D) -1

16. If $[a + b + c + d + e] + m = (a + b + c + d + e)$, which of the following is a possible value of m ?

- (A) 2 (B) 3 (C) 0 (D) 1

17. If $\$(x, y) = \text{L.C.M.}(x, y)$, $\Delta(x, y) = \text{H.C.F.}(x, y)$, $\nabla(x, y) = \text{A.M.}(x, y)$ and $\pi(x, y) = \text{remainder when } x \text{ is divided by } y$, then $\pi[\$(\Delta(\nabla(\$(240, 180), 360), 420), 600), 2] =$

18. If GOS (a, b, c) is the geometric mean of a, b and c , GOSS (a, b, c) is the geometric mean of squares of a, b and c , GOC (a, b, c) is the geometric mean of cubes of a, b , and c . Which of the following statements is not true for $a, b, c \geq 1$?

- (A) $\text{GOC}(a, b, c) \geq \text{GOSS}(a, b, c)$
 (B) $\text{GOSS}(a, b, c) \geq \text{GOS}(a, b, c)$
 (C) $\text{GOC}(a, b, c) \geq \text{GOS}(a, b, c)$
 (D) $\text{GOSS}(a, b, c) \geq \text{GOC}(a, b, c)$

19. If $avb = abe^a e^b$; $a^b = e^a e^b$, which of the following statements is true?

- (A) avb can never be negative.
 (B) a^b can never be less than 1.
 (C) $avb = av\wedge$; for $a = b$.
 (D) a^b is always positive.

Directions for questions 20 to 23: These questions are based on the following data.

P, Q and R are positive and distinct real numbers in the descending order.

$$\begin{aligned} a(P, Q, R) &= \min[(\max(P, Q), \max(Q, R), \max(P, R))] \\ b(P, Q, R) &= \max[(\min(P, Q), \min(Q, R), \min(P, R))] \\ c(P, Q, R) &= \max[(\min(P, Q), \min(Q, R), \min(P, R))] \\ d(P, Q, R) &= \min[(\max(P, Q), \max(Q, R), \max(P, R))] \\ e(P, Q, R) &= \min[(\max(P, Q), \max(Q, R), \min(P, R))] \\ f(P, Q, R) &= \min[(\min(P, Q), \min(Q, R), \min(P, R))] \end{aligned}$$

20. Which of the following must exceed 1?

- (A) $\frac{a(P, Q, R) + c(P, Q, R)}{d(P, Q, R) + e(P, Q, R)}$
 (B) $\frac{a(P, Q, R) - e(P, Q, R)}{b(P, Q, R) - f(P, Q, R)}$
 (C) $\frac{a(P, Q, R) + c(P, Q, R)}{e(P, Q, R) + f(P, Q, R)}$
 (D) More than one of the previous choices.

21. Which of the following expressions is undefined?

- (A) $\frac{a(P, Q, R) - b(P, Q, R)}{c(P, Q, R) - d(P, Q, R)}$
 (B) $\frac{a(P, Q, R) - c(P, Q, R)}{b(P, Q, R) - c(P, Q, R)}$
 (C) $\frac{a(P, Q, R) + b(P, Q, R)}{c(P, Q, R) - f(P, Q, R)}$
 (D) $\frac{a(P, Q, R) - b(P, Q, R)}{d(P, Q, R) - e(P, Q, R)}$

22. Which of the following is/are equal to 1?

- (A) $\frac{a(P, Q, R)}{c(P, Q, R)}$
 (B) $\frac{d(P, Q, R)}{e(P, Q, R)}$
 (C) $\frac{e(P, Q, R)}{f(P, Q, R)}$
 (D) All the previous choices

23. Which of the following is negative?

- (A) $\frac{b(P, Q, R) - a(P, Q, R)}{c(P, Q, R) - d(P, Q, R)}$
 (B) $\frac{c(P, Q, R) - b(P, Q, R)}{e(P, Q, R) - c(P, Q, R)}$
 (C) $\frac{c(P, Q, R) - b(P, Q, R)}{a(P, Q, R) - f(P, Q, R)}$
 (D) $\frac{a(P, Q, R) - e(P, Q, R)}{c(P, Q, R) - f(P, Q, R)}$

Directions for questions 24 to 26: These questions are based on the following data.

$$\begin{aligned} a(x, y) &= 3^{x+y} \\ b(x, y) &= 3^{x-y} \\ c(x, y) &= \log_3 xy \end{aligned}$$

$$d(x, y) = \log_3 \left(\frac{x}{y} \right)$$

24. Find the value of $d(a(-x, x), b(-x, -x))$.

25. Find the value of $c(a(2, 1), b(3, 2))$.

26. Find the value of $\frac{c[a(3, 4), b(5, 2)]}{d[a(3, 4), b(5, 2)]}$.

Directions for questions 27 and 28: These questions are based on the following data.

X and Y are two numbers. # and \oplus are defined as
 $X \# Y = \frac{XY}{4}$ and $X \oplus Y = X + Y + XY$

27. If $8(X \# Y) = X \oplus Y$ and both X and Y are integers, how many values can (X, Y) assume?

28. Find the value of $((1 \oplus 2) \# 4) \oplus 3$

Directions for questions 29 and 30: These questions are based on the following data.

For the expression $px^2 + qy^2 + rx + sy + t$, the quantities \oplus and \ominus are defined as follows.

$$\oplus = \sqrt{r^2 - pt}$$

$$\ominus = \sqrt{s^2 - qt}$$

29. Which of the following is/are true for the expression $x^2 + y^2 + 2x + 3y + 4$?

- (A) $\oplus^2 > \ominus^2$
 (B) $\oplus \ominus > 0$
 (C) $\oplus < \ominus$
 (D) Both (A) and (C)

30. Which of the following is/are true for the expression $ax^2 + by^2 + ax + by + c$ where $a, b > 0$?

- (A) If $\oplus = \ominus$, $c > 0$
 (B) If $\oplus^2 > 0$, $a > c$
 (C) If $\ominus^2 < 0$, $b > c$
 (D) Both (A) and (B)

Key

Concept Review Questions

- | | | | | |
|-------|--------|-------|--------|-------|
| 1. 27 | 4. D | 7. 32 | 10. B | 13. C |
| 2. B | 5. 100 | 8. D | 11. A | 14. D |
| 3. 7 | 6. C | 9. C | 12. 64 | 15. 0 |

Exercise – 8(a)

- | | | | | |
|---------|---------|--------|-------|-------|
| 1. 1008 | 6. 1 | 11. C | 16. 0 | 21. C |
| 2. A | 7. D | 12. B | 17. B | 22. A |
| 3. -2 | 8. C | 13. D | 18. B | 23. B |
| 4. 9 | 9. -260 | 14. D | 19. D | 24. C |
| 5. 0 | 10. A | 15. 25 | 20. C | 25. D |

Exercise – 8(b)

- | | | | | | |
|---------|-------|--------|-------|-------|---------|
| 1. 1 | 6. D | 11. D | 16. D | 21. D | 26. 2.5 |
| 2. D | 7. B | 12. D | 17. 0 | 22. D | 27. 2 |
| 3. D | 8. A | 13. 49 | 18. D | 23. C | 28. 23 |
| 4. 1339 | 9. A | 14. D | 19. D | 24. 0 | 29. C |
| 5. C | 10. 2 | 15. D | 20. D | 25. 4 | 30. B |