CHAPTER – 4

Comparison of Fractions

COMPARISON OF FRACTIONS:

Comparison of fractions will be required in a number of problems in Data Interpretation and Quantitative Ability. Let us study some of the common methods of identifying out the largest or smallest of a given set of fractions.

Type 1:

When two or more fractions have the same numerators and different denominators, the fraction with the largest denominator is the smallest.

4.01. Which of the following fractions is the smallest?

$\frac{3}{5}, \frac{3}{7}, \frac{3}{13}, \frac{3}{8}$

Sol. 13 is the largest denominator, hence 3/13 is the smallest fraction. 5 is the smallest denominator, hence 3/5 is the largest fraction.

Type 2:

When the numerators are different and the denominators are same, the fraction with the largest numerator is the largest.

4.02. Which of the following fractions is the smallest?

$\frac{7}{5}, \frac{9}{5}, \frac{4}{5}, \frac{11}{5}$

Sol. As 4 is the smallest numerator, the fraction 4/5 is the smallest.
As 11 is the largest numerator, the fraction 11/5 is the largest.

Type 3:

The fraction with the largest numerator and the smallest denominator is the largest.

4.03. Which of the following fractions is the largest?

$\frac{19}{16}, \frac{24}{11}, \frac{17}{13}, \frac{21}{14}, \frac{23}{15}$

Sol. As 24 is the largest numerator and 11 is the smallest denominator, $\frac{24}{11}$ is the largest fraction.

Type 4:

When the numerators of two fractions are the unequal, we try and equate them by suitably cancelling factors or by suitably multiplying the numerators. Thereafter we compare the denominators as in Type 1.

4.04. Which of the following fractions is the largest?

$\frac{64}{328}, \frac{28}{152}, \frac{36}{176}, \frac{49}{196}$

Sol. $\frac{64}{328} = \frac{32}{164} = \frac{16}{82} = \frac{8}{41} \sim \frac{1}{5}$

$\frac{28}{152} = \frac{14}{76} = \frac{7}{38} \sim \frac{1}{5.5}$

$\frac{36}{176} = \frac{18}{88} = \frac{9}{44} \sim \frac{1}{5}$ $\frac{49}{196} = \frac{7}{28} = \frac{1}{4}$

As all the numerators are 1 and the least denominator is 4, the fraction $\frac{49}{196}$ is the largest.

4.05. Which of the following fractions is the largest?

$\frac{71}{181}, \frac{214}{519}, \frac{429}{1141}$

Sol. $\frac{71}{181} = \frac{71 \times 6}{181 \times 6} = \frac{426}{1086}$

$\frac{214}{519} = \frac{214 \times 2}{519 \times 2} = \frac{428}{1038}$

The numerators are now all almost equal (426, 428 and 429). The smallest denominator is 1038.
Hence the largest fraction must be

$\frac{428}{1038}$ i.e. $\frac{214}{519}$

Type 5:

A: For a fraction less than 1.

If the difference between the numerator and the denominator is same, then the fraction with the larger values of numerator and denominator will be the largest.

4.06. Which of the following fractions is the largest.

$\frac{31}{37}, \frac{23}{29}, \frac{17}{23}, \frac{35}{41}, \frac{13}{19}$

Sol. The difference between the numerator and the denominator of each fraction is 6. ∴ The fraction with the larger numerals i.e., 35/41 is the greatest and the fraction with smaller numerals i.e., 13/19 is the smallest.

B: For a fraction greater than 1.

If the difference between the numerator and denominator is same, then the fraction with the smaller values will be the largest.

4.07. Which of the following fraction is largest?

$\frac{31}{27}, \frac{43}{39}, \frac{57}{53}, \frac{27}{23}, \frac{29}{25}$

Sol. As the difference between the numerator and the denominator is same, the fraction with the smaller values i.e., $\frac{27}{23}$, is the largest.

4.08. Which of the following fractions is the largest?

$\frac{15}{17}, \frac{23}{29}, \frac{31}{34}, \frac{11}{15}$

Sol. Comparing fractions

$\frac{15}{17}$ and $\frac{23}{29}$

The numerator of the fraction has increased from 15 to 23 i.e., $\frac{8}{15}$ i.e. a little more than 50%.

The denominator of the fraction has increased from 17 to 29 i.e. 12/17 i.e. well over 50%. As the percentage increase in the numerator is less than the percentage increase in the denominator, the fraction $\frac{15}{17} > \frac{23}{29}$. Now compare $\frac{15}{17}$ with $\frac{31}{34}$.

As the change in the numerator is more than double (15 to 31), and the change in the denominator is exactly double, the fraction $\frac{15}{17} < \frac{31}{34}$.

Now compare $\frac{11}{15}$ and $\frac{31}{34}$.

The numerator has almost tripled from 11 to 31 whereas the denominator has just over doubled from 15 to 34. Since the increase in numerator is greater than the increase in the denominator, $\frac{31}{34} > \frac{11}{15}$.

So, $\frac{31}{34}$ is the largest fraction.

We can also compare fractions as follows.

For example, to compare 5/13 and 9/20 make the numerator 1 for all the fractions by approximately dividing the denominator with the respective numerator (upto first decimal place).

$\therefore \frac{5}{13} = \frac{1}{2.6}$ and $\frac{1}{20} \cong \frac{1}{2.2}$

Now, clearly $\frac{1}{2.6} < \frac{1}{2.2}$ (from rule (ii) above)

$\Rightarrow \frac{5}{13} < \frac{9}{20}$

Another method of comparing fractions is by comparing the percentage changes in denominators and numerators. The important points to remember are that when two fractions are compared, if the percentage increase in the numerator is more than the percentage increase in the denominator (where the first fraction is taken as reference), then the second fraction is greater than the first fraction. Conversely, if the percentage increase in the denominator is greater than that in the numerator, then the second fraction is smaller than the first.

Exercise – 4(a)

Directions for questions 1 to 15: Find the greatest of the following fractions.

1. $\frac{21}{38}, \frac{31}{48}, \frac{41}{58}, \frac{61}{78}$

2. $\frac{12}{31}, \frac{17}{48}, \frac{21}{51}, \frac{34}{87}$

3. $\frac{79}{29}, \frac{91}{31}, \frac{129}{34}, \frac{206}{57}$

4. $\frac{7}{29}, \frac{11}{59}, \frac{21}{97}, \frac{18}{89}, \frac{28}{125}$

5. $\frac{19}{61}, \frac{27}{73}, \frac{31}{98}, \frac{11}{49}, \frac{61}{193}$

6. $\frac{123}{172}, \frac{81}{97}, \frac{79}{101}, \frac{41}{63}, \frac{53}{97}$

7. $\frac{13}{123}, \frac{18}{147}, \frac{7}{93}, \frac{11}{81}, \frac{16}{157}$

8. $\frac{39}{161}, \frac{21}{93}, \frac{17}{77}, \frac{28}{113}, \frac{32}{147}$

9. $\frac{163}{183}, \frac{103}{123}, \frac{151}{199}, \frac{127}{181}$

10. $\frac{7}{8}, \frac{13}{15}, \frac{104}{119}, \frac{122}{139}$

11. $\frac{19}{99}, \frac{228}{1201}, \frac{342}{1801}, \frac{304}{1599}$.

12. $\frac{26}{53}, \frac{21}{43}, \frac{89}{102}, \frac{99}{202}$

13. $\frac{65}{126}, \frac{78}{153}, \frac{16}{31}$

14. $\frac{79}{108}, \frac{43}{69}, \frac{17}{30}, \frac{102}{129}, \frac{221}{435}$

15. $\frac{8}{35}, \frac{17}{71}, \frac{43}{175}, \frac{91}{400}, \frac{113}{450}$

Directions for questions 16 to 30: Find the smallest of the following fractions.

16. $\frac{13}{86}, \frac{15}{71}, \frac{18}{67}, \frac{12}{87}$

17. $\frac{47}{21}, \frac{68}{43}, \frac{29}{11}, \frac{47}{27}, \frac{61}{44}$

18. $\frac{17}{70}, \frac{8}{51}, \frac{19}{96}, \frac{21}{141}, \frac{33}{193}$

19. $\frac{13}{14}, \frac{21}{23}, \frac{61}{93}, \frac{37}{43}, \frac{29}{49}$

20. $\frac{23}{57}, \frac{37}{83}, \frac{18}{41}, \frac{29}{71}, \frac{33}{92}$.

21. $\frac{15}{47}, \frac{7}{29}, \frac{11}{37}, \frac{18}{65}, \frac{22}{89}$

22. $\frac{47}{161}, \frac{68}{243}, \frac{51}{216}, \frac{91}{478}, \frac{78}{352}$

23. $\frac{18}{35}, \frac{31}{63}, \frac{12}{23}, \frac{16}{33}, \frac{25}{49}$

24. $\frac{47}{23}, \frac{613}{300}, \frac{1277}{625}$

25. $\frac{465}{226}, \frac{49}{22}, \frac{83}{40}, \frac{97}{47}, \frac{834}{398}$

26. $\frac{7}{9}, \frac{29}{39}, \frac{24}{29}, \frac{16}{19}$

27. $\frac{43}{90}, \frac{59}{100}, \frac{67}{120}, \frac{71}{140}, \frac{81}{160}$

28. $\frac{335}{1089}, \frac{493}{1584}, \frac{784}{3107}, \frac{524}{1725}$

29. $\frac{12}{29}, \frac{113}{225}, \frac{408}{825}, \frac{516}{1089}, \frac{47}{101}$

30. $\frac{29}{18}, \frac{23}{14}, \frac{17}{12}, \frac{9}{5}, \frac{11}{6}$

Directions for questions 31 to 40: Arrange the following in ascending order

31. $a = \frac{16}{11}, b = \frac{26}{21}, c = \frac{32}{27}, d = \frac{39}{34}$

32. $a = \frac{76}{129}, b = \frac{151}{257}, c = \frac{375}{640}, d = \frac{155}{261}$

33. $a = \frac{19}{1730}, b = \frac{29}{2639}, c = \frac{31}{2790}$

34. $\frac{21}{31}, \frac{17}{27}, \frac{37}{27}, \frac{27}{17}, \frac{31}{41}$

35. $\frac{67}{155}, \frac{39}{141}, \frac{71}{152}, \frac{83}{206}, \frac{47}{142}$

36. $\frac{17}{67}, \frac{25}{71}, \frac{31}{96}, \frac{23}{70}$

37. $\frac{28}{37}, \frac{15}{22}, \frac{17}{21}, \frac{13}{17}$

38. $\frac{21}{53}, \frac{33}{69}, \frac{18}{41}, \frac{53}{126}, \frac{17}{48}$

39. $\frac{12}{17}, \frac{17}{21}, \frac{29}{48}, \frac{31}{47}, \frac{41}{63}$

40. $\frac{48}{131}, \frac{51}{163}, \frac{24}{267}, \frac{25}{86}, \frac{39}{147}$

Directions for questions 41 to 45: Arrange the given fractions in descending order.

41. $a = \frac{22}{34}, b = \frac{2}{3}, c = \frac{54}{79}, d = \frac{78}{109}$

42. $a = \frac{16}{43}, b = \frac{27}{83}, c = \frac{42}{126}, d = \frac{23}{63}$

43. $a = \frac{19}{56}, b = \frac{17}{51}, c = \frac{21}{69}, d = \frac{24}{79}$

44. $a = \frac{23}{98}, b = \frac{28}{103}, c = \frac{45}{181}, d = \frac{51}{204}$

45. $a = \frac{25}{161}, b = \frac{93}{555}, c = \frac{44}{264}, d = \frac{11}{65}$

Exercise – 4(b)

Directions for questions 1 to 15: Find the largest of the following fractions.

1. $\frac{43}{18}, \frac{94}{45}, \frac{188}{91}, \frac{435}{289}, \frac{729}{361}$

2. $\frac{18}{75}, \frac{21}{85}, \frac{23}{95}, \frac{43}{185}$

3. $\frac{37}{14}, \frac{85}{41}, \frac{121}{49}, \frac{385}{169}, \frac{202}{81}$

4. $\frac{7}{12}, \frac{13}{15}, \frac{23}{25}, \frac{33}{35}, \frac{43}{45}$

5. $\frac{4}{9}, \frac{13}{22}, \frac{43}{89}, \frac{37}{78}, \frac{23}{47}$

6. $\frac{98}{41}, \frac{121}{53}, \frac{239}{105}, \frac{957}{356}$

7. $\frac{3}{4}, \frac{5}{7}, \frac{8}{11}, \frac{7}{8}$

8. $\frac{65}{128}, \frac{137}{360}, \frac{274}{445}, \frac{835}{1302}$

9. $\frac{17}{25}, \frac{21}{40}, \frac{11}{20}, \frac{23}{35}, \frac{30}{47}$

10. $\frac{27}{56}, \frac{37}{76}, \frac{47}{96}, \frac{57}{116}$

11. $\frac{24}{63}, \frac{36}{93}, \frac{42}{103}, \frac{45}{113}$

12. $\frac{23}{32}, \frac{34}{43}, \frac{45}{54}, \frac{56}{65}$

13. $\frac{67}{89}, \frac{24}{34}, \frac{74}{87}, \frac{63}{71}$

14. $\frac{28}{59}, \frac{38}{83}, \frac{35}{74}, \frac{43}{93}$

15. $\frac{23}{45}, \frac{4}{9}, \frac{131}{269}, \frac{42}{87}, \frac{11}{24}$

Directions for questions 16 to 30: Find the smallest of the following fractions:

16. $\frac{349}{72}, \frac{563}{120}, \frac{721}{175}, \frac{943}{204}$

17. $\frac{5}{12}, \frac{3}{7}, \frac{7}{16}, \frac{9}{20}, \frac{11}{23}$

18. $\frac{7}{15}, \frac{23}{35}, \frac{41}{25}, \frac{19}{20}, \frac{27}{45}$

19. $\frac{63}{25}, \frac{71}{31}, \frac{79}{34}, \frac{75}{39}$

20. $\frac{13}{20}, \frac{19}{39}, \frac{17}{36}, \frac{51}{89}$

21. $\frac{23}{16}, \frac{43}{26}, \frac{33}{22}$

22. $\frac{94}{108}, \frac{72}{78}, \frac{60}{68}$

23. $\frac{16}{21}, \frac{34}{41}, \frac{38}{51}, \frac{25}{31}$

24. $\frac{14}{63}, \frac{17}{72}, \frac{19}{94}, \frac{41}{201}$

25. $\frac{43}{167}, \frac{64}{198}, \frac{85}{345}, \frac{93}{467}$

26. $\frac{12}{15}, \frac{15}{18}, \frac{18}{22}, \frac{22}{27}$

27. $\frac{21}{45}, \frac{32}{65}, \frac{39}{79}, \frac{27}{54}$

28. $\frac{16}{39}, \frac{23}{49}, \frac{27}{59}, \frac{33}{69}$

29. $\frac{23}{45}, \frac{195}{394}, \frac{271}{539}, \frac{71}{151}$

30. $\frac{11}{43}, \frac{21}{73}, \frac{31}{123}, \frac{41}{163}$

Directions for questions 31 to 40: Arrange the given fractions in ascending order.

31. $\frac{35}{68}, \frac{47}{86}, \frac{29}{41}, \frac{19}{40}, \frac{41}{83}$

32. $\frac{71}{129}, \frac{58}{119}, \frac{32}{79}, \frac{87}{139}$

33. $\frac{121}{396}, \frac{183}{403}, \frac{97}{234}, \frac{107}{281}, \frac{141}{421}$

34. $\frac{18}{157}, \frac{27}{179}, \frac{41}{203}, \frac{32}{189}, \frac{69}{335}$

35. $\frac{102}{37}, \frac{228}{83}, \frac{11}{4}, \frac{1376}{499}$

36. $\frac{43}{51}, \frac{86}{101}, \frac{6}{7}$

37. $\frac{157}{120}, \frac{17}{13}, \frac{144}{110}$ and $\frac{209}{160}$

38. $\frac{176}{225}, \frac{98}{125}, \frac{39}{50}, \frac{453}{584}$

39. $\frac{269}{1336}, \frac{333}{1669}, \frac{397}{2002}, \frac{64}{333}$

40. $\frac{204}{101}, \frac{20406}{10201}, \frac{2607}{1300}, \frac{439743}{219700}$

Directions for questions 41 to 45: Arrange the given fractions in descending order.

41. $a = \frac{27}{18}, b = \frac{39}{25}, c = \frac{50}{29}, d = \frac{45}{31}$

42. $a = \frac{16}{51}, b = \frac{21}{72}, c = \frac{31}{94}, d = \frac{49}{183}$

43. $a = \frac{37}{12}, b = \frac{38}{13}, c = \frac{40}{14}, d = \frac{45}{17}$

44. $a = \frac{23}{82}, b = \frac{35}{96}, c = \frac{39}{108}, d = \frac{41}{138}$

45. $\frac{31}{139}, \frac{51}{193}, \frac{43}{169}, \frac{123}{647}$

Key

Exercise – 4(a)

1. $\frac{61}{78}$
2. $\frac{21}{51}$
3. $\frac{129}{34}$
4. $\frac{7}{29}$
5. $\frac{27}{73}$
6. $\frac{81}{97}$
7. $\frac{11}{81}$
8. $\frac{28}{113}$
9. $\frac{163}{183}$
10. $\frac{122}{139}$
11. $\frac{19}{99}$
12. $\frac{89}{102}$
13. $\frac{16}{31}$
14. $\frac{102}{129}$
15. $\frac{113}{450}$
16. $\frac{12}{87}$
17. $\frac{61}{44}$
18. $\frac{21}{141}$
19. $\frac{29}{49}$
20. $\frac{33}{92}$
21. $\frac{7}{29}$
22. $\frac{91}{478}$
23. $\frac{16}{33}$
24. $\frac{16}{33}$
25. $\frac{465}{226}$
26. 29/39
27. $\frac{43}{90}$
28. $\frac{784}{3107}$
29. $\frac{12}{29}$
30. $\frac{17}{12}$
31. $\frac{5}{11} > \frac{5}{21} > \frac{5}{27} > \frac{5}{34}$
32. $\frac{155}{261}$
33. $\frac{19}{1730} < \frac{29}{2639} < \frac{31}{2790}$
34. $\frac{17}{27}, \frac{21}{31}, \frac{31}{41}, \frac{37}{27}, \frac{27}{17}$
35. $\frac{39}{141}, \frac{47}{142}, \frac{83}{206}, \frac{67}{155}, \frac{71}{152}$
36. $\frac{17}{67}, \frac{31}{96}, \frac{23}{70}, \frac{25}{71}$
37. $\frac{15}{22}, \frac{28}{37}, \frac{13}{17}, \frac{17}{21}$
38. $\frac{17}{48}, \frac{21}{53}, \frac{53}{126}, \frac{18}{41}, \frac{33}{69}$
39. $\frac{29}{48}, \frac{41}{63}, \frac{31}{47}, \frac{12}{17}$ and $\frac{17}{21}$
40. $\frac{20406}{10201} < \frac{439743}{219700} < \frac{2607}{1300} < \frac{204}{101}$
41. $\frac{78}{109} > \frac{54}{79} > \frac{2}{3} > \frac{22}{34}$
42. $\frac{16}{43} > \frac{23}{63}, > \frac{42}{126} > \frac{27}{83}$
43. $\frac{19}{56} > \frac{17}{51} > \frac{21}{69} > \frac{24}{79}$
44. $\frac{28}{103} > \frac{51}{204} > \frac{45}{181} > \frac{23}{98}$
45. $\frac{11}{65} > \frac{93}{555} > \frac{44}{264} > \frac{25}{161}$

Exercise – 4(b)

1. $\frac{43}{18}$
2. $\frac{21}{85}$
3. $\frac{37}{14}$
4. $\frac{43}{45}$
5. $\frac{13}{22}$
6. $\frac{957}{356}$
7. $\frac{7}{8}$
8. $\frac{835}{1302}$
9. $\frac{17}{25}$
10. $\frac{57}{116}$
11. $\frac{42}{103}$
12. $\frac{56}{65}$
13. $\frac{63}{71}$
14. $\frac{28}{59}$
15. $\frac{23}{45}$
16. $\frac{721}{175}$
17. $\frac{5}{12}$
18. $\frac{7}{15}$
19. $\frac{75}{39}$
20. $\frac{17}{36}$
21. $\frac{23}{16}$
22. 94/108
23. 38/51
24. 19/94
25. 93/467
26. $\frac{21}{45}$
27. $\frac{16}{39}$
28. $\frac{16}{39}$
29. $\frac{71}{151}$
30. $\frac{41}{163}$
31. $\frac{19}{40}, \frac{41}{83}, \frac{35}{68}, \frac{47}{86}$ and $\frac{29}{41}$
32. $\frac{32}{79}, \frac{58}{119}, \frac{71}{129}$ and $\frac{87}{139}$
33. $\frac{121}{396}, \frac{141}{421}, \frac{107}{281}, \frac{97}{234}$ and $\frac{183}{403}$
34. $\frac{18}{157}, \frac{27}{179}, \frac{32}{189}, \frac{41}{203}$ and $\frac{69}{335}$
35. $\frac{228}{83} < \frac{11}{4} < \frac{102}{37} < \frac{1376}{499}$
36. $\frac{43}{51} < \frac{86}{101} < \frac{6}{7}$
37. $\frac{209}{160} < \frac{17}{13} < \frac{157}{120} < \frac{144}{110}$
38. $\frac{453}{584} < \frac{39}{50} < \frac{176}{225} < \frac{98}{125}$
39. $\frac{64}{333} < \frac{397}{2002} < \frac{333}{1669} < \frac{269}{1336}$
40. $\frac{439743}{219700} < \frac{2607}{1300} < \frac{20406}{10201} < \frac{204}{101}$
41. $\frac{50}{29} > \frac{39}{25} > \frac{27}{18} > \frac{45}{31}$
42. $\frac{31}{94} > \frac{16}{51} > \frac{21}{72} > \frac{49}{183}$
43. $\frac{37}{12} > \frac{38}{13} > \frac{40}{14} > \frac{45}{17}$
44. $\frac{35}{96} > \frac{39}{108} > \frac{41}{138} > \frac{23}{82}$
45. $\frac{51}{193}, \frac{43}{169}, \frac{31}{139}, \frac{123}{647}$