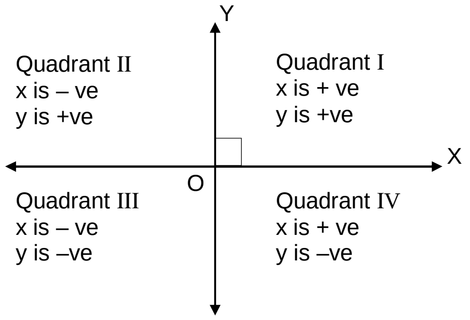
CHAPTER – 6

GRAPHS

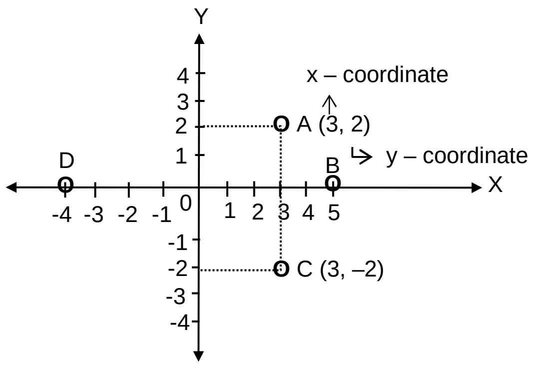
GRAPHS:

By a graph, we mean the set of points (x, y) that satisfy a relation of the form $f(x, y) = k$ or $f(x, y) \leq k$ or $f(x, y) \geq k$. The graphs in this section will be of those curves that can be drawn in a plane.
The rectangular coordinate system is shown below:



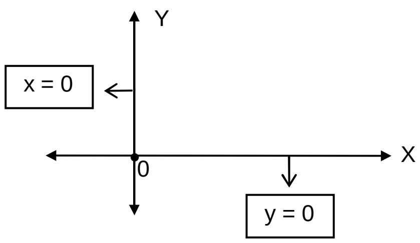
The rectangular coordinate system is divided into 4 quadrants by two mutually perpendicular lines called the coordinate-axes. The horizontal line is called the x-axis while the vertical line is called the y-axis. The point O, where the two lines meet is called the 'origin'.

Any point on the x-axis can be represented as (a, 0), while any point on the y-axis can be represented as (0, b).



The co-ordinates of the point A are (3, 2). If we want to specify the coordinates of the points, we can write A = (3, 2), B = (5, 0) etc and if we want to refer to the points we can write A (3, 2), B (5, 0) etc.

Line Graphs:

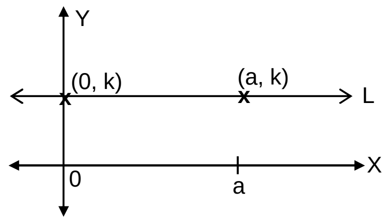


(1) We note that any point on the x-axis has the y-coordinate as 0.
So, y = 0 is the condition satisfied by any point on the line.
∴ The equation of the x-axis is y = 0

(2) The equation of the y-axis is x = 0.

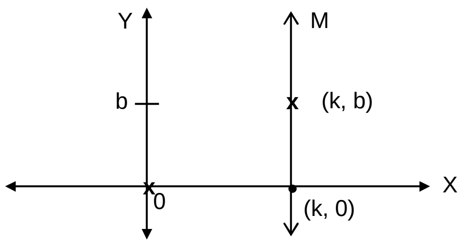
(3) The equation of a horizontal line or any line parallel to the x-axis is $\boxed{y = k \text{ (constant)}}$

Reason: We note that every point on the line L has the y-coordinate as k.

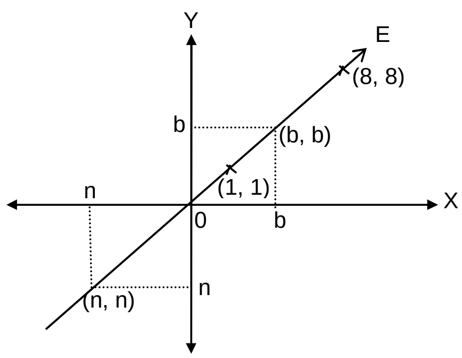


(4) The equation of a vertical line or any line parallel to the y-axis is $\boxed{x = k \text{ (constant)}}$

Reason: We note that every point on the line M has the x-coordinate as k.

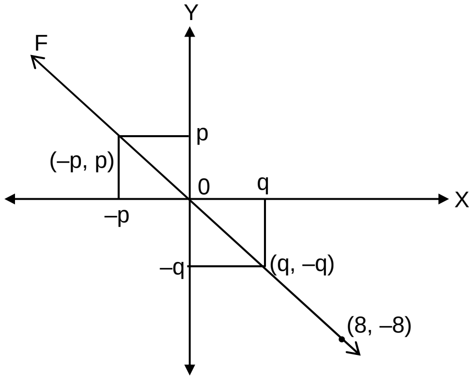


(5) On the line E shown in the figure, we find that x and y-co-ordinates are equal.
So, $\boxed{y = x}$ is the equation.

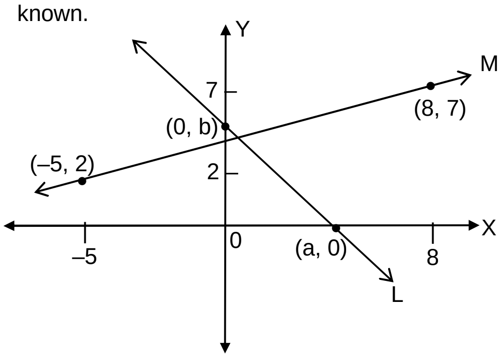


(6) On the line F, as shown in the figure, we find that x and y co-ordinates are equal in magnitude but opposite in sign.

So $\boxed{y = -x}$ is the equation.



(7) Obtaining the equation of a line if two points on it are known.



We apply the two-point formula for line equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (Refer to the coordinate geometry chapter).

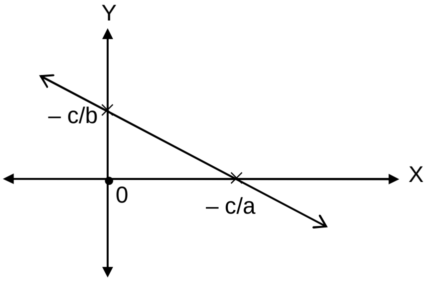
Accordingly, the equation of line L is

$y - 0 = \frac{b-0}{0-a}(x - a)$ or $\frac{x}{a} + \frac{y}{b} = 1$

The equation of M, is $y - 7 = \frac{2-7}{-5-8}(x - 8)$

or $5x - 13y + 51 = 0$.

(8) Plotting the graph of an inclined line:
For a line given as $ax + by + c = 0$; $ab \neq 0$
Step (i) Get the x-intercept as $x = -c/a$, by plugging $y = 0$.
Step (ii) Get the y-intercept as $y = -c/b$, by plugging $x = 0$.
Step (iii) Plot the points $(0, -c/b)$ and $(-c/a, 0)$ on the graph sheet.
Step (iv) Join these points by a line



(9) Graphing the regions bounded by lines
i. $x \geq 0$: The right half-plane (including the y-axis)
ii. $y \geq 0$: The upper half-plane (including the x-axis)
iii. $x < 0$: The left half-plane (excluding the y-axis)
iv. $y < 0$: The lower half-plane (excluding the x-axis)
v. $x \geq 0$ and $y \geq 0$: The first quadrant (with the boundaries)
vi. $xy \leq 0$: The second and fourth quadrants together (with the boundaries)
vii. $xy > 0$: The first and third quadrants together (without the boundaries)
viii. $ax + by + c \geq 0$ OR ≤ 0 : represents the half-planes demarcated by the line $ax + by + c = 0$ (with the line)

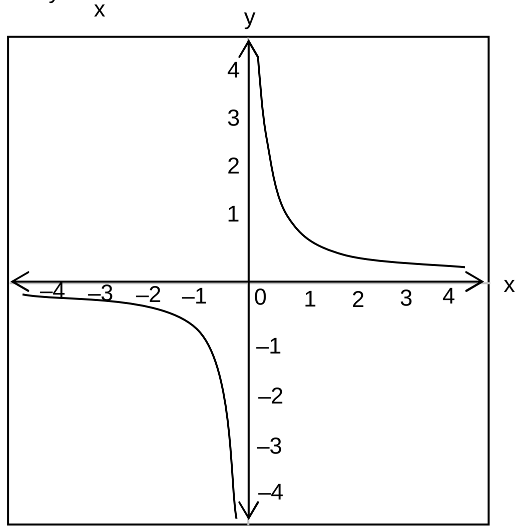
Shifting of graphs:

Consider the graph of f(x). To get the graph of f(x + a) (a > 0) move the graph a units to the left. Similarly to get the graph of f(x – a) (where a > 0) move the graph a units to the right.

Examples:

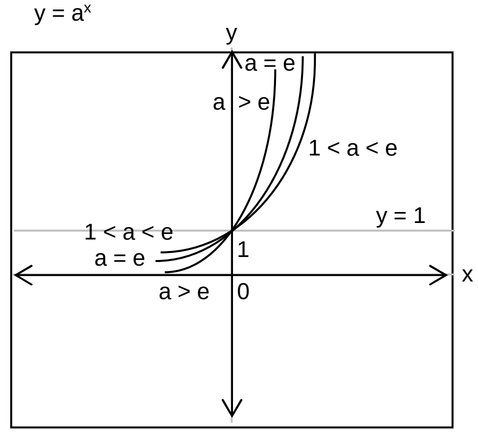
6.01. Draw the graph of $y = \frac{1}{x}$, $(x \neq 0)$

Sol: $y = \frac{1}{x}$



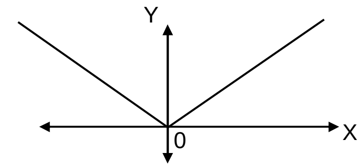
6.02. Draw the graph of $y = a^x$ for
(i) $1 < a < e$ (ii) $a > e$ (iii) $a = e$

Sol: $y = a^x$

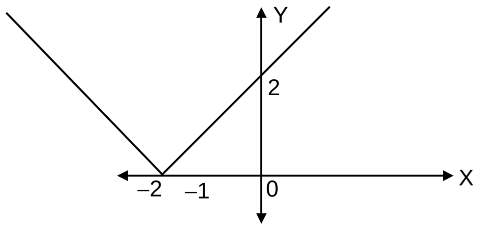


6.03. Draw the graph of y = |x + 2|

Sol: The graph of f(x) = |x| is shown below:

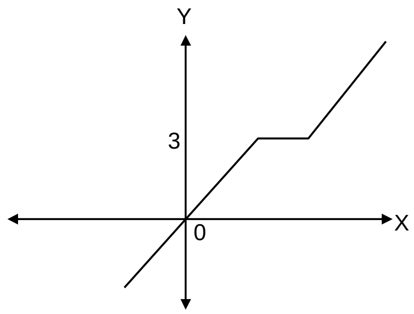


To get the graph of |x + 2|, move the above graph 2 units to the left. Then graph is as follows:



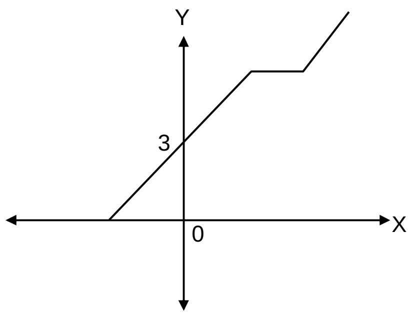
Note: Similarly, the graph of |x – 2| is obtained by shifting the graph of |x|, 2 units to the right.
To get the graph of f(x) + k, (where k>0) shift the graph of f(x), k units up and similarly to get the graph of f(x) – k, shift the graph of f(x) k units down.

6.04. The graph of the following function is shown below.
f(x) = x for x ≤ 2
= 2 for 2 < x ≤ 3
= x – 1 for 3 < x



Draw the graph of y = f(x) + 3.

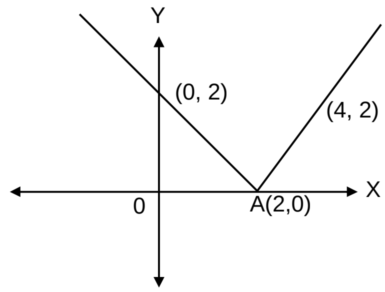
Sol:



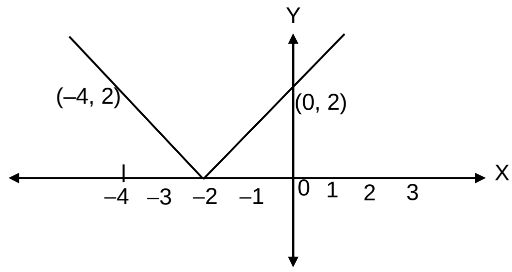
To get the graph of f(x) + 3, shift the graph of f(x), 3 units up.

6.05. Draw the graph of y = |x + 2| from the graph of y = |x – 2|

Sol.: The graph of f(x) = |x – 2| is as follows:

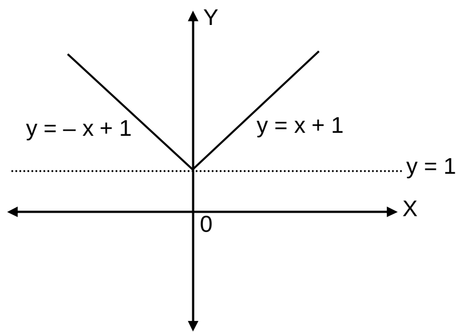


To get the graph of |x + 2|, shift this graph 4 units to the left. Then the graph is as follows:



6.06. Draw the graph of y = |x| + 1

Sol:



y = |x| + 1
The least value taken by y is 1, as no portion of the graph y = |x| + 1 is below y = 1.

The equation $\boxed{y = |x| + 1}$ can be written as

$$y = \begin{cases} x+1; & x \geq 0 \\ -x+1; & x < 0 \end{cases}$$

Graphs of commonly used functions

The table below shows how to get the graphs of some commonly used functions from the graph of a given function, say y = f(x).

Table showing the changes made to y = f(x) and the corresponding effect in the graph.

Function	Obtaining the graph from the graph of y = f(x)
(1) $y = \|f(x)\|$.	(1) unchanged when $f(x) \geq 0$ and reflected in the x-axis when $f(x) < 0$.
(2) $y = -\|f(x)\|$.	(2) unchanged when $f(x) \leq 0$ and reflected in the x-axis when $f(x) > 0$.
(3) $x = f(y)$.	(3) reflected in the line $y = x$.
(4) (i) $y = cf(x)$, $c > 1$, (ii) $y = cf(x)$, $0 < c < 1$.	(4) (i) vertical stretch, (ii) vertical shrink.
(5) (i) $f(cx)$; $c > 1$, (ii) $f(cx)$; $0 < c < 1$.	(5) (i) horizontal shrink, (ii) horizontal stretch.
(6) $y - k = f(x - h)$.	(6) translates 'h' units horizontally to the right and 'k' units vertically down.

Floor and Ceiling (or ceil) Functions:

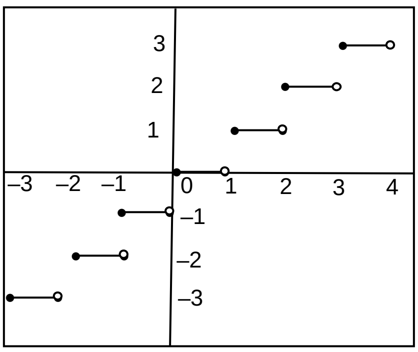
For any real number x, the greatest integer less than or equal to x is called the floor function. It is denoted as $\lfloor x \rfloor$.The least integer greater than or equal to x is called the ceiling (or ceil) function. It is denoted as $\lceil x \rceil$.

Thus $\lfloor 4.1 \rfloor = 4$, $\lfloor -1.2 \rfloor = -2$ and $\lfloor -7 \rfloor = -7$
while $\lceil 4.1 \rceil = 5$, $\lceil -1.2 \rceil = -1$ and $\lceil -7 \rceil = -7$

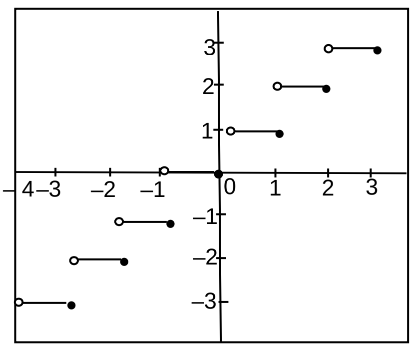
When the symbol [x] is used without any further explanation, it normally means $\lfloor x \rfloor$.

The graphs of these two functions are shown below

Floor function



Ceil function

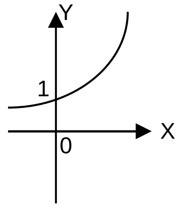


We note that for any integer n, $\lfloor n \rfloor = n = \lceil n \rceil$ and for any non-integer x, $\lfloor x \rfloor + 1 = \lceil x \rceil$

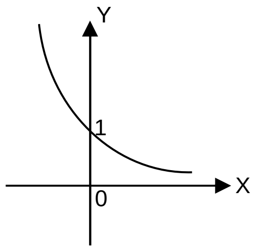
We also note that for the floor function, the endpoint on the left of each segment or 'step' is included, while the endpoint on the right is excluded. For the ceil function, it is vice-versa.

Exponential Function (e^{ax})

Case 1: a > 0

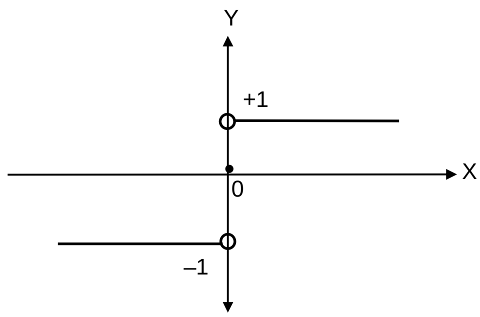


Case 2 : a < 0



Note: e = 2.718 (app)

(2) Signum function:



Definition:

$$sg(x) = \begin{cases} \dfrac{|x|}{x}; & x \neq 0 \\ 0 \;; & x = 0 \end{cases}$$

Examples: sg(7) = +1, sg(0) = 0
sg(–7) = –1.

6.07. Sketch the graph of the function
$y = x - 4;\ 0 \leq x < 5$

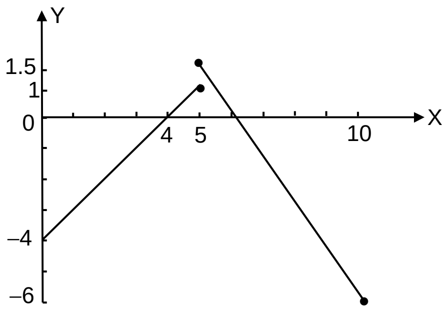
$9 - \frac{3}{2}x,\ 5 \leq x \leq 10$

Also determine the maximum and minimum values of y in the given domain.

Sol. The maximum and minimum values of y can be determined from the graph.
The max value occurs at x = 5.

$y = 9 - \frac{3}{2}x = 9 - \frac{3}{2}(5) = 1.5$

Minimum value of y occurs at x = 10 and it is –6.



6.08. Sketch the graph of $y = \dfrac{|x-9| - |x-5|}{2}$ and $x = \dfrac{|y-9| - |y-5|}{2}$.

Sol. Let us divide the possible values of x into three cases.

Case (i) $x \le 5$: If $x \le 5$, $|x - 5| = 5 - x$ and $|x - 9| = 9 - x$

$\therefore\ y = \dfrac{9 - x - (5 - x)}{2} = 2$

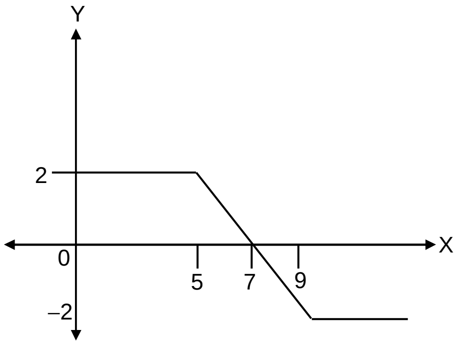
Case (ii) $5 \le x \le 9$: If $5 \le x \le 9$, $|x - 5| = x - 5$ and $|x - 9| = 9 - x$

$\therefore y = \dfrac{9 - x - (x - 5)}{2} = 7 - x$

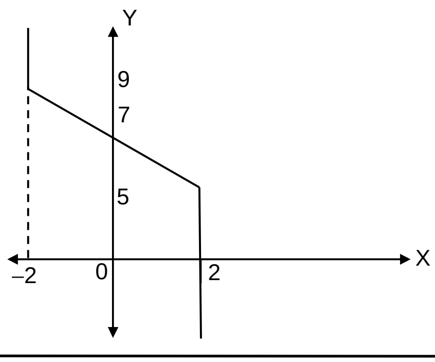
Case (iii) $x \ge 9$: If $x \ge 9$, $|x - 5| = x - 5$ and $|x - 9| = x - 9$

$\therefore\ y = \dfrac{x - 9 - (x - 5)}{2} = -2$

The graph can be represented as follows:

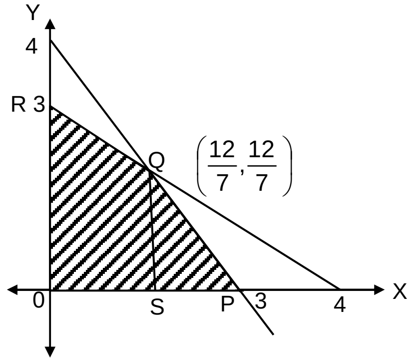


To get the graph of $x = \dfrac{|y-9| - |y-5|}{2}$ we need to reflect the graph discussed above in the line y = x. The graph is as follows.



6.09. Find the area enclosed by the region bounded by the graphs of $4x + 3y \le 12$, $3x + 4y \le 12$, $x \ge 0$ and $y \ge 0$.

Sol:



Sum of the areas

= Area of trapezium RQSO + Area of ΔQSP

$= \dfrac{1}{2}$ OS (OR + QS) + $\dfrac{1}{2}$ SP × QS

$= \dfrac{1}{2}\left(\dfrac{12}{7} + 3\right)\left(\dfrac{12}{7}\right) + \dfrac{1}{2}\left(\dfrac{12}{7}\right)\left(3 - \dfrac{12}{7}\right)$

$= \dfrac{36}{7}$ Sq units

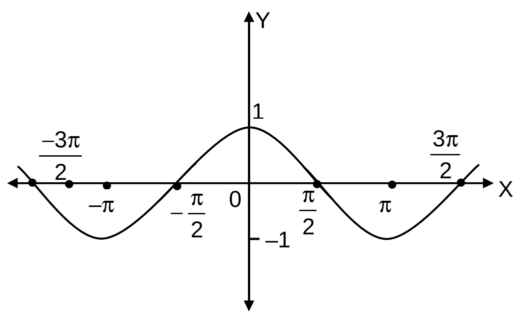
6.10. Two points A and B move in such a way that A is 8 units from the origin and B is 4 units from the origin. Find the area of the annular region in the first quadrant.

Sol. The set of points transversed by A will be the points on the area of the circle with radius 8 centered at the origin and that of B will be on the area of the circle with radius 4 centered at the origin and are in the first quadrant.

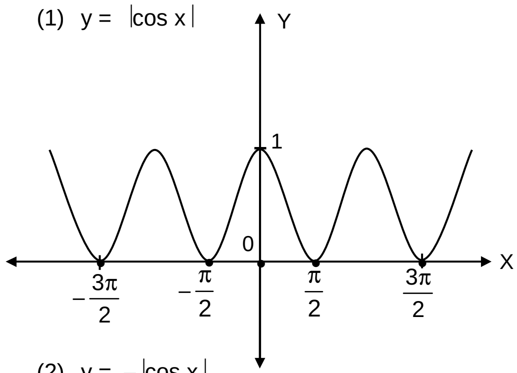
Area enclosed = $\dfrac{1}{4}[\pi (8)^2 - \pi(4)^2] = 12\pi$

6.11. Sketch the graph of
(1) y = |cos x|
(2) y = – |cos x|

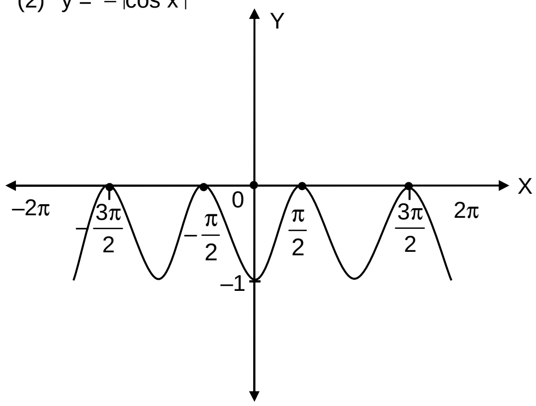
Sol: The graph of y = cos x is



(1) $y = |\cos x|$



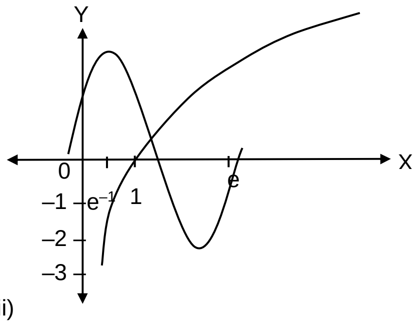
(2) $y = -|\cos x|$



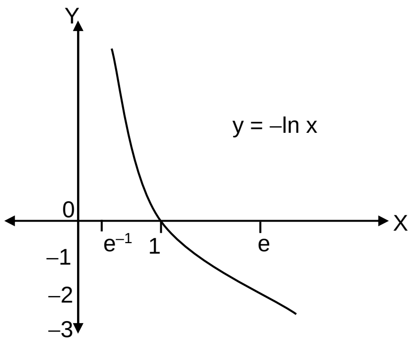
6.12. Sketch the graph of
(i) y = ln x
(ii) y = –ln x
(iii) y = |ln x|
(iv) y = ln (–x)

Sol: (i) y = ln x
Only when x > 0, y is defined.

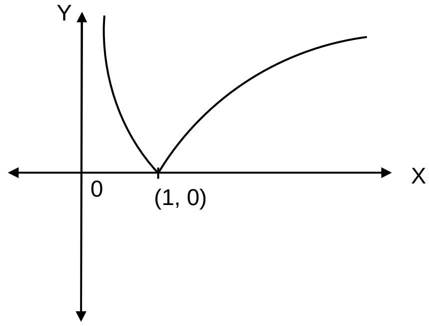
x	1	e	e^2	e^3	e^{-1}	e^{-2}	e^{-3}
y	0	1	2	3	–1	–2	–3



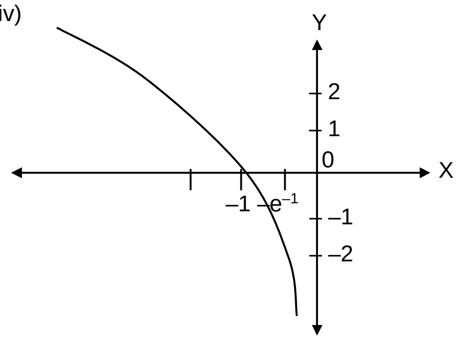
(ii)



(iii)

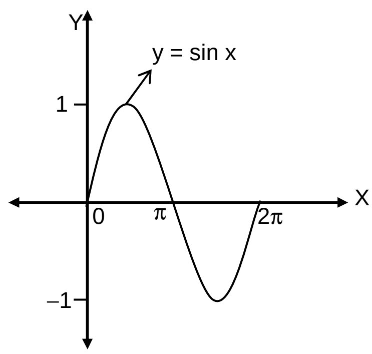


(iv)

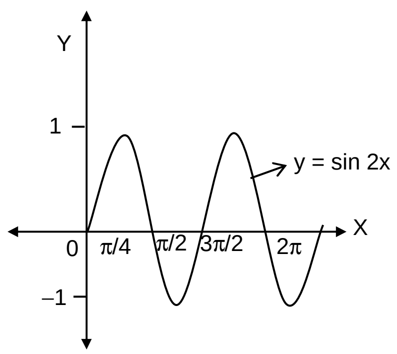


6.13. Sketch the graph of y = sinx and hence sketch the graph of
(i) y = sin2x
(ii) $y = \sin\frac{x}{2}$
(iii) y = 2sinx
(iv) $y = \frac{1}{2}\sin x$
In the interval [0, 2π]

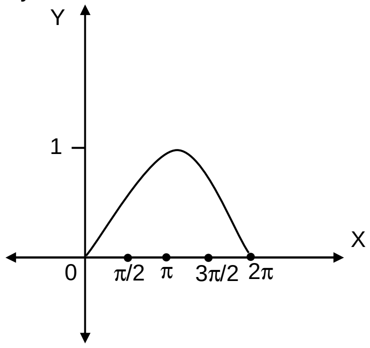
Sol:



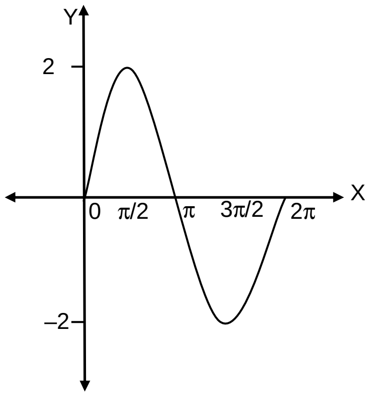
(i)



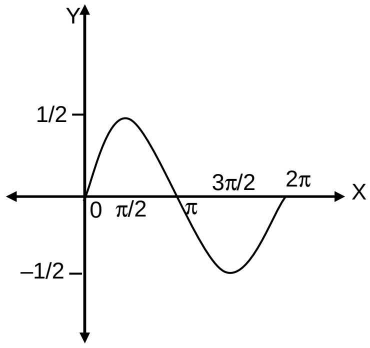
(ii) $y = \sin x/2$



(iii) $y = 2\sin x$

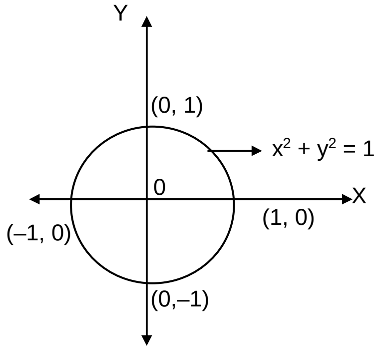


(iv) $y = \dfrac{1}{2}\sin x$

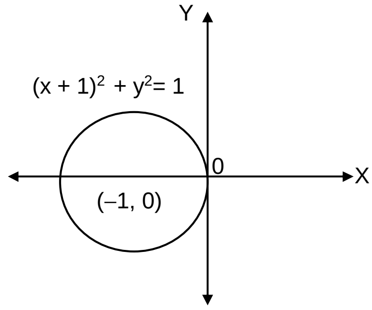


6.14. Sketch the graph of
(i) $(x + 1)^2 + y^2 = 1$
(ii) $x^2 + (y – 1)^2 = 1$
(iii) $(x + 1)^2 + (y + 1)^2 = 1$.

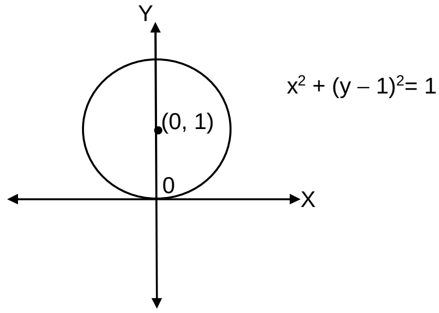
Sol. The graph of $x^2 + y^2 = 1$ is a circle of radius 1, centered at the origin.



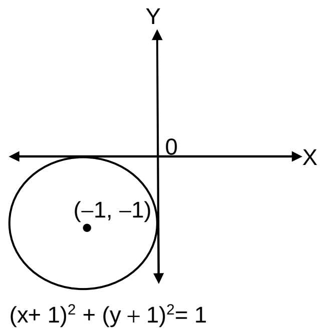
(i) The graph of $(x + 1)^2 + y^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ horizontally 1 unit to the left.



(ii) The graph of $x^2 + (y – 1)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically by 1 unit upwards.



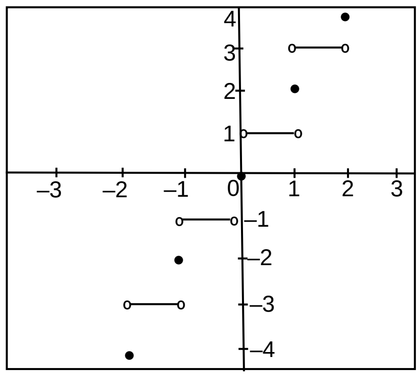
(iii) The graph of $(x + 1)^2 + (y + 1)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically downwards by 1 unit and horizontally by 1 unit to the left.



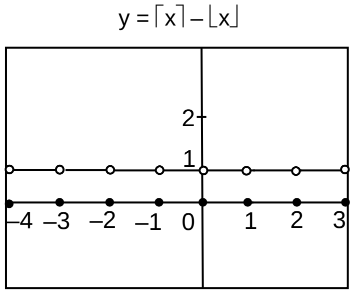
6.15. Sketch the graph of $y = \lceil x \rceil + \lfloor x \rfloor$

For integral values of x, $\lfloor x \rfloor = \lceil x \rceil = x$ and y = 2x, an even number. For other values of x, y is odd. Endpoints on both ends of the steps are excluded.

$y = \lceil x \rceil + \lfloor x \rfloor$

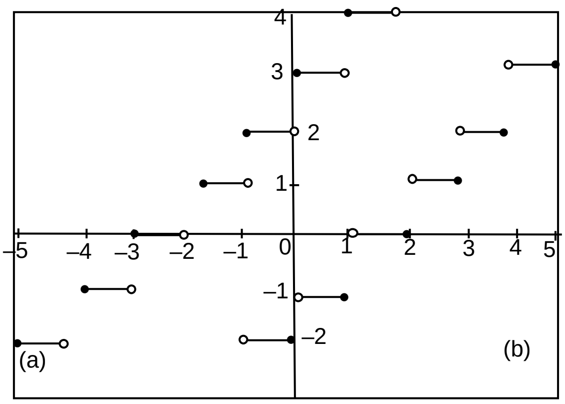


6.16. Sketch the graph of $y = \lceil x \rceil - \lfloor x \rfloor$.



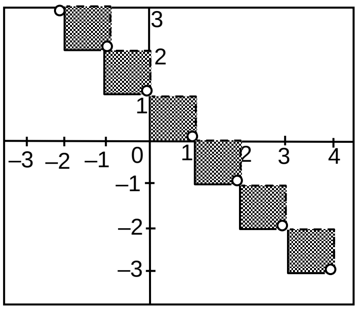
The graph is the line y = 1 (with excluded points represented by ∘) for nonintegral values of x and the isolated points given by y = 0 for integral values of x.

6.17. Sketch the graph of (a) $y = \lfloor x \rfloor + 3$ or $y = \lfloor x+3 \rfloor$ and (b) $y = \lceil x \rceil - 2$ or $y = \lceil x-2 \rceil$



We see that for all x, $\lfloor x+3 \rfloor = \lfloor x \rfloor + 3$ and for all x, $\lceil x-2 \rceil = \lceil x \rceil - 2$

6.18. Sketch the graph of $\lfloor x \rfloor + \lfloor y \rfloor = 0$



We get a sequence of squares, in each of which the left and lower sides are included, which is indicated by the solid line while the right and upper sides are excluded, which is indicated by the broken line

Concept Review Questions

Directions for questions 1 to 20: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. **(a)** The graph of x = 3 is a line
(A) parallel to the x– axis.
(B) parallel to the y – axis.
(C) passing through the origin.
(D) None of these

(b) The graph of y + 7 = 0 is a line
(A) parallel to the x – axis.
(B) parallel to the y – axis.
(C) passing through the origin.
(D) None of these.

(c) The graph 3x + y = 0 is a line
(A) parallel to the x – axis.
(B) parallel to the y – axis.
(C) passing through the origin.
(D) None of these

2. **(a)** The graph of 2x – 3y = 6 meets the x – axis at the point
(A) (0, 3). (B) (–2, 0). (C) (–3, 0). (D) (3, 0).

(b) The graph of $x^2 + y^2 = 9$ meets the negative y – axis at the point
(A) (–3, 0) (B) (0, –3) (C) (0, 3) (D) (3, 0)

3. The graph of $y = x^2$ lies entirely in the quadrants _____ .
(A) Q_1, Q_3 (B) Q_1, Q_4
(C) Q_1, Q_2 (D) Q_2, Q_3

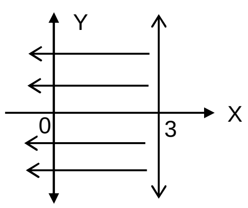
4. The graph of 3x + 4y = 12, passes through the quadrants
(A) Q_1, Q_2 and Q_3. (B) Q_2, Q_3 and Q_4.
(C) Q_1, Q_3 and Q_4. (D) Q_1, Q_2 and Q_4.

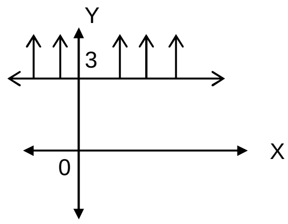
5. The graph of y = $|x|$ is symmetric with respect to
(A) the x – axis
(B) the y – axis
(C) both x and y – axes
(D) None of these

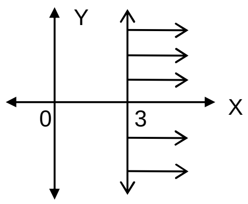
6. The graph y = $\log_e x$ crosses the x – axis at the point
(A) (1, 0) (B) (0, 1) (C) (2, 0) (D) (e, 0)

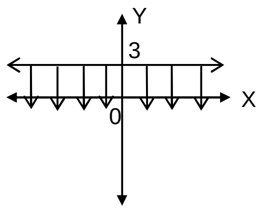
7. f(x) is a function and if f(x) = f(–x) then f(x) is symmetric about
(A) the x – axis
(B) the y – axis
(C) both the x and y axes
(D) None of these

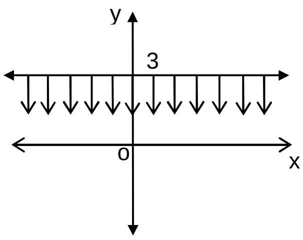
8. The graph of y = x ≥ 3 is

(A)


(B)


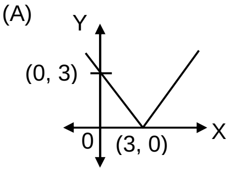
(C)


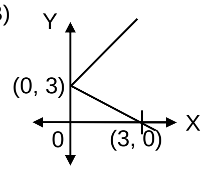
(D)


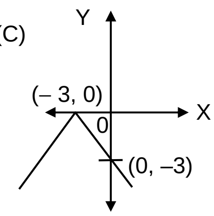
9. The region represented by the given graph is denoted by


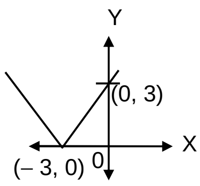
(A) x ≤ 3. (B) x ≥ 3. (C) y ≤ 3. (D) y ≥ 3.

10. The graph of the function y = $|x+3|$ is

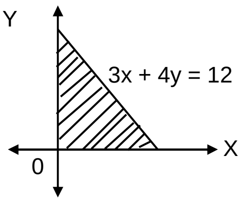
(A)


(B)


(C)


(D)


11. Which of the following points belong to the shaded region of the following graph?



(A) (1, 1) (B) (1, 2)
(C) (2, 1) (D) All the above

12. Area of the region represented by the graph of $x^2 + y^2 \le 49$ (in sq.units) is $\boxed{\quad}$ π.

13. If the curve $x^2 + y^2 - 2x + 3y + k = 0$ passes through the origin, then the value of k is $\boxed{\quad}$

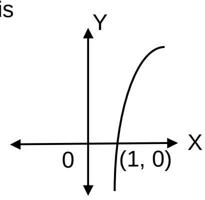
14. **(a)** The number of points in which the line x = 3 meets the graph of $y^2 = 12x$ is $\boxed{\quad}$

(b) The number of points in which the line y = 4 meets the graph of $x^2 + y^2 = 9$ is $\boxed{\quad}$

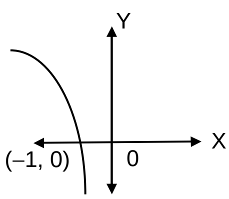
(c) The number of points in which the line y = 2 meets the graph of $y = -|x|$ is $\boxed{\quad}$

15. The line 2 + 3y + 5 = 0 divides the x – y plane into two regions. The region containing the origin is represented by ______ .
(A) $2x + 3y \le -5$ (B) $2x + 3y - 5 \le 0$
(C) $2x + 3y + 5 > 0$ (D) None of these

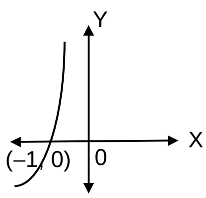
16. The graph obtained by reflecting the given graph in the X – axis is



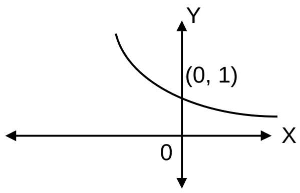
(A)



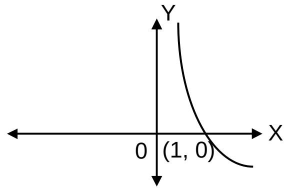
(B)



(C)



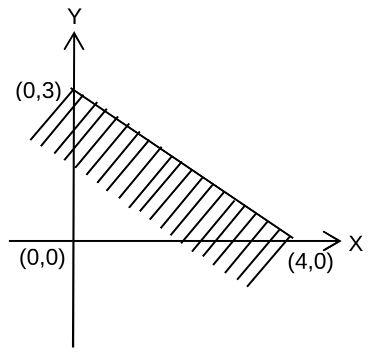
(D)



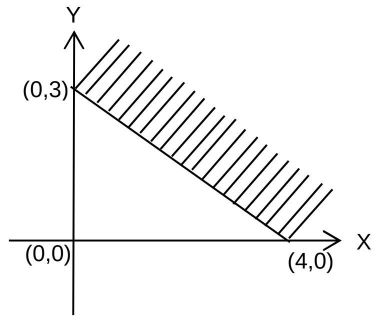
17. Let F be the graph of y = f(x). G is the graph of y = g(x), obtained by moving F 3 units to the right. Then g(x) =
(A) f(x + 3) (B) f(x – 3)
(C) f(x) + 3 (D) f(x) – 3

18. The graph of $3x + 4y \le 12$ is

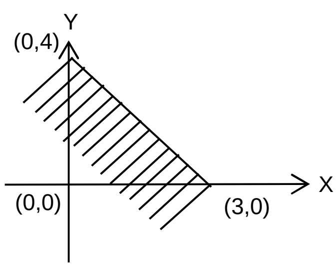
(A)



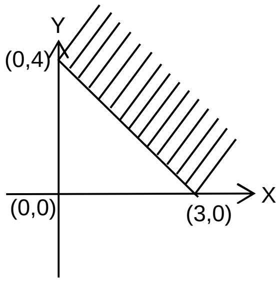
(B)



(C)



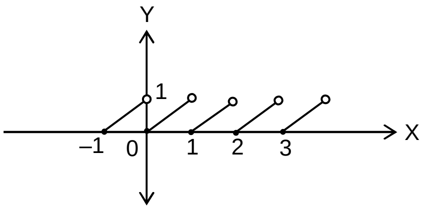
(D)



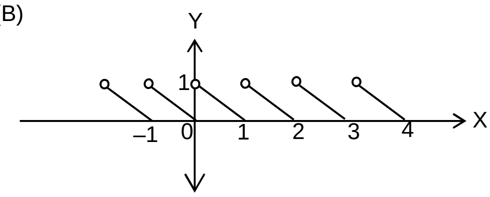
19. The image of (3, 2) w.r.t y = x is
(A) (3,–2) (B) (–3 – 2)
(C) (–2, 3) (D) (2, 3)

20. The graph of the function ⌈ x ⌉ – x where ⌈ x ⌉ represents the least integer greater than or equal to x is

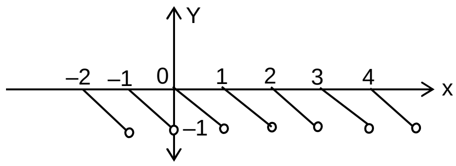
(A)



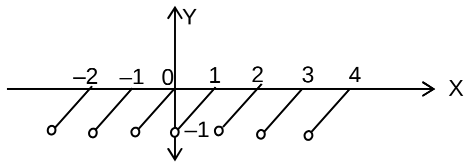
(B)



(C)



(D)

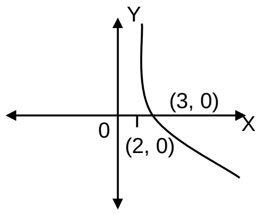


Exercise – 6(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

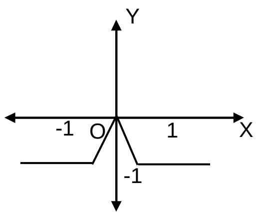
Directions for questions 1 to 4: In each of these questions a graph is given. Choose the relation that best describes the graph (the lines, curves or the shaded region) from the alternatives given.

1.



(A) $\log_{0.3}(x + 2)$ (B) $\log_{0.3}(x – 2)$
(C) $\log_3(x – 2)$ (D) $\log_{0.3}x$

2.



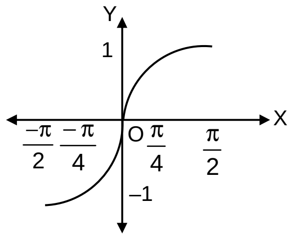
(A) $\left|\dfrac{|x-1|-|x+1|}{2}\right|$

(B) $\left|\dfrac{|x-1|+|x+1|}{2}\right|$

(C) $-\left|\dfrac{|x+1|-|x-1|}{2}\right|$

(D) $-\left||x+1|-|x-1|\right|$

3.



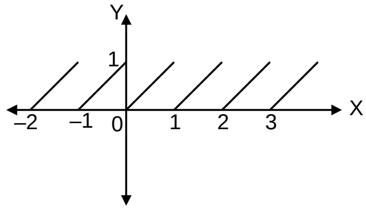
(A) $\tan x,\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(B) $\cot x,\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(C) $\cos x,\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(D) $\sin x,\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

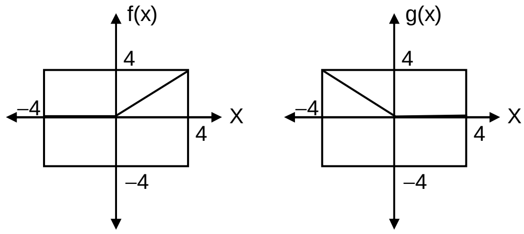
4.



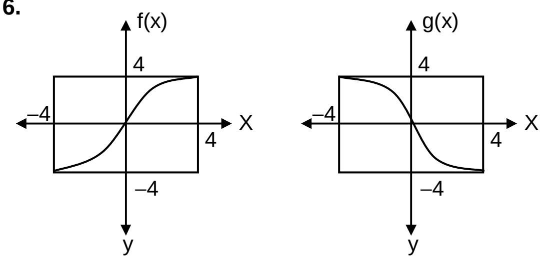
(A) 2x – [x] (B) [x] – x
(C) x – [x] (D) [x] – 2x

Directions for questions 5 to 8: The questions are based on the following data. In each of these questions, the graphs of two relations f(x), g(x) are given. The graphs are shown as solid curves
If for all x, g(x) = f(–x) and for some x, g(x) ≠ –f(x), choose (A).
If for all x, g(x) = –f(x) = f(–x), choose (B).
If for all x, g(x) = –f(–x), choose (C).
If none of the above conditions or more than one of them is true, choose (D).

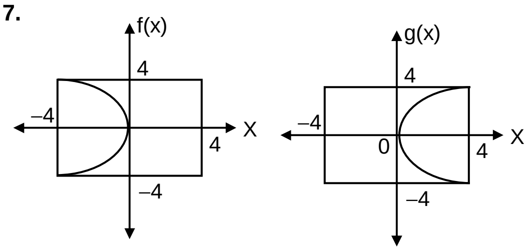
5.



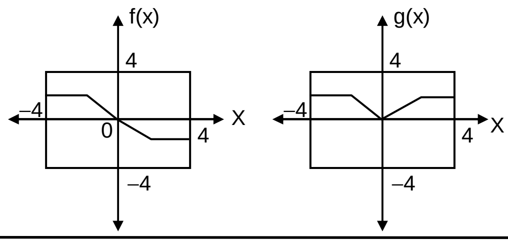
6.



7.

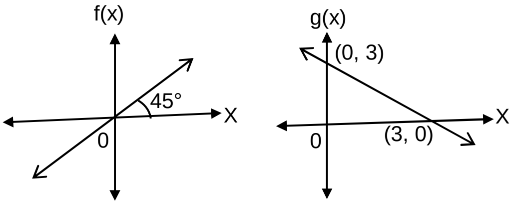


8.



Directions for questions 9 and 10: Select the correct alternative from the given choices.

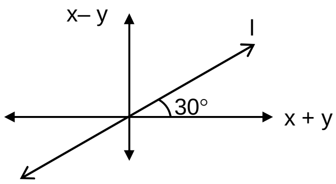
9.



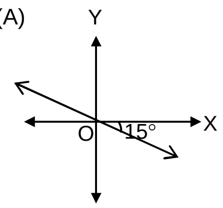
The relation between f(x) and g(x) is

(A) g(x) = f(3 + x)
(B) g(x) = f(3 – x)
(C) g(x) = f(x – 3)
(D) g(x) = f(x) + 3

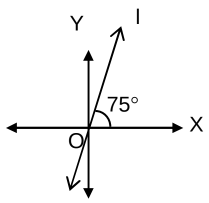
10. The graph of x – y against x + y is shown. Find the corresponding graph of y against x.



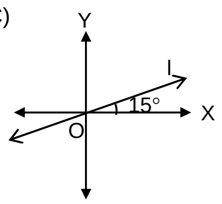
(A)



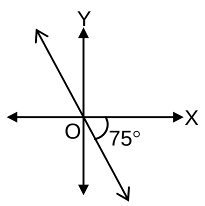
(B)



(C)



(D)



Directions for questions 11 to 14: These questions are based on the data given below. In each of these questions the relation satisfied by the points on the graph is given. Use this relation and select the answer as

(A) If a horizontal line intersecting the graph more than once exists
(B) If a vertical line intersecting the graph more than once exists
(C) If a horizontal line as well as a vertical line intersecting the graph more than once exist
(D) None of the above hold true.

11. $y = 2x^2$

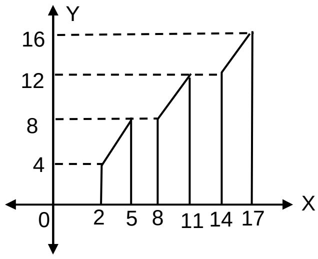
12. The graph consisting of the set of all points (x, y) that are at a distance of 3 units from the origin and xy > 0.

13. $|x| - |y| = 5$

14. $|y|\ x = 3$

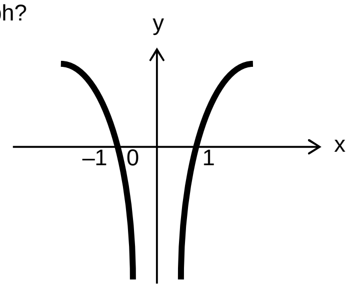
Directions for questions 15 to 23: Select the correct alternative from the given choices.

15. Find the sum of the perimeters of the trapeziums that are shown in the figure.



(A) 84 (B) 64 (C) 48 (D) 56

16. Which of the following relations represents the graph?



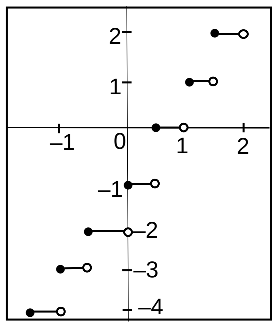
(A $x = \log|y|, y \neq 0$ (B) $y = \log|x|, x \neq 0$
(C) $y = |\log x|$ (D) $x = \log y$

17. Find the area enclosed between the curves $|x| + |y| \geq 3$ and $x^2 + y^2 \leq 9$.
(A) $9\pi - 4$ (B) $9(\pi - 4)$
(C) $9(\pi - 2)$ (D) $3\pi - 36$

18. Find the area of the region represented by the inequations $0 \leq |x - 3| \leq 7$ and $0 \leq |y - 5| \leq 9$.

19. Find the area enclosed by the coordinate axes and $y = |x| - 5$.

20. Which of the following relations represents the graph?



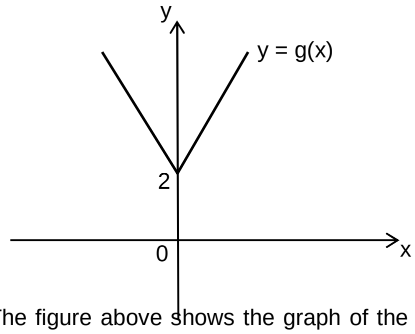
(A) $y = \lceil 2x - 1 \rceil$ (B) $y = \lceil 2x + 1 \rceil$
(C) $y = \lfloor 2x - 1 \rfloor$ (D) $y = \lfloor 2x + 1 \rfloor$

21. If $g(x) = 3 - \left|5 - |x|\right|$, which of the following statements hold true about the graph of g(x) versus x?

I. The graph has exactly one y intercept.
II. The graph has at least two x intercepts.
III. The graph has at most three x intercepts.

(A) I, II (B) I, III (C) II, III (D) I, II, III

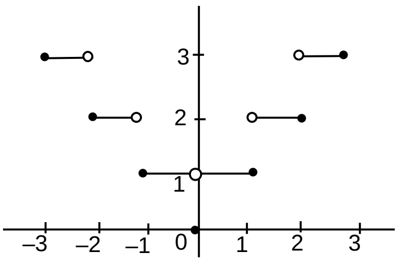
22.



The figure above shows the graph of the function g defined by g(x) = |3x|+ 2 for all numbers x. For which of the following functions, f(x), defined for all numbers x, does the graph of f intersect the graph of g at exactly two points?

(A) f(x) = 2x + 4
(B) f(x) = 3x + 5
(C) f(x) = 4x + 1
(D) All the above

23. Which of the following relations represents the graph shown below?

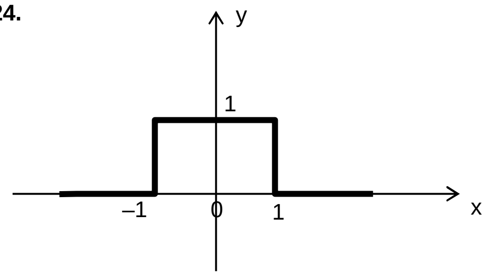


(A) $y = |\lfloor x \rfloor|$
(B) $y = |\lceil x \rceil|$
(C) $y = \lfloor |x| \rfloor$
(D) $y = \lceil |x| \rceil$

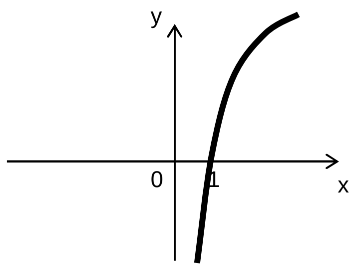
Directions for questions 24 and 25: Study the given graphs and mark your answer as

(A) if f(–x) = f(x)
(B) if f(–x) = –f(x)
(C) Both (A) and (B) are true
(D) Neither (A) nor (B) is true

24.



25.

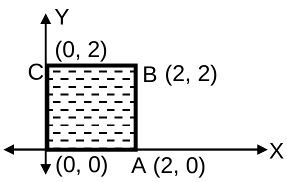


Exercise – 6(b)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

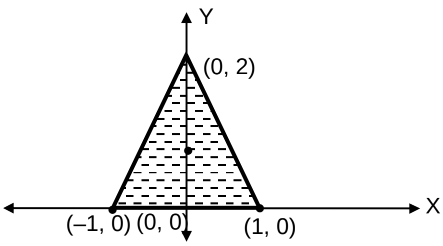
Directions for questions 1 to 5: In each of these questions a graph is given. Choose the relation that best describes the shaded region from the alternatives given.

1.



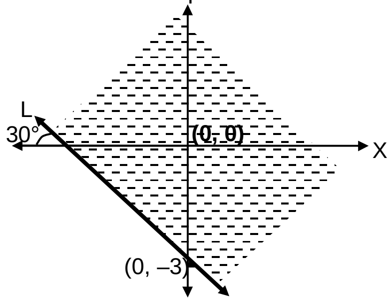
(A) |x| ≤ 2 or |y| ≤ 2
(B) |x| ≤ 2 and |y| ≤ 2
(C) 0 ≤ x ≤ 2 or 0 ≤ y ≤ 2
(D) 0 ≤ x ≤ 2 and 0 ≤ y ≤ 2

2.



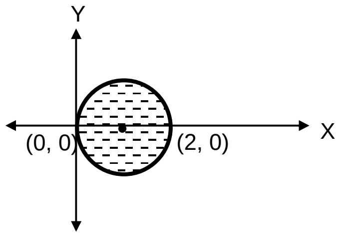
(A) 2x + |y| ≤ 2; y ≥ 0
(B) 2x + |y| ≥ 2; y ≥ 0
(C) 2 |x| + y ≤ 2; y ≥ 0
(D) 2|x| + y ≥ 2; y ≥ 0

3.



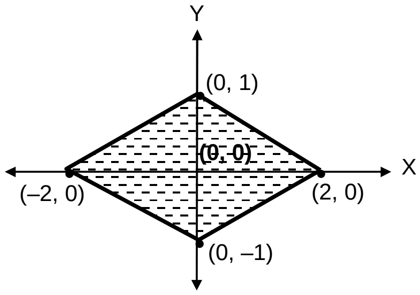
(A) $x - \sqrt{3}y - 3\sqrt{3} \geq 0$
(B) $x + \sqrt{3}y + 3\sqrt{3} \geq 0$
(C) $x - \sqrt{3}y + 3\sqrt{3} \leq 0$
(D) $x + \sqrt{3}y + 3\sqrt{3} \leq 0$

4.



(A) $x^2 + y^2 \le 2(x + y)$ (B) $x^2 + y^2 \le 2(x - y)$
(C) $x^2 + y^2 \le 2x$ (D) $x^2 + y^2 \le 2y$

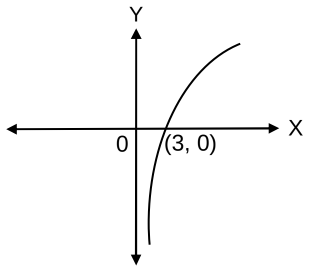
5.



(A) $|x| + 2|y| \le 2$ (B) $2|x| + |y| \le 2$
(C) $|x| + 2|y| \ge 2$ (D) $2\,|x| + y \ge 2$

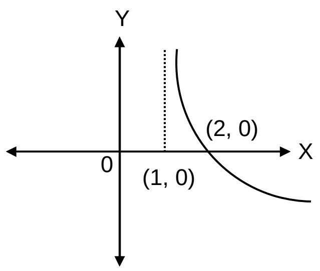
Directions for questions 6 to 8: Select the relation that best describes the given graph(s).

6.



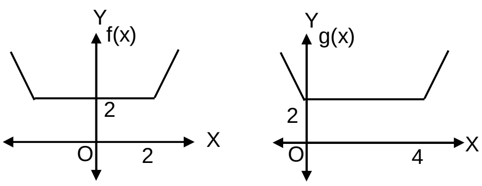
(A) $y = \log_e 3x$ (B) $y = \log_e(x/3)$
(C) $y = 3\log_e x$ (D) $y = \log_e(x + 3)$

7.



(A) $y = \log_2(x + 1)$ (B) $y = \log_{0.5}(x - 1)$
(C) $y = \log_2(x - 1)$ (D) $y = \log_{0.5}(x + 1)$

8.



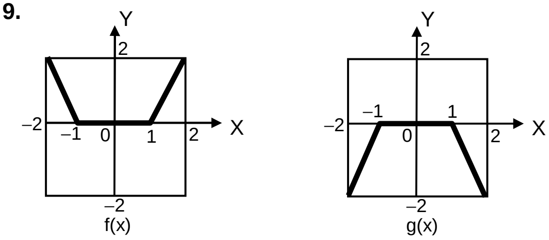
(A) $g(x) = f(x) + 2$ (B) $g(x + 2) = f(x - 2)$
(C) $f(x) = g(x - 2)$ (D) $g(x) = f(x - 2)$

Directions for questions 9 to 12: These questions are based on the following data.

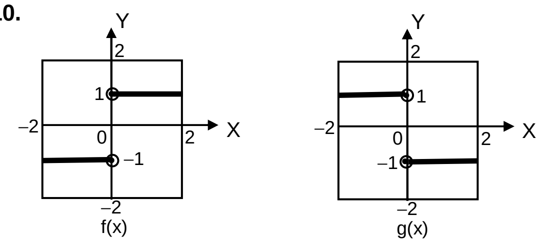
In each of these questions a pair of graphs f(x) and g(x) is given. The graphs are shown as solid curves in the domain $x \in (-2, 2)$.

If $f(x) = g(-x)$ choose the answer as choice (A).
If $f(x) = -g(x)$ choose the answer as choice (B).
If $f(x) = -g(-x)$ choose the answer as choice (C).
Choose the answer as choice (D), if f(x) and g(x) exhibit more than one of these relations.

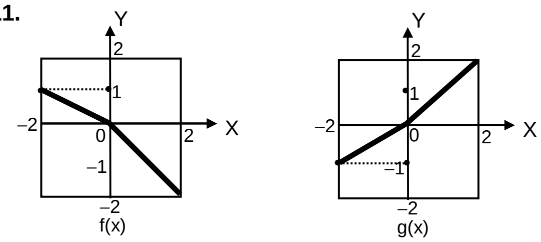
9.



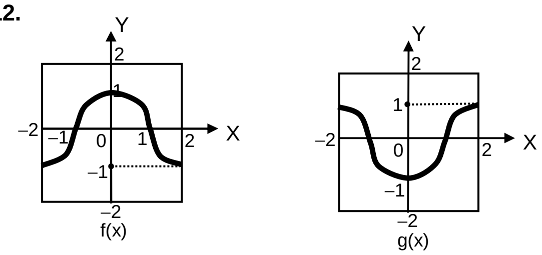
10.



11.



12.



Directions for questions 13 to 15: These questions are based on the data given below.

In each of these questions, the relation satisfied by the points on the graph is given. Use this relation and select the correct alternative as

(A) if a vertical line intersecting the graph more than once exists.
(B) if a horizontal line intersecting the graph more than once exists.
(C) if a vertical and a horizontal line intersecting the graph more than once exist.
(D) None of the above

13. $x|y| = 1$

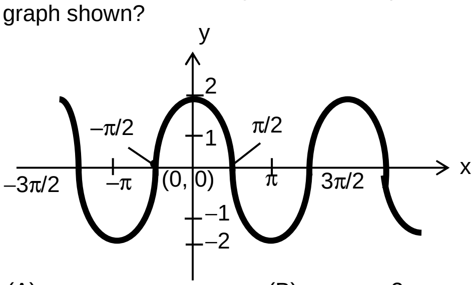
14. $ax + by + c = 0$, with $abc \ne 0$

15. The graph consisting of the set of all points (x, y) that are at a distance 2 units from the origin with xy < 0.

(A) $y = |x| + k$, k is an integer and $xy > 0$.
(B) $y = k|x|$, k is an integer and $xy < 0$.
(C) $|x| + |y| = k$, k is any integer and $xy < 0$.
(D) $x + y = k$, k is a non-zero integer and $xy > 0$

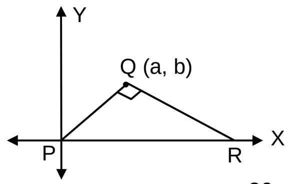
16. Find the area enclosed (in sq.units) between $|x| + |y| \geq 1$ and $x^2 + y^2 \leq 1$. $\pi - \boxed{}$

17. Which of the following relations represents the graph shown?



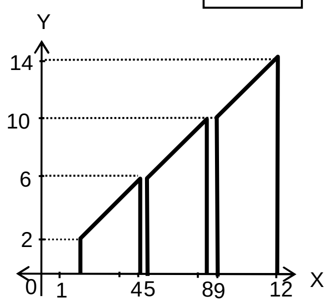
(A) y = cos x
(B) y = cos 2x
(C) y = 2 cos x
(D) y = cos (x/2)

18. If the area of the triangle shown in the figure given is 40 sq.units, find the coordinates of the point R.

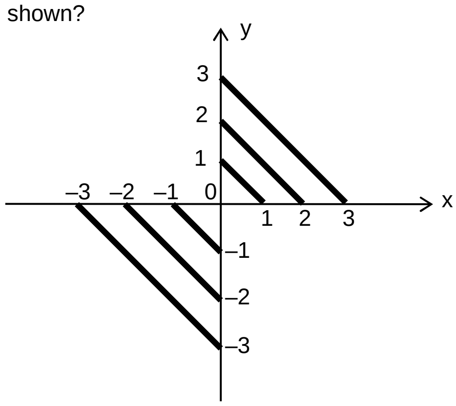


(A) $(a^2 + b^2, 0)$
(B) $(\frac{80}{a}, 0)$
(C) $(\frac{80}{b}, 0)$
(D) $(\frac{20}{b}, 0)$

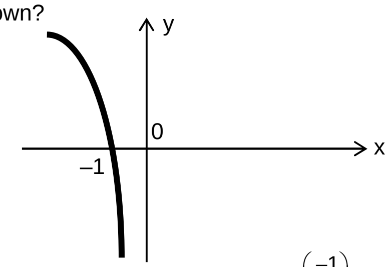
19. Find the sum of the perimeters of the trapeziums that are shown in the figure. $\boxed{}$



20. Which of the following relations represents the graph shown?



21. Which of the following relations represents the graph shown?



(A) $y = -\log x$
(B) $y = \log\left(\frac{-1}{x}\right)$
(C) $y = \log(-x)$
(D) $y = \log(-x)$

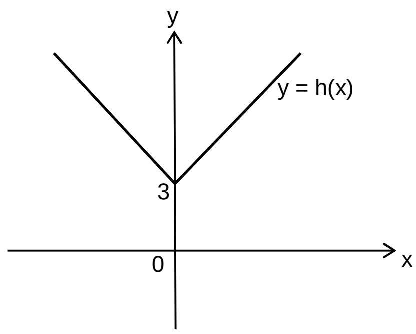
22. $h(x) = \big|2 - |x|\big|$

Which of the following statements holds true about the graph of h(x) versus x?

I. The sum of all the x-intercepts is zero
II. The only y-intercept of h(x) is 1
III. For all values of k, (–k, h(k)) is a point on the graph

(A) Only I and III
(B) Only II and III
(C) Only I and II
(D) I, II, III

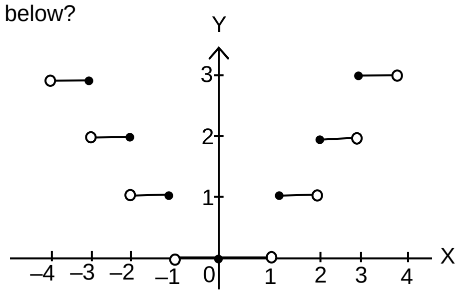
23.



The figure above shows the graph of the function h defined by $h(x) = |x| + 3$ for all real numbers x. For which of the following functions g, defined for all numbers x, do the graphs of g and h intersect only in the first quadrant?

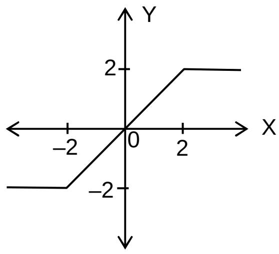
(A) $g(x) = x + 4$
(B) $g(x) = \frac{1}{2}x + 2$
(C) $g(x) = \frac{1}{2}x + 4$
(D) $g(x) = 2x + 2$

24. Which of the following represents the graph shown below?



(A) $y = |\lfloor x \rfloor|$
(B) $y = |\lceil x \rceil|$
(C) $y = \lfloor |x| \rfloor$
(D) $y = \lceil |x| \rceil$

25. Which of the following represents the graph shown below?



(A) $\frac{|x+2|-|x-2|}{2}$ (B) $|x+2|-|x-2|$ (C) $\frac{|x-2|-|x+2|}{2}$ (D) $|x-2|-|x+2|$

Key

Concept Review Questions

1. (a) B	3. C	8. C	13. 0	16. D
(b) A	4. D	9. C	14. (a) 2	17. B
(c) C	5. B	10. D	(b) 0	18. A
2. (a) D	6. A	11. D	(c) 0	19. D
(b) B	7. B	12. 49	15. C	20. B

Exercise – 6(a)

1. B	6. B	11. A	16. B	21. A
2. C	7. D	12. D	17. C	22. A
3. D	8. D	13. C	18. 252	23. D
4. C	9. B	14. B	19. 25	24. A
5. A	10. C	15. A	20. C	25. D

Exercise – 6(b)

1. D	6. B	11. B	16. 2	21. C
2. C	7. B	12. D	17. C	22. D
3. B	8. D	13. A	18. C	23. D
4. C	9. D	14. D	19. 72	24. C
5. A	10. D	15. D	20. D	25. A