20

PROBABILITY

Generally this chapter is considered as an extension of permutation and combination, since it is as logical in nature as permutation and combination. I hope that you must have been well-versed with the permutation and combination (previous chapter) which will enable you to solve the problems of this chapter quickly and easily. This chapter is important for XAT, SNAP and other MBA entrance tests. Over the past years CAT has shown no great interest in this chapter, still you should learn it heartly due to uncertain syllabus and irregular pattern of CAT. Remember that so far CAT has asked very simple and logical problems from this chapter which could be solved by applying just commonsense.

The literal meaning of probability is the chance of occurrence of an event. For example, if a person is standing at the crossing of two roads which direct towards North, South, East and West. Thus he has total four alternatives (*i.e,* four different directions) to proceed. Now if he wish to go towards a particular direction then the probability of completing his wish is $\frac{1}{4}$ since he can choose only one direction at a time out of four directions.

Consider another example, A person has two different cars *viz.*, Scorpio and Safari, which he uses randomly,then it can be said that the probability of using Scorpio is $\frac{1}{2}$ since out of two cars he can use any one car at a time. Similarly the probability of using Safari is also $\frac{1}{2}$. Thus we can say that the probability of using any one car at a time is $\frac{1}{2}$ *i.e.*, 50%. So there are chances that in 50% cases he can use Scorpio and in other 50% cases he can use Safari.

Hence from the above illustrations we can conclude that the probability of an event

$$= \frac{\text{No. of ways in which favourable (or desired event occurs.)}}{\text{(Total number of possible outcomes)}}$$

IMPORTANT ARTICLES/DEFINITIONS

1. Experiment
2. Random Experiment
3. Sample Space
4. Events
5. Elementary Events
6. Compound Events
7. Occurrence of Events
8. Impossible Events
9. Sure Events
10. Equally Likely Events
11. Favourable Events
12. Complementary Events
13. Algebra of Events
14. Mutually Exclusive Events
15. Mutually Exclusive and Exhaustive Events
16. Addition Theorems on Probability
17. Conditional Probability
18. Multiplication Theorems on Probability
19. Independent Events
20. The Law of Total Probability
21. Baye's Rule

Experiment : An operation which can produce some well-defined outcomes, is known as an experiment.

The various experiments, when repeated under identical conditions, results. (*i.e.*, outcomes) in each case are same *e.g.*, standard scientific experiments.

But there are some other experiment, when repeated under identical conditions, results in each case are different *e.g.*, rolling of a fair die, tossing of a coin etc.

Random Experiment : If in each trial of an experiment conducted under identical conditions, the outcome is not unique, but may be any of the possible outcomes then such an experiment is known as a random experiment. *e.g.*, rolling of an unbiased die, tossing of a fair coin, drawing of a card from a well shuffled pack of cards.

Sample Space : The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by *S*.

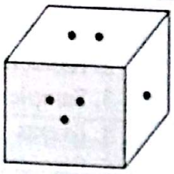
If $E_1, E_2, E_3 \dots E_n$ are the possible outcomes of a random experiment, then $S = \{E_1, E_2, \dots, E_n\}$. Also, each of the element of sample space 'S' is called a **sample point.**

EXAMPLE 1 In tossing of a fair coin, there are two possible outcomes, *viz.*, head (H) and tail (T). So, the sample space in this random experiment is $S = \{H, T\}$

EXAMPLE 2 When two fair coins are tossed together, the possible outcomes of the experiment are HH, HT, TH and TT. So the sample space is given by $S = \{HH, HT, TH, TT\}$

EXAMPLE 3 When unbiased die is thrown, it gives 6 possible outcomes *viz.*, 1, 2, 3, 4, 5 and 6. So, the sample space $S = \{1, 2, 3, 4, 5, 6\}$

NOTE A die is a cubical solid having 6 similar faces. In each of the six faces there are unique number of holes *viz.*, 1 hole, 2 holes, 3 holes, 4 holes, 5 holes and 6 holes.
Remember that the sum of number of holes in any two opposite faces is always 7 *viz.*, $1 + 6 = 2 + 5 = 3 + 4 = 7$.



- Die means one (single) die
- Dice means more than one die.
- In throwing a die, the outcome is the number of holes on the uppermost face

EXAMPLE 4 When two unbiased dice are rolled (or tossed) simultaneously then there are total $6 \times 6 = 36$ possible outcomes.

So, the sample space

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1) \dots\dots & & & & & (3,6) \\ (4,1) \dots\dots & & & & & (4,6) \\ (5,1) \dots\dots & & & & & (5,6) \\ (6,1) \dots\dots & & & & & (6,6) \end{Bmatrix}$$

EXAMPLE 5 When a die and a coin are tossed simultaneously, then there are total 12 possible outcomes. So the sample space

$$S = \begin{Bmatrix} (1,H), (2,H), (3,H), (4,H), (5,H), (6,H) \\ (1,T), (2,T), (3,T), (4,T), (5,T), (6,T) \end{Bmatrix}$$

EXAMPLE 6 A coin is tossed twice. If the second throw results in a tail, then a die is thrown. So in this random experiment sample space.

$$S = \begin{Bmatrix} HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, \\ TT1, TT2, TT3, TT4, TT5, TT6, \end{Bmatrix}$$

EXAMPLE 7 From a bag containing 2 black and 3 white balls we draw two balls. Let B_1, B_2 be the black balls and W_1, W_2 and W_3 be the white balls then the sample space

$$S = \begin{Bmatrix} B_1W_1, & B_1W_2, & B_1W_3 \\ B_2W_1, & B_2W_2, & B_2W_3 \\ B_1B_2, & W_1W_2, & W_2W_3 \\ W_1W_3 & & \end{Bmatrix}$$

Event : Any subset of a sample space is called an event.

EXAMPLE 1 In a single throw of a die, the event of getting an even number is given by $E = \{2, 4, 6\}$

Clearly, here the sample space $S = \{1, 2, 3, 4, 5, 6\}$

Hence $E \subset S$ *i.e.*, E is the subset of S.

EXAMPLE 2 Consider a random experiment of throwing two dice at a time. The sample space $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5) \dots (6,1), (6,2) \dots (6,6)\}$

Some different events associated with the above sample space are given below.

$E_1 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$E_2 = \{(1,8), (2,7), (3,6), (4,5), (5,4), (6,3), (7,2), (8,1)\}$

$E_3 = \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$

$E_4 = \{(2,2), (2,3), (2,5), (3,2), (3,3), (3,5), (5,2), (5,3), (5,5)\}$

Clearly $E_1 \subset S, E_2 \subset S, E_3 \subset S, E_4 \subset S$

Where E_1 is the event of getting a doublet and E_2 is the event of getting 9 as the sum.

E_3 is the event of getting odd number on the first die and an even number on the second die

E_4 is the event of getting only prime number on each of the two dice.

EXAMPLE 3 Consider a random experiment of tossing three coins at a time. The sample space $S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$

Following are the various events associated with the above sample space.

$E_1 = \{HHH, HHT, HTH, THH\}$

$E_2 = \{HHT, HTH, THH, TTH, HTT, THT\}$

$E_3 = \{HHH, TTT\}$

$E_4 = \{THH, TTH, THT, TTT\}$

Where E_1 is the event of getting atleast two heads and E_2 is the event of getting atleast one head and atmost two heads.

E_3 is the event of getting all the three coins identical and E_4 is the event of getting tail on the first coin

Elementary Events : An event containing only a single sample point is called an elementary event, or simple event.

EXAMPLE 1 In a simultaneous toss of two coins the sample space $S = \{HH, HT, TH, TT\}$

then $E_1 = \{HH\}$

$E_2 = \{TT\}$ are the elementary events

EXAMPLE 2 Consider a random experiment in which two dice rolled simultaneously, the sample space

$S = \{(1,1), (1,2) \dots (6,5), (6,6)\}$

then $E_1 = \{(1,1)\}$

$E_2 = \{(6, 6)\}$

$E_3 = \{(5, 5)\}$ are the elementary events.

Here E_1 is the event of getting the sum of two.

E_2 is the event of getting the sum of twelve

E_3 is the event of getting the product of 25.

Compound Events : Events which are not elementary are known as compound events or the events which contains more than one element are called compound or composite events.

Example 1 Consider a random experiment in which two dice are rolled simultaneously, the sample space

$$S = \{(1, 1), (1, 2) \ldots\ldots (6, 5), (6, 6)\}.$$

then $E_1 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

$E_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$E_3 = \{(1, 2), (2, 4), (3, 6)\}$

are the compound events.

Here E_1 is the event of getting the sum of 6

E_2 is the event of getting identical results

E_3 is the event of getting the twice number by the second die than that by first die.

Occurrence of Events : In a random experiment, let S be the sample space and E be the event such that $E \subseteq S$, Let w be an outcome of a trial such that $w \in A$, then we say that the event E has occurred. If $w \notin E$, we say that the event E has not occurred.

EXAMPLE 1 Consider the random experiment of throwing an unbiased die. Let E be the event of getting an odd number, then $E = \{1, 3, 5\}$

Now, in a trial, let the outcome be 3, Since $3 \in E$, so in this trial, the event E has occurred.

In another trial, let the outcome be 4.

Since $4 \notin E$ so in this trial, the event E has not occurred.

EXAMPLE 2 Consider the random experiment of throwing a die. Let the outcome of the trial is 6. Then we can say that the following events have occurred.

(i) Getting a number greater than or equal to 3, represented by the set $\{3, 4, 5, 6\}$

(ii) Getting an even number represented by the set $\{2, 4, 6\}$

We can also say that the following events have not occurred :

(i) Getting an odd number represented by the set $\{1, 3, 5\}$.

(ii) Getting a prime number represented by the set $\{2, 3, 5\}$

Impossible Events : Let S be a sample space associated with a random experiment,

Now, since $\phi \subseteq S$, so ϕ is an event, called an impossible event.

Sure (or certain) Event : Let S be a sample space associated with a random experiment.

Now, since $S \subseteq S$, so S is an event, called a sure or certain

EXAMPLE Consider the random experiment of throwing an unbiased die. The sample space $S = \{1, 2, 3, 4, 5, 6\}$.

Let E_1 be the event of getting a number less than 1.

and E_2 be the event of getting a number greater than 6.

Here E_1 and E_2 both are impossible events.

Let E_3 be the event of getting a number less than or equal to 7.

and E_4 be the event of getting a number multiple of 2 but less than 7.

Here E_3 and E_4 are the certain events.

Equally Likely Events : Events are said to be equally likely, if none of them is expected to occur in preference to the other.

EXAMPLE 1 If an unbiased die is rolled, then each outcome is equally likely to happen *i.e.*, all elementary events are equally likely to happen. If however, the die is so formed that a particular face occurs most often, then the die is biased. So in this case, the outcomes are not equally likely to happen.

Favourable Events : Let S be the sample space associated with a random experiment and let $E \subset S$. Then the elementary events belonging to E are known as the favourable events to A.

EXAMPLE 1 Consider a random experiment of throwing a die. Let E be the event of getting an even number, then $E = \{2, 4, 6\}$.

So there are three favourable events of event E *viz.* $\{2\}$, $\{4\}$ and $\{6\}$

EXAMPLE 2 Consider a random experiment of throwing a pair of dice. Let E be the event of getting the sum as 8, then

$E = \{(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1)\}$

So, there are 7 favourable events to event E.

Complementary Events : In a random experiment, let S be the sample space and let E be an event. Then $E \subseteq S$. Clearly, $E^c \subseteq S$. So E^c is also an event, called the complementary of E.

Sometimes E^c is denoted by $\overline{E}$ or E'.

Where $\overline{E}$ is called as "not-E"

Thus it is clear that $\overline{E}$ occurs only when E does not occur.

Also in a trial one and only of E and $\overline{E}$ must occur.

Algebra of Events : In a random experiment, let the sample space be S. Let $E \subseteq S$, $F \subseteq S$ and $G \subseteq S$, then we can define the following relations in the set theory mode.

(i) $\overline{E}$ is an event that occurs only when E does not occur.

(ii) $(E \cup F)$ is an event that occurs only when E occurs or F occurs or both occurs.

(iii) $(E \cap F)$ is an event that occurs only when each one of E and F occurs.

(iv) $(E \cap \overline{F})$ is an event that occurs only when E occurs but not F.

(v) $(\overline{E} \cap \overline{F}) = (\overline{E \cup F})$ is an event that occurs when neither E nor F occurs.

(vi) $(E \cup F \cup G)$ is an event that occurs when at least one of E, F or G occurs.

(vii) $(E \cap F \cap G)$ is an event that occurs when all three E, F and G occurs.

(viii) $(E \cap \bar{F}) \cup (\bar{E} \cap F)$ is an event that occurs when exactly one of E and F occurs.

(ix) $(E \cap F \cap \bar{G}) \cup (E \cap \bar{F} \cap G) \cup (\bar{E} \cap F \cap G)$ is an event that occurs when exactly two of E, F and G occurs.

Mutually Exclusive Events : Let S be the sample space associated with a random experiment and let E_1 and E_2 be the two events. Then E_1 and E_2 are mutually exclusive events if $E_1 \cap E_2 = \phi$

EXAMPLE 1 Consider a random experiment of throwing a die. Let E_1, E_2 and E_3 be three events such that

$E_1 = \{1, 3, 5\}$, the event of getting an odd number

$E_2 = \{2, 4, 6\}$, the event of getting an even number

$E_3 = \{2, 3, 5\}$, the event of getting a prime number.

Clearly $E_1 \cap E_2 = \phi$ and $E_1 \cap E_3 \neq \phi$, $E_2 \cap E_3 \neq \phi$ Hence E_1 and E_2 are mutually exclusive events but E_1 and E_3, E_2 and E_3 are not mutually exclusive.

MUTUALLY EXCLUSIVE & EXHAUSTIVE SYSTEM OF EVENTS

Let S be the sample space associated with a random experiment. Let $E_1, E_2, \ldots E_n$ be the subsets of S such that

(i) $E_i \cap E_j = \phi$ for $i \neq j$ and

(ii) $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$

then the set of events $E_1, E_2, E_3 \ldots E_n$ is said to form a mutually exclusive and exhaustive system of events.

EXAMPLE 1 In a sample space all the elementary events form a mutually exclusive and exhaustive system of events.

EXAMPLE 2 Let $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space when an unbiased die is rolled.

Let $E_1 = \{1, 3, 5\}$ and $E_2 = \{2, 4, 6\}$, then E_1 and E_2 form the mutually exclusive and exhaustive system of events.

Also, let $E_3 = \{1, 2\}$, $E_4 = \{4, 5\}$, then E_3 and E_4 do not form the mutually exclusive and exhaustive system of events since E_3 and E_4 do not include all the elementary events of S.

Again if $E_5 = \{3, 6\}$ and $E_6 = \{2, 4, 6\}$, then E_5 and E_6 do not form mutually exclusive and exhaustive system of events since $E_5 \cap E_6 \neq \phi$ and E_5, E_6 do not include all the elementary events of the sample space S.

- A pack of cards consists of 52 cards. There are four suits each containing 13 cards called as spades, clubs, hearts and diamonds.

All the spades and clubs are black faced cards while hearts and diamonds are red faced cards. The aces, kings, queens and jacks are known as face cards.

In each suit there is one ace, one king, one queen and one jack and rest 9 cards are numbered cards.

Definition of Probability : In a random experiment, let S be the sample space and let $E \subseteq S$.

where E is an event.

The probability of occurrence of the event E is defined as

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{\text{number of elements in } E}{\text{number of elements in } S} = \frac{n(E)}{n(S)}$$

$$= \frac{\text{number of elementary events in } E}{\text{number of elementary events in } S}$$

From the above definitions it is clear that

(i) $0 \leq P(E) \leq 1$

(ii) $P(\phi) = 0$

(iii) $P(S) = 1$

Also, $$P(\bar{E}) = \frac{\text{number of elementary events in } \bar{E}}{\text{number of elementary events in } S}$$

$$= \frac{n(S) - n(E)}{n(S)} = 1 - \frac{n(E)}{n(S)} = 1 - P(E)$$

$$\Rightarrow \quad P(\bar{E}) = 1 - P(E)$$

$$\therefore \quad P(E) + P(\bar{E}) = 1$$

Odds in Favour of An Event and Odds Against An Event

In m be the number of ways in which an event occurs & n be the number of ways in which it does not occur, then

(i) odds in favour of the event $= \frac{m}{n}$ (or $m:n$)

(ii) odds against the event $= \frac{n}{m}$ (or $n:m$)

INTRODUCTORY EXERCISE-20.1

1. Find the probability of getting a head in a throw of a coin.

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) none of these

Directions for question number 2-5 : Two fair coins are tossed simultaneously. Find the probability of

2. Getting only one head.

(a) 1/2 (b) 1/3

(c) 2/3 (d) 3/4

3. Getting atleast one head.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

4. Getting two heads.
(a) $\frac{2}{7}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{4}{5}$

5. Getting atleast two heads :
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 1

***Directions for question number 6-12 :** Three fair coins are tossed simultaneously. Find the probability of*

6. Getting one head.
(a) 0 (b) 3/4
(c) $\frac{5}{8}$ (d) $\frac{3}{8}$

7. Getting one tail.
(a) 1 (b) $\frac{1}{4}$
(c) $\frac{5}{8}$ (d) $\frac{3}{8}$

8. Getting atlest one head.
(a) $\frac{7}{8}$ (b) $\frac{1}{8}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

9. Getting two heads.
(a) $\frac{3}{5}$ (b) $\frac{3}{8}$
(c) $\frac{5}{8}$ (d) $\frac{2}{5}$

10. Getting atleast two heads.
(a) $\frac{3}{8}$ (b) $\frac{7}{8}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

11. Getting atleast one head and one tail.
(a) $\frac{2}{8}$ (b) $\frac{1}{2}$
(c) $\frac{3}{10}$ (d) $\frac{3}{4}$

12. Getting more heads than the number of tails.
(a) 2 (b) 7/8
(c) $\frac{5}{8}$ (d) $\frac{1}{2}$

***Directions for question number 13-16 :** An unbiased die is rolled. Find the probability of*

13. Getting a number less than 7 but greater than zero.
(a) 0 (b) 3/4
(c) 1 (d) $\frac{7}{8}$

14. Getting a multiple of 3.
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{5}{6}$ (d) none of these

15. Getting a prime number.
(a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) $\frac{5}{7}$ (d) $\frac{5}{8}$

16. Getting an even number.
(a) $\frac{1}{2}$ (b) $\frac{4}{5}$
(c) $\frac{2}{8}$ (d) $\frac{3}{4}$

***Directions for question number 17 and 18 :** A coin is tossed successively three times. Find the probability of*

17. Getting exactly one head or two heads.
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

18. Getting no heads.
(a) 0 (b) 1
(c) $\frac{1}{8}$ (d) $\frac{7}{8}$

***Directions for question number 19-27 :** Two dice are rolled simultaneously. Find the probability of*

19. Getting a total of 9.
(a) $\frac{1}{3}$ (b) $\frac{1}{9}$
(c) $\frac{8}{9}$ (d) $\frac{9}{10}$

20. Getting a sum greater than 9.
(a) $\frac{10}{11}$ (b) $\frac{5}{6}$
(c) $\frac{1}{6}$ (d) $\frac{8}{9}$

21. Getting a total of 9 or 11.
(a) $\frac{2}{99}$ (b) $\frac{20}{99}$
(c) $\frac{1}{6}$ (d) $\frac{1}{10}$

22. Getting a doublet.
(a) 1/12 (b) 0
(c) 5/8 (d) 1/6

23. Getting a doublet of even numbers.
(a) 5/8 (b) 1/12
(c) 3/4 (d) 1/4

24. Getting a multiple of 2 on one die and a multiple of 3 on the other.
(a) $\frac{15}{36}$ (b) $\frac{25}{36}$
(c) $\frac{11}{36}$ (d) $\frac{5}{6}$

25. Getting the sum of numbers on the two faces divisible by 3 or 4.
(a) $\frac{4}{9}$ (b) $\frac{1}{7}$
(c) $\frac{5}{9}$ (d) $\frac{7}{12}$

26. Getting the sum as a prime number.

(a) $\frac{3}{5}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

27. Getting atleast one '5'.

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$

(c) $\frac{5}{36}$ (d) $\frac{11}{36}$

***Directions for question number 28-35 :** One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that*

28. The card drawn is black.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{8}{13}$ (d) can't be determine

29. The card drawn is a queen.

(a) $\frac{1}{12}$ (b) $\frac{1}{13}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

30. The card drawn is black and a queen.

(a) $\frac{1}{13}$ (b) $\frac{1}{52}$

(c) $\frac{1}{26}$ (d) $\frac{5}{6}$

31. The card drawn is either black or a queen.

(a) $\frac{15}{26}$ (b) $\frac{13}{17}$

(c) $\frac{7}{13}$ (d) $\frac{15}{26}$

32. The card drawn is either king or a queen.

(a) $\frac{5}{26}$ (b) $\frac{1}{13}$

(c) $\frac{2}{13}$ (d) $\frac{12}{13}$

33. The card drawn is eithe a heart, a queen or a king.

(a) $\frac{17}{52}$ (b) $\frac{21}{52}$

(c) $\frac{19}{52}$ (d) $\frac{9}{26}$

34. The card drawn is neither a spade nor a king.

(a) 0 (b) $\frac{9}{13}$

(c) $\frac{1}{2}$ (d) $\frac{4}{13}$

35. The card drawn is neither an ace nor a king.

(a) $\frac{11}{13}$ (b) $\frac{1}{2}$

(c) $\frac{2}{13}$ (d) $\frac{11}{26}$

36. From a well shuffled pack of 52 cards, three cards are drawn at random. Find the probability of drawing an ace, a king and a jack.

(a) $\frac{16}{5525}$ (b) $\frac{16}{625}$

(c) $\frac{16}{3125}$ (d) none of these

37. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of same number.

(a) $\frac{17}{1665}$ (b) $\frac{1}{20825}$

(c) $\frac{7}{25850}$ (d) none of these

38. From a well shuffled pack of 52 playing cards, four cards are accidently dropped. Find the probability that one card is missing from each suit.

(a) $\frac{17}{20825}$ (b) $\frac{2197}{20825}$

(c) $\frac{197}{1665}$ (d) none of these

39. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of different numbers.

(a) $\frac{141}{4165}$ (b) $\frac{117}{833}$

(c) $\frac{264}{4165}$ (d) none of these

***Directions for question number 40-43 :** Four dice are thrown simultaneously. Find the probability that :*

40. All of them show the same face.

(a) $\frac{1}{216}$ (b) $\frac{15}{16}$

(c) $\frac{15}{36}$ (d) $\frac{1}{2}$

41. All of them show the different face.

(a) $\frac{3}{28}$ (b) $\frac{5}{18}$

(c) $\frac{15}{36}$ (d) $\frac{11}{36}$

42. Two of them show the same face and remaining two show the different faces.

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$

(c) $\frac{11}{18}$ (d) $\frac{7}{9}$

43. Atleast two of them show the same face.

(a) $\frac{37}{72}$ (b) $\frac{11}{36}$

(c) $\frac{47}{72}$ (d) $\frac{25}{36}$

44. What is the probability that a number selected from the numbers 1, 2, 3, ..., 20, is a prime number when each of the given numbers is equally likely to be selected?

(a) 7/10 (b) 2/15

(c) 2/5 (d) 3/5

45. Tickets are numbered from 1 to 18 are mixed together and then 9 ticket is drawn at random. Find the probability that the ticket has a number, which is a multiple of 2 or 3.

(a) $\frac{1}{3}$ (b) $\frac{3}{5}$
(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

46. In a lottery of 100 tickets numbered 1 to 100, two tickets are drawn simultaneously. Find the probability that both the tickets drawn have prime numbers.
(a) $\frac{2}{33}$ (b) $\frac{7}{50}$
(c) $\frac{7}{20}$ (d) $\frac{5}{66}$

47. In the previous question (number 46), find the probability that none of the tickets drawn has a prime number.
(a) $\frac{29}{66}$ (b) $\frac{17}{33}$
(c) $\frac{37}{66}$ (d) $\frac{17}{50}$

48. Find the probability that a leap year selected at random will contain 53 Sundays.
(a) $\frac{5}{7}$ (b) $\frac{3}{4}$
(c) $\frac{4}{7}$ (d) $\frac{2}{7}$

Directions for question number 49-53: *A bag contains 8 red and 4 green balls. Find the probability that*

49. The ball drawn is red when one ball is selected at random.
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

50. All the 4 balls drawn are red when four balls are drawn at random.
(a) $\frac{17}{32}$ (b) $\frac{14}{99}$
(c) $\frac{7}{12}$ (d) none of these

51. All the 4 balls drawn are green when four balls are drawn at random.
(a) $\frac{1}{495}$ (b) $\frac{7}{99}$
(c) $\frac{5}{12}$ (d) $\frac{2}{3}$

52. Two balls are red and one ball is green when three balls are drawn at random.
(a) $\frac{56}{99}$ (b) $\frac{112}{495}$
(c) $\frac{78}{495}$ (d) none of these

53. Three balls are drawn and none of them is red.
(a) $\frac{68}{99}$ (b) $\frac{7}{99}$
(c) $\frac{4}{495}$ (d) none of these

54. The odds in favour of an event are 2 : 7. Find the probability of occurrence of this event.
(a) $\frac{2}{9}$ (b) $\frac{5}{12}$
(c) $\frac{7}{12}$ (d) $\frac{2}{5}$

55. The odds against of an event are 5 : 7. find the probability of occurrence of this event.
(a) $\frac{3}{8}$ (b) $\frac{7}{12}$
(c) $\frac{2}{7}$ (d) $\frac{5}{12}$

56. If there are two children in a family, find the probability that there is atleast one girl in the family.
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) none of these

57. From a group of 3 men and 2 women, two persons are selected at random. Find the probability that atleast one woman is selected.
(a) $\frac{1}{5}$ (b) $\frac{7}{10}$
(c) $\frac{2}{5}$ (d) $\frac{5}{6}$

58. A box contains 5 defective and 15 non-defective bulbs. Two bulbs are chosen at random. Find the probability that both the bulbs are non-defective.
(a) $\frac{5}{19}$ (b) $\frac{3}{20}$
(c) $\frac{21}{38}$ (d) none of these

59. In the previous question (number 58), find the probability that atleast 3 bulbs are defective when 4 bulbs are selected at random.
(a) $\frac{31}{969}$ (b) $\frac{7}{20}$
(c) $\frac{1}{20}$ (d) none of these

IMPORTANT RESULTS

1. $P(E) \geq 0$
2. $P(\phi) = 0$
3. $P(S) = 1$

Addition Theorem 1 : If A and B are two events associated with a random experiment.

Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Corollary: If A and B are mutually exclusive events.

then $P(A \cap B) = 0$, therefore

$P(A \cup B) = P(A) + P(B)$

Addition Theorem 2 : If A, B and C three events associated with a random experiment,

then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Corollary : If A, B and C are the three mutually exclusive events, then

$$P(A \cap B) = P(B \cap C) = P(A \cap C)$$
$$= P(A \cap B \cap C) = 0$$

$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Theorem 3. If A and B be two events such that $A \subseteq B$, then $P(A) \leq P(B)$

Theorem 4. If E is an event associated with a random experiment, then

$$0 \leq P(E) \leq 1$$

Theorem 5. For any two events A and B

$$P(A - B) = P(A) - P(A \cap B)$$
$$P(B - A) = P(B) - P(A \cap B)$$
$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$
$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

SOME IMPORTANT RESULTS

(A) If A, B and C are three events, then

(i) P [Exactly one of A, B, C occurs]

$$= P(A) + P(B) + P(C) - 2[(A \cap B) + (B \cap C) + (A \cap C)] + 3P(A \cap B \cap C)$$

(ii) P [Exactly two of A, B, C occur]

$$= P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$$

(iii) P (Atleast two of A, B, C occur)

$$= P(A \cap B) + P(B \cap C) + P(A \cap C) - 2P(A \cap B \cap C)$$

(B) If A and B are two events, then

P(Exactly one of A, B occurs)

$$= P(A) + P(B) - 2P(A \cup B)$$
$$= P(A \cup B) - P(A \cap B)$$

INTRODUCTORY EXERCISE-20.2

1. The probability of occurrence of two events A and B are 1/4 and 1/2 respectively. The probability of their simultaneous occurrence is $\frac{7}{50}$. Find the probability that either A or B must occur.

(a) $\frac{61}{100}$ (b) $\frac{29}{100}$

(c) $\frac{39}{100}$ (d) $\frac{56}{99}$

2. In the previous question, find the probability that neither A nor B occurs.

(a) $\frac{25}{99}$ (b) $\frac{39}{100}$

(c) $\frac{61}{100}$ (d) $\frac{17}{100}$

3. If A and B be two events in a sample space such that $P(A) = \frac{3}{10}$ and $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{5}$. Find $P(A \cup B)$.

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

4. If A and B be two events in a sample space such that $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$, find $P(A \cap B)$.

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$

(c) $\frac{4}{7}$ (d) $\frac{4}{15}$

***Directions for question number 5-10 :** If A and B be two mutually exclusive events in a sample space such that, $P(A) = \frac{2}{5}$ and $P(B) = \frac{1}{2}$, then*

5. Find $P(\bar{A})$:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{6}{7}$

6. Find $P(\bar{B})$:

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{4}{5}$

7. Find $P(A \cup B)$:

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$

(c) $\frac{9}{10}$ (d) $\frac{1}{2}$

8. Find $P(\bar{A} \cap \bar{B})$:

(a) $\frac{4}{5}$ (b) $\frac{1}{10}$

(c) $\frac{8}{9}$ (d) $\frac{13}{20}$

9. Find $P(\bar{A} \cap B)$:

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$

(c) $\frac{4}{7}$ (d) $\frac{7}{15}$

10. Find $P(A \cap \bar{B})$:

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{4}{15}$, (d) $\frac{3}{10}$

11. If $P(\bar{A}) = 0{\cdot}65$, $P(A \cup B) = 0{\cdot}65$, where A and B are two mutually exclusive events, then find $P(B)$.

(a) 0·60 (b) 0·30

(c) 0·70 (d) none of these

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12. If A, B and C are three mutually exclusive and exhaustive events. Find $P(A)$, if $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$.
(a) $\frac{8}{13}$ (b) $\frac{5}{13}$
(c) $\frac{4}{13}$ (d) $\frac{9}{13}$

13. Two dice are tossed once. Find the probability of getting an even number on first die, or a total of 8.
(a) $\frac{4}{9}$ (b) $\frac{2}{3}$
(c) $\frac{5}{9}$ (d) $\frac{1}{3}$

14. A die is thrown twice, what is the probability that atleast one of the two throws comes up with the number 5?
(a) $\frac{11}{36}$ (b) $\frac{5}{6}$
(c) $\frac{15}{36}$ (d) none of these

15. In a single throw of two dice, find the probability that neither a doublet nor a total of 8 will appear.
(a) $\frac{7}{15}$ (b) $\frac{5}{18}$
(c) $\frac{13}{18}$ (d) $\frac{3}{16}$

16. A die is thrown twice, what is the probability that atleast one of the two numbers is 6?
(a) $\frac{11}{12}$ (b) $\frac{11}{36}$
(c) $\frac{1}{6}$ (d) $\frac{7}{36}$

17. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a heart or a king.
(a) $\frac{4}{13}$ (b) $\frac{9}{13}$
(c) $\frac{8}{13}$ (d) $\frac{11}{26}$

18. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are queens?
(a) $\frac{17}{112}$ (b) $\frac{55}{221}$
(c) $\frac{55}{121}$ (d) $\frac{33}{221}$

19. A card is drawn from a deck of 52 cards. Find the probability of getting a red card or a heart or a king.
(a) $\frac{6}{13}$ (b) $\frac{7}{13}$
(c) $\frac{11}{26}$ (d) $\frac{15}{26}$

20. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit.
(a) $\frac{5}{13}$ (b) $\frac{12}{65}$
(c) $\frac{44}{4165}$ (d) $\frac{44}{169}$

21. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability that the drawn card is a king or a queen.
(a) $\frac{2}{13}$ (b) $\frac{8}{13}$
(c) $\frac{11}{13}$ (d) none of these

22. Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are jacks?
(a) $\frac{65}{121}$ (b) $\frac{55}{221}$
(c) $\frac{17}{221}$ (d) none of these

23. A natural number is chosen at random from amongst the first 300. What is the probability that the number, so chosen is divisible by 3 or 5?
(a) $\frac{48}{515}$ (b) $\frac{4}{150}$
(c) 1/2 (d) none of these

24. A natural number is chosen at random from the first 100 natural numbers. What is the probability that the number chosen is a multiple of 2 or 3 or 5?
(a) $\frac{30}{100}$ (b) $\frac{1}{33}$
(c) $\frac{74}{100}$ (d) $\frac{7}{10}$

25. A box contains 5 red balls, 8 green balls and 10 pink balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?
(a) $\frac{13}{23}$ (b) $\frac{10}{23}$
(c) $\frac{11}{23}$ (d) $\frac{13}{529}$

26. A basket contains 10 apples and 20 oranges out of which 3 apples and 5 oranges are defective. If we choose two fruits at random, what is the probability that either both are oranges or both are non defective?
(a) $\frac{136}{345}$ (b) $\frac{17}{87}$
(c) $\frac{316}{435}$ (d) $\frac{158}{435}$

27. In a class 40% of the students offered Physics 20% offered Chemistry and 5% offered both. If a student is selected at random, find the probability that he has offered Physics or Chemistry only.
(a) 45% (b) 55%
(c) 36% (d) none of these

28. The probability that an MBA aspirant will join IIM is $\frac{2}{5}$ and that he will join XLRI is $\frac{1}{3}$. Find the probability that he will join IIM or XLRI.
(a) $\frac{4}{15}$ (b) $\frac{7}{15}$
(c) $\frac{11}{15}$ (d) $\frac{8}{15}$

29. In a given race, the odds in favour of horses H_1, H_2, H_3 and H_4 are $1:2, 1:3, 1:4, 1:5$ respectively. Find the probability that one of them wins the race.

(a) $\frac{57}{60}$ (b) $\frac{1}{20}$

(c) $\frac{2}{7}$ (d) $\frac{7}{60}$

CONDITIONAL PROBABILITY

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability and it is denoted by $P\left(\frac{A}{B}\right)$

Thus, $P\left(\frac{A}{B}\right)$ = Probability of occurrence of A given that B has already occurred.

Similarly, $P\left(\frac{B}{A}\right)$ = Probability of occurrence of B given that A has already occurred.

NOTE

(i) Sometimes $P\left(\frac{A}{B}\right)$ is used to denote the probability of occurrence of A when B occurs.

(ii) Similarly $P\left(\frac{B}{A}\right)$ is used to denoted the probability of occurrence of B when A occurs.

The above two cases happens due to the simultaneous occurrence of two events since the two events are the subsets of the same sample space.

EXAMPLE 1 *An urn contains 6 red and 9 blue balls. Two balls are drawn from the urn one after another without replacement. Find the probability of drawing a red ball when a blue ball has been drawn from the urn.*

SOLUTION Let A = drawing of a red ball in the second draw and B = drawing of a blue ball in the first draw

Now $P\left(\frac{A}{B}\right)$ = Probability of drawing a red ball in second draw when a blue ball has been drawn in the first draw.

Now, since there are only 14 balls after drawing a blue ball in first draw and out of these 14 balls, 6 balls are red.

Therefore $$P\left(\frac{A}{B}\right) = \frac{6}{14} = \frac{3}{7}$$

EXAMPLE 2 *A pair of dice is thrown simultaneously, find the probability that the sum is obtained 9 when there is an odd number on the first die.*

SOLUTION Let A be the event of getting a sum of 9 and B be the event of getting an odd number on the first die.

$\therefore$ $A = \{(3, 6)(4, 5), (5, 4), (6, 3)\}$

$B = \{(1, 1), (1, 2), (1, 3) \ldots (3, 1), (3, 2), (3, 3) \ldots (5, 1), (5, 2) \ldots (5, 6)\}$

$$P(A) = \frac{4}{36} = \frac{1}{9} \text{ and } P(B) = \frac{18}{36} = \frac{1}{2}$$

$\therefore P\left(\frac{A}{B}\right)$ = Probability of occurrence of A when B Occurs.

$\Rightarrow P\left(\frac{A}{B}\right)$ = Probability of getting 9 as the sum when there is an odd number on first die.

$$\Rightarrow \quad P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{2}{18} = \frac{1}{9}$$

[Here $(A \cap B) = (3, 6), (6, 3)]$

MULTIPLICATION THEOREM

Let A and B be two events associated with the same random experiment then

$$P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right) \quad \text{if } P(A) \neq 0 \ldots\text{(i)}$$

or $$P(A \cap B) = P(B)\, P\left(\frac{A}{B}\right), \quad P(B) \neq 0 \ldots\text{(ii)}$$

NOTE $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$ from (i)

and $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ from (ii)

In general, if $A_1, A_2, A_3 \ldots A_n$ are events associated with the same random experiment, then

$$P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1)\, P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) \ldots P\left(\frac{A_n}{A_1 \cap A_2 \cap \ldots \cap A_{n-1}}\right)$$

EXAMPLE 1 *Let A and B be the two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, find*

(i) $P\left(\frac{A}{B}\right)$ (ii) $P\left(\frac{B}{A}\right)$ (iii) $P(A \cup B)$ (iv) $P\left(\frac{\bar{A}}{\bar{B}}\right)$

SOLUTION (i) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$$

(iv) $P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= \frac{1 - \frac{7}{12}}{1 - \frac{1}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{8}$$

EXAMPLE 2 *An urn contains 6 red balls and 9 green balls. Two balls are drawn in succession without replacement. What is the probability that first is red and second is green.*

SOLUTION Let A be the event of drawing a red ball in first draw and B be the event of getting a green ball in the second draw.

$$\therefore \quad P(A) = \frac{^6C_1}{^{15}C_1} = \frac{6}{15} = \frac{2}{5}$$

$P\left(\frac{B}{A}\right)$ = Probability of getting a green ball in second draw when a red ball has been selected in first draw $= \frac{^9C_1}{^{14}C_1} = \frac{9}{14}$

$\therefore$ Required probability $= P(A \cap B) = P(A)\,P\left(\frac{B}{A}\right)$

$$= \frac{2}{5} \times \frac{9}{14} = \frac{9}{35}$$

Introductory Exercise-20.3

1. If A and B are two events such that $P(A) = 0\cdot5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$. Find $P\left(\frac{A}{B}\right)$.
   (a) $\frac{1}{3}$ (b) $\frac{1}{2}$
   (c) $\frac{1}{4}$ (d) none of these

2. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P\left(\frac{B}{A}\right) = 0.6$, find $P(A \cup B)$:
   (a) 0.24 (b) 0.96
   (c) 0.04 (d) none of these

3. Three fair coins are tossed. Find the probability that they are all tails, if one of the coins shows a tail.
   (a) $\frac{2}{7}$ (b) $\frac{5}{14}$
   (c) $\frac{1}{7}$ (d) none of these

4. A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If E is the event, "Both head and tail have appeared and F be the event, at most one tail is observed ; find $P\left(\frac{E}{F}\right)$ and $P\left(\frac{F}{E}\right)$.
   (a) $\frac{2}{3}, 1$ (b) $\frac{1}{3}, \frac{2}{3}$
   (c) $1, \frac{2}{3}$ (d) none of these

5. A coin is tossed twice if the coin shows head it is tossed again but if it shows a tail then a die is tossed. If 8 possible outcomes are equally likely. Find the probability that the die shows a number greater than 4, if it is known that the first throw of the coin results in a tail.
   (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
   (c) $\frac{2}{5}$ (d) $\frac{4}{15}$

6. A die is rolled. If the outcome is an odd number, what is the probability that it is a prime number?
   (a) $\frac{3}{8}$ (b) $\frac{7}{9}$
   (c) $\frac{2}{3}$ (d) none of these

7. A die is thrown twice and the sum of the numbers appearing is observed to be 9. What is the conditional probability that the number 4 has appeared atleast once?
   (a) $\frac{1}{2}$ (b) $\frac{2}{3}$
   (c) $\frac{3}{4}$ (d) none of these

8. Two dice are thrown. Find the probability that the sum is 8 or greater than 8, if 4 appears on the first die.
   (a) $\frac{3}{8}$ (b) $\frac{5}{8}$
   (c) $\frac{1}{2}$ (d) none of these

9. A die is rolled. If the outcome is an odd number, what is the probability that it is a number greater than 1 ?
   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
   (c) $\frac{3}{8}$ (d) $\frac{5}{6}$

10. In a class 45% students read English, 30% read French and 20% read both English and French. One student is selected at random. Find the probability that he reads English, if it is known that he reads French.
    (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
    (c) $\frac{5}{6}$ (d) none of these

11. In the previous question, find the probability that he reads French, if it is known that he reads English.
    (a) $\frac{4}{9}$ (b) $\frac{5}{9}$
    (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

12. A couple has two children. Find the probability that both are boys, if it is known that one of the children is a boy.
(a) $\frac{1}{9}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{4}{5}$

13. In the previous question find the probability that both are boys, if it is known that the older child is a boy.
(a) $\frac{3}{8}$ (b) $\frac{1}{2}$
(c) $\frac{5}{8}$ (d) $\frac{3}{4}$

***Directions for question number 14-17 :** A bag contains 3 red and 4 black balls and another bag has 4 red and 2 black balls. One bag is selected at random and from the selected bag a ball is drawn. Let E be the event that the first bag is selected, F be the event that the second bag is selected, G be the event that ball drawn is red.*

14. Find $P(E)$.
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$
(c) $\frac{1}{4}$ (d) $\frac{5}{8}$

15. Find $P(F)$.
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

16. Find $P\left(\frac{G}{E}\right)$.
(a) $\frac{5}{6}$ (b) $\frac{5}{14}$
(c) $\frac{3}{7}$ (d) none of these

17. Find $P\left(\frac{G}{F}\right)$.
(a) $\frac{2}{3}$ (b) $\frac{1}{9}$
(c) $\frac{5}{9}$ (d) $\frac{4}{5}$

18. Two balls are drawn from a bag containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that atleast one ball is red?
(a) $\frac{7}{12}$ (b) $\frac{5}{12}$
(c) $\frac{3}{10}$ (d) none of these

19. A bag contains 6 red and 9 blue balls. Two successive drawing of four balls are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 4 red balls and second draw gives 4 blue balls.
(a) $\frac{3}{715}$ (b) $\frac{7}{715}$
(c) $\frac{15}{233}$ (d) none of these

20. An urn contains 4 white 6 black and 8 red balls. If 3 balls are drawn one by one without replacement, find the probability of getting all white balls.
(a) $\frac{5}{204}$ (b) $\frac{1}{204}$
(c) 13/204 (d) none of these

21. Two numbers are selected at random from the integers 1 through 9. If the sum is even, find the probability that both numbers are odd.
(a) 5/8 (b) 3/8
(c) 3/10 (d) none of these

22. A box contains 25 tickets, numbered 1, 2, 3, .. 25. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show odd numbers.
(a) $\frac{37}{50}$ (b) $\frac{13}{50}$
(c) 13/25 (d) none of these

INDEPENDENT EVENTS

Events are said to be independent, if the occurrence of one does not depend upon the occurrence of the other.

Suppose an urn contains m red balls and n green balls. Two balls are drawn from the urn one after the other.

If the ball drawn in the first draw is not replaced back in the bag, then two events of drawing the ball are dependent because first draw of the ball determine the probability of drawing the second ball.

If the ball drawn in the first draw is replaced back in the bag, then two events are independent because first draw of a ball has no effect on the second draw.

Theorem 1. Two events A and B associated with the same sample space of a random experiment are independent if and only if

$$P(A\cap B)=P(A)\cdot P(B)$$

Theorem 2. If $A_1, A_2, A_3 ... A_n$ are independent events associated with a random experiment, then

$$P(A_1\cap A_2\cap A_3 ...\cap A_n)=P(A_1)P(A_2)...P(A_n)$$

Theorem 3. If $A_1, A_2 ... A_n$ are n independent events associated with a random experiment, then

$$P(A_1\cup A_2\cup ...\cup A_n)=1-P(\bar{A}_1)P(\bar{A}_2)...P(\bar{A}_n)$$

IMPORTANT RESULTS

If A and B are independent events then the following events are also independent.

(i) $A\cap\bar{B}$ (ii) $\bar{A}\cap B$ (iii) $\bar{A}\cap\bar{B}$

LAW OF TOTAL PROBABILITY

Let $E_1, E_2, ... E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$$P(A)=P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+...P(E_n)P\left(\frac{A}{E_n}\right)$$

Baye's Rule : Let $E_1, E_2, \ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\,P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}, \qquad i = 1, 2, \ldots n$$

EXAMPLE 1 *A coin is tossed twice and all 4 outcomes are equally likely. Let A be the event that first throw results in a head and B be the event that second throw results in a tail, then show that the events A and B are independent.*

SOLUTION

$$S = \{HH, HT, TH, TT\}$$
$$A = \{HH, HT\}$$
$$B = \{HT, TT\}$$
$$A \cap B = \{HT\}$$

$\therefore \quad P(A) = \frac{2}{4} = \frac{1}{2}, \; P(B) = \frac{2}{4} = \frac{1}{2}$

and $\quad P(A \cap B) = \frac{1}{4}$

Clearly $\quad P(A \cap B) = \frac{1}{4} = P(A) \cdot P(B)$

Hence A and B are independent events.

EXAMPLE 2 *A bag contains 10 red balls and 10 green balls. Two balls are drawn at random, one at a time, with replacement. Let A be the event that first ball is red, B be the event that second ball is green and C be the event that both balls are either red or green, then show that the events A, B and C are pair wise independent and A, B, C are mutually dependent.*

SOLUTION We have $P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{2},$

$$P(C) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$$

$P(A \cap B)$ = Probability that the first is red and second is green

$\Rightarrow \quad P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B \cap C)$ = Probability that both the balls are green

$\Rightarrow \quad P(B) \cdot P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

and $P(A \cap C)$ = Probability that both the balls are red

$\Rightarrow \quad P(A)\,P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Hence A, B, C are pairwise independent

Now, $P(A \cap B \cap C)$ = Probability that the first ball is red and second ball is green and the first and second are both red or green $= 0$

and

$$P(A)\,P(B)\,P(C) = \frac{1}{8}$$

$$P(A \cap B \cap C) \neq P(A)\,P(B)\,P(C)$$

Thus, A, B, C are not mutually independent.

EXAMPLE 3 *(For explanation of law of total probability) There are two bags. The first bag contains 4 white and 5 black balls and the second bag contains 5 white and 4 black balls. Two balls are drawn at random from the first bag and are put into the second bag without noticing their colours. Then two balls are drawn from the second bag. Find the probability that the balls are white and black.*

SOLUTION A white and a black ball can be drawn from the second bag in the following mutually exclusive ways :

(i) By transferring 2 white balls from the first bag to the second bag and then drawing a white and a black ball from it.

(ii) By transferring 2 black balls from the first bag to the second bag and then drawing a white and a black ball from it .

(iii) By transferring 1 white and 1 black ball from first bag to the second bag and then drawing a white and a black ball from it.

Let A, B, C and D be the events as defined below :

A = Two white balls are drawn from the first bag

B = Two black balls are drawn from the first bag.

C = One white and one black ball is drawn from the first bag.

D = Two balls drawn from the second bag are white and black.

We, have $\quad P(A) = \frac{{}^4C_2}{{}^9C_2} = \frac{6}{36} = \frac{1}{6}$

$$P(B) = \frac{{}^5C_2}{{}^9C_2} = \frac{10}{36} = \frac{5}{18}$$

$$P(C) = \frac{{}^4C_1 \times {}^5C_1}{{}^9C_2} = \frac{20}{36} = \frac{5}{9}$$

If A has already occurred, *i.e.* if two white balls have been transferred from the first bag to the second bag, then the second bag will contain 7 white and 4 black balls, therefore the probability of drawing a white and a black from the second bag is

$$P\left(\frac{D}{A}\right) = \frac{{}^7C_1 \times {}^4C_1}{{}^{11}C_2} = \frac{28}{55}$$

Similarly, $\quad P\left(\frac{D}{B}\right) = \frac{{}^5C_1 \times {}^6C_1}{{}^{11}C_2} = \frac{30}{55} = \frac{6}{11}$

and $\quad P\left(\frac{D}{C}\right) = \frac{{}^6C_1 \times {}^5C_1}{{}^{11}C_2} = \frac{30}{55} = \frac{6}{11}$

$\therefore$ By the law of total probability, we have

$$P(D) = P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right) + P(C)\,P\left(\frac{D}{C}\right)$$

$$= \frac{1}{6} \times \frac{28}{55} + \frac{5}{18} \times \frac{6}{11} + \frac{5}{9} \times \frac{6}{11}$$

$$= \frac{14}{165} + \frac{5}{33} + \frac{10}{33}$$

$$= \frac{14 + 25 + 50}{165}$$

$$= \frac{89}{165}$$

EXAMPLE 4 *(For explanation of Baye's rule) Three boxes contain 6 red, 4 black, ; 5 red, 5 black and 4 red, 6 black balls respectively. One of the box is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first bag.*

SOLUTION Let A, B, C and D be the events defined as follows :

A = first box is chosen
B = second box is chosen
C = third box is chosen
D = ball drawn is red.

Since there are three boxes and one of the three boxes is chosen at random, therefore

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

If A has already occurred, then first box has been chosen which contains 6 red and 4 black balls. The probability of drawing a red ball from it is $\frac{6}{10}$

So, $$P\left(\frac{D}{A}\right) = \frac{6}{10}$$

Similarly, $P\left(\frac{D}{B}\right) = \frac{5}{10}$ and $P\left(\frac{D}{C}\right) = \frac{4}{10}$

We are required to find $P\left(\frac{A}{D}\right)$ i.e., given that the ball drawn is red, what is the probability that it is drawn from the first box. By Baye's rule.

$$P\left(\frac{A}{D}\right) = \frac{P(A)\cdot P\left(\frac{D}{A}\right)}{P(A).\, P\left(\frac{D}{A}\right) + P(B)\cdot P\left(\frac{D}{B}\right) + P(C)\cdot P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{6}{10}}{\left(\frac{1}{3}\times\frac{6}{10}\right)+\left(\frac{1}{3}\times\frac{5}{10}\right)+\left(\frac{1}{3}\times\frac{4}{10}\right)} = \frac{2}{5}$$

INTRODUCTORY EXERCISE-20.4

Directions for question number 1-5 : Let A and B be independent events such that $P(A) = 0.6$ and $P(B) = 0.4$

1. Find $P(A \cap B)$.
(a) 0.24 (b) 0.76
(c) 0.56 (d) none of these

2. Find $P(A \cup B)$.
(a) 0.24 (b) 0.76
(c) 0.36 (d) none of these

3. Find $P(\bar{A} \cap B)$.
(a) 0.24 (b) 0.16
(c) 0.36 (d) none of these

4. Find $P(A \cap \bar{B})$.
(a) 0.24 (b) 0.56
(c) 0.36 (d) 0.76

5. Find $P(\bar{A} \cap \bar{B})$.
(a) 0.76 (b) 0.54
(c) 0.36 (d) 0.24

6. If A and B are two independent events such that $P(\bar{A}) = 0.65$, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p.
(a) $\frac{7}{13}$ (b) $\frac{6}{13}$
(c) $\frac{37}{65}$ (d) none of these

7. An unbiased die is tossed twice. Find the probability of getting a 1, 2, 3 or 4 on the first toss and a 4, 5 or 6 on the second toss.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

8. Two persons A and B throw a die alternatively till one of them gets a three and wins the game. Find their respective probabilities of winning.
(a) $\frac{6}{11}, \frac{5}{11}$ (b) $\frac{5}{11}, \frac{8}{11}$
(c) $\frac{3}{11}, \frac{7}{11}$ (d) $\frac{8}{11}, \frac{3}{11}$

9. Two persons A and B throw a coin alternatively till one of them gets head and wins the game. Find their respective probabilities of winning.
(a) $\frac{1}{3}, \frac{5}{6}$ (b) $\frac{3}{5}, \frac{4}{5}$
(c) $\frac{2}{3}, \frac{1}{3}$ (d) $\frac{1}{6}, \frac{5}{6}$

10. Three persons A, B, C throw a die in succession till one gets a six and wins the game. Find their respective probabilities of winning.
(a) $\frac{36}{91}, \frac{30}{91}, \frac{25}{91}$ (b) $\frac{10}{71}, \frac{16}{91}, \frac{22}{81}$
(c) $\frac{13}{61}, \frac{15}{61}, \frac{17}{61}$ (d) none of these

11. A and B take turn in throwing two dice; the first to throw 9 being awarded. Find the ratio of probabilities of their winning if A has the first throw.
(a) 7/8 (b) 9/8
(c) 8/7 (d) 9/10

12. From a pack of 52 cards, two are drawn one by one without replacement. Find the probabilities that both of them are kings.
(a) $\frac{11}{21}$ (b) $\frac{13}{121}$
(c) $\frac{1}{221}$ (d) $\frac{1}{121}$

13. Ashmit can solve 80% of the problem given in a book and Amisha can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?
(a) 0.60 (b) 0.06
(c) 0.94 (d) 0.56

14. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. What is the probability that the target will be hit, if each one of A and B shoots the target?
(a) $\frac{5}{6}$ (b) $\frac{3}{5}$
(c) $\frac{11}{15}$ (d) $\frac{1}{6}$

15. A problem is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{7}{12}$

16. The probabilities of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.
(a) $\frac{25}{52}$ (b) $\frac{25}{56}$
(c) 13/42 (d) none of these

17. A, B and C shoot to hit a target. If A hits the target 4 times in 5 trials, B hits it 3 times in 4 trials and C hits it 2 times in 3 trials. What is the probability that the target is hit by atleast 2 persons?
(a) $\frac{5}{6}$ (b) $\frac{3}{4}$
(c) $\frac{4}{5}$ (d) $\frac{1}{9}$

18. An airgun can take a maximum of 4 shots at a balloon at some distance. The probabilities of hitting the balloon at the first, second, third and fourth shot are 0.1, 0.2, 0.3 and 0.4 respectively. What is the probability that the balloon is hit?
(a) 0.6976 (b) 0.6576
(c) 0.786 (d) none of these

19. The probability that A can solve a problem is $\frac{2}{3}$ and the probability that B can solve the same problem is $\frac{3}{5}$. Find the probability that atleast one of A and B will be able to solve the problem.
(a) $\frac{12}{15}$ (b) $\frac{13}{15}$
(c) $\frac{19}{45}$ (d) none of these

20. In the previous question (number 19). Find the probability that none of the two will be able to solve the problem.
(a) $\frac{13}{15}$ (b) $\frac{4}{15}$
(c) $\frac{2}{15}$ (d) $\frac{23}{30}$

***Directions for question number 21-24 :** The probabilities that a student will receive an A, B, C or D grade are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability that a student will receive*

21. Not an A grade.
(a) 0.4 (b) 0.6
(c) 0.56 (d) none of these

22. At most a C grade.
(a) 0.3 (b) 0.7
(c) 0.36 (d) none of these

23. B or C grade.
(a) 0.2 (b) 0.5
(c) 0.8 (d) 0.6

24. Atleast B grade.
(a) 0.21 (b) 0.3
(c) 0.7 (d) none of these

***Directions for question number 25-28 :** Ajay and his wife Kajol appear in an interview for two vaccancies in the same post. The probability of Ajay's selection is $\frac{1}{7}$ and that of his wife Kajol's selection is $\frac{1}{5}$. What is the probability that*

25. Both of them will be selected?
(a) $\frac{1}{12}$ (b) $\frac{1}{35}$
(c) $\frac{13}{35}$ (d) $\frac{12}{35}$

26. Only one of them will be selected ?
(a) $\frac{5}{7}$ (b) $\frac{1}{5}$
(c) $\frac{2}{7}$ (d) $\frac{2}{35}$

27. None of them will be selected?
(a) $\frac{12}{35}$ (b) $\frac{6}{35}$
(c) $\frac{24}{35}$ (d) none of these

28. Atleast one of them will be selected?
(a) $\frac{11}{35}$ (b) $\frac{24}{35}$
(c) $\frac{2}{7}$ (d) $\frac{1}{35}$

29. A speaks truth in 60% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident?
(a) 44% (b) 36%
(c) 64% (d) 48%

✍ ***Directions for question number 30 and 31 :*** *The odds against a husband who is 50 years old, living till he is 70 are 7 : 5 and the odds against his wife who is now 40, living till she is 60 are 5 : 3. Find the probability that*

30. The couple will be alive 20 years hence.

(a) $\frac{21}{32}$ (b) $\frac{5}{32}$

(c) $\frac{15}{32}$ (d) $\frac{12}{35}$

31. Atleast one of them will be alive 20 years hence.

(a) $\frac{61}{96}$ (b) $\frac{31}{96}$

(c) $\frac{41}{70}$ (d) none of these

32. Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the critics. Find the probability that majority are in favour of the book.

(a) $\frac{108}{343}$ (b) $\frac{209}{343}$

(c) $\frac{1}{7}$ (d) $\frac{1}{243}$

33. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of the part X, 9 out of 100 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of the part Y. Calculate the probability that the assembled product will not be defective.

(a) 0.6485 (b) 0.6565

(c) 0.8645 (d) none of these

34. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is red.

(a) $\frac{23}{42}$ (b) $\frac{19}{42}$

(c) $\frac{7}{32}$ (d) $\frac{16}{39}$

35. In a toys making factory, machine A, B and C manufacture respectively 25%, 35% and 40% of the total toys. Of their output 5%, 4% and 2% respectively are defective toys. A toy is drawn at random from the product. What is the probability that the toy drawn is defective?

(a) 0.225 (b) 0.345

(c) 0.235 (d) none of these

36. A box contains 20 bulbs. The probability that the box contains exactly 2 defective bulbs is 0.4 and the probability that the box contains exactly 3 defective bulbs is 0.6. Bulbs are drawn at random one by one without replacement and tested till the defective bulbs are found. What is the probability that the procedure ends at the twelfth testing?

(a) 0 (b) 1

(c) can't be determined (d) none of these

37. In a toys making factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total toys of their output 5%, 4% and 2% respectively are defective toys. A toy is drawn at random from the product. If the toy drawn is found to be defective, what is the probability that it is manufactured by the machine B?

(a) $\frac{17}{69}$ (b) $\frac{28}{69}$

(c) $\frac{35}{69}$ (d) none of these

38. An architecture company built 200 bridges 400 hospitals and 600 hotels. The probability of damage due to earthquake of a bridge, hospital and hotel is 0.01 0.03 and 0.15 respectively. One of the construction gets damaged with earthquake. What is the probability that it is a bridge?

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$

(c) $\frac{7}{52}$ (d) none of these

39. There are 3 boxes each containing 3 red and 5 green balls. Also there are 2 boxes, each containing 4 red and 2 green balls. A green ball is selected at random. Find the probability that this green ball is from a box of the first group.

(a) $\frac{54}{61}$ (b) $\frac{45}{61}$

(c) $\frac{8}{31}$ (d) none of these

40. A man speaks truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$

(c) $\frac{7}{8}$ (d) $\frac{1}{12}$

41. A card from a pack of 52 cards is lost. From the remaining cards of pack, two cards are drawn and are found to be diamonds. Find the probability of the missing card to be diamond.

(a) $\frac{39}{50}$ (b) $\frac{11}{50}$

(c) $\frac{23}{25}$ (d) $\frac{3}{26}$

EXERCISE

LEVEL 1

1. The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in random order to form a nine digit number. Find the probability that this number is divisible by 4 :
(a) $\frac{4}{9}$ (b) $\frac{2}{9}$
(c) $\frac{17}{81}$ (d) none of these

2. A four digit number is formed with the digits 1, 3, 4, 5 without repetition. Find the chance that the number is divisible by 5 :
(a) $\frac{3}{4}$ (b) $\frac{1}{4}$
(c) $\frac{9}{16}$ (d) $\frac{1}{16}$

3. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability that all persons leaving at different floors :
(a) $\frac{365}{2401}$ (b) $\frac{360}{2401}$
(c) $\frac{35}{2410}$ (d) none of these

4. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :
(a) $\frac{13}{32}$ (b) $\frac{27}{32}$
(c) $\frac{19}{32}$ (d) none of these

5. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :
(a) $\frac{5}{12}$ (b) $\frac{1}{2}$
(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

6. There are four calculators and it is known that exactly two of them are defective. They are tested one by one , in a random order till both the defective calculators are identified. Then the probability that only two tests are required is :
(a) $\frac{5}{6}$ (b) $\frac{1}{2}$
(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

7. 20 girls, among whom are A and B sit down at a round table. The probability that there are 4 girls between A and B is :
(a) $\frac{17}{19}$ (b) $\frac{2}{19}$
(c) $\frac{13}{19}$ (d) $\frac{6}{19}$

8. Two integers x and y are chosen with replacement out of the set $\{0, 1, 2, 3, \ldots 10\}$. Then the probability that $|x - y| > 5$ is :
(a) $\frac{7}{11}$ (b) $\frac{40}{121}$
(c) $\frac{35}{121}$ (d) $\frac{30}{121}$

9. The probability that the birthdays of 4 different persons will fall in exactly two calendar months is :
(a) $\frac{77}{1728}$ (b) $\frac{17}{87}$
(c) $\frac{11}{144}$ (d) none of these

10. If 6 objects are distributed at random among 6 persons, the probability that atleast one of them will not get any thing is :
(a) $6\cdot(6!)$ (b) $\frac{5^6}{6!}$
(c) $\frac{6^6 - 6!}{6^6}$ (d) none of these

11. There is 4 volume encyclopaedia among 40 books arranged on a shelf in a random order. If the volumes are not necessarily kept side by side, the probability that they occur in increasing order from left to right is :
(a) $\frac{1}{24}$ (b) $\frac{1}{12}$
(c) $\frac{1}{10}$ (d) none of these

12. Four numbers are multiplied together. Then the probability that the product will be divisible by 5 or 10 is :
(a) $\frac{169}{625}$ (b) $\frac{369}{625}$
(c) $\frac{169}{1626}$ (d) none of these

13. 8 couples (husband and wife) attend a dance show 'Nach Baliye' in a popular TV channel ; A lucky draw in which 4 persons picked up for a prize is held, then the probability that there is atleast one couple will be selected is :
(a) $\frac{8}{39}$ (b) $\frac{15}{39}$
(c) $\frac{12}{13}$ (d) none of these

14. Three persons A, B and C are to speak at a function along with 4 other persons. If they all speak in random order, the probability that A speaks before B and B speaks before C is :

(a) $\frac{5}{6}$ (b) $\frac{1}{6}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

15. A bag contains 16 coins of which 2 coins are counterfeit with heads on the both sides. The rest are fair coins. One is selected at random from the bag and tossed. The probability of getting a head is :

(a) $\frac{3}{16}$ (b) $\frac{13}{16}$
(c) $\frac{9}{16}$ (d) $\frac{7}{16}$

16. A committee of five persons is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is :

(a) 4/9 (b) 5/9
(c) 13/18 (d) none of these

17. A speaks truth in 60% cases and B speaks truth in 80% cases. The probability that they will say the same thing while describing a single event is :

(a) 0.36 (b) 0.56
(c) 0.48 (d) 0.20

18. Two squares are chosen at random on a chessboard, the probability that they have a side in common is :

(a) $\frac{3}{32}$ (b) $\frac{1}{32}$
(c) 1/18 (d) none of these

19. An old person forgets the last two digits of a telephone number, remembering only that these are different dialled at random. The probability that the number is dialled correctly is :

(a) 1/90 (b) 81/91
(c) 2/99 (d) none of these

20. Three squares of a chessboard are chosen at random, the probability that two are of one colour and one of another is :

(a) $\frac{67}{992}$ (b) $\frac{16}{21}$
(c) $\frac{31}{32}$ (d) none of these

21. The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is :

(a) $\frac{17}{53}$ (b) $\frac{1}{53}$
(c) $\frac{3}{7}$ (d) none of these

22. In order to get atleast once a head with probability $P \geq 0.9$, the number of times a coin needs to be tossed is :

(a) 3 (b) 2
(c) 5 (d) 4

23. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that atleast one of the selected persons will be a woman is :

(a) $\frac{25}{39}$ (b) $\frac{31}{65}$
(c) $\frac{25}{69}$ (d) $\frac{5}{13}$

24. Nine squares are chosen at random on a chessboard. What is the probability that they form a square of size 3×3?

(a) $\frac{9}{^{64}C_9}$ (b) $\frac{36}{^{64}C_9}$
(c) $\frac{6}{^{64}C_9}$ (d) none of these

25. Seven digits from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order. The probability that this seven digit number is divisible by 9 is :

(a) $\frac{7}{9}$ (b) $\frac{1}{9}$
(c) $\frac{2}{91}$ (d) $\frac{4}{9}$

LEVEL 2

1. What is the probability that four S's come consecutively in the word MISSISSIPPI?

(a) $\frac{4}{165}$ (b) $\frac{4}{135}$
(c) $\frac{24}{165}$ (d) none of these

2. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing ordinary six faced die. Find the probability that the equation will have real roots.

(a) $\frac{34}{161}$ (b) $\frac{43}{216}$
(c) $\frac{25}{36}$ (d) none of these

3. A and B throw alternately a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. Find their respective chance of winning, if A begins.

(a) $\frac{13}{16}, \frac{31}{16}$ (b) $\frac{30}{61}, \frac{31}{61}$
(c) $\frac{31}{61}, \frac{41}{61}$ (d) $\frac{38}{61}, \frac{23}{61}$

4. A consignment of 15 wristwatches contains 4 defectives. The wristwatches are selected at random, one by one and examined. The ones examined are not put back. What is the probability that ninth one examined is the last defective?

(a) $\frac{11}{195}$ (b) $\frac{17}{195}$
(c) $\frac{8}{195}$ (d) $\frac{16}{195}$

5. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $1/6$. The probability that his answer is correct, given that he copied it, is $1/8$. Find the probability that he knew the answer to the question, given that he correctly answered it.

(a) $\frac{17}{39}$ (b) $\frac{13}{29}$
(c) $\frac{24}{29}$ (d) $\frac{24}{39}$

6. Given that the sum of two non-negative quantities is 200, the probability that their product is not less than $\frac{3}{4}$ times their greatest product value is :

(a) $\frac{99}{200}$ (b) $\frac{101}{200}$

(c) $\frac{87}{100}$ (d) none of these

7. A pack of cards consists of 9 cards numbered 1 to 9. Three cards are drawn at random with replacement. Then the probability of getting 1 even and 2 odd numbered cards is :

(a) $\frac{3}{143}$ (b) $\frac{100}{243}$

(c) $\frac{50}{343}$ (d) $\frac{7}{72}$

8. Three numbers are to be selected at random without replacement from the set of numbers $\{1, 2, \ldots n\}$. The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second is :

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{6}$ (d) $\frac{7}{12}$

9. Two numbers b and c are chosen at random with replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that $x^2 + bx + c > 0$ for all $x \in R$ is :

(a) $\frac{23}{81}$ (b) $\frac{7}{9}$

(c) $\frac{32}{81}$ (d) $\frac{65}{729}$

10. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects the student has 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly, two, which of the following relations are true.

(a) $p + m + c = \frac{27}{20}$ (b) $p + m + c = \frac{13}{20}$

(c) $pmc = \frac{1}{10}$ (d) both (a) and (c)

11. A student appears for tests A, B and C. The student is successful if he passes either in tests A and B or tests A and C. The probabilities of the student passing in tests A, B, C are p, q and 1/2 respectively. If the probability that the student is successful is $\frac{1}{2}$ then,

(a) $p = 1, q = 0$

(b) $p = 0, q = 1$

(c) $\frac{p}{q} = 1$

(d) infinitely many solutions

12. If $\frac{1 + 4p}{p}, \frac{1 - p}{4}, \frac{1 - 2p}{2}$ are probabilities of three mutually exclusive events then

(a) $p = \frac{1}{2}$ (b) $p = \frac{3}{4}$

(c) $p = \frac{1}{3}$ (d) none of these

13. A letter is takenout at random from 'ASSISTANT' and another is taken out from "STATISTICS'. The probability that they are the same letters is :

(a) $\frac{35}{96}$ (b) $\frac{19}{90}$

(c) $\frac{19}{96}$ (d) none of these

14. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is :

(a) $\frac{37}{87}$ (b) $\frac{47}{87}$

(c) $\frac{17}{29}$ (d) none of these

15. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is :

(a) ${}^{11}C_6(0.1)^{11}$ (b) ${}^{11}C_6(0.24)^5$

(c) ${}^{11}C_6(0.2)^{11}$ (d) none of these

16. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral equals :

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{10}$ (d) none of these

17. A man can take a step forward, backward, left or right with equal probability. Find the probability that after nine steps he will be just one step away from his initial position.

(a) $\frac{3696}{7^4}$ (b) $\frac{3969}{4^7}$

(c) $\frac{4^4}{10}$ (d) none of these

18. Urn A contains six red and four black balls and urn B has four red and six black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is transfered at random from urn B to urn A. If one ball is now drawn at random from urn A, find the probability that it is red.

(a) $\frac{32}{65}$ (b) $\frac{32}{55}$

(c) $\frac{23}{55}$ (d) $\frac{56}{65}$

19. The digits 1, 2, 3, ..., 9 are written in random order to form a nine digit number. Find the probability that this number is divisible by 11.

(a) $\frac{11}{63}$ (b) $\frac{11}{81}$

(c) $\frac{11}{126}$ (d) none of these

Answers

INTRODUCTORY EXERCISE- 20.1

1 (a)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (c)
11. (d)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (c)
31. (c)	**32.** (c)	**33.** (c)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (a)
41. (b)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (b)
51. (a)	**52.** (b)	**53.** (c)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (b)	**58.** (c)	**59.** (a)	

INTRODUCTORY EXERCISE- 20.2

1 (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (c)
21. (a)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (a)	

INTRODUCTORY EXERCISE- 20.3

1 (b)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)
11. (a)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (b)
21. (a)	**22.** (b)								

INTRODUCTORY EXERCISE- 20.4

1 (a)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (c)
21. (b)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (a)	**30.** (b)
31. (a)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (d)	**37.** (b)	**38.** (b)	**39.** (b)	**40.** (a)
41. (b)									

LEVEL-1

1 (b)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (b)
21. (c)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (b)					

LEVEL-2

1 (a)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (d)
11. (d)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (c)	

Hints & Solutions

INTRODUCTORY EXERCISE 20.1

1. Here the sample space $S = \{H, T\} \Rightarrow n(S) = 2$

Event of getting head $= \{H\} \quad \Rightarrow n(E) = 1$

$\therefore$ Probability of getting a head is given by $P(E) = \frac{1}{2}$

Solutions for question number 2-5 :

$$S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$$

2. $E = \{HT, TH\} \quad \Rightarrow \quad n(E) = 2$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

3. $E = \{HH, HT, TH\} \quad \Rightarrow \quad n(E) = 3$

$\therefore \quad P(E) = \frac{3}{4}$

4. $E = \{H, H\} \quad \Rightarrow \quad n(E) = 1$

$\therefore \quad P(E) = \frac{1}{4}$

5. $E = \{H, H\} \quad \Rightarrow \quad n(E) = 1$

$\therefore \quad P(E) = \frac{1}{4}$

Solutions for question number 6-12 :

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$\Rightarrow \quad n(S) = 8$

6. $E = \{HTT, THT, TTH\} \Rightarrow n(E) = 3$

$\therefore \quad P(E) = \frac{3}{8}$

7. $E = \{HHT, HTH, THH\} \Rightarrow n(E) = 3$

$\therefore \quad P(E) = \frac{3}{8}$

8. $E = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

$\Rightarrow \quad n(E) = 7$

$\therefore \quad P(E) = \frac{7}{8}$

9. $E = \{HHT, HTH, THH\} \Rightarrow n(E) = 3$

$\therefore \quad P(E) = \frac{3}{8}$

10. $E = \{HHH, HHT, HTH, THH\} \Rightarrow n(E) = 4$

$\therefore \quad P(E) = \frac{4}{8} = \frac{1}{2}$

11. $E = \{HHT, HTH, HTT, THH, THT, TTH\}$

$\Rightarrow \quad n(E) = 6$

$\therefore \quad P(E) = \frac{6}{8} = \frac{3}{4}$

12. $E = \{HHH, HHT, HTH, THH\} \Rightarrow n(E) = 4$

$\therefore \quad P(E) = \frac{4}{8} = \frac{1}{2}$

Solutions for question number 13-16 :

$$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$$

13. $E = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(E) = 6$

$\therefore \quad P(E) = \frac{6}{6} = 1$

14. $E = \{3, 6\} \quad \Rightarrow \quad n(E) = 2$

$\therefore \quad P(E) = \frac{2}{6} = \frac{1}{3}$

15. $E = \{2, 3, 5\} \quad \Rightarrow \quad n(E) = 3$

$\therefore \quad P(E) = \frac{3}{6} = \frac{1}{2}$

16. $E = \{2, 4, 6\} \quad \Rightarrow \quad n(E) = 3$

$\therefore \quad P(E) = \frac{3}{6} = \frac{1}{2}$

Solution for question number 17 and 18 :

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

$\Rightarrow \quad n(S) = 8$

17. $E = \{HHT, HTH, THH, TTH, THT, HTT\}$

$\Rightarrow n(E) = 6 \quad \therefore \quad P(E) = \frac{6}{8} = \frac{3}{4}$

18. $E = \{TTT\} \quad \Rightarrow \quad n(E) = 1$

$\therefore \quad P(E) = \frac{1}{8}$

Solutions for question number 19-27 :

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), \ldots, (6, 5), (6, 6)\}$$

$\Rightarrow \quad n(S) = 6 \times 6 = 36$

19. $E = \{(6, 3), (5, 4), (4, 5), (3, 6)\}$

$n(E) = 4$

$\therefore \quad P(E) = \frac{4}{36} = \frac{1}{9}$

20. $E = \{(6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)\}$

$n(E) = 6$

$\therefore \quad P(E) = \frac{6}{36} = \frac{1}{6}$

21. $E = \{(6, 3), (5, 4), (4, 5), (3, 6), (6, 5), (5, 6)\}$

$\Rightarrow \quad n(E) = 6$

$\therefore \quad P(E) = \frac{6}{36} = \frac{1}{6}$

22. $E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$n(E) = 6$

$\therefore \quad P(E) = \frac{6}{36} = \frac{1}{6}$

23. $E = \{(2, 2), (4, 4), (6, 6)\} \Rightarrow n(E) = 3$

$\therefore \quad P(E) = \frac{3}{36} = \frac{1}{12}$

24. $E = \{(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6)\ (3, 2), (6, 2), (3, 4), (6, 4), (3, 6)\}$

$\Rightarrow \quad n(E) = 11$

$\therefore \quad P(E) = \frac{11}{36}$

25. $E = \{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6), (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$\Rightarrow \quad n(E) = 20$

$\therefore \quad P(E) = \frac{20}{36} = \frac{5}{9}$

26. $E = \{(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$

$\Rightarrow \quad n(E) = 15$

$\therefore \quad P(E) = \frac{15}{36} = \frac{5}{12}$

27. $E = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\}$

$\Rightarrow \quad n(E) = 11$

$\therefore \quad P(E) = \frac{11}{36}$

Solutions for question number 28-35 : $S = \{52 \text{ cards}\}$

52 cards

(26) Red — *Black (26)*

Hearts (13) *Diamonds (13)* *Spades (13)* *Clubs (13)*

In each of the four suits there is one ace, one king, one queen and one jack (or knave) and rest 9 cards are numbered.

$\therefore \quad n(S) = 52$

28. $n(E) = 26$

$\therefore \quad P(E) = \frac{26}{52} = \frac{1}{2}$

29. $n(E) = 4$

$\therefore \quad P(E) = \frac{4}{52} = \frac{1}{13}$

30. Since drawn card must be black so there are only two queens.

Hence $\quad n(E) = 2$

$\therefore \quad P(E) = \frac{2}{52} = \frac{1}{26}$

31. There are 26 black cards (including two queens). Besides it there are two more queens (in red colours)

Thus $\quad n(E) = 26 + 2 = 28$

$\therefore \quad P(E) = \frac{28}{52} = \frac{7}{13}$

32. There are 4 kings and 4 queens

$E = K \cup Q$

$\therefore \quad n(E) = 4 + 4 = 8$

$\therefore \quad P(E) = \frac{8}{52} = \frac{2}{13}$

$\therefore$

33. There are 13 hearts (including one queen and one king). Besides it there are 3 queens and 3 kings in remaining 3 suits each.

Thus $\quad n(E) = 13 + 3 + 3 = 19$

$\therefore \quad P(E) = \frac{19}{52}$

34. There are 13 spades (including one king). Besides there are 3 more kings in remaining 3 suits).

Thus $\quad n(E) = 13 + 3 = 16$

Hence $\quad n(\bar{E}) = 52 - 16 = 36$

$\therefore \quad P(\bar{E}) = \frac{36}{52} = \frac{9}{13}$

35. There are 4 aces and 4 kings

$\therefore \quad n(E) = 4 + 4 = 8$

$\therefore \quad n(\bar{E}) = 52 - 8 = 44$

$\therefore \quad P(\bar{E}) = \frac{44}{52} = \frac{11}{13}$

36. There are 4 aces, 4 kings and 4 jacks and their selection can be made in following ways.

$^{12}C_1 \times {}^{8}C_1 \times {}^{4}C_1 = 12 \times 8 \times 4$

$n(E) = 12 \times 8 \times 4$

Total selection can be made $= {}^{52}C_3 = 52 \times 51 \times 50$

$P(E) = \frac{12 \times 8 \times 4}{52 \times 51 \times 50} = \frac{16}{5525}$

37. $E = \{(1, 1, 1, 1), (2, 2, 2, 2) \ldots\ldots (13, 13, 13, 13)\}$

$\therefore \quad n(E) = 13$

and $\quad n(S) = {}^{52}C_4 = 270725$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{13}{270725} = \frac{1}{20825}$

38. $n(E) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (13)^4$

$n(S) = {}^{52}C_4 = 270725$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{(13)^4}{270725} = \frac{2197}{20825}$

39. $n(E) = {}^{13}C_1 \times {}^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_1 = 13 \times 12 \times 11 \times 10$

$n(S) = {}^{52}C_4 = 270725$

$\therefore \quad P(E) = \frac{13 \times 12 \times 11\ 10}{270725} = \frac{264}{4165}$

Solutions for question number 40-43 :

$n(S) = 6 \times 6 \times 6 \times 6 = 6^4$

40. $n(E) = 6$

$\because \quad E = \{(1, 1, 1, 1), (2, 2, 2, 2), (3, 3, 3, 3) \ldots (6, 6, 6, 6)\}$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{6^4} = \frac{1}{6^3} = \frac{1}{216}$

41. $n(E) = {}^{6}C_1 \times {}^{5}C_1 \times {}^{4}C_1 \times {}^{3}C_1 = 360$

$\therefore \quad P(E) = \frac{360}{6^4} = \frac{5}{18}$

42. Select a number which occurs on two dice out of six numbers (1, 2, 3, 4, 5, 6). This can be done in ${}^{6}C_1$ ways. Now select two distinct number out of remaining 5 numbers which can be done in ${}^{5}C_2$ ways. Thus these 4 numbers can be arranged in $\frac{4!}{2!}$ ways.

So, the number of ways in which two dice show the same face and the remaining two show different faces is

$$^6C_1 \times {}^5C_2 \times \frac{4!}{2!} = 720$$

$\therefore$ $n(E) = 720$

$\therefore$ $P(E) = \frac{720}{6^4} = \frac{5}{9}$

43. There are 3 possible cases-

(i) 2 similar faces + 2 different faces
(ii) 3 similar faces + 1 different face
(iii) all 4 faces are similar

$\therefore$ Required number of ways

$$= \left({}^6C_1 \times {}^5C_2 \times \frac{4!}{2!}\right) + \left({}^6C_1 \times {}^5C_1 \times \frac{4!}{3!}\right) + \left({}^6C_1 \times \frac{4!}{4!}\right)$$

$$= (6 \times 10 \times 12) + (6 \times 5 \times 4) + 6 = 846$$

$\therefore$ $n(E) = 846$

and $n(S) = 6^4$

$\therefore$ $P(E) = \frac{846}{6^4} = \frac{47}{72}$

44. $E = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$\Rightarrow$ $n(E) = 8$

$S = \{1, 2, 3, 4, \ldots 20\}$

$\Rightarrow$ $n(S) = 20$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

45. $S = \{1, 2, 3, 4, \ldots. 18\} \Rightarrow n(S) = 18$

$E_1 = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$\Rightarrow$ $n(E_1) = 9$

$E_2 = \{3, 6, 9, 12, 15, 18\} \Rightarrow n(E_2) = 6$

$(E_1 \cap E_2) = E_3 = \{6, 12, 18\} \Rightarrow n(E_3) = 3$

$\therefore$ $E = E_1 \cup E_2 = E_1 + E_2 - E_3$

$n(E) = 9 + 6 - 3$

$n(E) = 12$

where $E = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{12}{18} = \frac{2}{3}$

NOTE For any problem refer set theory and number system.

46.

$$n(S) = {}^{100}C_2 = \frac{100 \times 99}{2} = 4950$$

$n(E) = {}^{25}C_2 = 300$

$\therefore$ $P(E) = \frac{300}{4950} = \frac{2}{33}$

NOTE There are total 25 prime numbers upto 100.

47.

$n(S) = {}^{100}C_2 = 4950$

$n(E) = {}^{75}C_2 = 2775$ $(100 - 25 = 75)$

$\therefore$ $P(E) = \frac{2775}{4950} = \frac{37}{66}$

48. $366 = 7 \times 52 + 2$

It means a leap year has 52 full weeks and 2 more days. These 2 days can be :

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Friday and Saturday
(vii) Saturday and Sunday

Clearly atleast there are 52 Sundays.

Now, for having 53 Sundays in the year, one of the above 2 consecutive, days must be Sunday.

Thus, out of the above 7 possibilities, 2 possibilities are in favour [(i) and (vii)] of the event that one of the two days is a Sunday.

$\therefore$ Required probability $= \frac{2}{7}$

49.

$n(S) = 8 + 4 = 12$

$n(E) = 8$

$\therefore$ $P(E) = \frac{8}{12} = \frac{2}{3}$

50.

$n(S) = {}^{12}C_4 = 495$

$n(E) = {}^8C_4 = 70$

$\therefore$ $P(E) = \frac{70}{495} = \frac{14}{99}$

51.

$n(S) = {}^{12}C_4 = 495$

$n(E) = {}^4C_4 = 1$

$\therefore$ $P(E) = \frac{1}{495}$

52.

$n(S) = {}^{12}C_4 = 495$

$n(E) = {}^8C_2 \times {}^4C_1 = 112$

$\therefore$ $P(E) = \frac{112}{495}$

53.

$n(S) = 495$

$n(E) = {}^4C_3 = 4$

$\therefore$ $P(E) = \frac{4}{495}$

54. Total number of outcomes $= 2 + 7 = 9$

Favourable number of cases $= 2$

$\therefore$ $P(E) = \frac{2}{9}$

55. Total number of outcomes $= 5 + 7 = 12$

Number of cases against the occurrence of event $= 5$

$\therefore$ Number of cases in favour of event $= 12 - 5 = 7$

$\therefore$ $P(E) = \frac{7}{12}$

56.

$S = \{GG, GB, BG, BB\}$

and $E = \{GG, GB, BG\}$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

57.

$n(S) = {}^5C_2 = 10$

$n(E) = ({}^2C_1 \times {}^3C_1) + ({}^2C_2) = 7$

$\therefore$ $P(E) = \frac{7}{10}$

58.

$n(S) = {}^{20}C_2 = 190$

$n(E) = {}^{15}C_2 = 105$

$\therefore$ $P(E) = \frac{105}{190} = \frac{21}{38}$

59.

$n(S) = {}^{20}C_4 = 4845$

$n(E) = ({}^5C_3 \times {}^{15}C_1) + ({}^5C_4) = 155$

$\therefore$ $P(E) = \frac{155}{4845} = \frac{31}{969}$

Introductory Exercise 20.2

1. $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{7}{50}$

$\therefore$ $P(A \text{ or } B)=P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{4}+\frac{1}{2}-\frac{7}{50}=\frac{61}{100}$

2. $P(\text{neither } A \text{ nor } B)=P(\bar{A} \text{ and } \bar{B})$

$=P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)$

$=1-\frac{61}{100}=\frac{39}{100}$

3. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{3}{10}+\frac{1}{2}-\frac{1}{5}=\frac{6}{10}=\frac{3}{5}$

4. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{3}{5}=\frac{2}{5}+\frac{1}{2}-P(A\cap B)$

$\Rightarrow$ $P(A\cap B)=\frac{3}{10}$

5. $P(\bar{A})=1-P(A)=1-\frac{2}{5}=\frac{3}{5}$

6. $P(\bar{B})=1-P(B)=1-\frac{1}{2}=\frac{1}{2}$

7. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{2}{5}+\frac{1}{2}-0=\frac{9}{10}$ $(\because P(A\cap B)=0)$

8. $P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)$

$=1-\frac{9}{10}=\frac{1}{10}$

9. $P(\bar{A}\cap B)=P(B)-P(A\cap B)$

$=P(B)=\frac{1}{2}$ $(\because P(A\cap B)=0)$

10. $P(A\cap\bar{B})=P(A)-P(A\cap B)=\frac{2}{5}$

11. $P(A\cup B)=P(A)+P(B)$

$0.65=0.35+P(B)$ $(\because P(A)=1-P(\bar{A}))$

$\Rightarrow$ $P(B)=0.30$

12. Let $P(A)=x$, then $P(B)=\frac{3}{2}x$ and $P(C)=\frac{1}{2}\times\frac{3}{2}x=\frac{3}{4}x$

$\therefore$ $P(A)+P(B)+P(C)=P(A\cup B\cup C)=P(S)=1$

$\therefore$ $x+\frac{3}{2}x+\frac{3}{4}x=1$

$\Rightarrow$ $x=\frac{4}{13}$

$\therefore$ $P(A)=\frac{4}{13}$

13. $n(S)=6\times 6=36$

Let A = Event of getting an even number on the first die

$\therefore$ $n(A)=3\times 6=18$

and B = Event of getting a total of 8

$n(B)=5$

$$A=\begin{Bmatrix}(2,1),(2,2),(2,3)\ldots(2,6)\\(4,1)\ldots\ldots\ldots\ldots(4,6)\\(6,1)\ldots\ldots\ldots\ldots(6,6)\end{Bmatrix}$$

$B=\{(6,2),(5,3),(4,4),(3,5),(2,6)\}$

$\therefore$ $n(A)=3\times 6=18$

and $n(B)=5$

$\therefore$ $(A\cap B)=\{(2,6),(6,2),(4,4)\}$

$\therefore$ $n(A\cap B)=3$

$P(A)=\frac{18}{36}=\frac{1}{2}$, $P(B)=\frac{5}{36}$

and $P(A\cap B)=\frac{3}{36}=\frac{1}{12}$

$\therefore$ P (Even number on first die or a total of 8)

$=p(A \text{ or } B)=P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{5}{36}-\frac{1}{12}=\frac{5}{9}$

14. $A=\{(5,1),)\ (5,2),(5,3),(5,4),(5,5),(5,6)\}$

$B=\{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$A\cap B=\{(5,5)\}$

Also $n(S)=36$

$\therefore$ $P(A)=\frac{6}{36}=\frac{1}{6}$

$P(B)=\frac{6}{36}=\frac{1}{6}$

and $P(A\cap B)=\frac{1}{36}$

$\therefore$ Required probability $=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{6}+\frac{1}{6}-\frac{1}{36}=\frac{11}{36}$

15. $n(S)=36$

$A=\{(1,1),)\ (2,2),(3,3),(4,4),(5,5),(6,6)\}$

$B=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$(A\cap B)=\{(4,4)\}$

$n(A)=6,\quad n(B)=5,\quad n(A\cap B)=1$

$\therefore$ Required probability $=P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{6}{36}+\frac{5}{36}-\frac{1}{36}$

$=\frac{10}{36}=\frac{5}{18}$

16. See the solution of question number 14

P (atleast one die shows 6) $=\frac{11}{36}$

17. $n(S)=52$

$A\rightarrow$ The event of getting a heart

$B\rightarrow$ The event of getting a king

then $A\cap B\rightarrow$ The event of getting a king of heart.

$\therefore$ $P(A)=\frac{13}{52}=\frac{1}{4}$, $P(B)=\frac{4}{52}=\frac{1}{13}$

and $P(A\cap B)=\frac{1}{52}$

$\therefore$ P (a heart or a king) $=P(A \text{ or } B)=P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{1}{4}+\frac{1}{13}-\frac{1}{52}=\frac{4}{13}$$

18. $n(S)={}^{52}C_2=1326$

Let A = event of getting both red cards

and B = event of getting both queens

then $A\cap B$ = event of getting two red queens

$$n(A)={}^{26}C_2=325,\ n(B)={}^4C_2=6$$

$$n(A\cap B)={}^2C_2=1$$

$$\therefore\quad P(A)=\frac{325}{1326},\quad P(B)=\frac{6}{1326}=\frac{1}{221}$$

$$P(A\cap B)=\frac{1}{1326}$$

$\therefore$ P (both red or both queens) $=P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{325}{1326}+\frac{1}{221}-\frac{1}{1326}=\frac{55}{221}$$

19. $n(S)=52$

Let A, B, C be the events of getting a red card, a heart and a king respectively.

then $n(A)=26,\quad n(B)=13,\quad n(C)=4$

Clearly $n(A\cap B)=13,\quad n(B\cap C)=1,$

$$n(A\cap C)=2,\quad n(A\cap B\cap C)=1$$

$$\therefore P(A)=\frac{26}{52}=\frac{1}{2},\quad P(B)=\frac{13}{52}=\frac{1}{4},\quad P(C)=\frac{4}{52}=\frac{1}{13}$$

$$P(A\cap B)=\frac{13}{52}=\frac{1}{4},\ P(B\cap C)=\frac{1}{52},$$

$$P(A\cap C)=\frac{2}{52}=\frac{1}{26}$$

$$P(A\cap B\cap C)=\frac{1}{52}$$

$\therefore$ P (a red card, or a heart or a king) $=P(A\cup B\cup C)$

$$=P(A)+P(B)+P(C)-(P(A\cap B)+P(B\cap C)+P(A\cap C)+P(A\cap B\cap C)$$

$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{13}-\left(\frac{1}{4}+\frac{1}{52}+\frac{1}{26}\right)+\frac{1}{52}=\frac{7}{13}$$

20. $n(S)={}^{52}C_4$

Let E_1, E_2, E_3, E_4 be the events of getting all spades, all clubs, all hearts and all diamonds respectively.

Then

$$n(E_1)={}^{13}C_4$$

$$n(E_2)={}^{13}C_4$$

$$n(E_3)={}^{13}C_4$$

$$n(E_4)={}^{13}C_4$$

$$P(E_1)=\frac{{}^{13}C_4}{{}^{52}C_4};\quad P(E_2)=\frac{{}^{13}C_4}{{}^{52}C_4},$$

$$P(E_3)=\frac{{}^{13}C_4}{{}^{52}C_4},\quad P(E_4)=\frac{{}^{13}C_4}{{}^{52}C_4}$$

Since E_1, E_2, E_3 and E_4 are mutually exclusive events.

P (getting all the 4 cards of the same suit)

$$P(E_1 \text{ or } E_2 \text{ or } E_3 \text{ or } E_4)=P(E_1)+P(E_2)+P(E_3)+P(E_4)$$

$$=4\times\left(\frac{{}^{13}C_4}{{}^{52}C_4}\right)=\frac{44}{4165}$$

21. $n(S)=52$

A = The event of getting a king

B = The event of getting a queen

$(A\cap B)$ = The event of getting a king and a queen both

$$\therefore\quad n(A)=4,\quad n(B)=4,\quad n(A\cap B)=0$$

$$P(A)=\frac{4}{52},\quad P(B)=\frac{4}{52}$$

$$\therefore\quad P(A\cup B)=P(A)+P(B)=\frac{4}{52}+\frac{4}{52}=\frac{2}{13}$$

22. $n(S)={}^{52}C_2=1326$

Let A be the event of getting two black cards

and B be the event of getting two jacks

and $(A\cap B)$ be the event of getting two black jacks.

$$\therefore\quad n(A)={}^{26}C_2,\quad n(B)={}^4C_2,\quad n(A\cap B)={}^2C_2$$

$$\therefore\quad P(A)=\frac{{}^{26}C_2}{{}^{52}C_2},\quad P(B)=\frac{{}^4C_2}{{}^{52}C_2},\quad P(A\cap B)=\frac{{}^2C_2}{{}^{52}C_2}$$

$\therefore$ Required probability $=P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{{}^{26}C_2}{{}^{52}C_2}+\frac{{}^4C_2}{{}^{52}C_2}-\frac{{}^2C_2}{{}^{52}C_2}$$

$$=\frac{325}{1326}+\frac{1}{221}-\frac{1}{1326}=\frac{55}{221}$$

23. $n(S)=300$

Let A be the event of getting a number divisible by 3 and B be the event of getting a number divisible by 5 and $(A\cap B)$ be event of getting a number divisible by 3 and 5 both

$$\therefore\quad n(A)=100,\quad n(B)=60,\quad n(A\cap B)=20$$

$$\therefore\quad P(A)=\frac{100}{300}=\frac{1}{3},\ P(B)=\frac{60}{300}=\frac{1}{5},\ P(A\cap B)=\frac{20}{600}=\frac{1}{30}$$

$$\therefore\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{1}{3}+\frac{1}{5}-\frac{1}{30}=\frac{1}{2}$$

24. $n(S)=100$

Let A be the event of getting a number divisible by 2 and B be the event of getting a number divisible by 3 and C be the event of getting of number divisible by 5.

$\therefore (A\cap B)$ be the event of getting a number divisible by both 2 and 3.

$(B\cap C)$ be the event of getting a number divisible by both 3 and 5.

$(A\cap C)$ be the event of getting a number divisible by both 2 and 5.

$(A\cap B\cap C)$ be the event of getting a number divisible by A, B and C.

Now, $n(A)=50,\quad n(B)=33,\quad n(C)=20,\quad n(A\cap B)=16,$

$$n(B\cap C)=6,\ n(A\cap C)=10,\ n(A\cap B\cap C)=3$$

$$\therefore\quad P(A)=\frac{50}{100}=\frac{1}{2},\quad P(B)=\frac{33}{100},$$

$$P(C)=\frac{20}{100}=\frac{1}{5},\quad n(A\cap B)=\frac{16}{100}$$

$$n(B\cap C)=\frac{6}{100},\quad n(A\cap C)=\frac{10}{100},$$

$$n(A\cap B\cap C)=\frac{3}{100}$$

Required probability $= P(A\cup B\cup C)$

$$= P(A)+P(B)+P(C)-[P(A\cap B)+P(B\cap C)+P(A\cap C)]+P(A\cap B\cap C)$$

$$=\frac{50}{100}+\frac{33}{100}+\frac{20}{100}-\left(\frac{16}{100}+\frac{6}{100}+\frac{10}{100}\right)+\frac{3}{100}$$

$$=\frac{74}{100}$$

25. $n(S)=5+8+10=23$

$n(A)=5$

$n(B)=8$ $\quad [n(A\cap B)=0]$

$\therefore\quad P(A)=\frac{5}{23}$ and $P(B)=\frac{8}{23}$ $\quad P(A\cap B)=0$

$\therefore\quad P(A\cup B)=P(A)+P(B)=\frac{5}{23}+\frac{8}{23}=\frac{13}{23}$

26. $n(S)={}^{30}C_2$

Let A be the event of getting two oranges and B be the event of getting two non-defective fruits. and $(A\cap B)$ be the event of getting two non-defective oranges

$\therefore\quad P(A)=\frac{{}^{20}C_2}{{}^{30}C_2},\ P(B)=\frac{{}^{22}C_2}{{}^{30}C_2}$

and $\quad P(A\cap B)=\frac{{}^{15}C_2}{{}^{30}C_2}$

$\therefore\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{{}^{20}C_2}{{}^{30}C_2}+\frac{{}^{22}C_2}{{}^{30}C_2}-\frac{{}^{15}C_2}{{}^{30}C_2}=\frac{316}{435}$$

HINT There are 20 oranges, and $30-(3+5)=22$ non-defective fruits and $20-5=15$ non-defective oranges.

27. $n(S)=100$

$n(A)=40,\quad n(B)=20,\quad n(A\cap B)=5$

$\therefore\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{40}{100}+\frac{20}{100}-\frac{5}{100}$$

$$P(A\cup B)=\frac{55}{100}=55\%$$

28. $P(A)=\frac{2}{5},\quad P(B)=\frac{1}{3}$

$\therefore\quad P(A\cup B)=\frac{2}{5}+\frac{1}{3}=\frac{11}{15}$

($\because$ A and B are mutually exclusive events)

29. $P(H_1)=\frac{1}{3},\ P(H_2)=\frac{1}{4},\ P(H_3)=\frac{1}{5},\ P(H_4)=\frac{1}{6}$

$\therefore P(H_1\cup H_2\cup H_3\cup H_4)=P(H_1)+P(H_2)+P(H_3)+P(H_4)$

$$=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{57}{60}$$

HINT Here, H_1, H_2, H_3 and H_4 are the mutually exclusive events.

INTRODUCTORY EXERCISE 20.3

1. $P\left(\frac{A}{B}\right)=\frac{1}{2}$ (see the example 1)

2. $P(A\cup B)=0.96$ (see the example 1)

3. Here $S=\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Let A be the event that one of the coins shows a tail

$\therefore\quad A=\{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore\quad P(A)=\frac{7}{8}$

Now, let B be the event that they are all tails

$\therefore\quad B=\{TTT\}$

$\therefore\quad P(B)=\frac{1}{8}$

$\therefore\quad (A\cap B)=\{TTT\}$

$\Rightarrow\quad P(A\cap B)=\frac{1}{8}$

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{1/8}{7/8}=\frac{1}{7}$$

4. Here sample space $S=\{HH, HT, TH, TT\}$

$E=\{HT, TH\}$

$F=\{HH, HT, TH\}$

$\therefore\quad E\cap F=\{HT, TH\}$

$\therefore\quad P(E)=\frac{2}{4}=\frac{1}{2}$ and $P(F)=\frac{3}{4}$

and $\quad P(E\cap F)=\frac{2}{4}=\frac{1}{2}$

$$\therefore\quad P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{2}}{\frac{3}{4}}=\frac{2}{3}$$

$$P\left(\frac{F}{E}\right)=\frac{P(E\cap F)}{P(E)}=\frac{\frac{1}{2}}{\frac{1}{2}}=1$$

5. Here sample space

$$S=\{HH, HT, T1, T2, T3, T4, T5, T6\}$$

Let A be the event that the die shows a number greater than and B be the event that the first throw of the coin results in tail then,

$$A=\{T5, T6\}$$
$$B=\{T1, T2, T3, T4, T5, T6\}$$

$$\therefore\text{ Required probability}=P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{2}{8}}{\frac{6}{8}}=\frac{1}{3}$$

6. $S=\{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting an odd number and B be the event of getting a prime number

$\therefore\quad A=\{1, 3, 5\},\quad B=\{2, 3, 5\}$

and $(A \cap B) = \{3, 5\}$

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}, \quad P(B) = \frac{3}{6} = \frac{1}{2}, \quad P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

$\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{1/3}{1/2} = \frac{2}{3}$

7. Let A be the event of getting the sum 9 and B be the event of getting atleast one 4.

Then $A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$

$B = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)\}$

then $A \cap B = \{(4, 5) (5, 4)\}$

$\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{n(A \cap B)}{n(A)} = \frac{2}{4} = \frac{1}{2}$

8. Let A = the event of getting 4 on the first die.

and B = the event of getting the sum 8 or greater

$\therefore A = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$

$B = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 2), (2, 6), (3, 5), (3, 6), (5, 3), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$\therefore A \cap B = \{(4, 4)\}, (4, 5), (4, 6)\}$

$\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{n(A \cap B)}{n(A)} = \frac{3}{6} = \frac{1}{2}$

9. Let A = event of getting an odd number

and B = the event of getting a number greater than 1.

$\therefore A = \{1, 3, 5\}, B = \{3, 5\}, A \cap B = \{3, 5\}$

$\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)} = \frac{2}{3}$

10. Let A be the event of reading English and

B be the event of reading French.

Then $P(A) = \frac{45}{100} = \frac{9}{20}, \quad P(B) = \frac{30}{100} = \frac{3}{10}$

and $P(A \cap B) = \frac{20}{100} = \frac{1}{5}$

$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{2}{3}$

11. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/5}{9/20} = \frac{4}{9}$

12. $S = (B_1B_2, B_1G_2, G_1B_2, G_1G_2)$

Where $B \to$ Boy and $G \to$ Girl

Let A be the event that both are boys

and B be the event that one of two is a boy.

Then $A = \{B_1B_2\}$ and $B = \{B_1B_2, B_1G_2, G_1B_2\}$

So $A \cap B = \{B_1B_2\}$

$\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{3}$

13. A be the event that both children are boys and B be the event that the other child is a boy.

then $A = \{B_1, B_2\}$ and $B = \{B_1B_2, B_1G_2\}$

So $(A \cap B) = \{B_1B_2\}$

$\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{2}$

14. $P(E) = \frac{1}{2}$

15. $P(F) = \frac{1}{2}$

16. $P\left(\frac{G}{E}\right)$ = Probability of drawing a red ball when first bag is selected

= Probability of drawing a red ball from first bag

$= \frac{3}{7}$

17. $P\left(\frac{G}{F}\right)$ = Probability of drawing a red ball from second bag

$= \frac{4}{6} = \frac{2}{3}$

18. Let A be the event of not getting a red ball in first draw and B be the event of not getting a red ball in second draw. Then

Required probability

= Probability that atleast one ball is red

= 1 – Probability that none is red

$= 1 - P(A \text{ and } B)$

$= 1 - P(A \cap B)$

$= 1 - P(A) \cdot P\left(\frac{B}{A}\right)$

$= 1 - \left(\frac{2}{3} \times \frac{5}{8}\right) = \frac{7}{12}$

Here $P(A) = \frac{6}{9} = \frac{2}{3}$

and $P\left(\frac{B}{A}\right) = \frac{5}{8}$ [There are 5 balls (excluding 3 red balls) after the selection of one non-red ball]

19. Let A be the event drawing 4 red ball in first draw and B be the event of drawing 4 blue balls in the second draw.

Then $P(A) = \frac{{}^6C_4}{{}^{15}C_4} = \frac{15}{1365} = \frac{1}{91}$

$P\left(\frac{B}{A}\right) = \frac{{}^9C_4}{{}^{11}C_4} = \frac{126}{330} = \frac{21}{55}$

Hence, the required probability $= P(A \cap B)$

$= P(A) P\left(\frac{B}{A}\right)$

$= \frac{1}{91} \times \frac{21}{55} = \frac{3}{715}$

20. Let A, B, C be the events of getting a white ball in first, second and third draw respectively, then

Required probability $= P(A \cap B \cap C)$

$= P(A) P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$

Now, $P(A)$ = Probability of drawing a white ball in first draw

$$=\frac{4}{18}=\frac{2}{9}$$

When a white ball is drawn in the first draw there are 17 balls left in the urn, out of which 3 are white.

$$\therefore \quad P\left(\frac{B}{A}\right)=\frac{3}{17}$$

Since the ball drawn is not replaced, therefore after drawing a white ball in the second draw there are 16 balls left in the urn, out of which 2 are white.

$$\therefore \quad P\left(\frac{C}{A\cap B}\right)=\frac{2}{16}=\frac{1}{8}$$

Hence the required probability $=\frac{2}{9}\times\frac{3}{17}\times\frac{1}{8}=\frac{1}{204}$

21. There are 4 even numbers and 5 odd numbers

Let A = the event of choosing odd numbers

B = the event of getting the sum an even number.

Then $A\cap B$ = The event of choosing odd numbers whose sum is even.

$$\therefore \quad n(B)={}^4C_2+{}^5C_2=16$$

and $\quad n(A\cap B)={}^5C_2=10$

$\therefore$ Required probability $=P\left(\frac{A}{B}\right)$

$$=\frac{n(A\cap B)}{n(B)}=\frac{10}{16}=\frac{5}{8}$$

22. Let A be the event of drawing an odd numbered ticket in the first draw and B be the event of drawing an odd numbered ticket in the second draw. Then

Required probability $=P(A\cap B)=P(A)\,P\left(\frac{B}{A}\right)$

$P(A)=\frac{13}{25}$, since there 13 odd numbers 1, 3, 5, … 25

Since the ticket drawn in the first draw is not replaced, therefore second ticket drawn is from the remaining 24 tickets, out of which 12 are odd numbered.

$$\therefore \quad P\left(\frac{B}{A}\right)=\frac{12}{24}=\frac{1}{2}$$

Hence, Required probability $=\frac{13}{25}\times\frac{1}{2}=\frac{13}{50}$

INTRODUCTORY EXERCISE 20.4

1. $P(A\cap B)=P(A)\cdot P(B)=0.6\times 0.4=0.24$

2. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=0.6+0.4-0.24=0.76$$

3. $P(\bar{A}\cap B)=\{1-P(A)\}\,P(B)$

$$=0.4\times 0.4=0.16$$

4. $P(A\cap\bar{B})=P(A)\{1-P(B)\}$

$$=0.6\times 0.6=0.36$$

5. $P(\bar{A}\cap\bar{B})=\{1-P(A)\}\{1-P(B)\}$

$$=0.4\times 0.6=0.24$$

6. $P(\bar{A})=0.65 \Rightarrow P(A)=0.35$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \quad P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

$\Rightarrow \quad 0.65=0.35+p-0.35p$

$\Rightarrow \quad 0.65p=0.30$

$\Rightarrow \quad p=\frac{6}{13}$

7. $S=\{1, 2, 3, 4, 5, 6\}$, for each case

Let A = event of getting a 1,2, 3 or 4 on the first toss

and B = event of getting a 4, 5 or 6 on the second toss

Then, clearly A and B are independent events.

$$\therefore \quad P(A)=\frac{4}{6}=\frac{2}{3} \quad\text{and}\quad P(B)=\frac{3}{6}=\frac{1}{2}$$

So, required probability $=P(A\cap B)$

$$=P(A)\cdot P(B)=\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$$

8. Let E = the event that A gets a three

and F = the event that B gets a three

Then, $\quad P(E)=\frac{1}{6},\quad P(F)=\frac{1}{6}$

$$P(\bar{E})=\frac{5}{6},\quad P(\bar{F})=\frac{5}{6}$$

Suppose A wins then, he gets a three in 1st or 3rd of 5th … throw etc.

$\therefore P(A \text{ wins})=P[E \text{ or } (\bar{E}\,\bar{F}\,E) \text{ or } (\bar{E}\,\bar{F}\,\bar{E}\,\bar{F}\,E) \text{ or} \ldots \infty]$

$=P[E \text{ or } (\bar{E} \text{ and } \bar{F} \text{ and } E) \text{ or } (\bar{E} \text{ and } \bar{F} \text{ and } \bar{E} \text{ and } \bar{F} \text{ and } E) \text{ or} \ldots]$

$=P(E)+P(\bar{E} \text{ and } \bar{F} \text{ and } E)+P(\bar{E} \text{ and } \bar{F} \text{ and } \bar{E} \text{ and } \bar{F} \text{ and } E)+\ldots\infty$

$=P(E)+P(\bar{E})P(\bar{F})P(E)+P(\bar{E})P(\bar{F})\cdot P(\bar{E})P(\bar{F})\cdot P(E)+\ldots\infty$

$$=\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\ldots\infty$$

$$=\frac{1}{6}+\frac{1}{6}\left(\frac{5}{6}\right)^2+\frac{1}{6}\left(\frac{5}{6}\right)^4+\ldots\infty$$

$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\infty\right]$$

$$=\frac{1}{6}\cdot\frac{1}{\left[1-\left(\frac{5}{6}\right)^2\right]}=\left(\frac{1}{6}\cdot\frac{36}{11}\right)=\frac{6}{11}$$

Thus, $P(A \text{ wins})=\frac{6}{11}$ and $P(B \text{ wins})=\frac{5}{11}$

9. We have, $P(H)=\frac{1}{2}$ and $P(T)=\frac{1}{2}$

Now, A wins if he throws a head in 1st, or 3rd or 5th or … draw.

$\therefore P(A \text{ wins})=P[H \text{ or } (T\,TH) \text{ or } (T\,T\,T\,TH) \text{ or } (T\,T\,T\,T\,T\,TH) \text{ or} \ldots]$

$=P(H)+P(T\,TH)+P(T\,T\,T\,TH)+\cdots$

$=P(H)+P(T)P(T)P(H)+P(T)P(T)P(T)P(T)P(H)+\cdots$

$$=\frac{1}{2}+\left(\frac{1}{2}\right)^2\frac{1}{2}+\left(\frac{1}{2}\right)^4\frac{1}{2}+\ldots\infty$$

$$=\frac{1}{2}+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^5+\ldots\infty$$

$$=\frac{1}{2}\left[1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^4+\ldots\infty\right]$$

$$=\frac{1}{2}\cdot\frac{1}{\left[1-\left(\frac{1}{2}\right)^2\right]}=\left(\frac{1}{2}\times\frac{4}{3}\right)=\frac{2}{3}$$

Thus $P(A \text{ wins})=\frac{2}{3}$ and $P(B \text{ wins})=\left(1-\frac{2}{3}\right)=\frac{1}{3}$

10. Let E be the event of 'getting a six'. Then $\bar{E}$ is the event of 'not getting a six'.

Then $\quad P(E)=\frac{1}{6}$ and $P(\bar{E})=\frac{5}{6}$

Now, A wins if he throws a six in 1st, 4th or 7th or … draw.

$\therefore P(A \text{ wins})=P(E \text{ or } (\bar{E}\,\bar{E}\,\bar{E}\,E) \text{ or } (\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E) \text{ or} \ldots]$

$=P(E)+P(\bar{E}\,\bar{E}\,\bar{E}\,E)+P(\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E)+\ldots$

$$=\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}+\ldots\infty$$

$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6+\ldots\infty\right]$$

$$=\frac{1}{6}\cdot\frac{1}{\left[1-\left(\frac{5}{6}\right)^3\right]}=\left(\frac{1}{6}\cdot\frac{216}{91}\right)=\frac{36}{91}$$

Now, B wins if he throws a six in 2nd or 5th or 8th or ..draw.

$\therefore P(B \text{ wins})=P[(\bar{E}E) \text{ or } (\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E) \text{ or } (\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E) \text{ or} \ldots]$

$=P(\bar{E}E)+P(\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E)+P(\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,\bar{E}\,E)+\ldots$

$$=\frac{5}{6}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^7\cdot\frac{1}{6}+\ldots\infty$$

$$=\frac{5}{36}\left[1+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6+\ldots\infty\right]$$

$$=\frac{5}{36}\frac{1}{\left[1-\left(\frac{5}{6}\right)^3\right]}=\left(\frac{5}{36}\,\frac{216}{91}\right)=\frac{30}{91}$$

Thus, $\quad P(A \text{ wins})=\frac{36}{91}$ and $P(B \text{ wins})=\frac{30}{91}$

$\therefore P(A \text{ wins or } B \text{ wins})=P(A \text{ wins})+P(B \text{ wins})$

$$=\frac{36}{91}+\frac{30}{91}=\frac{66}{91}$$

and $\quad P(C \text{ wins})=1-P\,(A \text{ wins or } B \text{ wins})$

$$=1-\frac{66}{91}=\frac{25}{91}$$

11. Let E = Event of getting a sum 9 on two dice.

Then, $\quad E=\{(3, 6), (4, 5), (5, 4), (6, 3)\}$

$\therefore \quad PE=\frac{4}{36}=\frac{1}{9}$ and $P(\bar{E})=\frac{8}{9}$

$\therefore \quad P(A \text{ wins})=P[E_1 \text{ or } \bar{E}_1\,\bar{E}_2E_3 \text{ or } \bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4E_5 \text{ or} \ldots]$

$$=\left[\frac{1}{9}+\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9}+\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{8}{9}\cdot\frac{1}{9}+\ldots\infty\right]$$

$$=\frac{9}{17}$$

$\therefore \quad P(B \text{ wins})=\left(1-\frac{9}{17}\right)=\frac{8}{17}$

$\therefore$ Required ratio $=\frac{9}{8}$

12. Required probability $=\frac{4}{52}\times\frac{3}{51}=\frac{1}{221}$

13. Let A = the event that Ashmit solves the problem.

and B = the event that Amisha solves the problem.

Clearly, A and B are independent events.

Now, $\quad P(A)=\frac{80}{100}=\frac{8}{10}$

and $\quad P(B)=\frac{70}{100}=\frac{7}{10}$

$\therefore \quad P(A\cap B)=P(A).\,P(B)=\frac{8}{10}\times\frac{7}{10}=\frac{56}{100}$

So $\quad P(A \text{ or } B)=P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{8}{10}+\frac{7}{10}-\frac{56}{100}$$

$$=\frac{94}{100}=0.94$$

14. Let A = the event that A hits the target

and B = the event that B hits the target

As given, we have $P(A)=\frac{1}{3}$ and $P(B)=\frac{2}{5}$

Clearly, A and B are independent events

$\therefore \quad P(A\cap B)=P(A).\,P(B)=\frac{1}{3}\times\frac{2}{5}=\frac{2}{15}$

$\therefore \quad P$ (target is hit) $=P(A$ hits or B hits)

$$=P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{1}{3}+\frac{2}{5}-\frac{2}{15}=\frac{3}{5}$$

15. Let A, B, C be the respective events of solving the problem and $\bar{A}, \bar{B}, \bar{C}$ be the respective events of not solving the problem. Then A, B, C are independent events

$\therefore \bar{A}, \bar{B}, \bar{C}$ are independent events

Now, $\quad P(A)=\frac{1}{2}\, P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$

$$P(\bar{A})=\frac{1}{2},\quad P(\bar{B})=\frac{2}{3}\quad\text{and}\quad P(\bar{C})=\frac{3}{4}$$

$\therefore P$(none solves the problem)

$=P$(not A) and (not B) and (not C)]

$=P(\bar{A}\cap\bar{B}\cap\bar{C})$

$=P(\bar{A})P(\bar{B})P(\bar{C}) \qquad (\because \bar{A}, \bar{B}$ and $\bar{C}$ are independent)

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{4}$$

Hence, P(the problem will be solved)

$=1-P$ (none solves the problem)

$$=1-\frac{1}{4}=\frac{3}{4}$$

16. Let A, B, C respectively.
Then $\bar{A}, \bar{B}, \bar{C}$ are the respective events of not solving the problem by them

Now, $P(A)=\frac{1}{3}$, $P(B)=\frac{2}{7}$ and $P(C)=\frac{3}{8}$

∴ $P(\bar{A})=\frac{2}{3}$, $P(\bar{B})=\frac{5}{7}$ and $P(\bar{C})=\frac{5}{8}$

∴ The probability that exactly one of them will solve it

$= P\{[A \cap (\text{not } B) \cap (\text{not } C)]$ or $[(\text{not } A) \cap B \cap (\text{not } C)]$ or $[(\text{not } A) \cap (\text{not } B) \cap C]\}$

$= P\{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)\}$

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$[\because (A \cap \bar{B} \cap \bar{C}) \cap (\bar{A} \cap B \cap \bar{C}) \cap (\bar{A} \cap \bar{B} \cap C) = \phi]$

$= P(A)P(\bar{B})P(\bar{C}) + P(\bar{A}).P(B).P(\bar{C}) + P(\bar{A}).P(\bar{B}).P(C)$

$= \left(\frac{1}{3} \times \frac{5}{7} \times \frac{5}{8}\right) + \left(\frac{2}{3} \times \frac{2}{7} \times \frac{5}{8}\right) + \left(\frac{2}{3} \times \frac{5}{7} \times \frac{3}{8}\right)$

$= \frac{25}{168} + \frac{5}{42} + \frac{5}{28} = \frac{25}{56}$

17. Let A, B, C be the events that A hits the target, B hits the target and C hits the target respectively.

Then, $P(A)=\frac{4}{5}, P(B)=\frac{3}{4}, P(C)=\frac{2}{3}$

$P(\bar{A})=\frac{1}{5}, P(\bar{B})=\frac{1}{4}, P(\bar{C})=\frac{1}{3}$

Case 1. $P(A, B,$ and C, all hit the target$) = P(A \cap B \cap C)$
$= P(A)P(B)P(C) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$

Case II. $P(A$ and B hit but not $C) = P[A \cap B \cap \bar{C}]$
$= P(A)P(B)P(\bar{C})$
$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{5}$

Case III. $P(A$ and C hit but not $B) = P(A \cap C \cap \bar{B})$
$= P(A)P(C)P(\bar{B})$
$= \frac{4}{5} \times \frac{2}{3} \times \frac{1}{4} = \frac{2}{15}$

Case IV. $P(B$ and C hit but not $A) = P(B \cap C \cap \bar{A})$
$= P(B)P(C)P(\bar{A})$
$= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} = \frac{1}{10}$

All the above cases being mutually exclusive, we have the required probability

$= \frac{2}{5} + \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{5}{6}$

18. Let $P_1 = 0.1, P_2 = 0.2, P_3 = 0.3, P_4 = 0.4$

∴ P (The balloon is hit) $= P$ (the balloon is hit atleast once)
$= 1 - P$(the balloon is hit in none of the shots)
$= 1 - (1-P_1)(1-P_2)(1-P_3)(1-P_4)$
$= 1 - (0.9)(0.8)(0.7)(0.6) = 0.6976$

19. $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$

Required probability $= P(A \text{ or } B) = P(A \cup B)$ be the events of solving the problem by A, B, C

$= P(A) + P(B) - P(A)P(B)$

$= \frac{2}{3} + \frac{3}{5} - \frac{2}{3} \cdot \frac{3}{5} = \frac{13}{15}$

20. $P(\text{none}) = 1 - P(\text{atleast one})$
$= 1 - P(A \cup B) = 1 - \frac{13}{15} = \frac{2}{15}$

21. $P(\text{not } A) = 1 - P(A) = 1 - 0.4 = 0.6$

22. $P(\text{at most } C) = P(D \text{ or } C) = P(D) + P(C)$
$= (0.1) + (0.2) = 0.3$

23. $P(B \text{ or } C) = P(B) + P(C)$
$= (0.3) + (0.2) = 0.5$

24. $P(\text{atleast } B) = P(B \text{ or } A) = P(B) + P(A)$
$= (0.3) + (0.4) = 0.7$

Solutions for question number 25-28 :
Let E = the event that Ajay is selected and F = the event that Kajol is selected
Clearly, E and F are independent events

Now, $P(E)=\frac{1}{7}$ and $P(F)=\frac{1}{5}$

∴ $P(\bar{E})=\frac{6}{7}$ *and* $P(\bar{E})=\frac{4}{5}$

25. $P(\text{Both of them will be selected}) = P(E \text{ and } F) = P(E \cap F)$
$= P(E)P(F) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$

26. P(only one of them will be selected)
$= P[(E \text{ and not } F) \text{ or } (F \text{ and not } E)]$
$= P[(E \cap \bar{F}) \cup (F \cap \bar{E})]$
$= P(E \cap \bar{F}) + P(F \cap \bar{E})$
$= P(E)P(\bar{F}) + P(F)P(\bar{E})$
$= \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = \frac{2}{7}$

27. P(none of them will be selected)
$= P[(\text{not } E) \text{ and } (\text{not } F)]$
$= P(\bar{E} \cap \bar{F}) = P(\bar{E}) \times P(\bar{F})$
$= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$

28. P(atleast one of them will be selected)
$= 1 - P$(none will be selected)
$= 1 - \frac{24}{35} = \frac{11}{35}$

29. Let E = the event that A speaks the truth
and F = the event that B speaks the truth
then $\bar{E}$ = the event that A tells a lie
and $\bar{F}$ = the event that B tells a lie.
Clearly, E and F are independent events, So, E and F as well as $\bar{E}$ and F are independent.

Now, $P(E)=\frac{60}{100}=\frac{3}{5}$, $P(F)=\frac{80}{100}=\frac{4}{5}$

∴ $P(\bar{E})=\frac{2}{5}, P(\bar{F})=\frac{1}{5}$

∴ $P(A$ and B contradict each other$)$
$= P(A$ speaks the truth and B tells a lie$)$

or $(A$ tells a lie and B speaks the truth$)$
$= P[(E \cap \bar{F}) \cup (\bar{E} \cup F)]$
$= P(E \cap \bar{F}) + P(\bar{E} \cap F)$
$= P(E)P(\bar{F}) + P(\bar{E})P(F)$
$= \frac{3}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{4}{5}$
$= \frac{11}{25} \Rightarrow 44\%$

So A and B contradict each other in 44% cases.

Solutions for question number 30 and 31 : *Let E = the event that the husband will be alive 20 years hence and F = the event that the wife will be alive 20 years hence.*

Then $P(E)=\frac{5}{12}$ *and* $P(F)=\frac{3}{8}$

∴ $P(\bar{E})=\frac{7}{12}$ *and* $P(\bar{F})=\frac{5}{8}$

Clearly, E and F are independent events.

30. P(Couple will be alive 20 years hence)
$= P(E \text{ and } F) = P(E \cap F)$
$= P(E).P(F) = \frac{5}{12} \times \frac{3}{8} = \frac{5}{32}$

31. P(atleast one of them will be alive 20 years hence)
$= 1 - P$(none will be alive 20 years hence)
$= 1 - P(\bar{E} \cap \bar{F})$
$= 1 - P(\bar{E})P(\bar{F})$ (∵ $\bar{E}$ and $\bar{F}$ are independent)
$= 1 - \left(\frac{7}{12} \times \frac{5}{8}\right) = \frac{61}{96}$

32. Let A, B, C denote the events of favouring the book by the first, the second and the third critic respectively.

Then $P(A)=\frac{5}{7}$, $P(B)=\frac{4}{7}$ and $P(C)=\frac{3}{7}$

∴ $P(\bar{A})=\frac{2}{7}$, $P(\bar{B})=\frac{3}{7}$ and $P(\bar{C})=\frac{4}{7}$

∴ Required probability
$= P$ (two favour the book or three favour the book)
$= P$ (two favour the book) $+ P$ (three favour the book)
$= P[\{A$ and B (not $C)\}$ or $\{A$ and (not B) and $C\}$ or $\{$(not A) and B and $C\}] + P(A$ and B and $C)$
$= P[(A \cap B \cap \bar{C}) \cup P(A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)] + P(A \cap B \cap C)$
$= P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(B)P(C) + P(A)P(B)P(C)$
$= \left(\frac{5}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{5}{7} \times \frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{2}{7} \times \frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$
$= \frac{209}{343}$

33. Required probability
$= P$ (X not defective and Y not defective)
$= P(\bar{X})P(\bar{Y})$
$= \{1 - P(X)\}\{1 - P(Y)\}$
$= \frac{91}{100} \times \frac{95}{100} = \frac{8645}{10000} = 0.8645$

34. A red ball can be drawn in two mutually exclusive ways
(i) Selecting bag I and then drawing a red ball from it.
(ii) Selecting bag II and then drawing a red bal form it.
Let E_1, E_2 and A denote the events defined as follows :
E_1 = selecting bag I,
E_2 = selecting bag II
A = drawing a red ball.
Since one of the two bags is selected randomly, therefore

$P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$.

Now, $P\left(\frac{A}{E_1}\right)$ = probability of drawing a red ball when the first bag has been chosen $= \frac{4}{7}$

$P\left(\frac{A}{E_2}\right)$ = probability of drawing a red ball when the second bag has been selected $= \frac{2}{6}$

Using the law of total probability, we have

$P(\text{red ball}) = P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$
$= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{2}{6} = \frac{19}{42}$

35. Let E_1, E_2, E_3 and A be the events defined as follows.
E_1 = the toy is manufactured by machine A
E_2 = the toy is manufactured by machine B
E_3 = the toy is manufactured by machine C
and A = the toy is defective then

$P(E_1)=\frac{25}{100}=\frac{1}{4}$, $P(E_2)=\frac{35}{100}=\frac{7}{20}$

$P(E_3)=\frac{40}{100}=\frac{2}{5}$

and $P\left(\frac{A}{E_1}\right)$ = Probability that the toy drawn is defective given the condition that it is a manufactured by machine $A = \frac{5}{100}$

Similarly, $P\left(\frac{A}{E_2}\right)=\frac{4}{100}$ and $P\left(\frac{A}{E_3}\right)=\frac{2}{100}$

Using law of total probability, we have

$P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$
$= \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}$
$= 0.345$

36. The testing procedure may terminate at the twelfth testing in two mutually exclusive ways
(i) When lot contain 2 defective bulbs.
(ii) When lot contains 3 defective bulbs.
Consider the following events :
A = testing procedure ends the twelfth testing
E_1 = lot contains 2 defective bulbs.
E_2 = lot contains 3 defective bulbs.

Required Probability $= P(A)$

$$= P(A \cap E_1) \cap P(A \cap E_2)$$
$$= P(A \cap E_1) + P(A \cap E_2)$$
$$= P(E_1)P\left(\frac{A}{E_1}\right) + P(A_2)P\left(\frac{A}{E_2}\right)$$

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that first 11 draws contain 10 non defective and one defective and 12[th] draw contains a defective article.

$$= \frac{^{18}C_{10} \times {}^{2}C_1}{^{20}C_{11}} \times \frac{1}{9}$$

and $P\left(\frac{A}{E_2}\right)$ = Probability that first 11 draws contain 9 non-defective and 2 defective articles and twelfth draw contains a defective article.

$$= \frac{^{17}C_9 \times {}^{3}C_2}{^{20}C_{11}} \times \frac{1}{9}$$

Hence, Required probability

$$= 0.4 \times \frac{^{18}C_{10} \times {}^{2}C_1}{^{20}C_{11}} \times \frac{1}{9} + 0.6 \times \frac{^{17}C_9 \times {}^{3}C_2}{^{20}C_{11}} \times \frac{1}{9}$$

37. Let E_1, E_2, E_3 and A be the events defined as follows :
E_1 = the toy is manufactured by machine A
E_2 = the toy is manufactured by machine B
E_3 = the toy is manufactured by machine C
A = the toy is defective

Then $P(E_1)$ = probability that the toy drawn is manufactured by machine $A = \frac{25}{100}$

Similarly, $P(E_2) = \frac{35}{100}$ and $P(E_3) = \frac{40}{100}$

$P\left(\frac{A}{E_1}\right)$ = probability that the toy drawn is defective given that it is manufactured by machine $A = \frac{5}{100}$

Similarly $P\left(\frac{A}{E_2}\right) = \frac{4}{100}$, $P\left(\frac{A}{E_3}\right) = \frac{2}{100}$

Now, required probability

$$= \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}} = \frac{28}{69}$$

38. Let E_1, E_2, E_3 and A be the events defined as follows :
E_1 = construction chosen is a bridge.
E_2 = construction chosen is a hospital
E_3 = construction chosen is a hotel.
A = construction gets damaged

Since there are 1200 constructions, therefore

$$P(E_1) = \frac{200}{1200} = \frac{1}{6}, \quad P(E_2) = \frac{400}{1200} = \frac{1}{3}$$

and $$P(E_3) = \frac{600}{1200} = \frac{1}{2}$$

It is given that $P\left(\frac{A}{E_1}\right)$ =Probability that a construction gets damaged is a bridge = 0.01

Similarly, $P\left(\frac{A}{E_2}\right) = 0.03$ and $P\left(\frac{A}{E_3}\right) = 0.15$

We are required to find $P\left(\frac{E_1}{A}\right)$ by Baye's rule.

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15}$$

$$= \frac{1}{1 + 6 + 45} = \frac{1}{52}$$

39. Let E_1, E_2 and A be the events defined as follows :
E_1 = selecting a box from the first group
E_2 = selecting a box from the second group and
A = ball drawn is green

Since there are 5 boxes out of which 3 boxes belong to first group and 2 boxes belong the second group. Therefore

$$P(E_1) = \frac{3}{5}, P(E_2) = \frac{2}{5}$$

If E_1 has already occurred, then a box from the first group is chosen. The box chosen contains 5 green balls and 3 red balls.

Therefore the probability of drawing a green ball from it is $\frac{5}{8}$

So $$P\left(\frac{A}{E_1}\right) = \frac{5}{8}$$

Similarly, $$P\left(\frac{A}{E_2}\right) = \frac{2}{6} = \frac{1}{3}$$

Now, we have to find $P\left(\frac{E_1}{A}\right)$

By Baye's rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5} \times \frac{5}{8}}{\frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{1}{3}} = \frac{45}{61}$$

40. Let E_1, E_2 and A be the events defined as follows :
E_1 = six occurs, E_2 = six does not occur
and A = the man reports that it is a six.

We have, $P(E_1) = \frac{1}{6}, P(E_2) = \frac{5}{6}$

Now $P\left(\frac{A}{E_1}\right)$ = probability that the man reports that there is a six on the die given that six has occurred on the die.

= probability that the man speaks truth $= \frac{3}{4}$

and $P\left(\frac{A}{E_2}\right)$ = probability that the man reports that there is six on the die given that six has not occurred on the die.

= probability that the man does not speak truth

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

We have to find $P\left(\frac{E_1}{A}\right)$

By Baye's rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

41. Let E_1, E_2, E_3, E_4 and A be the events as defined below :
E_1 = the missing card is diamond
E_2 = the missing card is heart
E_3 = the missing card is spade
E_4 = the missing card is club
A = drawing two diamonds cards from the remaining cards

Then $$P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = \frac{13}{52} = \frac{1}{4}$$
$$P(E_3) = \frac{13}{52} = \frac{1}{4}$$
and $$P(E_4) = \frac{13}{52} = \frac{1}{4}$$

$P\left(\frac{A}{E_1}\right)$ = probability of drawing two diamonds cards given that one diamond card is missing $= \frac{^{12}C_2}{^{51}C_2}$

$P\left(\frac{A}{E_2}\right)$ = probability of drawing two diamond cards given that one heart card is missing.

$$= \frac{^{13}C_2}{^{51}C_2}$$

Similarly, $$P\left(\frac{A}{E_3}\right) = \frac{^{13}C_2}{^{51}C_2}$$

and $$P\left(\frac{A}{E_4}\right) = \frac{^{13}C_2}{^{31}C_2}$$

By, Baye's rule

Required probability $= P\left(\frac{E_1}{A}\right)$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right) + P(E_4)P\left(\frac{A}{E_4}\right)}$$

$$= \frac{11}{50}$$

LEVEL 1

1. Total possible nine digit numbers = 9!
Out of these 9! numbers only those numbers are divisible by 4 which have their last digits as even natural number and the numbers formed by their last two digits are divisible by 4.
The possible numbers of last two digits are
12, 32, 52, 72, 92, 24, 64, 84, 16, 36, 56, 76, 96, 28, 48, 68.
Thus there are 16 ways of choosing the last two digits. Corresponding to each of these ways the remaining 7 digits can be arranged in 7! ways. Therefore, the total number of 9 digits numbers divisible by 4 is $16 \times 7!$.

Hence, required probability $= \frac{16 \times 7!}{9!} = \frac{2}{9}$

2. Total possible number of 4 digits = 4! = 24
The number is divisible by 5 if unit digit itself is 5. Therefore we fix 5 at unit place and then remaining 3 places can be filled up in 3! ways.

Hence, the required probability $= \frac{3!}{4!} = \frac{6}{24} = \frac{1}{4}$

3. Excluding ground floor there are 7 floors. A person can leave the cabin at any of the 7 floors. Therefore, the total number of ways in which each of the 5 persons can leave the cabin at any of the 7 floors $= 7^5$.

$\therefore$ Exhaustive number of ways $= 7^5$

Five persons can leave the cabin at five different floors in $^7P_5$ ways.

$\therefore$ The favourable number of cases $= {}^7P_5$

Hence, the required probability $= \frac{^7P_5}{7^5} = \frac{360}{2401}$

4.

Box 1	Box 2	Box 3
3W	2W	1W
1B	2B	3B

There can be three mutually exclusive cases of drawing 2 white balls and 1 black ball.

	Box 1	Box 2	Box 3
Case 1	1W	1W	1B
Case 2	1W	1B	1W
Case 3	1B	1W	1W

∴ Required probability

$= P(W_1 \cap W_2 \cap B_3) \cup (W_1 \cap B_2 \cap W_3) \cup (B_1 \cap W_2 \cap W_3)$

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{26}{64} = \frac{13}{32}$

5. Since coin is fair *i.e.* with equal probability a heads and a tail can be obtained. Also all the trials are independent so the probability that head appears on the fifth toss does not depend upon previous results of the tosses.

Hence, required probability $= \frac{1}{2}$

6. The total number of ways in which two calculators can be chosen out of four calculators is $^4C_2 = 6$. If only two tests are required to identify defective calculators, then in first two tests defective calculators are identified. This can be done in one way only.

∴ Required probability $= \frac{1}{6}$

7. 20 girls can be seated around a round table in 19! ways.
So, exhaustive number of cases = 19!
Excluding A and B, out of remaining 18 girls 4 girls can be selected $^{18}C_4$ ways which can be arranged in 4! ways.
Remaining $20 - (4 - 2) = 14$ girls can be arranged in 14! ways. Also A and B mutually can be arranged in 2! ways.

∴ Required number of arrangements $= {}^{18}C_4 \times 4! \times 2! \times 14!$
$= 18! \times 2$

∴ Required probability $= \frac{18! \times 2}{19!} = \frac{2}{19}$

8. Since x and y can take values from 0 to 10. So, the total number of ways of selecting x and y is $11 \times 11 = 121$

Now, $|x - y| > 5 \Rightarrow x - y < -5$ or $x - y > 5$

There are 30 pairs of values of x and y satisfying these two inequalities, so favourable number of ways = 30

Hence, required probability $= \frac{30}{121}$

9. Since a persons's birthday can fall in any of the 12 months.
So, total number of ways $= 12^4$.
Now, any two months can be chosen in $^{12}C_2$ ways. The 4 persons birthday can fall in these two months in 2^4 ways. Out of these 2^4 ways there are two ways when all of the four birthdays fall in one month.

So, favourable number of ways $= \frac{^{12}C_2 \times (2^4 - 2)}{12^4} = \frac{77}{1728}$

10. 6 objects can be distributed among 6 persons in 6^6 ways.

∴ Total number of ways $= 6^6$

The number of ways of distribution in which each one gets only one thing is 6!. So, the number of distribution in which atleast one of them does not get any thing is $6^6 - 6!$

Hence, required probability $= \frac{6^6 - 6!}{6^6}$

11. Total number of ways of arranging 40 books on a shelf = 40!
Out of 40 places, 4 places for the four volumes can be chosen in $^{40}C_4$ ways. In the remaining 36 places the remaining 36 books can be arranged in 36! ways. In the 4 places four volumes of encyclopedia can be arranged in increasing order in one way only.

So, favourable number of ways $= {}^{40}C_4 \times 36!$

Hence, required probability $= \frac{^{40}C_4 \times 36!}{40!} = \frac{1}{24}$

12. The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any of the 10 digits 0, 1, 2 ... 9. So, the total number of ways of selecting last digits of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$.
If the product of the 4 numbers is not divisible by 5 or 10. Then the number of choices for the last digit of each number is 8 (excluding 0 or 5).

So, favourable number of ways $= 8 \times 8 \times 8 \times 8 = 8^4$

∴ The probability that the product is not divisible by 5 or 10

$= \frac{8^4}{10^4} = \left(\frac{8}{10}\right)^4$

Hence, Required probability $= 1 - \left(\frac{8}{10}\right)^4 = \frac{369}{625}$

13. P(selecting atleast one couple) = 1 − P(selecting none of the couples for the prize)

$= 1 - \left(\frac{^{16}C_1 \times {}^{14}C_1 \times {}^{12}C_1 \times {}^{10}C_1}{^{16}C_4}\right) = \frac{15}{39}$

14. Total number of ways in which 7 (= 4 + 3) persons can speak is 7!.
The number of ways in which A, B, C speak in the given order is $^7C_3$ ways and remaining 4 persons can be arranged in 4! ways.

∴ Favourable number of ways $= {}^7C_3 \times 4!$

∴ Required probability $= \frac{^7C_3 \times 4!}{7!} = \frac{1}{6}$

15. Let A be the event of selecting a counterfeit coin and B the event of getting head, then

Required probability $= P(A \cap B) \cup (\bar{A} \cap B)$

$= P(A \cap B) + P(\bar{A} \cap B)$

$= P(A)P\left(\frac{B}{A}\right) + P(\bar{A})P\left(\frac{B}{\bar{A}}\right)$

$= \frac{2}{16} \times 1 + \frac{14}{16} \times \frac{1}{2} = \frac{9}{16}$

16. Total number of ways in which 5 people can be chosen out of 9 people $= {}^9C_5 = 126$

Number of ways in which the couple serves the committee

$= {}^7C_3 \times {}^2C_2 = 35$

Number of ways in which the couple does not serve the committee $= {}^7C_5 = 21$

∴ Favourable number of cases = 35 + 21 = 56

Hence, the required probability $= \frac{56}{126} = \frac{4}{9}$

17. E_1 = The event in which A speaks truth
E_2 = The event in which B speaks truth

Then $P(E_1) = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = \frac{80}{100} = \frac{4}{5}$

and $P(\bar{E}_1) = \frac{2}{5}$, $P(\bar{E}_2) = \frac{1}{5}$

∴ Required probability $= P[(E_1 \cap E_2) \cup (\bar{E}_1 \cap \bar{E}_2)]$

$= P(E_1 \cap E_2) + P(\bar{E}_1 \cap \bar{E}_2)$

$= P(E_1).P(E_2) + P(\bar{E}_1).P(\bar{E}_2)$

$= \left(\frac{3}{5} \times \frac{4}{5}\right) + \left(\frac{2}{5} \times \frac{1}{5}\right) = \frac{14}{25} = 0.56$

18. Two different squares can be chosen in 64×63 ways.
For each of the four corner squares, the favourable number of cases is 2.
For each of the 24 non-corner squares on all the four sides of the chessboard, the favourable number of cases is 3.
For each of the 36 remaining squares, the favourable number of cases is 4.
Thus, the total number of favourable cases

$= 4 \times 2 + 24 \times 3 + 36 \times 4 = 224$

Hence, the required probability $= \frac{224}{64 \times 63} = \frac{1}{18}$

19. The last two digits can be dialled in $^{10}P_2 = 90$ ways.
Out of these 90 cases only one case is favourable.

Hence, the required probability $= \frac{1}{90}$.

20. 3 squares on a chessboard can be chosen in $^{64}C_3$ ways.
Two squares of one colour and third square of different colour can be chosen in two mutually exclusive way.
(i) 2 white and one black (ii) 2 black and one white
Thus the favourable number of cases

$= {}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2$

$= 2({}^{32}C_2 \times {}^{32}C_1)$

Hence, the required probability $= \frac{2({}^{32}C_2 \times {}^{32}C_1)}{^{64}C_3} = \frac{16}{21}$

21. A leap year contains 366 days comprising of 52 full weeks and 2 extra days. Thus there can be following 7 possibilities for 2 extra days.

(i) Sunday, Monday, (ii) Monday, Tuesday
(iii) Tuesday, Wednesday (iv) Wednesday, Thursday
(v) Thursday, Friday (vi) Friday, Saturday
(vii) Saturday, Sunday

Let A be the event that the leap year contains 53 Sundays. and B be the event that leap year contains 53 Mondays.
Then, we have

$P(A) = \frac{2}{7}$, $P(B) = \frac{2}{7}$, $P(A \cap B) = \frac{1}{7}$

So, required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

22. Since, the probability of getting atleast one head in n times $= 1 - \left(\frac{1}{2}\right)^n$

Therefore, $1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$

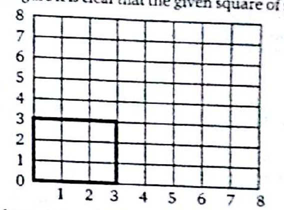
$\Rightarrow 2^n \geq 10 \Rightarrow n \geq 4$

Hence, the least value of n is 4.

23. Total number of ways in which 2 persons can be selected out of 13 persons is $^{13}C_2$

Now, favourable number of cases $= \frac{^5C_1 \times {}^8C_1 + {}^5C_2}{^{13}C_2} = \frac{25}{39}$

24. We can choose 9 squares out of 64 squares in $^{64}C_9$ ways.
Hence, exhaustive number of cases $= {}^{64}C_9$
From the figure it is clear that the given square of size 3 × 3



can be formed by using four consecutive horizontal and 4 consecutive vertical lines, which can be done in

$^6C_1 \times {}^6C_1 = 36$ ways

Basically you can make 6 squares of size 3 × 3 in vertical direction and 6 squares of the size 3 × 3 in horizontal direction. Hence total 6 × 6 = 36 squares can be chosen.

∴ The required probability $= \frac{36}{^{64}C_9}$

25. Total 7 digit numbers can be formed from the 9 digits $= {}^9P_7$
There are four exclusive cases of selecting 7 digits out of 9 digits which can form 7 digit numbers which are divisible by 9.

2, 3, 4, 5, 6, 7, 9 } 36 removing 1 and 8
1, 3, 4, 5, 6, 8, 9 } 36 removing 2 and 7
1, 2, 4, 5, 7, 8, 9 } 36 removing 3 and 6
1, 2, 3, 6, 7, 8, 9 } 36 removing 4 and 5

All the 7 numbers of each of the 4 sets can be arranged in 7! ways.
Hence the favourable number of numbers $= 4 \times 7!$

∴ Required probability $= \frac{4 \times 7!}{^9P_7} = \frac{1}{9}$

LEVEL 2

1. Total number of words that can be formed from the letters of the word MISSISSIPPI is $\frac{11!}{4!4!2!}$

When all the S's are together then the number of words can be formed $= \frac{8!}{4!2!}$

$$\text{Required probability} = \frac{\frac{8!}{4!2!}}{\frac{11!}{4!4!2!}} = \frac{4}{165}$$

2. Since each of the coefficients a, b and c can take values from 1 to 6. Therefore the total number of equations

$$= 6 \times 6 \times 6 = 216$$

Hence the exhaustive number of cases = 216
Now, the roots of the equation $ax^2 + bx + c = 0$ will be real if $b^2 - 4ac \geq 0 \Rightarrow b^2 \geq 4ac$
Following are the number of favourable cases

a	c	ac	$4ac$	b^2 $(\geq 4ac)$	b	Number of cases
1	1	1	4	4, 9, 16, 25, 36	2, 3, 4, 5, 6	$1 \times 5 = 5$
1 2	2 1	2	8	9, 16, 25, 36	3, 4, 5, 6	$2 \times 4 = 8$
1 3	3 1	3	12	16, 25, 36	4, 5, 6	$2 \times 3 = 6$
1 2 4	4 2 2	4	16	16, 25, 36	4, 5, 6	$3 \times 3 = 9$
1 5	5 1	5	20	25, 36	5, 6	$2 \times 2 = 4$
1 2 3 6	6 3 2 1	6	24	25, 36	5, 6	$4 \times 2 = 8$
2 4	4 2	8	32	36	6	$2 \times 1 = 2$
3	3	9	36	36	6	$1 \times 1 = 1$
					Total	= 43

Note → $ac = 7$ is not possible
Since $b^2_{(max)} = 36$ and $4ac \leq b^2$ hence $ac = 10, 11, 12 \ldots$ etc. is not possible.
Hence, total number of favourable cases = 43

So, the required probability $= \frac{43}{216}$.

3. 6 can be thrown with a pair of dice in the following ways (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)

So, probability of throwing a '6' $= \frac{5}{36}$

and probability of not throwing a '6' $= \frac{31}{36}$

And 7 can be thrown with a pair of dice in the following ways (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

so probability of throwing a '7' $= \frac{6}{36} = \frac{1}{6}$

and probability of not throwing a '7' $= \frac{5}{6}$

Let E_1 be the event of the throwing a '6' in a single throw of a pair of dice and
E_2 be the event of throwing a 7 in a single throw of a pair of dice.

Then $P(E_1) = \frac{5}{36}$, $P(E_2) = \frac{1}{6}$

and $P(\bar{E}_1) = \frac{31}{36}$, $P(\bar{E}_2) = \frac{5}{6}$

A wins if he throws '6' in first or third or fifth .. throws.
Probability of A throwing a 6 in first throw $= P(E_1) = \frac{5}{36}$
and probability of A throwing a 6 in third throw

$$= P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) = P(\bar{E}_1)P(\bar{E}_2)P(E_1)$$
$$= \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}$$

Similarly, probability of A throwing a '6' in fifth throw

$$= P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_1)P(\bar{E}_2)P(E_1)$$
$$= \left(\frac{31}{36}\right)^2 \times \left(\frac{5}{6}\right)^2 \times \frac{5}{36}$$

Hence, probability of winning of A

$$= P[E_1 \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_1) \cup (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1) \cup \ldots]$$
$$= P(E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1) \cdot$$
$$= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \ldots$$
$$= \frac{\frac{5}{36}}{1 - \left(\frac{31}{36}\right) \times \left(\frac{5}{6}\right)} = \frac{30}{61}$$

Thus, probability of winning of $B = 1 - \frac{30}{61} = \frac{31}{61}$

4. Let A be the event of getting exactly 3 defectives in the examination of 8 wristwatches.
And B be the event of getting nineth wristwatch defective.
Then

$$\text{Required probability} = P(A \cap B) = P(A)P\left(\frac{B}{A}\right)$$

Now, $P(A) = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}$

And $P\left(\frac{B}{A}\right)$ = Probability that the nineth examined wristwatch is defective given that there were 3 defectives in the first 8 pieces examined $= \frac{1}{7}$

Hence, required probability $= \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} \times \frac{1}{7} = \frac{8}{195}$

5. Let E_1, E_2, E_3 and A be the events defined as follows :
E_1 = the examinee guesses the answer
E_2 = the examinee copies the answer
E_3 = the examinee knows the answer and
A = the examinee answers correctly
We have $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{6}$

Since E_1, E_2 and E_3 are mutually exclusive and exhaustive events therefore

$$P(E_1) + P(E_2) + P(E_3) = 1$$
$$\Rightarrow \quad P(E_3) = \frac{1}{2}$$

If E_1 has already occurred, then the examinee guesses. Since there are four choices out of which only one is correct, therefore the probability that he answers correctly given that he has made a guess is $\frac{1}{4}$ i.e., $P\left(\frac{A}{E_1}\right) = \frac{1}{4}$

It is a given that $P\left(\frac{A}{E_2}\right) = \frac{1}{8}$ and $P\left(\frac{A}{E_3}\right)$ is the probability that he answers correctly given that he knew the answer = 1
By Baye's rule,

$$\text{Required probability} = P\left(\frac{E_3}{A}\right)$$
$$= \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$
$$= \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{24}{29}$$

6. Let x and y be the two non-negative integers since $x + y = 200$

$\therefore \quad (xy)_{max} = 100 \times 100 = 10000 \quad (xy_{max}$ at $x = y)$

Now, $\quad xy \nless \frac{3}{4} \times 10000 \Rightarrow xy \geq \frac{3}{4} \times 10000$

$\Rightarrow \quad xy \geq 7500 \Rightarrow x(200 - x) \geq 7500$

$\Rightarrow \quad x^2 - 200x + 7500 \leq 0$

$\Rightarrow \quad 50 \leq x \leq 150$

So favourable number of ways = 150 − 50 + 1 = 101
Total number of ways = 200

Hence, required probability $= \frac{101}{200}$

7. Let $E_i (i = 1, 2, 3$ etc.) denote the event of drawing an even numbered card in i^{th} draw and F_i $(i = 1, 2, 3)$ denote the event of drawing an odd numbered card in i^{th} draw, then required probability

$$= P[(E_1 \cap F_2 \cap F_3) \cup (F_1 \cap E_2 \cap F_3) \cup (F_1 \cap F_2 \cap E_3)]$$
$$= P(E_1)P(F_2)P(F_3) + P(F_1)P(E_2)P(F_3) + P(F_1)P(F_2)P(E_3)$$
$$= \frac{4}{9} \times \frac{5}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{5}{9} \times \frac{4}{9}$$
$$= 3 \times \frac{4 \times (5)^2}{(9)^3} = \frac{100}{243}$$

8. Consider the following events
A = The first number is less than the second number
B = The third number lies between the first and the second.
Now, we have to find $P\left(\frac{B}{A}\right)$.

Also, we have $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

Any 3 numbers can be chosen out of n numbers in ${}^nC_3$ ways. Let the selected numbers be x_1, x_2, x_3. Then they satisfy exactly one of the following inequalities.

$$x_1 < x_2 < x_3, \quad x_1 < x_3 < x_2, \quad x_2 < x_1 < x_3,$$
$$x_2 < x_3 < x_1, \quad x_3 < x_1 < x_2, \quad x_3 < x_2 < x_1$$

The total number of ways of selecting three numbers and then arranging them $= {}^nC_3 \times 3! = {}^nP_3$

$\therefore \quad P(A) = \frac{{}^nC_3 \times 3}{{}^nC_3 \times 3!}$

and $\quad P(A \cap B) = \frac{{}^nC_3}{{}^nC_3 \times 3!}$

Hence $\quad P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$

9. Since b and c each can assume 9 values from 1 to 9.
So, total number of ways of choosing b and c is $9 \times 9 = 81$
Now, $x^2 + bx + c > 0$ for all $x \in R$

$\Rightarrow \quad D < 0$

$\Rightarrow \quad b^2 - 4ac < 0 \Rightarrow b^2 - 4c < 0 \Rightarrow b^2 < 4c$

Alternatively : $\quad x^2 + bx + c > 0$

$\Rightarrow \quad \left(x + \frac{b}{2}\right)^2 + \frac{4c - b^2}{4} > 0$

$\Rightarrow \quad 4c - b^2 > 0$

$\Rightarrow \quad b^2 < 4c$

Now, the following table shows the possible values of b and c for which $b^2 < 4c$

c	b	Total
1	1,	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3,	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
		32

So, favourable number of cases = 32

Hence required probability $= \frac{32}{81}$

10. We have, $P(A \cup B \cup C) = \frac{3}{4}$

i.e., $\quad P(A) + P(B) + P(C) - P(A \cap B)$

$$- P(B \cap C) - P(A \cap C) + P(A \cup B \cup C) = \frac{3}{4}$$

and $\quad P(A \cap B) + P(B \cap C) + P(A \cap C) - 2P(A \cap B \cap C) = \frac{1}{2}$

and $\quad P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C) = \frac{2}{5}$

Solving the above equations (last two), we get

$$P(A \cap B \cap C) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$$

$$\Rightarrow \quad P(A)P(B)P(C) = \frac{1}{10} \quad \Rightarrow \quad pmc = \frac{1}{10}$$

Also, $\quad P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(A \cap C)] + P(A \cup B \cup C) = \frac{3}{4}$

$$p + m + c - \left(\frac{1}{2} + \frac{2}{10}\right) + \frac{1}{10} = \frac{3}{4}$$

$$\Rightarrow \quad p + m + c = \frac{27}{20}$$

11. Let E, F, G be the events that the student is successful in tests A, B and C respectively. Then the probability that the students is successful is

$= P[(E \cap F \cap \bar{G}) \cup (E \cap \bar{F} \cap G) \cup (E \cap F \cap G)]$

$= P(E \cap F \cap \bar{G}) + P(A \cap \bar{F} \cap G) + P(E \cap F \cap G)$

$= P(E)P(F)P(\bar{G}) + P(E)P(\bar{F})P(G) + P(E)P(F)P(G)$

$= pq\left(1 - \frac{1}{2}\right) + p(1-q)\left(\frac{1}{2}\right) + pq\left(\frac{1}{2}\right)$

$= \frac{pq + p - pq + pq}{2}$

$= \frac{p(1+q)}{2}$

But the probability that the student is successful $= \frac{1}{2}$

$$\therefore \quad \frac{P(1+q)}{2} = \frac{1}{2} \quad \Rightarrow \quad p(1+q) = 1$$

This is satisfied by $p = 1$, $q = 0$

Also there are other values (infinite numbers) of p, q for which the above relation is satisfied.

Hence, (d) is the correct option.

12. Since $\frac{1+4p}{p}, \frac{1-p}{4}, \frac{1-2p}{2}$ are the probabilities of 3 mutually exclusive events, therefore

$$0 \le \frac{1+4p}{p} \le 1, \quad 0 \le \frac{1-p}{4} \le 1, \quad 0 \le \frac{1-2p}{2} \le 1$$

and $\quad 0 \le \frac{1+4p}{p} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$

$$\Rightarrow \quad -\frac{1}{4} \le p \le \frac{3}{4}, \quad -1 \le p \le 1, \quad -\frac{1}{2} \le p \le \frac{1}{2}$$

and $\quad \frac{1}{2} \le p \le \frac{5}{2}$

$$\Rightarrow \max\left\{-\frac{1}{4}, -1, -\frac{1}{2}, \frac{1}{2}\right\} \le p \le \min\left\{\frac{3}{4}, 1, \frac{1}{2}, \frac{5}{2}\right\}$$

$$\Rightarrow \quad \frac{1}{2} \le p \le \frac{1}{2} \quad \Rightarrow \quad p = \frac{1}{2}$$

13. ASSISTANT $\rightarrow$ AA I N SSS TT

STATISTICS $\rightarrow$ A II C SSS TTT

Here N and C are not common and same letters can be A, I, S, T. Therefore

Probability of choosing $A = \frac{^2C_1}{^9C_1} \times \frac{^1C_1}{^{10}C_1} = \frac{1}{45}$

Probability of choosing $I = \frac{1}{^9C_1} \times \frac{^2C_1}{^{10}C_1} = \frac{1}{45}$

Probability of choosing $S = \frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{10}$

Probability of choosing $T = \frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{15}$

Hence, required probability

$$= \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$$

14. Out of 30 numbers 2 numbers can be chosen in $^{30}C_2$ ways. So, exhaustive number of cases $= {}^{30}C_2 = 435$

Since $a^2 - b^2$ is divisible by 3 iff either a and b are divisible by 3 or none of a and b is divisible by 3. Thus, the favourable numbers, of cases $= {}^{10}C_2 + {}^{20}C_2 = 235$

Hence, required probability $= \frac{235}{435} = \frac{47}{87}$

15. The man will be one step away from the starting point if (A) either he is one step ahead or (B) one step behind the starting point.

Therefore, required probability $= P(A) + P(B)$

The man will be one step ahead at the end of eleven steps if he moves six steps forward and five steps backward. The probability of this event $= {}^{11}C_6(0.4)^6\,(0.6)^5$

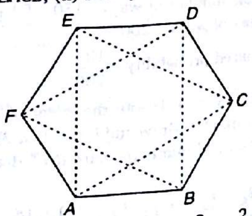
The man will be one step behind at the end of the eleven steps if he moves six steps backward and five steps forward. The probability of this event $= {}^{11}C_6(0.6)^6\,(0.4)^5$.

Hence, the required probability

$$= {}^{11}C_6(0.4)^6(0.6)^5 + {}^{11}C_6(0.6)^6\,(0.4)^5$$

$$= {}^{11}C_6(0.4)^5\,(0.6)^5\,(0.4 + 0.6) = {}^{11}C_6(0.24)^5$$

16. There are 6 vertices in a hexagon. Using 3 vertices out of 6 vertices we can form $^6C_3$ triangles. But there can be only two triangles out of $^6C_3$ triangles which are equilateral (see the figure, (i) ΔACE, (ii) ΔBDF)



Hence, the required probability $= \frac{2}{^6C_3} = \frac{2}{20} = \frac{1}{10}$

17. Let F, B, L and R denote the forward, backward, left and right steps (or movements) then the following mutually exclusive ways are possible.

$$\begin{array}{cccc} F & B & L & R \end{array}$$

$$\begin{bmatrix} 0 & 0 & 4 & 5 \\ 1 & 1 & 3 & 4 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 1 & 2 \\ 4 & 4 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 5 & 4 \\ 1 & 1 & 4 & 3 \\ 2 & 2 & 3 & 2 \\ 3 & 3 & 2 & 1 \\ 4 & 4 & 1 & 0 \end{bmatrix}$$

In this case he cancels out his forward and backward movements by moving equal steps in forward and backward directions each and he creates a difference of 1 step by moving one step extra either in right or left direction.

$$\begin{array}{cccc} F & B & L & R \end{array}$$

$$\begin{bmatrix} 4 & 5 & 0 & 0 \\ 3 & 4 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 0 & 1 & 4 & 4 \end{bmatrix} \quad \begin{bmatrix} 5 & 4 & 0 & 0 \\ 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 2 & 1 & 3 & 3 \\ 1 & 0 & 4 & 4 \end{bmatrix}$$

In this case he cancels out his left or right movement by moving equal number of steps in left and right directions each and he creates a difference of 1 step by moving one step extra either in forward or backward directions.

The number of permutations of these five arrangements is

$$4\left[\frac{9!}{5!4!} + \frac{9!}{1!1!3!4!} + \frac{9!}{2!2!2!3!} + \frac{9}{3!3!1!2!} + \frac{9!}{4!4!1!}\right]$$

$= 4(126 + 2520 + 7560 + 5040 + 630)$

$= 4 \times 15876$

But the total number of ways of arranging nine steps $= 4^9$.

$\therefore \quad$ The required probability $= \frac{4 \times 15876}{4^9} = \frac{3969}{4^7}$

18. Let E_{rr} denote that a red colour ball is transferred from urn A to urn B tourn then a red colour ball is transferred from urn B to urn A.

E_{rb} denote that a red colour ball is transferred from urn A to urn B then a black colour ball is transferred from urn B to urn A

E_{br} denote that a black colour ball is transferred from urn A to urn B then a red colour ball is transferred from urn B to urn A.

E_{bb} denote that a black colour ball is transferred from urn A to urn B then a black colour ball is transferred from urn B to urn A. Then

$$P(E_{rr}) = \left(\frac{6}{10}\right)\left(\frac{5}{11}\right) = \frac{3}{11},$$

$$P(E_{rb}) = \left(\frac{6}{10}\right)\left(\frac{6}{11}\right) = \frac{18}{55}$$

$$P(E_{br}) = \left(\frac{4}{10}\right)\left(\frac{4}{11}\right) = \frac{8}{55}$$

$$P(E_{bb}) = \left(\frac{4}{10}\right)\left(\frac{7}{11}\right) = \frac{14}{55}$$

Let A be the event of drawing a red colour ball after these transfers. Then

$$P\left(\frac{A}{E_{rr}}\right) = \frac{6}{10}, \; P\left(\frac{A}{E_{rb}}\right) = \frac{5}{10}$$

$$P\left(\frac{A}{E_{br}}\right) = \frac{7}{10}, \; P\left(\frac{A}{E_{bb}}\right) = \frac{6}{10}$$

Therefore, the required probability is

$$P(A) = P(E_{rr})\,P\left(\frac{A}{E_{rr}}\right) + P(E_{rb})\,P\left(\frac{A}{E_{rb}}\right) + P(E_{br})\,P\left(\frac{A}{E_{br}}\right) + P\,(E_{bb})P\left(\frac{A}{E_{bb}}\right)$$

$$= \left(\frac{3}{11}\right)\left(\frac{6}{10}\right) + \left(\frac{5}{10}\right)\left(\frac{18}{55}\right) + \left(\frac{8}{55}\right)\left(\frac{7}{10}\right) + \left(\frac{14}{55}\right)\left(\frac{6}{10}\right)$$

$$= \frac{90 + 90 + 56 + 84}{550} = \frac{32}{55}$$

19. A number is divisible by 11 only if the difference of the sum of the digits at odd places and sum of the digits at even places is divisible by 11 *i.e*, 0, 11, 22, 33 ....

Here the sum of all the 9 digits (1, 2, 3, ...9) is 45.

We cannot create the difference of zero since $x + y = 45$, which is odd hence cannot be broken into two equal parts in integers.

Now, we will look for the possibilities of 11 which are as follows :

{1, 2, 6, 8}{1, 2, 5, 9}{1, 3, 6, 7}
{1, 3, 5, 8} {1, 3, 4, 9}{1, 4, 5, 7}
{2, 3, 5, 7}{2, 3, 4, 8}{2, 4, 5, 6}
and {4, 7, 8, 9}{5, 6, 8, 9}

The above set of values either gives the sum of 17 or 28. Since if the sum of 4 digits at even places be 17 or 28 then the sum of rest of the digits (*i.e.*, digits at odd places) be 28 or 17 respectively and thus we can get the difference of 11.

Further we cannot get the difference of 22 or 33...

So there is only possible difference that can be created is 11 and there are only 11 set of values given above containing 4 digits which can be arranged in 4! ways and the remaining 5 digits can be arranged in 5! ways.

Thus the favourable number of numbers $= 11 \times 4! \times 5!$

But the total number of ways of arranging a nine digit number is $^9P_9 = 9!$

$\therefore \quad$ Exclusive number of cases = 9!

$\therefore \quad$ Required probability $= \frac{11 \times 4! \times 5!}{9!} = \frac{11}{126}$