CHAPTER – 5
CLOCKS

The hour hand and the minute hand of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle (> 180°).

For solving the problems on clocks, the following points will be helpful.

- Minute hand covers 360° in 1 hour, i.e., in 60 minutes. Hence, MINUTE HAND COVERS 6° PER MINUTE.

- Hour hand covers 360° in 12 hours. Hence, hour hand covers 30° per hour. Hence, HOUR HAND COVERS 1/2° PER MINUTE.

The following additional points also should be remembered. In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e., they coincide with each other), 11 times.
- 180° with each other (i.e., they lie on the same straight line), 11 times.
- 90° or any other angle with each other, 22 times.

Note: We can also solve the problems on clocks using the method of "Relative Velocity"

In 1 minute, Minute Hand covers 6° and Hour hand covers 1/2°.

Therefore, Relative Velocity = 6 – 1/2 = $5^{1}/_{2}$° per minute. Alternately, in 1 hour, the minute hand covers 60 minute divisions whereas the hour hand covers 5 minute divisions.

∴ Relative Speed = 60 – 5 = 55 minutes per hour.
However, adopting the approach of actual angles covered is by far the simplest and does not create any confusion.

Points to Note:

- Any angle is made 22 times in a period of 12 hours.
- In a period of 12 hours, there are 11 coincidences of the two hands, when the two hands are in a straight line facing opposite directions.
- The time gap between any two coincidences is 12/11 hours or $65^{5}/_{11}$ minutes.
- If the hands of a clock (which do not show the correct time) coincide every p minutes, then

If p > $65^{5}/_{11}$, then the watch is going slow or losing time.
If p < $65^{5}/_{11}$, then the watch is going fast or gaining time.

To calculate the angle 'θ' between the hands of a clock, we use the following formula (where m = minutes and h = hours)

(i) $\theta = \frac{11}{2}m - 30h \left(\text{when } \frac{11}{2}m > 30h \right)$

(ii) $\theta = 30h - \frac{11}{2}m \left(\text{when } 30h > \frac{11}{2}m \right)$

Worked out Examples

1. What is the angle between the minute hand and the hour hand of a clock at 3 hours 40 minutes?
   (A) 20° (B) 70° (C) 90° (D) 130° (E) 120°

Sol: The angle between the hands can be calculated
by $\theta = \left| \frac{11}{2}m - 30h \right|$, where m is minutes and h is
hours. Here, m = 40 and h = 3
∴ $\theta = \left| \frac{11}{2} \times 40 - 30 \times 3 \right|$
= |220 – 90| = 130°
The angle between the two hands is 130°.

Choice (D)

2. Find the time between 2 and 3 O'clock at which the minute hand and the hour hand
   (i) make an angle of 60° with each other.
   (ii) overlap.
   (iii) are perpendicular to each other.
   (iv) are on the same straight line but are facing opposite directions.

Sol: (i) In the formula $\theta = \left| \frac{11}{2}m - 30h \right|$,

θ = 60° and h = 2

∴ $60 = \frac{11}{2}m - 30 \times 2$

$\frac{11}{2}m = 120$

$m = \frac{240}{11} = 21\frac{9}{11}$ min past 2

or

$60 = 30 \times 2 - \frac{11}{2}m$

∴ $\frac{11}{2}m = 0$

m = 0
Therefore, the angle between the hour hand and the minute hand is 60°
at 2 O'clock and at $21\frac{9}{11}$ minutes
past 2 O'clock.

(ii) When the two hands overlap, the angle between them is 0°.

$$\theta = \left| \frac{11}{2} m - 30h \right|$$

$\therefore \theta = 0°$ and $h = 2$

$$\frac{11}{2} m = 30 \times 2$$

$$m = \frac{120}{11} = 10\frac{10}{11} \text{ min past 2.}$$

(iii) When two hands are perpendicular, $\theta = 90°$ and $h = 2$

$$\therefore \theta = \left(\frac{11}{2} m - 30h \right) \text{ or } \left(30h - \frac{11}{2} m \right)$$

$$90 = \frac{11}{2} m - 30 \times 2$$

$$\frac{11}{2} m = 150$$

$$m = \frac{300}{11} = 27\frac{3}{11} \text{ minutes past 2.}$$

or

$$90 = 30 \times 2 - \frac{11}{2} m$$

$$\frac{11}{2} m = -30$$

As m cannot be negative, this case is not possible.

So, the hands are perpendicular to each other only once i.e. at $27\frac{3}{11}$ minutes past 2 O'clock.

(iv) When two hands are pointing opposite directions and are on a straight line the angle between them would be 180°. i.e. $\theta = 180°$ and $h = 2$.

$$180° = \frac{11}{2} m - 30h$$

$$\frac{11}{2} m = 180 + 60 = 240$$

$$m = \frac{480}{11} = 43\frac{7}{11}$$

So, at $43\frac{7}{11}$ minutes past 2 O'clock the hands will be at 180°.

Exercise – 5

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. What is the angle covered by the minute hand in 22 minutes?
   (A) 66° (B) 110° (C) 121°
   (D) 132° (E) 88°

2. By how many degrees does an hour hand move in one quarter of an hour?
   (A) 5° (B) 7.5° (C) 10°
   (D) 12.5° (E) 9°

3. By how many degrees will the minute hand move, in the same time, in which the hour hand moves 6°?
   (A) 54° (B) 84° (C) 72°
   (D) 60° (E) 78°

4. What is the angle between the hands of the clock, when it shows 40 minutes past 6?
   (A) 42.5° (B) 70° (C) 80°
   (D) 90° (E) 40°

5. When the clock shows 3 hours 14 minutes, what is the angle between the hands of the clock?
   (A) 10° (B) 12° (C) 13°
   (D) 14° (E) 16°

6. What is the angle between the two hands of a clock when the time is 25 minutes past 7 O'clock?
   (A) $62^1/_2°$ (B) $66^1/_2°$ (C) $72^1/_2°$
   (D) $69^1/_2°$ (E) $70^1/_2°$

7. When the clock shows 20 minutes past 11 O'clock, what is the angle between the two hands of the clock?
   (A) 110° (B) 120° (C) 130°
   (D) 150° (E) 140°

8. At what time between 9 and 10 O'clock, will both the two hands of the clock coincide?
   (A) $43\,^3/_{11}$ minutes past 9 O'clock.
   (B) $45\,^6/_{11}$ minutes past 9 O'clock.
   (C) $49\,^1/_{11}$ minutes past 9 O'clock .
   (D) $49\,^6/_{11}$ minutes past 9 O'clock.
   (E) $43\,^7/_{11}$ minutes past 9 O'clock.

9. At what time between 4 and 5 O'clock are the hands of a clock in the opposite directions?
   (A) $52^3/_{11}$ minutes past 4 O'clock
   (B) $54^6/_{11}$ minutes past 4 O'clock
   (C) $51^7/_{11}$ minutes past 4 O'clock
   (D) $53^9/_{11}$ minutes past 4 O'clock
   (E) $52^8/_{11}$ minutes past 4 O'clock

10. The angle between the hands of a clock is 20° and the hour hand is in between 2 and 3. What is the time shown by the clock?
    (A) 2 hours $7^3/_{11}$ minutes
    (B) 2 hours $14^6/_{11}$ minutes
    (C) 2 hours $15^5/_{11}$ minutes
    (D) Both (A) and (B)
    (E) None of these

11. Which of the following can be the time shown by the clock, when the hour hand is in between 4 and 5 and the angle between the two hands of the clock is 60°?
    (A) $16^4/_{11}$ minutes past 4
    (B) $18^9/_{11}$ minutes past 4
    (C) $34^6/_{11}$ minutes past 4
    (D) $36^5/_{11}$ minutes past 4
    (E) $32^8/_{11}$ minutes past 4

12. How many times, the hands of a clock will be at 30° with each other in a day?
    (A) 36 (B) 40 (C) 44 (D) 48 (E) 42

13. How many times, the minute hand of a clock overlaps with the hour hand from 9:00 a.m. to 4:00 p.m. in a day?
    (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

14. A watch which gains uniformly was observed to be 1 minute slow at 8:00 a.m. on a day. At 6:00 p.m. on the same day it was 1 minute fast. At what time did the watch show the correct time?
    (A) 12:00 noon (B) 1:00 p.m.
    (C) 2:00 p.m. (D) 3:00 p.m.
    (E) None of these

15. A watch, which gains uniformly was observed to be 6 minutes slow at 9:00 a.m. on a Tuesday and 3 minutes fast at 12:00 noon on the subsequent Wednesday. When did the watch show the correct time?
    (A) 9:00 p.m. on Tuesday
    (B) 12:00 a.m. on Wednesday
    (C) 3:00 a.m. on Wednesday
    (D) 6:00 a.m. on Wednesday
    (E) 5:00 a.m. on Wednesday

16. A watch showed 10 minutes past 6 O'clock on Thursday morning when the correct time was 6 O'clock. It loses uniformly and was observed to be 15 minutes slow at 8 O'clock on Saturday morning. When did the watch show the correct time?
    (A) 1 O'clock on Friday afternoon
    (B) 12 O' clock noon on Friday
    (C) 4 O' clock on Friday evening
    (D) 4 O'clock on Friday morning
    (E) 2 O'clock on Friday morning

17. The minute hand of a clock overtakes the hour hand at intervals of 60 minutes of correct time. How much time does the clock gain or lose in one hour of correct time?
    (A) Gains $5^5/_{11}$ minutes (B) Loses $5^5/_{11}$ minutes
    (C) Gains $^5/_{11}$ minutes (D) Loses $^5/_{11}$ minutes
    (E) None of these

18. The minute hand of a clock overtakes the hour hand after every 70 minutes of correct time. How much time does the clock lose or gain in a day of normal time?
    (A) $93^{39}/_{77}$ minutes (B) $91^{31}/_{77}$ minutes
    (C) $92^{24}/_{77}$ minutes (D) $94^{56}/_{77}$ minutes
    (E) None of these

19. Two clocks are showing correct time at 4:00 p.m. One clock loses 3.5 minutes in an hour, while the other gains 2.5 minutes in one hour. At 10:00 p.m. on the same day, by how much time will the two clocks differ?
(A) 12 minutes   (B) 36 minutes
(C) 24 minutes   (D) 30 minutes
(E) 18 minutes

20. There are two clocks on a wall, both set to show the correct time at 5:00 p.m. The clocks lose 2 minutes and 3 minutes respectively in an hour. If the clock which loses 2 minutes in one hour shows the time as 9:50 p.m. on the same day, then what time does the other clock show?
(A) 9:30 p.m.   (B) 9:40 p.m.
(C) 9:45 p.m.   (D) 10:15 p.m.
(E) 10:00 p.m.

21. If the time in a clock is 10 hours 40 minutes, then what time does its mirror image show?
(A) 1 hour 25 minutes
(B) 1 hour 15 minutes
(C) 1 hour 10 minutes
(D) 1 hour 20 minutes
(E) 1 hour 40 minutes

22. The reflection of a wall clock in a mirror shows the time as 3 hours 40 minutes. What is the actual time?
(A) 8 hours 30 minutes
(B) 8 hours 15 minutes
(C) 8 hours 45 minutes
(D) 9 hours 20 minutes
(E) 8 hours 20 minutes

23. If the seconds hand moves by 240°, then by how many degrees does the minute hand move in the same time?
(A) 1° (B) 2° (C) 3° (D) 4° (E) 5°

24. When the time is 10:30, if the minute hand points towards south, the hour hand will point towards
(A) North-east   (B) North-west
(C) South-east   (D) South-west
(E) North

25. A clock strikes once at 1 O'clock, twice at 2 O'clock, three times at 3 O'clock and so on. If it takes 10 seconds to strike at 6 O'clock, find the time taken by it to strike at 12 O'clock.
(A) 18 seconds   (B) 22 seconds
(C) 24 seconds   (D) 26 seconds
(E) 30 seconds

Key

1. D	6. C	11. E	16. E	21. D
2. B	7. E	12. C	17. A	22. E
3. C	8. C	13. B	18. A	23. D
4. E	9. B	14. B	19. B	24. B
5. C	10. D	15. C	20. C	25. B