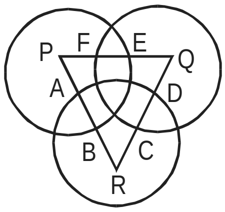
Actual CAT Problems 1998-2006
Geometry and Mensuration

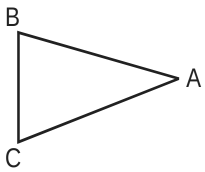
CAT 1998

1. Three circles, each of radius 20, have centres at P, Q and R. Further, AB = 5, CD = 10 and EF = 12. What is the perimeter of ΔPQR?



a. 120 b. 66 c. 93 d. 87

Direction for questions 2 and 3: Answer the questions based on the following information.
A cow is tethered at point A by a rope. Neither the rope nor the cow is allowed to enter ΔABC.



$\angle BAC = 30^\circ$

$l(AB) = l(AC) = 10$ m

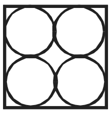
2. What is the area that can be grazed by the cow if the length of the rope is 8 m?

a. $\frac{134\pi}{3}$ sq. m b. 121π sq. m c. 132π sq. m d. $\frac{176\pi}{3}$ sq. m

3. What is the area that can be grazed by the cow if the length of the rope is 12 m?

a. $\frac{133\pi}{6}$ sq. m b. 121π sq. m c. 132π sq. m d. $\frac{176\pi}{3}$ sq. m

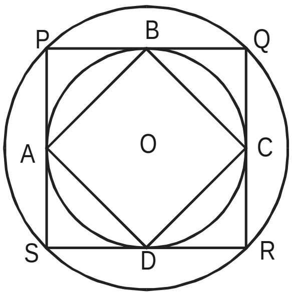
4. Four identical coins are placed in a square. For each coin the ratio of area to circumference is same as the ratio of circumference to area. Then find the area of the square that is not covered by the coins.



a. $16(\pi - 1)$ b. $16(8 - \pi)$ c. $16(4 - \pi)$ d. $16\left(4-\frac{\pi}{2}\right)$

CAT 1999

5. The figure below shows two concentric circles with centre O. PQRS is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points B, C, D and A. What is the ratio of the perimeter of the outer circle to that of polygon ABCD?



a. $\frac{\pi}{4}$ b. $\frac{3\pi}{2}$ c. $\frac{\pi}{2}$ d. π

6. There is a circle of radius 1 cm. Each member of a sequence of regular polygons S1(n), n = 4, 5, 6, …, where n is the number of sides of the polygon, is circumscribing the circle: and each member of the sequence of regular polygons S2(n), n = 4, 5, 6, … where n is the number of sides of the polygon, is inscribed in the circle. Let L1(n) and L2(n) denote the perimeters of the corresponding polygons of S1(n) and S2(n), then $\frac{\{L1(13) + 2\pi\}}{L2(17)}$ is

a. greater than $\frac{\pi}{4}$ and less than 1 b. greater than 1 and less than 2

c. greater than 2 d. less than $\frac{\pi}{4}$

7. There is a square field of side 500 m long each. It has a compound wall along its perimeter. At one of its corners, a triangular area of the field is to be cordoned off by erecting a straight-line fence. The compound wall and the fence will form its borders. If the length of the fence is 100 m, what is the maximum area that can be cordoned off?

a. 2,500 sq m b. 10,000 sq m c. 5,000 sq m d. 20,000 sq m

Directions for questions 8 and 9: Answer the questions based on the following information. A rectangle PRSU, is divided into two smaller rectangles PQTU, and QRST by the line TQ. PQ = 10 cm. QR = 5 cm and RS = 10 cm. Points A, B, F are within rectangle PQTU, and points C, D, E are within the rectangle QRST. The closest pair of points among the pairs (A, C), (A, D), (A, E), (F, C), (F, D), (F, E), (B, C), (B, D), (B, E) are $10\sqrt{3}$ cm apart.

8. Which of the following statements is necessarily true?

a. The closest pair of points among the six given points cannot be (F, C)

b. Distance between A and B is greater than that between F and C.

c. The closest pair of points among the six given points is (C, D), (D, E), or (C, E).

d. None of the above

9. AB > AF > BF ; CD > DE >CE ; and BF = 6 $\sqrt{5}$ cm. Which is the closest pair of points among all the six given points?

a. B, F b. C, D c. A, B d. None of these

CAT 2000

10. ABCD is a rhombus with the diagonals AC and BD intersecting at the origin on the x – y plane. The equation of the straight line AD is x + y = 1. What is the equation of BC?

a. x + y = – 1 b. x – y = – 1 c. x + y = 1 d. None of these

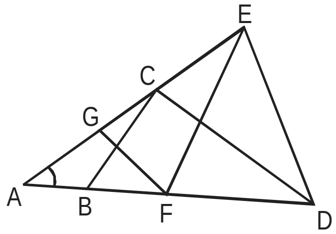
11. Consider a circle with unit radius. There are seven adjacent sectors, S_1, S_2, S_3, ..., S_7, in the circle such that their total area is $\frac{1}{8}$ of the area of the circle. Further, the area of the jth sector is twice that of the (j – 1)th sector, for j = 2, ..., 7. What is the angle, in radians, subtended by the arc of S_1 at the centre of the circle?

a. $\frac{\pi}{508}$ b. $\frac{\pi}{2040}$ c. $\frac{\pi}{1016}$ d. $\frac{\pi}{1524}$

12. If a, b and c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is

a. equilateral b. isosceles c. right-angled d. obtuse-angled

13.



In the figure above, AB = BC = CD = DE = EF = FG = GA. Then $\angle$DAE is approximately

a. 15° b. 20° c. 30° d. 25°

14. ABCDEFGH is a regular octagon. A and E are opposite vertices of the octagon. A frog starts jumping from vertex to vertex, beginning from A. From any vertex of the octagon except E, it may jump to either of the two adjacent vertices. When it reaches E, the frog stops and stays there. Let a_n be the number of distinct paths of exactly n jumps ending in E. Then what is the value of a_{2n-1}?

a. 0 b. 4 c. 2n – 1 d. Cannot be determined

15. A farmer has decided to build a wire fence along one straight side of his property. For this, he planned to place several fence-posts at 6 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 8 m apart. What is the length of the side of his property and how many posts did he buy?

a. 100 m, 15 b. 100 m, 16 c. 120 m, 15 d. 120 m, 16

CAT 2001

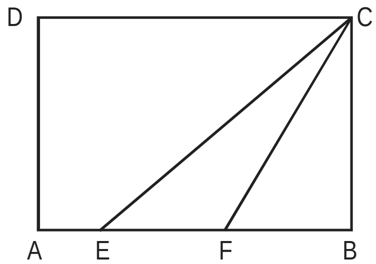
16. A square, whose side is 2 m, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in metres, is

a. $\frac{\sqrt{2}}{\sqrt{2}+1}$ b. $\frac{2}{\sqrt{2}+1}$ c. $\frac{2}{\sqrt{2}-1}$ d. $\frac{\sqrt{2}}{\sqrt{2}-1}$

17. A certain city has a circular wall around it, and this wall has four gates pointing north, south, east and west. A house stands outside the city, 3 km north of the north gate, and it can just be seen from a point 9 km east of the south gate. What is the diameter of the wall that surrounds the city?

a. 6 km b. 9 km c. 12 km d. None of these

18.



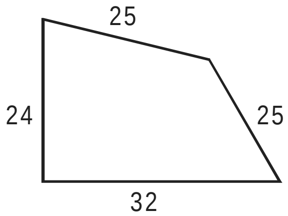
In the above diagram, ABCD is a rectangle with AE = EF = FB. What is the ratio of the areas of ΔCEF and that of the rectangle?

a. $\frac{1}{6}$ b. $\frac{1}{8}$ c. $\frac{1}{9}$ d. None of these

19. A ladder leans against a vertical wall. The top of the ladder is 8 m above the ground. When the bottom of the ladder is moved 2 m farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

a. 10 m b. 15 m c. 20 m d. 17 m

20. Two sides of a plot measure 32 m and 24 m and the angle between them is a perfect right angle. The other two sides measure 25 m each and the other three angles are not right angles.



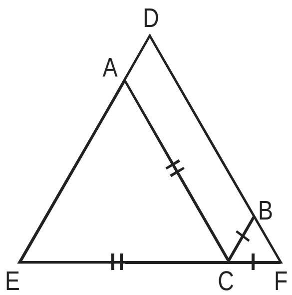
What is the area of the plot?

a. 768 m^2 b. 534 m^2 c. 696.5 m^2 d. 684 m^2

21. Euclid has a triangle in mind. Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?

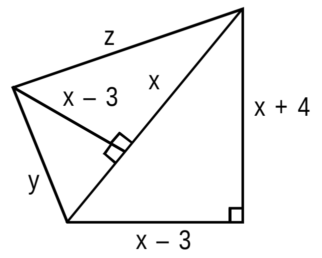
a. $\sqrt{260}$ b. $\sqrt{250}$ c. $\sqrt{240}$ d. $\sqrt{270}$

22. In DDEF shown below, points A, B and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. If $\angle D = 40^\circ$, then $\angle ACB$ =



a. 140 b. 70 c. 100 d. None of these

23. Based on the figure below, what is the value of x, if y = 10?



a. 10 b. 11 c. 12 d. None of these

24. A rectangular pool of 20 m wide and 60 m long is surrounded by a walkway of uniform width. If the total area of the walkway is 516 m^2, how wide, in metres, is the walkway?

a. 43 m b. 3 m c. 3 m d. 3.5 m

CAT 2002

25. In ΔABC, the internal bisector of $\angle A$ meets BC at D. If AB = 4, AC = 3 and $\angle A = 60^\circ$, then the length of AD is

a. $2\sqrt{3}$ b. $\frac{12\sqrt{3}}{7}$ c. $\frac{15\sqrt{3}}{8}$ d. $\frac{6\sqrt{3}}{7}$

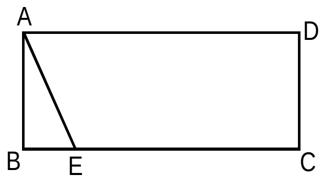
26. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is

a. 24 cm b. 25 cm c. 15 cm d. 20 cm

27. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area.

a. 22 m^2 b. 42 m^2 c. 84 m^2 d. 168 m^2

28. In the figure given below, ABCD is a rectangle. The area of the isosceles right triangle ABE = 7 cm^2; EC = 3(BE). The area of ABCD (in cm^2) is



a. 21 cm^2 b. 28 cm^2 c. 42 cm^2 d. 56 cm^2

29. The area of the triangle whose vertices are (a, a), (a + 1, a + 1) and (a + 2, a) is

a. a^3 b. 1 c. 2a d. $2^{\frac{1}{2}}$

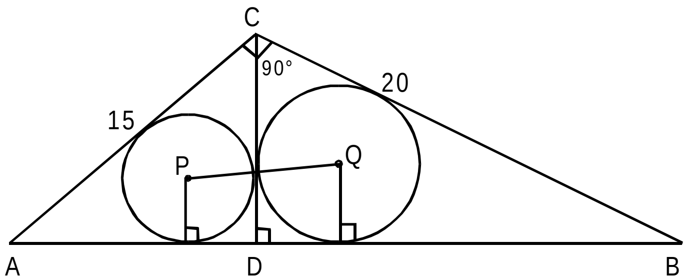
30. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is

a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{1}{4}$ d. $\frac{3}{4}$

31. Neeraj has agreed to mow a lawn, which is a 20 m × 40 m rectangle. He mows it with 1 m wide strip. If Neeraj starts at one corner and mows around the lawn toward the centre, about how many times would he go round before he has mowed half the lawn?

a. 2.5 b. 3.5 c. 3.8 d. 4

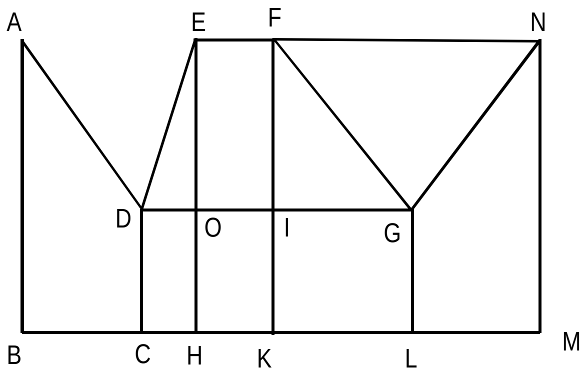
32.



In the above figure, ACB is a right-angled triangle. CD is the altitude. Circles are inscribed within the ΔACD and ΔBCD. P and Q are the centres of the circles. The distance PQ is

a. 5 b. $\sqrt{50}$ c. 7 d. 8

Directions for questions 33 and 34: Answer the questions based on the following diagram.



In the above diagram, $\angle ABC = 90° = \angle DCH = \angle DOE = \angle EHK = \angle FKL = \angle GLM = \angle LMN$

AB = BC = 2CH = 2CD = EH = FK = 2HK = 4KL = 2LM = MN

33. The magnitude of $\angle FGO$ =

a. 30° b. 45° c. 60° d. None of these

34. What is the ratio of the areas of the two quadrilaterals ABCD to DEFG?

a. 1 : 2 b. 2 : 1 c. 12 : 7 d. None of these

CAT 2003 Leaked

35. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

a. 85.5 b. 92.5 c. 90.5 d. 87.5

Direction for questions 36 to 38: Answer the questions on the basis of the information given below. A city has two perfectly circular and concentric ring roads, the outer ring road (OR) being twice as long as the inner ring road (IR). There are also four (straight line) chord roads from E1, the east end point of OR to N2, the north end point of IR; from N1, the north end point of OR to W2, the west end point of IR; from W1, the west end point of OR, to S2, the south end point of IR; and from S1 the south end point of OR to E2, the east end point of IR. Traffic moves at a constant speed of 30π km/hr on the OR road, 20π km/hr on the IR road, and $15\sqrt{5}$ km/hr on all the chord roads.

36. The ratio of the sum of the lengths of all chord roads to the length of the outer ring road is

a. $\sqrt{5} : 2$ b. $\sqrt{5} : 2\pi$ c. $\sqrt{5} : \pi$ d. None of the above.

37. Amit wants to reach N2 from S1. It would take him 90 minutes if he goes on minor arc S1 – E1 on OR, and then on the chord road E1 – N2. What is the radius of the outer ring road in kms?

a. 60 b. 40 c. 30 d. 20

38. Amit wants to reach E2 from N1 using first the chord N1 – W2 and then the inner ring road. What will be his travel time in minutes on the basis of information given in the above question?

a. 60 b. 45. c. 90 d. 105

Directions for Questions 39 and 40: Each question is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements together.

39. AB is a chord of a circle. AB = 5 cm. A tangent parallel to AB touches the minor arc AB at E. What is the radius of the circle?

A. AB is not a diameter of the circle.

B. The distance between AB and the tangent at E is 5 cm.

40. D, E, F are the mid points of the sides AB, BC and CA of triangle ABC respectively. What is the area of DEF in square centimeters?

A. AD = 1 cm, DF = 1 cm and perimeter of DEF = 3 cm

B. Perimeter of ABC = 6 cm, AB = 2 cm, and AC = 2 cm.

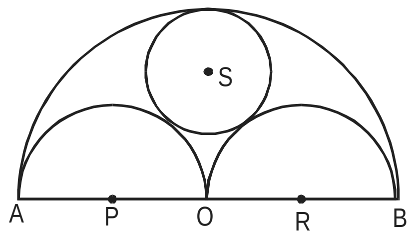
41. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be

a. 20 b. 0 c. 21 d. 22

42. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then the area (in square centimeters) of the triangle ABC would be

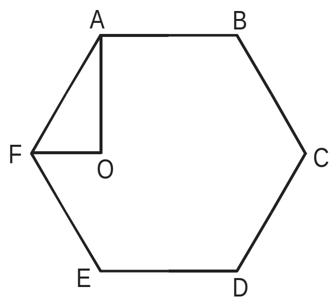
a. $\pi\sqrt{12}$ b. $\frac{9}{\pi}$ c. $\frac{9\sqrt{3}}{\pi}$ d. $\frac{6\sqrt{3}}{\pi}$

43. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with center at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centers at P and R respectively, and S is the center of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circles and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circle with diameter AB that cannot be grazed by the horses is nearest to



a. 20 b. 28 c. 36 d. 40

44. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90^\circ$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



a. $\frac{1}{12}$ b. $\frac{1}{6}$ c. $\frac{1}{24}$ d. $\frac{1}{18}$

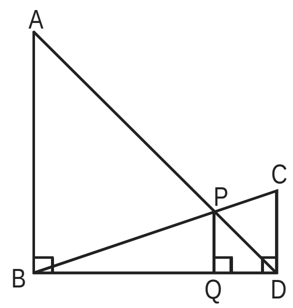
45. A vertical tower OP stands at the center O of a square ABCD. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60^\circ$ then the relationship between h and b can be expressed as

a. $2b^2 = h^2$ b. $2h^2 = b^2$ c. $3b^2 = 2h^2$ d. $3h^2 = 2b^2$

46. In the triangle ABC, AB = 6, BC = 8 and AC = 10. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with center B) is drawn. If the circle cuts AB and BC at P and Q respectively, the AP:QC is equal to

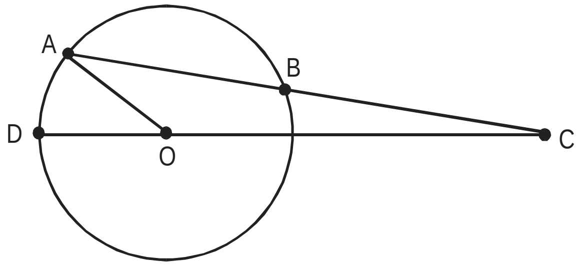
a. 1:1 b. 3:2 c. 4:1 d. 3:8

47. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90^\circ$. If AB:CD = 3:1, the ratio of CD: PQ is



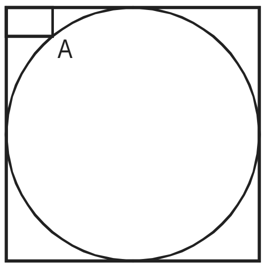
a. 1 : 0.69 b. 1 : 0.75 c. 1 : 0.72 d. None of the above.

48. In the figure below, AB is the chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD$ = y degrees and $\angle AOD$ = x degrees such that x = ky, then the value of k is



a. 3 b. 2 c. 1 d. None of the above.

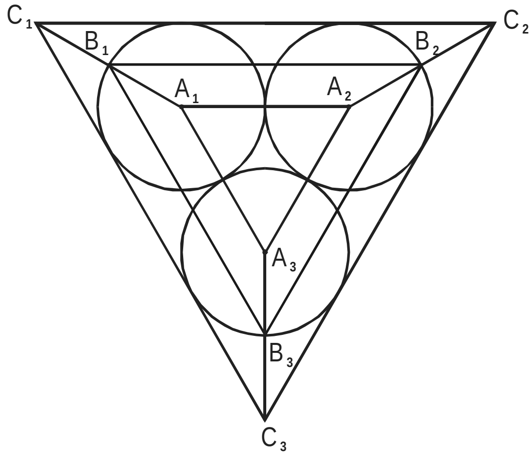
49. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



a. 10 cm b. 40 cm c. 50 cm d. None of the above.

CAT 2003 Retest

Directions for questions 50 to 52: Answer the questions on the basis of the information given below. Consider three circular parks of equal size with centres at A_1, A_2, and A_3 respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$,as shown. Three sprinters A, B, and C begin running from points A_1, B_1 and C_1 respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.



50. Let the radius of each circular park be r, and the distances to be traversed by the sprinters A, B and C be a, b and c respectively. Which of the following is true?

a. $b - a = c - b = 3\sqrt{3}\,r$
b. $b - a = c - b = \sqrt{3}\,r$
c. $b = \frac{a+c}{2} = 2(1 + \sqrt{3})\,r$
d. $c = 2b - a = (2 + \sqrt{3})\,r$

51. Sprinter A traverses distances A_1A_2, A_2A_3, and A_3A_1 at an average speeds of 20, 30 and 15 respectively. B traverses her entire path at a uniform speed of $\left(10\sqrt{3} + 20\right)$. C traverses distances C_1C_2, C_2C_3 and C_3C_1 at an average speeds of $\frac{40}{3}\left(\sqrt{3}+1\right)$, $\frac{40}{3}\left(\sqrt{3}+1\right)$ and 120 respectively. All speeds are in the same unit. Where would B and C be respectively when A finishes her sprint?

a. B_1, C_1
b. B_3, C_3
c. B_1, C_3
d. B_1, Somewhere between C_3 and C_1

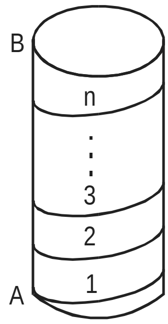
52. Sprinters A, B and C traverse their respective paths at uniform speeds of u, v and w respectively. It is known that $u^2:v^2:w^2$ is equal to Area A: Area B: Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$ respectively. Where would A and C be when B reaches point B_3?

a. A_2, C_3
b. A_3, C_3
c. A_3, C_2
d. Somewhere between A_2 and A_3, Somewhere between C_3 and C_1

Directions for questions 53 to 55: Answer the questions on the basis of the information given below.

Consider a cylinder of height h cm and radius $r = \frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns).

53. What is the vertical spacing between the two consecutive turns?



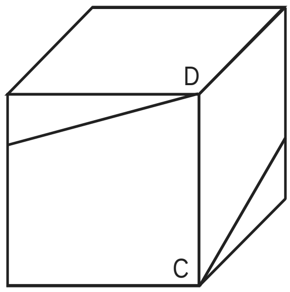
a. $\frac{h}{n}$ cm

b. $\frac{h}{\sqrt{n}}$ cm

c. $\frac{h}{n^2}$ cm

d. Cannot be determined

54. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string is



a. $\sqrt{2}\,n$ cm

b. $\sqrt{17}\,n$ cm

c. n cm

d. $\sqrt{13}\,n$ cm

55. In the set-up of the previous two questions, how is h related to n?

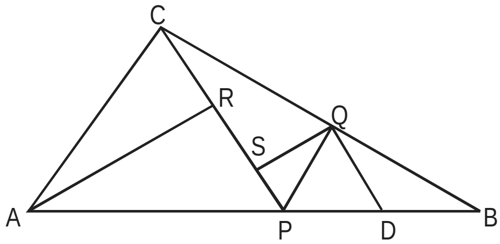
a. $h = \sqrt{2}\,n$

b. $h = \sqrt{17}\,n$

c. $h = n$

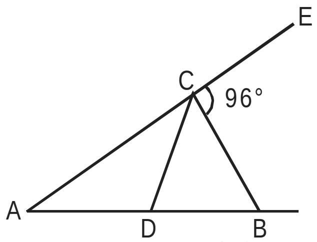
d. $h = \sqrt{13}\,n$

56. In the figure (not drawn to scale) given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, $\angle ARC = 90^\circ$, and in ΔPQS, $\angle PSQ = 90^\circ$. The length of QS is 6 cm. What is the ratio of AP : PD?



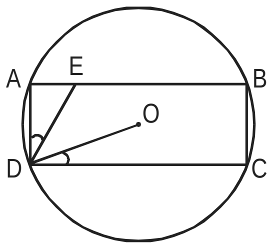
a. 10 : 3 b. 2 : 1 c. 7 : 3 d. 8 : 3

57. In the figure (not drawn to scale) given below, if AD = CD = BC and $\angle BCE = 96^\circ$, how much is the value of $\angle DBC$?



a. 32° b. 84° c. 64° d. Cannot be determined

58. In the figure below (not drawn to scale), rectangle ABCD is inscribed in the circle with centre at O. The length of side AB is greater than side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : \sqrt{3}$. The line segment DE intersects AB at E such that $\angle ODC = \angle ADE$. The ratio AE : AD is



a. $1 : \sqrt{3}$ b. $1 : \sqrt{2}$ c. $1 : 2\sqrt{3}$ d. 1 : 2

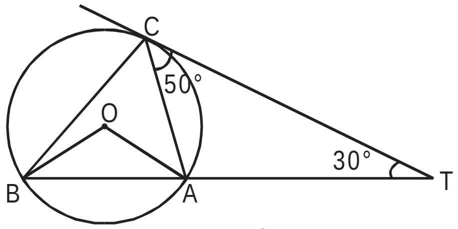
59. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s, respectively. Then,

a. s > t > c b. c > t > s c. c > s > t d. s > c > t

60. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio $\frac{P_1+P_2+P_3+\cdots}{A_1+A_2+A_3+\cdots}$ equals

a. $\frac{2\left(1+\sqrt{2}\right)}{a}$ b. $\frac{2\left(2-\sqrt{2}\right)}{a}$ c. $\frac{2\left(2+\sqrt{2}\right)}{a}$ d. $\frac{2\left(1+2\sqrt{2}\right)}{a}$

61. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If $\angle ATC = 30°$ and $\angle ACT = 50°$, then the angle $\angle BOA$ is



a. 100° b. 150°
c. 80° d. not possible to determine

62. Let ABCDEF be a regular hexagon. What is the ratio of the area of the Δ ACE to that of the hexagon ABCDEF?

a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. $\frac{2}{3}$ d. $\frac{5}{6}$

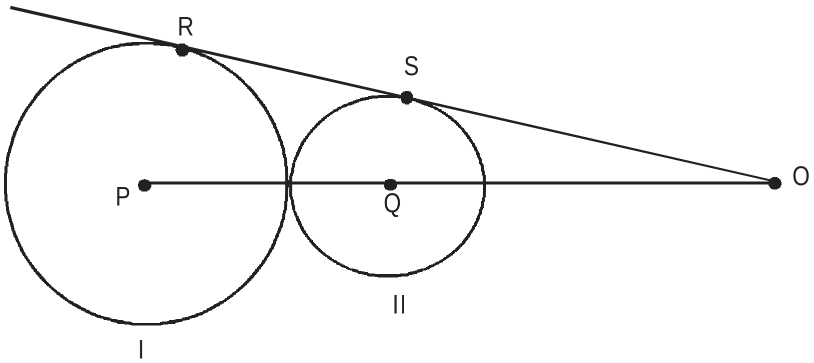
63. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?
a. 16.665 b. 16.565 c. 15.465 d. 14.365

CAT 2004

64. A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle?

a. $4\sqrt{2}$ b. $2\sqrt{2}$ c. $\sqrt{2}$ d. None of the above

Directions for questions 65 to 68: Answer the questions on the basis of the information given below.
In the adjoining figure I and II, are circles with P and Q respectively, The two circles touch each other and have common tangent that touches them at points R and S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4 : 3. It is also known that the length of PO is 28 cm.



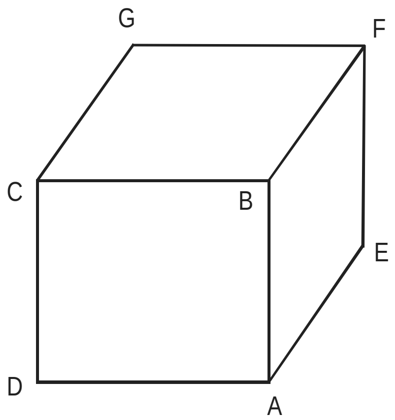
65. What is the ratio of the length of PQ to that of QO?
a. 1 : 4 b. 1 ; 3 c. 3 : 8 d. 3 : 4

66. What is the radius of the circle II?
a. 2 cm b. 3 cm c. 4 cm d. 5 cm

67. The length of SO is
a. $8\sqrt{3}$ cm b. $10\sqrt{3}$ cm c. $12\sqrt{3}$ cm d. $14\sqrt{3}$ cm

68. Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let C_1, C_2, C_3, ... be circles with diameters P_0P_1, P_1P_2, P_2P_3... respectively. Suppose the circles C_1, C_2, C_3, ... are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle is
a. 8 : 9 b. 9 : 10 c. 10 : 11 d. 11 : 12

69. If the lengths of diagonals DF, AG and CE of the cube shown in the adjoining figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be



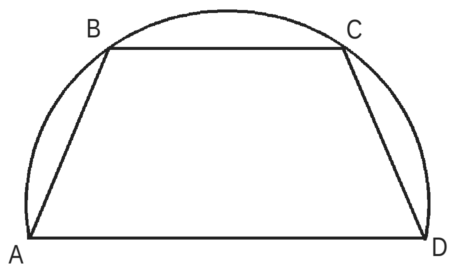
a. equal to the side of cube

b. $\sqrt{3}$ times the side of the cube

c. $\frac{1}{\sqrt{3}}$ times the side of the cube

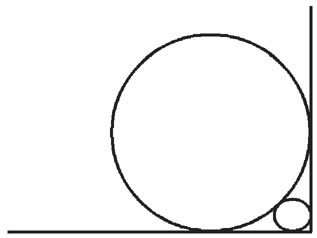
d. impossible to find from the given information.

70. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC?



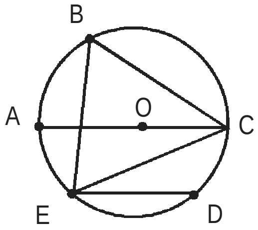
a. 7.5 b. 7 c. 7.75 d. None of the above

71. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



a. $3-2\sqrt{2}$ b. $4-2\sqrt{2}$ c. $7-4\sqrt{2}$ d. $6-4\sqrt{2}$

72. In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If $\angle CBE = 65^\circ$, then what is the value of $\angle DEC$?



a. 35° b. 55° c. 45° d. 25°

CAT 2005

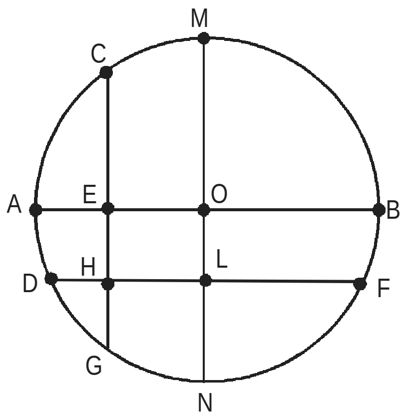
73. Four points A, B, C and D lie on a straight line in the X –Y plane, such that AB = BC = CD, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. the ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle is

a. $3\sqrt{2}$ b. $1 + \pi$ c. $\frac{4\pi}{3}$ d.5

74. Rectangular tiles each of size 70 cm by 30 cm must be laid horizontally on a rectangular floor of size 110 cm by 130 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is

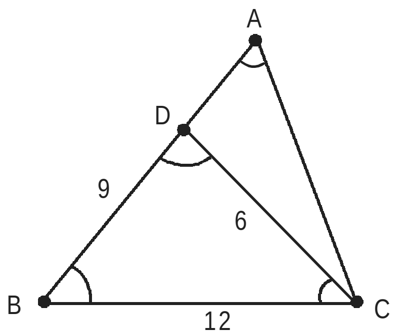
a. 4 b. 5 c. 6 d. 7

75. In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2, and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is



a. $2\sqrt{2}-1$ b. $\frac{(2\sqrt{2}-1)}{2}$ c. $\frac{(3\sqrt{2}-1)}{2}$ d. $\frac{(2\sqrt{2}-1)}{3}$

76. Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$



a. $\frac{7}{9}$ b. $\frac{8}{9}$ c. $\frac{6}{9}$ d. $\frac{5}{9}$

77. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral PQSR?

a. $2r\left(1+\sqrt{3}\right)$ b. $2r\left(2+\sqrt{3}\right)$ c. $r\left(1+\sqrt{5}\right)$ d. $2r+\sqrt{3}$

78. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is

a. 10 b. 12 c. 14 d. 16

79. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously form the point where one of the circular tracks touches the smaller side of the rectangular track. A jogs along the rectangular track, while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point?

a. 3.88% b. 4.22% c. 4.44% d. 4.72%

80. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?

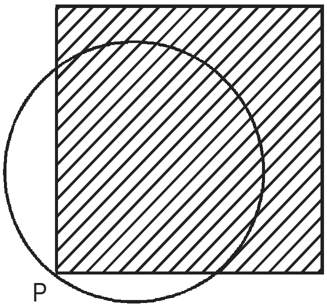
a. 1 or 7 b. 2 or 14 c. 3 or 21 d. 4 or 28

CAT 2006

81. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semi-circle at D. Given that AC = 2 cm and CD = 6 cm, the area of the semi-circle (in sq. cm) will be:

1. $32\ \pi$ 2. $50\ \pi$ 3. $40.5\ \pi$ 4. $81\ \pi$ 5. undeterminable

Directions for questions 82 and 83: Answer questions on the basis of the information given below:
A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal ofthe square.



82. The proportion of the sheet area that remains after punching is:

1. $\frac{(\pi+2)}{8}$ 2. $\frac{(6+\pi)}{8}$ 3. $\frac{(4+\pi)}{4}$ 4. $\frac{(\pi-2)}{4}$ 5. $\frac{(14-3\pi)}{6}$

83. Find the area of the part of the circle (round punch) falling outside the square sheet.

1. $\frac{\pi}{4}$ 2. $\frac{(\pi-1)}{2}$ 3. $\frac{(\pi-1)}{4}$ 4. $\frac{(\pi-2)}{2}$ 5. $\frac{(\pi-2)}{4}$

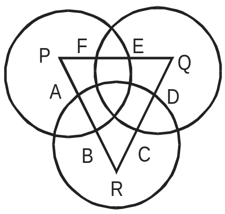
84. An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in degrees?

1. 75   2. 90   3. 120   4. 135   5. 150

Answers and Explanations

CAT 1998

1. c Since AB = 5, PA = PB – AB = 20 – 5 = 15
Similarly, RB = RA – AB = 20 – 5 = 15
Hence, PR = PA + AB + BR = 15 + 5 + 15 = 35
Since CD = 10, RC = RD – CD = 20 – 10 = 10
Similarly, QD = QC – DC = 20 – 10 = 10
Hence, QR = RC + CD + DQ = 10 + 10 + 10 = 30
And since EF = 12, PF = PE – EF = 20 – 12 = 8 and EQ = QF – EF = 20 – 12 = 8,
then PQ = PF + FE + EQ = 8 + 12 + 8 = 28
Hence, perimeter of ΔPQR = (35 + 30 + 28) = 93

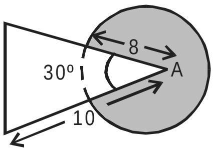


Shortcut:

Since PQ = QR = RP = 20 unit and it is given that AB = 5, CD = 10, and EF = 12 unit.
So perimeter of ΔPQR = 6 × 20 – (5 + 10 + 12) = 120 – 37 = 93 unit.

Hence option (c).

2. d



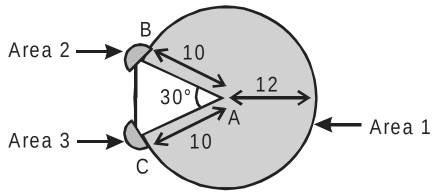
It can be seen that if the length of the rope is 8 m, then the cow will be able to graze an area equal to (the area of the circle with radius 8m) – (Area of the sector of the same circle with angle 30°). This can further be expressed as $\pi(8)^2 - \frac{30}{360}\pi(8)^2$

$= 64\pi - \frac{1}{12}(64\pi) = 64\pi\left(\frac{11}{12}\right) = \frac{176\pi}{3}$ sq. m

Shortcut:

Area grazed without restiriction is $64\pi\,m^2$ it should be less than 64π sq. m. with restriction. So choice (d).

3. a



If the length of the rope is 12 m, then the total area that can be grazed by the cow is as depicted in the diagram. Area 1 is (the area of the circle with radius 12) – (Area of the sector of the same circle with angle 30º)

So area 1 = $\pi(12)^2 - \frac{30}{360}\pi(12)^2 = 132\pi$

Since the length of the rope is higher than the sides of the triangle (viz. AB and AC), if the cow reaches point B or C, there would still be a part of the rope (12 – 10) = 2 m in length. With this extra length available the cow can further graze an area equivalent to some part of the circle with radius = 2 m from both points, i.e. B and C. This is depicted as area 2 and area 3 in the diagram. Hence, the actual area grazed will be slightly more than 132π. The only answer choice that supports this is (a).

4. c Let R be the radius of each circle. Then $\frac{\pi R^2}{2\pi R} = \frac{2\pi R}{\pi R^2}$

which implies that $\frac{R}{2} = \frac{2}{R}$, i.e. $R^2 = 4$, i.e. R = 2.

Then the length of the square is 8. Thus, the area of the square is 64, while the area covered by each coin is $\pi 2^2 = 4\pi$. Since there are four coins, the area covered by coins is $4(4\pi) = 16\pi$. Thus, the area not covered by the coins is $64 - 16\pi = 16(4 - \pi)$.

CAT 1999

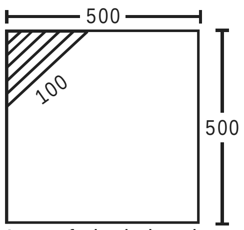
5. c Let the radius of the outer circle be x = OQ
Hence, perimeter of the circle $= 2\pi x$
But OQ = BC = x (diagonals of the square BQCO)
Perimeter of ABCD = 4x

Hence, ratio = $\frac{2\pi x}{4x} = \frac{\pi}{2}$.

6. c Following rule should be used in this case: The perimeter of any polygon circumscribed about a circle is always greater than the circumference of the circle and the perimeter of any polygon inscribed in a circle is always less than the circumference of the circle. Since, the circles is of radius 1, its circumference will be 2π. Hence, L1(13) > 2π and L2(17) < π.

So {L1(13) + 2 π } > 4 and hence $\frac{\{L1(13)+2\pi\}}{L2(17)}$ will be greater than 2.

7. a



Area of shaded region

$= \frac{1}{2} \times \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}}$ = 2,500 sq m

Area of a Δ is maximum when it is an isosceles Δ.

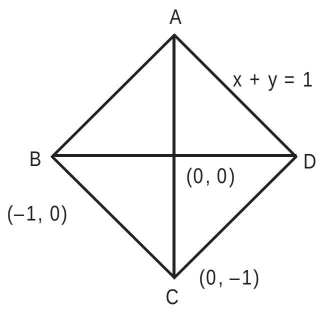
So perpendicular sides should be of length $\frac{100}{\sqrt{2}}$.

8. d We have not been given the distances between any two points.

9. d Since CD > DE, option (b) cannot be the answer. Similarly, since AB > AF, Option (c) cannot be the answer. We are not sure about the positions of points B and F. Hence, (a) cannot be the answer.

CAT 2000

10. a The gradient of the line AD is –1. Coordinates of B are (– 1, 0).



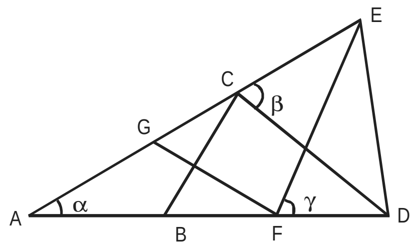
Equation of line BC is x + y = – 1.

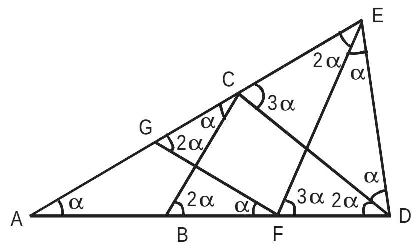
11. c Let the area of sector S_1 be x units. Then the area of the corresponding sectors shall be 2x, 4x, 8x,16x, 32x and 64x. Since every successive sector has an angle that is twice the previous one, the total area then shall be 127x units. This is $\frac{1}{8}$ of the total area of the circle.
Hence, the total area of the circle will be 127x × 8 = 1016x units. Hence, angle of sector S_1 is $\frac{\pi}{1016}$.

12. a We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 3ab + 3bc + 3ac$
Now assume values of a, b, c and substitute in this equation to check the options.

Short cut: $(a – b)^2 + (b – c)^2 + (c – a)^2 = 0$.
Hence, a = b = c.

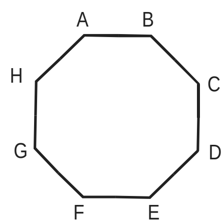
13. d





Let $\angle$EAD = α. Then $\angle$AFG = α and also $\angle$ACB = α.
Therefore, $\angle$CBD = 2α (exterior angle to ΔABC).
Also $\angle$CDB = 2α (since CB = CD).
Further, $\angle$FGC = 2α (exterior angle to ΔAFG).
Since GF = EF, $\angle$FEG = 2α. Now $\angle$DCE = $\angle$DEC = β (say). Then $\angle$DEF = β – 2α.
Note that $\angle$DCB = 180 – (α + β).
Therefore, in ΔDCB, 180 – (α + β) + 2α + 2α = 180 or β = 3α. Further $\angle$EFD = $\angle$EDF = γ (say).
Then $\angle$EDC = γ – 2α. If CD and EF meet at P, then $\angle$FPD = 180 – 5α (because β = 3α).
Now in ΔPFD, 180 – 5α + γ + 2α = 180 or γ = 3α.
Therefore, in ΔEFD, α + 2γ = 180 or α + 6α = 180 or α = 26 or approximately 25.

14. a

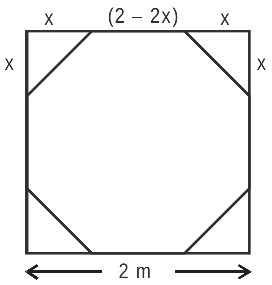


In order to reach E from A, it can walk clockwise as well as anticlockwise. In all cases, it will have to take odd number of jumps from one vertex to another. But the sum will be even. In simple case, if n = 4, then a_n = 2. For $a_{2n–1}$ = 7 (odd), we cannot reach the point E.

15. d Work with options. Length of wire must be a multiple of 6 and 8. Number of poles should be one more than the multiple.

CAT 2001

16. a

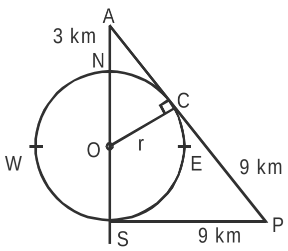


Let the length of the edge cut at each corner be x m. Since the resulting figure is a regular octagon,

$\therefore \sqrt{x^2 + x^2} = 2 - 2x \Rightarrow x\sqrt{2} = 2 - 2x$

$\Rightarrow \sqrt{2}\,x\ (1 + \sqrt{2}) = 2 \Rightarrow x = \dfrac{\sqrt{2}}{\sqrt{2}+1}$

17. b



ΔAPS and ΔAOC are similar triangles.
Where OC = r

$\therefore \dfrac{r}{r+3} = \dfrac{9}{\sqrt{81 + (2r + 3)^2}}$

Now use the options. Hence, the diameter is 9 km.

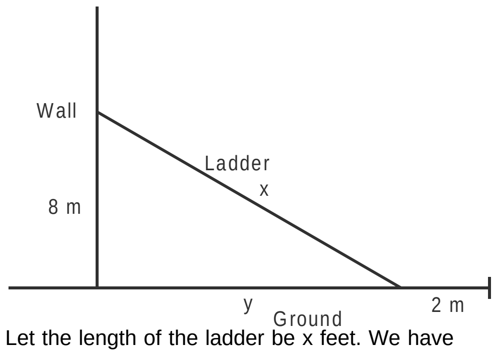
18. a Let BC = y and AB = x.
Then area of ΔCEF = Area(ΔCEB) – Area(ΔCFB)

$= \dfrac{1}{2}.\dfrac{2x}{3}.\,y - \dfrac{1}{2}.\dfrac{x}{3}.\,y = \dfrac{xy}{6}$

Area of ABCD = xy
$\therefore$ Ratio of area of ΔCEF and area of ABCD is

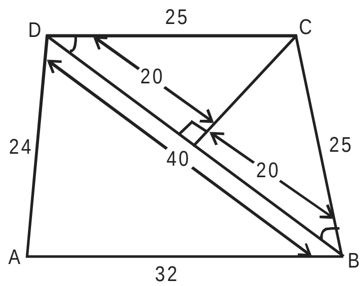
$\dfrac{xy}{6} : xy = \dfrac{1}{6}$

19. d



Let the length of the ladder be x feet. We have
$8^2 + y^2 = x^2$ and (y + 2) = x
Hence, $64 + (x - 2)^2 = x^2$
$\Rightarrow 64 + x^2 - 4x + 4 = x^2$
$\Rightarrow 68 = 4x \Rightarrow x = 17$

20. d



CE = $\sqrt{25^2 - 20^2} = 15$

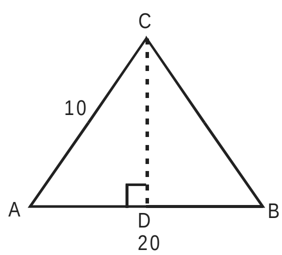
(Since DBC is isosceles triangle.)
Assume ABCD is a quadrilateral
where AB = 32 m, AD = 24 m, DC = 25 m, CB = 25 m
and $\angle$DAB is right angle.
Then DB = 40 m because ΔADB is a right-angled triangle and DBC is an isosceles triangle.

So area of Δ ADB $= \dfrac{1}{2} \times 32 \times 24 = 384$ sq. m

Area of ΔBCD $= 2 \times \dfrac{1}{2} \times 15 \times 20 = 300$ sq. m

Hence area of ABCD = 384 + 300 = 684 sq. m

21. a



Let's assume AB be the longest side of 20 unit and another side AC is 10 unit. Here CD $\perp$ AB.

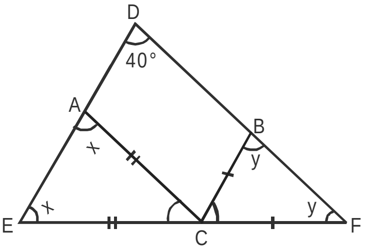
Since area of $\Delta ABC = 80 = \frac{1}{2} AB \times CD$

So $CD = \frac{80 \times 2}{20} = 8$. In ΔACD; $AD = \sqrt{10^2 - 8^2} = 6$

Hence DB = 20 – 6 = 14.

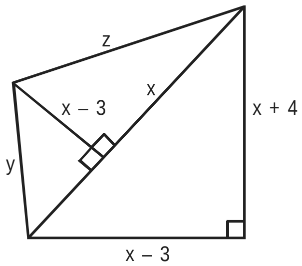
So $CB = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260}$ unit

22. c



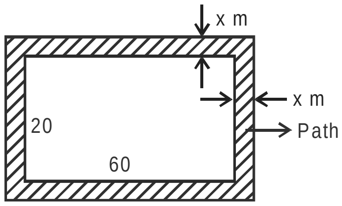
Here $\angle ACE = 180 - 2x$, $\angle BCF = 180 - 2y$
and $x + y + 40° = 180°$ (In ΔDEF)
So $x + y = 140°$
So $\angle ACB = 180° - \angle ACE - \angle BCF$
$= 180° - (180° - 2x) - (180° - 2y)$
$= 2(x + y) - 180°$
$= 2 \times 140 - 180 = 100°$

23. b



We can find the value of x, using the answer choices given in the question. We put (a), (b), (c) and (d) individually in the figure and find out the consistency of the figure. Only (b), i.e. 11 is consistent with the figure.

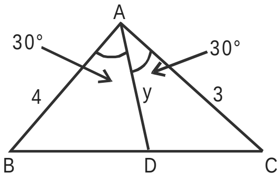
24. c



Let width of the path be x metres.
Then area of the path = 516 sq. m
$\Rightarrow (60 + 2x)(20 + 2x) - 60 \times 20 = 516$
$\Rightarrow 1200 + 120x + 40x + 4x^2 - 1200 = 516$
$\Rightarrow 4x^2 + 160x - 516 = 0 \Rightarrow x^2 + 40x - 129 = 0$
Using the answer choices, we get x = 3.

CAT 2002

25. b



Let BC = x and AD = y.

As per bisector theorem, $\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3}$

Hence, $BD = \frac{4x}{7}$; $DC = \frac{3x}{7}$

In ΔABD, $\cos 30° = \dfrac{(4)^2 + y^2 - \frac{16x^2}{49}}{2 \times 4 \times y}$

$\Rightarrow 2 \times 4 \times y \times \frac{\sqrt{3}}{2} = 16 + y^2 - \frac{16x^2}{49}$

$\Rightarrow 4\sqrt{3}y = 16 + y^2 - \frac{16x^2}{49}$ ... (i)

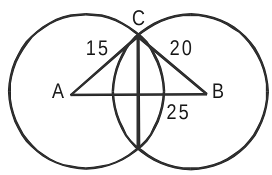
Similarly, from ΔADC, $\cos 30° = \dfrac{9 + y^2 - \frac{9x^2}{49}}{2 \times 3 \times y}$

$\Rightarrow 3\sqrt{3}y = 9 + y^2 - \frac{9x^2}{49}$ ... (ii)

Now (i) × 9 – 16 × (ii), we get

$36\sqrt{3}y - 48\sqrt{3}y = 9y^2 - 16y^2 \Rightarrow y = \frac{12\sqrt{3}}{7}$

26. a



Let the chord = x cm

$\therefore \frac{1}{2}(15 \times 20) = \frac{1}{2} \times 25 \times \frac{x}{2} \Rightarrow x = 24$ cm

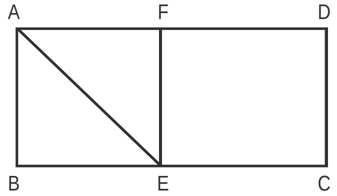
27. a Total area = 14 × 14 = 196 m^2

Grazed area = $\left(\frac{\pi \times r^2}{4}\right) \times 4 = \pi r^2 = 22 \times 7 (r = 7)$

$= 154$ m^2

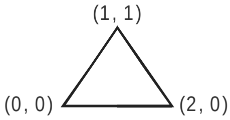
Ungrazed area is less than (196 – 154) = 42 m^2, for which there is only one option.

28. d



Area of ΔABE = 7 cm^2
Area of $\Delta ABEF$ = 14 cm^2
Area of $\Delta ABCD$ = 14 × 4 = 56 cm^2

29. b



Let a = 0

Hence, area = $\frac{1}{2}(2)\ (1) = 1$

Note: Answer should be independent of a and area of the triangle does not have square root.

30. d Check choices, e.g. $\frac{1}{2} \Rightarrow$ Diagonal = $\sqrt{5}$

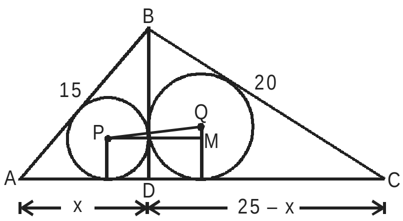
Distance saved = $3 - \sqrt{5} \approx 0.75 \neq$ Half the larger side. Hence, incorrect.

$\frac{3}{4} \Rightarrow$ Diagonal = 5

Distance saved = (4 + 3) – 5 = 2 = Half the larger side.

31. c Area = 40 × 20 = 800
If 3 rounds are done, area = 34 × 14 = 476
$\Rightarrow$ Area > 3 rounds
If 4 rounds $\Rightarrow$ Area left = 32 × 12 = 347
Hence, area should be slightly less than 4 rounds.

32. b



$(15)^2 - x^2 = (20)^2 - (25 - x)^2$

$\Rightarrow x = 9$

$\Rightarrow BD = 12$

Area of $\Delta ABD = \frac{1}{2} \times 12 \times 9 = 54$

$s = \frac{1}{2}(15 + 12 + 9) = 18$

$r_1 = \frac{\text{Area}}{s} \Rightarrow r_1 = 3$

Area of $\Delta BCD = \frac{1}{2} \times 16 \times 12 = 96$

$s = \frac{1}{2}(16 + 20 + 12) = 24$

$r_2 = \frac{\text{Area}}{s} \Rightarrow r_2 = 4$

In ΔPQM, $PM = r_1 + r_2 = 7$ cm
$QM = r_2 - r_1 = 1$ cm

Hence, PQ = $\sqrt{50}$ cm

33. d If KL = 1, then IG = 1 and FI = 2

Hence, $\tan\theta = \frac{2}{1} = 2$

Thus, θ none of 30, 45 and 60°.

34. c Area of quadrilateral ABCD $= \frac{1}{2}(2x + 4x) \times 4x = 12x$

Area of quadrilateral DEFG $= \frac{1}{2}(5x + 2x) \times 2x = 7x$

Hence, ratio = 12 : 7

CAT 2003 (Leaked)

35. d The surface area of a sphere is proportional to the square of the radius.

Thus, $\frac{S_B}{S_A} = \frac{4}{1}$ (S. A. of B is 300% higher than A)

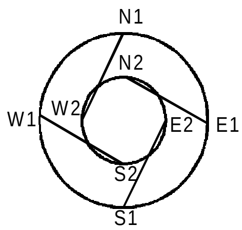
$\therefore \frac{r_B}{r_A} = \frac{2}{1}$

The volume of a sphere is proportional to the cube of the radius.

Thus, $\frac{V_B}{V_A} = \frac{8}{1}$

Or, V_A is $\frac{7}{8}$th less than B i.e. $\left(\frac{7}{8} \times 100\right)$ 87.5%

For questions 36 to 38:



If the radius of the inner ring road is r, then the radius of the outer ring road will be 2r (since the circumference is double).

The length of IR = 2π r, that of OR = 4π r and that of the chord roads are $r\sqrt{5}$ (Pythagoras theorem)
The corresponding speeds are
20π, 30π and $15\sqrt{5}$ kmph.
Thus time taken to travel one circumference of
IR = $\frac{r}{10}$ hr , one circumference of OR = $\frac{r}{7.5}$ hr hr.

and one length of the chord road = $\frac{r}{15}$ hr

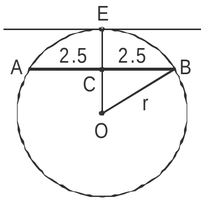
36. c Sum of the length of the chord roads = $4r\sqrt{5}$ and the length of OR = 4π r.
Thus the required ratio = $\sqrt{5} : \pi$

37. c The total time taken by the route given = $\frac{r}{30} + \frac{r}{15} = \frac{3}{2}$
(i.e. 90 min.)
Thus, r = 15 km. The radius of OR = 2r = 30 kms

38. d The total time taken = $\frac{r}{20} + \frac{r}{15} = \frac{7r}{60}$

Since r = 15, total time taken = $\frac{7}{4}$ hr. = 105 min.

39. a



We can get the answer using the second statement only. Let the radius be r.
AC = CB = 2.5 and using statement B, CE = 5, thus OC = (r – 5).
Using Pythagoras theorem, $(r - 5)^2 + (2.5)^2 = r^2$
We get r = 3.125

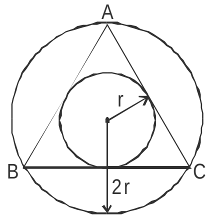
NOTE: You will realize that such a circle is not possible (if r = 3.125 how can CE be 5). However we need to check data sufficiency and not data consistency. Since we are able to find the value of r uniquely using second statement the answer is (a).

40. b The question tells us that the area of triangle DEF will be $\frac{1}{4}$ th the area of triangle ABC. Thus by knowing either of the statements, we get the area of the triangle DEF.

41. c In this kind of polygon, the number of convex angles will always be exactly 4 more than the number of concave angles.

NOTE : The number of vertices have to be even. Hence the number of concave and convex corners should add up to an even number. This is true only for the answer choice (c).

42. c



Since the area of the outer circle is 4 times the area of the inner circle, the radius of the outer circle should be 2 times that of the inner circle.
Since AB and AC are the tangents to the inner circle, they should be equal. Also, BC should be a tangent to inner circle. In other words, triangle ABC should be equilateral.
The area of the outer circle is 12. Hence the area of inner circle is 3 or the radius is $\sqrt{\frac{3}{\pi}}$. The area of equilateral triangle = $3\sqrt{3}$ r^2, where r is the inradius.

Hence the answer is $\frac{9\sqrt{3}}{\pi}$

43. b If the radius of the field is r, then the total area of the field = $\frac{\pi r^2}{2}$.
The radius of the semi-circles with centre's P and $R = \frac{r}{2}$.

Hence, their total area = $\frac{\pi r^2}{4}$
Let the radius if the circle with centre S be x.
Thus, OS = (r – x), OR = $\frac{r}{2}$ and $RS = \left(\frac{r}{2} + x\right)$.
Applying Pythagoras theorem, we get

$$(r - x)^2 + \left(\frac{r}{2}\right)^2 = \left(\frac{r}{2} + x\right)^2$$

Solving this, we get $x = \frac{r}{3}$.

Thus the area of the circle with centre $S = \frac{\pi r^2}{9}$.

The total area that can be grazed = $\pi r^2\left(\frac{1}{4} + \frac{1}{9}\right)$

$= \frac{13\pi r^2}{36}$

Thus the fraction of the field that can be grazed

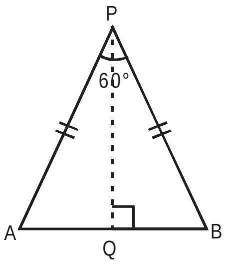
$= \frac{26}{36}$ $\left(\frac{\text{Area that can be grazed}}{\text{Area of the field}}\right)$

The fraction that cannot be grazed = $\frac{10}{36}$

= 28% (approx.)

44. a It is very clear, that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle.
Hence the required ratio = 1 : 12

45. b



Given $\angle APB = 60°$ and AB = b.

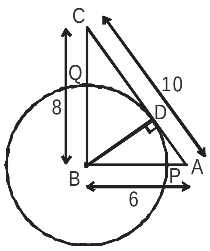
$\therefore PQ = \frac{b}{2} \times \sqrt{3}$

Next, $\frac{b}{2}$, h and PQ form a right angle triangle.

$\therefore \frac{b^2}{4} + h^2 = \frac{3b^2}{4}$

$\therefore 2h^2 = b^2$

46. d



Triangle ABC is a right angled triangle.

Thus $\frac{1}{2}$ × BC × AB = $\frac{1}{2}$ × BD × AC

Or, 6 × 8 = BD × 10. Thus BD = 4.8.
Therefore, BP = BQ = 4.8.
So, AP = AB – BP = 6 – 4.8 = 1.2 and CQ = BC – BQ = 8 – 4.8 = 3.2.
Thus, AP : CQ = 1.2 : 3.2 = 3 : 8

47. b Using the Basic Proportionality Theorem, $\frac{AB}{PQ} = \frac{BD}{QD}$

and $\frac{PQ}{CD} = \frac{BQ}{BD}$.

Multiplying the two we get, $\frac{AB}{CD} = \frac{BQ}{QD}$ = 3 : 1.

Thus CD : PQ = BD : BQ = 4 : 3 = 1 : 0.75

48. a If y = 10°,

$\angle BOC$ = 10° (opposite equal sides)

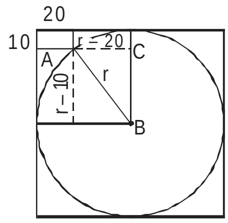
$\angle OBA$ = 20° (external angle of ΔBOC)

$\angle OAB$ = 20 (opposite equal sides)

$\angle AOD$ = 30° (external angle of ΔAOC)

Thus k = 3

49. c



Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have $(r – 10)^2 + (r – 20)^2 = r^2$
i.e. $r^2 – 60r + 500 = 0$. Thus r = 10 or 50.
It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

CAT 2003 (Retest)

For questions 50 to 52: $A_1A_2 = 2r$, $B_1B_2 = 2r + r\sqrt{3}$, $C_1C_2 = 2r + 2r\sqrt{3}$

Hence,
a = 3 × 2r

b = 3 × $(2r + r\sqrt{3})$

c = 3 × $\left(2r + 2r\sqrt{3}\right)$

50. a Difference between (1) and (2) is $3\sqrt{3}r$ and that between (2) and (3) is $3\sqrt{3}r$. Hence, (1) is the correct choice.

51. c Time taken by A = $\frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \left(\frac{2r \times 9}{60}\right) = \frac{3}{10}r$

Therefore, B and C will also travel for time $\frac{3}{10}r$.

Now speed of B = $\left(10\sqrt{3} + 20\right)$

Therefore, the distance covered

$= \left(10\sqrt{3} + 20\right) \times \frac{3}{10}r = \left(\sqrt{3} + 2\right) \times 10 \times \frac{3}{10}r$

$= \left(2r + \sqrt{3}r\right) \times 3 \; = B_1B_2 + B_2B_3 + B_3B_1$

$\therefore$ B will be at B_1.

Now time taken by for each distance are

$$\frac{C_1C_2}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_2C_3}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_3C_1}{120}$$

$$\frac{3}{40} \times \frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{3}{40} \times \frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{\left(2+2\sqrt{3}\right)r}{120}$$

i.e. $\frac{3}{40} \times 2r, \frac{3}{40} \times 2r, \frac{\left(1+\sqrt{3}\right)}{60}r$

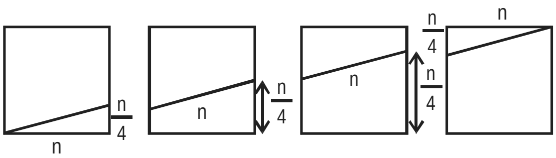
i.e. $\frac{3}{20}r, \frac{3}{20}r, \frac{\left(1+\sqrt{3}\right)}{60}r$

We can observe that time taken for C_1C_2 and C_2C_3 combined is $\frac{3}{20}r + \frac{3}{20}r = \frac{3}{10}r$, which is same as time taken by A. Therefore, C will be at C_3.

52. b In similar triangles, ratio of Area = Ratio of squares of corresponding sides.
Hence, A and C reach A_3 and C_3 respectively.

53. a The whole height h will be divided into n equal parts. Therefore, spacing between two consecutive turns $= \frac{h}{n}$.

54. b The four faces through which string is passing can be shown as



Therefore, length of string in each face

$= \sqrt{n^2 + \left(\frac{n}{4}\right)^2}$

$= \sqrt{n^2 + \frac{n^2}{16}} = \frac{\sqrt{17}n}{4}$

Therefore, length of string through four faces

$= \frac{\sqrt{17}n}{4} \times 4 = \sqrt{17}n$

55. c As h/n = number of turns = 1 (as given). Hence h = n.

56. c $PQ \parallel AC$

$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$

$QD \parallel PC$

$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

As $\frac{PD}{DB} = \frac{4}{3}$

$\therefore PD = \frac{4}{7}PB$

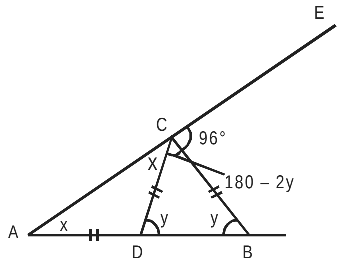
$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB}$

$= \frac{7}{4} \times \frac{AP}{PB}$

$= \frac{7}{4} \times \frac{4}{3}$

= 7 : 3

57. c



Using exterior angle theorem

$\angle A + \angle B = 96$

i.e. x + y = 96 … (i)

Also x + (180 – 2y) + 96 = 180°

$\therefore$ x – 2y + 96 = 0

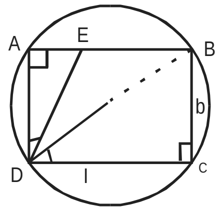
$\therefore$ x – 2y = –96 … (ii)

Solving (i) and (ii),

y = 64° and x = 32°

$\therefore \angle DBC = y = 64^\circ$

58. a



BD = 2r

$$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi r^2}{lb} = \frac{\pi}{\sqrt{3}}$$

$$\frac{r^2}{lb} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{d^2}{4}}{lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{d^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2 + b^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2 + b^2}{lb} = \frac{4}{\sqrt{3}}$$

$$\therefore \frac{l}{b} + \frac{b}{l} = \frac{4}{\sqrt{3}} \qquad \ldots \text{(i)}$$

Now Δ AEB $\sim$ Δ CBD

$$\therefore \frac{AE}{CB} = \frac{AD}{DC}$$

$$\therefore \frac{AE}{AD} = \frac{BC}{DC}$$

$$\therefore \frac{AE}{AD} = \frac{b}{l}$$

$\therefore$ We have to find $\frac{AE}{AD}$, i.e. $\frac{b}{l}$.

Let $\frac{b}{l} = x$

Therefore, from (i), we get

$$\frac{1}{x} + x = \frac{4}{\sqrt{3}}$$

$$\frac{1 + x^2}{x} = \frac{4}{\sqrt{3}}$$

$$\sqrt{3} + \sqrt{3}x^2 = 4x$$

$$\therefore \sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\therefore x = \frac{-(-4) \pm \sqrt{16 - 4\left(\sqrt{3}\right)\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2\sqrt{3}}$$

$$= \frac{4 \pm 2}{2\sqrt{3}}$$

$$= \frac{6}{2\sqrt{3}}$$

OR $\frac{2}{2\sqrt{3}}$

$$= \frac{\sqrt{3}}{1} \text{ OR } \frac{1}{\sqrt{3}}$$

From options, the answer is $\frac{1}{\sqrt{3}}$, i.e. $1:\sqrt{3}$.

59. c It's standard property among circle, square and triangle, for a given parameter, area of circle is the highest and area of the triangle is least whereas area of the square is in-between, i.e. c > s > t.

60. c $$\frac{P + \frac{P}{\sqrt{2}} + \cdots\infty}{A + \frac{A}{2} + \cdots\infty} = \frac{\frac{P}{1 - \frac{1}{\sqrt{2}}}}{2A} = \frac{P\sqrt{2}}{\left(\sqrt{2} - 1\right)} \times \frac{1}{2A}$$

$$= \frac{\sqrt{2}P\,(\sqrt{2} + 1)}{2A}$$

$$= \frac{\sqrt{2} \times 4a\left(\sqrt{2} + 1\right)}{2 \times a^2} = \frac{\sqrt{2} \times 2\left(\sqrt{2} + 1\right)}{a} = \frac{2\left(2 + \sqrt{2}\right)}{a}$$

61. a $\angle BAC = \angle ACT + \angle ATC = 50 + 30 = 80^\circ$

And $\angle ACT = \angle ABC$ (Angle in alternate segment)

So $\angle ABC = 50^\circ$

$$\angle BCA = 180 - \left(\angle ABC + \angle BAC\right)$$

$$= 180 - \left(50 + 80\right) = 50^\circ$$

Since $\angle BOA = 2\angle BCA = 2 \times 50 = 100^\circ$

Alternative Method:

Join OC

$\angle OCT = 90^\circ$ (TC is tangent to OC)

$\angle OCA = 90^\circ - 50^\circ = 40^\circ$

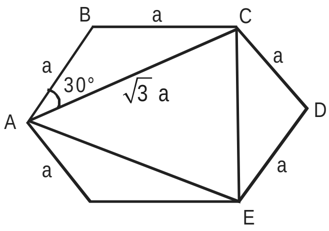
$\angle OAC = 40^\circ$ (OA = OC being the radius)

$\angle BAC = 50^\circ + 30^\circ = 80^\circ$

$\angle OAB = 80^\circ - 40^\circ = 40^\circ = \angle OBA$ (OA = OB being the radius)

$\angle BOA = 180^\circ - (\angle OBA + \angle OAB) = 100^\circ$

62. b



$\because \Delta$ ACE is equilateral triangle with side $\sqrt{3}$ a.

Area of hexagon = $\frac{\sqrt{3}}{4} a^2 \times 6$

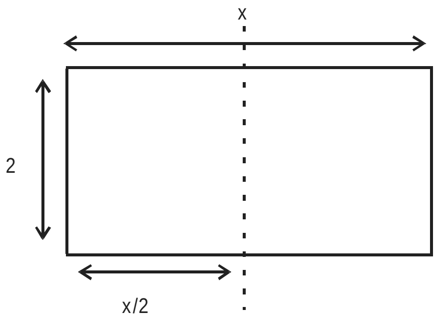
Area as $\Delta ACE = \frac{\sqrt{3}}{4} (\sqrt{3}a)^2$

Therefore, ratio = $\frac{1}{2}$

63. d The required answer is 34 × 0.65 × 0.65 = 14.365 Because we get two similar triangles and area is proportional to square of its side.

CAT 2004

64. b



In original rectangle ratio = $\frac{x}{2}$

In Smaller rectangle ratio = $\frac{2}{\left(\frac{x}{2}\right)}$

Given $\frac{x}{2} = \frac{2}{\frac{x}{2}} \Rightarrow x = 2\sqrt{2}$

Area of smaller rectangle = $\frac{x}{2} \times 2 = x = 2\sqrt{2}$ sq. units

65. b $\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

OP = 28
OQ = 21
PQ = OP – OQ = 7

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

66. b PR + QS = PQ = 7

$= \frac{PR}{QS} = \frac{4}{3}$

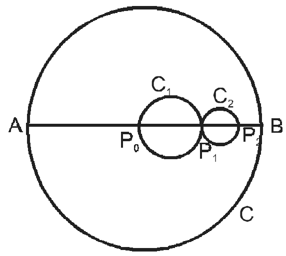
$\Rightarrow QS = 3$

67. c $SO = \sqrt{OQ^2 - QS^2}$

$= \sqrt{21^2 - 3^2}$

$= \sqrt{24 \times 18} = 12\sqrt{3}$

68. d



Circle	Radius
C	r
C_1	$\frac{r}{4}$
C_2	$\frac{r}{8}$
C_3	$\frac{r}{16}$
⋮	⋮

$\Rightarrow$ either $\frac{\text{Area of unshaded portion of C}}{\text{Area of C}}$

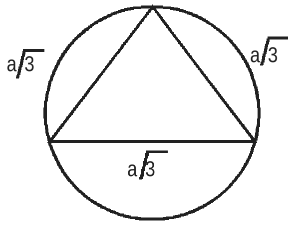
$= 1 - \frac{\text{Area of shaded portion}}{\text{Area of C}}$

$= 1 - \frac{\pi\left(\left(\frac{r}{4}\right)^2 + \left(\frac{r}{8}\right)^2 + \ldots\right)}{\pi r^2}$

$= 1 - \left(\frac{1}{4^2} + \frac{1}{8^2} + \ldots\right) = 1 - \frac{\frac{1}{16}}{1 - \frac{1}{4}}$

$= \frac{11}{12}$

69. a DF, AG and CE are body diagonals of cube.
Let the side of cube = a
Therefore body diagonal is $a\sqrt{3}$

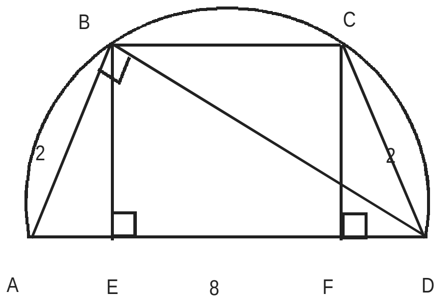


Circum radius for equilateral triangle

$= \dfrac{\text{side}}{\sqrt{3}}$

Therefore $\dfrac{a\sqrt{3}}{\sqrt{3}} = a$

70. b



$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$

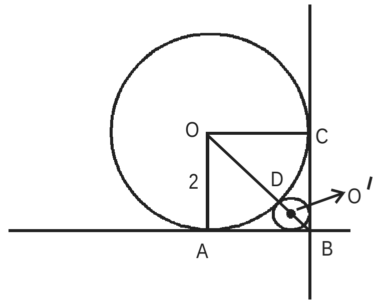
$2\sqrt{8^2 - 2^2} = 8 \times BE$

$BE = \dfrac{\sqrt{60}}{4} = \dfrac{\sqrt{15}}{2}$

$AE = \sqrt{2^2 - \left(\dfrac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \dfrac{15}{4}} = \dfrac{1}{2}$

$BC = EF = 8 - \left(\dfrac{1}{2} + \dfrac{1}{2}\right) = 7$

71. d



Let the radius of smaller circle = r

$\therefore O'B = r\sqrt{2}$

$\therefore OB = O'B + O'D + OD$

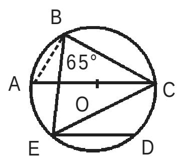
$= r\sqrt{2} + r + 2$

Also OB = $2\sqrt{2}$

$\Rightarrow r\sqrt{2} + r + 2 = 2\sqrt{2}$

$\Rightarrow r = 6 - 4\sqrt{2}$

72. d



In ΔABC,

$\angle B = 90°$ (Angles in semicircle)

Therefore $\angle ABE = 90 - 65 = 25°$

Also $\angle ABE = \angle ACE$ (angle subtended by same arc AE)

Also $\angle ACE = \angle CED$ [AC ∥ ED]

Therefore $\angle CED = 25°$

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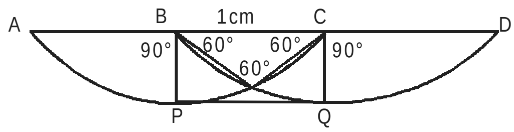
73. b



Drawn figure since it have not to be within distance of 1 cm so it will go along APQD.

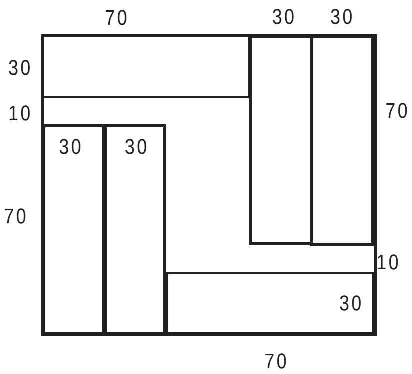
AP = $\dfrac{90}{360} \times 2\pi \times 1 = \dfrac{\pi}{2}$

Also AP = QD = $\dfrac{\pi}{2}$

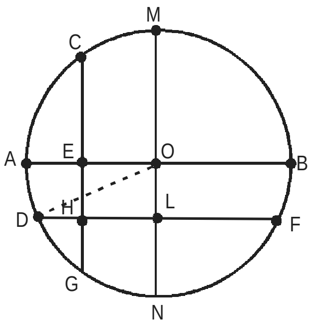
So the minimum distance = AP + PQ + QD =

$\dfrac{\pi}{2} + 1 + \dfrac{\pi}{2} = 1 + \pi$

74. c



75. b



$HL = OE = \frac{1}{2}$

$DL = DH + HL$

$DL = DH + \frac{1}{2}$

OB = AO = radius = 1.5

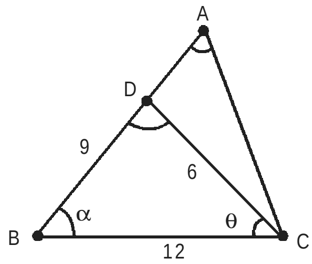
$DO^2 = OL^2 + DL^2$

$\left(\frac{3}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(DH + \frac{1}{2}\right)^2$

$\Rightarrow \left(DH + \frac{1}{2}\right)^2 = 2 \Rightarrow DH = \sqrt{2} - \frac{1}{2}$

Hence option (b)

76. a



Here $\angle ACB = \theta + 180 - (2\theta + \alpha) = 180 - (\theta + \alpha)$

So here we can say that triangle BCD and triangle ABC will be similar.

Hence from the property of similarity

$\frac{AB}{12} = \frac{12}{9}$ Hence AB = 16

$\frac{AC}{6} = \frac{12}{9}$ Hence AC = 8

Hence AD = 7

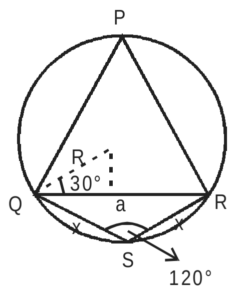
AC = 8

$S_{ADC} = 8 + 7 + 6 = 21$

$S_{BDC} = 27$

Hence $r = \frac{21}{27} = \frac{7}{9}$

77. a



Here $\cos 30° = \frac{a}{2r}$

$a = r\sqrt{3}$

Here the side of equilateral triangle is $r\sqrt{3}$

From the diagram $\cos 120° = \frac{x^2 + x^2 - a^2}{2x^2}$

$a^2 = 3x^2$

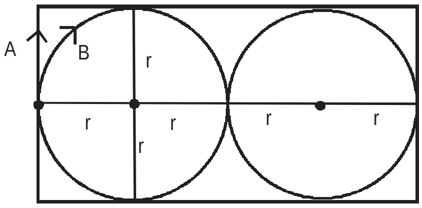
x = r

Hence the circumference will be $2r\left(1 + \sqrt{3}\right)$

Hence answer is (a).

78. b Let the rectangle has m and n tiles along its length and breadth respectively.
The number of white tiles
W = 2m + 2(n – 2) = 2 (m + n – 2)
And the number of Red tiles = R = mn – 2 (m + n – 2)
Given W = R $\Rightarrow$ 4 (m + n – 2) = mn
$\Rightarrow$ mn – 4m – 4n = – 8
$\Rightarrow$ (m – 4) (n – 4) = 8
$\Rightarrow$ m – 4 = 8 or 4 $\Rightarrow$ m = 12 or 8
$\therefore$ 12 suits the options.

79. d



A covers 2r + 2r + 4r + 4r = 12 r

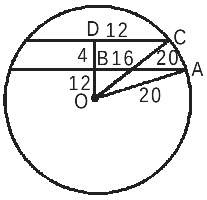
B covers $2\pi r + 2\pi r = 4\pi r$ distance

$$\frac{4\pi r}{S_B} = \frac{12r}{S_A} \Rightarrow S_B = \frac{\pi}{3}S_A$$

$$\frac{S_B - S_A}{S_A}\times 100 = \frac{\pi - 3}{3}\times 100 = 4.72\%$$

Hence Option (d)

80. d



$OB^2 = OA^2 – AB^2 = 20^2 – 16^2 = 144$
OB = 12
$OD^2 = 20^2 – 12^2 = 400 – 144 = 256$
OD = 16
BD = 4
Only one option contains 4 hence other will be 28. Hence option (d)

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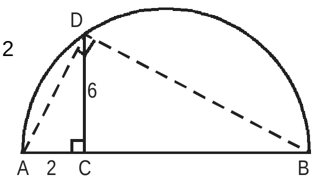
81. 2

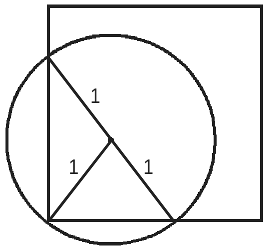


$\angle ADC = 90°$ ($\angle$ in semicircle)
$CD^2 = AC \times CB$
$(6)^2 = 2 \times CB$
36 = 2 × CB
CB = 18
Hence AB = AC + CB = 20

Area of semicircle = $= \frac{1}{2}\pi(10)^2 = 50\pi$

Option is (2).

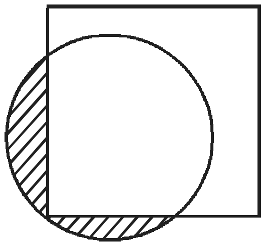
82. 2



Remaining area = $4 - \left(\frac{\pi}{2} + \frac{1}{2}\times 1\times 2\right) = \frac{6-\pi}{2}$

Remaining proportion = $= \frac{6-\pi}{8}$

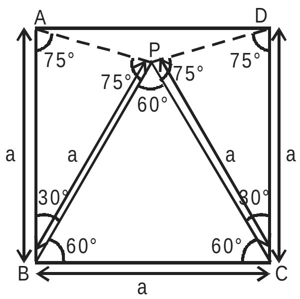
83. 4



Area = $\pi(1)^2 - \left(\frac{\pi}{2} + 1\right)$

$= \pi - \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

84. 5



$\angle PBC = \angle CPB = \angle BPC = 60°$ (L's of equilateral triangle)
PC = CD (both a)

Also $\angle CPD = \angle PDC = \frac{180° - 30°}{2} = 75°$

Similarly, $\angle BAP = \angle BPA = 75°$

$\angle APD = 360° - 75° - 75° - 60° = 150°$