CHAPTER – 5

FUNCTIONS

SETS

A set is a well defined collection of objects. The objects of the set are called its elements. Sets are usually denoted by capital letters and the elements of the set are denoted by lower case letters. If an element x belongs to set A, it is denoted by $x \in A$. If x is not an element of A, it is denoted by $x \notin A$.

A set, in general is represented in two forms:

1. ***Roster Form:*** In this form a set is described by actually listing out the elements. For example, the set of all even natural numbers less than 12 is represented by {2, 4, 6, 8, 10}
2. ***Set Builder Form:*** In this form a set is described by a characterising property. For example, the set of all even natural numbers less than 12 is represented by $\{x \in N \mid x < 12$ and x is even$\}$. The symbol | is read as "such that."

Some Definitions:

Null Set: A set is said to be a null set if it has no elements. It is also called an empty set or a void set and is denoted by ϕ.

Examples:

(i) $\{x \mid x$ is an integer, $1 < x < 2\}$
(ii) $\{x \mid x$ is a real number and $x^2 < 0\}$

Finite and Infinite sets: A set 'A' is said to be **finite** if it is either an empty set or contains a finite number of elements. Otherwise, it is said to be **infinite**.

Examples:

(i) The set of vowels in the English alphabet is finite.
(ii) The set of natural numbers is infinite.

Cardinality of a Set: The number of elements in a set is called the cardinality (or order) of the set. If a finite set A has n elements, the cardinality of the set is n and is denoted by O(A) or n(A). The cardinality of the empty set is 0.

Example: The cardinality of A = {x, y, z, t} is 4.

Singleton Set: A set consisting of a single element is called a singleton set, i.e. a set whose cardinality is 1 is a singleton set.

Examples: (i) {3} (ii) {a}

Equal Sets: Two sets A and B are said to be equal if they have the same elements, i.e. if every element of A is an element of B and every element of B is an element of A.

Subsets and Supersets:

Let A and B be two sets. If every element of A belongs to B, then A is said to be a subset of B. This is written as $A \subseteq B$.

$\therefore A \not\subseteq B$ means A is not a subset of B. If A is a subset of B and there is at least one element in B that is not there in A, A is said to be a proper subset of B. This is written as $A \subset B$.

$\therefore A \not\subset B$ means A is a not a proper subset of B.
Note: $A \subset B \Rightarrow A \subseteq B$. But the converse is not true.

If A is a subset of B we say that B contains A or B is a superset of A. This is written as $B \supseteq A$

Note:

(1) Every set is a subset of itself.
(2) The empty set is a subset of every set.
(3) If A is a finite set of cardinality n, then the total number of subsets of A is 2^n.

Power Set: If A is any set, then the set of all subsets of A is called the power set of A and is denoted by P(A), i.e. $P(A) = \{S \mid S \subseteq A\}$

Example: If A = {1, 2, 3}, then P(A) = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

Note:

(1) $A, \phi \in P(A)$.
(2) If A is a finite set having n elements, then the cardinality of P(A) is 2^n.

Universal Set: The set that contains all the elements in a given context is called the Universal Set. It is denoted by μ or U.

Examples:

(i) In Plane Geometry, the set of all points in the plane is the universal set.
(ii) In the context of divisibility tests, the set of natural numbers is the universal set.

Basic Set Operations

Union of Sets: If A and B are two sets, the union of A and B is the set of all those elements which belong to either A or B or both A and B. This is denoted by $A \cup B$.
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Note:

(1) If $A \subseteq B$, then $A \cup B = B$
(2) $A \cup \phi = A$
(3) $A \cup \mu = \mu$

Intersection of Sets:

Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B. It is denoted by $A \cap B$.
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Note:

(1) $A \subseteq B$, then $A \cap B = A$.
(2) $A \cap \phi = \phi$.
(3) $A \cap \mu = A$.

Disjoint Sets:

Two sets A and B are said to be disjoint if they have no element in common.

$\therefore$ If A and B are disjoint, then $A \cap B = \phi$.

Difference of Sets:

The difference of two sets is the set of all elements which are there in one set but not in the other.

Let A and B be two sets. A – B is the set of all those elements of A which do not belong to B.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Similarly, $B - A = \{ x / x \in B \text{ and } x \notin A\}$.

Example: A = {1, 2, 3, 4}, B = {3, 4, 8, 10}

A – B = {1, 2}

B – A = {8, 10}

Complement of a Set:

The complement of set A is the set of all those elements that do not belong to set A, i.e. the complement of a set A is the difference of the universal set and set A and is denoted by A' or A^c.

Example: If μ is the set of natural numbers, A is the set of even natural numbers, then A' is the set of odd natural numbers.

Symmetric Difference of Two Sets:

Let A and B be two sets. The symmetric difference of the sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.

Example: A = {1, 2, 3, 4}, B = {3, 4, 5, 6} then $A \Delta B$ = {1, 2, 5, 6}

Some Results:

(1) $A \cup A = A$; $A \cap A = A$

(2) $A \cup B = B \cup A$

(3) $A \cap B = B \cap A$

(4) $A \cup (B \cup C) = (A \cup B) \cup C$

(5) $A \cap (B \cap C) = (A \cap B) \cap C$

(6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(7) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(8) $C - (C - A) = C \cap A$

(9) $C - (A \cup B) = (C - A) \cap (C - B)$

(10) $C - (A \cap B) = (C - A) \cup (C - B)$

(11) $(A^c)^c = A$

(12) $(A \cup B)^c = A^c \cap B^c$

(13) $(A \cap B)^c = A^c \cup B^c$

Cartesian Product of Two Sets:

Let A and B be any two sets. Then the Cartesian Product of A and B is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. The product is denoted by $A \times B$

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

Example: A = {1, 2, 3} B = {a, b} then

$A \times B$ = {(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)}

$B \times A$ = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}

Note:

(1) If A and B are two sets such that n(A) = x and n(B) = y, then the number of ordered pairs in A x B is xy

(2) $A \times B$ is not necessarily equal to $B \times A$

(3) $n(A \times B) = n(B \times A)$

RELATIONS

Any subset of $A \times B$ is a relation from A to B. A relation pairs up elements of A with elements of B. If a from A is paired with b from B in a relation R, then we write $(a, b) \in R$ or aRb.

A subset of $A \times A$ is a relation in the set A.

Example: A = {1, 2, 3}, B = {a, b} then R = {(1, a) (3, b)} is a possible relation.

Domain and Range of a Relation:

The set of all first coordinates of the ordered pairs of a relation R is called the Domain of R and the set of all second coordinates of the ordered pairs of R is called the range of R, i.e. if R is a relation from A to B then Domain of R = $\{x/(x, y) \in R$ for some $y\}$ and Range of R = $\{y/ (x, y) \in R$ for some $x\}$. The domain and range of R are denoted as Dom R and Range R.

Inverse of a Relation:

Let A and B be two sets and let R be a relation from A to B. Then the inverse of R denoted by R^{-1} is a relation from B to A and is defined by

$R^{-1} = \{(b, a) \mid (a, b) \in R\}$

We note that Domain of R = Range of R^{-1} and Range of R = Domain of R^{-1}

Examples:

5.01. If A = {1, 2, 3, 4, 6, 8} and B = {1, 2, 3, 5, 7}, then find $A \cup B$, $A \cap B$, A – B, B – A, $A \Delta B$, $A \times B$ and $B \times A$

Sol:

- $A \cup B$ is the set having the elements that are in A or B or in both
  $\therefore A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8}
- $A \cap B$ = {1, 2, 3}. The elements that are present in both A and B
- A – B = {4, 6, 8}. The elements present in A but not in B.
- B – A = {5, 7}. The elements present in B but not in A.
- $A \Delta B = (A \cup B) - (A \cap B)$ = {4, 5, 6, 7, 8}
- $A \times B$ = {(1, 1), (1, 2), (1, 3), (1, 5), (1, 7),
  (2, 1), (2, 2), (2, 3), (2, 5), (2, 7)
  (3, 1), (3, 2), (3, 3), (3, 5), (3, 7),
  (4, 1), (4, 2), (4, 3), (4, 5), (4, 7),
  (6, 1), (6, 2), (6, 3), (6, 5), (6, 7),
  (8, 1), (8, 2), (8, 3), (8, 5), (8, 7)}

We have to list all the possible ordered pairs taking the first element from set A and second element from set B.

$B \times A$ =

- {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (1, 8),
  (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (2, 8)
  (3, 1), (3, 2), (3, 3), (3, 4), (3, 6), (3, 8),
  (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (5, 8)
  (7, 1), (7, 2), (7, 3), (7, 4), (7, 6), (7, 8)}

We have to listed all the possible ordered pairs taking the first element from set B and the second element from set A.

Note: In general $A \times B \neq B \times A$ but
$n (A \times B) = n (B \times A)$

5.02. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7\}$, then verify the following:
(i) $(A \cup B)^c = A^c \cap B^c$
(ii) $(A \cap B)^c = A^c \cup B^c$ when $\mu = \{1, 2, 3 \ldots 10\}$.

Sol: (i) $A \cup B = \{1, 2, 3, 4, 5, 7\}$
$(A \cup B)^c = \mu - (A \cup B)$
$= \{6, 8, 9, 10\}$ ——— (1)
$A^c = \mu - A = \{5, 6, 7, 8, 9, 10\}$
$B^c = \mu - B = \{2, 4, 6, 8, 9, 10\}$
$A^c \cap B^c = \{6, 8, 9, 10\}$ —— (2)
From (1) and (2), we notice that
$(A \cup B)^c = A^c \cap B^c$.

(ii) $A \cap B = \{1, 3\}$
$(A \cap B)^c = \{2, 4, 5, 6, 7, 8, 9, 10\}$ ---(3)
$A^c \cup B^c = \{2, 4, 5, 6, 7, 8, 9, 10\}$ —— (4)
From (3) and (4), we notice that $(A \cap B)^c = A^c \cup B^c$.

5.03. If $A = \{1, 2, 3, 4, 5\}$, then find the number of subsets of A which do not contain 3.

Sol: The number of subsets of A which do not contain 3 is 2^4.

5.04. If $A = \{a, b, c\}$; $R = \{(a, b), (b, c), (c, a)\}$ and $R_1 = \{(a, c), (b, c), (a, a)\}$, then find R^{-1}, $R \cup R_1$ and $R \cap R_1$.

Sol: (i) $R^{-1} = \{(b, a), (c, b), (a, c)\}$
(ii) $R \cup R_1 = \{(a, a), (a, b), (b, c), (c, a), (a, c)\}$
(iii) $R \cap R_1 = \{(b, c)\}$

FUNCTIONS

A relation f, which associates to each element of a set A exactly one element of set B is called a function from A to B and is denoted as $f: A \to B$ (read as f maps A to B). If B = A, then f is said to be a function from A to A (or in A)

If $(a, b) \in f$ then 'b' is called the image of 'a' under f. This is written as $b = f(a)$ and 'a' is called the pre-image of 'b'.

If $f: A \to B$ then A is called the domain of f and B is called the co-domain of f and the set $R = \{f(a)/a \in A\}$ is called the range of f. The range of f is also called the image of f and is denoted by Im(f) or f(A)

Note:
(1) Range $\subseteq$ Co-domain.
(2) $f \subseteq A \times B$.
(3) Every element of A has a unique f-image in B.
(4) Two or more elements of A can have the same f-image in B.
(5) There may be elements in B which are not f-images of any element of A.
(6) The number of functions from a set A containing m elements to another set B containing n elements is n^m.

One-One Function (Injection):

A function $f: A \to B$ is called a one-one function if distinct elements of A have distinct images in B. One- One functions are also called injective functions or injections.

i.e. $f: A \to B$ is one-one if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ equivalently $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Note:
(1) A one-one function is possible from A to B if $n(A) \leq n(B)$.
(2) If $n(A) = m$, $n(B) = n$ and $m \leq n$, then the number of one to one functions is ${}^nP_m$.
(3) If $n(A) = m$ and $n(B) = 2$, there are $2^m - 2$ onto functions from A to B.

Many-One Function:

A function which is not one-one is called a many-one function.

Onto Function (Surjection):

A function $f: A \to B$ is called an onto function if every element of B is an image of at least one element of A i.e. $f: A \to B$ is onto, if for each $y \in B$, there exists $x \in A$ such that $f(x) = y$. Onto functions are also called surjective functions or surjections.

Note:
(1) If f is onto, Range of f = co-domain of f.
(2) An onto function is possible from A to B if $n(A) \geq n(B)$.
(3) If $n(A) = m$, $n(B) = n$ then the number of onto functions is $n^m - {}^nC_1 (n - 1)^m + {}^nC_2 (n - 2)^m \ldots.+\ldots(n \leq m)$

Into Function:

A function which is not onto is called into function, i.e. $f: A \to B$ is into if Range of f is a proper subset of B.

Bijection:

If a function is both one-one and onto, then it is called a bijective function or bijection.

Note:
(1) A bijection from A to B is possible iff $n(A) = n(B)$.
(2) If A is a set with n elements, the number of bijections from A to A (or in A) is n!.

Constant Function:

A function $f: A \to B$ is said to be a constant function if $f(x) = k$, for all $x \in A$, where k is a fixed element of B.

Note: Range of f = {k}

Identity Function:

The function $f : A \to A$ defined by $f(x) = x$ is called the identity function, denoted by I_A.

Note:
(1) For the identity function, Range = Domain. Symbolically, $I_A(x) = x$ for $x \in A$.
(2) The identity function is a bijection.

Inverse Function:

If $f : A \to B$ is a bijective function then the function $f^{-1} : B \to A$, where $f^{-1} = \{(b, a)/(a, b) \in f\}$ is called the inverse of the function f.

Note:

(1) If $f : A \to B$ is bijective, $f^{-1} : B \to A$ is also bijective
(2) If $f(a) = b$, then $a = f^{-1}(b)$

Composition of functions:

If $f: A \to B$ and $g : B \to C$ are two functions, $g(f(a))$ for every $a \in A$ and is called the composite function of f and g. It is denoted as gof, which is read as g circle f.

Note:

(1) If $f : A \to A$, then $(fof^{-1})(x) = x$ or $fof^{-1} = I$.
(2) If A is a finite set $f: A \to A$ is injective, then f is bijective. But if A is infinite this need not be true.
(3) If $f: A \to B$ is bijective then $f^{-1}of = I_A$ and $fof^{-1} = I_B$.
(4) If $f: A \to B$ and $g: B \to A$ are two functions such that $gof = I_A$ and $fog = I_B$ then $g = f^{-1}$.
(5) If $f: A \to B$, $g: B \to C$ are two bijective functions then $(gof)^{-1} = f^{-1}og^{-1}$.
(6) If $f: A \to B$ and $g : B \to C$ are two functions then,
 (a) f and g are injective $\Rightarrow$ gof is injective.
 (b) f and g are surjective $\Rightarrow$ gof is surjective.
 (c) f and g are bijective $\Rightarrow$ gof is bijective.
 (d) If gof is injective then 'f' must be injective but 'g' need not be.
 (e) If gof is surjective then 'g' must be surjective but 'f' need not be.
 (f) If gof is bijective then 'f' must be injective and g' must be surjective.
(7) If $h: A \to B$, $g: B \to C$ and $f: C \to D$ be any three functions then $fo(goh) = (fog)oh$.

Real Function:

If A is a non-empty subset of **R,** then a function $f : A \to R$ is called a real valued function of a real number (or a real function)

Operations on Real Functions:

If $f: D_1 \to \mathbf{R}$ and $g: D_2 \to \mathbf{R}$ and $D = D_1 \cap D_2$ then we can define

(a) $f \pm g : D \to \mathbf{R}$ such that for all $x \in D$,
 $(f \pm g)(x) = f(x) \pm g(x)$
(b) $f \cdot g : D \to \mathbf{R}$ such that for all $x \in D$,
 $(fg)(x) = f(x) \cdot g(x)$
(c) $f/g: D \to \mathbf{R}$ such that for all $x \in D$,
 $f/g(x) = f(x)/g(x)$ provided $g(x) \neq 0$
(d) for some constant c, $(cf) : D_1 \to \mathbf{R}$ such that for all $x \in D_1$, $(cf)(x) = c.f(x)$.

Note : Domain of $f \pm g$, fg, f/g, is $D_1 \cap D_2$ where D_1 is the domain of f and D_2 is domain of g.

Ways of expressing Functions:

Explicit and Implicit Functions:

If the relation between the variables is of the form $y = f(x)$ then y is an explicit function of x. Similarly x is an explicit function of y if $x = f(y)$.

Examples:

(i) $x = y^2 + 3$
(ii) $y = \sin x + x^3 + 2$

A function which is expressed implicitly is called an implicit function. A function in two variables x, y is of the form $f(x, y) = 0$.

Examples: (i) $x^2 + e^{xy} \log y = \sin x$; (ii) $x^2 + y^2 = a^2$. Here, we can think of y as an implicit function of x or x as an implicit function of y.

Some properties of Functions

(1) Even and Odd Functions

A function f(x) is said to be an even function if for any number x in its domain, –x is also there in its domain and $f(-x) = f(x)$.

Examples : (i) $x^2 + 5$ (ii) cosx

A function f is said to be odd if for any number x in its domain, –x is also in its domain and $f(-x) = -f(x)$.

Examples : (i) x^3 (ii) sinx

Note :

Only certain functions are either even or odd. The others are neither even nor odd. This property (of being even or odd) is called parity of functions which has to be carefully distinguished from the property of parity of numbers. If f, g are two functions, the following table shows how the parity of $f \pm g$, fg, f/g gof and fog depends on the parity of f and g. (e denotes even and o denotes odd)

f	g	f+ g	f – g	fg	f/g	gof	fog
e	e	e	e	e	e	e	e
e	o	–	–	o	o	e	e
o	e	–	–	o	o	e	e
o	o	o	o	e	e	o	o

The student should construct the corresponding table for $x \pm y$, xy, x/y where x and y are integers and carefully note the points at which the two tables differ.

(2) Periodic Functions

A function f(x) is said to be periodic if there exists a non-negative real number T such that $f(x + T) = f(x)\ \forall\ x \in R$. If T is the smallest positive real number such that $f(x + T) = f(x)$ for all $x \in R$, then T is called the fundamental period of f(x). Any integral multiple of T is also a period. But when we speak of the period, without further description, we refer to the fundamental period.

Note:

(i) If f(x) is a periodic function with period T, then the function $f(ax + b)$, where $a > 0$ is periodic with period T/a. The period of both sin x and cos x is 2π and the period of tanx is π
(ii) The constant function is a periodic function without a fundamental period.

(3) Increasing and Decreasing Functions

Let f(x) be a function defined in a certain domain D. If for all a, b in D, where $a < b$, $f(a) \leq f(b)$, then f(x) is said to be an increasing function, and if $f(a) < f(b)$, then f(x) is said to be strictly increasing.
Consider the following two functions defined on [0, 5]
$f(x) = 2x$ for $0 \leq x \leq 5$
$g(x) = 2x$ for $0 \leq x \leq 3$
 $= 6$ for $3 \leq x \leq 5$

We see that f(x) is strictly increasing in [0, 5], while g(x) is an increasing function in [0, 5]. (It is strictly increasing in [0, 3])

Let f(x) be a function defined on a certain domain D. If for all a, b in D, where a < b, f(a) ≥ f(b), then f(x) is said to be a decreasing function and if f(a) > f(b), then f(x) is said to be a strictly decreasing function.

Let F(x) = – f(x), G(x) = – g(x), where f(x) and g(x) are the functions defined above. F(x) is a strictly decreasing function in [0, 5] while G(x) is a decreasing function. (It is strictly decreasing in [0, 3])

For most functions that we need to deal with, it will be possible to break up the domain into different parts so that throughout each part the function is either increasing or decreasing.

Some types of Functions

(1) Polynomial Functions

A function of the form $f(x) = a_0x^n + a_1x^{n-2} + a_2x^{n-2} + ... + a_n$ where x, a_0, a_1, a_2 .... a_n are real numbers, $a_0 \neq 0$ and $n \in N$ is called a polynomial function of degree n.
The domain and the range of f is R.

(2) Modulus Functions

The function defined by
f(x) = x if x ≥ 0 and –x if x < 0 is called the modulus function. This function is denoted as |x|. Similarly, we can define |g(x)|, where g(x) is an arbitrary function.

Here the domain of f = R, range of f = set of all non-negative real numbers

Example:

$|7| = 7; \ |-7| = 7$

(3) Signum Functions

The function f defined by $sg(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is called signum function.

Domain of f = R, Range of f = {–1, 0, 1}.
Similarly, we can define sg(g(x)), where g(x) is an arbitrary function.

(4) Trigonometric Functions

We give the domain and range of various trigonometric functions.

Function	Domain	Range
sin x	R	[–1, 1]
cos x	R	[–1, 1]
tan x	$R-\{(2n+1)\,\pi/2 \mid n\in Z\}$	R
cot x	$R - \{n\pi \mid n\in Z\}$	R
sec x	$R-\{(2n+1)\pi/2 \mid n\in Z\}$	$(-\infty, -1]\cup[1, \infty)$
cosec x	$R-\{n\pi \mid n \in Z\}$	$(-\infty,-1]\cup[1, \infty)$

Functional Equations

Equations involving unknown functions are called functional equations. Some basic functional equations and their solutions, i.e., the functions which satisfy these equations are given below.

	Equation	Solution
(1)	f(x + y) = f(x) + f(y)	f(x) = kx, $k\in R$
(2)	f(x + y) = f(x)f(y)	f(x) = 0 or $f(x) = a^x$, a > 0
(3)	f(xy) = f(x) + f(y)	f(x) = k lnx, $k\in R$
(4)	f(xy) = f(x)f(y)	f(x) = 0 or $f(x) = x^n$, $n\in R$

There are some unusual functions (sometimes called pathological functions) which may also satisfy the above conditions in addition to the solutions given but they are way beyond the scope of every MBA entrance exam in our country.

Examples:

5.05. Which of the following relations is a function on {1, 2, 3}?
(A) f_1 = {(2, 1), (2, 2), (3, 3)}
(B) f_2 = {(1, 2), (2, 1), (3, 3)}
(C) f_3 = {(1, 1), (1, 3), (2, 2)}
(D) f_4 = {(3, 1), (3, 2)}

Sol: Clearly f_2 is a function while in f_1, ‘2’ has more than one image. In f_3, 1 has more than one image and in f_4, 3 has more than one image.
∴ f_1, f_3, and f_4 are not functions.

5.06. Let f and g be two real valued functions defined on the domain {–2, –1, 0, 1}. If f = {(–2, –1), (–1, 0), (0, 1), (1, 2)} and g = {(–2, 0), (–1, 1), (0, 2), (1, 3)} then, find 2f + 3g, f^2 and fg.

Sol: (i) 2f(–2) + 3g(–2) = 2(–1) + 3(0) = –2
2f(–1) + 3g(–1) = 2(0) + 3(1) = 3
2f(0) + 3g(0) = 2(1) + 3(2) = 8
2f(1) + 3g(1) = 2(2) + 3(3) = 13
∴ 2f + 3g = {(–2, –2), (–1, 3), (0, 8), (1, 13)}
(ii) $f^2(x)$
$= \{f(x)\}^2$ = {(–2, 1), (–1, 0), (0, 1), (1, 4)}
(iii) fg(x) = f(x) . g(x)
f(–2) . g(–2) = (–1)(0) = 0, f(–1)g(–1) = 0,
f(0)g(0) = 1 . 2 = 2 and
f(1)g(1) = (2) (3) = 6
∴ fg = {(–2, 0), (–1, 0), (0, 2), (1, 6)}

5.07. If $f(x) = 2x^2 - 1$, g(x) = 3x + 1, then find the domain of $\sqrt{\dfrac{f}{g}}$.

Sol: $\sqrt{\dfrac{f(x)}{g(x)}} = \sqrt{\dfrac{2x^2-1}{3x+1}}$

$\dfrac{2x^2-1}{3x+1} \geq 0$

$(2x^2 - 1)(3x + 1) \geq 0$

Critical points are $\dfrac{-1}{\sqrt{2}}, \dfrac{-1}{3}, \dfrac{1}{\sqrt{2}}$

$\left(\sqrt{2}x-1\right)\left(\sqrt{2}x+1\right)(3x+1) \geq 0$

When x = 0; the above equation is not satisfied.

∴ The domain is $\left[\dfrac{-1}{\sqrt{2}}, \dfrac{-1}{3}\right) \cup \left[\dfrac{1}{\sqrt{2}}, \infty\right)$

5.08. If $f(x) = \frac{3x+2}{5x-3}$ and $x \neq \frac{3}{5}$, then find the relation between f and f^{-1}.

Sol: Let $\frac{3x+2}{5x-3} = y \Rightarrow 3x + 2 = 5xy - 3y$

$\Rightarrow x(5y-3) = 3y + 2$

$\Rightarrow x = \frac{3y+2}{5y-3}$

$f^{-1}(y) = \frac{3y+2}{5y-3} = f(y)$

$\therefore f^{-1}(x) = \frac{3x+2}{5x-3} = f(x) \Rightarrow \therefore f^{-1}(x) = f(x)$

5.09. If $f(x) = \frac{\operatorname{cosec} x \sec x}{\tan x + \cot x}$, then verify f(x) is even function or not.

Sol: $f(-x)$

$= \frac{\operatorname{cosec}(-x)\sec(-x)}{\tan(-x)+\cot(-x)} = \frac{-\operatorname{cosec} x \sec x}{-(\tan x + \cot x)} = f(x)$

$\therefore$ f(x) is even.

5.10. Find the domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$

Sol: $f(x) = \frac{1}{\sqrt{|x|-x}}$ is defined only if

$|x| - x > 0$

i.e. $|x| > x$ i.e. $x < 0$

$\therefore$ Domain is $(-\infty, 0)$.

5.11. Find the domain of the following functions.

(i) $f(x) = \frac{1}{x}\sqrt{x+6}$

(ii) $f(x) = \log_e(3x - 1)$

Sol: (i) $f(x) = \frac{1}{x}\sqrt{x+6}$

Division by 0 is not allowed.

so $x \neq 0$.

Square root is defined only for non-negative quantities.

$\therefore x + 6 \geq 0$ or $x \geq -6$.

$\therefore$ domain is $[-6, 0) \cup (0, \infty)$

(ii) $f(x) = \log_e(3x - 1)$.

As the logarithm is defined for positive numbers only, we must have $3x - 1 > 0$ or $x > \frac{1}{3}$.

Domain is $(\frac{1}{3}, \infty)$.

5.12. Find the range of the following functions

(1) $f(x) = x^2 + 4x + 10$; $x \in R$

(2) $f(x) = x - [x]$ $x \in R$, where [x] represents the greatest integer less than or equal to x.

Sol: (1) $f(x) = x^2 + 4x + 10$

$= (x + 2)^2 + 6$

Since $(x + 2)^2 \geq 0$

$\therefore (x + 2)^2 + 6 \geq 6$.

$\therefore f(x) \geq 6$.

The range of f(x) is $[6, \infty)$

(2) $f(x) = x - [x]$ or $x = [x] + f(x)$

Any real number x can be expressed as the sum of its integer part and the fractional part.

The fractional part lies between 0 and 1 where 1 is excluded.

In the above relation f(x) is the fractional part.

$\therefore$ The range of f(x) = [0, 1)

5.13. Find the inverse of the function $f(x) = \frac{3x+2}{4x-1}$ where $x \neq \frac{1}{4}$

Sol: Let $y = f(x) = \frac{3x+2}{4x-1}$, $x \neq \frac{1}{4}$

The inverse of a function is found by expressing x in terms of y. We think of y as the argument of the inverse function and x as the value of the inverse function.

$4xy - y = 3x + 2$

$\Rightarrow x(4y - 3) = y + 2$

$\Rightarrow x = \frac{y+2}{4y-3}$

$\therefore$ The inverse function $f^{-1}(y) = \frac{y+2}{4y-3}$, where $y \neq \frac{3}{4}$ or $f^{-1}(x) = \frac{x+2}{4x-3}$

5.14. Find the zeros of the function $f(x) = e^x(x - 1)(x + 2)$, $x \in R$.

Sol: To find the zeros of f(x), we equate f(x) to 0 and find the values of x satisfying

$e^x(x - 1)(x + 2) = 0$.

For all x, $e^x > 0$

$\therefore (x - 1)(x + 2) = 0$

$\therefore x = 1$ or -2.

5.15. If $f(x) = \frac{1}{x^3}$ and $g(x) = \sqrt{x-6}$, find f(g(x)) and also find its domain, given that it is a real function.

Sol: $f(g(x)) = f\left(\sqrt{x-6}\right) = \frac{1}{\left(\sqrt{x-6}\right)^3} = \frac{1}{\sqrt{(x-6)^3}}$

Domain is $x > 6$ since $\frac{1}{\sqrt{x-6}}$ would not be defined if $x = 6$ and would be complex if $x < 6$.

Domain $(6, \infty)$.

5.16. Find the range of the function $f(x) = |x - 6| - |x - 7|$.

Sol: Let us divide the possible values of x into three.

Case (i) $x \leq 6$

If $x \leq 6$, $|x - 6| = 6 - x$ and

$|x - 7| = 7 - x$

$\therefore f(x) = -1$

Case (ii) $6 \leq x \leq 7$.

If $6 \leq x \leq 7$,

$|x - 6| = x - 6$ and $|x - 7| = 7 - x$

$\therefore f(x) = 2x - 13$

$\therefore$ range of f(x) is $-1 \leq f(x) \leq 1$

Case (iii) $x \geq 7$: If $x \geq 7$, $|x - 6| = x - 6$ and

$|x - 7| = x - 7$

$\therefore f(x) = 1$.

$\therefore$ range of f(x) is $-1 \leq f(x) \leq 1$

Concept Review Questions

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The set builder form of the set
A = {4, 8, 12, 16, 20} is
(A) {x/x is a multiple of 4 less than 20}
(B) {x/x is a multiple of 4 less than or equal to 20}
(C) {x/x is a multiple of 4 less than 25}
(D) {x/x is a multiple of 4}

2. The roster form of the set A = {x/x is an odd prime number less than 20} is
(A) {7, 11, 17, 19, 2}
(B) {3, 5, 19, 23}
(C) {3, 5, 7, 11, 13, 17, 19}
(D) {2, 3, 5, 7, 11, 13, 17}

3. The number of elements in the set {5, {3, 6}, {7, 8}, 10, 11} is $\boxed{}$

4. Which of the following is a subset of the set {{3, 5}, 1, 4}?
(A) {1, {3}}
(B) {1, 2, 3, 5}
(C) {3, 5}
(D) {{3, 5}, 1}

5. If P is the set of all the letters of the word "MATHEMATICS", then cardinality of P is $\boxed{}$

6. If A = {1, 3, 5, 7} and B = {2, 4, 6, 8, 10}, then which of the following is true?
(A) A and B are equivalent sets.
(B) A and B are disjoint sets.
(C) A and B are equal sets.
(D) One set is the complement of the other set.

7. If A is the set of all factors of 72, B is the set of all multiples of 8, then $A \cap B$ =
(A) {6, 24, 72, 8}
(B) {3, 24, 2, 72}
(C) {8, 12, 48, 29, 72}
(D) {8, 24, 72}

8. **(a)** If n(A) = 8 and n(B) = 10, then the minimum number of elements in $A \cup B$ is
(A) 18 (B) 6 (C) 10 (D) 12

(b) If n(A) = 6 and n(B) = 4, then the maximum number of elements in $A \cap B$ is
(A) 4 (B) 6 (C) 8 (D) 10

9. If n(A) = 10 and n(B) = 13, then the maximum number of elements in $A \Delta B$ is $\boxed{}$

10. Which of the following cannot be the cardinality of power set of any set?
(A) 64 (B) 510 (C) 32 (D) 128

11. The number of non-empty proper subsets of a set 'A' is 62. The number of elements of set A is
$\boxed{}$

12. If A = {1, 2, 3, 4} and B = {a, b, c}, then the number of elements in the cartesian product of A and B is
$\boxed{}$

13. If $n(A \times B)$ = 48, then which of the following is not a value that n(A) can take?
(A) 12 (B) 6 (C) 14 (D) 8

14. If a set A has 6 elements and set B has 4 elements, then the number of relations defined from A to B is
(A) 2^{16} (B) 8^8 (C) 2^{10} (D) 8^{24}

15. If A = {a, e, i, o, u}, then the maximum number of elements in a relation is $\boxed{}$

16. If n(A) = 15, n(B) = 13 and $n(A \cup B)$ = 20, then the number of elements in $A \cap B$ is $\boxed{}$

17. In a group of people, 'c' persons like coffee, 't' persons like tea, and e persons like both coffee and tea. Then the number of persons who like either coffee or tea is
(A) t – e – c
(B) t + c – e
(C) t + c + e
(D) c + e – t

18. If X = {x : $x^2 - 5x - 6 = 0$} and Y = {y: $y^2 - 8y - 9 = 0$}, then Y – X =
(A) {–1, 9} (B) {–1, 6} (C) {9} (D) {6}

19. If A = {1, 5, 7, 9, 10}, B = {3, 4, 6, 8} and C = {3, 7, 9}, then $A - (B \cup C)$ =
(A) {1, 5, 10}
(B) {5, 10}
(C) {3, 4, 5}
(D) {1, 3, 5, 10}

20. If A, B and C are three mutually disjoint sets, then $(A - B) \cap (B - C)$ =
(A) A (B) ϕ (C) B (D) C

21. Which of the following is a function from $A \to B$ where A = {1, 2, 3} and B = {2, 3, 4, 5}?
(A) { (1, 2), (3, 4), (4, 5) }
(B) { (1, 1), (2, 3), (3, 4) }
(C) { (1, 2), (2, 4), (3, 5) }
(D) { (1, 1) (2, 2), (3, 3) }

22. Which of the following sets of ordered pairs does not represent a function from A to A, where A = {–3, 5, 2, –4}?
(A) {(–3, 5), (5, –3), (2, 2), (–4, 5)}
(B) {(5, –3), (2, –3), (–3, 5), (–4, –4)}
(C) {(–3, –3), (5, 5), (2, –4)}
(D) None of these

23. **(a)** The domain of the function {(3, 5), (4, 6), (7, 8), (1, 2)} is
(A) {3, 4, 1}
(B) {7, 1, 4}
(C) {5, 6, 7, 1}
(D) {1, 4, 7, 3}

(b) The range of the function {(1, 3), (2, 5), (3, 3), (4, 6)} is
(A) {3, 5, 6}
(B) {1, 2, 3, 5}
(C) {5, 6}
(D) {3, 4, 5}

24. If $f(n) = (-1)^{2n} + 3$, where n is any whole number, the range of f(n) is _____ .
(A) {3} (B) {4} (C) {2, 4} (D) {3, 4}

25. **(a)** The range of the function $f(x) = \frac{3x+2}{|3x+2|}$, $x \neq -2/3$ is
(A) {–1, 1} (B) {1} (C) {2, –2} (D) R

(b) The range of the function [x] – x is
(A) (–1, 0] (B) (0, 1)
(C) [0,1) (D) [–1, 0]

26. If f(x, y) = 3x – 2y, then f(4, f(3, –1)) is $\boxed{}$

27. If $f(x) = ax^3 - bx^2 + bx - a$, then $f\left(\frac{1}{x}\right) =$
(A) $\frac{-f(x)}{x^3}$ (B) f(x) (C) –f(x) (D) $\frac{f(x)}{x^3}$

28. If $f(x) = a^{x+p}$, then $\frac{f(k+\ell)}{f(k-\ell)} =$
(A) $f(2\ell + p)$ (B) $f(\ell + 2p)$
(C) $f(2\ell - p)$ (D) $f(\ell - 2p)$

29. **(a)** Which of the following functions is even?
(A) sinx + cosx – 2 (B) $e^{\sin x} + \cos x$
(C) cosx + 3 (D) $\cos^2 x + x^3$

(b) The function y = sin x is an
(A) even function (B) odd function
(C) both even and odd (D) None of these

(c) The function y = cos x is an
(A) even function (B) odd function
(C) both even and odd (D) None of these

30 For any function f(x) the function defined by $\frac{f(x)+f(-x)}{2}$ is _____ .
(A) even (B) odd
(C) both even and odd (D) None of these

31. The function which is both even as well as odd is _____ .
(A) constant function. (B) zero function.
(C) bijective function. (D) identity function.

32. The domain of the function $f(x) = \frac{1}{|x+2|}$ is
(A) R (B) (0, 2)
(C) R – {– 2} (D) None of these

33. The range of the function $f(x) = \frac{\operatorname{cosec}^2 x + \sec^2 x}{\operatorname{cosec}^2 x \sec^2 x}$ is
(A) (– 1, 1) (B) [0, 1]
(C) (0, 1] (D) {1}

34. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {8, 9}. The number of onto functions defined from A to B is
$\boxed{}$

35. Let A be a non-empty set and B contains 6 elements. f is a one-one function defined from A to B. The number of elements in A is
(A) at least 8. (B) at most 6.
(C) at most 8. (D) at least 6.

Exercise – 5(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 to 4: These questions are based on the following information.

Let A = {a, b, c, d, e, f, g, h}

1. The number of proper subsets of A is $\boxed{}$

2. The number of subsets of A that contain exactly 4 elements is
(A) 2^4 (B) 64 (C) 70 (D) 128

3. The number of subsets of A that contain at most 5 elements is
(A) 219 (B) 247 (C) 232 (D) 228

4. The number of subsets of A that contain a, f but not g is
(A) 8 (B) 16 (C) 32 (D) 4

5. The number of functions from set A to set B, where n(A) = 4 and n(B) = 3 is
(A) 64 (B) 27 (C) 81 (D) 12

6. The number of functions from set A to set B which are not one-one, where n (A) = 3 and n (B) = 5 is
$\boxed{}$

7. The number of onto functions (surjections), where n(A) = 4 and n (B) = 3 is
(A) 81 (B) 64 (C) 36 (D) 33

8. The number of bijections from set A, to set A, where n(A) = 4 is $\boxed{}$

9. If F(a, b, c, d) = ab – cd; and F(y, y + 4, – 4 , 6) = F (8, 17.5, 4, 5), then the possible values of y are
(A) 12, – 4 (B) 16 , –8 (C) –12, 8 (D) –16, 4

10. If $g_1(x) = 9x^2$ and $g_2(x) = x^2 – 12x + 27$, and $g_1(m+1) = g_2(3m)$, then the value of m is
(A) 3 (B) 9 (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

11. If f(x) = $\log\left(\frac{1-x}{1+x}\right)$, then f(x) + f(y) =
(A) f(x y + 1) (B) (x + y) .f(xy + 1)
(C) $f\left(\frac{x+y}{xy+1}\right)$ (D) f(x + y)

12. g(x) = 3x and h(x) = $3|x|$; If $[g(x) + f(x)]^2 + [h(x) + f(x)]^2 = 0$, then for those values of x for which f(x) is defined, f(x) =
(A) $3|x|$ (B) 3x
(C) –3x (D) None of these

13. If $4f(x) – 5f\left(\frac{1}{x}\right) = x^3$, then find f (0.2). $\boxed{}$

Directions for questions 14 and 15: These questions are based on the following information

$f_1(x) = x$ for $–1 \le x < 0$
$= –1$ for $x \le –1$
$= 0$ otherwise
and $f_2(x) = f_1(–x)$ for all x
$f_3(x) = –f_2(x)$ for all x
$f_4(x) = f_3(–x)$ for all x

14. How many of the following expressions is/are always 0 for every x?
f_1f_2; f_2f_3; f_3f_4;f_2f_4
(A) 0 (B) 1 (C) 2 (D) 3

15. Which of the following is true?
(A) $f_1(x) = f_4(x)$ (B) $f_1(x) = f_3(–x)$
(C) $f_2(x) = f_4(x)$ (D) $f_1(–x) = f_3(x)$

16. A function f(x) is an odd function and the function g(x) is an even function. What can be said about the nature of the composite functions fog (x) and gof(x)?
(A) Both are even.
(B) Both are odd.
(C) fog (x) is even and gof(x) is odd.
(D) fog (x) is odd and gof(x) is even.

17. Find the domain of the function
$f(x) = \log(x^2 – 4) + \frac{1}{\sqrt{9 - x^2}}$.
(A) (–3, –2) (B) $(–3, 2) \cup (2, 3)$
(C) $(–3, –2) \cup (2, 3)$ (D) (–3, 3)

18. Find the range of the function f(x) = |x + 7| + |x – 9| + 12
(A) $(33,\infty)$ (B) $(12,\infty)$ (C) $(28, \infty)$ (D) $[28,\infty)$

19. The domain of $\frac{1}{3\,|\,x-[x]\,|}$ where [x] is the greatest integer less than or equal to x is
(A) R (B) Z (C) R –Z (D) R^+

20. Let $f(x) = \min\{|2-x|, |x|, |x+2|\}$. Find the range of x for which f(x) = x –2.
(A) $(–\infty , 2)$ (B) (–2, 2)
(C) $[2,\infty)$ (D) None of these

21.

x	1	2	3	4	5
f(x)	4	5	1	2	3

The table above defines f(x) for x = 1, 2, 3, 4 and 5.

For $x \ge 5$, f(x + 1) = f(f(x)). Find f(499). $\boxed{}$

22. If f(x + y) = f(x) + f(y) and f (3) = 29, then find f(27).
(A) 261 (B) 216 (C) 621 (D) 612

23. If f(x + y) = f(x)f(y) and f (4) = 4096, then find f(10)
(A) 2^{16} (B) 2^{64} (C) 2^{30} (D) 2^{32}

24. If $f(x) = \frac{x-1}{x+1}$, $x \neq -1$, then find $f\left(\frac{x-1}{x+1}\right)$.

(A) x (B) $-\frac{1}{x}$ (C) $f(x)$ (D) $f\left(\frac{1}{x}\right)$

25. If $f(xy) = f(x)\,f(y)$ for all $x \in R$ and $f(3) = 27$, then find $\sum_{n=1}^{30} f(n)$.

(A) 465 (B) $\frac{3}{2}(3^{30} - 1)$

(C) $\frac{3^{30}-1}{2}$ (D) 465^2

26. If $f(xy) = f(x) + f(y)$ for all positive values of x and $f(3) = 1$, then $(f(243) - f(81)) / (f(27) - f(9))$ = $\boxed{}$

27. If $f(x) = \frac{x+1}{x-1}$, then for what value(s) of x is $f^{-1}(x)$ not defined?

(A) 1 (B) –1 (C) 0 (D) –1 and 1

28. If $f(x) = \frac{5x+3}{4x-9}$, $x \neq 9/4$, then $f^{-1}(x)$ =

(A) $\frac{9x+3}{4x-5}$ (B) $\frac{5x+3}{4x-9}$

(C) $\frac{9x-3}{4x+5}$ (D) $\frac{5x-3}{9-4x}$

29. Let A = {1, 2, 3, 4 …30}. A relation R maps an element x in A to the element 2x + 5, also in A. Find the number of elements in R. $\boxed{}$

30. If $f(2x - 1) = 8x^2 - 10x + 6$, then find f(0).

$\boxed{}$

31. If $f(x) + f(2 - x) = 4$, then the value of

$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \ldots\ldots.. + f\left(\frac{199}{100}\right) =$

$\boxed{}$

32. If f (x) = ax + b where a and b are constants and f (f (f (x))) = 125x +217 then find the value of 7a – 5b.

(A) 0 (B) 1 (C) 5 (D) –1

33. What is the maximum value of min(x + 2, 4 – 3x)?

$\boxed{}$

34. P = {1, 2, 3,.....100}. Q is a non-empty subset of P. If the product of all the elements of Q is even, how many such subsets Q are possible?

(A) $2^{100} - 2^{50} - 2$ (B) $2^{100} - 2^{50}$

(C) $2^{100} - 2^{40}$ (D) $2^{100} - 2^{40} - 2$

35. If $f(x) = ax^6 + bx^4 - cx^2 + 3x + 7$ and $f(9) = 26$, find f(–9). $\boxed{}$

Exercise – 5(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 to 5: These questions are based on the following information.

Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

1. The number of non-empty subsets of A is

(A) 256 (B) 511 (C) 512 (D) 255

2. The number of non-empty proper subsets of A is

(A) 510 (B) 511 (C) 255 (D) 254

3. The number of subsets of A that contain at least 2 elements is

(A) 503 (B) 502 (C) 500 (D) 474

4. The number of subsets of A that contain 1, 2, 3 is

(A) 48 (B) 128 (C) 64 (D) 32

5. The number of subsets of A that do not contain 7, 8, 9 is

(A) 64 (B) 32 (C) 128 (D) 48

6. The number of injections (one-one functions) from set A to set B, where n (A) = 5 and n (B) = 6 is

$\boxed{}$

7. The number of onto functions (surjections) from set A to set B, where n(A) = 5 and n(B) = 2 is

(A) 30 (B) 32 (C) 10 (D) 20

8. The number of injections from set A to set B, where n(A) = 6 and n(B) = 4 is

(A) ${}^6P_4$ (B) 0 (C) 4^6 (D) 6^4

9. The number of proper subsets of set A is 127. The number of subsets of A that contain exactly 2 elements of set A but do not contain a particular element of set A is

(A) 20 (B) 18 (C) 16 (D) 15

10. The number of possible bijections from set A to set B, where n(A) = 5 and n(B) = 5 is

(A) 0 (B) 24 (C) 25 (D) 120

11. The number of one-one and onto functions from set A to set B, where n(A) = 6 and n(B) = 5 is

(A) 0 (B) ${}^6P_5$ (C) 5^6 (D) 6^5

12. If H(m, n, p, q) = mq + np and H(x, 8, 9, x + 12) = H(12, 16, 7, 50), then the possible values of x are

(A) –18, 40 (B) –10, 64

(C) – 40, 16 (D) –32, 20

13. Let h(0) = –2, h(1) = 3 and h(x + 2) = 2h(x +1) – h (x), where x is any rational number. Find the remainder when h(8) h(13) is divided by 17.

$\boxed{}$

14. If g(x) = 8x –9, h(x) = 9x + 8 and 2(goh (a)) = hog (a), then a =

(A) $\frac{-183}{72}$ (B) $\frac{-72}{167}$ (C) $\frac{-72}{183}$ (D) $\frac{-167}{72}$

15. If f (2x + 3) = $4x^2 + 14x + 14$, then f(x) =

(A) $x^2 - x + 2$ (B) $x^2 - x - 2$
(C) $x^2 + x - 2$ (D) $x^2 + x + 2$

16. If h(x –2) = $4(h(x))^2 - 5$, find h(x – 4).

(A) $64(h(x))^4 - 120h(x))^2 + 105$
(B) $64 (h(x))^4 - 200(h(x))^2 + 125$
(C) $64(h(x))^4 - 140(h(x))^2 + 95$
(D) $64(h(x))^4 - 160(h(x))^2 + 95$

17. If $f_1(x) = 16x^2$, $f_2 (x) = x^2 - 18x - 48$; and $f_1(k - 3) = f_2(4k)$, then k = $\boxed{}$

18. If f(x) = $\log\left(\frac{1+x}{1-x}\right)$, then f(x) – f(y) =

(A) $f\left(\frac{1-xy}{x-y}\right)$ (B) $f\left(\frac{1-xy}{y-x}\right)$

(C) $f\left(\frac{x-y}{1-xy}\right)$ (D) $f\left(\frac{y-x}{1-xy}\right)$

19.

x	1	2	3	4	5	6
g(x)	5	6	4	1	2	3

The table above defines g(x) for x = 1, 2, 3, 4, 5 and 6. For x > 6, g(x) = g(g(x – 1)). The value of g(899)is

(A) 6 (B) 5 (C) 1 (D) 2

20. A function f(x) is defined as f(x) = $\frac{5}{\sqrt[3]{|x|+x}}$, then domain of f(x) is

(A) R (B) R^-
(C) R^+ (D) R – R^-

21. If f:R → R is a function defined as f(α – f(α)) = 5f(α) and f(1) = 7, then find f(–6). $\boxed{}$

22. The domain of $\frac{1}{2(x-|x|)}$ is

(A) R^+ (B) R^- (C) Z^- (D) Z^+

23. If f(x) = $\frac{3}{\sqrt{x(x-3)(x+2)}}$ is a real valued function, then the domain of f(x) is

(A) (–2, 3) (B) (–2, 0) ∪ (3, ∞)
(C) (4, ∞) (D) (–∞ , 0)

24. F(x) = max $\{|1-x|, |x+1|, |x|\}$ The range of x for which f(x) = x + 1 is

(A) (–∞ , 0) (B) $\left(\frac{1}{2}, \infty\right)$
(C) (–2, 0) ∪ (0, ∞) (D) (0, ∞)

25. If f(x +y) = f(x) + f(y) and f(3) = 9, then f(20) = $\boxed{}$

26. Find the domain of the function f(x) = log |x| + $\frac{1}{x+3}$.

(A) (–3, ∞) (B) R – {0, 3}
(C) R – {0} (D) R – {0, –3}

27. Find the domain of the function, f(x) = $\sqrt{1-\sqrt{1-x^2}}$.

(A) [0, 1] (B) [–1, 1]
(C) [0, ∞) (D) [–1, 0]

28. Find the range of the function f(x) = |x – 2| – |x|, x ∈ R.

(A) (–∞, 2] (B) [–2, 2]
(C) [2, ∞) (D) R

29. If the cost of parking a car in a parking lot at the railway station is ₹5 for the first hour or any part thereof, plus ₹2 for each additional hour or part thereof. Which of the relations best represents the cost C as a function of time (t) in hours? $\lceil x \rceil$ is the least integer greater than or equal to x and $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

(A) C = $\lceil 2t \rceil$ + 5 (B) C = 5$\lfloor t \rfloor$ + 3
(C) C = 5$\lfloor t \rfloor$ + 5 (D) C = 2$\lceil t - 1 \rceil$ + 5

30. Let f(x) = $\begin{cases} 2; \text{ when } x \text{ is a rational number,} \\ -2; \text{ when } x \text{ is an irrational number.} \end{cases}$

Find the value of the expression

$f(|\sqrt{2}|) + |(f\sqrt{2})| + \sqrt{|f(2)|} + |\sqrt{f(2)}|$.

(A) $\sqrt{2}$ (B) 2 (C) $-\sqrt{2}$ (D) $2\sqrt{2}$

31. f(x) = $\begin{cases} 1+|x|; & x < -2 \\ [x]-1; & x \geq -2, \end{cases}$

where [x] is the greatest integer less than or equal to x.

Find the value of f(f(–2.6)). $\boxed{}$

32. If f(x) + f(1–x) = 4, then find the value of $f\left(\frac{1}{30}\right) + f\left(\frac{2}{30}\right) + ... + f\left(\frac{29}{30}\right)$

(A) 56 (B) 58 (C) 29 (D) 26

33. If 3f(x) + 2f(1 – x) = x^2 + 4, find f(3).

(A) $\frac{1}{9}$ (B) $\frac{23}{5}$ (C) $\frac{5}{9}$ (D) $\frac{9}{5}$

34. The domain of the function f(x) = log | $2x^2 - 11x - 30$ | is

(A) $R - \left\{\frac{15}{2}, -2\right\}$ (B) $\left(-2, \frac{15}{2}\right]$

(C) $\left(-2, \frac{15}{2}\right)$ (D) $\left[-2, \frac{15}{2}\right)$

35. If f : R → R and f(x) = $\frac{2}{1+x^2}$, then f is ______ .

(A) one-one, but not onto.
(B) onto, but not one-one
(C) bijective
(D) neither one-one nor onto

36. The number of surjections possible from set A to set B, where n(A) = 4 and n(B) = 6 is ______ .

(A) 4^6 (B) 6^4 (C) ${}^6P_4$ (D) 0

37. If n(A) = p, n(B) = 2 and the number of onto functions from A to B is 1022, then p is $\boxed{}$

38. If f(x) = 3x + 4 and g(x) = 4x – 3; then fog (x) + gof (x) =
(A) 24x + 12 (B) 24x + 8
(C) 24x – 8 (D) 24x – 12

39. If $\frac{1}{x}f(x) - 3f\left(\frac{1}{x}\right) = \frac{3}{2}$ for all $x \neq 0$, then f (3) =
(A) $\frac{-8}{9}$ (B) $\frac{8}{9}$ (C) $\frac{-9}{8}$ (D) $\frac{9}{8}$

40. If f (x) = 2x + 5 and g (x) = 3x – 4 then find the value of $(fog)^{-1}(-9)$
(A) 1 (B) –1 (C) 0 (D) $\frac{1}{9}$

41. A function g(x) is defined as $g(x) = \frac{2}{\sqrt[3]{\lfloor x \rfloor - x}}$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x. The domain of g(x) is
(A) R (B) Z (C) Q (D) R–Z

42. If f(x + y) = f(x)f(y) for all x, $y \in R$ and f(a) = 3, then the value of f(20a) is
(A) $(243)^3$ (B) 3^{10} (C) 3^{20} (D) 3^5

43. If $f(x) = 8x^4$ and $g(x) = \sqrt[3]{f(x)}$, find the value of $\log_2(fog(64))$
(A) 39 (B) 15 (C) 28 (D) 32

44. If $f(4x - 5) = \frac{x+2}{x}$, then find $f^{-1}(2)$. $\boxed{}$

45. If f: R → R defined as f(x) = 3x – 5, then $f^{-1}(\{-1, -2, 1, 2\})$ =
(A) $\left\{1, \frac{4}{3}, \frac{7}{3}\right\}$ (B) $\left\{-1, 2, \frac{-4}{3}\right\}$
(C) $\left\{1, 2, \frac{4}{3}, \frac{7}{3}\right\}$ (D) {1, 2, –1, –2}

Key

Concept Review Questions

1. B	10. B	20. B	28. C
2. C	11. 6	21. C	29. (a) C
3. 5	12. 12	22. C	(b) B
4. D	13. C	23. (a) D	(c) A
5. 8	14. B	(b) A	30. A
6. B	15. 25	24. B	31. B
7. D	16. 8	25. (a) A	32. C
8. (a) C	17. B	(b) A	33. D
(b) A	18. C	26. –10	34. 126
9. 23	19. A	27. A	35. B

Exercise – 5(a)

1. 255	10. C	19. C	28. A
2. C	11. C	20. C	29. 12
3. A	12. C	21. 5	30. 3
4. C	13. –69.448	22. A	31. 398
5. C	14. D	23. C	32. A
6. 65	15. B	24. B	33. 2.5
7. C	16. A	25. D	34. B
8. 24	17. C	26. 1	35. –28
9. C	18. D	27. A	

Exercise – 5(b)

1. B	13. 14	25. 60	37. 10
2. A	14. A	26. D	38. B
3. B	15. D	27. B	39. C
4. C	16. D	28. B	40. B
5. A	17. 8	29. D	41. D
6. 720	18. C	30. D	42. C
7. A	19. A	31. 2	43. A
8. B	20. C	32. B	44. 3
9. D	21. 35	33. B	45. C
10. D	22. B	34. A	
11. A	23. B	35. D	
12. D	24. D	36. D	