CHAPTER – 1

SIMPLE EQUATIONS

There will be linear equations of one or two unknowns invariably in every problem. A linear equation is one where each variable occurs only in its first power and not in any higher powers. Some times we get three equations in three unknowns. In general, we need as many equations as the variables we will have to solve for. So, for solving for the values of two unknowns, we need two equations (or two conditions given in the problem) and for solving for the values of three unknowns, we need three equations (and hence the problem should give three conditions from which we can frame three equations). Solving the equations by itself is not a difficult task. The most important part of the problem is framing the equation/equations. Once the equations are framed, solving them is very easy. In this chapter, we will deal with problems involving as many equations (of first degree) as the number of unknowns. Later on, we will look at equations of second degree (Quadratic Equations) and linear equations where the number of equations will be less than that of the number of variables (under the chapter Special Equations).

ONE EQUATION IN ONE UNKNOWN

An equation like $2x + 4 = 26$ is an equation in one unknown. We have only one variable x whose value we have to find out. The steps in solving this are:

Step I: Take all quantities added to (or subtracted from) the x term (term with the unknown) to the right side with a change of sign.
i.e., $2x = 26 – 4 = 22$.

Step II: Take the co-efficient of x from left hand side and divide right hand side with this term to get the value of x.
i.e., $x = 22/2 = 11$. Therefore, $x = 11$.

TWO EQUATIONS IN TWO UNKNOWNS

A set of equations like

$2x + 3y = 8$ ..... (1)
$5x + 4y = 13$ ..... (2)

is called a system of simultaneous equations in two unknowns. Here, we have two variables (or unknowns) x and y whose values we have to find out. This can be done using the two given equations. The steps for this are as follows:

Step I: Using both the equations we first eliminate one variable (so that we can then have one equation in one unknown).
For this purpose, we multiply equation (1) with 5 (the co-efficient of x in the second equation) and multiply equation (2) with 2 (the co-efficient of x in the first equation) to eliminate x. Thus we have
(1) $\times 5 \Rightarrow 10x + 15y = 40$ ..... (3)
(2) $\times 2 \Rightarrow 10x + 8y = 26$ ..... (4)
Now, subtracting equation (4) from equation (3) we have $7y = 14$ ..... (5)
This is one equation in one unknown.

Step II: Solve for the value of one variable from the equation (in one unknown) obtained from Step I above. Therefore, $y = 2$.

Step III: Substitute this value of the variable in one of the two equations to get the value of the second variable.

Substituting the value of y in equation (1) or equation (2), we get $x = 1$. Therefore the values of x and y that satisfy the given set of equations are $x = 1$ and $y = 2$.

THREE EQUATIONS IN THREE UNKNOWNS

A set of equations like
$x + 2y + 3z = 14$ ..... (1)
$2x + y + 2z = 10$ ..... (2)
$3x + 3y + 4z = 21$ ..... (3)
is a system of three equations in three unknowns.

Here we have three unknowns x, y and z which we have to solve for from the three given equations. The procedure for the same is as follows:

Step I: Take two out of the three equations [say, eqn. (1) and (2)] and eliminate one variable (say x) so that we get an equation in two unknowns (y and z in this case).

For this purpose, take equations (1) and (2). Multiply equation (1) by 2 and subtract equation (2) from it.

Equation (1) $\times 2 \Rightarrow$ $2x + 4y + 6z = 28$
$2x + y + 2z = 10$

$3y + 4z = 18$..... (4)

Step II: Repeat Step I for two other equations [say equations (2) and (3)] and eliminate the same variable (x in this case) so that we get one more equation in two unknowns (y and z).

For this purpose, take equations (2) and (3). Multiply equation (2) by 3 and from that subtract equation (3) multiplied by 2.

Equation (2) x 3 $\Rightarrow$ $6x + 3y + 6z = 30$
Equation (3) x 2 $\Rightarrow$ $6x + 6y + 8z = 42$

$–3y – 2z = –12$ .... (5)

Step III: Now the equations in two unknowns that have been obtained from the above two steps have to be solved as discussed previously (in TWO EQUATIONS IN TWO UNKNOWNS) to get the values of two of the three variables (y and z in this case).

In this case, solving equations (4) and (5), we get $y = 2$ and $z = 3$.

Step IV: Substitute these values of the two variables in one of the three equations to get the value of the third variable.
Substitute the value of y and z in equation (1) to get the value of x = 1.
Thus the values of the three variables x, y and z that satisfy the three given equations are x = 1; y = 2 and z = 3

Examples

1.01. The cost of 3 tables and 4 chairs is ₹2500. The cost of 4 tables and 3 chairs is ₹2400. Find the costs of each table and each chair.

Sol: Let the cost of each table be ₹x.
Let the cost of each chair be ₹y.
3x + 4y = 2500 --- (1)
4x + 3y = 2400 --- (2)

Method 1:
Multiplying (1) by 3 and subtracting it from (2) multiplied by 4, we get
7x = 2100
x = 300
Substituting x = 300 in (1),
y = 400

Method 2:
Adding both the equations (1) and (3), we get
7 (x + y) = 4900
x + y = 700 --- (3)
subtracting (2) from (1),
– x + y = 100 --- (4)
Adding (3) and (4), 2y = 800
y = 400
Substituting y = 400 in either (3) or (4), x = 300

1.02. Raju bought 6 pens, 5 erasers and 4 sharpeners for ₹32. Had he bought 4 pens, 3 erasers and 5 sharpeners, his total expenditure would have been ₹23. Had he bought 7 pens, 2 erasers and 6 sharpeners, his total expenditure would have been ₹31. Find the cost of 1 pen, 1 eraser and 2 sharpeners.

Sol: Let the prices of each pen, each eraser and each sharpener be ₹p, ₹e and ₹s respectively.
6p + 5e + 4s = 32 --- (1)
4p + 3e + 5s = 23 --- (2)
7p + 2e + 6s = 31 --- (3)
Multiplying (1) by 2 and subtracting from (2) multiplied by 3, –e + 7s = 5 --- (4)
Multiplying (3) by 4 and subtracting it from (2) multiplied by 7, 13e + 11s = 37 --- (5)
Multiplying (4) by 13 and adding it to (5), 102s = 102
s = 1
Substituting s = 1 in (4),
e = 2
Substituting values of e and s in (1), p = 3

1.03. In a two digit number, the digits differ by 2. 10 times the number exceeds 5 times the sum of the number formed by reversing its digits and the sum of its digits by 90. Find the number.

Sol: Let the number be xy. Hence the value of the number is 10x + y.
x – y = 2 or y – x = 2 --- (1)
10 (10x + y) – 5 (10y + x + x + y) = 90
90x – 45y = 90
2x – y = 2
2x – (x ± 2) = 2
x = 4 or 0
As x cannot be 0, x = 4
∴ y = 6
∴ the number is 46

1.04. The age of Fahim 4 years ago was 4 times of Sachin's age. Fahim's age three years ago was thrice Sachin's age. How many times Sachin's age was Fahim's age 5 years ago?

Sol: Let the present ages of Fahim and Sachin be f years and s years respectively.
f – 4 = 4 (s – 4) ⇒ f = 4s – 12
f – 3 = 3 (s – 3) ⇒ f = 3s – 6
f = 4s – 12 = 3s – 6
s = 6, f = 12
f – 5 = 7 (s – 5)
∴ 5 years ago Fahim's age was 7 times Sachin's age.

1.05. The age of a man 15 years ago was 5 times his son's age. His age 10 years ago was thrice his son's age. After how many years from now will their combined age become 80 years?

Sol: Let the present age of the man and his son be f years and s years respectively.
f – 15 = 5 (s – 15) ⇒ f = 5s – 60
f – 10 = 3 (s – 10) ⇒ f = 3s – 20
f = 5s – 60 = 3s – 20
s = 20, f = 40
Their combined present age is 60 years. For the combined present age to become 80 years, the age of each of them must increase by 10 years.
∴ Their combined age will become 80 years after 10 years.

1.06. If the numerator and the denominator of a fraction are both increased by 1, the fraction becomes $\frac{3}{5}$. If both are decreased by 1, it becomes $\frac{5}{9}$. Find the fraction.

Sol: Let the fraction be $\frac{x}{y}$.

$\frac{x+1}{y+1} = \frac{3}{5}$

⇒ 5x + 5 = 3y + 3
5x + 2 = 3y --- (1)

$\frac{x-1}{y-1} = \frac{5}{9}$

⇒ 9x – 9 = 5y – 5
9x – 4 = 5y --- (2)
Multiplying (1) by 5 and subtract it from (2) after multiplying by 3, 5 (5x + 2) = 3 (9x – 4)
⇒ x = 11
substituting x = 11 in (1), y = 19
∴ The fraction = $\frac{11}{19}$

1.07. Find the values of x and y satisfying the equations below.

$$\frac{35}{x+y}+\frac{18}{x-y}=11$$

$$\frac{28}{x+y}+\frac{33}{x-y}=15$$

Sol: Let $\frac{1}{x+y}=p$ and

$$\frac{1}{x-y}=q$$

$35p + 18q = 11$ --- (1)
$28p + 33q = 15$ --- (2)
Multiplying (1) by 4 and subtracting it from (2) multiplied by 5, $93q = 31$

$\Rightarrow q = \frac{1}{3}$

Substituting $q = \frac{1}{3}$ in (1),

$p = \frac{1}{7}$

$\therefore \frac{1}{x-y}=\frac{1}{3}$ and $\frac{1}{x+y}=\frac{1}{7}$

$\therefore x - y = 3$ and $x + y = 7$.
Solving these equations, $x = 5$ and $y = 2$.

ADDITIONAL CASES IN LINEAR EQUATIONS

(1) If the number of equations is less than the number of unknowns, then we say the variables are 'indeterminate' or we have an "indeterminate" system of equations. Here, we cannot uniquely determine the values of all the variables. There will be infinite sets of solutions that satisfy the equations.

For example, if we take the following two equations in three unknowns,

$x + y + 2z = 8$
$2x – y + 3z = 13$

this system of equations have infinite number of solutions and no unique solution is possible. For any value we take for x, we can find a corresponding set of values for y and z.

(2) However, even in case of indeterminate equations, say, of three variables, it is possible that the value of one of the variables may be uniquely determined, i.e., if we have two equations and three unknowns, we may be still able to determine the value of one variable uniquely but the other two variables will have infinite number of values. This will happen if the ratio of the coefficients of two variables in one equation is the same as the ratio of the coefficients of the same two variables in the second equation.

This depends on the equations given. Example 1.08 will clarify this aspect.

1.08. Tarun bought 2 shirts, 4 trousers and 5 pairs of shoes for ₹3600. Had he bought 6 shirts, 5 trousers and 15 pairs of shoes, his total expenditure would have been ₹8700. Find the price of each trouser.

Sol: Let the prices of each shirt, each trouser and each pair of shoes be ₹x, ₹y and ₹z respectively.
$2x + 4y + 5z = 3600$ --- (1)
$6x + 5y + 15z = 8700$ --- (2)
Multiplying (1) by 3 and subtracting (2) from it, $7y = 2100$
$\therefore y = 300$

(3) Even in case of indeterminate equations, when some additional conditions are either implicitly built into the problem or explicitly imposed by specifying some constraints on the values of the variables, we may some times be able to determine the values of the variables uniquely or find out a finite set of values that the variables may take. Such problems are separately considered under the chapter "SPECIAL EQUATIONS."

(4) Sometimes, even if we have equations less in number than the number of variables (i.e., indeterminate equations), while we cannot find out the values of ALL the variables uniquely, it may be possible to find out the value of some specific combination of the variables.

1.09. The cost of 3 dosas, 5 idlis and 7 vadas is ₹154. The cost of 5 dosas, 8 idlis and 11 vadas is ₹246. Find the total cost of one idli, one dosa and one vada.

Sol: Let the cost of each dosa, each idli and each vada be ₹d, ₹i and ₹v respectively.
$3d + 5i + 7v = 154$ --- (1)
$5d + 8i + 11v = 246$ --- (2)
Multiplying (1) by 3 and subtracting it, from twice (2), $d + i + v = 30$

(5) Sometimes, even if we have three equations in three unknown, we may not be able to uniquely determine the values of the variables if the equations are not "INDEPENDENT," i.e., one of the given equations can be written as a "linear combination" of the other two equations.

For example, let us take the following system of three equations in three unknowns.
$3x + 5y + 7z = 12$ ----- (1)
$x – 3y + 9z = 16$ ----- (2)
$9x + 8y + 31z = 54$ ----- (3)

If we try to solve these equations, we will find that we cannot get a unique solution. That is because these equations are not independent. In this case, equation (3) can be obtained by multiplying equation (1) by 2.5 and equation (2) by 1.5 and adding them.

If there are three equations l_1, $l_2$and l_3 in three unknowns, we say that they are linearly dependent if one of the three equations can be written as a linear combination of the other two, i.e., **$l_3 = l_1 + kl_2$ where k is any constant**.

In such a case, the system of equations will have infinite number of solutions.

If it is not possible to write the three equations in the form above, then they are linearly independent and the system of equations will have a unique solution.

(6) Sometimes, we can have "inconsistent" equations. For example, if we know that $x + 2y = 4$, then the value of $2x + 4y$ has to be 8. The expression $(2x + 4y)$ cannot take any other value. If it is given any other value, there will be inconsistency in the data because then we will effectively be saying that $x + 2y = 4$ and at the same time $x + 2y \neq 4$.
So, if we have the system of equations

$x + 2y = 4$ and $2x + 4y = k$, this system of equations will be consistent ONLY If the value of $k = 8$. For any other value of k, the system of equations will be INCONSISTENT.
In the above system of equations, when $k = 8$, there will be infinite number of solutions (and not a unique solution).

1.10. Find the value of k for which the following system of equations will be consistent.
$2x - 5y = 10$ and $6x - 15y = k$

Sol: In the given system of equations, the ratio of the coefficients of x equals the ratio of the coefficients of y.
$\therefore$ They would be consistent only if this ratio equals the ratio of the constant terms.
$\therefore$ If $\frac{10}{k} = \frac{2}{6} = \frac{-5}{-15}$ i.e.,
if $k = 30$, the given system of equations would be consistent.

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Thrice a number is 24 more than one-third of it. Find it.

2. Four added to half of one-third of one-sixth of a number is equal to one-twelfth of the number. What is the number?
(A) 48 (B) 36 (C) 72 (D) 24

3. The sum of two numbers is 18. The difference of the numbers is 4. Find the numbers.
(A) 11, 7 (B) 12, 8 (C) 16, 2 (D) 8, 10

4. How many pairs (x, y) satisfy the equations $3x + 6y = 12$ and $4x + 8y = 16$?
(A) 0 (B) 1 (C) 2 (D) ∞

5. How many pairs (x, y) satisfy the equations $3x + 9y = 21$ and $6x + 18y = 45$?
(A) 0 (B) 1 (C) 2 (D) ∞

6. How many pairs (x, y) satisfy the equations $3x + 5y = 21$ and $4x + 7y = 29$?
(A) 0 (B) 1 (C) 2 (D) ∞

7. If $8x + y = 10$ and $4x + 2y = 13$, then the respective values of x and y are
(A) 3, 4/3 (B) 2/3, 4/3
(C) 3/4, 2 (D) 7/12, 16/3

8. Find the value of x, if $3x + 9y + 12z = 18$ and $2x + 3y + 4z = 8$.
(A) 2 (B) 3 (C) 1 (D) 4

9. Five puffs and two samosas cost ₹48. The cost of a puff is twice the cost of a samosa. Find the cost of a puff (in ₹).

10. Four pens and five erasers cost ₹32. Five pens and four erasers cost ₹31. Find the cost of each pen (in ₹).
(A) 2 (B) 3 (C) 4 (D) 5

11. Four pens and three erasers cost ₹17. Four erasers and 3 pens cost ₹18. Find the total cost of a pen and a eraser.

12. The cost of ten mangoes and nine apples is ₹104. The cost of twenty-seven mangoes and twenty-five apples is ₹285. What is the cost of three mangoes and two apples?
(A) ₹24 (B) ₹30 (C) ₹27 (D) ₹32

13. Two chocolates, three milk shakes and four cakes cost ₹190. Four chocolates and eight cakes cost ₹320. Find the cost of a milkshake. (in ₹).
(A) 10 (B) 20
(C) 30 (D) Cannot be determined

14. A two digit number has a tens digit of T and a units digit of U. The digit 4 is placed immediately to the right of U forming a three digit number. The new number is equal to _____.
(A) 10T + U + 4 (B) 100T + 10U + 4
(C) 1000T + 10U + 4 (D) T + U + 4

15. A two-digit number is such that twice the tens digit added to eleven times the units digit is equal to the number itself. Find the number.

16. The digits of a two digit number differ by 3. Find the difference of the number and the number formed by reversing its digits.
(A) 18 (B) 27 (C) 36 (D) 45

17. The difference between a three-digit number and the number formed by reversing its digits is 198. Find the difference of its first and last digits.

18. The greatest number that always divides the difference between a three digit number and the number formed by reversing its digits is

19. A fraction is such that the numerator is five less than the denominator. Also four times the numerator is one more than the denominator. Find the fraction.
(A) 4/9 (B) 3/8 (C) 2/7 (D) 7/12

20. The denominator of a fraction is two more than its numerator. If one is added to the denominator, the fraction becomes 3/4. Find the fraction.
(A) 7/9 (B) 5/7
(C) 11/13 (D) 9/11

21. Three consecutive even integers are such that one-third of the second number is equal to one-fourth of the third number. Find the three numbers.
(A) 4, 6, 8 (B) 8, 10, 12
(C) 12, 14, 16 (D) 2, 4, 6

22. Nalini has an amount of ₹20 in coins of denominations of 50 paise and ₹1. If she has a total of 30 coins with her, how many ₹1 coins does she have?

23. Shankar has an amount of ₹7,000 in the denominations ₹500 and ₹100. How many ₹500 notes does he have, if he has a total of 22 notes with him?

24. Amar, Bhavan, Chetan and Dinesh have a total of ₹150 with them. Amar has one-fourth of the total amount with the others. Find the amount with Amar (in ₹).
(A) 20 (B) 25 (C) 30 (D) 37.5

25. P, Q and R had a total of ₹1250 among them. P had three-seventh of the total share with Q and R. Find the amount that P had (in ₹).
(A) 300 (B) 450 (C) 525 (D) 375

26. Twenty years from now, Sreedhar will be six times as old as he was twenty years ago. What is the present age of Sreedhar?
(A) 20 years (B) 24 years
(C) 28 years (D) 32 years

27. Ramesh is thrice as old as Suresh. Two years hence, Ramesh will be twice as old as Suresh. Find Ramesh's present age (in years).
(A) 2 (B) 3 (C) 4 (D) 6

28. Four years ago, Ashok's age, was four times that of Bala. Three years hence, Ashok's age will be thrice the age of Bala. Find Ashok's present age. (in years).
(A) 56 (B) 63 (C) 60 (D) 66

29. Mrudhula gave birth to twins, when she was twenty years old. After x years, the sum of the ages of Mrudhula and her twin children is 50 years. What is the value of x?

30. Twenty seven years ago, a man's age was half his wife's present age. Sum of their present ages is 90 years. What are the respective present ages (in years) of the man and his wife?
(A) 48, 42 (B) 52, 38
(C) 54, 36 (D) 46, 44

Exercise – 1(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Solve
$7(x + 2) + 3(y - 2) = 8$
$3(x - 2) + 7(y + 2) = 8$

(A) $x = 2, y = 0$ (B) $x = 0, y = 2$
(C) $x = 0, y = 0$ (D) $x = -2, y = 2$

2. $2x + 3y - 6z = 18$
$2y - \frac{4}{3}x + 4z = 12$
Solve for y.

3. If $x + 2y + 3z = 14$,
$2x + 3y + z = 11$ and
$3x + y + 2z = 11$,
find $(x + y + z)$.
(A) 6 (B) 5 (C) 4 (D) 0

4. If $3x + y - 3z = 11$ and $2x - 2z + 5y = 29$ what is the value of $x + y - z$?
(A) 7 (B) 14 (C) 9 (D) 8

5. What is the value of p, if the equations are consistent and $x \neq 0 \neq y$?
$5x + 4y = 32$
$2py + 15x = 96$
(A) 5 (B) 6 (C) 4 (D) 3

6. For which of the following values of k will $3x + (k + 3)y = 1$ and $kx + 6y = 4$ have a unique solution?
(A) 3
(B) -6
(C) 6
(D) Any value except 3 and -6

7. Six times the denominator of a fraction is two less than eight times the numerator and ten times the numerator equals 20 less than ten times the denominator. The reciprocal of the fraction is
(A) 6/7 (B) 3/8 (C) 7/9 (D) 9/7

8. John covers 10 km per hour more than Peter while driving. On doubling his speed, Peter covers 15 km per hour more than John who is driving at his normal speed. What is John's speed? (in km/hr)

9. Classes A and B have 35 students each. If seven girls shift from class A to class B, then the number of girls in the classes would interchange. If four girls shift from class B to class A, then the number of girls in class A would be twice the original number of girls in class B. What is the number of boys in Class A and in Class B?
(A) 18 and 11
(B) 24 and 17
(C) 18 and 27
(D) 17 and 24

10. Nitya and Satya have some marbles with them. Nitya says to Satya, "If you give one marble to me, we will have equal number of marbles. Satya says to Nitya, "If you give me one marble, I will have twice the number of marbles you have". How many marbles do Nitya and Satya have respectively?
(A) 4, 6 (B) 5, 7 (C) 6, 4 (D) 7, 5

11. Gopi, Murthy and Hari had some amount of money. Gopi gives half his amount to Murthy, who then gives half of what he now has to Hari. Hari gives half of what he now has to Gopi, who, now has exactly what he started with. If the sum of Murthy's initial amount and twice Hari's initial amount is ₹45, what was the amount (in rupees) Gopi started with?

12. A shopkeeper has a few 100 gm weights and a few 500 gm weights. He can weigh a maximum of 8 kg in one weighing. If he has 20 pieces of weights, what is the maximum weight that he can weigh with only 100 gm weights?
(A) 800 gm (B) 600 gm
(C) 450 gm (D) 500 gm

13. A child went to a shop to buy a pen, a pencil and a ruler where costs are integral values (in ₹) and are in decreasing order. Each item costs at least ₹4. The total cost is ₹15 and the cost of a pencil is ₹5. How many pencils can he purchase with the amount required to purchase ten rulers?

14. A housewife, with a given amount, can buy either 10 apples or 15 oranges or 2 watermelons. Find the maximum number of oranges which she can buy with six times the initial amount such that she gets each of the three varieties of fruits.
(A) 75 (B) 81 (C) 60 (D) 72

15. Ten years ago, the age of a man was 35 years more than twice his son's age. After how many years from now will the man be
(i) twice his son's age?
(A) 15 (B) 20
(C) 25 (D) Cannot be determined

(ii) thrice his son's age?
(A) 10 (B) 15
(C) 20 (D) Cannot be determined

16. Runs scored by Bhangar in a match are 10 more than the balls faced by Karthik. The number of balls faced by Bhangar is 5 more than the number of runs scored by Karthik. Together they have scored 50 runs and Bhangar has faced 15 balls less than Karthik. What is the number of runs scored by Bhangar?

17. With the data from the above question what is the number of runs scored by Karthik per ball?
(A) 2.67 (B) 2.33 (C) 0.5 (D) 0.33

18. An amount of ₹9000 is divided among four people A, B, C and D. The sum of the shares of A, C and D is four times the share of B. The sum of the shares of B and D is equal to four-fifths the sum of shares of A and C. Find the share of D.
(A) ₹1,800 (B) ₹2,400
(C) ₹2,200 (D) Cannot be determined

19. P, Q and R completed a certain project working together. The number of hours for which P worked on the project was $\frac{2}{3}^{rd}$ of that for which Q worked on the project, which in turn was $\frac{2}{5}^{th}$ of that for which R worked on the project. The number of hours for which R worked is what fraction of the total number of hours for which all three worked?
(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{7}{10}$ (D) $\frac{1}{2}$

20. The cost of four apples, six bananas and eight oranges is p. The cost of five apples, eight bananas and eleven oranges is q. The cost of eight apples, fourteen bananas and twenty oranges is r. Which of the following is always true?
(A) $3p + 4q = \frac{3r}{4}$ (B) $-3p + 4q = \frac{r}{2}$
(C) $-4q + 3p = \frac{r}{2}$ (D) $-3p + 4q = r$

21. While adding ten two-digit numbers the digits of one of the numbers were interchanged. As a result the sum of all the ten numbers increased by a value which was four less than that number. Three times the sum of the digits of that original number is ten less than the number. What is the product of the digits of that number?
(A) 56 (B) 18 (C) 36 (D) 42

Directions for questions 22 and 23: Answer these questions based on the information below.

A shopkeeper sold a certain number of toys. The number of toys as well as the price of each toy (in ₹) was a two digit number. By mistake, he reversed the digits of both the number of toys he sold and the price of each toy. As a result, he found that his stock account at the end of the day showed 81 items more than it actually was.

22. Find the actual number of toys sold.
(A) 92 (B) 81 (C) 90 (D) 29

23. If the faulty calculations show a total sale of ₹882, find the actual selling price of each toy (in ₹).
(A) 89 (B) 98 (C) 97 (D) 79

24. There are ten children (aged 1 to 10 years) who have equal amounts of money. In the first transaction the eldest child gives one rupee to every child younger to him. In the second transaction, the second eldest child gives one rupee to every child younger to him. This type of distribution continues for the next two transactions. In the end the total sum with the children who have given money to other children is half the sum of money with the children who did not give any money. What was the original amount with each child?
(A) ₹36 (B) ₹48 (C) ₹50 (D) ₹32

25. A three-digit number is eleven times the two-digit number formed by using the hundreds and the units digit of the three-digit number respectively, in the tens and units place of the two-digit number. If the difference between the digit in tens place and the digit in hundreds place is 1, then what is the digit in the units place?

[ ]

26. In addition to the data in the above question, if the difference between the digit in tens place and that in the units place of the three-digit number is also 1, then what is the number?

[ ]

27. A grocer uses a weighing balance in which one pan weighs 0.5 kg and the other 0.75 kg. He puts a certain quantity of food grains in 0.5 kg pan and finds the weight (in kg) as a two-digit number. However, as the customer insists, he puts it in 0.75 kg pan. Now the indicated weight is 9.5 kg more than the weight which is obtained by reversing the digits of previous weight. Which of the following cannot be the actual weight (in kg) of the food grains?
(A) 43.25
(B) 36.25
(C) 41.5
(D) More than one of the above

28. Using the data of the above question, which of the following can be the weight as indicated in the first case?
(A) 65 kg (B) 36 kg (C) 64 kg (D) 85 kg

29. In t minutes from now, the time will be noon. 44 minutes ago, the time was 7t minutes past 9 a.m. Find the present time. :

[ ] : [ ] a.m.

30. In an exam, there are 35 questions. Each correct answer, each wrong answer and each unanswered question carries 5 points, 0 points and 1 point respectively. Had each correct answer, each wrong answer and each unanswered question carried 4, – 1 and 0 points respectively, the number of points a student would score in the exam is 25. Find the actual number of points that he would score in the exam.

[ ]

Exercise – 1(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. (a) Solve

$$x+\frac{x}{2}+\frac{x}{3}=\frac{x}{4}+5$$

(A) 60/19 (B) 4 (C) 12/5 (D) 19/60

(b) Solve
99x + 101y = 400
99y + 101x = 400

(A) x = 2, y = 1 (B) x = 1, y = 2
(C) x = 1, y = 1 (D) x = 2, y = 2

(c) $\frac{6}{(2x-y)}+\frac{7}{(3y-x)}=7$

$\frac{4}{(2x-y)}+\frac{14}{(3y-x)}=6$

(A) x = 2, y = 3 (B) x = 3, y = 2
(C) x = 4, y = 7 (D) x = 7, y = 13

2. Divide 1 kg weight into two parts such that the sum of the parts is 5/4th the difference.
(A) 550 gm, 450 gm
(B) 200 gm, 800 gm
(C) 900 gm, 100 gm
(D) 400 gm, 600 gm

3. P, Q and R are successive even natural numbers in ascending order. Five times R is eight more than seven times P. Find Q.

4. A is greater than B by 1/3rd the sum of A and B. If B is increased by 40, it becomes greater than twice A by 10. Find A, B.
(A) 30, 20 (B) 60, 30
(C) 20, 10 (D) 20, 40

5. In three hours Meena makes one basket more than what Reena makes in two hours. In five hours, Meena makes one basket less than what Reena makes in four hours. How many baskets can Meena make in an hour?
(A) 4 (B) 2 (C) 3 (D) 6

6. Ajay was asked to find (2/9)th of a number. He instead multiplied the number by (9/2) and obtained an answer which was 4235 more than the correct answer. Find the number.

7. Ashok and Balu have some coins. If Ashok gives 20 coins to Balu, he would have thrice as many coins as Balu. If Ashok gives 30 coins to Balu, he would have twice as many coins as Balu. Find the number of coins with Balu.
(A) 8 (B) 10 (C) 12 (D) 14

8. Ashok has a total of 30 notes in denominations of ₹20 and ₹5. The total value of the notes with him is ₹300. Find the number of ₹20 notes with him.
(A) 5 (B) 10 (C) 8 (D) 6

9. The cost of printing the first fifteen hundred copies of a book, is ₹1500. It costs y rupees to print each subsequent copy. The cost of printing the first 7500 copies of the book is ₹7080. Find y.

10. The cost of four pens, five erasers and six sharpeners is ₹47. The cost of six pens, seven erasers and nine sharpeners is ₹69. Find the cost of an eraser (in ₹).
(A) 3 (B) 6 (C) 5 (D) 7

11. The cost of five pens, eight erasers and eleven sharpeners is ₹54. The cost of three pens, five erasers and seven sharpeners is ₹34. Find the cost of one pen, one eraser and one sharpener (in ₹).

12. Cost of two pens, five pencils and seven erasers is ₹37. Cost of seven pens, one eraser and two pencils is ₹49. What is the cost of nine pencils and forty seven pens?
(A) ₹184 (B) ₹276
(C) ₹284 (D) None of these

13. The cost of two pens, one eraser and three sharpeners is ₹23. The cost of six pens, three erasers and one sharpener is ₹45. The cost of fourteen pens, seven erasers and twenty one sharpeners is ₹161. Find the cost of each pen (in ₹).
(A) 3 (B) 4
(C) 5 (D) Cannot be determined

14. Anand bought a total of 30 white and brown pens for a total of ₹32. The cost of each white pen is 70 paise more than the cost of each brown pen. Which of the following represents a possible value of the cost of each brown pen (in paise)?
(A) 40 (B) 35 (C) 45 (D) 60

Directions for questions 15 to 17: Answer these questions based on the information below.

Rakesh went to a stationery shop to purchase a total of 38 pens, erasers and sharpeners. He purchased at least 11 items of each. He purchased more sharpeners than erasers and more erasers than pens.

15. How many pens did he purchase?
(A) 11 (B) 12 (C) 13 (D) 14

16. If each pen cost ₹2, each eraser cost ₹3 and each sharpener cost ₹4, find the minimum expenditure be could have incurred on the items (in ₹).
(A) 116 (B) 118 (C) 117 (D) 119

17. If in the previous question the condition that he purchased more sharpeners than erasers and more erasers than pens is removed, find the minimum expenditure he could have incurred on the items (in ₹).
(A) 108 (B) 109 (C) 110 (D) 111

18. An amount of ₹5,600 is divided among A, B and C. The sum of the shares of B and C is equal to thrice the share of A. The sum of the shares of A and C is equal to nine-fifths the share of B. What is the share of C?
(A) ₹1,400 (B) ₹2,400 (C) ₹2,200 (D) ₹2,000

19. Aswin went on a tour to three cities. In each city, he spent ₹2 more than half of the amount he had with him when he arrived in the city. At the end of the tour he had ₹150. Find the amount that he started with (in ₹).

20. If both the numerator and denominator of a fraction are increased by one, the fraction formed is 2/3. If both the numerator and denominator are decreased by two, the fraction formed is 3/5. What is the fraction formed, if both numerator and denominator are increased by four?
(A) 7/10 (B) 9/11 (C) 5/7 (D) 3/4

21. Find the value of k if the equations $3x + (k/3+ 2)y = 1$ and $kx + 2ky = 4$ have infinite solutions.

22. Four times the sum of the digits of a two-digit number is 18 less than the number and is also 9 less than the number formed by reversing its digits. Find the product of its digits.
(A) 12 (B) 20 (C) 30 (D) 42

23. The sum of a two-digit number and its reverse is 99. How many two-digit numbers satisfy this condition?

24. The sum of the digits of a three-digit number is eight. The middle digit is thrice the sum of the other two digits. The difference between the number and the number obtained by reversing the order of the digits is zero. What is the number?
(A) 161 (B) 242 (C) 192 (D) 125

25. Five three-digit numbers including N, were to be added. While adding, the reverse of N was added by mistake instead of N. Hence, the sum increased by 11 times the sum of the digits of N. Eight times the difference of N's units and hundreds digits is 6 more than twice its hundreds digit. Find its tens digit.
(A) 4 (B) 6 (C) 8 (D) 2

26. In a four-digit number, the sum of the middle two digits is twice the units digit. The sum of the hundreds digit and six times the thousands digit is twice the sum of the other two digits. The sum of the units digit and five times the thousands digit is twice the hundreds digit. How many values can the four-digit number assume?

27. A test had 200 questions. Each correct answer carried 2 marks. Each wrong answer carried – 1/2 mark and unanswered questions carried no mark. Ajay attempted all the questions in the test and scored 360 marks. What would his marks be, if for each correct answer he got only 1/2 mark and for each wrong answer he lost 2 marks?
(A) 80 (B) 60 (C) 40 (D) 100

28. Ten children are standing in a line. Each child has some chocolates with him. If the first child attempted to double the number of chocolates with each of the others he would fall short by two chocolates. If the second child took two chocolates from each of the remaining, he would have three chocolates less than what the first child initially had. Find the total number of chocolates with the third to the tenth child.

29. Ashok went to a casino to play a card game. In each round he happened to double the amount with himself and in each round he gave ₹x to his friend. At the end of three rounds he was left with no money. If the amount he gave to his friend in each round was ₹20 more than the amount he started with, find the amount (in ₹) that he started with.
(A) 110 (B) 120 (C) 130 (D) 140

30. Six years ago, Ram's age was four times Shyam's age. Six years hence, Ram's age will be thrice Shyam's age. After how many years from now will their combined age be 150 years?
(A) 21 (B) 9 (C) 36 (D) 18

31. Ashok told his son, "My age x years ago was twice your age. My age 5x years ago was thrice your age." If the difference of their present ages is 24 years, find the sum of their present ages (in years).
(A) 91 (B) 65 (C) 78 (D) 52

32. The sum of two numbers is 4 and the reciprocal of one exceeds the reciprocal of the other by twice the product of their reciprocals. What is the product of the reciprocals of the two numbers?
(A) 3 (B) 2 (C) 1/3 (D) 1/2

33. The sum of two numbers is 250. The difference of their squares is 12500. Find the larger number.
(A) 130 (B) 140
(C) 150 (D) 160

34. Dinesh, Eswar, Ganesh and Harish had a total of 120 marbles. The number of marbles with Dinesh was half the total number of marbles with the others. The number of marbles with Eswar was one-third the total number of marbles with the others. The number of marbles with Ganesh one fourth the total number of marbles with the others. Find the number of marbles with Harish.

35. Amar, Bhavan, Chetan and Dinesh each had some money. Dinesh doubled the amounts with the others. Chetan then doubled the amounts with the others. Bhavan then doubled the amounts with the others. Amar then doubled the amounts with the others. At this stage, each of them has ₹80. Find the initial amount with Chetan (in ₹).
(A) 45 (B) 65 (C) 95 (D) 85

Directions for questions 36 to 40: Each question is followed by two statements I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone
Mark (D) if the question cannot be answered even using I and II together.

36. Find the value of 2x + 3y.
I. $\frac{x}{2} + \frac{3y}{4} = 1$.
II. 9y + 6x = 12.

37. What is the difference between the costs of 12 shirts and 36 pairs of pants?
I. Total cost of 12 shirts and 36 pairs of pants is ₹1200.
II. If the cost of each shirt is doubled, 12 shirts will cost the same as that of 72 pairs of pants.

38. How many questions did Aarti attempt in the Data Sufficiency test?
I. There were 25 questions in the test.
II. Aarti got 21 marks in the test, in which every correct answer fetched her 1 mark and for every incorrect answer 1/3 mark was deducted from the total.

39. Is the age of a person equal to the sum of the ages of his son and wife?
I. The person is two years older than his wife.
II. The age of his son is twelve years, who was born when his wife was 30 years old.

40. Which car gives more mileage, Uno or Zen?
I. Uno gives a mileage of 45 miles per 10 litres of petrol, which is three-fourths of the mileage of Alto.
II. The mileage of Palio is four-fifths of the mileage of Alto, which is two miles less than that of Zen.

Key

Concept Review Questions

1. 9	7. D	13. A	19. C	25. D
2. C	8. A	14. B	20. D	26. C
3. A	9. 8	15. 54	21. A	27. D
4. D	10. B	16. B	22. 10	28. C
5. A	11. 5	17. 2	23. 12	29. 10
6. B	12. C	18. 99	24. C	30. A

Exercise – 1(a)

1. C	9. D	16. 40	24. A
2. 6	10. B	17. D	25. 1
3. A	11. 30	18. C	26. 121
4. A	12. D	19. B	27. D
5. B	13. 8	20. D	28. A
6. D	14. B	21. C	29. 11, 43
7. D	15. (i) C	22. C	30. 60
8. 35	(ii) D	23. A	

Exercise – 1(b)

1. (a) A	6. 990	13. D	20. C	27. B	34. 26
(b) D	7. B	14. D	21. 12	28. 23	35. D
(c) A	8. B	15. A	22. B	29. D	36. B
2. C	9. 0.93	16. C	23. 9	30. B	37. A
3. 8	10. A	17. B	24. A	31. C	38. D
4. C	11. 6	18. C	25. B	32. C	39. C
5. C	12. D	19. 1228	26. 2	33. C	40. C