Solutions for SM1001910

Chapter – 1
(Special equations)

Concept Review Questions

Solutions for questions 1 to 15:

1. (3, 1) satisfies the equation 7x + 2y = 23 and (1, 8) also satisfies the equation
$\therefore$ Both (A) and (C) are solutions of the equation
Choice (D)

2. Given 3x + 7y = 84 is $y = \frac{84-3x}{7}$ when x = 0, y = 12 the remaining values of x can be obtained by adding 7 to the first x value i.e. 0
$\therefore$ The possible values of x and corresponding values of y are listed below
x = 0, 7, 14, 21, 28
y = 1, 9, 6, 3, 0 $\therefore$ The number of possible values of (x, y) is five
Ans: (5)

3. Given 5x + 4y = 150
$x = \frac{150-4y}{5}$ $x \in z$ only when y is a multiple of 5
$\therefore$ y values are multiples of 5
Choice (D)

4. Given 7x + 4y = 102
$y = \frac{102-7x}{4}$
Possible values of x are 2, 6, 10, 14
Corresponding values of y are 22, 15, 8, 1
$\therefore$ The number of solutions is 4
Ans: (4)

5. Given 31p - 11q = 187
$p = \frac{187+11q}{31}$ When q = 14, p = 11
$\therefore$ The possible values of q and corresponding values of p are listed below
q = 14, 45, 76, - - - - (obtained by adding 31 to each)
p = 11, 22, 33, - - - - (obtained by adding 11 to each)
Choice (C)

6. Given remainder of $\left(\frac{9Q}{11}\right) = 6$
When Q = 8 remainder is 6 and the remaining values of Q can be obtained by adding 11 to 8 and so on.
Choice (D)

7. 8a + 13b = 452
$a = \frac{452-13b}{8}$ b must be an even number
The possible values of b are 4, 12, 20, 28 when b > 28 then a < 0 corresponding values of a are 50, 37, 24, 11
$\therefore$ Number of solutions is 4
Choice (A)

8. Given Remainder $\left(\frac{13x}{24}\right) = 13$

Rem $\left(\frac{13}{24}\right)$. Rem $\left(\frac{x}{24}\right) = 13$

$\Rightarrow$ 13. Rem $\left(\frac{x}{24}\right) = 13$

$\Rightarrow$ Rem $\left(\frac{x}{24}\right) = 1.$

$\therefore$ x = 24 k + 1 , $k \in z$.
Choice (C)

9. Let the numbers of books purchased by Ramesh, of type i, ii, iii are x, y and z respectively
He spent an amount of ₹34 on them
$\Rightarrow$ 8x + 4y + 2z = 34
We purchased at least one of each variety
I.e. 8 + 4 + 2 = ₹14
Amount left = 20
the total number of books becomes maximum
If he spent ₹20 on purchasing variety iii books he will get 10 books
$\therefore$ Maximum number of books he can purchase = 10 + 3 = 13
Ans: (13)

10. Let a, b, c be the number of apples purchased of varieties X, Y and Z respectively. Total amount spent = ₹74
$\therefore$ 11a + 10b + 5c = 74
Only if a = 4, the amount spent on purchasing Y and Z varieties can be a multiple of 5
$\therefore$The maximum number of apples of variety X he could have bought is 4. The greatest (or the least) number in a single ton set is the number itself.
Ans: (4)

11. Let the parts into which 38 is divided be 8a, 7b, 3c (where a, b, c are positive integers)
$\Rightarrow$ 8a + 7b + 3c = 38
If a = 1 $\Rightarrow$ 7b + 3c = 30 $\Rightarrow$ $b = \frac{30-3c}{7}$
When c = 3, b = 3 and when c = 10, b = 0 we will not consider this
If a = 2, 7b + 3c = 22
$\Rightarrow$ If b = 4, c < 0.
$\therefore$ There are 2 possibilities (a, b, c) = (1, 3, 3) or (2, 1, 5)
Choice (B)

12. Given 17A = 19B
$\Rightarrow A = \frac{19}{17} B$
Obviously B is a multiple of 17. The possible values are listed below

A	B	C
19	17	144
38	34	108
57	51	72
76	68	36

When b > 85 the A + B > 180 not possible
$\therefore$ Number of triangles possible is 4
Choice (A)

13. Let x, y, z be the number of 8, 5 and 3 marks questions Kumar answered respectively to secure 53 marks
$\Rightarrow$ 8x + 5y + 3z = 53
If x = y, 13x + 3z = 53
$\therefore$ (x, z) = (2, 9)
If x = z, 11x +5y = 53
$\therefore$ (x, y) = (3, 4)
$\therefore$(x, y, z) = (2, 2, 9) or (3, 4, 3).
But as x + y + z = 10
(x, y, z) = (3, 4, 3). He answered 4, 5 mark questions.
Choice (D)

14. Let the number of persons in the group be x
And the weight of the person who joined the group be w kg
Average of x boys = 42kg
When two boys with weights 38 kg and 43kg left and the boy with weight w kg joined the group there is no change in the average
i. e $\frac{42x-38-43+w}{x-1} = 42$
$\Rightarrow$ 42x – 81 + w = 42x – 42 $\therefore$ w = 39 kg
Ans: (39)

15. Let date of birth be D and month of birth be M
Given 13D + 21M = 441
Dividing by 13 both sides we have

$D + M + \frac{8M}{13} = \frac{441}{13} = 33 + \frac{12}{13}$

$\frac{8M-12}{13}$ = K (K = 33 – D – M is an integer)

$M = \frac{13k+12}{8}$

When k = 4 then $M = \frac{64}{8} = 8$ is the only value that satisfies $M \le 12$

$\therefore$ Month of birth = 8 = August Choice (B)

Exercise – 1(a)

Solutions for questions 1 to 11:

1. Suppose D is the date and M is the month of my birth. Clearly D and M are positive integers with $D \le 31$ and $M \le 12$. Given that 12D + 31M = 531. The equation can be solved in two different ways.
(1) By the method based essentially on numbers-based reasoning.
(2) By the method discussed in the introduction of the chapter.

Method 1:

Consider the equation 12D + 31M = 531
Since L.H.S. = R.H.S., the remainder when either side of the equation is divided by any number should be the same. We divide the equation by the least coefficient i.e., 12.

$\text{Rem}\left(\frac{12D+31M}{12}\right) = \text{Rem}\left(\frac{531}{12}\right)$

$\Rightarrow \text{Rem}\left(\frac{7M}{12}\right) = 3$

By inspection, we obtain the least possible positive integral value of M as 9. The subsequent values of M are obtained by adding multiples of 12 to 9 viz., 21, 33, …. etc.
But these values can be ignored as M is the month of a year. Hence M = 9. Hence I am born in September.

Method 2:

Divide the equation 12D + 31M = 531 by the least coefficient i.e.12, and collect all fractions on left and all integers on right. We get,

$\frac{7M}{12} - \frac{3}{12} = 44 - D - 2M.$

Denote the R.H.S. by k, where k is an integer. This gives

$\frac{7M-3}{12} = k$ i.e., $M = \frac{12k+3}{7}$

By trial and error we get k = 5 which gives M = 9. Now, for the next possible value of k i.e., 12, we get M = 21. Since M denotes the month of a year, M has to be 9. Hence I was born in September. Choice (C)

2. The equation 3x + 11y = k has exactly 3 solutions in which both x and y are positive integers. The min values of x are 1, 12, 23 and the corresponding min values of y are 7, 4, 1 respectively. These values give k = 3(1) + 11(7) = 80.
We can increase each value in the x – triplet successively by 1 and get other values of k. they are 83, 86, 89, 92, 95 and 198. ($\because$ k is a 2 digit no). if we increase each value in the y – triplet by 1. we get k = 3x + 11y = 3(1) + 11(8) = 91.
The values of x and the corresponding values of k are tabulated below for the two possible sets of values of y.

x	y	K
1, 2, 3, 4, 5, 6, 7	7	
12, 13, 14, 15, 16, 17, 18	4	80, 83, 86, 89, 92, 95, 98
23, 24, 25, 26, 27, 28, 29	1	
1, 2, 3	8	
12, 13, 14	5	91, 94, 97
23, 24, 25	2	

We can now see all the possible values of x and y which satisfy the two conditions – that there are exactly 3 positive roots for (x, y) and that k < 100. Among the choices, x can't be 19. Choice (D)

3. Let the number of days on which Kishan made a non-defective pot be x.
Let the number of days on which Kishan made a defective pot be y.
Then we have 80x – 18y = 1518 i.e. 40x – 9y = 759
We divide the equation by the least coefficient i.e., 9.

$\Rightarrow \text{Rem}\left(\frac{40x-9y}{9}\right) = \text{Rem}\left(\frac{759}{9}\right)$

$\Rightarrow \text{Rem}\left(\frac{4x}{9}\right) = 3$

By inspection we obtain the least possible positive integral value of x as 3.
Hence, x = 3, 12, 21, 30, ………
For x = 3 and x = 12, the corresponding values of y are negative. x = 21 gives y = 9 while for x = 30, y = 49. As y is the number of days in a month its value cannot exceed 31.
Hence, only one solution is possible i.e., x = 21, y = 9.
$\therefore$The number of defective pots made = 9
Ans: (9)

4. Let the number of marbles with Hary and Lary be H and L respectively and let x be the number of marbles exchanged. Now,
H + x = 8 (L – x) $\Rightarrow$ 8L – H = 9x
H – x = 2 (L + x) $\Rightarrow$ H – 2L = 3x
Eliminating x, we get, $H = \frac{7L}{2}$
Clearly L has to be a multiple of 2. As the number of marbles between them is more than 40 and less than 50, the only possible value of L is 10 and corresponding value of H is 35. Hence Hary has 25 marbles more than Lary. Choice (B)

5. Total weight of the present group = 64n kg.
Let the weight of the person who leaves be (50 + x) kg where $0 \le x \le 10$.
New total weight after 2 persons join and 1 person leaves is [64n + (68 + 67) – (50 + x)] kg.
As the average weight goes up by 2 kgs, new average weight = 64 + 2 = 66 kg.

$\therefore$ New average = $\frac{64n+85-x}{n+1} = 66$

$\Rightarrow$ 2n + x = 19
Given n is a multiple of 5
n = 5
$\Rightarrow$ x = 9 and n = 10
$\Rightarrow$ x = –1 (not possible)
$\therefore$ Weight of the man who left the group is 50 + 9 = 59 kg.
Ans: (59)

6. Let c, r and m be the number of cords, resistors and microchips purchased.
c + r + m = 43
10c + 5r + 2m = 229

Eliminating m from the equations, we get 8c + 3r = 143
Divide the equation by the least coefficient i.e.,3

$\Rightarrow$ Rem $\left(\frac{8c+3r}{3}\right)$ = Rem $\left(\frac{143}{3}\right)$

$\Rightarrow$ Rem $\left(\frac{2c}{3}\right)=2$

By inspection, the least possible integral value of c = 1
The subsequent values being 4, 7, 10, 13, 16, 19, …..
Given c > r and as 8c + 3r = 143.
We have 11c > 143 or c > 13.
Hence we consider c = 16, 19, ….. for possible values of c.
c = 16 gives r = 5 and m = 22
c = 19 gives negative values of r.
Hence Pasha bought 22 microchips Choice (A)

7. Suppose Pratyush buys f flower pots, r rockets and s sparkers. Then 8f + 6r + 4s = 64.
Since Pratyush buys a minimum of 3 of each of the items, he spends a minimum of 18 × 3 = 54/- rupees on these items. Now with ₹10 left, he has the only option of buying a rocket and a sparkler. Hence he can buy the crackers in a unique combination of 3 flower pots, 4 rockets and 4 sparklers i.e., 11 crackers totally. Ans: (11)

8. Let x, y and z be the number of paper weights of prism, aesthetic and oval varieties purchased by Tina.
Then 60x + 72y + 15z = 336. Clearly y < 4, for if y = 4, 60x + 15z = 48 which is not possible, since Tina has to buy at least one paper weight of each variety.
When y = 3, 60x + 15z = 120 which in turn gives x = 1, z = 4. y = 1 and y = 2 are not possible since in these cases 60x + 15y whose value should be a multiple of 15 is 264 and 192 respectively.
Hence Tina purchased 1, 3 and 4 of prism, aesthetic and oval varieties respectively i.e., a total of 8 paper weights. Choice (C)

9. Let x, y and z be the number of employees in groups A, B and C respectively.
Hence x + y + z = 12 → (1)
and 9000x + 8000y + 4000z = 79,000
$\Rightarrow$ 9x + 8y + 4z = 79 → (2)
Eliminating z from (1) and (2) we get, 5x + 4y = 31
Divide the equation by the least coefficient i.e.,4

$\Rightarrow$ Rem $\left(\frac{5x+4y}{4}\right)$ = Rem $\left(\frac{31}{4}\right)$

$\Rightarrow$ Rem $\left(\frac{x}{4}\right)=3$

By inspection, the least possible positive integral value of x = 3.
Hence, x = 3, 7, 11, 15, …..
But x cannot exceed $\left[\frac{31}{5}\right]=6$, as in the event of which y would take negative values.
∴The only possible combination is x = 3, y = 4 and z = 5.
Hence the number of employees in group B is 4.
Ans: (4)

10. Let A, B, C be the angles in the triangle. We have 15A = 17B $\Rightarrow$ A = $\frac{17B}{15}$
As the angles are all whole numbers, B is a multiple of 15 and accordingly A is a multiple of 17. Further, each of the angles is acute. Hence

A	B	C	Possibility
17	15	>90	×
34	30	>90	×
51	45	84	✓
68	60	52	✓
85	75	20	✓

The largest possible angle is 85° Choice (C)

11. 10A + 2B + C = 100
Further C = 4B
∴10A + 6B = 100 or 5A + 3B = 50
5A = 50 – 3B
$\Rightarrow$ B should be a multiple of 5.
For A to be maximum, B has to be minimum
∴ B = 5
Hence 5A = 35 i.e., A = 7 Ans: (7)

Solutions for questions 12 and 13:

12. Suppose Rahul purchased 'x' number of staplers and 'y' number of pens.
We have 50x + 10y = 2000 i.e., y = 200 – 5x
$\Rightarrow x \le 39$ → (1).
Now instead, if he purchased y staplers and x pens, he spends 50y + 10x. But 50y + 10x < 1000 (since he now spends less than half the amount earlier)
i.e., 5y + x < 100 $\Rightarrow$ 5(200 – 5x) + x < 100 i.e.,
x > $\frac{900}{24}$ = 37.5 $\Rightarrow x \ge 38$ → (2)
(1) and (2) together give x = 38 or 39. Hence Rahul can buy the items in two combinations. Choice (B)

13. When x = 38, y = 10
When x = 39, y = 5
Now imposing the condition that he has bought at least 10 of each variety, we conclude that Rahul purchased 38 staplers and 10 pens. Hence he purchased a total of 48 items. Choice (B)

Solutions for questions 14 and 15:

Let f, s, g be the number of French, Spanish and German magazines. Then f + s + g = 40 → (1) and
120f + 250s + 150g = 7100
$\Rightarrow$ 12f + 25s + 15g = 710 → (2)
Eliminating f from (1) and (2) we get 13s + 3g = 230
Divide the equation by the least coefficient i.e.3

$\Rightarrow$ Rem $\left(\frac{13s+3g}{3}\right)$ = Rem $\left(\frac{230}{3}\right)$

$\Rightarrow$ Rem $\left(\frac{s}{3}\right)=2$

By inspection, the least possible positive integral value of s = 2.

Hence, s = 2, 5, 8, 11, 14, 17, ……

But s cannot exceed $\left[\frac{230}{13}\right]=17$, in the event of which g would take negative values.
The values of s = 2, 5, 8 can be ignored as these values give the corresponding values of g to be greater than 40, which is the total number of magazines purchased.
Now s = 11 gives g = 35 which means that Spanish and German magazines together are 46, which is more than the number of total magazines.
s = 14 gives g = 16 and f = 10
s= 17 gives g = 3 and f = 20

14. If Pradyumna decides to buy minimum of German magazines, then he is buying 3 of them and 20 of the French magazines. Choice (A)

15. If Pradyumna decides to buy maximum number of German magazines, he then buys 14 Spanish magazines.
Choice (C)

Solutions for questions 16 and 17:

G + K + W = 27 → (1) and
15000G +20000K + 25000W = 600000
$\Rightarrow$ 3G + 4K + 5W = 120 → (2)
Eliminating G from (1) and (2) we get K + 2W = 39
$\Rightarrow$ K = 39 – 2W

$\therefore$ W ≤ 17 (given W > 3)
From (1) we get G = 27 – (39 – 2W) – W
$\Rightarrow$ G = W –12
$\therefore$ W ≥ 16
Hence W = 16 or 17
When W = 17, G = 5, K = 5
This is not possible as the dealer does not have the same number of refrigerators of any two companies.
Hence W = 16 which gives G = 4 and K = 7

16. Hence Godrej and Kelvinator together are 11. Ans: (11)

17. Whirlpool is stocked in maximum number. Choice (C)

Solutions for questions 18 and 19:

Let x, y and z be the number of levels with 4 points, 2 points and 1 point respectively.
Given x + y + z = 36 → (1)
4x + 2y + z = 78 → (2)
Eliminating z from (1) and (2) we get 3x + y = 42
$\Rightarrow$ y = 42 – 3x
Substituting for y in (1) we get z = 2x – 6
we are also given that
x – y ≤ 2 and z – y ≥ 7
Thus x – (42 – 3x) ≤ 2 and (2x – 6) – (42 – 3x) ≥ 7
4x ≤ 44 and 5x ≥ 55 $\Rightarrow$ x ≤ 11 and x ≥ 11
$\therefore$ x = 11. Consequently y = 9 and z = 16

18. Number of 2 point levels are 9. Ans: (9)

19. Given, Kapil completes 2 four point levels, he now has 9 of these left. Hence he now has equal number of four point levels and two point levels left. Choice (C)

Solutions for questions 20 to 30:

20. Let x be the number greater than 10.
Let y be the number less than 10.
Given xy < 100. Also (x – 4) (y + 3) = xy
$\Rightarrow$ 3x – 4y = 12
$\Rightarrow y = \frac{3x-12}{4}$
Clearly x is a multiple of 4 and greater than 10.
Hence x = 12, 16, 20,....
when x = 12, y = 8
when x = 16, y = 9. But this is not possible as xy < 100
Hence x = 12 and y = 6 is the only possibility and the difference of the numbers is 6. Ans: (6)

21. Let x be the number of bacteria of type I (which doubles every 10 seconds) and y be the number of bacteria of type II (which triples every 10 seconds). We have

	type I	type II
at the beginning	x	Y
at the end of 10 secs	2x	3y
at the end of 20 secs	4x	9y
at the end of 30 secs	8x	27y
at the end of 40 secs	16x	81y

Now 16x + 81y = 337
Clearly y ≤ 4
when y = 4, 16x = 13 not possible
when y = 3, 16x = 94 not possible
when y = 2, 16x = 175 not possible
when y = 1, 16x = 256 $\Rightarrow$ x = 16
Hence total number of bacteria at the beginning is x + y = 17. Ans: (17)

22. Let x, y and z be the one-rupee, 50 paise and 25 paise stamps with Deepthi.
Given x + y + z = 49 → (1)
$x + \frac{1}{2}y + \frac{1}{4}z = 23\frac{1}{2}$ i.e., 4x + 2y + z = 94 → (2)
Eliminating z from (1) and (2), we get 3x + y = 45.
i.e., y = 45 – 3x.
Further, given that z > x + y and x < y.
Hence 4 + 2x > x + 45 – 3x and x < 45 – 3x
i.e., 4x > 41 and 4x < 45
$\Rightarrow$ x > 10.25 and x < 11.29
$\Rightarrow$ x ≥ 11 and x ≤ 11.
Hence x = 11, y = 12 and z = 26
Hence Deepti has got 15 of 25 - paise stamps more than one-rupee stamps. Choice (A)

23. Let the number of 50-rupee notes and the 500-rupee notes that the teller intended to give be x and y. He made a mistake and gave y 50-rupee notes and x 500-rupee notes.
$\therefore$ 50y + 500x – 50 = 3(50x + 500y)
$\Rightarrow$ 350x – 1450y = 50 $\Rightarrow$ 7x – 29y = 1
Rem $\frac{29y}{7}$ = 6 $\Rightarrow$ Rem $\frac{y}{7}$ = 6
This and the other values of (x, y) are listed below. The corresponding values of 50x + 500y are also listed.

x	y	50x + 500y
25	6	4250
54	13	9200
83	20	14,150
112	27	19,100

We see that the amount on the cheque could be more than 14,000 and less than 16,000. Choice (C)

24. $\frac{7x}{12} - \frac{5y}{12} = 1$
7x – 5y = 12
By trial and error x = 6 and y = 6 satisfies the above equation.
$\therefore$ The solutions are

x	6	11	16	….
y	6	13	20	….

$\therefore$ The maximum value of x – y is 0. Choice (A)

25. $q = \frac{40+p}{p-3}$
$= \frac{43+p-3}{p-3}$
$q = \frac{43}{p-3} + 1$
Since q and p are integers, p – 3 can be 1, 43, –1 or – 43.
$\therefore$ (p, q) = (4, 44), (46, 2) (2, –42), (-40, 0)
The number of pairs is 4 Choice (D)

26. Given $\frac{1}{x}+\frac{1}{y}=\frac{1}{8}$
$\Rightarrow$ 8y + 8x = xy
$\Rightarrow$ xy – 8x – 8y = 0
$\Rightarrow$ x (y – 8) – 8 (y – 8) = 64
$\Rightarrow$ (x – 8) (y – 8) = 64
$\Rightarrow$ (x – 8) (y – 8) = 2^6
(x – 8) (y – 8) can be expressed as the product of two factors in seven ways.
$\therefore$ The number of ordered pairs that satisfy the above equation is 7. Choice (B)

27. $\frac{1}{x}+\frac{3}{y}=\frac{1}{29}$ $\Rightarrow$ 29y + 87x = xy
$\Rightarrow$ xy – 29y – 87x = 0
$\Rightarrow$ y (x – 29) – 87 (x – 29) = 87.29
$\Rightarrow$ (x – 29) (y – 87) = 3 $(29)^2$

The number of factors of 3 $(29)^2$ is 6. We can also take negative integers; so the number of ordered pairs (x, y) is 12.
But x > 0; x – 29 > – 29;
i.e., x – 29 cannot be – 29, $–(29)^2$, –(3)(29) or – $3(29)^2$
∴ The total number of solutions is 12 – 4, i.e.,8.
Choice (C)

28. $\frac{7}{x}-\frac{3}{y}=\frac{1}{4}$ ⇒ 28y – 12x = xy ------ (1)
⇒ xy + 12x – 28y = 0
⇒ x (y + 12) – 28 (y + 12) = – 28 (12)
⇒ (x – 28) (y + 12) = – 2^4 (3) (7) ------ (2)
The number of factors is 5(2) (2) i.e.,20. Since negative integers can also be considered, the total number of solutions for (2) is 40. One of these solution is x – 28 = –28, y + 12 = 12. This is not a solution of (1). But every other solution of (2) is also a solution of (1)
∴ The number of solutions of (1) is 39. Choice (C)

29. $x^2 – y^2$ = 273 ⇒ (x – y) (x +y) = (3) (7) (13)
The number of factors of (3)(7)(13) is 8. But x – y must be less than x + y. So half the values satisfy this condition. The total number of solutions is 4. Choice (D)

30. $x^2 – 4y^2$ = 980
(x + 2y) (x – 2y) = $2^2\ 7^2$ 5.
Since x and y are integers, x + 2y and x – 2y are either both even integers or both odd integers (∵ they differ by an ever number). They cannot be both odd (as the two 2's have to be factors). So both (x + 2y) and (x – 2y) are even.
i.e., Each of x + 2y and x – 2y is multiple 2. The other factors are 7^2 5. The one 5 can be assigned to either expression. The two 7's can be assigned in 3 ways (7^2, 7^0) (7, 7) or (0, 7^2). Therefore, the number of possible values of (x + 2y) and (x – 2y) is 6.
Since we can also consider negative integers, the total number of solutions for the equation is 12.
Choice (A)

Exercise – 1(b)

Solutions for questions 1 to 4:

1. Let's say Avinash purchased x pencils and y pens. According to the problem,
6x + 9y = 105
2x + 3y = 35
Dividing throughout by the least coefficient, we get y = 1 as a possible value
when y = 1, x = 16
To obtain the remaining solutions the value of y is increased successively by 2 and the value of x is successively decreased by 3.
∴ The remaining solutions are
(16, 1), (13, 3), (10, 5), (7, 7), (4, 9), (1, 11)
Since x > y, The number of solutions that satisfy the equation is 3. Choice (D)

2. Let the date of birth be x. ∴ 1 ≤ x ≤ 31.
and the month of birth be y. ∴ 1 ≤ y ≤ 12
12x + 31y = 316 (Given)
Dividing throughout by 12 (least coefficient) we get
7y = 4 + 12k
y = 4 satisfies the above equation
∴ when y = 4, x = 16
The month that I was born in is April. Choice (C)

3. Say, heads turns up x times and tails turns up y times, according to the problem
9x + 5y = 182
Dividing throughout by 5, we get
Rem $\left(\frac{9x}{5}\right)$ = Rem $\left(\frac{182}{5}\right)$ = 2
⇒ Rem $\frac{4x}{5}$ = 2
⇒ x = 3, 8, 13, 18, ...
⇒ y = 31, 22, 13, 4, ...
These values are listed in the following table.

X	3	8	13	18
Y	31	22	13	4

∴ Thus, Vijay can win the game in 4 ways. Choice (A)

4. From the table, the maximum number of times he could have tossed the coin is 34. Choice (B)

Solutions for questions 5 to 7:

5. Let the number of correct and wrong answers be x and y respectively and the number of unattempted questions be z.
∴ x + y + z = 150 → (1)
4x – 2y – z = 90 → (2)
(1) + (2) ⇒ 5x – y = 240
The values of x, y which satisfy this equation and the corresponding value of z that satisfies (1) are tabulated below.

5x – y =	240	z
5(48) – (0)		102
5(49) – (5)		96
– – –		–
5(62) – (70)		18
5(63) – (75)		12
5(64) – (80)		6
5(65) – (85)		6

We can see that (x, y, z) has 18 values and when z = 18, y = 70 and if x = y, 5x – y = 4x = 240 ⇒ x = 60 and z = 150 – 120 = 30 Ans: (18)

6. From the above table when x = 18 corresponding value of y = 70 Ans: (70)

7. From the table when x = 60, y = 60, z = 30
Choice (A)

Solutions for questions 8 to 30:

8. 5x + 7y = k
Dividing throughout by 5, we get
Rem $\left(\frac{7y}{5}\right)$ = Rem $\left(\frac{k}{5}\right)$
Rem $\left(\frac{2y}{5}\right)$ = Rem $\left(\frac{k}{5}\right)$
y = p satisfies the above equation and the corresponding value of x is q.
∴ for remaining values of y and x, y values are increased by 5 and x values are decreased by 7. This gives the following table.

x	q	q – 7	q – 14	q – 21	q – 28	q – 35	q – 42
y	p	p + 5	p + 10	p +15	p + 20	p + 25	p + 30

The number of solutions is 7.
∴ for the minimum value of k, q = 42 ; p = 0
k = 5(42) + 7 (0) = 210
The minimum value of k = 210 Choice (A)

9. Let x be the number of pencils and y be the number of pens. Then we have 4x + 7y = 115.

Method 1:

We divide the equation 4x + 7y = 115 by the least coefficient i.e.,4.

⇒ Rem $\left(\frac{4x+7y}{4}\right)$ = Rem $\left(\frac{115}{4}\right)$

⇒ Rem $\left(\frac{3y}{4}\right)$ = 3

By inspection the least possible positive integral value of y = 1.

The subsequent values of y being 5, 9, 13, 17, …..

But y cannot exceed $\left[\frac{115}{7}\right] = 16$, as in the event of which x would take negative values.

$\therefore$y = 1, 5, 9, 13. Hence there are 4 different ways in which Raghav can purchase the items.

Note: [x] indicates indicates greatest integer less than or equal to x.

Method 2:

Dividing the equation 4x + 7y = 115 with the least coefficient i.e., 4 and retaining all fractions on the left and taking all whole numbers to the right, we get

$\frac{3y}{4} - \frac{3}{4} = k$; where k is an integer.

Multiplying the equation with 3 (a number which makes the coefficient of y, 1 more than the denominator). We get

$\frac{y}{4} - \frac{1}{4} = k$ i.e., y = 4k + 1

Using this in 4x + 7y = 115, we get
x = 27 – 7k
y = 4k + 1 gives k ≥ 0 while
x = 27 – 7k gives k ≤ 3
Hence k = 0, 1, 2, 3 given that there are 4 different ways in which Raghav can purchase the items. Ans: (4)

For subsequent questions, we discuss the solution by Method 1 alone, i.e., by the method of numbers-based reasoning.

10. Let the measures of the 3 angles be x°, y°, z°,
Lets say 14x = 19y
The possible values of x, y, z are tabulated below.

x	y	z
19	14	147
38	28	114
57	42	81
76	56	48

As the triangle is acute-angled the measure of the largest possible angle is 81°. There are 2 sets of values f or {x, y, z} Ans: (81)

11. The commission on each item of the 3 types and the number of items of each kind are tabulated below.

	A	B	C
Com.	10	5	2
No	y	3x	x

$\therefore$ 2x + 5(3x) + 10y = 400 ⇒ 17x + 10y = 400

The values are listed below.

x	y
0	40
10	23
20	6

As the salesman sold at least 1 of each type, the maximum number of items of type A is 23. Choice (D)

12. The salaries of the employees (in thousands) and the number of employees in the 3 categories are tabulated below.

	P	Q	R
Sal.	13	9	6
No.	p	q	r

Given p + q + r = 18 → (1) and
13p + 9q + 6r = 127 → (2)
As we want r, we can eliminate q (or p)

(2) – 9(1) ⇒ 4p – 3r = – 35

As Rem $\left(\frac{-35}{3}\right)$ = 1 (–35 = – 36 + 1), Rem $\frac{4p}{3}$ = 1

⇒ Rem $\frac{p}{3}$ = 1.

By trial (p, r) = (1, 13)
This and the other value(s) are listed below.

p	r	q
1	13	4
4	17	-3

As p, q, r are positive r = 13 Choice (A)

13. Suppose Rakesh makes x non-defective pieces and y defective pieces.
Given,
90x – 25y = 1895
⇒ 18x – 5y = 379
Dividing the equation with 5, we get

Rem $\left(\frac{18x}{5}\right)$ = Rem $\left(\frac{379}{5}\right)$

Rem $\left(\frac{3x}{5}\right)$ = Rem $\left(\frac{4}{5}\right)$

x = 3 satisfies the above equation. The corresponding value of y is – 65. $\therefore$ The various solutions are

x	3	8	13	18	23
y	– 65	– 47	– 29	– 11	7

$\therefore$ Rakesh makes 23 good items and 7 defective items in a month. Choice (D)

14. The data (and conclusions) are tabulated below

	Vijay	Ajay
	37	11
After Ajay gives	42	6
After Vijay gives	32	16

After Ajay gives some marbles, Vijay has 7 times as many as Ajay has, i.e., the total number is a multiple of 8. The only multiple of 8 between 45 and 55 is 48. $\therefore$ Vijay has 42 and Ajay has 6. Instead, if Vijay gives, he would have twice as many as Ajay, i.e., he would have 32 and Ajay would have 16.
$\therefore$ Initially Vijay had 37, Ajay had, 11, i.e. a total of 48.
Ans: (48)

15. The cost and the number of each type of cracker is tabulated below.

	Flower pots	Rockets	Sparklers
Cost	12	9	6
No.	4 + x	4 + y	4 + z

Given 12(4 + x) + 9(4 + y) + 6(4 + z) = 123
⇒ 12x + 9y + 6z = 15
The possible value(s) are listed below.

0 1 1

$\therefore$ The total number of crackers that Ravi bought is 4 + 5 + 5 or 14. Choice (B)

16. Let 7x + 9y = 228

$\therefore$ Rem $\left(\frac{9y}{7}\right)$ = Rem $\left(\frac{228}{7}\right)$ = 4

⇒ Rem $\left(\frac{2y}{7}\right)$ = 4

y = 2 satisfies the equation and the corresponding value of x is 30. The other values of x and y are given below.

x	30	21	12	3
y	2	9	16	23

$\therefore$ The number can be divided in the required way in 4 ways.
Choice (D)

17. Let the number of 5 Ns, 10 Ns, 20 Ns coins be x, y, z respectively.
$5x + 10y + 20z = 105$
When z = 0, $5x + 10y = 105 \Rightarrow x + 2y = 21$
The number of solutions is 11
When z = 1, $5x + 10y = 85 \Rightarrow x + 2y = 17$
The number of solutions is 9.
When z = 2, $5x + 10y = 65$
The number of solutions is 7
When z = 3, and 4 and 5
The number of solutions is respectively 5, 3 and 1
$\therefore$ total number of solutions is
$11 + 9 + 7 + 5 + 3 + 1 = 36$ Choice (D)

18. The cost and the number of bars of the 3 varieties are tabulated below.

	Vanilla	Strawberry	Pineapple
Cost	10	12	15
No.	X	Y	Z

Given $x + y + z = 20 \rightarrow (1)$
and $10x + 12y + 15z = 240 \rightarrow (2)$
$(2) - 10(1) \Rightarrow 2y + 5z = 40$
As Rem $\left(\frac{40}{2}\right) = 0$, z = 0 satisfies the equation. This and the other values are given below.
$2(20) + 5(0)$
$= 2(15) + 5(2)$
$= 2(10) + 5(4)$
$= 2(5) + 5(6)$
$= 2(0) + 5(8)$
As $z > 5$ and $y \geq 1$,
(y, z) = (5, 6) and (x, y, z) = (9, 5, 6).
$\therefore$ He bought 5 strawberry bars. Choice (B)

19. Let x, y and z be the number of boxes of dust-free, low-dust and regular varieties of chalk purchased.
Then $y = 2x$ and $x \geq z + 1$. Further the cost of regular variety is ₹10, low dust is ₹20 and dust free is ₹30
Hence $30x + 20y + 10z = 470$ i.e.,
$3x + 2y + z = 47$
$\Rightarrow 7x + z = 47$ (using y = 2x) i.e., $z = 47 - 7x$
Hence $x \leq 6$
$x \geq 47 - 7x + 1$ (using $x \geq z + 1$)
i.e., $8x \geq 48$ or $x \geq 6$
which gives x = 6 and consequently z = 5 and y = 12
Hence Raman purchased 23 boxes in all. Ans: (23)

20. $5x + 8y + 13z = 72$
The possible values are listed below.

0	9	0
8	4	0
7	3	1
6	2	2
5	1	3
4	0	4

Thus 72 can be divided into 3 parts in the required way in 3 ways Choice (A)

21. The number and the cost of the 3 types of pens are tabulated below.

	Ball Point	Fountain	Gel
No.	x	y	z
Cost	7	9	12

$7x + 9y + 12z = 130$
Let x = 2 + a, y = 2 + b and y = 2 + c, where a, b, c $\geq$ 0.
$\therefore 7a + 9b + 12c = 74$
The solutions are listed below.

8	2	0
5	3	1
2	4	2

We take z = 0, 1, 2 respectively and work out what Rem $\frac{74 - 12c}{7}$ is and from that we can get a value of b.
Once we get one set of values for x, y we can get all the other. In the example above, for each value of z, there is only one set of positive values for (x, y).
The maximum number of fountain pens that Anil could have bought is 2 + z = 6. Ans: (6)

22. The different outcomes and the corresponding points are tabulated below.

Outcomes	2, 3, 5	4, 6	1
Points	5	7	9

Let the number of times. Sreedhar throws a prime, a composite and 1 be x, y, z respectively.
$\therefore 5x + 7y + 9z = 74$
To get the maximum value of x + y + z, we set z = 0 (the variable with the greatest coefficient) and then find the minimum value of y.
We get (x, y, z) = (12, 2, 0), this would correspond to the maximum value of x + y + z.
$\therefore$ This maximum value is 14. Ans: (14)

23. Let the number of members in Group A and Group B be x and y respectively. Let the number of people who change their group be k.
According to the given conditions,
$x + k = 3(y - k) \Rightarrow x - 3y = -4k \rightarrow (1)$
$x - k = y + k \Rightarrow x - y = 2k \rightarrow (2)$
solving (1) and (2) we get x = 5k and y = 3k
Given $35 < x + y < 45$
$\Rightarrow 35 < 5k + 3k < 45 \Rightarrow 35 < 8k < 45$
$\Rightarrow 4\frac{3}{8} \leq k \leq 5\frac{5}{8}$. As k is an integer k = 5
$\therefore$ x = 25 and y = 15
$\therefore$ total number of members in Group B is 15.

Alternate Solution:

The data (and the conclusions) are tabulated below.

	A	B
Initial	5x	3x
After some join A	6x	2x
After some join B	4x	4x

After some change from B to A, A has 3 times as many members as A.
After the same number change from A to B, the two groups have an equal number, i.e., 4x.
$\therefore$ Initially A had 5x, B had 3x, i.e., the total is multiple of 8.
The only multiple of 8 greater than 35 and less than 45 is 40 i.e., 8x = 40.
$\therefore$ The initial number of people in B = 3x = 3(5) = 15.
Choice (A)

24. Let the number of apples, bananas and oranges Renu brought be x, y, and z respectively.
$x + y + z = 20 \rightarrow (1)$
$10x + 2y + 5z = 83 \rightarrow (2)$
$(1) \times 10 \Rightarrow 10x + 10y + 10z = 200 \rightarrow (3)$
(3) – (2) gives,
$8y + 5z = 117$
Dividing throughout with 5 we get
Rem $\left(\frac{8y}{5}\right)$ = Rem $\left(\frac{117}{5}\right)$
Rem $\left(\frac{3y}{5}\right) = 2$
y = 4 satisfies the above equation the corresponding value of z = 17
The solutions are (4, 17), (9, 9) (14, 1)
The number of bananas he purchased is 9 or 14.
Choice (D)

25. Let's say Nikil purchased x erasers, y sharpeners and z pencils.
$2x + 3y + 5z = 35$
If y is maximum only then z is minimum
Put z = 1; $2x + 3y = 30$
y = 0; x = 5 is one of the solutions.
The other values are tabulated below.

y	0	2	4	6	8
z	15	12	9	6	3

We see that, the maximum number of sharpeners that Nikil could have bought is 8. Ans: (8)

26. $\frac{1}{a}+\frac{1}{b}=\frac{1}{5} \Rightarrow 5a + 5b = ab$
$\Rightarrow ab - 5b - 5a = 0 \Rightarrow b(a-5) - 5(a-5) = 25$
$\Rightarrow (a-5)(b-5) = 5^2$
$\therefore$ The number of factors is 3.
Hence number of solutions is also 3. Choice (D)

27. $\frac{2}{a}+\frac{3}{b}=\frac{1}{4}$
$\Rightarrow (2b + 3a)\,4 = ab \Rightarrow ab - 12a - 8b = 0$
$\Rightarrow a(b-12) - 8(b-12) = 96$
$\Rightarrow (a-8)(b-12) = 2^5\ 3$
The number of positive factors is 12. As b > 0,
$(b-12) > -12$.
$\therefore$ (b – 12) can also be –1, –2, –3, –4, –6, or –8
The total number of solutions is 12 + 6, i.e., 18. Choice (B)

28. Given $\frac{5}{a}-\frac{7}{b}=\frac{1}{11} \Rightarrow 55b - 77a = ab$ . . . . (1)
$\Rightarrow ab + 77a - 55b = 0$
$\Rightarrow a(b+77) - 55(b+77) = -(55)(77)$
$\Rightarrow (a-55)(b+77) = -(11)^2\ 5\ (7)$ . . . . (2)
The number of positive factors is 12. Since negative integers can also be considered, the total number of solutions for 2 is 24. One of these is (a – 55) = –55 (b + 77) = 77. But this does not satisfy (1). Every other solution of (2) also satisfies(1).
$\therefore$ The number of solutions of (1) is 23. Choice (D)

29. Given $a^2 - b^2 = 987 \Rightarrow (a-b)(a+b) = 3\ (7)\ (47)$
The number of factors is 8. Negative integers can also be considered. The total number of solutions is 16. Choice (A)

30. Given $a^2 - b^2 = 140$
$\Rightarrow (a-b)(a+b) = 2^2\ (7)\ (5)$
Since a and b are positive integers, (a – b) and (a + b) must be positive and both are even or both are odd. Since even factors are there, both have to be even.
Also a + b must be greater than a – b.
$\therefore$ The number of solutions is 2
($\because$ (7) (5) has only four factors). Choice (D)

Chapter – 2
(Quadratic Equations)

Concept Review Questions

Solutions for questions 1 to 25:

1. $12x^2 + 23x + 5$
$= 12x^2 + 20x + 3x + 5$
$= 4x(3x + 5) + 1(3x + 5)$
$= (4x + 1)(3x + 5)$ Choice (B)

2. (a) $x^2 - x - 20 = 0$
$x^2 - 5x + 4x - 20 = 0$
$\Rightarrow (x-5)(x+4) = 0$
$\Rightarrow x = 5, -4$ Choice (C)

(b) $2x^2 - 5x - 3 = 0$
$2x^2 - 6x + x - 3 = 0$
$\Rightarrow (2x+1)(x-3) = 0$
$\Rightarrow x = -\frac{1}{2}, 3$ Choice (D)

3. A quadratic equation in x with certain roots is of the form
x^2 – (sum of the roots) x + product of the roots = 0
Given: that, the sum of the roots = 7 and the product of the roots = 12
$\therefore$ The quadratic equation would be $x^2 - 7x + 12 = 0$
Choice (B)

4. $x^2 - 12x + 13 = 0$
$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(13)}}{2} = \frac{12 \pm \sqrt{92}}{2}$
$= 6 \pm \sqrt{23}$ Choice (C)

5. **(a)** Let the equation be
$x^2 - 13x + 30 = 0$
$x^2 - 10x - 3x + 30 = 0$
$x(x-10) - 3(x-10) = 0$
$(x-10)(x-3) = 0$
x = 10 or 3 Choice (A)

(b) Sum $= -\frac{25}{\sqrt{5}} = -5\sqrt{5}$

Product $= \frac{2\sqrt{5}}{\sqrt{5}} = 2$ Choice (A)

6. **(a)** Let the two consecutive positive integers be (x – 1), x.
then, $(x-1)^2 + x^2 + x(x-1) = 331$
$\Rightarrow 3x^2 - 3x - 330 = 0$
$\Rightarrow x^2 - x - 110 = 0$
$\Rightarrow (x-11)(x+10) = 0$
$\Rightarrow x = 11$ and $x = -10$
As x cannot be negative, the integers are 10, 11
(or) alternatively substitute the options and check.
Choice (B)

(b) Let the three consecutive positive integers be (x – 1), x,
(x + 1) then, $(x-1)^2 + x^2 + (x+1)^2 = 869$
$\Rightarrow 3x^2 + 2 = 869$
$\Rightarrow x = \pm 17$
As x cannot be negative, x = 17
The numbers are 16, 17, 18 Choice (C)

7. Let the integers be x,
then $x - \frac{1}{x} = \frac{143}{12}$
$\Rightarrow 12x^2 - 143x - 12 = 0$
$\Rightarrow (12x-1)(x-12) = 0$
$\Rightarrow x = \frac{-1}{12}, 12$
As x is integer, x = 12 Ans: (12)

8. **(a)** $2x^2 - 7x + 2 = 0$
Discriminant $= (-7)^2 - 4 \times 2 \times 2 = 33 > 0$ but 33 is not perfect square.
$\therefore$ the roots are irrational. Choice (D)

(b) Discriminant $= 6^2 - 4(2)(-5) = 76$.
This is positive but not a perfect square.
$\therefore$ The roots are conjugate surds. Choice (C)

(c) Let the equation be $ax^2 + bx + c = 0$
$\left(-\frac{b}{a}\right)^2 = 4\frac{c}{a}$
$\therefore b^2 = 4ac$
$\therefore$ The discriminant is 0.
$\therefore$ The roots are real and equal.

Alternate method:
If α, β are the roots $(\alpha+\beta)^2 = 4\alpha\beta \Rightarrow (\alpha-\beta)^2$
$\Rightarrow \alpha = \beta$ Choice (B)

9. Discriminant = $7^2 - 4(3)(2) = 25$ Ans: (25)

10. The number of roots is given by the degree. The degree of the equation $(x^n - a)^2 = 0$ is 2 n.
$\therefore$ There are 2n roots. Choice (C)

11. The given equation has the sum of its roots as –1 and the product of its roots as –420. As the sum of the roots as well as the product of the roots are negative, the roots are of opposite signs with the numerically larger root being negative. Choice (D)

12. **(a)** The equation whose roots are m more than the roots of $ax^2 + bx + c = 0$ is given by a $(x-m)^2 + b(x-m) + c = 0$.
$\therefore$ The required equation is
$(x-2)^2 + 9(x-2) + 10 = 0$ i.e. $x^2 + 5x - 4 = 0$.
Choice (A)

(b) The equation whose roots are reciprocals of the roots of the equation $ax^2 + bx + c = 0$ is given by $cx^2 + bx + a = 0$.
$\therefore$ The required equation is $5x^2 + 8x + 2 = 0$.
Choice (A)

(c) The equation whose roots are p times the roots of $ax^2 + bx + c = 0$ is given by $a\left(\frac{x}{p}\right)^2 + b\left(\frac{x}{p}\right) + c = 0$.
Here p = 1/3.
$\therefore$ The required equation is $(3x)^2 + 6(3x) + 10 = 0$
i.e. $9x^2 + 18x + 10 = 0$ Choice (C)

13. A quadratic equation whose sum of the roots is S and whose product of the roots is P has the sum of the squares of its roots given by $S^2 - 2P$.
The sum of the squares of the roots = $33^2 - 2(90) = 909$.
Ans: (909)

14. Let the equation be $ax^2 + bx + c = 0$.
As the roots are reciprocals of each other, the product of the roots is 1.
$\therefore \frac{c}{a} = 1 \Rightarrow c = a \quad b = 2a$
$\therefore ax^2 + 2ax + a = 0$
$a(x+1)^2 = 0$
As $a \neq 0$, x = –1, –1
$\therefore$ The sum of the squares of its roots is $(-1)^2 + (-1)^2 = 2$
Ans: (2)

15. A quadratic equation whose sum of the roots is 5 and whose product of the roots is P has the difference of its roots given by $\sqrt{S^2 - 4P}$.
The difference of the roots is $\sqrt{19^2 - 4(90)} = 1$
Ans: (1)

16. Let one root be α, other root is 3α.
Sum = $4\alpha = 2$
$\Rightarrow \alpha = \frac{1}{2}, 3\alpha = \frac{3}{2}$
Product = $\frac{k}{4} = \frac{1}{2} \times \frac{3}{2} \Rightarrow k = 3$ Ans: (3)

17. From the quadratic equation
Sum = $\frac{2m}{m-k+l}$
Check out with each of the options as for which of them has the same sum and product.
Option (A):
Sum = $1 + \frac{\ell + m - k}{k + m - \ell} = \frac{2m}{k + m - \ell}$
Sum not satisfied
Option (B):
Sum = $1 + \frac{2m}{\ell + m - k\ell} = \frac{\ell + 3m - k}{\ell + m - k}$
Sum not satisfied
Option (C):
Sum = $1 + \frac{k + m - \ell}{\ell + m - k} = \frac{2m}{\ell + m - k}$
Sum satisfied
Product = $1 \times \frac{k + m - \ell}{\ell + m - k} = \frac{k + m - \ell}{\ell + m - k}$
Choice (C)

18. Let E be $ax^2 + bx + c = 0$.
Given : $\left(-\frac{b}{a}\right)^2 = 8\frac{c}{a}$
$b^2 = 8ac$
The equation whose roots are the reciprocals of the roots of E is $cx^2 + bx + a = 0$
As $b^2 = 8ac$
$\frac{b^2}{c^2} = \frac{8ac}{c^2}$
$\therefore \left(-\frac{b}{c}\right)^2 = 8\frac{a}{c}$
$\therefore \frac{\left(-\frac{b}{c}\right)^2}{\frac{a}{c}} = 8$
We don't need the coefficients of either of the equations.
Let the roots be $\alpha.\beta$.
Given : $(\alpha+\beta)^2 = 8\alpha\beta$.
For the second equation, the roots are $1/\alpha$, $1/\beta$. We need to evaluate
$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 \frac{1}{(1/\alpha)(1/\beta)} = \frac{(\alpha+\beta)^2}{(\alpha\beta)^2}(\alpha\beta) = \frac{(\alpha+\beta)}{\alpha\beta} = 8$ Ans: (8)

19. $x^2 + 10x + 24 = 0 \Rightarrow x = -4, -6$
$x^2 + 14x + 48 = 0 \Rightarrow x = -6, -8$
$\therefore$ The common root of both equations is –6.
Ans: (–6)

20. **(a)** The expression $\frac{4ac - b^2}{4a}$ represents the maximum value of the quadratic expression $ax^2 + bx + c$ when $a < 0$ and represents the minimum value of the quadratic expression $ax^2 + bx + c$ when $a > 0$.
$\therefore$ Neither (A) nor (B) is true. Choice (D)

(b) The maximum/minimum value of the quadratic expression $ax^2 + bx + c$ occurs at $x = \frac{-b}{2a}$.
Choice (A)

(c) When $a < 0$, $ax^2 + bx + c$ has a maximum value which is $\frac{4ac - b^2}{4a}$.
$\therefore$ The maximum value of $-3x^2 + 4x + 5$ is
$\frac{(4)(-3)(5) - 16}{4(-3)} = \frac{19}{3}$ Choice (A)

21. (a) The given equation can be written as

$x^4 + \frac{b}{a}x^3 + \frac{c}{a}x^2 + \frac{d}{a}x + \frac{e}{a} = 0.$

The sum of the roots

$\alpha + \beta + \gamma + \delta = -$ coefficient of $x^3 = \frac{-b}{a}$

Choice (B)

(b) Product of the roots $= \alpha\beta\gamma\delta =$ constant term $= \frac{e}{a}$.

Choice (D)

22. As the coefficients are real the complex roots occur in conjugate pairs but as the coefficients are not necessarily rational, the irrational roots need not occur in pairs. $\sqrt{3}$ and 3 + 2i are the roots, Therefore,

$\Rightarrow 3 - 2i$ is also root, but $-\sqrt{3}$ need not be.

$\therefore$ The lowest degree of the equation is three.

Ans: (3)

23. $(x^3 - 3)^2 - 6x^5 = 0$

$x^6 - 6x^3 + 9 - 6x^5 = 0$

The degree of an equation in a variable is the index of the highest power of that variable in that equation.

$\therefore$ The degree of the equation above in x is 6.

Choice (B)

24. The number of sign changes in the given polynomial is 3.

Ans: (3)

25. The equation f(x) = 0 is said to be a reciprocal equation if $f\left(\frac{1}{x}\right) = 0$

$f(\alpha) = 0 \Rightarrow f\left(\frac{1}{\alpha}\right) = 0$. For all α.

All the given choices satisfy this condition.

Choice (D)

Exercise – 2(a)

Solutions for questions 1 to 40:

1. (i) $x^4 - 35x^2 + 196 = 0$, Substituting a for x^2,

we have, $a^2 - 35a + 196 = 0$

Hence, $a^2 - 28a - 7a + 196 = 0$

$a(a - 28) - 7(a - 28) = 0$; $(a - 28)(a - 7) = 0$

a = 28 or 7

Hence $x^2 = 28$ or $x^2 = 7$

thus $x = \pm\sqrt{28}$ or $\pm 2\sqrt{7}$ or $x = \pm\sqrt{7}$ Choice (A)

(ii) $2(3^2 \cdot 3^{2x}) - 4(3^2 \cdot 3^x) + 10 = 0$

$2(9)(3^{2x}) - 4(9)(3^x) + 10 = 0$

$18(3^{2x}) - 36(3^x) + 10 = 0;$

Dividing by 2 and substituting a for 3^x,

we have $9a^2 - 18a + 5 = 0$

$9a^2 - 3a - 15a + 5 = 0;$

$3a(3a - 1) - 5(3a - 1) = 0$

a = 1/3 or 5/3

Hence $3^x = 1/3 = 3^{-1}$ or 5/3, thus $x = -1$ or $\log_3 (5/3)$

Choice (A)

2. If the given equation has equal roots, the discriminant is zero.

$(K + 12)^2 - 4(K + 12)(-2)$ must be equal to 0.

$(K + 12)(K + 12 + 8) = 0$; $(K + 12)(K + 20) = 0$

$\Rightarrow K = -12$ or $K = -20$

When k = –12, k + 12 = 0; and the given equation becomes invalid. Hence, k = –20 is the only solution.

Ans: (–20)

3. If the total number of children in the school is x, we have $\frac{5}{2}\sqrt{x} + \frac{1}{4}x + 28 = x$, where $\frac{5}{2}\sqrt{x}$ play football, $\frac{1}{4}x$ play tennis and 28 play basketball.

$\frac{5}{2}\sqrt{x} = x - \frac{1}{4}x - 28 = \frac{3}{4}x - 28$.

Substituting $\sqrt{x}$ = a, the equation becomes,

$\frac{5a}{2} + \frac{a^2}{4} + 28 - a^2 = 0, \Rightarrow -\frac{3a^2}{4} + \frac{5a}{2} + 28 = 0$

Multiplying with (–4), the equation becomes,

$3a^2 - 10a - 112 = 0$, $(3a + 14)(a - 8) = 0$

$\Rightarrow a = 8 \Rightarrow \sqrt{x} = 8 \Rightarrow x = 64$ Ans: (64)

4. Let $x + \frac{1}{x} = a$

$x^2 + \frac{1}{x^2} + 2 = a^2$

$x^2 + \frac{1}{x^2} = a^2 - 2$

Substituting these values, we get

$(a^2 - 2) - 4a + \frac{23}{4} = 0.$

$\Rightarrow 4a^2 - 16a + 15 = 0.$

$(2a - 3)(2a - 5) = 0$

a = 3/2 or a = 5/2.

$x + \frac{1}{x} = \frac{3}{2}$ or $x + \frac{1}{x} = 5/2$

As $x + \frac{1}{x} \geq 2$, $x + \frac{1}{x} \neq \frac{3}{2}$

$\therefore x + \frac{1}{x} = \frac{5}{2}$ Ans: (2.5)

5. $(k^2 - 3k + 2)(k^2 - 7k + 12) = 120$

$(k - 1)(k - 2)(k - 3)(k - 4) = 120 \rightarrow (1)$

= (5) (4) (3) (2)

Comparing the two sides, k – 1 = 5

k = 6 Ans: (6)

6. The product of the roots of the given equation is 2(2R – 2) and the sum of the roots $= -\left(\frac{-(R+7)}{1}\right) = R + 7$.

Hence 2(2R – 2) = 3(R + 7)

4R – 4 = 3R + 21; R = 25 Ans: (25)

7. Since A copied the coefficient of x wrongly he copied the constant term correctly, hence constant term = $12 \times 6 = 72$. B copied the constant term wrongly and hence he copied the coefficient of x correctly, hence coefficient of x = –(1 + 26) = –27. Hence, the equation is $x^2 - 27x + 72 = 0$

Thus $\Rightarrow (x - 24)(x - 3) = 0$

Thus x = 24 or x = 3 Choice (C)

8. $\sqrt{5x-4} - \sqrt{2x+1} = 1$. Going by the options, we have option (A) as x = 4

Substituting x = 4 in the equation given, we have $\sqrt{5(4)-4} - \sqrt{2(4)+1} = \sqrt{16} - \sqrt{9} = 4 - 3 = 1$

which is equal to the right hand side of the given equation.

Choice (A)

9. $(x - k_1)(x - k_2) = -1;$

Given that roots are integers.

$\Rightarrow$ x has integer values.

given that k_1, k_2 are integers.
Hence, $(x - k_1)$ as well as $(x - k_2)$ are integers.
The only way –1 can be resolved into the product of integers is $-1 = 1 \times -1$. Hence,

if, $(x - k_1) = +1$
then $(x - k_2) = -1$ } Case I
or
if, $(x - k_1) = -1$
then $(x - k_2) = +1$ } Case II

Case I gives
$k_1 + 1 = k_2 - 1$
or, $k_2 - k_1 = 2$

Case II gives
$k_1 - 1 = k_2 + 1$ or, $k_1 - k_2 = 2$ Choice (D)

10. Let the length of the playground be l and the breadth be b.
lb = 153.
l – 4 = b + 4, since the rectangle becomes a square when its length is decreased by 4 m and breadth is increased by 4 m.
l = b + 4 + 4 = b + 8
$\Rightarrow (b + 8)b = b^2 + 8b = 153$
$b^2 + 8b - 153 = 0$; $(b + 17)(b - 9) = 0$
$b \neq -17$. Hence b = 9.
The side of the square = b + 4 = 9 + 4 = 13 m
Ans: (13)

11. The given equation is $3x^2 + 17x + 6 = 0$
The roots are in the ratio p : q
Let $p\alpha$ and $q\alpha$ be the roots.
The sum of the roots = $\alpha(p + q) = -17/3$ → (1)
The product of the roots = $\alpha^2 pq = 6/3 = 2$ → (2)
Squaring (1) and then dividing by (2),

$$\frac{\alpha^2(p+q)^2}{\alpha^2 pq} = \frac{(-17/3)^2}{2} = \frac{289}{18}$$

$$\Rightarrow \frac{p+q}{\sqrt{pq}} = \pm\frac{17}{3\sqrt{2}} \quad \Rightarrow \sqrt{p/q} + \sqrt{q/p} = \pm(17/3\sqrt{2})$$

As $\sqrt{p/q}$ as well as $\sqrt{q/p}$ are positive, the required result is positive.

i.e., $\sqrt{p/q} + \sqrt{q/p} = +(17/3\sqrt{2}) = +(17\sqrt{2})/6$
Choice (A)

12. x + y = 4
$x^2 + y^2 = x^2 + (4 - x)^2 = 2x^2 - 8x + 16$
Coefficient of x^2 = 2 which is positive
∴Minimum value exists.

Minimum value = $\frac{4ac - b^2}{4a}$

As, a = 2, b = –8 and c = 16, the minimum value is $\frac{128 - 64}{8} = 8$ Choice (A)

13. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\left(-\frac{b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2}{a^2} - \frac{2ca}{a^2} = \frac{b^2 - 2ca}{a^2}$$

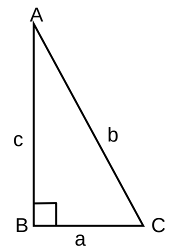
$\alpha^2\beta^2 = (\alpha\beta)^2 = \left(\frac{c}{a}\right)^2$

Hence, the required equation is
x^2 – (sum of the roots)x + product of roots = 0.

i.e. $x^2 - \left(\frac{b^2 - 2ca}{a^2}\right)x + \frac{c^2}{a^2} = 0$ or

$a^2x^2 - (b^2 - 2ca)x + c^2 = 0$
$a^2x^2 - b^2x + 2cax + c^2 = 0$ Choice (C)

14.



Let the sides be as represented in the figure.
From the data, b + a = 2c → (1)

and $c = 9 + \frac{b}{2}$; $\Rightarrow 2c = 18 + b$ → (2)

From (1) and (2); a = 18 → (3)
Applying the theorem of Pythogoras,
$b^2 = c^2 + a^2$ and substituting a = 2c – b,
$b^2 = c^2 + (2c - b)^2$; $\Rightarrow b^2 = c^2 + 4c^2 - 4bc + b^2$,
$\Rightarrow 5c^2 - 4bc = 0$; $\Rightarrow 5c = 4b$ → (4)
Substituting in (1)

$b + a = 2c \Rightarrow b + 18 = \left(\frac{4b}{5}\right)^2$

$\Rightarrow 5b + 90 = 8b \quad \Rightarrow b = 30$

Alternate method:
From the first two equations, a = 18 is obtained.
(3 : 4 : 5) is a ratio which satisfies the conditions b + a = 2c, because (5 + 3) = 2(4).
Hence, 3 parts of the ratio of (3 : 4 : 5) is 18.
Hence, 5 parts of the ratio is 5(18/3) = 30;
i.e., b = 30. Ans: (30)

15. As the equations given in the options contain coefficients a, b, and c, consider the equivalent of $px^2 + qx + r = 0$ with co-efficients a, b and c.
$px^2 + qx + r = 0$ is equivalent to
$a(x - k)^2 + b(x - k) + c = 0$ --- (I), because the equation whose roots are 'k' more than those of equation (I) is obtained by replacing; "x" by "x – k".
$a(x - 2k)^2 + b(x - 2k) + c = 0$ Choice (B)

16. $x^2 - 11x + 24 = 0$
$(x - 3)(x - 8) = 0$
x = 3 or x = 8

If $\frac{1}{\alpha} - \frac{1}{\beta}$ is positive, α should be less than β; hence α = 3, β = 8.

$$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3} - \frac{1}{8} = \frac{8}{24} - \frac{3}{24} = \frac{8-3}{24} = \frac{5}{24}$$ Choice (A)

17. $x^2 - 10x + 16 = 0$
$(x - 8)(x - 2) = 0$
x = 8 or x = 2.
If x = 8 is the root of $x^2 - 10x + 16$ which is half of one root (α) of $x^2 - 4Rx + 8 = 0$ then $\alpha = 2 \times 8 = 16$.

Then the other root β of $x^2 - 4Rx + 8 = 0$ would be $\frac{8}{16} = \frac{1}{2}$.

Since this is not an integral root this value is not acceptable. If x = 2 which is the root of $x^2 - 10x + 16 = 0$, which is half of one root (α) of $x^2 - 4Rx + 8 = 0$ then, the second root β is $\frac{8}{4} = 2$.

The sum of the roots = $-\frac{(-4R)}{1} = 4R = 4 + 2 = 6$,

$R = \frac{6}{4} = \frac{3}{2}$ Ans: (1.5)

18. For the equation $x^2 + 4x + p = 0$ to have real roots, discriminant $(b^2 - 4ac) = 16 - 4p$, must be positive or equal to 0. $16 \geq 4p \Rightarrow 4 \geq p$.

Hence p = 1, 2, 3 or 4 i.e. there are four equations of the form $x^2 + 4x + p = 0$ that have real roots and p is a positive integer. Choice (C)

19.

	N	P	A
Ranjan	x	p_a	$x . p_a$
Raman	y	p_b	$y . p_b$

When N, P and A stand for the number, the price and the amount respectively.

Given x + y = 108

$y P_a = 722 \quad \to \quad$ (I)

$x P_b = 578 \quad \to \quad$ (II)

$\Rightarrow \frac{y p_a}{x p_b} = \frac{722}{578} = \frac{361}{289}$ and $xp_a = yp_b$

$\therefore \frac{p_a}{p_b} = \frac{y}{x}$ or $\left(\frac{y}{x}\right)^2 = \left(\frac{19}{17}\right)^2 \quad \frac{y}{x} = \frac{19}{17}$

The number of floppies with Ranjan = $\frac{17}{36} \times 108 = 51$

Ans: (51)

20. For the equation $2x^2 + 8x + p = 0$ to have rational roots, $b^2 - 4ac = 8^2 - 4(2)(p) = 64 - 8p$, should be a perfect square.

64 – 8p is a perfect square.

$\Rightarrow$ 8 (8 – p) is a perfect square.

$\Rightarrow$ 8 (8 – p) = 0, 1, 4, 9, 16, 25, 36, 49, 64.

As p is an integer, the value selected should be an integer such that 8(8 – p) is divisible by 8. i.e. 8 (8 – p) = 0, 16, 64, ...

When 8(8 – p) = 0; p = 8

8(8 – p) = 16; 8 – p = 2, p = 6

8(8 – p) = 64, 8 – p = 8, p = 0; not admissible.

8(8 – p) = 144, 8 – p = 18, p = –10

Hence, p = 8 or 6 are the possible values.

Choice (D)

21. The equation whose roots would be reciprocals of the roots of the equation $ax^2 + bx + c = 0$ is

$$a\left(\frac{1}{x}\right)^2 + b\left(\frac{1}{x}\right) + c = 0 \Rightarrow \frac{a}{x^2} + \frac{b}{x} + c = 0$$

$\Rightarrow a + bx + cx^2 = 0$ i.e. $cx^2 + bx + a = 0$; since this only differs from $ax^2 + bx + c = 0$ in the way that constant term and coefficient of x^2 are swapped, it also obeys the relation $64ac = 15b^2$ as does $ax^2 + bx + c = 0$. Choice (B)

22. $c^3 + abc + a^3 = 0$

Dividing each side by a^3, $\left(\frac{c}{a}\right)^3 - \left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) + 1 = 0$

$(\alpha\beta)^3 - (\alpha\beta)(\alpha + \beta) + 1 = 0$

dividing each side by $\alpha\beta$,

$(\alpha\beta)^2 - (\alpha + \beta) + \frac{1}{\alpha\beta} = 0$

$\Leftrightarrow (\alpha^2 - \frac{1}{\beta})(\beta^2 - \frac{1}{\alpha}) = 0$

$\Leftrightarrow \alpha^2 = \frac{1}{\beta}$ or $\beta^2 = \frac{1}{\alpha}$ Choice (A)

23. Let the common root be a.

$x^2 - 9x + 18 = 0$

x = 6, 3

(6 + a) (3 + a) = 40

(a – 2) (a + 11) = 0 as a > 0, a = 2 Ans: (2)

24. The common root satisfies $x^2 = -px - q = qx - p$

$\Rightarrow px + q - qx - p = 0$

or (x – 1) (p – q) = 0

$\Rightarrow$ x = 1 or p = q

If p = q, the two equations are identical and both roots would be common.

$\therefore p \neq q$. x = 1

$x^2 + px + q = 1 + p + q = 0$

and as $p \neq q$, $p \neq -1/2$ Choice (D)

25. Let the roots of E be α and β

$a = \alpha + \beta$

$b = \alpha\,\beta$

Required equation has its roots as

$\alpha\,\beta^2$ and $\beta\,\alpha^2$

$\alpha\,\beta^2 + \beta\,\alpha^2 = \alpha\beta\,(\alpha + \beta)$

$(\alpha\,\beta^2)(\beta\alpha^2) = (\alpha\,\beta)^3$

required equation is $x^2 - abx + b^3 = 0$ Choice (A)

26. $|x|^2 + 6|x| - 55 = 0$

$(|x| + 11)(|x| - 5) = 0$

$|x|$ cannot be negative. $\therefore |x| = 5$. $\therefore x = \pm 5$

$\therefore (\alpha, \beta) = (5, -5)$ or $(-5, 5)$

Let the roots of the second equation be y_1 and y_2.

Let $y_1 = \alpha\beta y_2$

$y_1 + y_2 = (\alpha\beta + 1)\,y_2$ and $y_1y_2 = \alpha\beta\,y_2^2$

$$\frac{(y_1 + y_2)^2}{y_1y_2} = \frac{(\alpha\beta + 1)^2}{\alpha\beta}$$

$$\Rightarrow \frac{\left(\frac{-q}{p}\right)^2}{\frac{r}{p}} = \frac{(-25+1)^2}{-25}$$

$$\frac{q^2}{rp} = \frac{-576}{25}$$

Choice (A)

27. $x^2 - px + q = 0$. The sum of the roots = p = 36 (given)

The product of the roots = q = 12 (given) Choice (B)

28. $ax^2 + bx + c = 0$ and $a, b, c \in Q$

$\therefore$ If $3 + \sqrt{7}$ is a root, so is $3 - \sqrt{7}$

$\therefore$ Sum of roots = $\frac{-b}{a} = 6$ and product of roots = $\frac{c}{a} = 2$

i.e., b = –6a and c = 2a

$\therefore$ A possible value of (a, b, c) is (1, –6, 2) Choice (D)

29. Let *the* roots be a – 1, a and a + 1.

(x – (a – 1)) (x – a) (x – (a + 1)) = 0

The coefficient of x on the L.H.S. of the equation above is $3a^2 - 1$.

Only choice (D) is not in this form. Choice (D)

30. Let $\alpha = a - d$, $\beta = a$ and $\gamma = a + d$ be the roots of the equations $x^3 - 15x^2 + 71x - 105 = 0$

$\Rightarrow a - d + a + a + d = 15 \Rightarrow a = 5$

The product is (a - d) a (a + d) = 105

$(25 - d^2) = 21$

$d^2 = 4 \Rightarrow d = \pm 2$

$\therefore$ the roots are 5 – 2, 5, 5 + 2, i.e., $\alpha = 3$, $\beta = 5$ and $\gamma = 7$

The required difference = 7 – 3 = 4 Ans: (4)

31. $x^3 - 4x^2 + x + 6 = 0 \quad \to \quad (1)$

$x^3 - 3x^2 - 4x + k = 0 \quad \to \quad (2)$

if α is a common root, $\alpha^3 - 4\alpha^2 + \alpha + 6 = 0 \quad \to \quad (1')$

and $\alpha^3 - 3\alpha^2 - 4\alpha + k = 0 \quad \to \quad (2')$

$(2') - (1') \Rightarrow \alpha^2 - 5\alpha + (k - 6) = 0 \quad \to \quad (3)$

$\Rightarrow \alpha^3 - 5\alpha^2 + (k - 6)\,\alpha = 0 \quad \to \quad (3')$

$(1') - (3') \Rightarrow \alpha^2 + (7 - k)\,\alpha + 6 = 0 \quad \to \quad (4)$

$(4) - (3) \Rightarrow (12 - k)\,\alpha + (12 - k) = 0$

$\Rightarrow \alpha = -1$ or k = 12

If –1 is a root of (2), k = 0, or else k = 12

Choice (C)

32. The given equation is $x^3 - 11x^2 + 37x - 35 = 0 \quad \to \quad (1)$

One root is $3 - \sqrt{2}$. As the coefficients are rational,

The irrational roots occur in pairs, $3 + \sqrt{2}$ is also a root of (1)

$\therefore$ Sum of the roots = –(–11) = 11 from (1)

Sum of two of the roots is $3+\sqrt{2}+3-\sqrt{2}=6$

$\therefore$ The third root is 11 – 6 = 5

$\therefore$ The roots of the equation are $3\pm\sqrt{2}$ and 5.

Choice (C)

33. F(x) must have the form (x – 7) (x – 6) (x – 5) …..(x – 1) + G(x)
F(7) – F(6) = F(6) – F(5)=………=F(2) – F(1) = (2 i.e, a constant. $\therefore$ G(x) must be a linear function of x. $\therefore$ F(x) must have the form (x – 7) (x – 6) (x – 5)………(x – 1) + ax + b where a and b are constants.
F(1) = a + b = 5 and F(2) = 2a + b = 7
$\therefore$ a = 2 and b = 3
F(x) = (x – 7) (x – 6) (x – 5)………(x – 1) + 2x + 3
$\therefore$ F(8) = 1 (2) (3)……(7) + 19 = 7! + 19 = 5040 + 19 = 5059

Choice (C)

34. Given the equation f(x) = 5, x + $15x^4 + 85x^3 + 225x^2$ + 274x + a – 119 = 0 has exactly 5 negative roots.
$\Rightarrow$ f(x) should have 5 sign changes
$\Rightarrow$ a – 119 > 0 $\Rightarrow$ a > 119 Choice (C)

35. Let f(x) = $x^3 - 6x^2$ – ax – 6 = 0.
As f(x) has all positive roots, f(x) should have three sign changes so, a < 0.
Hence, going from the options a = –11. Choice (D)

36. x^2 + 16x – q = 0 has real roots
$\therefore$ $(16)^2 - 4q \geq 0$
$\Rightarrow$ 4q ≤ 256 $\Rightarrow$ q ≤ 64 $\longrightarrow$ (1)
x^2 – 11q x + 25 = 0 has real roots
$\therefore$ $(-11q)^2 - 4(25) \geq 0$
$(11q)^2 \geq 100$

$\Rightarrow q^2 \geq \frac{100}{121}$

$\Rightarrow q^2 \geq \left(\frac{10}{11}\right)^2$

$\Rightarrow q^2 \leq \frac{10}{11}$ or $q^2 \geq \frac{10}{11}$ $\longrightarrow$ (2)

From (1) and (2) $\frac{10}{11} \leq q \leq 64$.

$\therefore$ positive integer value of q is 64 Choice (D)

37. Given $x^3 - 2x^2$ – 2x – 3 = 0. By trial and error method clearly we can see x – 3 is a factor of the above equation.
$\therefore$ $(x^3 - 2x^2 - 2x - 3) = (x - 3)(x^2 + x + 1) = 0$
$\therefore$ The roots of x^2 + x + 1 = 0 are non - real since discriminant is less than zero.
$\therefore$ Hence the given equation has two non - real roots.

Ans: (2)

38. $x+\frac{1}{x}=\sqrt{2}$

squaring on both sides

$x^2+\frac{1}{x^2}+2=2$

$x^2+\frac{1}{x^2}=0$

$\Rightarrow x^4 + 1 = 0$
$\Rightarrow x^4 = -1$
$x^{80} + x^{76} + x^{72} + x^{68} + x^{64} + 4$
$= (x^4)^{20} + (x^4)^{19} + (x^4)^{18} + (x^4)^{17} + (x^4)^{16} + 4$
$= (-1)^{20} + (-1)^{19} + (-1)^{18} + (-1)^{17} + (-1)^{16} + 4$
= 1 – 1+ 1– 1+ 1+ 4 = 5 Choice (D)

39. We know that when f(x) is divided by x – a, the remainder is f(a).
Let f(x) = $x^{2030} - x^3$ + x + 1.
Remainder f(1) = $(1)^{2030} - 1^3$ + 1 + 1 = 2 Choice (C)

40. Let f(x) = $x^{87} + x^{69} + x^{51} + x^{33} + x^{15}$.
Let ax^2 + bx + c be the remainder when f(x) is divided by x^3 - x, By division algorithm f(x) = Q(x) . $(x^3 - x)$ + R(x)
where Q(x) is quotient and R(x) is remainder
f(x) = Q(x) x (x –1) (x +1) + ax^2 + bx + c
put x = 0
0 = c
put x = 1
1 + 1 + 1 + 1 + 1 = a + b + c = a + b = 5 $\longrightarrow$ (2)
put x = –1
– 1 – 1 – 1 – 1 – 1 – 1 = a $(-1)^2$ + b (– 1) + 0
–5 = a – b $\longrightarrow$ (3)
Solving (2) and (3) a = 0; b = 5
$\therefore$ Required remainder = 5x Choice (A)

Exercise – 2(b)

Solutions for questions 1 to 45:

1. (k + 1) (k + 2) (k + 3) (k + 4) = 360
(k + 1) (k + 4) (k + 2) (k + 3) = 360
$(k^2 + 5k + 4)(k^2 + 5k + 6) = 360$
If k^2 + 5k = y; (y + 4) (y + 6) = 360
$\Rightarrow$ y^2 + 10y + 24 – 360 = 0; y^2 + 10y – 336 = 0;
(y + 24) (y – 14) = 0
y = –24 or 14 Hence k^2 + 5k = –24
k^2 + 5k + 24 = 0 or k^2 + 5k – 14 = 0
k(k + 7) – 2(k + 7) = 0
(k is complex, if k^2 + 5k + 24 = 0)
(k + 7) (k – 2) – 0
$\Rightarrow$ k = –7 or 2
As k is positive, k = 2 is the solution.

Alternate method:
Given : k is a positive integer, k + 1, k + 2, k + 3, k + 4 are the 4 consecutive positive integers.
$\Rightarrow$ Product of 4 consecutive positive integers = 360.
360, by factorisation, is equal to : 3 x 4 x 5 x 6.
Hence, k + 1 = 3, k = 2. Ans: (2)

2. $2x^2$ – 15x + 18 = 0

The equation has sum of its roots = $-\left(\frac{-15}{2}\right)=\frac{15}{2}$ and

product of the roots = $\frac{18}{2}$ = 9

The equation whose roots are thrice the roots of the above equation will have the sum of the roots being thrice its sum of roots and have $3\left(\frac{15}{2}\right)=\frac{45}{2}$ as sum of roots. It will have the product of its roots as nine times the product of the roots of the original equation and hence $9 \times 9 = 81$ as product of roots.
Hence, the required equation whose roots are thrice the roots of $2x^2$ – 15x + 18 = 0 is $x^2-\frac{45x}{2}+81=0$

or $2x^2$ – 45x + 162 = 0

Alternate method:
In order to find out the equation whose roots are thrice the roots of the given equation, substitute $\frac{x}{3}$ for x, in the given equation.

$$2\left(\frac{x}{3}\right)^2-15\left(\frac{x}{3}\right)+18=0$$

$\Rightarrow$ $2x^2$ – 45x + 162 = 0 Choice (D)

3. x^2 + ax + b = 0 $\rightarrow$ (I)
x^2 + bx + a = 0 $\rightarrow$ (II)
Let 'k' be a common root, then
k^2 + ak + b = 0
k^2 + bk + a = 0
$\Rightarrow$ k (a – b) + (b – a) = 0 (on subtraction),
(k – 1) (a – b) = 0

$a - b \neq 0$, ∵ if a = b the two equations become identical and they will have two common roots.
∴ k = +1; ⇒ $1^2 + a + b = 0$, ⇒ a + b + 1 = 0.
Choice (C)

4. Assume that the person bought x oranges for ₹70. Hence price of each orange is $\frac{70}{x}$. If he bought 4 more oranges for ₹70, the price of each orange would be $\frac{70}{x+4}$ which is 2 less than $\frac{70}{x}$.

Hence $\frac{70}{x+4} = \frac{70}{x} - 2 \Rightarrow \frac{70}{x} - \frac{70}{x+4} = 2$;

$\frac{70(x+4)-70x}{x(x+4)} = \frac{70x+280-70x}{x(x+4)} = 2$; x(x + 4)

$= 140 \Rightarrow x^2 + 4x - 140 = 0$; (x + 14) (x – 10) = 0
Hence, x = –14 or x = 10.
Since the number of oranges bought cannot be –ve, x cannot be –14, so x = 10. Hence 10 oranges were bought originally. Ans: (10)

5. $x^2 - 2x - 8 = 0$ The sum of the roots = $\left(-\frac{2}{1}\right)$ = 2 and the product of the roots = –8. Since the product of the roots is negative, one of the roots is positive and the other negative. Since the sum of the roots is positive, the numerically larger root is positive. Choice (A)

6. Given p, q are integers, and one root of the equation is $2 + \sqrt{3}$, the equation must have the conjugate $2 - \sqrt{3}$ as the second root.
The product of the roots = $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$
Hence the product of the roots = p = 1 Ans: (1)

7. Let the required original number be x.
$(x + 3)^2 = 23 + x$.
Hence $x^2 + 6x + 9 = 23 + x$
$\Rightarrow x^2 + 5x - 14 = 0$. (x + 7) (x – 2) = 0
⇒ x = –7 or x = 2.
Since the original number is positive, x = 2. Ans: (2)

8. The given equation is $4kx^2 + 4\sqrt{k}\,x - k = 0$
The statements presented as options relate to the nature of the roots. Hence, discriminant is to be considered.
Discriminant (Δ) = $(4\sqrt{k})^2 - 4(4k)(-k) = 16k + 16k^2 = 16k(k + 1)$
As per data, k is a perfect square, i.e.+ $k \geq 0$.
Hence $\Delta \geq 0$. Hence, roots are definitely real. It is to be decided whether the roots are rational or irrational.
Δ can be equal to zero, if and only if either k = 0 or k = –1
As k is the coefficient of x^2, it cannot be zero; i.e. $k \neq 0$.
As k is given to be a perfect square, it cannot be equal to (–1); i.e. $k \neq -1$.
Hence $\Delta \neq 0$. ⇒ Δ > 0
As k and (k + 1) are two consecutive numbers and k is a perfect square, (k + 1) cannot be a perfect square. Hence, Δ is positive, but not a perfect square. Hence, the roots are always irrational. Choice (C)

9. If one of the roots is α, the other root is α^2.
Hence the product of the roots = $\alpha(\alpha^2)$.
$\alpha^3 = 64 \Rightarrow \alpha = \sqrt[3]{64} = 4$ and $\alpha^2 = 4^2 = 16$
The sum of roots = $-\left(\frac{6R}{1}\right) = -6R = 4 + 16 = 20$
$R = \left(\frac{20}{-6}\right) = -\frac{10}{3}$ Choice (A)

10. If the roots of $x^2 + x(14 - k) - 14k + 1 = 0$ are equal, $(14 - k)^2 - 4(-14k + 1) = 0$;
$\Rightarrow 196 - 28k + k^2 + 56k - 4 = 192 + 28k + k^2 = 0$.
Hence (k + 16) (k + 12) = 0 ⇒ k = –12 and –16
To check which of the values of k, leads to equal, integer roots, substitute the value of k = –12, we get the equation as $x^2 + 26x + 169 = 0$
$(x + 13)^2 = 0$
Both the roots are integers and equal.
If k = –16, the equation is $x^2 + 30x + 225 = 0$
$(x + 15)^2 = 0$
Both the roots are integers and equal. Choice (B)

11. If the roots of the given equation are α and $\frac{1}{\alpha}$;
$(\alpha)\left(\frac{1}{\alpha}\right) = 1$, $\frac{2(2m-1)}{2} = 2m - 1 = 1$
⇒ 1 + 1 = 2m, ⇒ m = 1 Ans: (1)

12. Since the roots of the given equation are equal, the discriminant $b^2 - 4ac = 0$
$b^2 - 4ac = 8^2 - 4(2^m)(64)^m = 2^6 - 2^2 2^m 2^{6m}$
$\Rightarrow 2^6 - 2^{2+7m} = 0$, Hence $2^6 = 2^{2+7m}$ i.e., 2 + 7m = 6 and hence 7m = 6 – 2 = 4; m = $\frac{4}{7}$ Choice (D)

13. Dividing both sides of the given equation by a + b,
$x^2 + \frac{2abx}{a+b} + \frac{(a+b)^2}{16} = 0$
Discriminant = $\left(\frac{2ab}{a+b}\right)^2 - \frac{4(a+b)^2}{16} = \left(\frac{2ab}{a+b}\right)^2 - \left(\frac{a+b}{2}\right)^2$
Shown below is the proof that this is always non-positive provided a and b are positive.
$(a - b)^2 \geq 0 \Rightarrow a^2 + b^2 + 2ab \geq 4ab$
dividing both sides by 2 (a + b)
$\frac{a+b}{2} \geq \frac{2ab}{a+b}$
As the expressions on both sides of the inequality are positive, $(\frac{a+b}{2})^2 \geq (\frac{2ab}{a+b})^2$
∴ Δ < 0 or Δ = 0
If Δ =0, the roots are real and equal.
If Δ< 0, the roots are non-real and distinct. Choice (D)

14. For the equation $x^2 + 2(p + 1)x + 2p = 0$
$b^2 - 4ac = [2(p + 1)]^2 - 4(2p)] = 4p^2 + 8p + 4 - 8p$
$= 4p^2 + 4$ which is always positive. Hence the roots of the equation are always real and unequal. Choice (C)

15. $6x^4 - 6x^3 - 24x^2 - 6x + 6 = 0$
Divide the equation by x^2 to get
$6x^2 - 6x - 24 - \frac{6}{x} + \frac{6}{x^2} = 0$
$6\left(x^2 + \frac{1}{x^2}\right) - 6\left(x + \frac{1}{x}\right) - 24 = 0$
Substitute y for $x + \frac{1}{x}$; $\left(x + \frac{1}{x}\right)^2 = y^2$;
$x^2 + \frac{1}{x^2} + 2 = y^2$; $x^2 + \frac{1}{x^2} = y^2 - 2$
$6(y^2 - 2) - 6(y) - 24 = 0$ $y^2 - y - 6 = 0$;
(y – 3) (y + 2) = 0 ⇒ y = +3 or –2
$\Rightarrow x + \frac{1}{x} = 3$ or $x + \frac{1}{x} = -2$
If $x + \frac{1}{x} = 3$, multiplying both sides of the equation by x, we get $x^2 + 1 = 3x$; $x^2 - 3x + 1 = 0$

$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(1)}}{2} = \frac{3\pm\sqrt{9-4}}{2} = \frac{3\pm\sqrt{5}}{2}$

If $x+\frac{1}{x} = -2$, multiplying both sides of the equation by x, we get $x^2 + 1 = -2x$

$x^2 + 2x + 1 = 0$; $(x + 1)^2 = 0$ x = –1, –1.

Hence the roots are –1 and $\frac{3\pm\sqrt{5}}{2}$

Alternate method:

By observing the co-efficients of the equation, it can be said that the sum of the co-efficients of x^4, x^2 and the independent term = +6 – 24 + 6 = –12.

The sum of the co-efficients of x^3 and x is (–6) + (–6) = –12.

As the two sums are equal, (x + 1) is a factor;

$\Rightarrow$ x = –1. There is only one option with (–1) as root.

Choice (B)

16. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}-2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+2\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta}-2\frac{(\beta+\alpha)}{\alpha\beta}+2\alpha\beta$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}-\frac{2(\beta+\alpha)}{\alpha\beta}+2\alpha\beta$

$= \frac{\left(\frac{-b}{a}\right)^2-2\frac{c}{a}}{\frac{c}{a}}-\frac{2\left(-\frac{b}{a}\right)}{\frac{c}{a}}+2\frac{c}{a}$

$= \frac{b^2}{a^2}\times\frac{a}{c}-2+\frac{2b}{c}+\frac{2c}{a} = \frac{b^2}{ac}+\frac{2b}{c}+\frac{2c}{a}-2$ Choice (D)

17. Let the initial number of books in dozens = b

Let initial price (in ₹) of books per dozen be p.

pb = 30,000. → (I)

(50 + b) (p – 20) = 30,000

50p – 1000 + pb – 20 b = 30,000

or, 50p – 20b = 1000

5p – 2b = 100. → (II)

From (I) and (II) $5p - \frac{60{,}000}{p} = 100$

5p2 – 100p – 60,000 = 0

5p2 – 600p + 500p – 60,000 = 0

5p(p – 120) + 500 (p – 120) = 0 $\Rightarrow$ p = 120

The price of each book = $\frac{120}{12} = 10$ Ans: (10)

18. If 31 is split into parts a and b,

31 = a + b and 481 = $a^2 + b^2$

$31^2 = (a + b)^2$ Hence $a^2 + b^2 + 2ab = 961$

2ab = 961 – 481 = 480

4ab = 480 × 2 = 960

$a^2 + b^2 + 2ab - 4ab = a^2 + b^2 - 2ab$

$= (a - b)^2 = 961 - 960 = 1 \Rightarrow (a - b) = \sqrt{1} = \pm1$

The difference of (a – b) = |(a – b)|= 1 Ans: (1)

19. (i) $x^4 - 42x^2 + 216 = 0$

$\Rightarrow (x^2) - 42x^2 + 216 = 0$

$\Rightarrow (x^2 - 6)(x^2 - 36) = 0$

$\Rightarrow x^2 - 6 = 0$ or $x^2 - 36 = 0$

$\Rightarrow x = \pm\sqrt{6}, \pm 6$ Choice (A)

(ii) $16(3^{2x+1}) - 32(3^x) + 4 = 0$

$\Rightarrow 48(3^x)^2 - 32(3^x) + 4 = 0$

$\Rightarrow 12(3^x)^2 - 8(3^x) + 1 = 0$

$\Rightarrow 12(3^x)^2 - 6(3^x) - 2(3^x) + 1 = 0$

$\Rightarrow 6(3^x)(2(3)^x - 1) - 1(2(3^x) - 1) = 0$

$\Rightarrow [2(3^x) - 1][6(3^x) - 1] = 0 \Rightarrow 3^x = \frac{1}{2}$ or $\frac{1}{6}$

$a = -\log_3 2$ or $-\log_3 6$ Choice (A)

Note- : (i) and (ii) can be solved using the choices

20. Let the strength be x. The number of students who play basketball = 8

The number of students who play football = x – 8 = $7\sqrt{x}$

Substituting the choices in place of x in the equation above, only choice (C) satisfies it. Choice (C)

21. $\left(x-\frac{1}{x}\right)^2+2-2\left(x-\frac{1}{x}\right)-\frac{5}{4}=0$

Substituting the choices in place of $x - \frac{1}{x}$ in the equation above, we see that only choice (B) satisfies it

Choice (B)

22. Squaring on both sides, $2x + 3 + 4x + 13 - 8^2$

$= -2\sqrt{2x+3}\sqrt{4x+13}$

$\frac{6x-48}{2} = \sqrt{2x+3}\sqrt{4x+13}$

Squaring on both sides,

$9x^2 - 144x + 576 = 8x^2 + 38x + 39$

$x^2 - 182x + 537 = 0$

(x – 179) (x – 3) = 0

x = 179 or 3 Choice (C)

23. For the equation, whose roots are twice the roots of the equation A : $3x^2 - 7x + 4 = 0$, the sum of the roots is twice the sum of the roots of A and the product of the roots is 4 times the product of the roots of A.

The required equation is $x^2 - \left(2\left(\frac{7}{3}\right)\right)x + 4\left(\frac{4}{3}\right) = 0$

i.e., $3x^2 - 14x + 16 = 0$ Choice (D)

24. Let the length and the breadth of the playground be l m and b m respectively.

lb = 247

Side of the square = l – 2 = b + 4 $\Rightarrow$ l = b + 6

$\therefore$ (b + 6) b = 247

(b + 19) (b – 13) = 0

as b > 0, b = 13. The side of the square is b + 4 = 17

Ans: (17)

25. Let ℓ and b be the length and breadth in cm.

Given that ℓ = b + 1

Also given that diagonal = 29 cm

$\Rightarrow \sqrt{\ell^2+b^2} = 29$

By squaring on both sides, $(b + 1)^2 + b^2 = 29^2$

$\Rightarrow 2b^2 + 2b - 840 = 0$

$\Rightarrow b^2 + b - 420 = 0$

$\Rightarrow$ (b + 21) (b – 20) = 0

$\therefore$ b = 20 Ans: (20)

26. Let the length of the sides containing the right angle be a cm and b cm, where a > b

The length of the hypotenuse = $\sqrt{a^2+b^2}$

a = b + 8

$a + b - \sqrt{a^2+b^2} = 16$ → (1)

(1), (2) $\Rightarrow 2b - 8 = \sqrt{(b+8)^2+b^2}$ → (2)

Squaring on both sides and simplifying,

b (b – 24) = 0

as b > 0, b = 24 Ans: (24)

27. Both the roots of the given equation are negative, ie, a < 0, b < 0.

$\therefore$ a/b > 0 and $\sqrt{a/b} + \sqrt{b/a} > 0$

$\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{\frac{|a|}{|b|}} + \sqrt{\frac{|b|}{|a|}}$

$= \dfrac{|a|+|b|}{\sqrt{|ab|}} = \dfrac{-(a+b)}{\sqrt{ab}}$

$= \dfrac{\text{sum of the roots}}{\sqrt{\text{product of the roots}}} = \dfrac{14}{4}\sqrt{\dfrac{4}{3}} = \dfrac{7\sqrt{3}}{3}$

Choice (C)

28. $x^2 - 3x - 108 = 0 \Rightarrow (x - 12)(x + 9) = 0$
x = 12 or – 9
As $|\alpha| > |\beta|$, $\alpha = 12, \beta = -9$, $\alpha - \beta = 12 - (-9) = 21$

Choice (D)

29. As P copies only the coefficient of x wrongly, he must have obtained the correct product.
Correct product = 108
Similarly, the correct sum must have been obtained by Q and this is 24.
The correct equation is $x^2 - 24x + 108 = 0$
The roots are 18 and 6. Choice (C)

30. $(4\sqrt{A})^2 - 4(3B) \geq 0$

$\dfrac{4}{3} A \geq B$

As A is a single digit prime number, A can be 2, 3, 5 or 7.
If A = 2, B has 2 possibilities. If A = 3, B has 4 possibilities.
If A = 5, B has 6 possibilities. If A = 7, B has 9 possibilities.
A total of 21 equations are possible. Ans: (21)

31. Let the roots of the equation $ax^2 + bx + c = 0$ be α and β.
The roots of $ez^2 + fz + y = 0$ are $\alpha + d$ and $\beta + d$.
The roots of $py^2 + qy + p = 0$ are $\alpha + 2d$ and $\beta + 2d$
y = x + 2d Choice (D)

32. The given equation is $3x^3 - 23x^2 + 72x - 70 = 0$

Dividing by 3, we have $x^3 - \dfrac{23}{3}x^2 + \dfrac{72}{3}x - \dfrac{70}{3} = 0 \rightarrow (1)$

Given $3 - \sqrt{5}i$ is a root of (1) $\Rightarrow$ $3 + \sqrt{5}i$ is also root of (1) as complex roots occurs in conjugate pairs

$\therefore$ Sum of the two roots = $3 - \sqrt{-5} + 3 + \sqrt{-5} = 6$

Sum of the roots of (1) = $\dfrac{23}{3}$

$\therefore$ The third root of the equation is $\dfrac{23}{3} - 6 = \dfrac{5}{3}$.

Choice (B)

33. Let the roots of $x^2 - px + q = 0$ be 5a and 5(a + 1)
$\therefore$ p = 5a + 5(a + 1) = 5(2a + 1)
and q = 5a[5(a + 1)] = 25a(a + 1)
and $p^2 - 4q = 25(4a^2 + 4a + 1) - 100a(a + 1) = 25$

Ans: (25)

34. Let $f(x) = 2x^7 - ax^5 - 3x^4 - bx^2 + 7 = 0 \quad \rightarrow \quad (1)$
There are two sign changes in (1) $\therefore$ The number of positive roots, is 2 or 0.
Now $f(-x) = -2x^7 + ax^5 - 3x^4 - bx^2 + 7 = 0 \quad \rightarrow \quad (2)$
There are three change of signs in (2) $\Rightarrow$ The number of negative roots of f(x) = 0 is 1 or 3.
$\therefore$ f(x) = 0 has 0 + 1, 0 + 3, 2 + 1 or 2 + 3 real roots.
$\therefore$ Thus the equation (1) has five real roots and two imaginary roots or 3 real roots and 4 imaginary roots.

Choice (B)

35. Let $f(x) = x^7 - ax^4 + bx^3 - 8 = 0 \quad \rightarrow \quad (1)$
$f(-x) = -x^7 - ax^4 - bx^3 - 8 = 0$
The number of changes of signs in (1) is 3, while (2) does not have any sign changes
$\therefore$ f(x) = 0 has 1 or 3 positive roots and 0 negative roots.
As f(x) = 0 has 6 or 4 complex roots. Choice (C)

36. Let the roots of the equation be $\alpha - \beta, \alpha, \alpha + \beta$
Sum of the roots $3\alpha = 6 \Rightarrow \alpha = 2$
Since α is the root of the equation f(2) = 0
$\Rightarrow (2)^3 - 6(2)^2 + 3(2) + k = 0$
$\Rightarrow$ k = 10. Ans: (10)

37. Let the numbers be x and x + 7.
Given $x(x + 7) + 84 = (x + 7)^2$
$\Rightarrow x^2 + 7x + 84 = x^2 + 14x + 49$
$\Rightarrow 7x = 35 \Rightarrow x = 35$
$\therefore$ The numbers are 5 and 12. Choice (B)

38. The roots of $27x^2 - 87x + k = 0$ are α and 8/3.

$\therefore \alpha + \dfrac{8}{3} = \dfrac{87}{27} = \dfrac{29}{9}$

$\Rightarrow \alpha = \dfrac{5}{9}$

and $\dfrac{8\alpha}{3} = \dfrac{k}{27}$

$\Rightarrow k = 72\alpha = 72\left(\dfrac{5}{9}\right) = 40$ Choice (A)

39. Let the three consecutive even numbers be a – 2, a, a + 2.
The sum of their squares = $(a^2 - 4a + 4) + a^2 + (a^2 + 4a + 4)$
$= 3a^2 + 8 = 440$ (given)

$\therefore a^2 = \dfrac{440 - 8}{3} = 144$.

As a > 0, a = 12.
The three even numbers are 10, 12, 14. Choice (D)

40. The given equation is $3x^2 - 8x + 4 = 0$ ……… (1)

Let 1/x = y ($\therefore x = \dfrac{1}{y}$)

If we express x in terms of y in (1) above, we will get an equation whose roots are y, the reciprocal of x.

i.e., $3\left(\dfrac{1}{y}\right)^2 - 8\left(\dfrac{1}{y}\right) + 4 = 0$

$\Rightarrow 4y^2 - 8y + 3 = 0$
i.e., required equation is $4x^2 - 8x + 3 = 0$

Choice (C)

41. When x = 1, E = a + b + c < 0
When x = – 1, E = a – b + c > 0
As a + b + c < 0 and 0 < a – b + c,
a + b + c < (a – b + c) $\Rightarrow$ b < 0
c, a can be positive or negative. Choice (C)

42. The complex roots for an equation with real coefficients, occur in conjugate pairs. The roots of the given equation are $\alpha = 2 - i\sqrt{5}$, $\beta = 2 + i\sqrt{5}$ and $\gamma = 3$

$\therefore \alpha + \beta + \gamma = 2 - i\sqrt{2} + 2 + i\sqrt{5} + 3 = 7$

$\alpha\beta + \alpha\gamma + \beta\gamma = (4 + 5) + 6 + 3i\sqrt{5} + 6 - 3i\sqrt{5} = 21$

$\alpha\beta\gamma = \left(2 - i\sqrt{5}\right)\left(2 + i\sqrt{5}\right)(3) = 27$

$\therefore$ The required equation is $x^3 - 7x^2 + 21x - 27 = 0$.

Choice (A)

43. Since f(x) = 0 has 3 negative roots, by Descartes rule f(–x) has 3 or 5 or 7 or 9 sign changes. Choice (C)

44. To determine a first degree expression, 2 conditions are needed, for a quadratic, 3 conditions, are needed and for a cubic, 4 conditions are needed. In this example 4 conditions have been given. Therefore we should be able to determine f(x).
But the sequence 5, 8, 11 suggests the linear expression 3x + 2
$\therefore$ let g(x) = (x–1)(x –2)(x –3) + 3x + 2
This function satisfies all the 4 conditions. As such a polynomial has to be unique, f(x) = g(x)
$\therefore$ f(4) = (4 –1)(4 –2)(4 –3) + 14
= 3(2)(1) + 14 = 20 Ans : (20)

45. Let $f(x) = 6x^5 + 11x^4 - px^3 - 33x^2 + qx + 6$
$2x^2 + 5x - 3 = (2x - 1)(x + 3)$ are factors of f(x)

$\Rightarrow f(-3) = 0 = f\left(\frac{1}{2}\right)$

$f(-3) = 6(-3)^5 + 11(-3)^4 - P(-3)^3 - 33(-3)^2 + q(-3) + 6 = 0$
$\Rightarrow 9p - q = 286$ ………..(1)

$$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^5 + 11\left(\frac{1}{2}\right)^4 - P\left(\frac{1}{2}\right)^3 - 33\left(\frac{1}{2}\right)^2 + q\left(\frac{1}{2}\right) + 6 = 0$$

$\Rightarrow p - 4q = -11$ ………..(2)
Solving (1) and (2), we have p = 33 and q = 11.
Choice (C)

Solutions for questions 46 to 55:

46. From statement I,
$x^2 - 4x + 3 = 0, \Rightarrow (x - 3)(x - 1) = 0.$
So x is either 3 or 1. (i.e., x is not equal to 2)
So statement I alone is sufficient.
From statement II,
$x^2 - x + 2 = 0$, which has no real solution.
So, we can answer the question with statement II alone also. Choice (B)

47. For $ax^2 + bx + c = 0$, sum of the roots is $\frac{-b}{a}$ and product is $\frac{c}{a}$.

From statement I, $\frac{c}{a} = \frac{-b}{a}$

From statement II, $\frac{-a}{c} = 2, \Rightarrow \frac{a}{c} = -2$

Combining both the statements, we can say that sum of the roots is $\frac{c}{a} = -\frac{1}{2}$ Choice (C)

48. $ax^2 + bx + c = 0$ has a real solution if $b^2 - 4ac \geq 0$.
Either of the statements alone is not sufficient as the information about a, c and b is given in different statements.
Combining statements I and II
$0 < a < c < 1 \Rightarrow 4ac < 4.$
$b > 3 \Rightarrow b^2 > 9.$, So $b^2 - 4ac > 5$
$\therefore$ It has a real solution. Choice (C)

49. Let the equation be $ax^2 + bx + c = 0$

Using statement I, we get, $\left(\frac{-b}{a}\right)^2 \leq 4\frac{c}{a}$

$b^2 - 4ac \leq 0$
If $b^2 - 4ac = 0$, the roots are real.
If $b^2 - 4ac < 0$, the roots are not real.
I is not sufficient.

Using statement II, we get, $\left(-\frac{b}{a}\right)^2 \geq 4\frac{c}{a}$, $b^2 - 4ac \geq 0$.

$\therefore$ The roots are always real. II is sufficient.
Choice (A)

50. Using statement I, we get
$A(1)^2 + B(1) + C > 0$ and $A(-1)^2 + B(-1) + c < 0$
i.e. $A + B + C > 0$ and $A - B + C < 0$
$\therefore A + B + C - (A - B + C) > 0, \therefore B > 0.$
$\therefore$ A or C is negative.
I is not sufficient.
Using statement II, we know that A or B is negative. II is not sufficient.
Using both statements, we know that A is negative. Both statements taken together are required to answer the question. Choice (C)

51. Discriminant $= B^2 - 4(A)(4A) = B^2 - 16A^2$
As the roots are real, $B^2 - 16A^2 \geq 0$
$\therefore (B - 4A)(B + 4A) \geq 0$
A and B are natural numbers
$\therefore B + 4A \geq 0$
$\therefore B - 4A \geq 0$ i.e. $B \geq 4A$ —— (1)
From statement A, $B < 5$ ——— (2)
$A \geq 1$
$\therefore (1) \Rightarrow B \geq 4$ ——— (3)
From (2) and (3), B = 4
From (1), $A \leq 1$, $\therefore$ A = 1
I is sufficient.
From statement II, $1 \leq A \leq 2$
$\therefore$ A = 1 or 2
II is not sufficient. Choice (A)

52. From statement I, discriminate $= Q^2 - 4PQ \geq 0$
$Q(Q - 4P) \geq 0$
$Q \geq 0$
$\therefore Q - 4P \geq 0$ i.e. $Q \geq 4P$ —— (1)
$\frac{P}{Q}$ is not unique.
From statement II, discriminate $= (4P)^2 - 4(Q)(P) \geq 0$
$4P(4P - Q) \geq 0$
$4P \geq 0$
$\therefore 4P - Q \geq 0$ i.e. $Q \leq 4P$ ——— (2)
$\frac{P}{Q}$ is not unique.
II is not sufficient
Using both statements, from (1) and (2), we get
Q = 4 P
$\therefore \frac{P}{Q} = \frac{1}{4}$
The two statements together are required to answer the question. Choice (C)

53. Let Q be $ax^2 + bx + c$
Maximum value of Q

$$= \frac{4ac - b^2}{4a} = -\frac{(\text{Discriminant of } Q = 0)}{4(\text{Coeffecient of } x^2 \text{ in } Q)}$$

As the discriminant of Q = 0 and the coefficient of x^2 are given in different statements, the question can be answered only by combining the two statements. Choice (C)

54. Let the roots be p and q.
$p + q = -a$ and $pq = b$
Using statement I, we know $a + b = 0 \Rightarrow -a = b$
$\therefore p + q = pq$
$pq - p - q = 0$
$pq - p - q + 1 = 1$
$p(q - 1) - 1(q - 1) = 1$
$(p - 1)(q - 1) = 1$
The roots are integers. $\therefore$ p – 1 and q – 1 are factors of 1.
$\therefore p - 1 = q - 1 = \pm 1$
$\therefore$ p = q = 2 or 0
I is sufficient.
Using statement II, p and q are reciprocal to each other.
$\therefore p = \frac{1}{q}$ i.e. pq = 1.
Also p and q are integers.
$\therefore$ p and q are factors of 1.
$\therefore$ p = q = 1 or – 1.
II is sufficient.
Either of the statements is sufficient. Choice (B)

55. Discriminant $= [2(ab + bc)]^2 - 4(a^2 + b^2)(b^2 + c^2)$
$= 4[a^2b^2 + 2ab^2c + b^2c^2 - (a^2b^2 + a^2c^2 + b^4 + b^2c^2)]$
$= -4(b^2 - ac)^2$
$\Rightarrow$ The discriminant is (0) or negative ———— (1)
Using statement I, we know that p and q are real, i.e. the discriminant is non - negative. ——— (2).
From (1) and (2)

discriminant is 0.

$\therefore$ $b^2 - ac = 0$,

$\therefore$ $\frac{b^2}{ac} = 1$

I is sufficient.

Using statement II, $\frac{b^2}{ac} = 1$

II is sufficient.

Either of the statements is sufficient. Choice (B)

Chapter – 3
(Inequalities and Modulus)

Concept Review Questions

Solutions for questions 1 to 30:

1. **(a)** The values lying between 3 and 4 are represented by (3, 4). Choice (C)

(b) The values lying between 5 and 6 including 5 is written as [5, 6). Choice (B)

(c) The real numbers from 4 to 6 means 4 and 6 included, it can be represented as [4, 6]. Choice (A)

2. When $x < 0$, $|x| = -x$ Choice (D)

3. When $a > b$ and $c > 0$
$\Rightarrow a + c > b + c$ is true
$a - c > b - c$ is also true
$ac > bc$ is also true. Choice (D)

4. When $a < b$ and $c < 0$
Then $ac > bc$ is true. Choice (C)

5. AM of x and $\frac{1}{x}$ is $\frac{x+\frac{1}{x}}{2}$

GM of x and $\frac{1}{x}$ is $\sqrt{x.\frac{1}{x}} = 1$

We know that when $x > 0$
AM $\geq$ GM

$\therefore$ $\frac{x+\frac{1}{x}}{2} \geq 1$ or $x + \frac{1}{x} \geq 2$

$\therefore$ The minimum value is 2. Ans: (2)

6. For all $x > 0$, the value of $\left(1+\frac{1}{x}\right)^x$ always lies between 1 and e. $\therefore$ Among the options only the numbers in (1, 2) can be expressed as $\left(1+\frac{1}{x}\right)^x$. Choice (C)

7. For any two real numbers, p, q

(1) $\frac{p}{q} < 1 \Rightarrow p < q$ is not true always.

(2) $p > 0$, $q > 0$ and $\frac{p}{q} > 1$
$\Rightarrow p > q$ is true.

(3) if $\frac{p}{q} > 1 \Rightarrow p > q$ is not true always. Choice (B)

8. $x \leq 5 \Rightarrow x \in (-\infty, 5]$. Choice (A)

9. $-3 \leq x \leq 8$ can be written as [–3, 8] Choice (C)

10. $3x - 7 \leq 5$
$\Rightarrow 3x \leq 12$
$x \leq 4$.
$\therefore$ Maximum value of x is 4. Ans: (4)

11. $7x - 4 \geq 31$
$\Rightarrow 7x \geq 35$
$x \geq 5$ i.e.,
$\therefore$ The minimum value of x is 5 Ans: (5)

12. $8 - 12x \geq -16$
$\Rightarrow -12x \geq -16 - 8$
$x \leq \frac{-24}{-12}$
$\therefore$ $x \leq 2$
$\therefore$ Maximum value of x is 2 Ans: (2)

13. $-7x + 5 \geq 5x - 19 \Rightarrow -12x \geq -19 - 5$
$\Rightarrow x \leq \frac{-24}{-12} \Rightarrow x \leq 2$ Choice (C)

14. $-9x - 5 < 7x + 27 \Rightarrow -9x - 7x < 27 + 5$
$\Rightarrow -16x < 32 \Rightarrow x > -2$
i.e., $x \in (-2, \infty)$ Choice (C)

15. Given, $-2x \geq 8$
$\Rightarrow x \leq -4$ Choice (D)

16. **(a)** Given, $2x + 7 \leq 9x$
$\Rightarrow 2x - 9x \leq -7$
$-7x \leq -7$
$x \geq 1$ Choice (C)

(b) Given, $4x + 34 > 7x + 31$
$4x - 7x > 31 - 34$
$-3x > -3$
$x < 1$ Choice (B)

(c) Given, $5x - 17 \geq 2x - 15$
$5x - 2x \geq 17 - 15$
$3x \geq 2$
$x \geq \frac{2}{3}$ Choice (C)

17. **(a)** $5x + 3 > 7x - 9$
$\Rightarrow 5x - 7x > -9 - 3$
$-2x > -12$
$x < 6$ Choice (C)

(b) $4x + 3 \geq 3x - 12$
$4x - 3x \geq -15$
$x \geq -15$ Choice (D)

18. Given $5x - 8 < 2x + 9$
$\Rightarrow 3x < 17$;
$x < \frac{17}{3}$; $\rightarrow$ (1)
$4x + 7 > 7x - 8$
$\Rightarrow 15 > 3x$
$\Rightarrow x < 5 \rightarrow$ (2)
The common solution for (1) and (2) is $x < 5$; $(-\infty, 5)$
Choice (B)

19. $\frac{1}{3x+1} \geq \frac{1}{3}$
Since $3x + 1$ is positive the above can be expressed as $3 \geq 3x + 1$
i.e. $3x + 1 \leq 3$ or $1 < 3x + 1 \leq 3$ Choice (B)

20. $2x - 3 \geq 7 \Rightarrow 2x \geq 10 \Rightarrow x \geq 5. \rightarrow$ (1)
$5x - 7 < 3 \Rightarrow x < 2$ $\rightarrow$ (2)
From (1) and (2), there are no common values of x.
$\therefore$ The solution is the empty set. Choice (A)

21. $-2 > -5$ but $(-2)^2 < (-5)^2$. Option (A) is not true always.
$5 > 2$ but $(5)^2 > (2)^2$. Option (B) is not true always.
But when $x > 0$, $y > 0$, and $x > y \Rightarrow x^2 > y^2$.
Option (C) is always true. Choice (C)

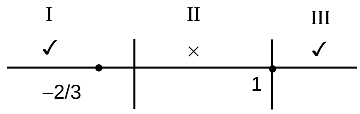
22. **(a)** We know that $|x|$ is ≥ 0 for all $x \in R$.
$\therefore$ No real value of x satisfies $|2x + 1| < 0$
$\therefore$ The solution set is the null set. Choice (D)

(b) $|2x - 3| \geq 0$
We know $|x|$ always ≥ 0 for $x \in R$
$\therefore |2x - 3| \geq 0 \ \forall\ x \in R$ Choice (C)

23. $x^2 - 9x - 36 < 0$
$\Rightarrow (x - 12)(x + 3) < 0$
$\Rightarrow x \in (-3, 12)$ Choice (A)

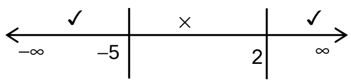
24. $\left|x^2 - 16\right| = 0$
$\Rightarrow x^2 - 16 = 0$
$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
$\therefore \{-4, 4\}$ Choice (D)

25. **(a)** Given $5x^2 - 3x - 2 \geq 0$
$(x - 1)(5x + 2) \geq 0$
The critical points are, 1, $\frac{-2}{5}$



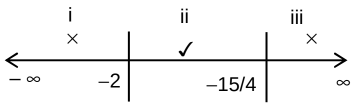
When x = 0, the inequality is not true
I and II regions satisfies the inequality
$\therefore$ solution set is $(-\infty, \frac{-2}{5}] \cup [1, \infty)$
$\therefore R - \left(\frac{-2}{5}, 1\right)$ Choice (B)

(b) $\frac{x+5}{x-2} \geq 0$
$\Rightarrow (x + 5)(x - 2) \geq 0$
Critical points are – 5, 2



When x = 0, the inequation is not satisfied.
The solution set is $R - (-5, 2]$
$\therefore$ The integral values of x that do not satisfy the inequation is – 4, – 3, –2, –1, 0, 1, 2.
$\therefore$ i.e., 7 Ans: (7)

(c) $4x^2 - 7x - 30 < 0$
$4x^2 + 8x - 15x - 30 < 0$
$4x(x + 2) - 15(x + 2) < 0$
$(x + 2)(4x - 15) < 0$
Critical points are – 2, $\frac{15}{4}$



When x = 0 the equation is satisfied.
$\therefore$ only II region satisfies
$\therefore$ solution set is $-2 < x < \frac{15}{4}$
$\therefore x \in \left(-2, \frac{15}{4}\right)$ Choice (A)

26. Given $\left|x-5\right| < 9$
We know that if $\left|x\right| < a \Rightarrow -a < x < a$
$\left|x-5\right| < 9 \Rightarrow -9 < x - 5 < 9$
$\Rightarrow -4 < x < 14$ Choice (B)

27. Given $\left|5x-7\right| = 12$
$5x - 7 = \pm 12$
$5x - 7 = 12$ or $5x - 7 = -12$
$5x = 19$ or $5x = -5$
$x = \frac{19}{5}$ or $x = -1$
$\because x > 0 \Rightarrow x = \frac{19}{5} = 3.8$ Ans: (3.8)

28. Given $(x + 5)(x + 9)(x + 3)^2 < 0$
Since $(x + 3)^2$ is always positive
$(x + 5)(x + 9) < 0$
$\Rightarrow -9 < x < -5$
Solution set is $(-9, -5)$ Choice (B)

29. The given options are properties of modulus,
$\therefore$ all are true. Choice (D)

30. $6x + 8 > 7x - 9 \Rightarrow 17 > 7x - 6x$
$\Rightarrow 17 > x$
$\Rightarrow x < 17$
$4x - 7 < 6x - 3 \Rightarrow 4x - 6x < 7 - 3$
$\Rightarrow -2x < 4$
$\Rightarrow x > -2$
$\therefore$ Solution set is $(-2, 17)$. Choice (C)

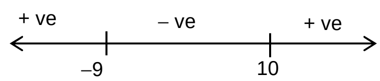
Exercise – 3(a)

Solutions for questions 1 to 30:

1. $3x + 4 \geq -5 \Rightarrow 3x \geq -5 - 4$
$\Rightarrow 3x \geq -9$
$\Rightarrow x \geq -3 \quad \rightarrow \quad (1)$
$8x - 13 \leq 19$
$\Rightarrow 8x \leq 19 + 13$
$\Rightarrow 8x \leq 32 \Rightarrow x \leq 4 \quad \rightarrow \quad (2)$
From (1) and (2), the common solution is
$-3 \leq x \leq 4 \Rightarrow x \in [-3, 4]$ Choice (B)

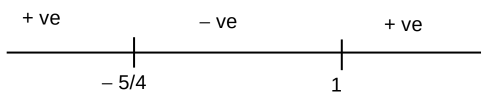
2. $8 - 3x \leq 5 \Rightarrow 3x \geq 8 - 5$
$\Rightarrow 3x \geq 3 \Rightarrow x \geq 1 \quad \rightarrow \quad (1)$
$4x + 5 \leq -7 \Rightarrow 4x \leq -7 - 5$
$\Rightarrow 4x \leq -12 \Rightarrow x \leq -3 \quad \rightarrow \quad (2)$
There is no x that satisfies both (1) and (2)
$\therefore$ The range is ϕ, the empty set. Choice (D)

3. $-x^2 + x + 90 > 0$
$\Rightarrow x^2 - x - 90 < 0$
$\Rightarrow (x - 10)(x + 9) < 0$



When x belongs to (–9, 10), the sign of the expression is negative. Choice (C)

4. $4x^2 + x - 5 > 0$
$\Rightarrow 4x^2 + 5x - 4x - 5 > 0$
$\Rightarrow (x - 1)(4x + 5) > 0$



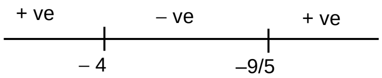
When x belongs to $(-\infty, -5/4) \cup (1, \infty)$, the sign of the expression is positive. Choice (D)

5. $\frac{2x-3}{x+4} < -3$ i.e., $\frac{2x-3}{x+4} + 3 < 0$
$\Rightarrow \frac{5x+9}{x+4} < 0$

$\Rightarrow \frac{5x+9}{x+4} \cdot \frac{x+4}{x+4} < 0$

$\Rightarrow (5x + 9)\ (x + 4) < 0$

As the denominator $(x + 4)^2 > 0$

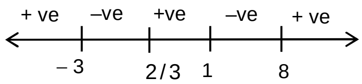


Hence the solution is (–4, –9/5) Choice (C)

6. $\frac{3x^2+7x-6}{x^2-9x+8} < 0$

$\Rightarrow \frac{(3x-2)(x+3)}{(x-8)(x-1)} < 0$

By multiplying and dividing the expression by $(x – 8)(x – 1)$ we get $(3x – 2)\ (x + 3)\ (x – 8)\ (x – 1) < 0$



Hence x belongs to $\left(-3, \frac{2}{3}\right) \cup (1, 8)$. Choice (A)

7. For any $x \geq 1$ we have $2 \leq \left(1+\frac{1}{x}\right)^x < 2.8$

Consider choice (A)

$\frac{31^{30}}{30^{31}} = \left(\frac{31}{30}\right)^{30} \cdot \frac{1}{30} = \left(\frac{30+1}{30}\right)^{30} \cdot \frac{1}{30}$

$= \left(1+\frac{1}{30}\right)^{30} \cdot \frac{1}{30} < \frac{2\cdot 8}{30} < 1$

i.e., $\frac{31^{30}}{30^{31}} < 1$ i.e., $31^{30} < 30^{31}$.

Choice (A) is false

Similarly, it can be seen that (2) is false

Consider choice (C)

$\frac{(155)^{29}}{(150)^{30}} = \left(\frac{155}{150}\right)^{30} \cdot \frac{1}{155}$

$= \left(1+\frac{1}{30}\right)^{30} \cdot \frac{1}{155} < \frac{2\cdot 8}{155} < 1$

$\therefore \frac{(155)^{29}}{(150)^{30}} < 1$ i.e., $(155)^{29} < (150)^{30}$.

$\therefore$ Choice (C) is true. Choice (C)

8. 6 + |4 – 7x| will have the minimum value when |4 – 7x| has the minimum value which is 0. Hence the minimum value of 6 + |4 – 7x| is 6. Ans: (6)

9. $-|x - 3| + \frac{21}{2}$ has the maximum value of $\frac{21}{2}$ when $|x – 3| = 0$

i.e. when x = 3. Ans: (3)

10. The maximum value of 3 – |2x – 1| is 3 as the minimum value of |2x – 1| is 0. Ans: (3)

11. $|2x + 3| \geq 7$

Using the basic definition of modulus, we have

$|2x + 3| = \begin{cases} 2x+3 \text{ if } x \geq \frac{-3}{2} \\ -2x-3 \text{ if } x < \frac{-3}{2} \end{cases}$

Case 1: If $x \geq \frac{-3}{2}$ the inequality is $2x + 3 \geq 7$.

i.e. $x \geq 2$ which falls within the range of $x \geq \frac{-3}{2}$

Hence $x \geq 2$ is an admissible range of values of x.

Case 2: If $x < \frac{-3}{2}$ then $-2x - 3 \geq 7$ i.e., $2x \leq -10$

$\Rightarrow x \leq -5$

$x \leq -5$ is also admissible as it agrees with $x < -3/2$.

Hence the solution set is $(-\infty, -5] \cup [2, \infty)$.

Choice (D)

12. $|3x + 5| = \begin{cases} 3x+5 \text{ if } x \geq \frac{-5}{3} \\ -3x-5 \text{ if } x < \frac{-5}{3} \end{cases}$

Case 1: If $x \geq \frac{-5}{3}$.

The inequality is $3x + 5 < 5x – 11$ i.e., $2x > 16$

$\Rightarrow x > 8$

$\therefore$ x > 8 is a possible set of solutions

Case 2: If $x < \frac{-5}{3}$.

The inequality is $-3x - 5 < 5x - 11$

$\Rightarrow 8x > 6 \Rightarrow x > \frac{3}{4}$

This is not a possible set of solutions as this contradicts the assumption, $x < \frac{-5}{3}$

$\therefore$ The solution set is $(8, \infty)$. Choice (A)

13. We make use of the fact that for any two positive numbers, their Arithmetic mean (A.M) ≥ Geometric mean (G.M) ≥ Harmonic mean (H.M)

Consider choice (A): $\frac{x+y}{2} \geq \sqrt{xy}$ (A.M ≥ G.M)

$x + y \geq 2\sqrt{xy} \Rightarrow y + z \geq 2\sqrt{yz} \Rightarrow x + z \geq 2\sqrt{xz}$

Multiplying the 3 equations, we get

$(x + y)\ (y + z)\ (z + x) \geq 8\sqrt{x^2y^2z^2}$

i.e. $(x + y)\ (y + z)\ (z + x) \geq 8xyz$.

Consider choice (B)

$\frac{x^2y^2+y^2z^2}{2} \geq \sqrt{x^2y^4z^2}$

(As A.M. of x^2y^2 and y^2z^2 ≥ G.M. of x^2y^2 and y^2z^2)

i.e. $x^2y^2 + y^2z^2 \geq 2xy^2z$

similarly $y^2z^2 + z^2x^2 \geq 2xyz^2$

$x^2z^2 + x^2y^2 \geq 2x^2yz$

Adding the three inequalities, we get

$2(x^2y^2 + y^2z^2 + z^2x^2) \geq 2xyz\ (x + y + z)$

i.e. $(xy + yz + zx)^2 – 2xyz(x + y + z) \geq xyz(x + y + z)$

Thus $(xy + yz + zx)^2 \geq 3xyz(x + y + z)$

Hence option (B) is also correct.

Similarly we can prove option (C) is true Choice (D)

14. We know that $\frac{a^2+c^2}{2} \geq ac$ (A.M of a^2, c^2 compared to G.M of a^2, c^2) i.e., $a^2 + c^2 \geq 2ac$

Similarly $b^2 + d^2 \geq 2bd$

Adding both these inequalities we get

$a^2 + b^2 + c^2 + d^2 \geq 2(ac + bd)$

But ac = bd = 2

$\therefore a^2 + b^2 + c^2 + d^2 \geq 2(2 + 2) = 8$

$\therefore$ The minimum value of $a^2 + b^2 + c^2 + d^2$ is 8.

Ans: (8)

15. Given: x , y > 0 and x + y = 3

Now $\frac{x+y}{2} \geq \sqrt{xy}$ i.e., $(x + y)^2 \geq 4xy$

But x + y = 3

$\therefore 4xy \leq 9$ or $xy \leq \frac{9}{4}$ i.e., $xy \leq 2.25$. Choice (C)

16. We now consider 3 cases

Case 1: $x \leq 1$

$|x - 5| + |x - 1|$

$= 5 - x + 1 - x = 6 - 2x$

$\therefore 6 - 2x < 2$ i.e., $x > 2$

This is inconsistent, so no solution is possible.

Case 2: $1 < x < 5$

$|x - 5| + |x - 1|$

$= 5 - x + x - 1 < 2$

$\Rightarrow 4 < 2$.

This is again inconsistent. Hence no solution is possible.

Case 3: $x \geq 5$

$|x - 5| + |x - 1|$

$= x - 5 + x - 1$

$= 2x - 6 < 2$

$\Rightarrow 2x < 8$

$\Rightarrow x < 4$

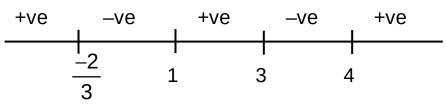
This is also inconsistent. Hence no solution is possible. Hence 0 solutions are possible. Ans: (0)

17. $(3x^2 - 7x - 6)(x^2 - 5x + 4) < 0$

$(3x^2 - 9x + 2x - 6)(x^2 - 4x - x + 4) < 0$

$(3x + 2)(x - 3)(x - 4)(x - 1) < 0$

Putting the roots on the number line in the order of increasing numbers,



Hence the inequality holds good for all real values in $(-2/3, 1) \cup (3, 4)$.

The integer values that satisfy the inequality is x = 0. Hence one integer value satisfies the inequalities.

Choice (A)

18. $\frac{1}{|2x-7|} > \frac{2}{9}$

$\Rightarrow |2x - 7| < \frac{9}{2} = 4{\cdot}5$

$-4.5 < 2x - 7 < 4.5$

$2.5 < 2x < 11.5$

$\Rightarrow 1.25 < x < 5.75$

$\Rightarrow 2 \leq x \leq 5$.

The difference between the greatest and the least integer in this range is 5 – 2 = 3. Ans: (3)

19. **Case (i):**- $x \geq 16$

$\therefore |x - 16| = x - 16$

So the relation becomes

$x - 16 > x^2 - 7x + 24$

$\Rightarrow x^2 - 8x + 40 < 0$

$\Rightarrow x^2 - 8x + 16 + 24 < 0$

$\Rightarrow (x - 4)^2 + 24 < 0$

But $(x - 4)^2 + 24$ is positive for all real x.

So there cannot be any solution in this domain.

Case (ii):- $x < 16$

$|x - 16| = 16 - x$

The relation becomes

$16 - x > x^2 - 7x + 24$

$\Rightarrow x^2 - 6x + 8 < 0 \Rightarrow (x - 4)(x - 2) < 0$

$\Rightarrow x \in (2, 4)$

which is consistent with x < 16

Hence the range is (2, 4). Choice (D)

20. Let $a = x^2 + x$, $b = x^3 + 1$

We need to find a range where a < b

i.e., $x^2 + x < x^3 + 1$.

$\Rightarrow x^3 - x^2 - x + 1 > 0$ or $x^2(x - 1) - 1(x - 1) > 0$

$\Rightarrow (x^2 - 1)(x - 1) > 0$ or $(x - 1)^2 (x + 1) > 0$

as $(x - 1)^2 > 0$ except at x = 1, x + 1 > 0 i.e., x > –1.

Hence, the range is $(-1, 1) \cup (1, \infty)$. Choice (C)

21. Given $a^2 b^3 = (540) . (35)^2 = 2^2 3^3 5^3 7^2$

We have to minimize 5a + 7b

Consider

$\left(\frac{5a}{2}\right)^2 \left(\frac{7b}{3}\right)^3 = \left(\frac{5}{2}\right)^2 \left(\frac{7}{3}\right)^3 . a^2b^3 = 5^5 7^5$

The product of the 5 factors on the left is constant.

$\therefore$ The sum of the 5 factors, rtz, $2\left(\frac{5a}{2}\right) + 3\left(\frac{7b}{3}\right)$

or 5a + 7b has its minimum value when

$\left(\frac{5a}{2}\right) = \left(\frac{7b}{3}\right) = 5 (7)$.

This minimum value is 2(5) (7) + 3 (5) (7) = 175.

Choice (C)

22. Given a + b + c = 24

consider the product

$\left(\frac{a}{2}\right)^2 \left(\frac{b}{3}\right)^3 \left(\frac{c}{1}\right)^1$

$= \frac{a}{2}.\frac{a}{2}.\frac{b}{3}.\frac{b}{3}.\frac{b}{3} . c$

The sum of above factors is $2.\frac{a}{2} + 3.\frac{b}{3} + c$

= a + b + c = 24

$\therefore$ The product is maximum, when all the 6 factors are equal (and hence equal to 4)

$\frac{a}{2} = \frac{b}{3} = c = 4$

$\Rightarrow$ a = 8; b = 12; c = 4

$\therefore$ The value of $a^2 b^3 c$ is $8^2 12^3 4$

$= (2^3)^2 (2^2 . 3)^3 . 2^2 = 2^{14} 3^3$ Choice (D)

23. Given, $\frac{1}{a^3 b^3 c^3}$ $[(a^3 + b^3)^2 c^3 + (b^3 + c^3)^2 a^3 + (a^3 + c^3)^2 b^3] = \frac{(a^3+b^3)^2}{a^3 b^3} + \frac{(b^3+c^3)^2}{b^3 c^3} + \frac{(a^3+c^3)^2}{a^3 c^3}$

As a, b, c are positive, AM $\geq$ HM

$\Rightarrow \frac{a^3+b^3}{2} \geq \frac{2a^3 b^3}{a^3+b^3} \Rightarrow \frac{(a^3+b^3)^2}{a^3 b^3} \geq 4$

similarly $\frac{(b^3+c^3)^2}{b^3 c^3} \geq 4$ and $\frac{(a^3+c^3)^2}{a^3 c^3} \geq 4$

by adding all the above results we have

$\frac{(a^3+b^3)^2}{a^3 b^3} + \frac{(b^3+c^3)^2}{b^3 c^3} + \frac{(c^3+a^3)^2}{c^3 a^3} \geq 4 + 4 + 4 = 12$

$\therefore$ The required minimum value = 12 Choice (B)

24. $x^2 - 14x + 56 < 0$

$x^2 - 14x + 49 + 7 < 0$

$(x - 7)^2 + 7 < 0$

Since $(x - 7)^2$ is always positive for any value of x, the above inequation is not true.

$\therefore$ Solution is an empty set. Choice (C)

25. $|5x+3| > 14$

$\Rightarrow 5x + 3 < -14$ or $5x + 3 > 14$

$\Rightarrow 5x < -17$ or $5x > 11$

$x < \frac{-17}{5}$ or $x > \frac{11}{5}$

Since $x > 0$, $x > \frac{11}{5}$ Ans: (2.2)

26. $x^2 - 15|x| + 56 = 0$

$|x|^2 - 15|x| + 56 = 0$

$(|x| - 7)(|x| - 8) = 0$

$|x| = 7$ or $|x| = 8$

$\therefore x = \pm 7$ or $x = \pm 8$

The number of solutions are = 4 Ans: (4)

27. Given $|2x - |5x - 3|| = 18$

Case-1: $5x - 3 > 0; \Rightarrow x > \frac{3}{5}$

Then $|2x - 5x + 3| = 18$

$3 - 3x = \pm 18$

$3x - 3 = \pm 18$

$3x = -15$ or 21

$\therefore x = -5$ or 7

Since $x > 3/5$, $x = 7$ is the solution

Case-2: $5x - 3 < 0 \Rightarrow x < \frac{3}{5}$ then

$\Rightarrow |2x + 5x - 3| = 18$

$|7x - 3| = 18$

$7x - 3 = 18$ or $7x - 3 = -18$

$7x = 21 \Rightarrow x = 3$; or $7x = -15 \Rightarrow x = \frac{-15}{7}$

$\because x < 3/5$; $x = -15/7$ is the solution

$\therefore$ The solutions are 7, $\frac{-15}{7}$

$\therefore$ number of solutions = 2 Ans: (2)

28. Consider $4x^2 + x + 4 = x\left(4\left(x + \frac{1}{x}\right) + 1\right) \geq 9x \left(\because x + \frac{1}{x} \geq 2\right)$

similarly $5y^2 + y + 5 \geq 10y$ and $7z^2 + z + 7 \geq 15z$

$\therefore \frac{(4x^2 + x + 4)(5y^2 + y + 5)(7z^2 + z + 7)}{xyz} \geq$

$\frac{(9x)(11y)(15z)}{xyz} = 1485$ Choice (D)

29. The value of the quadratic expression is positive

$\Rightarrow b^2 - 4ac < 0 \Rightarrow (-6)^2 - 4a(9) < 0 \Rightarrow 36 - 36a < 0$

$a > 1$

There for the range of values for a is $(1, \infty)$

Choice (B)

30. $2x^2 - 5x - 8 \leq |2x^2 + x|$ ----- (1)

$2x^2 + x = 0 \Rightarrow x = 0$ or $x = -1/2$

If $x \leq -1/2$ or $x \geq 0$, then $2x^2 + x \geq 0$ and $|2x^2 + x| = 2x^2 + x$.

If $-1/2 < x < 0$, then $2x^2 + x < 0$ and $|2x^2 + x| = -2x^2 - x$

Let $x \leq -1/2$ or $x \geq 0$

(1) $\Rightarrow 2x^2 - 5x - 8 \leq 2x^2 + x$

$\Rightarrow 6x + 8 \geq 0 \Rightarrow x \geq -4/3$

$\therefore x \in [-4/3, \alpha) - (-1/2, 0)$ ------ (A)

Let $-1/2 < x < 0$

(1) $\Rightarrow 2x^2 - 5x - 8 \leq -2x^2 - x$

$\Rightarrow 4x^2 - 4x - 8 \leq 0 \Rightarrow x^2 - x - 2 \leq 0$

$\Rightarrow (x - 2)(x + 1) \leq 0 \Rightarrow -1 \leq x \leq 2$

$\therefore x \in (-1/2, 0)$ ------ (B)

From (A), (B) $x \in [-4/3, \infty)$ Choice (A)

Exercise – 3(b)

Solutions for questions 1 to 40:

1. $\frac{x-5}{x+7} > 4$

$\frac{x-5}{x+7} - 4 > 0$

$\Rightarrow \frac{x - 5 - 4x - 28}{x+7} > 0$

$\Rightarrow \frac{-3x - 33}{x+7} > 0$

$\Rightarrow \frac{x+11}{x+7} < 0$

$\Rightarrow \frac{(x+11)(x+7)}{(x+7)^2} < 0$

$\Rightarrow (x + 7)(x + 11) < 0$

When $-11 < x < -7$, the above inequation is true.

The number of integral values between $-11$and -7 is 3.

$\therefore$ Hence the number of integral solutions is 3.

Ans: (3)

2. For positive numbers, the GM is less than or equal to the AM.

$\therefore \sqrt[3]{(xy)(yz)(zx)} \leq \frac{xy + yz + zx}{3}$

i.e., $xy + yz + yx \geq 3(xyz)^{2/3} = 3(216)^{2/3} = 108$

$\therefore$ 96 is not a possible value. Choice (D)

3. Given: $1 \leq x \leq 3$, $2 \leq y \leq 5$.

$\frac{x}{y}$ is minimum when x is minimum and y is maximum,

$\therefore$ The minimum value of $\frac{x}{y} = \frac{1}{5}$

The minimum value of $\frac{x+y}{y} = 1 + \frac{x}{y}$

$= 1 + \frac{1}{5} = \frac{6}{5}$ Ans: (1.2)

4. Given: $3x + 2 < |2x + 5| < 8x + 9$

Case: 1, $x > \frac{-5}{2}$, $|2x + 5| = 2x + 5$

$3x + 2 < 2x + 5 < 8x + 9$

consider $3x + 2 < 2x + 5$

$x < 3$ → (1)

$2x + 5 < 8x + 9$

$\Rightarrow -6x < 4$

$x > \frac{-4}{6}$ → (2)

From (1) and (2) $\frac{-2}{3} < x < 3$ → I

Case: $2x < \frac{-5}{2}$ then $|2x + 5| = -(2x + 5)$

$3x + 2 < -(2x + 5) < 8x + 9$

Consider $3x + 2 < -(2x + 5)$

$3x + 2x < -5 - 2$

$5x < -7$

$x < -7/5$ → (3)

$-(2x + 5) < 8x + 9$

$-10x < 9 + 5$

$-10x < 14$

$\Rightarrow x > \frac{-14}{10}$

$x > {-7}/{5}$

$x < {-7}/{5}$ and $x > {-7}/{5}$ such a value of x does not exist.

$\therefore$ The solution set of the inequation is $\frac{-2}{3} < x < 3$

Choice (A)

5. Given: $2x + 3y = 10$
consider the expression

$$\left(\frac{2x}{3}\right)^3 \cdot \left(\frac{3y}{2}\right)^2$$

$$3\left(\frac{2x}{3}\right) + 2\left(\frac{3y}{2}\right) = 2x + 3y = 10$$

$\therefore$ The sum is constant.

The product $\left(\frac{2x}{3}\right)^3\left(\frac{3y}{2}\right)^2$ is maximum when $\frac{2x}{3} = \frac{3y}{2}$

$$\frac{2x}{3} = \frac{3y}{2} = \frac{2x+3y}{3+2}$$

$$\frac{2x}{3} = \frac{3y}{2} = 2$$

$$\frac{2x}{3} = 2 \Rightarrow x = 3;\ \frac{3y}{2} = 2$$

$y = {4}/{3}$

$\therefore$ The maximum value of the product $x^3\,y^2$

is $3^3 \cdot \left(\frac{4}{3}\right)^2 = 27 \cdot \frac{16}{9} = 48.$ Ans: (48)

6. Given:

$$\frac{x^2-7x+10}{x^2+6x-40} < 1$$

$$\Rightarrow \frac{x^2-7x+10}{x^2+6x-40} - 1 < 0$$

$$\Rightarrow \frac{x^2-7x+10-x^2-6x+40}{x^2+6x-40} < 0$$

$$\Rightarrow \frac{-13x+50}{x^2+6x-40} < 0$$

$$\Rightarrow \frac{(13x-50)(x^2+6x-40)}{(x^2+6x-40)^2} > 0$$

$\Rightarrow (13x - 50)(x + 10)(x - 4) > 0$

The critical points are $\frac{50}{13}$, -10, 4

X	✓	X	✓
-10		$\frac{50}{13}$	4

when $x = 0$ the inequation is true
$\therefore$ The values of x lying in the 2nd and fourth regions satisfy the above inequations.

Solution set is $\left(-10, \frac{50}{13}\right) \cup (4, \infty)$ Choice (B)

7. $x = |a|b$ and $5 \le |b|$

$\therefore xb = |a|b^2 \ge 25\,|a|$
Consider $a - xb$
From a, we are subtracting a quantity that is greater than or equal to 25 |a|. If a = 0, this could be 0 or negative.
But if $a \neq 0$, this would be negative.
$\therefore a - xb \le 0.$ Choice (D)

8. Given; $\left|x - |x-2|\right| = 6$

When $x < 2$, $|x-2| = -(x-2)$

$\therefore |x-(-(x-2| = 6 \Rightarrow (2x-2) = \pm 6$
$x - 1 = \pm 3$
$x = 4$ or -2
since $x < 2$; so $x = -2$ is the only solution.
when $x > 2$ the equation is not true. Ans: (1)

9. GM $(a^3, b^3, c^3) \le$ AM (a^3, b^3, c^3)

$\therefore abc \le \frac{a^3 + b^3 + c^3}{3} = 9$

$\therefore$ The maximum value of abc is 9. Ans: (9)

10. Given: $x \le 4$, $\rightarrow$ (1) and $y \ge -2$
$y \ge -2 \Rightarrow -y \le 2$ $\rightarrow$ (2)
adding (1) and (2) we get
$x - y \le 4 + 2 = x - y \le 6$ [$\therefore$ (C) is true]
If $x = -10$, $y = 1$, $xy = -10$ [$\therefore$ (A) and (B) are false]

Choice (C)

11. If the product of several numbers is constant, their sum is minimum when they are all equal.

As $a_1a_2 \ldots a_{3n} = 1$, the minimum value of $a_1 + a_2 + \ldots + a_{3n}$ is $1 + 1 + \ldots + 1$ (3n times) = 3n. Choice (D)

12. We know that
AM (a, b, c, d) $\ge$ HM (a, b, c, d)

$$\frac{a+b+c+d}{4} \ge \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \ge 16$$

$\therefore$ The minimum value is 16. Ans: (16)

13. Given abcde = 32
We know that
AM $(1 + a, 1 + b, \ldots 1 + e) \ge$ GM $(1 + a, \ldots 1 + e)$

$$\frac{(1+a)+(1+b)+\ldots+(1+e)}{5} \ge \sqrt[5]{(1+a)(1+b)\ldots(1+e)}$$

$$(1+a)(1+b)\ldots(1+e) \le \left(\frac{5+a+b+c+d+e}{5}\right)^5$$

Since abcde = 32, the minimum value of a + b + c + d + e = 10
$\therefore$ The minimum value of

$(1 + a)(1 + b)(1 + c)(1 + d)(1 + e)$ is $\left(\frac{15}{5}\right)^5 = 243$

Ans: (243)

14. Given: $a + b + c = 12$
AM $(a + b, b + c, c + a) \ge$ GM $(a + b, b + c, c + a)$

$$\frac{a+b+b+c+c+a}{3} \ge \sqrt[3]{(a+b)(b+c)(c+a)}$$

$$\sqrt[3]{(a+b)(b+c)(c+a)} \le \frac{2}{3}(a+b+c)$$

$$\sqrt[3]{(a+b)(b+c)(c+a)} \le \frac{2}{3}12$$

$(a + b)(b + c)(c + a) \le 512$ Choice (D)

15. $\dfrac{x^2y+y^2z+z^2x+xy^2+yz^2+zx^2}{xyz}$

$= \dfrac{x^2y+x^2z+y^2z+y^2x+z^2x+z^2y}{xyz}$

$= \dfrac{x^2(y+z)+y^2(z+x)+z^2(x+y)}{xyz}$

$= \dfrac{x(y+z)}{yz}+\dfrac{y(z+x)}{xz}+\dfrac{z(x+y)}{xy}$

$= \dfrac{x}{z}+\dfrac{x}{y}+\dfrac{y}{x}+\dfrac{y}{z}+\dfrac{z}{y}+\dfrac{z}{x}$

$= \left(\dfrac{x}{z}+\dfrac{z}{x}\right)+\left(\dfrac{x}{y}+\dfrac{y}{x}\right)+\left(\dfrac{z}{y}+\dfrac{y}{z}\right)$

Since the sum of a number and its reciprocal is ≥ 2.
$\therefore$ The minimum value of the sum is = 2 + 2 + 2 = 6
Ans: (6)

16. The 5 expressions in the options involve x, x^2, y, y^2.
$\therefore$ We first determine the range of values for these 4 quantities.
$-7 \leq y \leq -3$ and $2 \leq x \leq 5$
$\therefore 9 \leq y^2 \leq 49 \quad \therefore 4 \leq x^2 \leq 25$
We note that $xy^2 \geq 0$ and $2x^2y < x^2y$. $\therefore$ We need to look at only choices (C), (D), (E). The range of possible values for these are tabulated below.
$-35 \leq xy \leq -6$
$-490 \leq -2xy^2 \leq -36 \because 18 < xy^2 < 245$
$-350 \leq 2x^2y \leq -24$
$\therefore$ The minimum value of $-2xy^2$ is the least.
Choice (D)

17. $y^{2/3} - 3y^{1/3} - 10 \leq 0$

Let $y^{1/3} = a$

$\Rightarrow a^2 - 3a - 10 \leq 0$
$(a - 5)(a + 2) \leq 0$
$\Rightarrow -2 \leq a \leq 5$
$-2 \leq y^{1/3} \leq 5$
or $-8 \leq y \leq 125$
Choice (C)

18. Given: $|x-3|+|x-4| \leq 7$

Case 1: when $x \geq 4$, $|x-3| = (x-3)$ and $|x-4| = x - 4$

$|x-3|+|x-4| \leq 7 \Rightarrow x - 3 + x - 4 \leq 7$
$2x \leq 14 \Rightarrow x \leq 7$
$\therefore x \in [4, 7]$
Case 2: when $x \leq 3$, $|x-3| = 3 - x$ and $|x-4| = 4 - x$

$|x-3|+|x-4| \leq 7 \Rightarrow 3 - x + 4 - x \leq 7$
$-2x \leq 0$
$x \geq 0 \therefore x\ [0, 30$
When $x \in (3, 4)$, $|x-3| = x - 3$; $|x-4| = 4 - x$

$|x-3|+|x-4| \leq 7 \Rightarrow x - 3 + 4 - x \leq 7$
$\Rightarrow 1 \leq 7$ is always true
$\therefore$ The solution set = $x \in [0, 7]$
The number of integral solutions of the given inequation is 8.
Ans: (8)

19. Given: $x^2 - 5x + 6 > 0$
$(x - 2)(x - 3) > 0$
$x < 2$ or $x > 3$
$x^2 - 3x + 2 > 0$; $(x - 2)(x - 1) > 0$
$x < 1$ or $x > 2$
$\therefore$ The common solution is $x < 1$ or $x > 3$ Choice (B)

20. Given: $\dfrac{x-3}{x+2} < 0$

$\Rightarrow \dfrac{(x-3)(x+2)}{(x+2)^2} < 0 \Rightarrow (x + 2)(x - 3) < 0$
$-2 < x < 3$
The number of integral values of x that satisfy is 4.
Ans: (4)

21. Given: $3x + 17 < 5x - 19$ and $4x + 15 > 9x + 21$
$3x + 17 < 5x - 19$
$3x - 5x < -19 - 17$
$-2x < -36$
$x > 18 \quad \rightarrow \quad (1)$
$4x + 15 > 9x + 21$
$4x - 9x > 21 - 15$
$-5x > 6$
$x < -6/5 \quad \rightarrow \quad (2)$
From (1) and (2) we see that there is no common solution.
The solution set is { } Choice (C)

22. Let $f(x) = |x+3|+|x-5| + 7$

When f (x) = x > 5, $|x+3| = x + 3$ and

$|x-5| = x - 5$
$\therefore f(x) = x + 3 + x - 5 + 7 = 2x + 5$
when $x < -3$, $|x+3| = -(x+3)$ and $|x-5| = -(x-5)$
$\therefore f(x) = -x - 3 - x + 5 + 7 = 9 - 2x$
when $-3 < x < 5$, $|x+3| = x + 3$

$|x-5| = -(x-5)$
$f(x) = x + 3 - x + 5 + 7 = 15.$
$\therefore$ The minimum value of f (x) is 15.
f (x) is minimum when $x \in [-3, 5]$ Choice (A)

23. Let $f(x) = 10 - |3x+5|$

We know that $|3x+5|$ is always positive

$10 - |3x+5| \leq 10$

$\therefore$ The maximum value of f(x) = 10 Ans: (10)

24. Given: $9 - 4x - 5x^2 \geq 0$
$5x^2 + 4x - 9 \leq 0$
$(x - 1)(5x + 9) \leq 0$
$\therefore$ Critical points are $\dfrac{-9}{5}$, 1

x	✓	x
$\dfrac{-9}{5}$		1

when x = 0 the above inequation is true.

The solution is $\left[\dfrac{-9}{5}, 1\right]$

The number of integral values of 'x' is 3. Choice (D)

25. Given xy = 27
$\Rightarrow (3x)(4y) = 2^2\, 3^4 = 18^2$
The minimum value of 3x + 4y occurs when 3x = 4y.
$\therefore$ The minimum value of 3x + 4y = 18 + 18 = 36
Ans: (36)

26. $|2x - 7| - 8$ is minimum when $|2x - 7|$ is minimum. This happens when $x = \dfrac{7}{2}$. Choice (B)

27. It may be noted that $(n!)^2 \geq n^n$ for any natural number n, the equality being valid for n = 1 or 2.

$\therefore (12!)^2 > (12)^{12}$. Choice (B)

28. $|x + 4| < 3x - 7$.
As the modulus of any quantity is non-negative and $3x - 7$ being more than the modulus of a quantity would imply that.
$3x - 7 > 0$
$\Rightarrow x > \frac{7}{3}$ or 2.33
Now when $x > 2.33$
$|x + 4| = x + 4$
So the relation is reduced to $x + 4 < 3x - 7$
$\Rightarrow 2x > 11$
$\Rightarrow x > \frac{11}{2}$
Thus the range of x is $\left(\frac{11}{2}, \infty\right)$. Choice (A)

29. $X + Y + Z = k$
Squaring both sides, $X^2 + Y^2 + Z^2 + 2(XY + YZ + ZX) = k^2$
As X, Y and Z are positive, X^2, Y^2, Z^2 are also positive.
AM $(X^2,Y^2) \geq$ GM (X^2,Y^2). $\therefore \frac{X^2 + Y^2}{2} \geq \sqrt{X^2.Y^2}$
$\therefore X^2 + Y^2 \geq 2\,XY$. Similarly $Y^2 + Z^2 \geq 2YZ$
and $X^2 + Z^2 \geq 2\,XZ$.
$\therefore (X^2 + Y^2 + Z^2) \geq 2(XY + YZ + ZX)$
$\therefore X^2 + Y^2 + Z^2 + 2\,(XY + YZ + ZX) \geq 3\,(XY + YZ + ZX)$
$\therefore XY + YZ + ZX \leq \frac{K^2}{3}$ Choice (A)

30. Given $3x - 7 \leq 6x + 8$; $2x - 5 \geq 7x + 10$
$3x - 6x \leq 8 + 7$; $2x - 7x \geq 10 + 5$
$-3x \leq 15$; $-5x \geq 15$
$x \geq -5$; $x \leq -3$
$\therefore$ The common solution set is $[-5, -3]$ Choice (A)

31. The variable x appears in the base a well as the index. In general, it may be difficult to trace the graph of g(x). But we have to focus on the options. The important points are 0,1,2,3. We should evaluate g(x) for these values.
Also, The graph of the function between these points is continuous.

x	0	1	2	3	4
g(x)	1	1	0	1	16

We can now consider the options.
(A) If $0 < a < b < 1$, then $g(a) < g(b)$ False
(B) If $1 < a < b < 2$, then $g(a) > g(b)$ Probably true
(C) If $2 < a < b < 3$, then $g(a) > g(b)$ False
(D) If $1 < a < b < 3$, then $g(a) < g(b)$ False
We can go with Choice (B) Choice (B)

Note: We should be aware of the difference between a rigorous proof and an examination approach. While the solution above is not a rigorous proof, it servers the purpose of deciding our response in an exam.

32. When $c > 0$ and $a > b \Rightarrow ac > bc$
$c < 0$ and $a < b \Rightarrow ac > bc$
$\therefore$ I statement is not true.
When $a > b$; $\Rightarrow a - c > b - c$
II statement is always true. Choice (B)

33. $\left|3x - |5x+7|\right| = 10$
Case-1: $5x + 7 > 0 \Rightarrow |5x+7| = 5x + 7$
$\left|3x - |5x+7|\right| = 10$
$\Rightarrow \left|3x - (5x+7)\right| = 10 \Rightarrow |2x+7| = 10$
$\Rightarrow 2x + 7 = 10$ or $2x + 7 = -10$
$\Rightarrow x = 3/2$ or $x = -17/2$

Since $x > -7/5$ $\therefore x = 3/2$
$5x + 7 < 0$, $|5x+7| = -(5x + 7)$
$\left|3x - |5x+7|\right| = 10$
$\Rightarrow |3x+5x+7| = 10 \Rightarrow |8x+7| = 10$
$\Rightarrow 8x + 7 = 10$ or $8x + 7 = -10$
$\Rightarrow x = 3/8$ or $x = \frac{-17}{8}$
Since $x < -7/5$ $\Rightarrow x = -17/8$
$\therefore$ The number of solutions are 2. Ans: (2)

34. $-3 < x < 5 \Rightarrow -9 < 3x < 15$ ....(1)
$-7 < y < 12 \Rightarrow -28 < 4y < 48$
$\Rightarrow -48 < -4y < 28$ ....(2)
(1), (2) $\Rightarrow -57 < 3X - 4Y < 43$ Choice (A)

35. $x\,|x-5| = 6$
case-1: $x > 5$, $|x-5| = x - 5$
$x\,|x-5| = 6 \Rightarrow x\,(x - 5) = 6$
$x^2 - 5x - 6 = 0 \Rightarrow (x - 6)(x + 1) = 0$
$x = 6$ or -1
Since $x > 5$; $x = 6$
Case 2: $x < 5$ $|x-5| = 5 - x$
$x\,|x-5| = 6 \Rightarrow x\,(5 - x) = 6$
$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0$
$x = 3$ or 2
$\therefore$ the solution set for the equation is = {2,3, 6}
Choice (A)

36. Consider $2p^2 + p + 2 = p\left(2\left(p + \frac{1}{p}\right) + 1\right) = p(2(\geq 2) + 1) \geq 5p$
$3q^2 + q + 3 \geq 7q$ and $r^2 + r + 1 \geq 3r$
$\frac{(2p^2 + p + 2)(3q^2 + q + 3)(r^2 + r + 1)}{15pqr} \geq \frac{5p \times 7q \times 3r}{15pqr} = 7$
$\therefore$ x cannot be 6. Choice (D)

37. Given $-x^2 + 3kx + 5k + 1 < 0$
$\Rightarrow x^2 - 3kx - 5k - 1 > 0$
The expression is always positive, if $b^2 - 4ac < 0$
$\Rightarrow 9k^2 + 4(5k + 1) < 0 \Rightarrow 9k^2 + 20k + 4 < 0$
$\Rightarrow 9k^2 + 18k + 2k + 4 < 0$
$(k + 2)(9K + 2) < 0$
$k \in \left(-2, \frac{-2}{9}\right)$ Choice (C)

38. $|x^2 + x - 2| \leq x^2 - x$ ----- (1)
$x^2 + x - 2 = (x + 2)(x - 1)$
If $x \leq -2$ or $x \geq 1$, then $x^2 + x - 2 \geq 0$ and $|x^2 + x - 2| = x^2 + x - 2$.
If $-2 < x < 1$, then $x^2 + x - 2 < 0$ and $|x^2 + x - 2| = -x^2 - x + 2$.
Let $x \leq -2$ or $x \geq 1$.
(1) $\Rightarrow x^2 + x - 2 \leq x^2 - x \Rightarrow x - 1 \leq 0$
$\therefore x \in (-\alpha, -2]$ ----- (A)
Let $-2 < x < 1$
(1) $\Rightarrow -x^2 - x + 2 \leq x^2 - x \Rightarrow -2x^2 + 2 \leq 0$
$\Rightarrow 2x^2 - 2 \geq 0 \Rightarrow x^2 \geq 1$
$\Rightarrow x \leq -1$ or $x \geq 1$
$\therefore x \in (-2, -1]$ ------ (B)
From (A), (B), $x \in (-\infty, -1]$ Choice (B)

39. We should evaluate $g(x) = |3-x|^x$ at the points mentioned in the options, i.e, 0,2,3 and 4. We can also include two negative values, say -2 and -1 and 1 (to achieve some kind of completeness)

x	–2	–1	0	1	2	3	4
g(x)	1/25	1/4	1	2	1	0	1

We can now consider the options

(A) If $a < b < 0$, then $g(a) > g(b)$ False

(B) If $0 < a < b < 2$, then $g(a) < g(b)$ False

(C) If $2 < a < b < 4$, then $|g(a) – g(b)| < 1$. True
When $2 < a < b < 4$, $g(a)$ and $g(b)$ are two non negative numbers less than 1. $\therefore$ $|g(a) – g(b)| < 1$

(D) If $3 < a < b$, then $g(a) > g(b)$. False

Choice (C)

40. Min $(x + 6, x – 3) = x – 3$, for all values of x.
Min $(x + 5, x – 7) = x – 7$, for all values of x.
$\therefore$ Required value = max$(x – 3, x – 7)$
$= x – 3$ for all values of x Choice (C)

Solutions for questions 41 to 50:

41. Using statement I,
Suppose $y = 4$
$x^2 > 64$
$x > 8$ or $x < – 8$
If $x > 8$, $x > y$
If $x < – 8$, $x < y$
I is not sufficient.
Using statement II, suppose
$x = 9$
$y^2 > 729$
$y > 27$ or $y < – 27$
If $y > 27$, $x < y$
If $y < – 27$, $x > y$
II is not sufficient.
Using both statements,

Suppose $x = y = \frac{1}{2}$

$x^2 > y^3$ and $y^2 > x^3$ would hold true.

Suppose $x = -\frac{1}{2}$ and $y = \frac{1}{2}$

$x^2 > y^3$ and $y^2 > x^3$ would hold true. In this case $x < y$
We cannot answer the question. Both statements even when taken together are not sufficient to answer the question. Choice (D)

42. Using statement I, $x^2 – x^3 > 0$
$x^2 (1 – x) > 0$
As $x^2 > 0$, $1 – x > 0$
$x < 1$
I is sufficient.
Using statement II, $x^3 – x > 0$
$x (x^2 – 1) > 0$
$x > 0$ and $x^2 – 1 > 0$, (i.e. $x > 1$) or $x < 0$ and $x^2 – 1 < 0$ (i.e. $–1 < x < 0$)
We cannot answer the question.
II is not sufficient. Choice (A)

43. Neither of the statements alone is sufficient. Combining the two statements, we get $|x - 2y| \le 5$ and $|y + 3z| \le 9$

$|x - 2y| \le 5 \Rightarrow – 5 \le x – 2y \le 5$ _______ (1)

$|y + 3z| \le 9 \Rightarrow – 9 \le y + 3z \le 9$ ______ (2)

$(2) \times 2 \Rightarrow – 18 \le 2y + 6z \le 18$ ______ (3)

adding (1) and (3) we get
$– 5 – 18 \le x + 6z \le 23$
$– 23 \le x + 6z \le 23$

$\therefore$ The maximum value of $|x + 6z|$ is 23

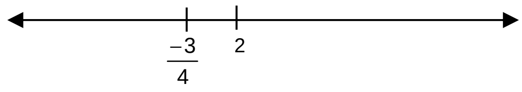
by combining both statements we can answer the question. Choice (C)

44. $|3x + 5| < 5x + 1$

$– (5x + 1) < 3x + 5 < 5x + 1$
$– 5x – 1 < 3x + 5 < 5x + 1$
$– 5x – 1 < 3x + 5$ and $3x + 5 < 5x + 1$
$– 8x < 6$ and $3x – 5x < – 4$

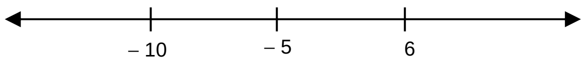
$x > -\frac{6}{8}$ and $– 2x < – 4$

$x > -\frac{3}{4}$ and $x > 2$



When $x > 2$ $|3x + 5| < 5x + 1$ and $x < 2$ $|3x + 5| > (5n + 1)$

Now from statement I,
$(x + 5) (x + 10) (x – 6) > 0$



$x \in (– 10, – 5) \cup (6, \infty)$
$\therefore$ Statement I is not sufficient to answer the question
From statement II
$8x^2 – 10x – 12 > 0$
$4x^2 – 5x – 6 > 0$
$4x^2 – 8x + 3x – 6 > 0$
$4x (x – 2) + 3 (x – 2) > 0$
$(4x + 3) (x – 2) > 0$

$x \in \left(-\infty, \frac{-3}{4}\right) \cup (2, \infty)$

$\therefore$ Statement II is also not sufficient to answer the question.
By combining both the statements, $x \in (–10, –5) \cup (6, \infty)$.
$\therefore$ The question cannot be answered even by combining both the statements also. Choice (D)

45. From statement I, $x = 2$ or $– 9$

When $x = 2$, $\left|\frac{x+5}{x+7}\right| = 7/9$

and when $x = –9$, $\left|\frac{x+5}{x+7}\right| = 2$

$\therefore$ Statement I alone is not sufficient.
From statement II,

$x = \frac{-19}{3}$ or $– 9$. In either case, $\left|\frac{x+5}{x+7}\right| = 2$.

$\therefore$ Statement II alone is sufficient to answer the question.
Choice (A)

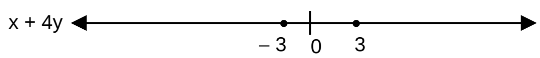
46. Either of the statements alone is not sufficient to answer the question.
Now from statement I

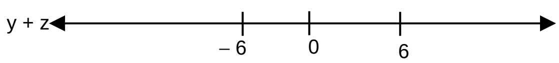
$|x + 4y| < 3 \Rightarrow – 3 < x + 4y < 3 \Rightarrow x + 4y > – 3$ and $x + 4y < 3$

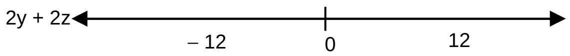
From statement II

$|y + z| > 6 \Rightarrow y + z < – 6$ or $y + z > 6 \Rightarrow 2y + 2z < – 12$ or $2y + 2z > 12$

We can represent the possible values $x + 4y$, $y + z$ and $2y + 2z$ on the number line as shown below.







The minimum value of $|(x+4y)-(2y+2z)|$ is 9, and it is occurs when x + 4y = – 3 and 2y + 2z = – 12 or x + 4y = 3 and 2y + 2z = 12

$\therefore |x+2y-2z|$ is always greater than 9.

By combining the two statements, we can answer the question. Choice (C)

47. Using statement I, $x - x^2 > 0 \Rightarrow x(1 - x) > 0$
$x > 0$ and $1 - x > 0$ or $x < 0$ and $1 - x < 0$
If $x > 0$ and $1 - x > 0$, $0 < x < 1$
If $x < 0$ and $1 - x < 0$, x has no possible value.
$\therefore 0 < x < 1$
I is sufficient.
Using statement II, $x^2 - x4 > 0$
$x^2 (1 - x^2) > 0$
As $x^2 > 0$, $1 - x^2, > 0$
$-1 < x < 1$
We can't say whether $x < 0$ or not
II is not sufficient. Choice (A)

48. Using statement I, $x > \sqrt{x} \geq 0$ ___ (1)
$\therefore 1 < x$. $\therefore$ I is sufficient.
Using statement II,
$\Rightarrow x > 1$. $\therefore$ II is sufficient
Either of the statements is sufficient. Choice (B)

49. Using statement I, $(x^2)^2 - 1 > 0$
$(x^2 - 1)(x^2 + 1) > 0$
$\therefore x^2 - 1 > 0$; $\therefore x^2 > 1$
I is sufficient.
Using statement II,
$\sqrt[4]{x}\ (\sqrt[4]{x} - 1) > 0$
$\sqrt[4]{x}$,> 0 and $\sqrt[4]{x} - 1 > 0$
$\therefore x > 1$ and $x^2 > 1$
$\therefore$ Statement II is sufficient Choice (B)

50. $|x + 1|$ represents the distance of x from –1 on the number line.
$|x + 4|$ is the distance of x from –4. If $x \geq -1$, the expression E = $|x + 1| - |x + 4|$ is –3. If $x \leq -4$, E = 3.
If $-4 < x < -1$, $x + 1 < 0$ and $|x + 1| = -x - 1$, while $x + 4 > 0$ and $|x + 4| = x + 4$.
$\therefore E = (-x - 1) - (x + 4) = -2x - 5 = -(2x + 5)$
$\therefore -3 \leq E \leq$ and hence $10 \leq 13 + E \leq 16$.
From statement I, 13 + E could be 16 or 10 (or some intermediate value). We can't answer the question.
From statement II, $E_{min} = 3$ and $13 + E_{min} = 16$. We can answer the question. Choice (A)

Chapter – 4
(Sequences and Series)

Concept Review Questions

Solutions for questions 1 to 35:

1. Here a = 20;d = 1/3
$T_{22} = a + 21d$
= 20 + 21 (1/3) = 27 Choice (A)

2. a = 2; d = 4
let the term equal to 106 be n^{th} term
$\Rightarrow T_n = 2 + (n - 1) 4 = 106 \Rightarrow n = 27$ Ans: (27)

3. Let the first term and the common difference of the arithmetic progression be a and d respectively.
n^{th} term = a + (n – 1) d
a + 3d = 7 $\rightarrow$ (1)
a + 16d = 72 $\rightarrow$ (2)
Solving (1) and (2), d = 5 and a = –8
10^{th} term = a + 9d = 37 Choice (B)

4. Let the first term and the common difference of the series be a and d respectively.
Then, $T_6 = a + 5d = 30 \rightarrow (1)$
$T_{11} = a + 10d = 55 \rightarrow (2)$
Solving (1) and (2)
a = 5, d = 5
$T_{21} = a + 20d \Rightarrow 5 + 20(5) = 105$ Choice (B)

5. Let the first term and the common difference of the arithmetic progression be a and d respectively.
$13 \times t_{13} = 7 \times t_7$
$\Rightarrow 13(a + 12d) = 7(a + 6d) \Rightarrow a = -19d$
$T_{20} = a + 19d = -19d + 19d = 0$ Ans: (0)

6. Let the first term of the arithmetic progression be a.
a + 2a = 9
a = 3
15^{th} term = a + 14 a
15 a = 45 Choice (C)

7. x + 4, 6x – 2 and 9x – 4 are three consecutive terms in an A.P.
$\Rightarrow 6x - 2 - (x + 4) = 9x - 4 - (6x - 2)$
$6x - 2 - x - 4 = 9x - 4 - 6x + 2$
$5x - 6 = 3x - 2 \Rightarrow 2x = 4$
x = 2 Choice (A)

8. a = 32, d = –4
$t_n = 32 + (n - 1)(-4) = 4$
$\Rightarrow n = 8$
sum of the series = $\frac{8(32+4)}{2}$ = 144 Choice (A)

9. Sum of the first n terms of an arithmetic progression whose first term is a and common difference is d = $\frac{n}{2}$ [2a + (n – 1) d]
Sum of the first 31 terms of the arithmetic progression
= $\frac{31}{2}$ [2 (6) + (31 – 1) $\left(\frac{8}{3}\right)$] = $\frac{31}{2}$ [92] = 1426
Ans: (1426)

10. Let the number of terms be n.
101 = 3 + 7 (n – 1)
15 = n
Sum of the terms = $\frac{15}{2}$ [3 + 101] = 780 Choice (C)

11. Sum to n terms = $\frac{n}{2}\left[a+\ell\right]$
Sum of the terms = $\frac{21}{2}$ [–9 + 51] = 441 Choice (B)

12. **(a)** Let the three terms of the arithmetic progression be a – d, a, a + d.
sum = a – d + a + a + d = 36
$\Rightarrow d = \pm 6$
The terms of the arithmetic progression be 6, 12, 18
Or alternatively substitute the options and check.
Choice (B)

(b) Let the five terms of the arithmetic progression be a – 2d, a – d, a, a + d, a + 2d.
sum = a – 2d + a – d + a + a + d + a + 2d = 70
$\Rightarrow 5a = 70 \Rightarrow a = 14$
Product of extremes = (14 – 2d) (14 + 2d) = 132
$\Rightarrow 196 - 4d^2 = 132 \Rightarrow d = \pm 4$
The five terms are 6, 10, 14, 18, 22
or alternatively, substitute the options and check.
Choice (C)

13. $S_n = 5n^2 + 2n$
$\Rightarrow S_{n-1} = 5n^2 + 2n - 1$
n^{th} term = $S_n - S_{n-1}$
$= 5n^2 + 2n - \{5(n - 1)^2 + 2(n - 1)$
= 10n – 3 Choice (B)

14. The two digit numbers which leave a remainder of 1 when divided by 4 are 13, 17, 21, …. 97

let the last term be n^{th} term
$\Rightarrow T_n = 13 + (n+1)\,4 = 97 \Rightarrow n = 22$
sum of the terms = 22/2 (13 + 97) = 1210 Choice (C)

15. **(a)** Let the first term and the common difference of the arithmetic progression be a and d respectively. The sum of the first 71 terms is 0, i.e.,

$\frac{71}{2}[2a + 70\,d] = 0$

$71[a + 35\,d] = 0$

$\therefore$ The 36^{th} term of the arithmetic progression must be 0. Choice (C)

(b) Let the n^{th} term be t_n.
Given that
$(t_1 + t_2 + \ldots + t_{30}) = t_1 + t_2 + t_3 + \ldots + t_{31} + \ldots + t_{60}$

$\Rightarrow t_{31} + t_{32} + \ldots + t_{60} = 0 = \frac{30}{2}\left[t_{31} + t_{60}\right]$

$\therefore$ Sum of the 31^{st} and the 60^{th} terms of the arithmetic progression is 0. Choice (A)

16. $a = 4$, $r = \sqrt{2}$

let $64\sqrt{2}$ be the n^{th} term of the geometric progression.

$T_n = 4\,(\sqrt{2}\,)^{n-1} = 64\sqrt{2}$
$\Rightarrow 2^{\,n-1} = 2^{9/2} \Rightarrow n = 10$ Ans: (10)

17. Putting n = 1, 2, 3, 4 we can get the terms
$t_1 = 4(-5)^1 = -20$
$t_2 = 4(-5)^2 = 100$
Finding two terms is enough to get the answer from options. Choice (A)

18. Sixth term = $2\,(3)^5 = 486$ Choice (B)

19. The series is a geometric progression with a = 4, r = 3

Sum = $\frac{4(3^n - 1)}{(3-1)} = 4372$

$\Rightarrow 3^n = 2187 = 3^7$
$\Rightarrow n = 7$ Ans: (7)

20. Let the first term and the common ratio be a and r respectively.
Fourth term $T_4 = ar^3 = 3 \rightarrow (1)$
Eighth term $T_8 = ar^7 = 1/27 \rightarrow (2)$
Solving (1) and (2)
$r^4 = 1/81$
r = 1/3, a = 81
Twelfth term = $T_{12} = 81\,(1/3) = 1/3^7 = 1/2187$
Choice (C)

21. Let the first term and the common ratio of the geometric progression be a and r respectively.
n^{th} term = ar^{n-1}
Given that ar = 9 $\rightarrow$ (1) and
$ar^5 = 729$ $\rightarrow$ (2)
dividing (2) by (1),
$r^4 = 81 \therefore r = \pm 3 \Rightarrow r^2 = 9$
$\therefore 4^{th}$ term = $ar^3 = (ar)(r^2) = 9 \times 9 = 81$ Choice (A)

22. **(a)** The sum of the first n terms of a geometric progression whose first term is a and whose common ratio is r is given by $\frac{a\left(r^n - 1\right)}{r - 1}$

Sum of the first 4 terms = $\frac{6\left(2^4 - 1\right)}{2 - 1} = 90$

Choice (A)

(b) Let the common ratio of the geometric progression be r.
$r^3 = 8$
$r = 2$

Sum of the first terms = $\frac{1\left(2^7 - 1\right)}{2 - 1} = 127$

Choice (C)

23. $a = 5$ $r = \sqrt{5}$

Sum = $\frac{a(r^n - 1)}{r - 1}$

$155 + 155\sqrt{5} = \frac{5\left(\left(\sqrt{5}\right)^n - 1\right)}{\left(\sqrt{5} - 1\right)}$

$155\,(\sqrt{5} + 1) = \frac{5(5^{\frac{n}{2}} - 1)}{(\sqrt{5} - 1)}$

$\Rightarrow 5^{n/2} = 5^3 \Rightarrow n = 6$ Ans: (6)

24. Let the common ratio be r, then

sum of the series = $\frac{(r \times \text{last term}) - \text{first term}}{r - 1}$

$\Rightarrow 3279 = \frac{2187r - 3}{r - 1} \Rightarrow r = 3$ Ans: (3)

25. Let the three terms of geometric progression be a/r, a, ar
product = $a/r \times a \times ar = 216$
$\Rightarrow a = 6$
sum $6/r + 6 + 6r = 26 \Rightarrow r = 30$ or $1/3$

$\therefore$ the terms are 2, 6, 18 Choice (B)

26. The given series represents a geometric progression whose first term is 1 and common ratio is $\frac{3}{4}$.

The sum to infinity of a geometric progression whose first term is a and whose common ratio is $r = \frac{a}{1 - r}$ ($|\,r\,| < 1$)

The sum to infinity = $\frac{1}{1 - \frac{3}{4}} = 4$ Ans: (4)

27. n = 7, common ratio = 1/n + 1 = 1/8 Choice (C)

28. **(a)** Arithmetic mean = $\frac{136}{8} = 17$

Alternate method:

If n terms are in arithmetic progression and n is even, their arithmetic mean is given by the average of the n/2 th term and the next term i.e., the average of the middle terms.
The arithmetic means will be the average of the 4^{th} and 5^{th} terms i.e., $\frac{15 + 19}{2} = 17$ Ans: (17)

(b) Geometric mean of n terms = (Their product)$^{1/n}$
Geometric mean

$\sqrt[4]{(3)\,(9)\,(27)\,(81)} = \sqrt[4]{3\;3^2\;3^3\;3^4} = 3^{\frac{10}{4}} = 9\sqrt{3}$

Choice (B)

29. **(a)** Let A be an arithmetic progression whose first term is a and whose common difference is d.
x^{th} term of A = a + (x – 1) d
y^{th} term of A = a + (y – 1) d
z^{th} term of A = a + (z – 1) d
As x, y and z are in arithmetic progression, (x – 1) d, (y – 1) d and (z – 1) d are in arithmetic progression….the x^{th} term of A, the y^{th} term of A and the z^{th} term of A are in arithmetic progression.
Choice (A)

(b) Let G be a geometric progression whose first term is a and whose common ratio is r.
x^{th} term of G = ar^{x-1}
y^{th} term of G = ar^{y-1}
z^{th} term of G = ar^{z-1}
As x, y and z are in arithmetic progression, $y = \frac{x+z}{2}$

$\therefore ar^{y-1} = ar^{\frac{x+z-2}{2}}$

$(ar^{y-1})^2 = a^2 r^{x-1+z-1}$

$= (ar^{x-1})(ar^{z-1})$
$\therefore$ The x^{th} term, the y^{th} term and z^{th} term of G are in geometric progression. Choice (B)

30. Suppose the numbers in geometric progression are p, pr and pr^2.
log pr = log p + log r
log pr^2 = log p + 2 log r
As log pr^2 – log pr = log pr – log p = log r log p, log pr and log pr^2 are in arithmetic progression. Choice (A)

Exercise – 4(a)

Solutions for questions 1 to 45:

1. The 67^{th} term of the arithmetic progression is a + 66d and the 4^{th} term of the series is a + 3d.
Thus a + 66d = 15(a + 3d)

21d = 14a, $a = \frac{3}{2}d$

The 11^{th} term is a + 10d = 23,

$\frac{3}{2}d + 10d = 11\frac{1}{2}d = 23 \Rightarrow d = 2$ and $a = \frac{3}{2}(2) = 3$
The 21^{st} term is a + 20d = 3 + 20(2) = 43. Ans: (43)

2. Let the three numbers in A.P. be a – d, a and a + d; a – d + a + a + d = 3a = 48

$\Rightarrow a = \frac{48}{3} = 16$. Given that $16^2 - d^2 = 252$

$256 - 252 = d^2$ $4 = d^2$

Thus, $d = \pm\sqrt{4} = \pm 2$

Hence, the smallest of the three numbers is 16 – 2 = 14, Even if d = –2 is considered, the smallest number will be 14 only. Choice (C)

3. If the first term of the A.P. is a and the common difference is d, we have the tenth term as
a + 9d = 40 $\rightarrow$ (1)
and the twelfth term as a + 11d = 44 $\rightarrow$ (2)
Subtracting (1) from (2), we have 2d = 44 – 40,
d = 2
Substituting the value of d in (1) we get, a = 22.
The sum of n terms of the A.P.

$= \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2(22) + (n-1)2]$

$= \frac{n}{2}[44 + 2n - 2] = \frac{n}{2}[42 + 2n] = 21n + n^2$

Choice (D)

4. Sum of the terms of an A.P. = (number of terms) $\times$ (the middle term of the A.P.), if number of terms (n) is odd. Since, n is 37 (odd), we have sum of the terms of the A.P. = 703 = 37(middle term of the A.P.). Middle term of the A.P = (703/37) = 19. Ans: (19)

5. The first term is positive and the common difference is negative, and is equal to 2. Hence, from a certain term onwards, the term becomes negative. Hence, the maximum sum is the sum of the terms before the first negative term occurs.
The last term in the series which leads to a maximum sum of 20, 18, 16, …… is 2. The eleventh term is 0 and the twelfth term is negative. There are a total of 10 positive terms. Hence, the required value n = 10. Choice (B)

6. If the first term is a and the common difference is d, we have the sum of the fifth, thirteenth and eighteenth terms as
(a + 4d) + (a + 12d) + (a + 17d)
$\Rightarrow$ 3a + 33d = 0.
Dividing by 3, we have a + 11d = 0. Hence, the 12^{th} term of the A.P. is 0. Choice (C)

7. The first number between 450 and 950 which is divisible by both 3 and 7 i.e., 21 is 462. The last number between 450 and 950 which is divisible by 21 is 945. Hence, from 21×22 to 21×45, there are a total of 24 terms which are divisible by both 3 and 7.
Solving using the concept of Progressions:
First term between 450 and 950 divisible by 21 (both 3 and 7); a = 462 (between 450 and 950). The last term divisible by 21 between 450 and 950 (ℓ) = 945. The common difference between successive terms which are divisible by 3 and 7 is 21.

Number of terms required = $\frac{945-462}{21} + 1 = \frac{483}{21} + 1$
= 23 + 1 = 24. Choice (B)

8. The least multiple of 9 greater than 300 is 306 = 9(34)
The greatest multiple of 9 less than 600 is 594 = 9(66)
$\therefore$ There are 66 – 33 or 33 multiples of 9 between 300 and 600. Choice (A)

9. The first two digit number which leaves a remainder 3 when divided by 7 is 10 = $(7 \times 1 + 3)$ and the last such two digit numbers are 94 = $(7 \times 13 + 3)$.

$94 = 10 + \overline{(n-1)}7$, n – 1 = 84/7 = 12, n = 13.

Hence, the number of two digit numbers which when divided by 7 leave a remainder of 3 are 13. Hence, sum of all such two-digit numbers is

$\frac{13}{2}(10 + 94) = \frac{13}{2}(104) = 676$. Ans: (676)

10. Let the first term and the common difference of first A.P. be a and d and that of the second A.P. be a_1 and d_1. Hence, ratio of the 21^{st} terms of the two series

$= \frac{a + (21-1)d}{a_1 + (21-1)d_1} = \frac{a + 20d}{a_1 + 20d_1}$

$= \frac{2a + 40d}{2a_1 + 40d_1} = \frac{\frac{41}{2}(2a + 40d)}{\frac{41}{2}(2a_1 + 40d_1)}$

$= \frac{7(41) - 17}{4(41) + 16} = \frac{270}{180} = \frac{3}{2}$.
Hence, ratio of the 21^{st} terms of the two A.P. s = 3 : 2.
Choice (A)

11. Let the first term and the common difference be a and d respectively.
$(a + 3d)^2 = (a + 2d)^2 + (a + d)^2$
$a^2 + 6ad + 9d^2 = 2a^2 + 6ad + 5d^2 \Rightarrow a = \pm 2d$
As all the terms are positive, a = 2d
a + a + d + a + 2d + a + 3d = 14 $\Rightarrow$ d = 1
Ans: (1)

12. Let the A.P., 7, 11, 15……….497 be called P_1 and let the A.P., 1, 6, 11, 16….501 be called P_2. Let P_3 be the A.P. containing all the terms common to P_1 and P_2. 11, which is the first of the values common to P_1 and P_2 is the first term of P_3 $\rightarrow$ (1)
Common difference of P_3 = LCM of (common difference of P_1 and P_2)
= LCM of 4 and 5, equal to 20 $\rightarrow$ (2)

The last term of P_3 cannot be greater than 497, which is the lesser of the last terms of P_1 and P_2 → (3)
P_3 is : 11, 31, 51,…………………and $t_n \le 497$, when t_n is the last term of P_3.
$\Rightarrow 11 + (n – 1) 20 \le 497; \Rightarrow (n – 1) 20 \le 486;$
$\Rightarrow (n – 1) \le 24.3$
$\Rightarrow n – 1 = 24$ or $n = 25$ Ans: (25)

13. Sum of the first 30 terms of the A.P. = $\frac{30}{2}$ [2 (10) + 29 (5)]
= 15 [20 + 145] = 15 [165] = 2475.
Sum of the first 10 terms of the A.P. = $\frac{10}{2}$ [2 (10) + 9(5)]
= 5[20 + 45] = 5 [65] = 325. Ratio of the sum of the first 30 terms of the A.P. to the sum of the last 20 terms of the A.P.
= (2475) : (2475 – 325) = 2475 : 2150
= 5 [495] : 5 [430] = 495 : 430 = 99 : 86
Choice (D)

14. Let the first term and the common difference be a and d respectively.
$$\frac{\frac{20}{2}[2a+19d]}{\frac{10}{2}[2a+9d]} = \frac{2(a+d)}{a}$$
(2a + 19d) as = (2a + 9d) (a + d)
d (8a – 9d) = 0
d = 0 or 8 a = 9d → (1)
as all the terms are distinct $d \ne 0$.
8 a = 9d [$\therefore$ 8 is a factor of d say d = 8k]
Sum of all its terms = $\frac{30}{2}$ [2a + 29d]
= $\frac{1875d}{4} = 3750k$, where k is an integer from (1)
Only choice (D) satisfies this condition. Choice (D)

15. The sum of n terms of an A.P. with first term a and common difference d is $n\left[\frac{2a+(n-1)d}{2}\right]$
$$\therefore n\left[\frac{2(2)+2(n-1)}{2}\right] = 156$$
$\Rightarrow n(n + 1) = 12(13) = (–13)(–12)$
As n > 0, n = 12 Choice (B)

16. With the usual notation,
73(a + 72d) = 37(a + 36d)
$\Rightarrow$ 36a = [36(37) – 72(73)]d
= [1332 – 5256]d = –3924d
$\Rightarrow$ a + 109d = 0
i.e., the 110th term is 0. Choice (D)

17. Let the terms be 15 – 3d, 15 – d, 15 + d, 15 + 3d.
(The sum is 60)
$$\frac{(15-3d)(15+d)}{(15-d)(15+3d)} = \frac{3}{8}$$
$\Rightarrow 8(–3d^2 – 30d + 225) = 3(–3d^2 + 30d + 225)$
$\Rightarrow 15d^2 + 330d – 1125 = 0$
$\Rightarrow d^2 + 22d – 75 = 0$
$\Rightarrow (d + 25) (d – 3) = 0$
i.e., d = 3 or –25
$\therefore$ The progression could be 6, 12, 18, 24 or 90, 40, –10, –60.
In either case the ratio of t_1t_3 and t_2t_4 is 3 : 8.
But as all the terms have to be positive, the progression is 6, 12, 18, 24. Choice (B)

18. $\frac{30}{2}\left[2\times 72000+(30-1)3600\right]$ = 37,26,000
Ans: (37,26,000)

19. Sum of the integers divisible by 3 from 1 to 300
= $\frac{100}{2}$ [3 + 300] = 50[303] = 15150.
Sum of the integers divisible by 5 from 1 to 300
= $\frac{60}{2}$ [5 + 300] = 30 [305] = 9150.
Sum of the integers divisible by both 3 and 5
= $\frac{20}{2}$ [15 + 300] = 10 [315] = 3150.
Thus the sum of all the integers that are divisible either by 3 or 5 from 1 to 300 = 15150 + 9150 – 3150
= 15150 + 6000 = 21150. Choice (A)

20. $\log_2x + \log_2x^2 + \log_2x^3 + \log_2x^4 + \ldots\ldots + \log_2x^{10}$
$= \log_2x + 2\log_2x + 3\log_2x + 4\log_2x + \ldots\ldots +10\log_2x$
$= (\log_2x)(1 + 2 + 3 + 4 + \ldots\ldots + 10)$
$= (\log_2x)\left(\frac{(10)\ (11)}{2}\right) = 55\log_2x = 220.$
Hence $\log_2x = \frac{220}{55} = 4$
Thus, $x = 2^4 = 16$.
Alternative method:
$\log_2x^{55} = 220$
$x^{55} = 2^{220}$
$x^{55} = (2^4)^{55} \Rightarrow x = 2^4$ or x = 16 Choice (B)

21. Let the three terms in GP be $\frac{a}{r}$, a and ar
$\therefore \frac{(a)}{r}(a)(ar) = 1728;\ a^3 = 1728$
$\Rightarrow a = 12$
$$\frac{a}{r}(a) + a\,(ar) + \frac{a}{r}(ar) = a^2\left(\frac{1}{r}+r\right) + a^2 = 1032$$
$\because a^2 = 144;\ 144\left(\frac{1}{r}+r\right) + 144 = 1032$
$144\left(\frac{1}{r}+r\right) = 888,\ \ \frac{1}{r}+r = \frac{888}{144} = \frac{111}{18} = \frac{37}{6}.$
$\frac{1}{r}+r = \frac{37}{6} = 6+\frac{1}{6} \Rightarrow r = 6$ or $\frac{1}{6}$
So, 2, 12, 72 or 72, 12, 2 is the G.P. and 2 is the smallest number. Ans: (2)

22. If the first term of the G.P is a and the common ratio is r, we have $\frac{a(r^8-1)}{r-1} = 510$ → (1)
and $\frac{a(r^4-1)}{r-1} = 30$ → (2)
Dividing (1) by (2) we have $\frac{a(r^8-1)}{r-1} / \frac{a(r^4-1)}{r-1}$
$= r^4 + 1 = \frac{510}{30} = 17.$
$r^4 = 17 – 1 = 16$ $\quad r = \pm\sqrt[4]{16} = \pm 2$
First term of the G. P, $a = \frac{510\,(r-1)}{r^8-1}$
As first term is positive, r = 2 is taken.
$a = \frac{510\,(2-1)}{2^8-1} = \frac{510\,(1)}{255} = 2.$
Choice (A)

23 Given, a = 4.
Let r be the common ratio.
$a = 3\frac{ar}{1-r}$

$\therefore 1 - r = 3r$ or $r = \frac{1}{4}$

$\therefore$ Fifth term $= 4\left(\frac{1}{4}\right)^4 = \frac{1}{64}$

Choice (B)

24. Let the first term and the common ratio be a and r respectively $\frac{a}{1-r} = 12 = \left(\frac{a}{1-r}\right)^2 = 12^2 \to (1)$

$\frac{a^2}{1-r^2} = 48 \quad \to \quad (2)$

in neither equation, r = 1
dividing (1) by (2),

$\frac{1-r^2}{(1-r)^2} = 3 \Rightarrow \frac{1+r}{1-r} = 3 \Rightarrow 1 + r = 3 - 3r \Rightarrow r = \frac{1}{2}$

$\Rightarrow a = 12\ (1 - r) = 6$ (from (1)) Ans: (6)

25. The sum of terms of a G.P with first term a and common ratio r is $\frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{(1-r)}$. For the given series, it is

$\frac{5\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = \frac{5115}{512}$ (given)

$\Rightarrow 1 - \frac{1}{2^n} = \frac{511.5}{512} = \frac{1023}{1024}$

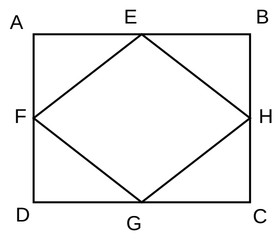
$\Rightarrow \frac{1}{2^n} = 1 - \frac{1023}{1024} = \frac{1}{2^{10}} \quad \therefore n = 10$ Ans: (10)

26. $ar^6 = 5^{7.5}$ …… (1); $ar^{12} = 5^{13.5}$ …….. (2)

$\frac{(2)}{(1)} \Rightarrow r^6 = 5^6 \quad \Rightarrow r = \pm 5$

As $r < 0$, $r = -5$ and $a = 5^{1.5}$ Choice (A)

27.



Let the square T_1, (ABCD), be of side a

AE = $\frac{a}{2}$ and AF = $\frac{a}{2}$. Hence $EF^2 = AE^2 + AF^2$

$= \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{4} + \frac{a^2}{4} = \frac{a^2}{2}$ EF $= \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$.

Hence area of EFGH $= \left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^2}{2}$.

Let us denote the sum of the areas, of all such squares formed by repeating the process indefinitely, as A.
As area of ABCD (T_1) = a(a) = a^2,

$A = a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \ldots\ldots\ldots \infty$

$= a^2 (1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots.. \infty) = \frac{a^2}{1/2} = 2a^2$.

It is given that a = 16 cm.
Hence, A = 2 $(16)^2$
= 2 × 256 = 512 square cm. Ans: (512)

28. 2(y + 3) – 2y = 2y + 6 – 2y = 6 = Common difference.
As per the G.P. given,

$\frac{5y-1}{2(y+1)} = \frac{2(y+1)}{x+2} \Rightarrow (5y - 1)(x + 2) = [2(y + 1)]^2$

$(5y - 1)(6 - 2y + 2) = 4(y + 1)^2$
$\Rightarrow 14y^2 - 34y + 12 = 0 \quad \Rightarrow y = 2.$ Choice (A)

29. Let the first term of either progression be a.
Let the common difference of the arithmetic progression be d.

$\frac{a+6d}{a} = \frac{a+11d}{a+6d}$

d (a + 36d) = 0
As a and d have opposite signs, d ≠ 0.
a + 36d = 0
37th term = 0 Choice (B)

30. 2(5x + 1) = 8x + 5 + x, x = 3.

If the third number is divided by 6, the result is $\frac{8x}{6} = \frac{8 \times 3}{6} = 4$

Hence, 4, 5 + x and 5x + 1 are in G.P.
i.e. 4, 5 + 3 = 8 and 5(3) + 1 = 16 are in G.P.

Common ratio of G.P = $\frac{16}{8}$ = 2. Ans: (2)

31. If p, q and r are in G.P and the common ratio for the G.P is x, we have q = px and r = px (x) = px^2
pqr = 512 ⇒ (p) (px) (px^2) = p^3x^3 = 512
$(px)^3 = 512 = 8^3$. Thus, px = 8. If p is increased by 14 and r is decreased by 8, we'll have the resulting values of q, p and r in A.P.; i.e., q, (p + 14), (r – 8) an in A.P.
(14 + p) –q = (r – 8) – (p + 14)
14 + p – q = r – 8 – p – 14
p + p + 14 + 8 – q + 14 = r
28 + 2p = r → (1)
pqr = 512 and as q = px = 8,

$pr = \frac{512}{q} = \frac{512}{8} = 64 \quad \to \quad (2)$

Solving (1) and (2), r = 32 Choice (B)

32. In an arithmetic progression with an odd number of terms, the middle term is the arithmetic mean of all the terms. In a geometric progression with an odd number of terms, the middle term is the geometric mean of all the terms.

7th term of A = middle term of A = $\frac{26}{13}$ = 2

7th term of G = middle term of G = $\sqrt[13]{8192}$ = 2

required sum = 4 Choice (B)

33. Let $S = 2 + 3x + 4x^2 + 5x^3 + \ldots.. \quad \to \quad (1)$
Then, $Sx = 2x + 3x^2 + 4x^3 + \ldots\ldots \quad \to \quad (2)$
Subtracting (2) from (1),
$S(1 - x) = 2 + x + x^2 + x^3 + \ldots\ldots$

$S(1 - x) = 2 + \frac{x}{1-x}$

$S(1 - x) = \frac{2(1-x) + x}{1-x} = \frac{2-x}{1-x}$

$S = \frac{2-x}{(1-x)^2}$ Choice (A)

34. Given series is $1 + \frac{.9}{11} + \frac{.99}{(11)^2} + \frac{.999}{(11)^3} + \ldots.$

i.e., $1+\frac{1-0.1}{11}+\frac{1-0.01}{(11)^2}+\frac{1-0.001}{(11)^3}+.....$

$= 1+\frac{1}{11}+\frac{1}{11^2}+\frac{1}{11^3}+...-\left[\frac{1}{110}+\frac{1}{(110)^2}+\frac{1}{(110)^3}+...\right]$

$=\frac{1}{1-\frac{1}{11}}-\left[\frac{1/110}{1-\frac{1}{110}}\right]$

[$\because$ S_∞ of the GP = $\frac{a}{1-r}$ when $|r|<|$] = $\frac{11}{10}-\frac{1}{109}=\frac{1189}{1090}$

Choice (D)

35. Given X =

$\frac{1}{80\times41}+\frac{1}{79\times42}+\frac{1}{78\times43}+....+\frac{1}{42\times79}+\frac{1}{41\times80}$

$=\frac{1}{121}\left[\left(\frac{1}{80}+\frac{1}{41}\right)+\left(\frac{1}{79}+\frac{1}{42}\right)+\left(\frac{1}{18}+\frac{1}{43}\right)+..............+\left(\frac{1}{42}+\frac{1}{79}\right)+\left(\frac{1}{41}+\frac{1}{80}\right)\right]$

$=\frac{1}{121}\left[\frac{2}{41}+\frac{2}{42}+\frac{2}{43}+....+\frac{2}{80}\right]$

$=\frac{2}{121}\left[\frac{1}{41}+\frac{1}{42}+\frac{1}{43}+.....+\frac{1}{80}\right]$

Given Y = $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+......+\frac{1}{79}-\frac{1}{80}$

$= 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{80}-2\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+.......+\frac{1}{80}\right\}$

$= 1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{80}-\left\{1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{40}\right\}$

$=\frac{1}{41}+\frac{1}{42}+\frac{1}{43}+.......+\frac{1}{80}$

$\therefore \frac{Y}{X}=\frac{\frac{1}{41}+\frac{1}{42}+\frac{1}{43}+.....+\frac{1}{80}}{\frac{2}{121}\left[\frac{1}{41}+\frac{1}{42}+\frac{1}{43}+.....+\frac{1}{80}\right]}=\frac{121}{2}=60.5$

Choice (C)

36. $S = 3(2)^2 + 4(3)^2 + 5(4)^2 + ...$ 10 terms.

$= (2 + 1)2^2 + (3 + 1)3^2 + (4 + 1)4^2 + ...$ 10 terms

$= (2^3 + 2^2) + (3^3 + 3^2) + (4^3 + 4^2) +.....$ 10 terms

$= (2^3 + 3^3 + 4^3 + ...+ 11^3) + (2^2 + 3^2 + 4^2......+11^2)$

$= (1^3 + 2^3 + 3^3 ...+ 11^3) + (1^2 + 2^2 + 3^2 +...+ 11^2) – 1^3 – 1^2$

$= 4355 + 505 = 4860$. Ans: (4860)

37. $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}=\sqrt{\frac{4+4+1}{2^2}}=\frac{3}{2}=2-\frac{1}{2}$

$\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}=\sqrt{\frac{36+9+4}{36}}=\frac{7}{6}$

Now, $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}=\frac{3}{2}+\frac{7}{6}=\frac{16}{6}=\frac{8}{3}=3-\frac{1}{3}$

Similarly,

$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}=\frac{3}{2}+\frac{7}{6}+\frac{13}{12}$

$=\frac{18+14+13}{12}=\frac{45}{12}=4-\frac{1}{4}$

$\therefore \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+....+\sqrt{1+\frac{1}{(n-1)^2}+\frac{1}{n^2}}=n-\frac{1}{n}$

$\therefore \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+...+\sqrt{1+\frac{1}{10^2}+\frac{1}{11^2}}=11-\frac{1}{11}$

$=\frac{120}{11}=10\frac{10}{11}$ Choice (D)

38. $T_1 = 2 = 3(2^{1-1}) – 1$

$T_2 = 2 (T_1) + (2 – 2) = 2 (2) + (2 – 2) = 4 = 3 (2^{2-1}) – 2$

$T_3 = 2 (T_2) + (3 – 2) = 2 (4) + 1 = 9 = 3(2^{3-1}) – 3$

Proceeding similarly

$T_n = 3(2^{n-1}) – n$

$\therefore T_{100} = 3 (2)^{99} – 100$. Choice (D)

39. Let S_n represent the sum of the first n terms and t_n the n^{th} term of the given series.

$S_n = 1 + 10 + 23 + 40 + 61 + 86 + ... t_n$ ----(1)

$S_n = \quad 1 + 10 + 23 + 40 + 61 +.....t_{n-1} + t_n$ ----(2)

Subtracting equation (2) from equation (1), we get

0 = 1 + 9 + 13 + 17 + 21 + 25 + ... upto (n – 1) terms - t_n

$\therefore t_n$ = 1 + (9 + 13 + 17 + 21+ 25 +....upto n – 1 terms)

$t_n = 1+\frac{n-1}{2}[2(9)+(n-2)4] = 1+\frac{(n-1)}{2}[10+4n]$

$= 1 + (n – 1)(2 n + 5)= 2 n^2 + 3 n – 4$

Therefore $s_N = \sum_{n=1}^{N} t_n=\sum_{n=1}^{N}\left(2n^2+3n-4\right)$

$=\frac{2N(N+1)(2N+1)}{6}+\frac{3N(N+1)}{2}-4N$

Therefore, $S_{20}=\frac{2(20)(21)(41)}{6}+\frac{3(20)(21)}{2}-4(20)$

= 5740 + 630 – 80 = 6290 Choice (A)

40. Value of $50 \times 1 + 49 \times 2 + ... 1 \times 50$ is $\sum_{i=1}^{n=50}(51-n)n = \sum_{i=1}^{n=50}(51n-n^2)$

[Note: See explanation below]

$=\frac{51(50)(51)}{2}-\frac{1}{6}(50)(51)(101)$

$= (50)(51)\left[\frac{51(3)}{6}-\frac{101}{6}\right]=(50)(51)\frac{(153-101)}{6}$

$=\frac{(50)(51)(52)}{6}=25\times17\times52=\frac{100}{4}\times17\times52$

$= 17 \times 13 \times 100 = 22100$.

Alternative method:

The first elements of the terms form the series,

50, 49, 48………..1.

This is an A.P., where a = 50, d = –1, n = 50.

General term of series = 50 + (n – 1) (–1)

= (51 – n) $\rightarrow$ (1)

The second elements of the terms form the series, 1, 2, 3, 4 ……….. 50

This is an A.P., with a = 1 and d = 1

General term of series = 1 + (n – 1) (1) = n $\rightarrow$ (2)

From (1) and (2), the general term, t_n = (51 – n) n

$= 51n – n^2$ $\rightarrow$ (3)

By giving values n = 1, 2, 3……….50 the following are obtained.

$t_1 = 51 \times 1 – 1^2$

$t_2 = 51 \times 2 – 2^2$

$t_3 = 51 \times 3 – 3^2$

$t_{50} = 51 \times 50 – 50^2$

Hence, sum = $51 (1 + 2 + 3 +…50) – (1^2 + 2^2 + 3^2 + …50^2)$

$= 51\times\frac{50\times51}{2}-\frac{50\times51\times101}{6}$

$= (25 \times 51 \times 51) – 25 \times 17 \times 101$

$= 25 \times 17\ (3 \times 51 - 101) = 25 \times 17 \times 52$
$= 25 \times 884 = 22100$ Ans: (22100)

41. $\frac{1}{2} + \frac{1}{2+4} + \frac{1}{2+4+6} + \ldots + \frac{1}{2+4+6+\ldots400}$

$= \frac{1}{2} + \frac{1}{2(1+2)} + \frac{1}{2(1+2+3)} + \ldots\ \frac{1}{2(1+2+3+\ldots200)}$

$\therefore$ n^{th} term $t_n = \frac{1}{2(1+2+\ldots n)}$

$= \frac{1}{2\left(\frac{n(n+1)}{2}\right)}$

$= \frac{1}{n(n+1)}$

$= \frac{1}{n} - \frac{1}{n+1}$

$\therefore \sum_{n=1}^{200} t_n = \sum_{n=1}^{200} \frac{1}{n} - \frac{1}{n+1}$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$ -------- $+ \left(\frac{1}{199} - \frac{1}{200}\right)$

$= 1 - \frac{1}{200} = \frac{199}{200}$ Choice (B)

42. $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots \frac{31}{1+2^2+3^2+\ldots15^2}$

The n^{th} term of above series is $t_n = \frac{2n+1}{1^2+2^2+3^2\ldots+n^2}$

$t_n = \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$

$= \frac{6}{n} - \frac{6}{n+1}$

$= 6\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{15} - \frac{1}{16}\right]$

$= 6\left[1 - \frac{1}{16}\right] = 6\ \frac{15}{16} = \frac{45}{8}$ Choice (B)

43. Given a, b, 2a + b, 2a – 3b – 7 are in A. P.
$\therefore$ b – a = 2a + b –b; = 2a – 3b – 7 – (2a + b)
$\Rightarrow$ b – a = 2a
$\Rightarrow$ b = 3a $\longrightarrow$ (1)
2a = – 4b – 7 $\longrightarrow$ (2)
Substitute the value of b in (2)
2a = – 4 (3a) – 7
14a = – 7

$\Rightarrow a = \frac{-7}{14} = \frac{-1}{2}$

$b = 3a = 3\left(\frac{-1}{2}\right) = \left(\frac{-3}{2}\right)$

$\therefore$ Common difference = b – a

$= \frac{-3}{2} - \left(\frac{-1}{2}\right) = -1$

$\therefore t_{97} = a + 96d$

$= \frac{-1}{2} + 96\ (-1) = \frac{-1-192}{2}$

$= \frac{-193}{2}$ Choice (A)

44. As the first five integers are N, N – 2, N – 4, N – 6 and N – 8, their average is N – 4. Given, N – 4 is 1594 more than the Nth term.
$\therefore$ N – 4 = 1594 + $T_N \Rightarrow T_N$ = N – 1598 = N – 2 (799).
$\therefore$ T_N is the 80^{th} term and N = 800. Choice (B)

45. Let S = 1 + 5 + 11 + 19 + 29 + ...t_n
S = 1 + 5 + 11 + 19 +.... t_{n-1} + t_n
0 = 1+ (4 + 6 + 8 + 10 + ... $t_n - t_{n-1}$) – t_n
$\therefore$ t_n = 1+ (4 + 6 + 8 + 10 + ... (n –1) terms)

$t_n = 1 + \frac{n-1}{2}[2(4) + (n-2)2]$ [$\because$ sum of n –1 terms]

= 1 + (n –1) (4 + n –2)
t_n = 1 + (n –1) (n + 2)
$\therefore$ t_{100} = 1 + 99 (102) = 10099 Ans: (10099)

Exercise – 4(b)

Solutions for questions 1 to 55:

1. If a is the first term of an arithmetic progression and the common difference is d, the n^{th} term of the progression is given by a + (n – 1)d. The n^{th} term is given as 250, a as 6 and d as 4.
Hence, 250 = 6 + (n – 1)4.
(n – 1)4 = 244
61 = n – 1.
Hence, n = 61 + 1 = 62. Ans: (62)

2. Let the three numbers in A. P. be a – d, a and a + d,
a – d + a + a + d = 39

$a = \frac{39}{3} = 13$; $(a-d)^2 + a^2 + (a+d)^2 = 515$

$3a^2 + 2d^2 = 515$, $2d^2 = 515 - 3(13)^2 = 515 - 507$
d = ±2 and the smallest number is 13 – 2 = 11
Choice (C)

3. Sum of the squares of the first 10 even natural numbers
$= 2^2 + 4^2 + 6^2 + 8^2 + 10^2 \ldots\ldots + 20^2$
$= 2^2\ (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots\ldots\ldots + 10^2)$

$= \frac{4(10)(11)(21)}{6}$

[Applying the formula $\frac{n(n+1)(2n+1)}{6}$ for the sum of the squares of first n natural numbers]

$= \frac{4 \times 210 \times 11}{6} = 140 \times 11 = 1540.$ Ans: (1540)

4. $1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots$ is an infinite series in geometric progression.
a = 1, r = 1/2

Sum to infinity = $\frac{a}{1-r} = \frac{1}{1-1/2} = 2$

Hence, required value is 7^2 = 49. Ans: (49)

5. If the middle term is M, first term would be M–3d where d is the common difference and the last term would be M + 3d.
Hence, (M – 3d) (M + 3d) = 595
$M^2 = 595 + 9d^2 = 595 + 9(3)^2$
Hence, M = $\sqrt{676}$ = ±26. Since all the answer choices are positive, sum of the terms of the A.P.
= 7 × 26 = 182. Choice (D)

6. In the first hour, the distance covered by the athlete = 16(1) = 16 km. In the second hour, distance covered by the athlete = $\frac{1}{2}$ (16) (1) = 8 km.
Assuming that the person travelled for a total of t hours, we have $\frac{16\left(1 - \left(\frac{8}{16}\right)^t\right)}{1 - \frac{8}{16}} = 31.5$

$$= \frac{16\left(1-\left(\frac{8}{16}\right)^t\right)}{1-\frac{1}{2}} = 31.5$$

$$1 - \left(\frac{1}{2}\right)^t = \frac{31\cdot5}{32}$$

$$\left(\frac{1}{2}\right)^t = 1-\frac{31\cdot5}{32} = \frac{32-31\cdot5}{32} = \frac{0\cdot5}{32} = \frac{1}{64}$$

$$\Rightarrow \left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^6 \Rightarrow t = 6 \text{ hours.}$$

Alternative method:
The sum required is 31.5. First term is 16 and r is 1/2. Hence, writing down the terms upto the value $\frac{1}{2}$, can be a good method of solving.
The terms are 16, 8, 4, 2, 1, 1/2, 1/4………………
S_6 = 31.5; hence t = 6 hours. Ans: (6)

7. 5x + 8 – 3x = 2x + 8 is equal to 10x + 4 – (5x + 8) = 5x – 4. Hence, 2x + 8 = 5x – 4. Thus, x = 12/3 = 4. The first term (call it a) = 3x = 3 (4) = 12 and the common difference = 2x + 8 = 2 (4) + 8 = 16.
Sum of the first 10 terms of the series
= $\frac{10}{2}$ [2 (12) + 9 (16)]
= 5 [24 +144]= 5 [168] = 840. Ans: (840)

8. A (N) = $\frac{N(N+1)}{2}$

B(N) = $\frac{N(N+1)(2N+1)}{6}$

$\frac{B(N)}{A(N)} = \frac{2N+1}{3}$

for $\frac{B(N)}{A(N)}$ to be an integer, N must be in the form 3k + 1 where k is a whole number, i.e., 6p + 1 or 6p + 4
∴ N – 1 or N + 2 would be divisible by 6.
Choice (C)

9. If the sum of the terms of the series 2, 6, 18, ….. shall exceed 500, then $\frac{2(3^n-1)}{3-1} > 500$
$\Rightarrow 3^n - 1 > 500$
$3^n > 501$. Minimum value of n, satisfying the above inequality is n = 6 (which gives 3^6 = 729). Ans: (6)

10. The given series 40, 38, 36,…is cm A.P. with common difference of –2.
The sum of the first 20 terms (s_{20}) is 2 (20) (21)/2 = 420
The 21^{st} term is 0. ∴ s_{21} = 420
For n > 21 the terms would be negative Sn < 420.
∴ The maximum value of Sn = 420. Choice (B)

11. Sum upto the first 37 terms is $\frac{37}{2}$ [2a + (37 – 1) d]
= $\frac{37}{2}$ [2a + 36d] where a is the first term of A.P and d is the common difference.
= 37[a + 18d] = 703.
a + 18d = $\frac{703}{37}$ = 1 + 18d = 19 $\Rightarrow$ d = 1
Sum of the first 10 terms of the A.P
= $\frac{10}{2}$ [2 (1) + (10– 1) 1] = +5 [2 + 9] = 55.
Choice (A)

12. Let the four terms be a – 3d, a – d, a + d and a + 3d
a – 3d + a – d + (a + d) + a + 3d = 160
4a = 160 $\Rightarrow$ a = 40;
Given (a – 3d) (a + 3d) = 1564; $a^2 - 9d^2$ = 1564;
$9d^2 = a^2 - 1564 = 40^2 - 1564$
$d^2 = \frac{36}{9}$ = 4; d = ±2.
Since the A.P is ascending d = 2.
Smallest number = a – 3d = 40 – 3(2) = 34.
Choice (B)

13. $-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 - 19^2 + 20^2$
= $(-1^2 - 3^2 - 5^2 \ldots\ldots -19^2) + (2^2 + 4^2 + \ldots\ldots + 20^2)$
= $-(1^2 + 3^2 + 5^2 + \ldots\ldots 19^2) + (2^2 + 4^2 + \ldots\ldots + 20^2)$
= $-(1^2 + 2^2 + 3^2 \ldots\ldots + 20^2) + (2^2 + 4^2 + \ldots\ldots + 20^2) + (2^2 + 4^2 + \ldots\ldots + 20^2)$
= $2(2^2 + 4^2 + \ldots\ldots + 20^2) - (1^2 + 2^2 + 3^2 + \ldots + 20^2)$
= $2[2^2(1^2 + 2^2 + \ldots + 10^2] - (1^2 + 2^2 + 3^2 + \ldots + 20^2)$
= $\frac{8(10)(11)(21)}{6} - \frac{(20)(21)(41)}{6}$
= $\frac{210(88-82)}{6} = \frac{210(6)}{6}$ = 210.

Alternate method:
$-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 + \ldots\ldots.15^2 + 20^2$
= (–1 + 4) + (–9 + 16) + (–25 + 36) + ...(–361 + 400)
$= 3 + 7 + 11 + \ldots\ldots..39$
This is in arithmetic progression.
a = 3, n^{th} term = 39
Number of terms = 20/2 = 10 (as the given terms are grouped into pairs)
Sum of these 10 terms = $\frac{10}{2}(3+39) = 5 \times 42 = 210$
Ans: (210)

14. We get that $1^2 + 2^2 + \ldots\ldots + n^2$ is a multiple of 385 and $(n+1)^2 + (n+2)^2 + \ldots\ldots. + (2m)^2$ is a multiple of 2485, Moreover, $1^2 + 2^2 + \ldots\ldots. + n^2$ = 385a
$(n+1)^2 + (n+2)^2 + \ldots..(2m)^2$ = 2485a, where a is the quotient of $1^2 + 2^2 + \ldots\ldots\ n^2$ divided by 385 and also quotient of $(n+1)^2 + \ldots\ldots\ (2m)^2$ divided by 2485.
Also it follows that $1^2 + 2^2 + \ldots.. + n^2 + (n+1)^2 + (n+2)^2 + \ldots.(2m)^2 = 1^2 + 2^2 + \ldots\ldots. + (2m)^2$ = 385a + 2485a = 2870a.
By trial and error, we find that the above equation is satisfied when m = 10.
[2m(2m + 1) (4m + 1)/6] = 2870a Choice (A)

15. $1^3 + 2^3 + 3^3 + \ldots\ldots.. + m^3 = \left(\frac{m(m+1)}{2}\right)^2 = 55^2$
Hence, $\frac{m(m+1)}{2}$ = 55; m^2 + m = 110
m^2 + m – 110 = 0
Hence, m = –11 or m = 10. The number of terms cannot be negative. Thus, m = 10. Choice (C)

16. If the first term of an A.P. is a and the common difference of the A.P. is d, we have the sum equal to $\frac{n}{2}$ [2a + (n – 1) d]
= $2n^2$ + 6n = $\frac{n}{2}$ (4n + 12).
Hence, we have d = 4.
Alternate method:
Given $S_n = 2n^2 + 6n$

$a = t_1 = S_1 = 2(1^2) + 6(1) = 8$
$S_2 = 2(2)^2 + 6(2) = 20$
$t_2 = S_2 – S_1 = 20 – 8 = 12$
Common difference = $t_2 – t_1 = 12 – 8 = 4$ Ans: (4)

17. Let the second term and the common difference be a and d respectively.
First term = a – d
Third term = a + d
$(a – d)^2 + (a + d)^2 + a^2 = 365$
$3a^2 – 2d^2 = 365 \quad \to \quad (1)$
$(a – d)(a + d) = 120$
$a^2 – d^2 = 120 \quad \to \quad (2)$
solving (1) and (2),
$a^2 = 121$ and $d^2 = 1$
$a^2 + d^2 = 122$ Choice (C)

18. Both progressions are A.P. s. The series of common terms of two A.P. s is also an A.P. Its common difference is the L.C.M of the common differences of the two progressions.
First common term of the two progressions is 14.
The N^{th} common term of the two progressions
= 14 + (N – 1) L.C.M. (6, 4) – 14 + (N – 1) 12
14 + (N – 1) 12 < the smaller of the last terms of the two A .Ps
= 14 + (N – 1) 12 = 98
$\Rightarrow$ N = 8 Choice (C)

19. Let the first term and the common difference of S_1 be a_1 and d_1 respectively. Let the first term and the common difference of S_2 be a_2 and d_2 respectively.

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{11n-17}{5n-21}$$

$$= \frac{a_1+\frac{n-1}{2}d_1}{a_2+\frac{n-1}{2}d_2} = \frac{11n-17}{5n-21}$$

The ratio of the 16 terms of S_1 and S_2 = $\frac{a_1+15d_1}{a_2+15d_2}$

The 16^{th} term is the average of the first 31 terms. The ratio of 16 terms in equal ratio of the sum of the 31 times of the two series.
$\therefore$ The ratio of the 16^{th} terms of S_1 and S_2

$$= \frac{11(31)-17}{5(31)-21} = \frac{341-17}{155-21} = \frac{324}{134} = \frac{162}{67}$$ Choice (B)

20. The general form of the three-digit numbers satisfying the given condition is 8k + 1 where k is a natural number satisfying 100 < 8k + 1 < 1000
or $12\frac{3}{8} < k < 124\frac{7}{8}$
Thus, k can have any integral value from 13 to 124
The required sum = 8(13) + 1 + 8 (14) + 1 +…. 8 (124) + 1
= 8 (13 + ….. 124) + 112

$$= 8\left[\frac{112}{2}(13+124)\right] + 112 = 61488$$ Choice (C)

21. Required number of three-digit numbers = number of three-digit numbers less than 500
Number of three-digit numbers which are divisible by at least one of 4 and 6 = number of three-digit numbers divisible by 4 + number of three-digit numbers divisible by 6 – number of three-digit numbers divisible by both 4 and 6.
The three-digit numbers divisible by 4 are : 100 = 4 (25), 104 = 4 (26), ….. 496 = 4(124).
Suppose there are n such numbers.
4(124) = 4(25) + (n – 1) (4)
n = 100
Similarly it can be shown that there are 67 three digit numbers divisible by 6.
The three-digit numbers divisible by both 4 and 6 must be divisible by L.C.M. (4, 6) = 12.
The number of three digit numbers divisible by both 4 and 6 = 33
$\therefore$ 100 + 67 – 33 = 134 numbers are divisible by either 4 or 6 and 400 – 134 = 266 numbers are divisible by neither 4 nor 6
Ans: (266)

22. The salary of the person during the last month of the 1^{st} year is 5000 + (12 – 1) 200
= 5000 + 11 (200) = ₹7200
The salary of the person during the last month of the second year is ₹7200 + 11(400) = ₹11600. The salary of the person during the last month of the third year is 11600 + 11 (600) = 11600 + 6600 = 18200.
Hence, total salary the person has earned in four years

$$= \frac{12}{2}[2(5000) + 11(200)] + \frac{12}{2}[2(7200) + 11(400)] + \frac{12}{2}[2(11600) + 11(600)] + \frac{12}{2}[2(18200) + 11(800)]$$

= ₹6.36 lakh. Choice (D)

23. $\frac{7}{12}, -2m, \frac{12}{7}$ are in G.P.

$\therefore (-2m)^2 = \left(\frac{7}{12}\right)\left(\frac{12}{7}\right) = 1$ or $m = \pm\frac{1}{2}$ Choice (B)

24. The terms of the series are in the form x (21 – x)
Required sum = $\sum_{x=1}^{20} x(21-x)$

$= \frac{(21)(20)(21)}{2} - \frac{1}{6}(20)(21)(41) = 1540$ Choice (C)

25. $\log_3 x + \log_3\frac{1}{3}x + \log_3\frac{1}{5}x + \ldots + \log_3\frac{1}{23}x = 432$
$\log_3 x + 3\log_3 x + \log_3 x + \ldots + 23\log_3 x = 432$
$\Rightarrow 144\log_3 x = 432 \Rightarrow x = 27$ Ans: (27)

26. If the number of bacteria present initially is a, a $(2)^5$ is the number of bacteria present after 5 minutes.
= 32a = 1024, a = $\frac{1024}{32}$ = 32. Ans: (32)

27. Let the first term and the common ratio be a and r respectively.
Second term = ar and Third term = ar^2
$a + ar + ar^2 = 38$ and $(a)(ar)(ar^2) = 1728$
$(ar)^3 = 12^3 \Rightarrow ar = 12$
The first 3 terms are $\frac{12}{r}$, 12, 12r

$\frac{12}{r} + 12 + 12r = 38 \Rightarrow 6r^2 – 13r + 6 = 0 \Rightarrow r = \frac{2}{3}$ or $\frac{3}{2}$

if r = $\frac{2}{3}$, the numbers are 18, 12 and 8.

If r = $\frac{3}{2}$, the same numbers are obtained in the reverse order.
In either case, 8 is the smallest number Choice (D)

28. Let the common ratio be r. 0 < r < 1,
$\therefore r > r^2$. The first 3 terms are 18, 18r, $18r^2$
$18r – 18r^2 = 4 \Rightarrow r = \frac{2}{3}$ or $\frac{1}{3}$ Choice (D)

29. Let the number be abc. Let the common ratio be r. b = ar and c = ar^2
as $c \le 9$ and $a \ge 1$, $\frac{c}{a} = r^2 \le 9$.
r = 2 or 3

If a = 1, abc = 124 or 139
If a = 2, abc = 248
Thus, abc has three possibilities Ans: (3)

30. As a, c and b are in geometric progression,
$c^2 = ab$ $c^4 = a^2b^2$
As $a^2 + b^2$, $a^2 + c^2$ and $b^2 + c^2$ are in geometric progression,
$(a^2 + c^2)^2 = (a^2 + b^2)(b^2 + c^2)$
$a^4 + 2a^2c^2 + c^4 = a^2b^2 + a^2c^2 + b^2c^2 + b^4$
$\Rightarrow a^4 + a^2c^2 + c^4 = a^2b^2 + b^2c^2 + b^4$
since $c^4 = a^2b^2$
$a^2(c^2 + a^2 + b^2) = b^2(a^2 + b^2 + c^2)$
$a^2 = b^2 \Rightarrow$ a, b have the same sign.
$\therefore$ a = b and c = ±a Choice (A)

31. Let the first term and the common ratio be a and r respectively
First term = sum of all the terms following it
$a = \frac{a}{1-r} - a$
a (1 – 2r) = 0
As all the terms are positive, a ≠ 0.
1 – 2r = 0
$r = \frac{1}{2}$
a = 32 (1 – r) = 16 Choice (A)

32. Side of $S_2 = \sqrt{\left(\frac{\text{side of } S_1}{2}\right)^2 (2)} = \frac{1}{\sqrt{2}}$ (side of S_1)

It follows that side of $S_{n+1} = \frac{1}{\sqrt{2}}$ (side of S_n)
Where n is any natural number
Sum of the perimeters = 4 $\left(32 + \frac{32}{\sqrt{2}} + \frac{32}{2} + \ldots\right)$

$= \frac{4(32)}{1-\frac{1}{\sqrt{2}}} = 128\sqrt{2}(\sqrt{2} + 1)$ cm Choice (B)

33. If the first term of a G.P is a and the common ratio of the G.P is r, second, third and first terms of the G.P are ar, ar^2 and a. Since, these terms are in A.P., $2ar^2 = a + ar$
$2r^2 = 1 + r$ and $2r^2 - r - 1 = 0$
$\Rightarrow (r - 1)(2r + 1) = 0$
r = 1 or r = –1/2
$r = \frac{-1}{2}$, since the G.P has sum to infinity, | r | < 1.

Thus, $\frac{a}{1-\left(\frac{-1}{2}\right)} = 36$

$a = 36 \times \left(1+\frac{1}{2}\right) = 36 \times \frac{3}{2} = 54.$ Ans: (54)

34. Let the first term and the common difference of the arithmetic progression be a and d respectively.
$\frac{a+d}{a-2} = \frac{a+2d+10}{a+d}$
$(a + d)^2 = (a + 2d + 10)(a - 2)$
d (d + 4) = 8a – 20 = 4 (2a – 5) = 4 (odd number)
d (d + 4) must be divisible by 4 but not by 8.
This is possible only if d (and hence d + 4) is divisible by 2 but not 4. Choice (D)

35. The smallest multiple of 11 greater than 250 is 253 = 11(23)
The greatest multiple of 11 less than 750 is 748 = 11(68)
$\therefore$ There are 68 – 22 or 46 multiples of 11 in the given range. Ans: (46)

36. $S = 2 + 4x + 6x^2 + 8x^3 + \ldots \to (1)$
Multiplying by x both sides
$Sx = 2x + 4x^2 + 6x^3 + \ldots \to (2)$
Subtracting (2) from (1),
$S(1 - x) = 2 + 2x + 2x^2 + 2x^3 + \ldots$
$S = \frac{\frac{2}{1-x}}{1-x} = \frac{2}{(1-x)^2}$ Choice (C)

37. $S = \frac{1}{21} + \frac{1}{22} + \frac{1}{23} + \ldots + \frac{1}{40}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{40} - \left\{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{20}\right\}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{40} - 2\left\{\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{40}\right\}$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{39} - \frac{1}{40}$ Which is given in choice (A)

$S = \frac{1}{21} + \frac{1}{22} + \frac{1}{23} + \ldots + \frac{1}{40}$

$= \frac{1}{2}\left[\frac{2}{21} + \frac{2}{22} + \frac{2}{23} + \ldots + \frac{2}{40}\right]$

$= \frac{1}{2}\left[\left(\frac{1}{21} + \frac{1}{40}\right) + \left(\frac{1}{22} + \frac{1}{39}\right) + \left(\frac{1}{23} + \frac{1}{38}\right) + \ldots\ldots\ldots . + \left(\frac{1}{39} + \frac{1}{22}\right) + \left(\frac{1}{40} + \frac{1}{21}\right)\right]$

$= \frac{61}{2}\left[\frac{1}{21\times40} + \frac{1}{22\times39} + \frac{1}{23\times38} + \ldots + \frac{1}{39\times22} + \frac{1}{40\times21}\right]$

Which gives in choice (B)
From Choice (C), we get
$\frac{1}{31} + \frac{1}{32} + \frac{1}{33} + \ldots + \frac{1}{60}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{60} - 2\left\{\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{60}\right\}$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{59} - \frac{1}{60}$, which is not equal to S.
Thus, the answer is both choice (A) or Choice (B)
Choice (D)

38. $S = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \ldots + \frac{1}{9999}$

$= \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \frac{1}{7\times9} + \ldots + \frac{1}{99\times101}$

$= \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} + \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \ldots + \left(\frac{1}{99} - \frac{1}{101}\right)\right]$

In the above series all the terms except the first and the last cancel out

$= \frac{1}{2}\left[1 - \frac{1}{101}\right] = \frac{50}{101}$ Choice (A)

39. Method 1
Let the first term and the common ratio of the progression be a and r respectively.
$S_{2013} = 300$ and $S_{4026} = 540$
$\frac{a(r^{2013} - 1)}{r - 1} = 300$ and $\frac{a(r^{4026} - 1)}{r - 1} = 540$

$\frac{S_{4026}}{S_{2013}} = 1.8$

$\frac{r^{4026} - 1}{r^{2013} - 1} = 1.8$

$r^{2013} + 1 = 1.8$
$r^{2013} = 0.8$

Sum of the first 6039 terms of the progression

$$= \frac{a(r^{6039}-1)}{r-1} = \frac{a((r^{2013})^3-1)}{r-1}$$

$$= \frac{a(r^{2013}-1)((r^{2013})^2+r^{2013}+1}{r-1}$$

$= 300(0.8^2 + 0.8 + 1) = 300(2.44) = 732$

Method 2

The first 2013 terms (A), the next 2013 terms (B) and the next 2013 terms (C) together form the first 6039 terms of the progression.

Then the term of B is r^{2013} times the nth term of A.

$\therefore$ The sum of the terms of B will also be r^{2013} times that of A.

The nth term of C will be r^{2013} times that of B

$\therefore$ The sum of the terms of C will also be r^{2013} times that of B.

Sum of the terms of B = 540 − 300 = 240

$r^{2013} = \frac{240}{300} = 0.8$

Sum of the terms of C = r^{2013} (Sum of the terms of B)

= (0.8)(240) = 192

Sum of the first 6039 terms of the progression

= S_{4020} + Sum of the terms of C

= 540 + 192 = 732 Ans: (732)

40. $S = \frac{3}{4}+\frac{5}{36}+\frac{7}{144}+......\text{upto nine terms}$

In the above series, the general term ,$t_n = \frac{(n+1)^2-(n)^2}{n(n+1)^2}$

$$= \frac{1}{n^2}-\frac{1}{(n+1)^2}$$

$$\therefore \frac{3}{4}=1-\frac{1}{2^2}, \frac{5}{36}=\frac{1}{2^2}-\frac{1}{3^2}, \frac{7}{144}=\frac{1}{3^2}-\frac{1}{4^2}$$

. .

. .

. .

$$\frac{19}{8100}=\frac{1}{9^2}-\frac{1}{10^2}$$

Adding, we get,

$$\frac{3}{4}+\frac{5}{6}+\frac{7}{144}+...+\frac{19}{8100}=1-\frac{1}{10^2}=\frac{99}{100}$$ Choice (D)

41. With the usual notation, $S_n = \frac{a(r^n-1)}{r-1}$

$\therefore 6560 = \frac{2(3^n-1)}{2}$

$\Rightarrow 3^n = 6561 \Rightarrow n = 8$ Choice (D)

42. With the usual notation, $S_n = n\left[\frac{2a+(n-1)d}{2}\right]$

$\therefore 2883 = n\left[\frac{6+(n-1)6}{2}\right] \Rightarrow 3n^2 = 2883$

$\Rightarrow n = 31$ Ans: (31)

43. $S_n = 1 + 3 + 7 + 13 + 21 + 31 + 43 +....+t_n$

$S_n = \quad 1+ 3 + 7 + 13 + 21 + 31 +... + t_{n-1} + t_n$

Subtracting, we get

$0 = 1 + [2 + 4 + 6 + 8 + 10 +$ .... upto n – 1 terms] – t_n

$\Rightarrow t_n = 1 + [2 + 4 + 6 + 8 +.....$ up to n – 1 terms]

$\Rightarrow t_n = 1 + \frac{n-1}{2}[2(2)+(n-2)2] = 1 + \frac{(n-1)}{2}[2n] = 1 + n^2 - n$

$$\therefore S_N = \sum_{n=1}^{N} t_n = \Sigma(n^2-n+1) = \Sigma n^2 - \Sigma n + \Sigma 1$$

$$= \frac{N(N+1)(2N+1)}{6}-\frac{N(N+1)}{2}+N$$

$$\therefore S_{12} = \frac{12(13)(25)}{6}-\frac{12(13)}{2}+12$$

= 650 – 78 + 12 = 584 Ans: (584)

44. Let $S = 3^2(1) + 4^2(2) + 5^2(3) + 6^2(4) +... + 12^2(10)$

$\therefore S = 3^2(3-2) + 4^2(4-2) + 5^2(5-2) + 6^2(6-2)+......+ 12^2(12-2)$

$= 3^3 + 4^3 + 5^3 +.... + 12^3 - 2(3^2 + 4^2 + 5^2 +.....+ 12^2)$

$= [(1^3 + 2^3 + 3^3 +....12^3) - (1^3 + 2^3)] - 2[(1^2 + 2^2 +...+12^2) - (1^2 + 2^2)]$

$$= \left[\left\{\frac{(12)(13)}{2}\right\}^2-9\right]-2\left[\frac{12(13)(25)}{6}-5\right]$$

= [6084 – 9] – 2 [650 – 5]

= 6075 –1290= 4785 Choice (B)

45. $S_1 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} = \sqrt{\frac{4+4+1}{2^2}} = \frac{3}{2} = 2-\frac{1}{2}$

$$S_2 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} = \sqrt{\frac{4+4+1}{4}}+\sqrt{\frac{36+9+4}{36}}$$

$$= \frac{3}{2}+\frac{7}{6} = 3-\frac{1}{3}$$

$$S_3 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}$$

$$= \frac{3}{2}+\frac{7}{6}+\frac{13}{12} = 4-\frac{1}{4}$$

Proceeding similarly,

$$S_4 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\sqrt{1+\frac{1}{4^2}+\frac{1}{5^2}}$$

$$= 5-\frac{1}{5}$$

$$S_5 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+....+\sqrt{1+\frac{1}{5^2}+\frac{1}{6^2}} = 6-\frac{1}{6}$$

$$\therefore S_1+S_2+S_3+S_4+S_5 = 2-\frac{1}{2}+3-\frac{1}{3}+4-\frac{1}{4}+5-\frac{1}{5}+6-\frac{1}{6}$$

$$= 2+3+4+5+6-\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$$

$$= 20-\frac{30+20+15+12+10}{60} = 20-\frac{87}{60} = 20-\frac{29}{20}$$

$= \frac{371}{20} = 18\frac{11}{20}$ Choice (C)

46. $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{9}}+..+\frac{1}{\sqrt{119}+\sqrt{121}}$

$$= \frac{\sqrt{3}-1}{(\sqrt{3}-1)(\sqrt{3}+1)}+\frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}+\frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})}+ .....+\frac{\sqrt{121}-\sqrt{119}}{(\sqrt{121}-\sqrt{119})(\sqrt{121}+\sqrt{119})}$$

$$= \frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+....+\frac{\sqrt{121}-\sqrt{119}}{2}$$

$= \frac{\sqrt{121}-1}{2} = 5$ Choice (B)

47. In the given G.P., $t_{11} = ar^{10} = 3^5(2)\sqrt{6}$

$t_{16} = ar^{15} = 3^7(2)\sqrt{6}\sqrt{3}$

$\therefore r^5 = 3^{2.5} \Rightarrow r = \sqrt{3}$

$t_{19} = t_{16}(r^3) = 3^7(2)\sqrt{6}\sqrt{3}\left(3\sqrt{3}\right) = 3^9(2)\sqrt{6}$

Choice (D)

48. $T_1 = 3$
$T_2 = 3(3) - 2 = 7 = 2(3^{2-1}) + 1$
$T_3 = 3(7) - 2 = 19 = 2(3^{3-1}) + 1$
$T_4 = 3(19) - 2 = 55 = 2(3^{4-1}) + 1$
Proceeding similarly
$T_n = 2(3^{n-1}) + 1$
Therefore, $T_{200} = 2(3^{200-1}) + 1$
$= 2(3^{199}) + 1$ Choice (C)

49. 2 + 22 + 222 + ……. n terms

$= \frac{2}{9}$ (9 + 99 + 999 + …….. nterms)

$= \frac{2}{9}\left[(10-1)+(10^2-1)+......+(10^n-1)\right]$

$= \frac{2}{9}\left[10+10^2+........+10^n-n\right]$

$= \frac{2}{9}\left[\frac{10(10^n-1)}{9}-n\right]$ Choice (B)

50. The heights to which the ball rises on successive rebounds are in G.P. Each term of the G.P., except the first occurs twice.

$\therefore$ The total distance covered, $S = \frac{2a}{1-r} - a$

$= \frac{2(1250)}{1-\frac{4}{5}} - 1250 = 12500 - 1250 = 11250$

Ans: (11250)

51. $10 + (15 - 1)d = 80 \Rightarrow d = 5$
The 4^{th} mean is $10 + (5 - 1)d = 30$ Ans: (30)

52. $\frac{4}{2}\left[2a+(4-1)d\right] = 74;$

$\frac{4}{2}\left[a+8d+a+11d\right] = -22$

Solving, we get a = 23, d = –3 Choice (A)

53. Given series is 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4)…
$\therefore$ The n^{th} bracket is

1 + 2 + 3 + 4 + …. + n i.e. $= \frac{n(n+1)}{2}$

$S_n = \Sigma t_n = \Sigma \frac{n(n+1)}{2} = \frac{1}{2}\left[\Sigma n^2 + \Sigma n\right]$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$S_n = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$

$\therefore S_{20} = \frac{(20)(21)(41)}{12} + \frac{(20)(21)}{4}$

= 1435 + 105 = 1540 Choice (A)

54. Given series is
1 + 2 + 3 – 4, 2 + 3 + 4 – 5, 3 + 4 + 5 – 6, …….
i.e. 2, 4, 6, …. 100 terms

Sum $= \frac{100}{2}$ [2(2) + 99 (2)] = 10100 Ans: (10100)

55. $S = 1 + 4x + 9x^2 + 16x^3 + 25x^4 + 36x^5 + 49x^6 + \ldots$ (1)
$Sx = x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + 36x^6 + \ldots$ (2)
(1) – (2) : $S(1-x) = 1 + 3x + 5x^2 + 7x^3 + 9x^4 + 11x^5 + 13x^6 + \ldots$ (3)
$Sx(1-x) = x + 3x^2 + 5x^3 + 7x^4 + 9x^5 + 11x^6 + \ldots$ (4)
(3) – (4): $S(1-x)^2 = 1 + 2x + 2x^2 + 2x^3 + 2x^4 + 2x^5 + 2x^6 + \ldots$

$= 1 + \frac{2x}{1-x}$ ($\because |x| < 1$)

$S = \frac{1}{(1-x)^2} + \frac{2x}{(1-x)^3}$ Choice (A)

Solutions for questions 56 to 65:

56. From statement I, nothing can be concluded.
From statement II, sum of the first n terms of the AP = n/2(2a + (n – 1)d) where a is the first term and d is the common difference.
10/2(2a + 9d) = 15/2(2a + 14d) $\Rightarrow$ 2a = –24d.
Sum of first 25 terms of AP = 25/2(2a + 24d)
$\Rightarrow$ 25/2(–24d + 24d) = 0
So statement II alone is sufficient. Choice (A)

57. Let the common ratio be r.
If the first is the least, the terms are 1, r and r^2. If the last is the least, the terms are $\frac{1}{r^2}, \frac{1}{r}$ and 1

Using statement I,
$1 + r + r^2 = 21$
$r^2 + r - 20 = 0$
$(r + 5)(r - 4) = 0$
$\Rightarrow r = -5$ or 4. If r = –5, the numbers 1, –5 and 25. But the least here is –5. $\therefore r \neq -5$. Hence r = 4 (we may check for the consistency here also)
I is sufficient.
Using statement II,
$(1)(r)(r^2) = 64$
$\Rightarrow r^3 = 64 \Rightarrow r = 4$
$\therefore$ The middle term is 4
$\therefore$ II is sufficient. Choice (B)

58. From statement I
If x = 5, y = 15 and z = 45
then x, y and z are in G.P.
If x = 4, y = 16 and z = 44,
then x, y and z are not in G.P.
So statement I alone is not sufficient.
From statement II, we do not know about z so the second statement alone is not sufficient.
Using both the statements, y/x = –2 $\Rightarrow$ y = –2x
x + y = 20 $\Rightarrow$ x = –20 so y = 40
y + z = 60 $\Rightarrow$ z = 20
So x, y and z are not in geometric progression.
Choice (C)

59. From statement I, $a_1 = 1$
From statement II, $a_{n+1} = (a_n + 1)^2$
Combining statements I and II, we can answer the question. Choice (C)

60. From statement I, $\sqrt{xy} = 4 \Rightarrow xy = 16$
but we can't find the arithmetic mean of x and y as we do not know the values of x and y.
Statement I alone is not sufficient.

From statement II, $\frac{x+y+4+8}{4} = 5 \Rightarrow x + y = 20 - 12 = 8$

$\frac{x+y}{2} = 4.$

Statement II alone is sufficient. Choice (A)

61. Let the first term of G and its common ratio be a and r respectively. Let the number of its terms be n.

$$\frac{a(r^n-1)}{r-1}=\frac{3^8-1}{2} \text{ ____ (1)}$$

(a) (ar) (ar^2) -------- (ar^{n-1}) = 3^{28}

Using statement I, $r\ \frac{a(r^n-1)}{r-1}=\frac{3(3^8-1)}{2}$ _____ (2)

Dividing (2) by (1), r = 3. Hence I is sufficient.
Using statement II, n = 8
r^8 [(a) (ar) (ar^2) --------- (ar^7)] = 3^{36} _____ (3)
Dividing (3) by (2),
$r^8 = 3^8 \Rightarrow r = \pm 3$

When r = 3, sum of the terms of G = $\frac{a(3^8-1)}{3-1} = \frac{3^8-1}{2}$

(given) $\Rightarrow$ a = 1 and the product of the terms of G = 3^{28}.
$\therefore$ r = 3

When r = –3, sum = $\frac{a((-3)^8-1)}{-3-1} = \frac{3^8-1}{2} \Rightarrow a = -2$

But the product of the terms = $2^8 \times 3^{28}$; $\therefore r \neq -3 \Rightarrow r = 3$
II is also sufficient Choice (B)

62. Let the first term and the common ratio of the progression be a and r respectively.

$\frac{a}{1-r}$ = 8 ___ (1)

Using statement I, $a^2 + (a\,r^2)+(ar^2)^2$ + ------ = $\frac{64}{3}$

$a^2(1 + r^2 + (r^2)^2$ + ---------) = $\frac{64}{3}$

$\frac{a^2}{1-r^2}=\frac{64}{3}$ ____ (2)

(1) $\Rightarrow \frac{a^2}{(1-r)^2}$ = 64 ____ (3)

Dividing (3) by (2), $\frac{(1-r)(1+r)}{(1-r)(1-r)}$ = 3

I is sufficient.
Using statement II, first term = sum of all terms following it.
$\therefore$ a = sum to infinity of the progression – a

2a = $\frac{a}{1-r}$

a (2 (1 – r) – 1) = 0 a > 0)

$\therefore$ 2 (1 – r) – 1 = 0 ; r = $\frac{1}{2}$

II is sufficient.
Either of the statements is sufficient. Choice (B)

63. Let the first term and the common difference of the arithmetic progression be a and d respectively.
Using statement I, Sum of the odd numbered terms
= a + a + 2d + a + 4d + a + 6d + a + 8d + a + 10d = 6a + 30d
Let 6a + 30d = 11x
6a + 36d = 14x
11x + 14 x = 100
x = 4
14x – 11x = 6d
$\therefore$ 2 = d
I is sufficient.
Using statement II,
6a + 36d – (6a + 30d) = 12
d = 2
II is sufficient.
Either of the statements is sufficient. Choice (B)

64. Let the middle term of the AP be a and the common difference be d. So, the terms are a – 5d, a – 4d, a – 3d, a – 2d, a – d, a, a + d, a + 2d, a + 3d, a + 4d, a + 5d.

From statement I, $\frac{11a+0}{11}$ = 63 $\Rightarrow$ a = 63
So the middle term of the AP is 63

From statement II,
$\frac{6a-15d}{6}$ = 60 $\Rightarrow$ 6a – 15d = 360…….(1)

$\frac{6a+15d}{6}$ = 66 $\Rightarrow$ 6a + 15d = 396…….(2)

Solving the equations (1) and (2) we can get the value of a which is the middle term of the AP Choice (B)

65. From statement I,
If a = 4 and the common ratio is ½ then b < a
If a = 6 and the common ratio is –2 then b < a
So statement I alone is not sufficient.
From statement II,
abc > ac, so b > 1 as a and c must be of same sign.
But using this alone we can't say whether the common ratio is negative or not. (eg. a, b, c can be 8, 4 and 2 or –8, 4 and –2)
Using both the statements,
b > 1 and b < a so a must be positive.
a and b are positive so the common ratio also positive.
Choice (C)

Chapter – 5
(Functions)

Concept Review Questions

Solutions for questions 1 to 35:

1. Given, A = {4, 8, 12, 16, 20}
A = {x/x is multiple of 4 less than or equal to 20}
Choice (B)

2. Given, A = {x/x is an odd prime number less than 20}
= {3, 5, 7, 11, 13, 17, 19}. Choice (C)

3. Given, set A = {5, {3, 6}, {7, 8}, 10, 11}
Total number of distinct elements in the set A = 5.
Ans: (5)

4. Every element in {{3, 5}, 1} is also an element in {{3, 5}, 1, 4}.
$\therefore$ {{3, 5}, 1} is a subset. Choice (D)

5. A = {m, a, t, h, e, i, c, s}
n(A) = 8 Ans: (8)

6. A = {1, 3, 5, 7} and B = {2, 4, 6, 8, 10}
$A \cap B = \phi$
$\therefore$ A and B are disjoint sets. Choice (B)

7. A = set of all factors of 72.
= {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}
B = set of all multiples of 8.
= {8, 16, 24, 32, 40, 48, 56, 64, 72 ….}
$A \cap B$ = {8, 24, 72}. Choice (D)

8. **(a)** When $A \subseteq B$, then $A \cup B$ have minimum number of elements.
Here, n(A) = 8; n(B) = 10.
$\therefore$ The minimum number of elements of $A \cup B$ is = 10.
Choice (C)

(b) Given, n(A) = 6, n(B) = 4
Since n(B) < n(A)
The maximum number of elements in $A \cap B$ is n(B) = 4. Choice (A)

9. $A \Delta B = (A \cup B) - (A \cap B)$
If $A \Delta B$ contain maximum number of elements, then $A \cap B = \phi$.

$\therefore$ The maximum number of elements in A Δ B = the number of elements in $A \cup B$.
$n(A \cup B) = n(A) + n(B)$
$= 10 + 13 = 23$ Ans: (23)

10. The number of elements in any power set can be expressed in the form of 2^n. The number in option 'B' can not be expressed in the form of 2^n. Choice (B)

11. We know that if n(A) = m, then the number of non-empty proper subsets of A is $2^m - 2$.
Given, $2^m - 2 = 62$
$2^m = 64$
m = 6
$\therefore$ Number of elements in set A = 6. Ans: (6)

12. Given, n(A) = 4, and n(B) = 3
The number of elements in $A \times B$ is
$= n(A \times B) = n(A) \,.\, n(B) = 4.3 = 12$. Ans: (12)

13. $n(A \times B) = 48 \Rightarrow$ n(A) and n(B) must be factors of 48.
From option, 14 is not a factor of 48. Choice (C)

14. Given, n(A) = 6 and n(B) = 4
We know that the number of relations defined from A to B is $2^{n(A).n(B)}$.
$\therefore$ The number of relations defined from A to B is $2^{6 \times 4}$
$= 2^{24} = (2^3)^8 = 8^8$. Choice (B)

15. The maximum number of elements in a relation is equal to the number of elements in $A \times A$.
Since, n(A) = 5, $n(A \times A) = 25$
The maximum number of elements in a relation is 25.
Ans: (25)

16. Given, n(A) = 15; n(B) = 13; $n(A \cup B) = 20$;
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 15 + 13 - 20 = 8$ Ans: (8)

17. n(T) = t, n(C) = c
and $n(T \cap C) = e$
$\therefore n(T \cup C) = n(T) + n(C) - n(T \cap C)$
$= t + c - e$ Choice (B)

18. Given, $X = \{x : x^2 - 5x - 6 = 0\} = \{-1, 6\}$
$Y = \{y : y^2 - 8y - 9 = 0\} = \{-1, 9\}$
$\therefore Y - X = \{9\}$ Choice (C)

19. Given, A = {1, 5, 7, 9, 10}
B = {3, 4, 8, 6}, C = {3, 7, 9}
$B \cup C$ = {3, 4, 6, 7, 8, 9}
$A - (B \cup C)$ = {1, 5, 10} Choice (A)

20. Given, $A \cap B = \phi$; $B \cap C = \phi$; and $A \cap C = \phi$
A – B = A; B – C = B
$\therefore (A - B) \cap (B - C) = A \cap B = \phi$ Choice (B)

21. Choice (A) is not a function from A to B as $4 \notin A$.
Choice (B) is not a function from A to B as $1 \notin B$
Choice (C) is a function from A to B as for $x \in A$, $y \in B$ and $x \in A$ there is only one image in B
i.e., (x = k) Choice (C)

22. Option (C) does not represent a function.
Since –4 of A is not having an image. Choice (C)

23. **(a)** The set of first coordinates in a function is known as domain.
$\therefore$ domain = {3,4,7,1} . Choice (D)

(b) The set of second coordinates in a function is known as range.
$\therefore$ range = {3,5,6} . Choice (A)

24. $f(x) = (-1)^{2n} + 3$,
for any $n \in w$; 2n is even.
$\therefore (-1)^{2n} = 1$.
$\therefore f(x) = (-1)^{2n} + 3 \Rightarrow 1 + 3 = 4 \ \forall \ n \in w$
Range= {4}. Choice (B)

25. **(a)** $f(x) = \dfrac{3x+2}{|3x+2|}$
we know that when 3x + 2 > 0
$|3x + 2| = 3x + 2$ and
3x + 2 < 0 then
$|3x + 2| = -(3x + 2)$
$\therefore$ when 3x + 2 > 0
$f(x) = \dfrac{3x+2}{3x+2} = 1$ and
when 3x + 2 < 0, $f(x) = \dfrac{3x+2}{-(3x+2)} = -1$
$\therefore$The range of f(x) = {–1, 1} Choice (A)

(b) Given f(x) = [x] – x
We know that x – [x] is always [0, 1)
[x] – x is always belongs to (–1, 0]
$\therefore$ The range of [x] – x is (–1, 0] Choice (A)

26. f(x, y) = 3x – 2y
f(3, –1) = 3(3) – 2(–1) = 11
f(4, f(3, –1)) = f(4, 11) = 12 – 2(11) = –10.
Ans: (–10)

27. $f(x) = ax^3 - bx^2 + bx - a$
$f\left(\dfrac{1}{x}\right) = \dfrac{a}{x^3} - \dfrac{b}{x^2} + \dfrac{b}{x} - a$
$= \dfrac{a - bx + bx^2 - ax^3}{x^3} = \dfrac{-(ax^3 - bx^2 + bx - a)}{x^3}$
$= -\dfrac{f(x)}{x^3}$ Choice (A)

28. $f(x) = a^{x+p}$
$f(k + \ell) = a^{k+\ell+p}$
$f(k - \ell) = a^{k-\ell+p}$
$\dfrac{f(k+\ell)}{f(k-\ell)} = \dfrac{a^{k+\ell+p}}{a^{k-\ell+p}} = a^{2\ell} = f(2\ell - p)$ Choice (C)

29. **(a)** If f(–x) = f(x), then f(x) is an even function.
Choice (C)

(b) A function is even if f(–x) = f(x) and odd if f(–x) = – f(x)
here f(–x) = sin(–x) = –sinx = –f(x)
$\therefore$ it is an odd function. Choice (B)

(c) f(x) = y = cosx
$\Rightarrow$ f(–x) = cos(–x) = cosx = f(x)
$\therefore$ y = cosx is an even function. Choice (A)

30. Let $g(x) = \dfrac{f(x)+f(-x)}{2}$, $g(-x) = \dfrac{f(x)+f(-x)}{2} = g(x)$
g(–x) = g(x)
$\therefore$ g(x) is an even function. Choice (A)

31. The only function which is both even and odd is the zero function i.e. $f(x) \equiv 0$ Choice (B)

32. Given $f(x) = \dfrac{1}{|x+2|}$
it is not defined only when x = – 2
$\therefore$ Domain is R – {–2} Choice (C)

33. Given $f(x) = \dfrac{\operatorname{cosec}^2 x + \sec^2 x}{\operatorname{cosec}^2 x . \sec^2 x}$
$= \dfrac{1}{\operatorname{cosec}^2 x . \sec^2 x}\left[\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}\right]$

$$= \frac{1}{\operatorname{cosec}^2 x.\sec^2 x}\left[\frac{\cos^2 x+\sin^2 x}{\sin^2 x \cos^2 x}\right]$$

$$= \frac{1}{\operatorname{cosec}^2 x.\sec^2 x}.\frac{1}{\sin^2 x}.\frac{1}{\cos^2 x} = 1$$

f(x) = 1 which is a constant function. Choice (D)

34. Given A = {1, 2, 3, 4, 5, 6, 7}
B = {8, 9}
We know that, if n(A) = m, n(B) = 2, then the number of onto functions defined from A to B is $2^m - 2$.
Here m = 7
$\therefore$ number of onto functions from A to B is
$= 2^7 - 2 = 128 - 2 = 126$ Ans: (126)

35. Given n(B) = 6
Since f is one-one function, from A to B
$n(A) \le n(B)$
$\therefore$ the number of elements in A is at most 6.
Choice (B)

Exercise – 5(a)

Solutions for questions 1 to 4:

1. The number of proper subsets of A = $2^8 - 1 = 255$
Ans: (255)

2. The number of subsets of A that contain exactly 4 elements $= {}^8C_4 = 70$ Choice (C)

3. The number of subsets of A that contain at most 5 elements
$= {}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$ (or) $2^8 - ({}^8C_6 + {}^8C_7 + {}^8C_8)$
= 256 – (28 + 8 + 1)= 219 Choice (A)

4. The subset must contain a, and f but not g $\therefore$ for each of the other 5 elements we can make a choice – include or exclude. $\therefore$ The number of subsets is 2^5 or 32.
Choice (C)

Solutions for questions 5 to 13:

5. The number of functions from set A to set B is given by $\{n(B)\}^{n(A)}$
Here n (A) = 4 and n(B) = 3 $\therefore$ The number of functions is
$3^4 = 81$ Choice (C)

6. The number of functions from set A to set B is $\{n(B)\}^{n(A)}$ $= 5^3 = 125$
The number of one - one functions from set A to set B is ${}^5P_3 = 60$
$\therefore$ The number of functions which are not one - one
125– 60 = 65. Ans: (65)

7. The number of onto functions from set A to set B, if n(A) = p and n(B) = q and $p \ge q$ is
$q^p - {}^qC_1(q-1)^p + {}^qC_2(q-2)^p + \ldots.. + {}^qC_{q-1}[q-(q-1)]^p$
Here P = 4 and q = 3
$\Rightarrow 3^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4 = 81 - 3(2)^4 + 3 = 36$
Choice (C)

8. The number of bijections from set A to set A when n(A) = n is n!. Here n = 4 $\therefore$ The number of bijections is 4! or 24
Ans: (24)

9. Given F(a,b,c,d) = ab – cd
F(y, y + 4 , – 4, 6) = y (y + 4) – (– 4) (6)
$= y^2 + 4y + 24$
F(8, 17.5, 4, 5) = (8) (17.5) – (4)(5) = 140 – 20 = 120
$y^2 + 4y + 24 = 120 \Rightarrow y^2 + 4y - 96 = 0$
$\Rightarrow (y + 12)(y - 8) = 0 \Rightarrow y = -12$ or 8
Choice (C)

10. $g_1(m + 1) = 9(m + 1)^2 = 9m^2 + 18m + 9$ and
$g_2(3m) = (3m)^2 - 12(3m) + 27 = 9m^2 - 36m + 27$
$\therefore 9m^2 + 18m + 9 = 9m^2 - 36m + 27$
$\Rightarrow 54m = 18 \Rightarrow m = \frac{1}{3}$ Choice (C)

11. Given f(x) = log $\frac{(1-x)}{1+x}$

$$f(x) + f(y) = \log\left(\frac{1-x}{1+x}\right)+\log\left(\frac{1-y}{1+y}\right)$$

$$= \log\left[\frac{(1-x)(1-y)}{(1+x)(1+y)}\right]$$

$$\log\left(\frac{1-x-y+xy}{1+x+y+xy}\right) = \log\left(\frac{1+xy-(x+y)}{1+xy+(x+y)}\right)$$

$$= \log\left(\frac{1-\left(\frac{x+y}{1+xy}\right)}{1+\left(\frac{x-y}{1+xy}\right)}\right) = f\left(\frac{x+y}{1+xy}\right)$$ Choice (C)

12. Whenever a and b are any two real numbers satisfying $a^2 + b^2 = 0$, a = b = 0
$\therefore$ The given equation implies g(x) + f(x) = h(x) + f(x) = 0
$\therefore$ g(x) = h(x) i.e $3x = 3|x|$ i.e, $x = |x|$ $\therefore x \ge 0$
$\therefore$ f(x) = –h(x) = –3x Choice (C)

13. Given $4f(x) - 5f\left(\frac{1}{x}\right) = x^3$ $\rightarrow$ (1)

Put $x = \frac{1}{x}$, $4f\left(\frac{1}{x}\right) - 5f(x) = \frac{1}{x^3}$ $\rightarrow$ (2)

$4(1) + 5(2) = 16f(x) - 25f(x) = 4x^3 + \frac{5}{x^3}$

$-9f(x) = 4x^3 + \frac{5}{x^3}$

$$f(x) = -\frac{1}{9}\left[4x^3 + \frac{5}{x^3}\right]$$

$$f(0.2) = -\frac{1}{9}\left[4(0.2)^3 + \frac{5}{(0.2)^3}\right]$$

$= -\frac{1}{9}[0.032 + 625]$

$= -\frac{1}{9}[625.032] = -69.448$ Ans: (–69.448)

Solutions for questions 14 and 15:

14. The values of the four functions for 4 sets of values of x (less than –1, between –1 and 0, between 0 and 1 and greater than 1) are tabulated below.

	x < –1	–1 ≤ x < 0	0 ≤ x < 1	1 < x
f_1	–1	–x	0	0
f_2	0	0	–x	–1
f_3	0	0	x	1
f_4	1	–x	0	0

We can see that f_1f_2, f_3f_4 and f_2f_4 are identically zero, but f_2f_3 is not, i.e 3 of the expressions are identically 0.
Choice (D)

15. We can consider the option.
Choice (A) $f_1(x) = f_4(x)$. False for x < – 1
Choice (C) $f_2(x) = f_4(x)$. False for x < – 1

Choice (D) $f_1(-x) = f_3(x)$. False for $0 \le x < 1$
Choice (B) can be seen to be true.

Choice (B)

Solutions for questions 16 to 35:

16. Given f(x) is an odd function and g(x) is an even function, i.e., $f(-x) = -f(x)$ and $g(-x) = g(x)$.
Now, fog $(-x) = f(g(-x)) = f(g(x)) = fog(x)$
$\therefore$fog (x) is an even function.
Also, $gof(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = gof(x)$.
$\therefore$gof(x) is an even function. Choice (A)

17. $f(x) = \log(x^2 - 4) + \dfrac{1}{\sqrt{9 - x^2}}$

log f(x) is defined only when f(x) > 0
$\log(x^2 - 4)$ is defined when $x^2 - 4 > 0$:
$x < -2$ or $x > 2$ $\rightarrow$ (1)
We know $\sqrt{f(x)}$ is defined only when $f(x) \ge 0$

$\dfrac{1}{\sqrt{9 - x^2}}$ is defined when $9 - x^2 > 0$

$9 - x^2 > 0$
$\Rightarrow x^2 - 9 < 0$
$\Rightarrow -3 < x < 3$ $\rightarrow$ (2)
$\therefore$ From (1) and (2) the domain of f(x) is $(-3, -2) \cup (2, 3)$
Choice (C)

18. $f(x) = |x + 7| + |x - 9| + 12$
When $x \ge 9$
$|x - 9| = x - 9$
$|x + 7| = x + 7$
$f(x) = x + 7 + x - 9 + 12$
$= 2x + 10$
The minimum of f(x) = 28
When $x < -7$, $|x + 7| = -(x + 7)$
$|x - 9| = -(x - 9)$
$f(x) = -(x + 7) - (x - 9) + 12$
$= 14 - 2x$
$\therefore$ f(x) is always greater 28.
When $-7 < x < 9$
$|x + 7| = x + 7$
$|x - 9| = -(x - 9)$
$f(x) = |x + 7| + |x - 9| + 12$
$= x + 7 - x + 9 + 12 = 28$
$\therefore$ The minimum value of f(x) is 28
$\therefore$ The range of f(x) is $[28, \infty)$ Choice (D)

19. Let $f(x) = \dfrac{1}{3\,|x - \lfloor x \rfloor|}$

For all integers, $x - [x] = 0$
$\therefore$ The domain of f(x) is R – Z Choice (C)

20. Consider a simpler function, say g(x) = min { |x– a|, |x –b| } where a < b. On the number line, every number corresponds to a point, say 'a' to point A, 'b' to point B and 'x' to point X.(As a < b, A is to the left of B)
|x – a| is the distance from X to A.
|x – b| is the distance from X to B.
g(x) is the distance from X to the closer of the two points A and B.
Let M be the midpoint of AB.
When X is to the left of M, g(x) = | x –a|
When X = M, g(x) = |x –a| = | x – b|
When X is to the right of M, g(x) = |x – b|
Now consider the given function f(x) = min { |x+2|,|x|,|x–2| }
Let the points on the number line be A = –2, B = 0, C = 2, X= x f(x) is the distance from X to the closest of the 3 points A, B and C.
Let M, be the midpoint of BC. When X = M, or X is to the right of M, the closest point (or one of the closest points) to X is C, ie f(x) = |x – 2|.

But we want f(x) to be equal to x–2 and not |x–2|. This will be true if $x - 2 \ge 0$ or $x \ge 2$. $\therefore$ The required range is $[2, \infty)$
Choice (C)

21. $f(x + 1) = f(f(x))$ when $x \ge 5$

x	f(x)	x	f(x)
1	4	6	1
2	5	7	4
3	1	8	2
4	2	9	5
5	3	10	3

$\therefore$ f(6) = f(f(5))= f(3) = 1
f(7) = f(f(6) = f(1) = 4
f(8) = f(4) = 2
f(9) = f(2) = 5
f(10) = f(5) = 3
f(11) = f(3) = 1= f(6)
After this the values will repeat
We have a cycle of 5 for the values of f(x) when $x \ge 6$.
f(9) = f(14) = f(19) = f(24) = f(29) ………..= f(499).
$\therefore$ f(499) = 5 Ans: (5)

22. f(x + y) = f(x) + f(y)
Only when f(x) is linear function (with the contant term equal to 0) the above condition is satisfied
Let f(x) = kx
Given f(3) = 29
$3k = 29 \Rightarrow k = \dfrac{29}{3}$

$\therefore f(x) = \dfrac{29}{3}x$

$f(27) = \dfrac{29}{3} \times 27 = 261$ Choice (A)

23. Given f(x + y) = f(x).f(y)
An exponential function satisfies the above condition
Let $f(x) = k^x$
$f(4) = 4096 = 2^{12}$
As $f(2 + 2) = 2^{12}$
$\Rightarrow f(2)\,f(2) = 2^{12}$
$\Rightarrow f(2) = 2^6$
Similarly $f(1) = 2^3$
$\Rightarrow f(10) = f(1 + 1 + \quad 1) = [f(1)]^{10} = 2^{30}$ Choice (C)

24. $f(x) = \dfrac{x-1}{x+1}, x \ne -1$

$$f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{x-1-x-1}{x+1} \times \frac{x+1}{x-1+x+1}$$

$= \dfrac{-2}{2x} = -\dfrac{1}{x}$. Choice (B)

25. Given f(xy) = f(x).f(y). The function which satisfies the above condition is in the form $f(x) = x^n$.
Given f(3) = 27
$\Rightarrow 3^n = 27 \Rightarrow n = 3$.
$\therefore f(x) = x^3$
$\sum_{n=1}^{30} f(n) = f(1) + f(2) + f(3) + \ldots f(30)$
$= 1^3 + 2^3 + \ldots + 30^3$
$= \left[\dfrac{30(31)}{2}\right]^2 = 465^2$ Choice (D)

26. f(xy) = f(x) + f(y). The only function possible is one whose form is $f(x) = \log_a x$. Given $f(3) = 1 \Rightarrow \log_a 3 = 1 \Rightarrow a = 3$
$\therefore f(x) = \log_3 x$.

$$\frac{f(243)-f(81)}{f(27)-f(9)} = \frac{\log_3 243 - \log_3 81}{\log_3 27 - \log_3 9}$$

$$= \frac{\log_3 \frac{243}{81}}{\log_3 \frac{27}{9}} = 1.$$

Alternative Method:
$f(x^2) = f(x) + f(x) = 2f(x)$
$f(x^3) = f(x^2) + f(x) = 3f(x)$
In generalises that $f(x^N) = Nf(x)$ where N is any positive integer.

$$\frac{f(243)-f(81)}{f(27)-f(9)} = \frac{f(3^5)-f(3^4)}{f(3^3)-f(3^2)} = \frac{5f(3)-4f(3)}{3f(3)-2f(3)} = 1$$

Ans: (1)

27. $f(x) = \frac{x+1}{x-1}$. Let $f^{-1}(x) = y \Rightarrow f(y) = x$

$\frac{y+1}{y-1} = x \Rightarrow \frac{2y}{2} = \frac{x+1}{x-1} \Rightarrow y = \frac{x+1}{x-1}$

$\therefore f^{-1}(x) = \frac{x+1}{x-1}, x - 1 \neq 0$

i.e., f^{-1} is defined for $x \neq 1$
$f^{-1}(x)$ is not defined for $x \neq 1$ Choice (A)

28. $f(x) = \frac{5x+3}{4x-9}$

$f(x) = y \Rightarrow x = f^{-1}(y)$

$\frac{5x+3}{4x-9} = y$

$5x + 3 = 4xy - 9y$
$3 + 9y = 4xy - 5x$
$3 + 9y = x(4y - 5)$

$\frac{9y+3}{4y-5} = x$

$\frac{9y+3}{4y-5} = f^{-1}(y)$

$f^{-1}(x) = \frac{9x+3}{4x-5}$ Choice (A)

29. $x, R(x), \in A \therefore R(x) \leq 30$

$2x + 5 \leq 30 \Rightarrow x \leq \frac{25}{2} \Rightarrow x \leq 12.5$

When $x \leq 12$, $R(x) \in A$.
$\therefore$ The number of elements in the relation R is 12.

Ans: (12)

30. $f(2x - 1) = 8x^2 - 10x + 6$
$= 2(4x^2 - 4x + 1) - 2x + 1 + 3$
$f(2x - 1) = 2(2x - 1)^2 - (2x - 1) + 3$
$\therefore f(x) = 2x^2 - x + 3$
$f(t) = 2t^2 - t + 3$ (replacing $2x - 1$ by t)
$f(0) = 2(0)^2 - 0 + 3 = 3.$

Alternate method:
Given,
$f(2x - 1) = 8x^2 - 10x + 6$

$2x - 1$ is zero, if $x = \frac{1}{2}$

Put $x = \frac{1}{2}$ in the given equation, we have

i.e., $f\left(2\left(\frac{1}{2}\right)-1\right) = 8\left(\frac{1}{2}\right)^2 - 10\left(\frac{1}{2}\right) + 6$

$= 2 - 5 + 6$

$\therefore f(0) = 3$ Ans: (3)

31. Let $x = \frac{1}{100}$, $f\left(\frac{1}{100}\right) + f\left(2 - \frac{1}{100}\right) = 4$

$\Rightarrow f\left(\frac{1}{100}\right) + f\left(\frac{199}{100}\right) = 4$

Let $x = \frac{2}{100}$, $f\left(\frac{2}{100}\right) + f\left(2 - \frac{2}{100}\right) = 4$

$\Rightarrow \left(\frac{2}{100}\right) + f\left(\frac{198}{100}\right) = 4$

Let $x = \frac{99}{100} \Rightarrow f\left(\frac{99}{100}\right) + f\left(2 - \frac{99}{100}\right) = 4$

$\Rightarrow f\left(\frac{99}{100}\right) + f\left(\frac{101}{100}\right) = 4$

Let $x = \frac{100}{100} \Rightarrow f\left(\frac{100}{100}\right) + f\left(2 - \frac{100}{100}\right) = 4$

$\Rightarrow f\left(\frac{100}{100}\right) + f\left(\frac{100}{100}\right) = 4 \Rightarrow f\left(\frac{100}{100}\right) = 2$

$\therefore$ The value of $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + \ldots\ldots\ldots + f\left(\frac{199}{100}\right)$

$= 4 \times 99 + 2 = 396 + 2 = 398.$ Ans: (398)

32. Given $f(x) = ax + b$
$f(f(x)) = a(ax + b) + b = a^2x + ab + b$
$f(f(f(x))) = a^2(ax + b) + ab + b = a^3x + a^2b + ab + b$
given $f(f(f(x))) = 125x + 217$
$\Rightarrow a^3 x + a^2b\ \ ab + b = 125x + 217$
$\Rightarrow a^3 x = 125x$ and $a^2b + ab + b = 217$
$\Rightarrow a^3 = 125$ and $25b + 5b + b = 217$
$a = 5$ $31b = 217$

$b = \frac{217}{31} = 7$

$\therefore 7a - 5b = 7 \times 5 - 5 \times 7 = 0$ Choice (A)

33. Consider the two expressions $x + 2$ and $4 - 3x$
$x + 2 = 4 - 3x \Rightarrow x = 1/2$
For $x < 1/2$, $x + 2 < 4 - 3x$
For $x = 1/2$, $x + 2 = 4 - 3x$
For $x > 1/2$, $x + 2 > 4 - 3x$
$\therefore$ For $x \leq 1/2$, min $(x + 2, 4 - 3x) = x + 2$
And for $x > 1/2$, min$(x + 2, 4 - 3x) = 4 - 3x$
The minimum value of this occurs when $x = 1/2$ and this value is 2.5. Ans: (2.5)

34. If product of elements in Q is even possible if the set contain at least one even number.
$\therefore$ The number of subsets are formed with no even numbers present is

${}^{50}C_1 + {}^{50}C_2 + \ldots. + {}^{50}C_{50}$
$= 2^{50} - 1$
Hence the number of subsets that contain at least one even number
= total number of subsets – the number of subsets that contain no even number
$= (2^{100} - 1) - (2^{50} - 1)$
$= 2^{100} - 2^{50}$
$= 2^{50}(2^{50} - 1)$ Choice (B)

35. Given $f(x) = ax^6 + bx^4 - cx^2 + 3x + 7$
$f(9) = a9^6 + b9^4 - 9^2c + 3(9) + 7$
$26 = 9^6 a + 9^4 b - \overline{81}c + 34$
$-8 = 9^6 a + 9^4 b - 81c$
put $x = -9$
$f(-9) = a(-9)^6 + b(-9)^4 + c(-9)^2 + 3(-9) + 7$
$= 9^6 a + 9^4 b - 9^2 c - 20$
$= -8 - 20$
$= -28$ Ans: (–28)

Exercise – 5(b)

Solutions for questions 1 to 5:

1. The number of non-empty subsets of A
$= 2^9 - 1 = 511$ Choice (B)

2. The number of non-empty proper subsets of A
$= 2^9 - 2 = 510$ Choice (A)

3. The number of subsets of A that contain at least two elements S
$= 2^9 - \left({}^9C_1 + {}^9C_0\right) = 512 - (9+1) = 502$ Choice (B)

4. The number of subsets of A that contain 1, 2, and 3 is given by $2^{9-\text{(the number of elements included)}} = 2^{9-3} = 2^6 = 64$
Choice (C)

5. The number of subsets of A that do not contain is $= 2^{9-3}$ $= 2^6 = 64$ Choice (A)

Solutions for questions 6 to 45:

6. The number of one - one functions (or) injections from set A to set B is given by ${}^bP_a$, where n(A) = a, n(B) = b
Here b = 6, a = 5
$\therefore$ The required number of injections is ${}^6P_5 = 720$
Ans: (720)

7. The number of onto functions from set A to set B when n(A) = p and n(B) = 2 is $2^p - 2$
Here p = 5, $\therefore$ $2^5 - 2 = 30$ Choice (A)

8. The number of injections from set A to set B when n(A) > n(B) is zero Choice (B)

9. Let n(A) = p
Then the number of proper subsets of A is $2^p - 1 = 127$
$2^p = 128 = 2^7 \Rightarrow p = 7$
$\Rightarrow$ n(A) = p = 7
Number of subsets that contain exactly two elements but not a particular element of A is ${}^6C_2 = 15$
Choice (D)

10. The number of bijections when n(A) = n(B) = P is P!
Here P = 5 i.e, 5! = 120 Choice (D)

11. The number of bijections from set A to set B when n(A) ≠ n(B) is zero Choice (A)

12. Given H (m,n,p,q) = mq + np
$H(x, 8, 9, x + 12) = x(x + 12) + 8(9) = x^2 + 12x + 72$
$H(12, 16, 7, 50) = 12(50) + 16(7) = 712$
$x^2 + 12x + 72 = 712 \Rightarrow x^2 + 12x - 640 = 0$
$\Rightarrow (x + 32)(x - 20) = 0 \Rightarrow x = -32$ or 20 Choice (D)

13. Given h(x + 2) = 2h(x + 1) – h(x)
h(x +2) – h(x + 1) = h(x + 1) –h(x)
$\therefore$ h(x), h(x + 1) and h(x + 2) are in AP
$\therefore$ h(0) , h(1), h(2)……………h(13) are in AP.
The first term of this AP is h(0) = –2 and the common difference is 3 – (–2) = 5
h(2) = 3 + 5 = 8
h(8) – h(2) = 6(5) = 30 and h(13) – h(2) = 11(5) = 55
$\Rightarrow$ h(8) = 30 + 8 h(13) = 55 + 8 ($\because$h(2) = 8)
$\therefore$ h(8) = 38 and h(13) = 63
$\text{Rem}\left(\frac{h(8)h(13)}{17}\right) = \text{Rem}\left(\frac{(38)(63)}{17}\right)$ $\text{Rem}\,\frac{4(12)}{17} = 14$
Ans: (14)

14. goh(a)= g[h(a)] = g(9a+8)= 8(9a + 8) – 9 = 72a + 55
2(goh(a)) = 144a + 110
hog(a)=h[g(a)] = h[8a –9]= 9(8a – 9) + 8 = 72a – 73
Given 2 goh(a) = hog(a)
$\therefore 144a + 110 = 72a - 73 \Rightarrow 72a = -183 \Rightarrow a = \frac{-183}{72}$
Choice (A)

15. $f(2x + 3) = 4x^2 + 14x + 14$
$= (2x)^2 + 2\cdot2x\cdot3 + (3)^2 + 2x + 3 + 2$
$f(2x + 3) = (2x + 3)^2 + (2x + 3) + 2$
replacing 2x + 3 by t we have $\Rightarrow f(t) = t^2 + t + 2$
$\therefore f(x) = x^2 + x + 2$ Choice (D)

16. Let h(x) = h, h(x–2) = h_2 and h(x– 4) = h_4
$h_2 = 4h^2 - 5$ and $h_4 = 4h_2^2 - 5 = 4(4h^2 - 5)^2 - 5 =$
$4(16h^4 - 40h^2 + 25) - 5 = 64h^4 - 160h^2 + 95$
Choice (D)

17. $f_1(k-3) = 16(k-3)^2 = 16k^2 - 96k + 144$
And $f_2(4k) = (4k)^2 - 18(4k) - 48 = 16k^2 - 72k - 48$
$\therefore 16k^2 - 96k + 144 = 16k^2 - 72k - 48 \Rightarrow 192 = 24k$
$\Rightarrow k = 8$ Ans: (8)

18. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$f(x) - f(y) = \log\left(\frac{1+x}{1-x}\right) - \log\left(\frac{1+y}{1-y}\right)$

$= \log\left(\frac{\frac{1+x}{1-x}}{\frac{1+y}{1-y}}\right)$

$= \log\left[\left(\frac{1+x}{1-x}\right)\left(\frac{1-y}{1+y}\right)\right] = \log\left[\frac{(1+x)(1-y)}{(1-xy-(x-y))}\right]$

$= \log\left[\frac{1+x-y-xy}{1-x+y-xy}\right] = \log\left[\frac{1-xy+x-y}{1-xy-(x-y)}\right]$

$= \log\left[\frac{1+\frac{x-y}{1-xy}}{1-\frac{x-y}{1-xy}}\right]$ (Dividing both numerator and denominator of the argument by 1–xy)

$= f\left(\frac{x-y}{1-xy}\right)$ Choice (C)

19. g(x) = g(g(x– 1)) when x ≥ 6
g(7) = g(g(6) = g(3) = 4
g(8) = g(4) = 1
g(9) = g(1) = 5
g(1) = g(5) = 2
g(11) = g(2) = 6
g(12) = g(6) = 3
g(13) = g(3) = 4 = g(7)
$\therefore$ We have a cycle of 6 for the values of g(x) when x ≥ 6
($\therefore$ Each of 11, 17, 23……..899 have the from 6k - 1)
$\therefore$ g(899) = 6 Choice (A)

20. $f(x) = \frac{5}{\sqrt[3]{|x| + x}}$
f(x) is not defined when
$|x| + x = 0$
For any negative value of x, f(x) is not defined.
$\therefore$ The domain of f(x) is R^+ Choice (C)

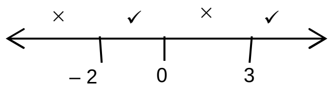
21. Given, $f: R \rightarrow R$
$f(\alpha - f(\alpha)) = 5f(\alpha)$ and f(1) = 7.
Consider,
$f(\alpha - f(\alpha)) = 5f(\alpha)$

If $\alpha = 1$, then
$f(1 - f(1)) = 5f(1)$
$f(1 - 7) = 5(7)(\because f(1) = 7)$
$\therefore f(-6) = 35$. Ans: (35)

22. For $x \ge 0$, $x - |x| = 0$
$\therefore$ The domain of $\dfrac{1}{x-|x|}$ is R^-, i.e., $x < 0$

Choice (B)

23. $f(x) = \dfrac{3}{\sqrt{x(x-3)(x+2)}}$
The above function is defined when $x(x-3)(x+2) > 0$
$x = 1$ does not satisfy the inequality above



$\therefore$ The above inequality is satisfied only when
$x \in (-2, 0)$ or $(3, \infty)$
$\therefore$ The domain of the function is $(-2, 0) \cup (3, \infty)$

Choice (B)

24. $f(x) = \max\{|1-x|, |x+1|, |x|\}$, = max { |x+1|,|x|,|x–1| }
Consider the 3 points x = –1, 0, 1 on the number line. f(x) is the distance of the point x from the point which is farthest to it. For x > 0, the farthest point is – 1. For x = 0, – 1 and 1 are equally far.
f(0) = 1 and f(a) where a>0 = a+1
$\therefore$ For $x \ge 0$, $f(x) = x + 1$. Choice (D)

25. Given $f(x + y) = f(x) + f(y)$
If any function satisfies the above condition, then it must be of the form kx where k is a constant
$\therefore$ Let $f(x) = kx$
Given $f(3) = 9 \Rightarrow 3k = 9 \Rightarrow k = 3$
$\therefore$ $f(x) = 3x$ and $f(20) = 3(20) = 60$ Ans: (60)

26. The function log|x|, is undefined at x = 0 and the function $1/(x + 3)$ is undefined at $x = -3$.
So $x \ne -3$ and $x \ne 0$
So the domain is $R - \{0, -3\}$.

Choice (D)

27. Starting with the innermost square root, we get
$1 - x^2 \ge 0 \Rightarrow -1 \le x \le 1$
The outer square root $1-\sqrt{1-x^2} \ge 0$, holds for the above values of x.
Hence the domain is $[-1, 1]$. Choice (B)

28. We split the domain R into 3 cases and define the function.
(i) $x \le 0$ (ii) $0 \le x \le 2$ and (iii) $x \ge 2$

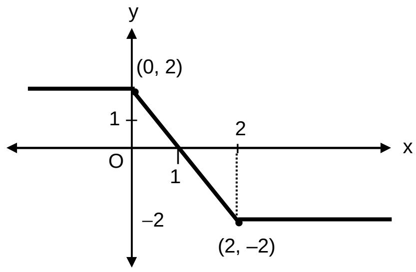
$$f(x) = \begin{cases} (2-x) - (-x); & \text{when } x \le 0 \\ (2-x) - (x); & \text{when } 0 \le x \le 2 \\ (x-2) - (x); & \text{when } x \ge 2 \end{cases}$$

$$\text{So } f(x) = \begin{cases} 2 & ;\text{ when } x \le 0 \\ 2-2x & ;\text{ when } 0 \le x \le 2 \\ -2 & ;\text{ when } x \ge 2 \end{cases}$$

Thus $-2 \le f(x) \le 2$
Range is $[-2, 2]$. Choice (B)

Note:

The graph of the above function is given below:



29. Cost of parking the car for the first hour or for any amount of time less than that = ₹5.
For every additional hour or a part thereof, the parking cost = ₹2. i.e., in case of parking the car for $2^1/_2$ hours or $2^3/_4$ hours or 3 hours, we pay the same amount, which is equal to ₹5 for the first hour and ₹2 for every additional hour (or a part of thereof).
$\therefore$the cost of parking is ₹($5 + 2 \times 2$) i.e. ₹9 for $2^1/_2$ hours or $2^3/4$ hours or 3 hours.
We find that option (D): $2\lceil t - 1\rceil + 5$, satisfies this, as $\lceil 2.5 - 1\rceil = \lceil 2.75 - 1\rceil = \lceil 3 - 1\rceil = 2$ Choice (D)

30. $\sqrt{2}$ is an irrational number and 2 a rational number, applying the given definitions, we have
$\Rightarrow f(\,|\sqrt{2}\,|\,) = f(\sqrt{2}) = -2$,
$|\,f(\sqrt{2})\,| = |-2| = 2$,
$\sqrt{|\,f(2)\,|} = \sqrt{2}$ and
$|\sqrt{f(2)}\,| = |\sqrt{2}\,| = \sqrt{2}$.
On adding the above values we get $2\sqrt{2}$. Choice (D)

31. As, $-2.6 < -2$, So $f(-2.6) = 1 + |-2.6| = 1 + 2.6 = 3.6$
So $f(f(-2.6)) = f(3.6) = [3.6] - 1$
$= 3 - 1 = 2$ (as $[3.6] = 3$) Ans: (2)

32. Given $f(x) + f(1 - x) = 4$
$f\left(\frac{1}{30}\right) + f\left(\frac{2}{30}\right) + \ldots.. + f\left(\frac{29}{30}\right)$
Consider $f\left(\frac{1}{30}\right) + f\left(\frac{29}{30}\right) = f\left(\frac{1}{30}\right) + f\left(1-\frac{1}{30}\right) = 4$
Similarly $f\left(\frac{2}{30}\right) + f\left(\frac{28}{30}\right) = f\left(\frac{2}{30}\right) + f\left(1-\frac{2}{30}\right) = 4$
$f\left(\frac{14}{30}\right) + f\left(\frac{16}{30}\right) = f\left(\frac{14}{30}\right) + f\left(1-\frac{14}{30}\right) = 4$
Given $f(x) + f(1 - x) = 4$
Put $x = \frac{1}{2}$, $f\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right) = 4$
$f\left(\frac{1}{2}\right) = 2$
$f\left(\frac{1}{30}\right) + f\left(\frac{29}{30}\right) + f\left(\frac{2}{30}\right) + f\left(\frac{28}{30}\right) + \ldots.. + f\left(\frac{14}{30}\right) + f\left(\frac{16}{30}\right) + f\left(\frac{15}{30}\right)$
$= 4 + 4 + 4 + \ldots\ldots$ (14 times) $+ 2 = 56 + 2 = 58$

Choice (B)

33. $3f(x) + 2f(1 - x) = x^2 + 4 \quad \rightarrow \quad (1)$
Put $x = 1 - x \Rightarrow 3f(1 - x) + 2f(x) = (1 - x)^2 + 4 \quad \rightarrow \quad (2)$
$3(1) - 2\times(2)$
$\Rightarrow 5f(x) = 3x^2 + 12 - 2(1 - x)^2 - 8$
$5f(x) = 3x^2 - 2(1 - x)^2 + 4$
$\therefore$ $5f(3) = 3(3^2) - 2(1 - 3)^2 + 4$
$= 27 - 8 + 4 = 23$
$\therefore$ $f(3) = \dfrac{23}{5}$ Choice (B)

34. Given: The function is not defined when $2x^2 - 11x - 30 = 0$

$\Rightarrow (2x - 15)(x + 2) = 0 \Rightarrow x = \frac{15}{2}$, $x = -2$

The domain is $R - \left\{\frac{15}{2}, -2\right\}$. Choice (A)

35. Clearly, $\frac{2}{1+x^2}$ is positive $\forall$ x $\in$ R. Hence, it is not onto and x = –1 and 1 have the same image.
$\therefore$ f(x) not one-one and also it is not onto.

$\therefore \frac{2}{1+x^2}$ is not bijective. Choice (D)

36. The number of Surjectious from set A to set B when n(A) < n(B) is zero. Choice (D)

37. Given, n(A) = p and n(B) = 2
$\therefore$The number of onto functions from A to B is $2^p - 2$
$\therefore 2^p - 2 = 1022 \Rightarrow 2^p = 1024 = 2^{10}$
$\therefore$p = 10 Ans: (10)

38. fog(x)= f(g(x)) = f(4x–3) = 3(4x – 3) +4 = 12x – 5
gof (x) g[f(x)] = g(3x+4) = 4(3x + 4) –3 = 12x + 13
$\therefore$ fog (x) + gof (x) = 24x + 8 Choice (B)

39. $\frac{1}{x}f(x) - 3f\left(\frac{1}{x}\right) = \frac{3}{2}$

Put x = 3 and x = $1/3$ we get

$\frac{1}{3}f(3) - 3f\left(\frac{1}{3}\right) = \frac{3}{2}$ and $3f\left(\frac{1}{3}\right) - 3f(3) = \frac{3}{2}$

Adding these, $\frac{-8}{3}$ f(3) = 3$\Rightarrow$ f(3) = $\frac{-9}{8}$ Choice (C)

40. f(x) = 2x+5 and g (x) = 3x – 4
fog(x) = f[g (x)] = f (3x – 4) = 2 (3x – 4) + 5
$\therefore$ (fog) (x) = 6x – 3
Let h (x) = 6x – 3
Then $h^{-1}(x) = \frac{x+3}{6}$

$h^{-1}(x) = \frac{x+3}{6}$

$h^{-1}(-9) = -1$
$\therefore (fog)^{-1}(-9) = h^{-1}(-9) = -1$ Choice (B)

41. $\lfloor 1.1 \rfloor = 1 \lfloor 1 \rfloor = 1 \lfloor -1.1 \rfloor = -2$
g(x) is defined when its denominator $\neq$ 0 i.e, when $(\lfloor x \rfloor - x)^{1/3} \neq 0$ i.e, when $\lfloor x \rfloor - x \neq 0$ i.e, when $\lfloor x \rfloor \neq x$ i.e, when x is not an integer.
$\therefore$Domain of g(x) = R –Z Choice (D)

42. Given f(a) = 3 and f(a + y) = f(a)f(y)
f(2a) = f(a + a) = f(a) + (a) = 3 (3) = (3^2)
f(3a) = f(a + 2a) = f(a)f(2a) = 3(3^2) = 3^3
f(4a) = f(a + 3a) = f(a)f(3a) = 3(3^3) = 3^4
In this manner, it follows that f(5a) = 3^4, f(6a) = 3^6, ….. f(19a) = 3^{19}, f(20a) = 3^{20} Choice (C)

43. f(x) = $8x^4$ and g (x) = $\sqrt[3]{f(x)}$

fog (x) = $f\left[\sqrt[3]{f(x)}\right] = f\left[\sqrt[3]{8x^4}\right] = f\left(2x^{\frac{4}{3}}\right)$

$= 8\ (2x^{4/3})^4 = 2^7 x^{16/3}$

fog(64) = $2^7.\ (64)^{\frac{16}{3}} = 2^7.4^{16} = 2^{39}$
$\log_2$[(fog) (64)] = $\log_2 2^{39}$ = 39 Choice (A)

44. f(4x –5) = $\frac{x+2}{x}$

Put 4x – 5 = t $\Rightarrow$ x = $\frac{t+5}{4}$

$f(t) = \frac{\frac{t+5}{4}+2}{\frac{t+5}{4}}$

$f(t) = \frac{t+13}{t+5}$
Let f (t) = y $\Rightarrow$ t = f^{-1} (y)
$\frac{t+13}{t+5} = y$
t + 13 = yt + 5y
t (1 – y) = 5y – 13
$t = \frac{5y-13}{1-y}$

$\therefore f^{-1}(y) = \frac{5y-13}{1-y}$
Put y = 2
$f^{-1}(2) = \frac{5(2)-13}{1-2}$

$= \frac{-3}{-1} = 3$ Ans: (3)

45. Given, f (x) = 3x – 5
Let $f^{-1}(x) = y$
x = f(y)
x = 3y – 5
$\frac{x+5}{3} = y$

$\therefore\ f^{-1}(x) = \frac{x+5}{3},\ f^{-1}(-1) = \frac{-1+5}{3} = \frac{4}{3}$

$f^{-1}(-2) = \frac{-2+5}{3} = 1,\ f^{-1}(1) = 2, f^{-1}(2) = \frac{7}{3}$

$\therefore\ f^{-1}(\{-1, -2, 1, 2\}) = \left\{1, 2, \frac{4}{3}, \frac{7}{3}\right\}$ Choice (C)

Chapter – 6
(Graphs)

Concept Review Questions

Solutions for questions 1 to 4:

1. **(a)** x = 3 represents a line parallel to the y – axis. Choice (B)

(b) y + 7 = 0 $\Rightarrow$ y = – 7 i.e. (y = k) is a line parallel to x – axis. Choice (A)

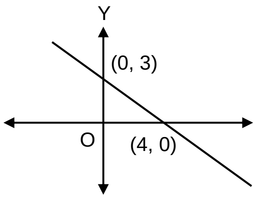
(c) The graph of 3x + y = 0 is a line passing through the origin. Choice (C)

2. **(a)** The line 2x – 3y = 6 meets x – axis at (x, y) where y = 0 and $2x_1 - 3y_1 = 6 \Rightarrow x_1 = 3$
$\therefore$ The required point is (3, 0) Choice (D)

(b) The graph of $x^2 + y^2 = 9$ meets the y axis at (x_1, y_1) where $x_1 = 0$ and $x_1^2 + y_1^2 = 9 \Rightarrow y_1 = \pm 3$
$\therefore$The required point is (0, –3) Choice (B)

3. $y = x^2 \Rightarrow y$ is always positive for any x and we know y is positive in Q_1 and Q_2
$\therefore$ The graph $y = x^2$ lies entirely in Q_1 and Q_2
Choice (C)

4. When $x = 0 \Rightarrow 3x + 4y = 12 \Rightarrow y = 3$ and when $y = 0 \Rightarrow 3x + 4y = 12 \Rightarrow x = 4$
$\therefore$ The graph meets the x – axis at (4, 0) and the y – axis at (0, 3).
$\therefore$ The graph (as shown in figure) passes through Q_1, Q_2 and Q_4.



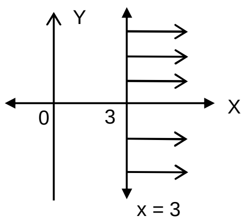
Choice (D)

5. Here $f(x) = |x|$ and $f(-x) = |-x| = x$
$\therefore$ f(x) is even and even function is symmetric about the y-axis. Choice (B)

6. $y = \log_e x$ where the graph crosses the x – axis, y = 0
$\Rightarrow \log x = 0 \Rightarrow x = 1$
$\therefore$ The required point is (1, 0). Choice (A)

7. If $f(x) = f(-x)$ then it is an even function.
An even function is symmetric to the y – axis
Choice (B)

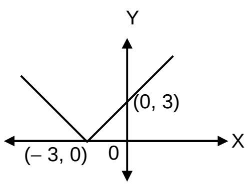
8. The graph of $x \geq 3$ is shown below



Choice (C)

9. The given line is parallel to the x – axis and lies at a distance of 3 units from the origin.
$\therefore$ Its equation is y = 3
$y \leq 3$ satisfies origin and origin side region is shaded.
$\therefore$ The required inequation of the graph is $y \leq 3$
Choice (C)

10. $y = |x+3|$ when x = – 3, then y = 0
$\Rightarrow$ the graph meets X – axis at (– 3, 0) and is always positive.
$\therefore$ The graph lies entirely in Q_1 and Q_2
$\therefore$ The required graph is



Choice (D)

11. The given shaded region is represented by $3x + 4y \leq 12$
(1, 1), 3(1) + 4(1) = 7 < 12 true
(1, 2), 3(1) + 4(2) = 11 < 12 true
(2, 1), 3(2) + 4(1) = 10 < 12 true Choice (D)

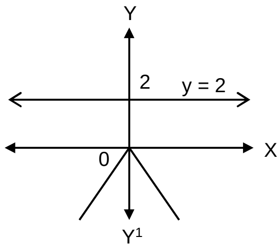
12. The given function represents a circle with radius 7 units.
Area of the circle = $\pi r^2 = \pi$ (49) Ans: (49)

13. If the curve $x^2 + y^2 - 2x + 3y + k = 0$ passes through the origin, k = 0 Ans: (0)

14. **(a)** Put x = 3 in $y^2 = 12x$ and we get $y^2 = 12(3) \Rightarrow y = \pm 6$.
$\therefore$ x = 3 meets $y^2 = 12x$ at (3, 6) and (3, – 6) i.e., at two points. Ans: (2)

(b) Put y = 4 in $x^2 + y^2 = 9$ and we get $x^2 + 16 = 9$
$\Rightarrow x^2 = -7$, i.e. x is not real
$\therefore$ y = 4 does not meet $x^2 + y^2 = 9$ Ans: (0)

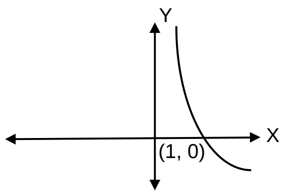
(c) The graph of $y = -|x|$ and y = 2 are as shown in the figure below. They do not meet each other.



Ans: (0)

15. When we substitute the coordinates of the origin in $2x + 3y = -5$ we get $0 + 0 > -5$, this statement is true when $2x + 3y > -5$
$\therefore$ The region containing origin is given by $2x + 3y > -5$
Choice (C)

16. When the given graph is reflected in the x – axis, the graph obtained is as shown below.



Choice (D)

17. When F is moved 'a' units to the right, then the equation of the new graph is $y = f(x - a)$.
$\therefore g(x) = f(x - 3)$. Choice (B)

18. Clearly the option (A) satisfies the given inequation.
Choice (A)

19. We know that the image of (a, b) w.r.t to y = x is (b, a).
$\therefore$ The image of (3, 2) w.r.t to y = x is (2, 3).
Choice (D)

20. If x = 0, $\lceil x \rceil - x = 0$.
If x = 1/4, $\lceil x \rceil - x = 1 - 1/4 = 3/4$
If x = 1, $\lceil x \rceil - x = 0$
Only B satisfies all these conditions. Choice (B)

Exercise – 6(a)

Solutions for questions 1 to 4:

1. When x > 3 the value of the function is negative and $x \in (2, 3)$ the value of the function is positive
$\therefore$ From options the graph represents the equation $\log_{0.3}(x - 2)$ Choice (B)

2. graph satisfies the equation
$$f(x) = -\left|\frac{|x+1| - |x-1|}{2}\right| \ \forall\ x$$ Choice (C)

3. given graph represents the equation y = sin x when
$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Choice (D)

4. equation of the graph is y = x – [x] Choice (C)

Solutions for questions 5 to 8:

The implications of the directions are worked out below.

1	2	3	4	5
We have to select	If	f is even	f is odd	f is neither even nor odd
A	g(x) = f(–x) For some x, g(x) ≠ –f(x)	g = f	f can't be odd	Refer to Col 2
B	g(x) = –f(x) = f(–x)		f has to be odd	Refer to Col 2
C	g(x) = –f(–x)	g = –f	g = f	Refer to Col 2

5. f is neither even nor odd and g(x) = f(–x). Choice (A)

6. f is odd and g(x) = –f(x). Also g(x) = f(–x). Choice (B)

7. f(x) is not odd (It is not even either).
We have to consider choices A and C.
g(x) = f(–x). Also g(x) = –f(–x). Both A and C are applicable.
Choice (D)

8. f(x) is odd. We can consider B and C. g(x) ≠ –f(x) for all x. B is not applicable.
g(x) ≠ –f(–x) for all x. C is not applicable. Choice (D)

Solutions for questions 9 and 10:

9. The graph g(x) is obtained as follows. We find the refection of f(x) in the y – axis then move the graph 3 units to the right.
The reflection r(x) = f(– x)
The given function g(x) = r(x – 3) = f(3 – x) Choice (B)

10. From the graph tan30° = $\frac{x-y}{x+y}$

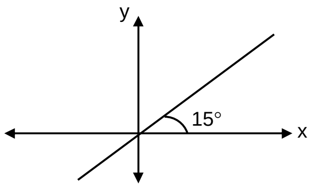
$\Rightarrow \frac{\tan 30^\circ + 1}{\tan 30^\circ - 1} = \frac{-x}{y}$

$\Rightarrow \frac{1-\tan 30^\circ}{1+\tan 30^\circ} = \frac{y}{x}$

$\Rightarrow \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} = \frac{y}{x}$

$\Rightarrow \tan(45^\circ - 30^\circ) = \frac{y}{x} \Rightarrow \tan 15^\circ = \frac{y}{x}$

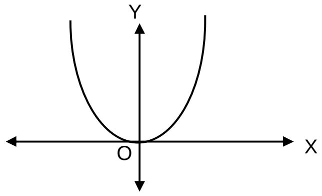
The graph is as follows



Choice (C)

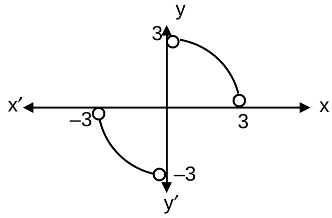
Solutions for questions 11 to 14:

11. The graph of y = $2x^2$ is as follows



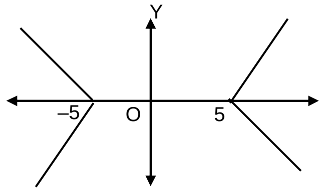
There are many horizontal lines that cut the graph at two points Choice (A)

12. xy > 0 ⇒ x > 0, y > 0 or x < 0, y < 0
∴ The given graph represents parts of the circle in first and third quadrants as shown in the below figure.



Any horizontal line and any vertical line cut the graph at only 0 or 1 points Choice (D)

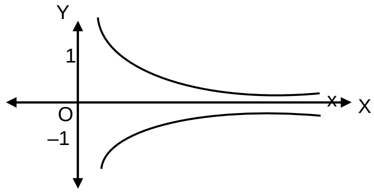
13. |x| – |y| = 5 the graph is as follows



There are many horizontal lines and many vertical lines that cut the graph at two points Choice (C)

14. |y| x = 3 ⇒ y = ± $\frac{3}{x}$ (for x > 0)

The graph is as follows



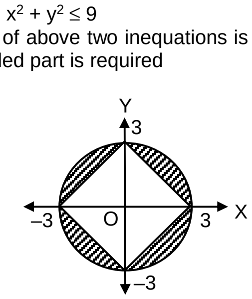
There are many vertical lines that cut the graph at two points. Choice (B)

Solutions for questions 15 to 23:

15. ∴ Perimeter of the smallest trapezium = 3 + 8 + 5 +4 = 20
Perimeter of the second trapezium = 3 + 12 + 5 +8 = 28
Perimeter of the biggest trapezium = 3 + 16 + 5 +12 = 36
∴ Sum of the perimeters = 20 + 28 + 36 = 84
Choice (A)

16. The part of graph in the I and IV quadrant is y = logx and that in II and III quadrant is y = log (–x). Hence, the equation y = log|x| ; x ≠ 0. Choice (B)

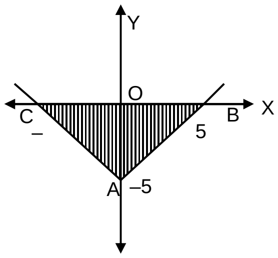
17. |x| + |y| ≥ 3; $x^2 + y^2 \le 9$
The graph of above two inequations is as follows and area of the shaded part is required



∴ Required area = area of circle – area of square
$= \pi 3^2 - (3\sqrt{2})^2$
= 9π – 18 = 9(π – 2) sq units Choice (C)

18. The region described by the relations is a rectangle of breadth 14 (parallel to the x axis) and length 18 (parallel to the y axis). Its area is 14(18) = 252. Ans: (252)

19. The graph of $y = |x| - 5$ is shown below.



Area of ΔABC = 2 (area of ΔOAB)

$= 2\left(\frac{1}{2}\right)$ OA (OB) = 5(5) = 25 Ans: (25)

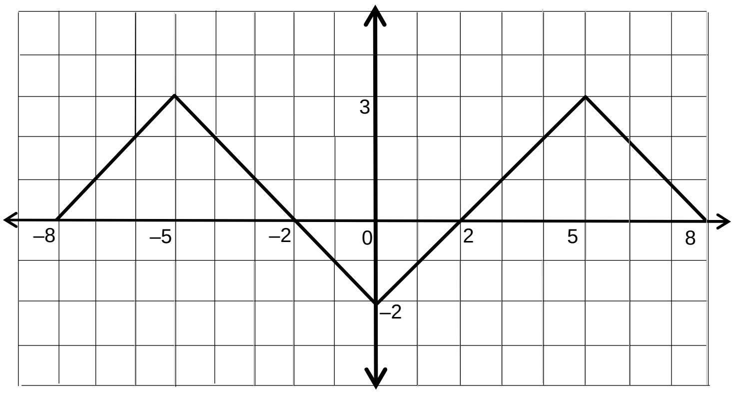
20. The given graph has a heavy dot (an included point) on the left endpoint of each step. $\therefore$ It is a floor function. We should consider C and D

Consider C, $y\left(\frac{1}{2}\right) = \left\lfloor 2\left(\frac{1}{2}\right) - 1\right\rfloor = 0$. The point $\left(\frac{1}{2}, 0\right)$ lies on the graph. For choice D, $y\left(\frac{1}{2}\right) = \left\lfloor 2\left(\frac{1}{2}\right) + 1\right\rfloor = 2$.

The point $\left(\frac{1}{2}, 2\right)$ does not lie on the graph.

Choice (C)

21. The graph of $3 - |5 - |x||$ is as follows.



From the graph given we can observe, options A, B are true. Choice (A)

22. When $x \geq 0$, $g(x) = 3x + 2$ and when $x < 0$, $g(x) = -3x + 2$ where the graphs intersect, $f(x) = g(x)$

Choice A: $2x + 4 = 3x + 2 \Rightarrow x = 2$

$2x + 4 = -3x + 2$

$\Rightarrow x = -\frac{2}{5}$

$\therefore$ The two graphs intersect at $x = 2$ and $x = -\frac{2}{5}$

Choice B: $3x + 5 = 3x + 2$ is not possible.

$\therefore$ The graph of f does not intersect the graph of g when $x \geq 0$

$\therefore$ there cannot be two intersection points.

Choice C: when $x = 0$, $f(x)$ is 1

When $x < 0$, $f(x)$ is less than 1. And $g(x)$ is greater than 2.

$\therefore$ the graph of f does not intersect the graph of g when $x < 0$.

$\therefore$ There cannot be two intersection points.

Choice (A)

23. Consider the options. We have mod of floor or ceil and ceil or floor of mod. The first two choices should have segment lying on the x – axis. The given graph has only the isolated point (the origin) on the x – axis.

If we take the mod and then the ceil, we would get the given graph (if we take $\lfloor |x| \rfloor$ we would have heavy dots on the left end points). Choice (D)

Solutions for questions 24 and 25

24. Since the graph is symmetrical about y-axis, $f(x) = f(-x)$. Choice (A)

25. Since the graph is symmetrical neither about the x-axis nor about the y-axis, $f(x) \neq f(-x)$ and $f(x) \neq -f(x)$. Choice (D)

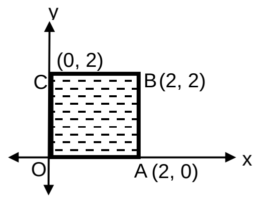
Exercise – 6(b)

Solutions for questions 1 to 5:

1. The given figure describes all points within and on the square OABC.

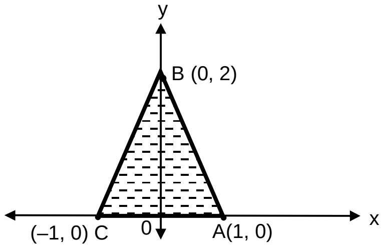
Equation of OA : y = 0 Equation of AB : x = 2

Equation of BC : y = 2 Equation of OC : x = 0



$\therefore$The given region is described by the intersection of regions $0 \leq y \leq 2$ and $0 \leq x \leq 2$. Choice (D)

2.



Equation of AB: $2x + y = 2$ → (1)
Equation of BC: $-2x + y = 2$ → (2)
Required region is the intersection of the regions bounded by the lines (1) and (2) such that they include the origin and $y \geq 0$ i.e., the intersection of the regions $2x + y \leq 2$ and $-2x + y \leq 2$ and $y \geq 0$
$\therefore$Required region: $2|x| + y \leq 2$. Choice (C)

3. The line makes an angle of 30° with the x-axis in the clockwise direction. So it makes 150° with the x-ax is in anti clockwise direction.

Hence slope = $\tan 150° = \dfrac{-1}{\sqrt{3}}$

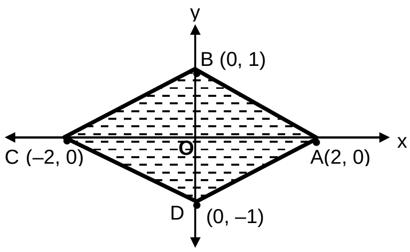
Further it passes through (0, –3)
Equation of line 'L':

$(y + 3) = -\left(\dfrac{1}{\sqrt{3}}\right)(x) \Rightarrow x + \sqrt{3}y + 3\sqrt{3} \geq 0$

Also the origin lies in this region
$\therefore$Required region: $x + \sqrt{3}y + 3\sqrt{3} \geq 0$ Choice (B)

4. Diameter of the circle = 2 units
$\Rightarrow$ Radius = 1 unit and centre = (1, 0)
Clearly, the required region is the region bounded by the circle $S \equiv (x - 1)^2 + y^2 \leq 1$
$\Rightarrow$ Required region : $(x - 1)^2 + y^2 \leq 1$
$\Rightarrow x^2 - 2x + y^2 \leq 0$ or $x^2 + y^2 \leq 2x$. Choice (C)

5.

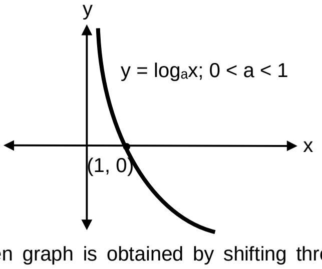


Equation of AB: $x + 2y = 2$ (using intercept form)
Equation of BC: $-x + 2y = 2$
Equation of CD: $x + 2y = -2$ and
Equation of AD: $x - 2y = 2$
Also the region includes the origin:
Hence it can be best described by $|x| + 2|y| \leq 2$.
Choice (A)

Solutions for questions 6 to 8:

6. Clearly, y is defined only for x > 0 and as $x \to 0$; $y \to -\infty$
Also y = 0 at x = 3.
$\therefore$The given curve represents the function
$y = \log_e(x/3)$ as $\log_e(3/3) = 0$ Choice (B)

7. The graph of $\log_a x$ when $0 < a < 1$ is as follows:



The given graph is obtained by shifting through 1 unit in positive x-axis direction.
$\therefore$Required equation is $y = \log_a(x - 1)$
[Here a = 0.5] Choice (B)

8. The graph of f(x) is moved 2 units to the right to get the graph of g(x)
$\therefore$ g(x) = f(x – 2) Choice (D)

Solutions for questions 9 to 12:

9. The graph g(x) can be obtained from the graph of f(x) in two ways;
(i) reflecting f(x) in the x-axis, i.e., f(x) = –g(x)
OR
(ii) by double reflecting f(x), i.e., reflecting f(x) in x-axis followed by a reflection in y-axis or vice versa.
$\therefore$f(x) = –g(–x) Choice (D)

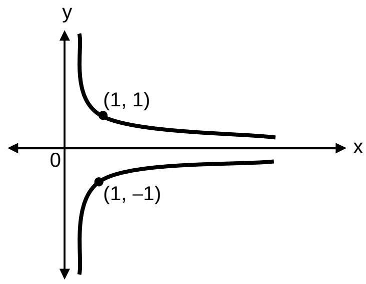
10. The graph g(x) can be considered as the reflection of f(x) in x-axis i.e., f(x) = –g(x) or a reflection of f(x) in y-axis i.e., f(x) = g(–x). Choice (D)

11. The graph g(x) can be obtained by reflecting f(x) in x-axis alone. Hence, f(x) = –g(x). Choice (B)

12. g(x) can be obtained by double reflection (both in the x-axis and the y-axis) or by reflection in the x-axis alone.
$\therefore$f(x) = –g(x) = –g(–x) Choice (D)

Solutions for questions 13 to 15:

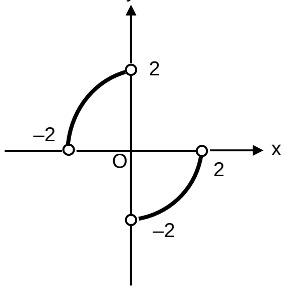
13. $x|y| = 1 \Rightarrow x = \dfrac{1}{|y|}, y \neq 0$
This means x is positive. We can guess a couple of points such as (1, 1) and (1, –1). We also note that $x, y \neq 0$.



The line x = 2 cuts the graph more than once.
Choice (A)

14. The graph represents an inclined line not passing through the origin. Choice (D)

15. The graph is of 2 arcs in the 2nd and the 4th quadrants as xy < 0. The sketch is as follows.

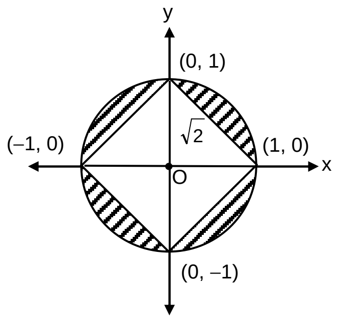


The points on the coordinate axes are to be excluded as xy < 0. Thus no line can be drawn as required.
Choice (D)

Solutions for questions 16 to 25:

16. Consider the equation |x| + |y| = 1; we discuss this equation in the following 4 cases:

Case 1: $x \geq 0, y \geq 0; L_1 \equiv x + y = 1$
Case 2: $x \geq 0, y \leq 0; L_2 \equiv x - y = 1$
Case 3: $x \leq 0, y \geq 0; L_3 \equiv -x + y = 1$
Case 4: $x \leq 0, y \leq 0; L_4 \equiv x - y = 1$
We have a set of four lines L_1, L_2, L_3 and L_4 representing a square.
The equation $x^2 + y^2 = 1$ represents a circle with its centre at origin and the radius as 1 unit.
Plotting the inequations we have:



The area of the shaded region can be obtained as
= area of the circle – area of the square
$= \pi(1)^2 - (\sqrt{2})^2 = (\pi - 2)$ sq.units Ans: (2)

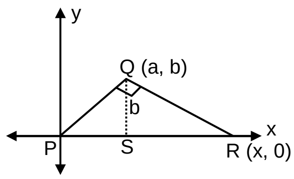
17. The given graph is a sinusoidal curve. At x = 0, y = 2.
$\therefore$ We should consider y = 2cosx.
This equation is satisfied by all points on the curve (For example, at $x = \pi/2$, y = 0 and at $x = \pi$, y = –2).
Choice (C)

18. Say the coordinates of the point R are given by (x, 0).
The area of the triangle

$PQR = \frac{1}{2} PR \times SQ$

$= \frac{1}{2} xb = 40$

$\Rightarrow x = \frac{80}{b}$



Hence the coordinates of point R are $\left(\frac{80}{b}, 0\right)$.
Choice (C)

19. Perimeter of the 1st trapezium:
The slanting part of the 1st trapezium $= \sqrt{3^2 + 4^2} = 5$
$\therefore$ Perimeter = 3 + 6 + 5 + 2 = 16 units
Perimeter of the 2nd trapezium:
The slanting part of the 2nd trapezium $= \sqrt{3^2 + 4^2} = 5$
Perimeter = 3 + 10 + 5 + 6 = 24 units.
Perimeter of the 3rd trapezium:
= 3 + 14 + 5 + 10 = 32 units.
Total perimeter = 32 + 24 + 16 = 72 units. Ans: (72)

20. Since the graph exists only in the I and III quadrants xy > 0. Also each line segment can be represented by the general equation x + y = k. Choice (D)

21. The given graph is the reflection of y = logx graph in y-axis. Hence y = log(–x); x < 0. Choice (C)

22. When $|x| \leq 2$, h (x) = 2 – |x| – 1 = 1 – |x|
When |x| > 2, h(x) = – (2 –|x|) – 1 = |x| –3
Statement I
h(x) is 0 when x = ±1, ±3 $\therefore$ statement I is true
Statement II
When x = 0, h(x) = 1
$\therefore$ The only y-intercept of h(x) is 1
$\therefore$ statement II is true
Statement III
For all values of x, h(x) = h(–x)
$\therefore$ statement III is true. Choice (D)

23. When $x \geq 0$, h (x) = x + 3 and when x < 0, h(x) = – x + 3
In the first quadrant, $x \geq 0$ and $h(x) \geq 0$
$\therefore$ h(x) = x + 3
Choice A: For every value of x in the first quadrant, x + 4 > x + 3, i.e., g(x) > h (x).
$\therefore$ Intersection is not possible (in the first quadrant)
Choice B: For every value of x in the first quadrant, $x > \frac{x}{2}$
Also 3 > 2. $\therefore$ h(x) > g(x).
$\therefore$ Intersection is not possible.
Choice C: In the first quadrant, h(x) = x + 3
But when x < 0, h(x) = –x + 3 and $-x + 3 = \frac{x}{2} + 4$
$\Rightarrow$ x = –2/3
$x + 3 = \frac{x}{2} + 4 \Rightarrow \frac{x}{2} = 1 \Rightarrow x = 2$
$\therefore$ Intersection occurs in Q_3 as well as Q_1.
Choice D: Intersection occurs only in the first quadrant at x = 1. Choice (D)

Solutions for questions 24 and 25:

24. Consider the options. We have to first take the floor or ceil and then the mod or vice versa. If we first take the floor/ceil, the heavy dots are all on one side (left for floor, right for ceil). Taking the mod after that would not change this feature. But the given graph has all heavy dots not on one side (right or left) but on the left in the first quadrant and on the right in the second quadrant. We get this feature if we first take the mod and then the floor. Choice (C)

25. From the options, $f(x) = \frac{|x+2| - |x-2|}{2}$ satisfies all the conditions Choice (A)

Chapter – 7
(Indices and Surds)
Concept Review Questions

Solutions for questions 1 to 35:

1. (a) $\left[\frac{3^5}{4^5}\right]^{\frac{-2}{5}} \times \left[\frac{12^2}{7^2}\right]^{\frac{-1}{2}} \times \left[\frac{8}{343}\right]^{\frac{2}{3}}$

$\frac{16}{9} \times \frac{7}{12} \times \frac{4}{49} = 2^4 \times 3^{-3} \times 7^{-1}$ Choice (C)

(b) $\left[\frac{7^{16}}{3^{16}}\right]^{\frac{-3}{16}}$

$\left[\frac{7}{3}\right]^{-3} = \frac{27}{343}$ Choice (B)

(c) $\frac{(-2)^2 \times (-3)^2}{(-3)^{-4}} = 2916$ Ans : 2916

(d) $\frac{x^{-4} \times y^6}{z^{-8}} \times \frac{x^6 \times y^3}{z^{-6}} \times \frac{x^{-12} \times y^7}{z^{-8}}$
$= \frac{x^{-4+6-12} . y^{6+3+7}}{z^{-8-6-8}} = x^{-10}. y^{16}. z^{22}$ Choice (D)

(e) $\left[\frac{5^5 \times x^{-15}}{10^5 \times y^{-20}}\right]^{\frac{-2}{5}}$

$= \frac{5^{-2} \times x^6}{10^{-2} \times y^8} = 2^2 \times x^6 \times y^{-8} = 4x^6 y^{-8}$ Choice (C)

(f) $\frac{5^{2a-5} \times (5^2)^{\frac{a}{2}} \times (5^3)^{a+3}}{(5^5)^{\frac{3a}{5}} \times (5^4)^{a+1} \times 5^{-a}}$

$= \frac{5^{2a-5} \times 5^a \times 5^{3a+9}}{5^{3a} \times 5^{4a+4} \times 5^{-a}} = 5^0 = 1$ Ans : 1

(g) $(3^6)^{1/3} - (3^5)^{3/5} + \frac{(3^4)^{\frac{3}{2}}}{27}$

$= 9 - 27 + \frac{729}{27} = 9$ Choice (A)

(h) $\frac{11^{-5} \times (11^2)^3}{(11^3)^{-4} \times (11^4)^{\frac{11}{4}}} = \frac{11^{-5} \times 11^6}{11^{-12} \times 11^{11}} = 121$

Choice (C)

(i) $\frac{1 - \left[1 - \left\{1 - \frac{1}{1+y}\right\}\right]}{(1-y)} = \frac{1 - \left(1 - \frac{y}{(1+y)}\right)}{(1-y)}$

$= \frac{\left(1 - \frac{1}{1+y}\right)}{(1-y)} = \frac{y}{(1+y)(1-y)} = \frac{y}{1-y^2}$ Ans : 41503

(j) $(11^{18} \times 7^{27})^{1/9} = 11^2 \times 7^3 = 41503$ Choice (D)

2. (a) $(a - b)(a^2 + ab + b^2) = a^3 - b^3$

Similarly the other powers of x become $b^3 - c^3$ and $c^3 - a^3$

$x^{a^3 - b^3} \times x^{b^3 - c^3} \times x^{c^3 - a^3}$

$= x^{a^3 - b^3 + b^3 - c^3 + c^3 - a^3} = x^0 = 1$ Choice (B)

(b) $\frac{x^{2ac}.x^{2ab}.x^{2bc}}{x^{ac+bc}.x^{ba+ca}.x^{bc+ab}}$

$= \frac{x^{2ac}.x^{2ab}.x^{2bc}}{x^{ac+bc+ba+ca+bc+ab}} = \frac{x^{2ac+2ab+2bc}}{x^{2ab+2ac+2bc}} = 1$

Choice (C)

3. $50 \times 2^{x-4} + 25 \times 2^{x-5}$

$= 25 \times 2^{x-5}(2 \times 2^1 + 1) = 5^3 \times 2^{x-5}$

Required value $= \frac{5^3 \times 2^{x-5}}{10^{x+3}} = \frac{5^3 \times 2^{x-5}}{(10^x)(10^3)}$

$= \left(\frac{5^3}{2^x.5^x}\right)\left(\frac{2^x}{2^5.2^3.5^3}\right) = \frac{1}{5^x.2^8}$ Choice (D)

4. $343^{0.12} \times 2401^{0.08} \times 49^{0.01} \times 7^{0.1}$

$= (7^3)^{0.12} \times (7^4)^{0.08} \times (7^2)^{0.01} \times 7^{0.1}$

$= 7^{0.36} \times 7^{0.32} \times 7^{0.02} \times 7^{0.1}$

$= 7^{0.36+0.32+0.02+0.1} = 7^{0.8} = 7^{8/10} = 7^{4/5}$ Choice (B)

5. The given expression equals $y^{\frac{p+q+r}{p+q+r}} = y$

Choice (A)

6. **(a)** $5^{2x} = 5^4$

$\Rightarrow 2x = 4$

$\therefore x = 2$ Ans: (2)

(b) $3^{x^x} = 81 = 3^{2^2}$

Comparing the two sides, x = 2. Choice (A)

7. $10^{\frac{1}{3}} - 9^{\frac{1}{3}}$ is in the form $a^{\frac{1}{3}} - b^{\frac{1}{3}}$.

$a - b = \left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} + (ab)^{\frac{1}{3}} + b^{\frac{2}{3}}\right)$

$\therefore 10^{\frac{2}{3}} + 90^{\frac{1}{3}} + 9^{\frac{2}{3}}$ is a rationalizing factor of

$10^{\frac{1}{3}} - 9^{\frac{1}{3}}$ Choice (C)

8. $3^x 7y = 21^2 = (3.7)^2 = 3^2 7^2 \Rightarrow 3^{x-2} = 7^{2-y}$

As x – 2 and 2 – y are integers, each has to be 0.

i.e. x = 2, y = 2

$\therefore x - y = 0$ Choice (A)

9. **(a)** $5^{\frac{1}{2}} 5^{\frac{3}{2}} 5^{\frac{5}{2}} 5^{\frac{7}{2}} 5^{\frac{9}{2}}$

$= 5^{\frac{1+3+5+7+9}{2}} = 5^{\frac{25}{2}} = 25^x = (5^2)^x$

$\therefore 5^{\frac{25}{2}} = 5^{2x}$ Comparing the two sides, $2x = \frac{25}{2}$

$\Rightarrow x = 6.25.$ Ans: (6.25)

(b) Since the bases are equal, powers will be equal.

3x + 4 = 4x + 2

$\Rightarrow x = 2$ Ans: (2)

(c) $(3^6)^{x+1} = 3^{4x-3}$

$\Rightarrow 3^{6x+6} = 3^{4x-3}$

Equating powers

6x + 6 = 4x – 3

$\Rightarrow x = -\frac{9}{2}$ Choice (A)

(d) $3^{2(2x+1)} = 3^{3(5x-3)}$

$\Rightarrow 2(2x + 1) = 3(5x - 3)$

$\Rightarrow 4x + 2 = 15x - 9$

$\Rightarrow 11x = 1 \quad \Rightarrow \quad x = 1$ Choice (A)

10. Let

$p^a = q^b = r^c = s^d = k$

$\Rightarrow p = k^{1/a}$

$q = k^{1/b}$

$r = k^{1/c}$, $s = k^{1/d}$

given $\frac{p}{q} = \frac{r}{s}$; $\frac{k^{1/a}}{k^{1/b}} = \frac{k^{1/c}}{k^{1/d}}$.

$k^{\frac{1}{a} - \frac{1}{b}} = k^{\frac{1}{c} - \frac{1}{d}}$

Equating powers of k on both sides, we get

$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ Choice (C)

11. (a) 7^{125} ; 2^{375}

7^{125} : $(2^3)^{125}$

7^{125} ; 8^{125}

$7^{125} < 8^{125}$

i.e. $7^{125} < 2^{375}$ Choice (A)

(b) 2^{51} ; $4^{13} \times 32^4$

2^{51} ; $2^{26} \times 2^{20}$

2^{51} ; 2^{46}

$2^{51} > 2^{46}$

$\Rightarrow 2^{51} > 4^{13} \times 32^4$ Choice (B)

(c) $(343)^5$, $(49)^7$, 7^{16}

7^{15} , 7^{14} , 7^{16}

$7^{14} < 7^{15} < 7^{16}$

$(49)^7 < (343)^5 < 7^{16}$ Choice (D)

(d) 27^{10} , 5^{20}, 2^{40}

27^{10} , $(5^2)^{10}$, $(2^4)^{10}$

$27^{10} > 25^{10} > (16)^{10}$

$27^{10} > 25^{10} > 16^{10}$

i.e. $27^{10} > 5^{20} > 2^{40}$ Choice (D)

(e) 7^{75}, $5^{75} \times 3^{25}$, 200^{25}
$(7^3)^{25}$, $(5^3)^{25} \times 3^{25}$, 200^{25}
343^{25}, $(125 \times 3)^{25}$, 200^{25}
343^{25}, 375^{25}, 200^{25}
$(375)^{25} > (343)^{25}$, $(200)^{25}$
i.e. $5^{75} \times 3^{25} > 7^{75} > (200)^{25}$ Choice (B)

12. $6^{1/2}$, $7^{1/3}$, $8^{1/4}$, $9^{1/5}$ take the LCM of denominators of the powers of the numbers. LCM = 60
Raise the numbers with this LCM
$(6^{1/2})^{60}$, $(7^{1/3})^{60}$, $(8^{1/4})^{60}$, $(9^{1/5})^{60}$
6^{30}, 7^{20}, 8^{15}, 9^{12}
Between 6^{30} and 7^{20}
$(6^3)^{10}$ and $(7^2)^{10}$
216^{10} and 49^{10}
$216^{10} > 49^{10}$ i.e. $6^{30} > 7^{20}$
Between 6^{30} and 8^{15}
$(6^2)^{15}$ and 8^{15}
$36^{15} > 8^{15}$ i.e. $6^{30} > 8^{15}$
Between 6^{30} and 9^{12}
$(6^5)^6$ and $(9^2)^6$
$(7776)^6 > 81^6$
i.e. $6^{30} > 9^{12}$
$\therefore$ $6^{1/2}$ is largest in value Choice (A)

13. $$\frac{9}{6^{\frac{2}{3}} - 18^{\frac{1}{3}} + 3^{\frac{2}{3}}} = \frac{\left(6^{\frac{1}{3}}\right)^3 + \left(3^{\frac{1}{3}}\right)^3}{\left(6^{\frac{1}{3}}\right)^2 - 18^{\frac{1}{3}} + \left(3^{\frac{1}{3}}\right)^2}$$
$= 6^{\frac{1}{3}} + 3^{\frac{1}{3}}$ Choice (A)

14. The conjugate of a mixed quadratic surd is retained by changing the sign of the irrational term.
$\therefore$ The conjugate of $\sqrt{7} - 2$ is $-\sqrt{7} - 2$
Choice (B)

15. (a) $$\frac{50}{\left(\sqrt{15}-\sqrt{10}\right)} \times \frac{\left(\sqrt{15}+\sqrt{10}\right)}{\left(\sqrt{15}+\sqrt{10}\right)}$$
$$= \frac{50\left(\sqrt{15}+\sqrt{10}\right)}{5}$$
$= 10\left(\sqrt{15}+\sqrt{10}\right) = 10\sqrt{15} + 10\sqrt{10}$ Choice (D)

(b) $$\frac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)} \times \frac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}$$
$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{2} = \frac{8-2\sqrt{15}}{2} = 4 - \sqrt{15}$$ Choice (C)

16. $$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2} = \sqrt{5} - \sqrt{2}$$
$$\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{5}\right)^2} = \sqrt{6} - \sqrt{5}$$
$$\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{1}{\sqrt{6}+\sqrt{5}} = \sqrt{6} - \sqrt{2}$$ Choice (A)

17. Method 1
$$\frac{6^{\frac{3}{4}} \times \sqrt[4]{6^{10}}}{\sqrt[4]{6^9}}$$
$6^{\frac{3}{4}+\frac{10}{4}-\frac{9}{4}} = 6^{\frac{4}{4}} = 6$ Choice (A)

18. $$2\sqrt{\frac{5}{2}} - 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000}$$
$= \sqrt{2}\sqrt{5} - \sqrt{5}\sqrt{2} + \sqrt{10} + 10\sqrt{10}$
$= 11\sqrt{10}$ Choice (D)

19. $$\frac{\sqrt{p} - \sqrt[4]{pq} = \sqrt[4]{p}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}{\sqrt[4]{pq} - \sqrt{q} = \sqrt[4]{q}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}$$
Required value = $\left(\frac{\sqrt[4]{p}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}{\sqrt[4]{q}\left(\sqrt[4]{p} - \sqrt[4]{q}\right)}\right)^{-4}$
$\left(\sqrt[4]{\frac{p}{q}}\right)^{-4} = \left(\left(\frac{p}{q}\right)^{\frac{1}{4}}\right)^{-4} = \left(\frac{p}{q}\right)^{-4\times\frac{1}{4}} = \left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$ Choice (D)

20. $\sqrt{324+2\sqrt{323}} = \sqrt{\left(\sqrt{323}+1\right)^2} = \sqrt{323}+1$
$\sqrt{324-2\sqrt{323}} = \sqrt{\left(\sqrt{323}-1\right)^2} = \sqrt{323}-1$
Required value = $\sqrt{323} + 1 - \left(\sqrt{323} - 1\right)$
$= \sqrt{323} + 1 - \sqrt{323} + 1 = 2$ Choice (A)

21. Given $y = 12 + 2\sqrt{35} = \left(\sqrt{7}+\sqrt{5}\right)^2$
$\therefore$ $\sqrt{y} = \sqrt{7}+\sqrt{5}$, $\frac{1}{\sqrt{y}} = \frac{1}{\sqrt{7}+\sqrt{5}} = \frac{\sqrt{7}-\sqrt{5}}{2}$
$\sqrt{7}+\sqrt{5} - \left(\frac{\sqrt{7}-\sqrt{5}}{2}\right) = \frac{\sqrt{7}+3\sqrt{5}}{2}$
$\therefore$ $\sqrt{y} - \frac{1}{\sqrt{y}} = \frac{3\sqrt{5}+\sqrt{7}}{2}$ Choice (D)

22. $y = \frac{9-\sqrt{77}}{2}$
$\frac{1}{y} = \frac{2}{9-\sqrt{77}} = \frac{2\left(9+\sqrt{77}\right)}{\left(9-\sqrt{77}\right)\left(9+\sqrt{77}\right)}$
$= \frac{2\left(9+\sqrt{77}\right)}{9^2-\left(\sqrt{77}\right)^2} = \frac{9+\sqrt{77}}{2}$
$\Rightarrow y^2 + \frac{1}{y^2} = \left(y+\frac{1}{y}\right)^2 - 2$
$= (9)^2 - 2 = 81 - 2 = 79$ Ans: (79)

23. $a = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}$
$= \frac{\left(\sqrt{6}-\sqrt{5}\right)^2}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} = 11-2\sqrt{30}$
$b = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}$
$= \frac{\left(\sqrt{6}+\sqrt{5}\right)^2}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} = 11+2\sqrt{30}$
a + b = 22
ab = 1
$a^2 - ab + b^2 = (a+b)^2 - 3ab$
$= (22)^2 - 3(1) = 484 - 3 = 481.$ Ans: (481)

24. (a) $14 - 6\sqrt{5}$

$= 14 - 2\sqrt{45}$

$= \left(\sqrt{2}\right)^2 + \left(\sqrt{5}\right)^2 - 2\sqrt{9x5}$

$= \left(\sqrt{9} - \sqrt{5}\right)^2 = \left(3 - \sqrt{5}\right)^2$

$\sqrt{14 - 6\sqrt{5}} = 3 - \sqrt{5}$

Choice (B)

(b) $18 + \sqrt{308}$

$= 18 + 2\sqrt{77}$

$= \left(\sqrt{7}\right)^2 + \left(\sqrt{11}\right)^2 + 2\sqrt{7 \times 11}$

$= \left(\sqrt{7} + \sqrt{11}\right)^2$

$= \sqrt{18 + \sqrt{308}} = \sqrt{7} + \sqrt{11}$ Choice (C)

25. **(a)** $a^2 = 13 + 2\sqrt{22}$

$b^2 = 13 + 2\sqrt{42}$

$c^2 = 13 + 2\sqrt{30}$

$d^2 = 13 + 2\sqrt{40}$

$\Rightarrow a^2 < c^2 < d^2 < b^2$

$\therefore$ a < c < d < b Choice (C)

(b) $a^2 = 22 + 2\sqrt{40}$

$b^2 = 30 + 2\sqrt{144}$

$c^2 = 26 + 2\sqrt{88}$

$d^2 = 34 + 2\sqrt{208}$

$\therefore d^2 > b^2 > c^2 > a^2$

(the rational parts as well as the irrational parts are in the same order when arranged in the ascending or descending order).

$\Rightarrow$ d > b > c > a Choice (D)

(c) Given a = $\sqrt{13} + \sqrt{11}$, b = $\sqrt{15} + \sqrt{9}$,

c = $\sqrt{18} + \sqrt{6}$ and d = $\sqrt{7} + \sqrt{17}$.

In the given surds, the sum of the terms of each of the surds is the same at 24. Then the surd containing the terms as close as possible is the greatest, and the surd containing the terms as far as possible is the smallest.

Hence a > b > d > c. Choice (A)

(d) Given p = $\sqrt{26} - \sqrt{23}$, q = $\sqrt{18} - \sqrt{15}$,

r = $\sqrt{11} - \sqrt{8}$ and s = $\sqrt{24} - \sqrt{21}$.

In the given surds, the difference between the terms of each of the surds is the same at 3. Then the surd containing the greater terms is the smallest and smaller terms is the greatest.

Hence p < s < q < r. Choice (D)

Exercise – 7(a)

Solutions for questions 1 to 35:

1. $[(81)^{4a}]^a \cdot [27^{8b}]^a \cdot [(243)\,(9^2)]^{b^2}$

$$3^{16a^2}.3^{24ab}.3^{9b^2} = 3^{(4a)^2+24ab+(3b)^2} = 3^{(4a+3b)^2}$$

Choice (D)

2. Let each be equal to "k"

Consider $(0.125)^y = [(0.5)^3]^y = (5/10)^{3y} = 10^{3z}$

$\therefore (5/10)^{3y} = k = 10^{3z}$

$\Rightarrow (5/10) = 10^{3z/3y}$ ——— (1)

Now consider $5^x = 10^{3z}$

$\Rightarrow 5^x/10^x = 10^{3z}/10^x \Rightarrow (5/10)^x = 10^{3z-x}$

$\Rightarrow (5/10) = 10^{(3z-x)/x}$ ——— (2)

From (1) and (2) we get $10^{\frac{3z}{3y}} = 10^{\frac{3z-x}{x}}$

$\Rightarrow \dfrac{3z}{3y} = \dfrac{3z - x}{x}$

$\Rightarrow$ xz = 3yz – xy

x(z + y) = 3yz

[3/x = 1/y + 1/z]

Alternate method:

Given that $5^x = (0.125)^y = 10^{3z}$

Let each equal k.

Then, $5^x = k$, $\Rightarrow 5 = k^{1/x}$ --------- (1)

$(0.125)^y = k$; $\Rightarrow (0.5)^{3y} = k$; $0.5 = k^{1/3y}$ --------- (2)

$10^{3z} = k$; $10 = k^{1/32}$ --------- (3)

As 5 = 0.5 x 10, from (1), (2) and (3)

$K^{1/x} = k^{1/3y} \cdot k^{1/3z}$

$\Rightarrow \dfrac{1}{x} = \dfrac{1}{3y} + \dfrac{1}{3z};$

$\Rightarrow \dfrac{3}{x} = \dfrac{1}{y} + \dfrac{1}{z}$ Choice (B)

3. $\dfrac{1}{p} = \dfrac{1}{a^{-x^3}} = a^{x^3}$, similarly, $\dfrac{1}{q} = a^{y^3}$ and $\dfrac{1}{r} = a^{z^3}$

$\therefore \dfrac{1}{pqr} = a^{x^3}.a^{y^3}.a^{z^3}$

$\Rightarrow \dfrac{1}{pqr} = a^{\left(x^3+y^3+z^3\right)} = a^{3xyz}$ (since x + y + z = 0)

$\Rightarrow \dfrac{1}{pqr} = a^{-3x\frac{1}{3}} = a^{-1} = (prq)^{-1} \therefore$ a = pqr

Choice (B)

4. Let $5^x = a$, $2^y = b$. 3a + 4b = 107; 5a + 8b = 189

Solving we get $a = 5^x = 25 = 5^2$

$\therefore$x = 2

$b = 2^y = 8 = 2^3$. $\therefore$ y = 3 Choice (D)

5. $(35)^3\,(25)^{3/2} = (7 \times 5)^3\,(5)^3$

$\left(\sqrt{7}\right)^6 (5)^6 = \left(5\sqrt{7}\right)^6 = \left(5\sqrt{7}\right)^{5x-4} \therefore 5x - 4 = 6$

x = 10/5 = 2 Ans: (2)

6. Let $3^x = a$ and $4^{y-2} = b$

Given 3a + 4b = 73 --- (1)

4a + 3b = 60 --- (2)

4 (1) $\Rightarrow$ 12a + 16b = 292

3 (2) $\Rightarrow$ 12a + 9b = 180

– –

Subtracting 7b = 112

$\Rightarrow$ b = 16

From (2), 4a = 60 – 3b = 60 – 48 $\Rightarrow$ a = 3

$\therefore 3^x = 3$ and $4^{y-2} = 16$

$\Rightarrow$ x = 1 and y – 2 = 2

$\therefore$ x + y = 5 Choice (A)

7. Given $5^{x+3} - 5^{x-3} = 78120$

$\Rightarrow 5^x \left[5^3 - \dfrac{1}{5^3}\right] = 78120$

$\Rightarrow 5^x\,[5^6 - 1] = 78120 \times (5^3)$

$\Rightarrow 5^x\,[15624] = 78120 \times (5^3)$

$\Rightarrow 5^x = 5^4 \therefore$ x = 4 Ans: (4)

8. $\dfrac{2^{a^2+b^2+c^2}}{2^{-2ab-2bc-2ca}} = 8$

$= 2^{a^2+b^2+c^2+2ab+2bc+2ca} = 2^{(a+b+c)^2} = 8$
$(a + b + c)^2 = 3$
$\therefore a + b + c = \pm\sqrt{3}$
From the choices, $(a + b + c) = -\sqrt{3}$ Choice (A)

9. $\dfrac{1}{m^{xy}\left[m^{-m} + m^{-yz} + m^{-3x}\right]} + \dfrac{1}{m^{yz}\left[m^{-yz} + m^{-3x} + m^{-xy}\right]} + \dfrac{1}{m^{zx}\left[m^{-3x} + m^{-xy} + m^{-yz}\right]}$

$= \dfrac{m^{-xy} + m^{-yz} + m^{-zx}}{m^{-xy} + m^{-yz} + m^{-zx}} = 1$ Choice (C)

10. $x = k^{(1/y)}$ $y = k^{(1/z)}$ $z = k^{(1/x)}$
$\therefore xyz = k^{1/x + 1/y + 1/z}$
$\therefore k = (xyz)^{(xyz)/(xy+yz+zx)}$ Choice (D)

11. $a^a \cdot b^b \cdot c^c = a^b \cdot b^c \cdot c^a$
$\Rightarrow a^{a-b} \cdot b^{b-c} \cdot c^{c-a} = 1$
Since a, b, c are positive integers > 1
$\Rightarrow a - b = 0$, $b - c = 0$ and $c - a = 0 \Rightarrow a = b = c$.
Choice (A) can be true for a = b = c = 2
Choice (B) can never be true for any of the posible values of a, b, c, since $a + b + c = 3a$ and $3a \neq 8$ for any integral value of 'a'.
Similarly abc = 27 for a = b = c = 3
and a + b + c = 27 for a = b = c = 9 Choice (B)

12. Given that
$y^x + y^{x+1} + y^{x+2} = 14; \Rightarrow y^2 (1 + y + y^2) = 2 \times 7$
From the given options, when y is given that value 2, the above equation becomes,
$2^x (1 + 2 + 2^2) = 14;$
$\Rightarrow 7 \times 2^x = 14$; $2^x = 2$; and x = 1, and this is a natural number.
$\Rightarrow$ x = 1, y = 2 is a set of values that satisfies the given conditions and the equation.
The values given under the other option, i.e. 7 and 14 are greater than 2.
As minimum value of y is required, y = 2. Ans: (2)

13. $x^2 - 4x + 1 = 0$ or dividing by x,
$x - 4 + \dfrac{1}{x} = 0$
$\therefore x + 1/x = 4$
$x^3 + 1/x^3 + 3(x + 1/x) = 64$
$x^3 + 1/x^3 + 12 = 64$
$x^3 + 1/x^3 = 52$ Ans: (52)

14. If the given expression is represented by x, then,
$x = \dfrac{2}{2 + x}$
$x^2 + 2x = 2$
$x = -1 + \sqrt{3}$ ($-1 - \sqrt{3}$ is not admissible as it is negative)
Choice (C)

15. Considering the 3 values
$9^{11} + 3^{12} = 3^{22} + 3^{12}$;
$81^7 + 7^{12} = 3^{28} + 7^{12}$
$27^8 + 5^{12} = 3^{24} + 5^{12}$.
Comparing the first parts of all three, we get 3^{28} the highest then come 3^{24} and 3^{22} in descending order. Similarly, 7^{12}, 5^{12}, 3^{12} are in descending order.
$\therefore$ The answer is $3^{22} + 3^{12}$; $3^{24} + 5^{12}$; $3^{28} + 7^{12}$
Hence, the order is a, c and b. Choice (A)

16. $3^{1/5}\ 4^{1/6}\ 5^{1/7}\ 6^{1/8}$
$3^{1/5} = (3^6)^{1/30} = (729)^{1/30}$
$4^{1/6} = (4^5)^{1/30} = (1024)^{1/30}$
$\therefore 3^{1/5} < 4^{1/6}$
$3^{1/5} = (3^7)^{1/35} = (2187)^{1/35}$
$5^{1/7} = (5^5)^{1/30} = (3125)^{1/35}$
$\therefore 3^{1/5} < 5^{1/7}$
$4^{1/6} = (4^7)^{1/42} = (16384)^{1/42}$
$5^{1/7} = (5^6)^{1/7} = (15625)^{1/42}$
$\therefore 3^{1/5} < 5^{1/7} < 4^{1/6}$
$3^{1/5} = (3^8)^{1/40} = (6561)^{1/40}$
$6^{1/8} = \left(6^5\right)^{\frac{1}{8}} = (7776)^{1/40}$
$3^{1/5} < 6^{1/8}$
$6^{\frac{1}{8}} = (6^7)^{1/56}$
$5^{1/7} = (5^8)^{1/56}$ $6^{1/8} < 5^{1/5}$ ($\because 6^7 < 5^8$)
$\therefore$ The ascending order is
$3^{1/5} < 6^{1/8} < 5^{1/7} < 4^{1/6}$ Choice (A)

17. $16^{\frac{7}{12}} = \left(2^4\right)^{\frac{7}{12}} = 2^{\frac{7}{3}}$
$81^{\frac{3}{8}} = \left(3^4\right)^{\frac{3}{8}} = 3^{\frac{3}{2}}$
$625^{\frac{2}{3}} = \left(5^4\right)^{\frac{2}{3}} = 5^{\frac{8}{3}}$
Let us first compare $2^{\frac{7}{3}}$ and $3^{\frac{3}{2}}$
When we raise both numbers to their sixth powers, we get 2^{14} and 3^9
$2^{14} = 2^{10} (2^4) = 1024(16) = 16384$
$3^9 = 3^6$, $3^3 = 729(27)$ which is more than 700(27) i.e., 18900
$3^9 > 2^{14}$ $\therefore$ $3^{\frac{3}{2}} > 2^{\frac{7}{3}}$
Let us now compare $3^{\frac{3}{2}}$ and $5^{\frac{8}{3}}$
When we raise both to their sixth powers, we get 3^9 and 5^{16}. $3^9 < 5^{16}$, since 3 is a lower base and 9 is a lower index.
$\therefore$ $3^{\frac{3}{2}} < 5^{\frac{8}{3}}$. Alternatively, we directly see that
$3 < 5$ and $\dfrac{3}{2} < \dfrac{8}{3}$. $\therefore$ $3^{\frac{3}{2}} < 5^{\frac{8}{3}}$
$2^{\frac{7}{3}} < 3^{\frac{3}{2}} < 5^{\frac{8}{3}}$ Choice (A)

18. $\dfrac{1}{a^{r/pq}} \cdot \dfrac{1}{a^{p/qr}} \cdot \dfrac{1}{a^{q/rp}} = \dfrac{1}{a^{\frac{p^2+q^2+r^2}{pqr}}} = \dfrac{1}{(a)^{1/pqr}}$,since $p^2 + q^2 + r^2 = 1$ Choice (A)

19. $\left(\dfrac{a}{\sqrt{b} - \sqrt{c}} + \dfrac{a}{\sqrt{b} + \sqrt{c}}\right)^2$
$= a^2 \left(\dfrac{\sqrt{b} + \sqrt{c} + \sqrt{b} - \sqrt{c}}{b - c}\right)^2$
$= \dfrac{a^2\ (2\sqrt{b})^2}{(b-c)^2} = \dfrac{4a^2 b}{(b-c)^2}$ Choice (C)

20. It can be noticed that $11 + 4\sqrt{6}$
$= (2\sqrt{2} + \sqrt{3})^2 = x^2$
$\therefore$ $\dfrac{11 + 4\sqrt{6}}{2\sqrt{2} - \sqrt{3}} = \dfrac{x^2}{2\sqrt{2} - \sqrt{3}}$;
Multiplying and dividing $2\sqrt{2} + \sqrt{3}$, it is equal to
$\dfrac{2\sqrt{2} + \sqrt{3}}{2\sqrt{2} + \sqrt{3}} \cdot \dfrac{x^2}{2\sqrt{2} - \sqrt{3}} = x^3/5$ Choice (D)

21. $x = \sqrt[3]{55+12\sqrt{21}}$

$x^3 = 55+12\sqrt{21} = \sqrt{3025}+\sqrt{3024}$

$\frac{1}{x^3} = \frac{\left(\sqrt{3025}\right)^2-\left(\sqrt{3024}\right)^2}{\sqrt{3025}+\sqrt{3024}} = \sqrt{3025}-\sqrt{3024}$

$x^3+\frac{1}{x^3} = 2(55) = 110$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-x.\frac{1}{x}\right)$

$110 = \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2-3\right)$ . . . . (1)

<u>Method 1</u>

Only choice B satisfies this equation.

<u>Method 2</u>

x is positive . $x+\frac{1}{x}$ is positive. The R.H.S of (1) increases with an increase in $x+\frac{1}{x}$. ∴ Exactly one solution exists for $x+\frac{1}{x}$. When $x+\frac{1}{x}$ = 5, (1) is satisfied.

∴ This is the only solution. Ans: (5)

22. A = $16\sqrt[5]{81}$, B = $5\sqrt[4]{27}$, C = $12\sqrt[6]{243}$, D = $8\sqrt[3]{9}$

A = $2^4\, 3^{\frac{4}{5}}$, B = $3^{\frac{3}{4}}\, 5^1$, C = $2^2\, 3^1\, 3^{\frac{5}{6}}$, D = $2^3\, 3^{\frac{2}{3}}$

A > B and C > D.

We have to compare only A and C

A	C
$2^4\, 3^{\frac{4}{5}}$	$2^2\, 3^1\, 3^{\frac{5}{6}}$
$\frac{4}{3}$	$3^{\frac{5}{6}-\frac{4}{5}} = 3^{\frac{1}{30}}$
$\left(\frac{4}{3}\right)^{30}$	3

$\left(\frac{4}{3}\right)^4 = \frac{256}{81} > 3$

∴ A > C. A is the greatest. Choice (A)

23. a = $\sqrt{5}+2$, $\frac{1}{a} = \frac{\sqrt{5}-2}{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)} = \sqrt{5}-2$ and

$a^2 - \frac{1}{a^2} = \left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right) = \left(2\sqrt{5}\right)4 = 8\sqrt{5}$

Choice (C)

24. Given x = $5-\sqrt{21} \Rightarrow x = \frac{1}{2}\left(10-2\sqrt{21}\right)$

$\sqrt{x} = \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

$32 - 2x = 22 + 2\sqrt{21}$ ∴ $\sqrt{32-2x} = 1 + \sqrt{21}$

Consider $\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}} = \frac{\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)}{\sqrt{21}+\sqrt{1}-\sqrt{21}}$

$= \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$ Choice (C)

25. $a^2 = 22 + 2\sqrt{117}$; $b^2 = 22 + 2\sqrt{57}$;

$c^2 = 22 + 2\sqrt{85}$; $d^2 = 22 + 2\sqrt{120}$

∴We get $d^2 > a^2 > c^2 > b^2$

∴Descending order is d, a, c, b. Choice (D)

26. $\sqrt{16+2\sqrt{55}} = \sqrt{11+5+2\sqrt{11}\sqrt{5}} = \sqrt{11}+\sqrt{5}$

Choice (D)

27. $\sqrt{(a+b+c)+2\sqrt{ac+bc}}$

$= \sqrt{(a+b) + c + 2\sqrt{c(b+a)}}$

$= \sqrt{\left(\sqrt{a+b}+\sqrt{c}\right)^2} = \sqrt{a+b}+\sqrt{c}$ Choice (B)

28. Let x = 23 + $4\sqrt{10}-10\sqrt{2}-8\sqrt{5}$

$\Rightarrow$ x = 23 + 2 $\sqrt{40}-2\sqrt{50}-2\sqrt{80}$

$\Rightarrow$ x = (5 + 8 + 10)

+ 2 $\sqrt{(5)(8)}$ – 2 $\sqrt{(5)(10)}$ – 2 $\sqrt{(8)(10)}$

$\Rightarrow x = \left(\sqrt{5}+\sqrt{8}-\sqrt{10}\right)^2$

∴ $\sqrt{x} = \sqrt{5}+\sqrt{8}-\sqrt{10}$ Choice (D)

29. $\sqrt{x\sqrt{x\sqrt{x.......\infty}}} = 11^x$; squaring, $x[11^x] = 11^{2x}$

$x = 11^x$ ($11^2 = 0$ is ignored)

$\sqrt[x]{x} = 11$ Choice (A)

30. Given x = $\sqrt{\frac{6}{4+\frac{6}{2+\sqrt{\frac{6}{4+\frac{6}{2+.....\infty}}}}}}$

$\Rightarrow x = \sqrt{\frac{6}{4+\frac{6}{2+x}}}$

$\Rightarrow x = \sqrt{\frac{6(2+x)}{4(2+x)+6}} = \sqrt{\frac{6(2+x)}{4x+14}}$

Squaring both sides

$x^2 = \frac{6x+12}{4x+14}$

$\Rightarrow 4x^3 + 14x^2 - 6x - 12 = 0$

By observation x = 1 satisfies the above equation.

$\Rightarrow (x - 1)(4x^2 + 18x + 12) = 0$

$\Rightarrow$ x = 1 (or) $4x^2 + 18x + 12 = 0$

If $4x^2 + 18x + 12 = 0$, then the value of x must be negative. But from the given question it is clear that x cannot be negative.

Hence x = 1 is the only solution. Choice (B)

31. Multiply and divide with $x^{-1/2}$ we get

$\frac{x^{-1/2}}{x^{-1/2}}\sqrt{x+\sqrt{x^2+\sqrt{x^4+.........\infty}}}$

$= \frac{\sqrt{x^{-1}x+x^{-1}\sqrt{x^2+\sqrt{x^4+.......\infty}}}}{(x)^{-1/2}}$

$= \frac{\sqrt{1+\sqrt{x^{-2}x^2+x^{-2}\sqrt{x^4+\sqrt{x^8.....\infty}}}}}{(x)^{-1/2}}$..

$= \dfrac{\sqrt{1+\sqrt{1+\sqrt{1+.......\infty}}}}{(x)^{-1/2}}$

Consider the numerator $\sqrt{1+\sqrt{1+\sqrt{1+......\infty}}}$ = z

1 +z = z^2 $\Rightarrow z^2 - z - 1 = 0$

$z = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

As Z is positive, the answer is $\sqrt{x}\left(\dfrac{1+\sqrt{5}}{2}\right)$

Choice (A)

32. $\sqrt{11-2\sqrt{30}} = \sqrt{6} - \sqrt{5}$ and $\sqrt{10+2\sqrt{27}} = \sqrt{7} + \sqrt{3}$

$\therefore$ The given expression = $\dfrac{1}{\sqrt{6}} + \dfrac{1}{\sqrt{7}} = \dfrac{\sqrt{6}+\sqrt{7}}{\sqrt{42}}$

$= \dfrac{6\sqrt{7}+7\sqrt{6}}{42}$ Choice (B)

33. Given $\dfrac{1}{\sqrt{x}+\sqrt{x+1}} + \dfrac{1}{\sqrt{x+1}+\sqrt{x+2}}$

$+\dfrac{1}{\sqrt{x+2}+\sqrt{x+3}} + \ldots\ldots \dfrac{1}{\sqrt{x+98}+\sqrt{x+99}} = 9$

Simplifying the expression, we get

$\left(\sqrt{x+1} - \sqrt{x}\right) + \left(\sqrt{x+2} - \sqrt{x+1}\right) +$

$\left(\sqrt{x+3} - \sqrt{x+2}\right) + \ldots\ldots \left(\sqrt{x+99} - \sqrt{x+98}\right) = 9$

$\Rightarrow \sqrt{x+99} - \sqrt{x} = 9$

$\Rightarrow \left(\sqrt{x+99}\right)^2 = \left(9+\sqrt{x}\right)^2$ (squaring both sides of the equation) = 81 + x + $18\sqrt{x}$

$\Rightarrow$ x + 99 = 81 + x + 18 $\sqrt{x}$ $\Rightarrow$ 18 = 18 $\sqrt{x}$

$\Rightarrow$ x = 1 Choice (B)

34. $x = \dfrac{\sqrt{19}+3\sqrt{2}}{\sqrt{19}-3\sqrt{2}} = 37 + 6\sqrt{38}$

$y = \dfrac{\sqrt{19}-3\sqrt{2}}{\sqrt{19}+3\sqrt{2}} = 37 - 6\sqrt{38} = \dfrac{1}{x}$

$\therefore x^2 - 3xy + y^2 = (x-y)^2 - xy$

$= \left(12\sqrt{38}\right)^2 - 1 = 5471$ Choice (B)

35. Given

$\left(\sqrt{7}+\sqrt{3}+2\right)\left(a\sqrt{21}+b\sqrt{3}+c\right) = 48$

$\Rightarrow a\sqrt{21}+b\sqrt{3}+c = \dfrac{48}{\sqrt{7}+\left(\sqrt{3}+2\right)}$

$= \dfrac{48\left(\sqrt{7}-\left(\sqrt{3}+2\right)\right)}{\left(\sqrt{7}+\left(\sqrt{3}+2\right)\right)\left(\sqrt{7}-\left(\sqrt{3}+2\right)\right)}$

$= \dfrac{48\left(\sqrt{7}-\sqrt{3}-2\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}+2\right)^2}$

$= \dfrac{48\left(\sqrt{7}-\sqrt{3}-2\right)}{7-\left(3+4+4\sqrt{3}\right)}$

$= \dfrac{48\left(\sqrt{7}-\sqrt{3}-2\right)}{-4\sqrt{3}}$

$= 4\sqrt{3}\left(2+\sqrt{3}-\sqrt{7}\right)$

$= 8\sqrt{3} + 12 - 4\sqrt{21}$

$\therefore a\sqrt{21} + b\sqrt{3} + c = -4\sqrt{21} + 8\sqrt{3} + 12$

$\therefore$ a = – 4; b = 8; c = 12

$\therefore$ 2a + 3b + 4c

= 2(– 4) + 3(8) + 4(12)

= – 8 + 24 + 48 = 64 Ans: (64)

Exercise – 7(b)

Solutions for questions 1 to 35:

1. a x 5^2 = 2020.20

a $\times$ $(10/2)^2$ = 2020.20

a x $(10)^2$ = 8080.80

a = 8080.80/100 = 80.8080

$\dfrac{a\,x\,10^{-3}}{10^4} = \dfrac{a}{10^4 x 10^3} = \dfrac{a}{10^7}$

$= \dfrac{80.8080}{10^7}$ = 0.00000808080 Choice (B)

2. Let, $3^x = 2^y = 6^z = k$

$\Rightarrow 3 = k^{1/x}$; $2 = k^{1/y}$; $6 = k^{1/z}$

$\Rightarrow (3 \times 2) = k^{1/x} \times k^{1/y}$

$\Rightarrow 6 = k^{(1/x + 1/y)} = k^{1/z}$

$\Rightarrow$ 1/z = 1/x + 1/y

In the case of the given data, k $\neq$ 0, k $\neq$ 1, k $\neq$ –1.

$\Rightarrow z = \dfrac{xy}{x+y}$

Hence, as bases are equal, equating the powers, we get

$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$ Choice (B)

3. $8^3 . 8^3 = 8^6 = (8^2)^3$

Since m = $(m^p)^q = (64)^3 = (2^6)^3 = [(2^3)^2]^3$

Comparing with $[(a^3)^b]^3$

We get a = 2, b = 2 as one set of values. Choice (A)

4. $3^{2x} + 2.3^x + 1 = 100$

$(3^x + 1)^2 = (10)^2$

$3^x + 1 = 10$ or –10

$3^x = 9 = 3^2$

$\therefore$x = 2

$3^x + 1 = -10$, $\Rightarrow 3^x = -11$ is ignored. Ans: (2)

5. $\dfrac{225}{16} = \dfrac{3^2.5^2}{2^4} = \dfrac{3^2.5^2}{\left(2^2\right)^2}$ $\left(\dfrac{3.5}{2^2}\right)^2 = \left(\dfrac{a.c}{b^2}\right)^x$

$\therefore$ x = 2

Since a, b, c are given to be prime numbers we have reduced 225 and 16 into the prime factors. Hence the answer is deducible. Choice (C)

6. $3^{x+3} - 3^{x-3} = 6552$

$3^x\left[3^3 - \dfrac{1}{3^3}\right] = 6552$

$3^x\left[\dfrac{728}{27}\right] = 6552$

$3^x = 243 = 3^5$

$\Rightarrow$ x = 5

$\therefore x^2 = 5^2 = 25$ Ans: (25)

7. Given xyz = 1

$\Rightarrow xy = \dfrac{1}{z}, \dfrac{1}{xy} = z$ --- (1)

Given expression,

$$\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$$

$$= \frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{1}{1+\frac{1}{xy}+\frac{1}{x}} \quad \text{(from (1))}$$

$$= \frac{y}{y+xy+1}+\frac{1}{1+y+xy}+\frac{xy}{xy+1+y}$$

$$= \frac{y+1+xy}{1+xy+y}=1 \qquad \text{Choice (A)}$$

8. $a^b + b^a = 1025 \Rightarrow a^b + b^a = (1024)^1 + (1)^{1024}$
$\therefore$ a + b = 1024 + 1 = 1025 Ans: (1025)

9. $$\frac{81^{a^2+b^2+c^2}}{81^{[-2bc-2ca-2ab]}}=81^{a^2+b^2+c^2+2ab+2bc+2ca}$$

$$= 81^{(a+b+c)^2} = 3 = 81^{\frac{1}{4}}$$

$$\Rightarrow a+b+c \quad = \pm\frac{1}{2} \qquad \text{Choice (C)}$$

10. Let A = $6\sqrt[3]{5}$, B = 9 – $\sqrt[3]{2}$, C = 15 – $\sqrt[4]{3}$

$1 < \sqrt[3]{5} < 2 \Rightarrow 6 < A < 12$

$1 < \sqrt[3]{2} < 2 \Rightarrow -2 < -\sqrt[3]{2} < -1 \Rightarrow 7 < B < 8$

$1 < \sqrt[4]{3} < 2 \Rightarrow -2 < -\sqrt[4]{3} < -1 \Rightarrow 13 < C < 14$

$\therefore$ C is the greatest. Choice (C)

11. Given x = $$\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\ldots\infty}}}}$$

$$\Rightarrow x = \frac{1}{4+\frac{1}{3+x}} \Rightarrow x = \frac{3+x}{4(3+x)+1}$$

$$\Rightarrow x = \frac{3+x}{4x+13}$$

$\Rightarrow 4x^2 + 12x - 3 = 0$

$$\Rightarrow x = \frac{-12\pm\sqrt{144+48}}{8}$$

$$\Rightarrow x = \frac{4\left(-3\pm\sqrt{12}\right)}{8}$$

$$\Rightarrow x = \frac{-3\pm 2\sqrt{3}}{2}$$

Since x > 0,

$$x = \frac{-3}{2}+\sqrt{3} \qquad \text{Choice (B)}$$

12. $x = 2^{55} = (2^5)^{11} = 32^{11}$
$y = 17^{14}$
$z = 31^{11}$
Clearly z < x
We have $16^{14} < 17^{14}$
$\Rightarrow (2^4)^{14} < 17^{14}$
$\Rightarrow 2^{56} < 17^{14}$
$\Rightarrow 2^{55} < 17^{14}$
$\therefore$ x < y
Hence z < x < y Choice (B)

13. A = 8^{88^8} B = $8^{8^{88}}$

C = 8^{888} D = $8^{8^{8^8}}$

Since the base of all the numbers is 8, the number power with highest index is the greatest number. Clearly ‘C’ has the lowest value.

Consider A = 8^{88^8} and B = $8^{8^{88}}$.

Consider the indices is 88^8 and 8^{88}

$(88)^8$ and $(8^{11})^8$
Since $8^{11} > 88$
$8^{88} > 88^8$
$\therefore$ B > A

Also, among the four powers the greatest power is 8^{8^8} . Hence D is the largest number.
$\therefore$ the ascending order is CABD. Choice (B)

14. Let $a^x = b^y = c^z = k$

$$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}} \text{ and } z = k^{\frac{1}{z}}$$

$$(abc)^{\left(\frac{xyz}{xy+yz+zx}\right)}$$

$$= \left(k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right)^{\frac{xyz}{xy+yz+zx}}$$

$$= \left(k^{\frac{xy+yz+zx}{xyz}}\right)^{\frac{xyz}{xy+yz+zx}} = k \qquad \text{Choice (D)}$$

15. $x = \sqrt{3}+\sqrt{2}$

$$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{3}-\sqrt{2}$$

$$\therefore\ x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$= \left(2\sqrt{3}\right)\left(2\sqrt{2}\right) = 4\sqrt{6} \qquad \text{Choice (D)}$$

16. $12345678900000 \times 10^k$ is less than one when k is less than or equal to–14
i.e. $12345678900000 \times 10^{-14} = 0.123456789$
The maximum value of k is –14 Ans: (–14)

17. $x^{\frac{1}{3}} \times x = (8 \text{ x } 49)\ (2 \text{ x } 49)$

$x^{4/3} = 16 \text{ x } 49 \text{ x } 49$

$\left(x^2\right)^{\frac{2}{3}} = 4^2 \text{ x } 49^2$

$x^{2/3} = (4 \text{ x } 49)^{2\text{x}\frac{1}{2}} = 196$

Choice (D)

18. Given, $(x-y)\left[\frac{1}{\sqrt{x}+\sqrt{y}}+\frac{1}{\sqrt{x}-\sqrt{y}}\right] = 12$

$$\Rightarrow (x-y)\left[\frac{\sqrt{x}-\sqrt{y}+\sqrt{x}+\sqrt{y}}{x-y}\right] = 12$$

$\Rightarrow 2\sqrt{x} = 12$ and y can take any value.

Choice (D)

19. $\sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\cdot^{\cdot^{\infty}}}}} = \frac{1}{4}$

$\Rightarrow \sqrt{x}^{\frac{1}{4}} = \frac{1}{4}$

$\Rightarrow \sqrt{x} = \left(\frac{1}{4}\right)^4 \Rightarrow x = \left(\frac{1}{4}\right)^8 = \frac{1}{2^{16}}$ Choice (C)

20. Given $\frac{3-\sqrt{a+5}}{a-4} = \frac{-1}{16}$

Rationalising the numerator on the left hand side, we have

$\frac{\left(3-\sqrt{a+5}\right)\left(3+\sqrt{a+5}\right)}{(a-4)\left(3+\sqrt{a+5}\right)} = \frac{-1}{16}$

$\Rightarrow \frac{(9-a-5)}{(a-4)\left(3+\sqrt{a+5}\right)} = \frac{-1}{16}$

$\Rightarrow \frac{4-a}{(a-4)\left(3+\sqrt{a+5}\right)} = \frac{-1}{16}$

$\Rightarrow \frac{1}{3+\sqrt{a+5}} = \frac{1}{16}$

$\Rightarrow 3 + \sqrt{a+5} = 16$

$\Rightarrow a + 5 = 169$

$a = 164$ Ans: (164)

21. $\sqrt{7-3\sqrt{5}} = \sqrt{\frac{14-2\sqrt{45}}{2}}$

$= \sqrt{\frac{9+5-2\sqrt{9}\sqrt{5}}{2}}$

$= \frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}} = \frac{3-\sqrt{5}}{\sqrt{2}}$ Choice (B)

22. $2-\sqrt{3} = a \quad 26-15\sqrt{3} = b$

Mean proportion $= \sqrt{ab} = \sqrt{\left(2-\sqrt{3}\right)\left(26-15\sqrt{3}\right)}$

$\sqrt{52-26\sqrt{3}-30\sqrt{3}+45}$

$= \sqrt{97-56\sqrt{3}} = \sqrt{97-2 \text{ x } 28\sqrt{3}}$

$= \sqrt{97-2\sqrt{28 \text{ x } 28 \text{ x } 3}} = \sqrt{97-2\sqrt{7 \text{ x } 4 \text{ x } 7 \text{ x } 4 \text{ x } 3}}$

$= \sqrt{49+48-2\sqrt{49 \text{ x } 48}} = 7-4\sqrt{3}$ Choice (A)

23. Let the other surd be "a".

$\frac{a+1+12\sqrt{2}}{2} = 5+9\sqrt{2}$

$a + 1 + 12\sqrt{2} = 10 + 18\sqrt{2}$

$a = 9 + 6\sqrt{2} = 9 + 2\sqrt{18} = \left(\sqrt{6}+\sqrt{3}\right)^2$

$\therefore \sqrt{a} = \left(\sqrt{6}+\sqrt{3}\right) = \sqrt{3}\left(\sqrt{2}+1\right)$ Choice (C)

24. $\frac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}} = \frac{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}+\sqrt{7}+\sqrt{13}\right)}\frac{(1)}{\left(\sqrt{6}+\sqrt{7}-\sqrt{13}\right)}$

$= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}+\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{13+2\sqrt{42}-13} = \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}}$

$\frac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}} = \frac{1\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{\left(\sqrt{6}-\sqrt{7}-\sqrt{13}\right)\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}$

$= \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{\left(\sqrt{6}-\sqrt{7}\right)^2-\left(\sqrt{13}\right)^2} = \frac{\sqrt{6}-\sqrt{7}+\sqrt{13}}{13-2\sqrt{42}-13}$

$= \frac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$

Required value $= \frac{\sqrt{6}+\sqrt{7}+\sqrt{13}}{2\sqrt{42}} + \frac{-\left(\sqrt{6}-\sqrt{7}+\sqrt{13}\right)}{2\sqrt{42}}$

$= \frac{2\sqrt{7}}{2\sqrt{42}} = \frac{1}{\sqrt{6}}$ Choice (B)

25. $x = 3-\sqrt{5} = \frac{6-2\sqrt{5}}{2} = \frac{\left(\sqrt{5}-1\right)^2}{2}$

$\sqrt{x} = \frac{\left(\sqrt{5}-1\right)}{\sqrt{2}}$,

Since $\sqrt{x} > 0$, $\sqrt{x} = \frac{\left(\sqrt{5}-1\right)}{\sqrt{2}}$

$\therefore \frac{\sqrt{x}}{\sqrt{2}+\sqrt{3x-2}} = \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\sqrt{2}+\sqrt{9-3\sqrt{5}-2}}$

$= \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\sqrt{2}+\sqrt{7-3\sqrt{5}}}$

$= \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\sqrt{2}+\sqrt{\frac{14-2\sqrt{45}}{2}}} = \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\sqrt{2}+\frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}}}$

$= \frac{\frac{\sqrt{5}-1}{\sqrt{2}}}{\frac{2+\sqrt{9}-\sqrt{5}}{\sqrt{2}}} = \frac{\sqrt{5}-1}{\left(5-\sqrt{5}\right)}$

$= \frac{\sqrt{5}-1}{\sqrt{5}\left(\sqrt{5}-1\right)} = \frac{1}{\sqrt{5}}$ Choice (C)

26. $a = \sqrt{2}\left(\sqrt{3}+2\right); 2\sqrt{2} = \left(\sqrt{2}\right)^3$ is needed in the answer, which can be had by cubing the given equation

$a^3 = \left[\sqrt{2}\left(\sqrt{3}+2\right)\right]^3 = 2\sqrt{2}\left(\sqrt{3}+2\right)^3$

$2\sqrt{2}\left[\left(\sqrt{3}\right)^3+(2)^3+3\left(\sqrt{3}\right)(2)\left(2\sqrt{3}\right)\right]$

$= 2\sqrt{2}\left[3\sqrt{3}+8+6\sqrt{3}\left(2+\sqrt{3}\right)\right]$

$= 2\sqrt{2}\left(26+15\sqrt{3}\right)$ Choice (D)

27. $a = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$; $b = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$7b^2 + 11ab - 7a^2 = 7(b + a)(b - a) + 11(a)(b)$

$= 7(2 + \sqrt{3} + 2 - \sqrt{3})(2 + \sqrt{3} - 2 + \sqrt{3}) + 11(2 - \sqrt{3})(2 + \sqrt{3}) = 7(4)(2\sqrt{3}) + 11(4 - 3)$

$= 56\sqrt{3} + 11$ Choice (B)

28. $a^2 = 16 + 2\sqrt{48}$

$b^2 = 19 + 2\sqrt{48}$

$c^2 = 14 + 2\sqrt{48}$

$d^2 = 26 + 2\sqrt{48}$

$\Rightarrow c^2 < a^2 < b^2 < d^2$

$\therefore c < a < b < d$ Choice (D)

29. The given function is $1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{324}+\sqrt{323}}$

$= 1 + \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots + \frac{\sqrt{324}-\sqrt{323}}{324-323}$ (on rationalizing the denominator of each term)

$= 1 + \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots + \sqrt{324} - \sqrt{323}$

$= \sqrt{324} = 18$ ($\because$ all terms cancel off except $\sqrt{324}$)

Hence, the square root of the given expression is $\sqrt{18} = 3\sqrt{2}$. Choice (A)

30. $\frac{1}{a^{\frac{2}{3}} + (ab)^{\frac{1}{3}} + b^{\frac{2}{3}}} = \frac{a^{\frac{1}{3}} - b^{\frac{1}{3}}}{\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} + (ab)^{\frac{1}{3}} + b^{\frac{2}{3}}\right)}$

$= \frac{a^{\frac{1}{3}} - b^{\frac{1}{3}}}{a-b}$

$\therefore \frac{1}{1 + (2)^{\frac{1}{3}} + 2^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}} - 1}{\left(2^{\frac{1}{3}} - 1\right)\left(2^{\frac{2}{3}} + (2)^{\frac{1}{3}} + 1\right)} = \frac{2^{\frac{1}{3}} - 1}{1}$

$\frac{1}{3^{\frac{2}{3}} + 6^{\frac{1}{3}} + 2^{\frac{2}{3}}} = \frac{3^{\frac{1}{3}} - 2^{\frac{1}{3}}}{\left(3^{\frac{1}{3}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{2}{3}} + 6^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)} = \frac{3^{\frac{1}{3}} - 2^{\frac{1}{3}}}{1}$

Proceeding similarly,

$\frac{1}{27^{\frac{2}{3}} + 702^{\frac{1}{3}} + 26^{\frac{2}{3}}} = \frac{27^{\frac{1}{3}} - 26^{\frac{1}{3}}}{\left(27^{\frac{1}{3}} - 26^{\frac{1}{3}}\right)\left(27^{\frac{2}{3}} + 702^{\frac{1}{3}} + 26^{\frac{2}{3}}\right)}$

$\therefore = \frac{27^{\frac{1}{3}} - 26^{\frac{1}{3}}}{1}$

$\frac{1}{1 + 2^{\frac{2}{3}} + 2^{\frac{2}{3}}} + \frac{1}{2^{\frac{2}{3}} + 6^{\frac{1}{3}} + 3^{\frac{2}{3}}} + \frac{1}{3^{\frac{2}{3}} + 12^{\frac{1}{3}} + 4^{\frac{2}{3}}} + \ldots + \frac{1}{26^{\frac{2}{3}} + 702^{\frac{1}{3}} + 27^{\frac{2}{3}}}$

$= \frac{2^{\frac{1}{3}} - 1}{1} + \frac{3^{\frac{1}{3}} - 2^{\frac{1}{3}}}{1} + \frac{4^{\frac{1}{3}} - 3^{\frac{1}{3}}}{1} + \ldots + \frac{27^{\frac{1}{3}} - 26^{\frac{1}{3}}}{1}$

$= 27^{\frac{1}{3}} - 1 = 3 - 1 = 2$ Ans: (2)

31. $x = \sqrt[3]{2} + \sqrt[3]{4}$; $x - \sqrt[3]{2} = \sqrt[3]{4}$

$x^3 - 2 - 3x\sqrt[3]{2}\left(x - \sqrt[3]{2}\right) = 4$

$x^3 - 2 - 3x\sqrt[3]{2}\left(\sqrt[3]{4}\right) = 4$

$x^3 - 2 - 3x\sqrt[3]{8} = 4$

$x^3 - 2 - 6x = 4$

$x^3 - 6x = 6$

Alternate method:

$x = \sqrt[3]{2} + \sqrt[3]{4}$; on cubing both sides,

$x^3 = \left(\sqrt[3]{2}\right)^3 + \left(\sqrt[3]{4}\right)^3 + (3)\left(\sqrt[3]{2}\right)\left(\sqrt[3]{4}\right)\left(\sqrt[3]{2} + \sqrt[3]{4}\right)$

$\Rightarrow x^3 = 2 + 4 + (3)\sqrt[3]{8}\ (x)$

$\Rightarrow x^3 = 6 + 6x; \Rightarrow x^3 - 6x = 6$ Ans: (6)

32. $x = 3 + \sqrt{5}$

$\Rightarrow (x - 3) = \sqrt{5}$

$\Rightarrow (x - 3)^3 = 5\sqrt{5}$

$\Rightarrow x^3 - 9x^2 + 27x - 27 = 5\sqrt{5} = 5(x - 3)$

$\Rightarrow x^3 - 9x^2 + 22x = 12$ Ans: (12)

33. $\sqrt{5 - 2\sqrt{6}} = \sqrt{3} - \sqrt{2}$ and $\sqrt{8 - 2\sqrt{15}} = \sqrt{5} - \sqrt{3}$

$\therefore$ The given expression $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} = \frac{\sqrt{2} + \sqrt{3}}{\sqrt{6}}$

$= \frac{2\sqrt{3} + 3\sqrt{2}}{6}$ Choice (A)

34. $a - b = (a^{1/3})^3 - (b^{1/3})^3$

$= (a^{1/3} - b^{1/3})(a^{2/3} + a^{1/3}b^{1/3} + b^{2/3})$

$a - b = (a^{1/3} - b^{1/3})\left(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}\right)$

$\frac{a-b}{\sqrt[3]{a^2} + \sqrt[3]{ab^2} + \sqrt[3]{b^2}} = a^{1/3} - b^{1/3}$ ——— (1)

Similarly we get

$\frac{a+b}{\sqrt[3]{a^2} - \sqrt[3]{ab^2} + \sqrt[3]{b^2}} = a^{1/3} + b^{1/3}$ ——— (2)

Subtracting (2) from (1) we get the answer as $-2b^{1/3}$

Choice (A)

35. $x = \frac{\sqrt{13} + 2\sqrt{13}}{\sqrt{13} - 2\sqrt{13}} = 25 + 4\sqrt{39}$

$y = \frac{\sqrt{13} - 2\sqrt{3}}{\sqrt{13} + 2\sqrt{13}} = 25 - 4\sqrt{39} = \frac{1}{x}$

$\therefore x^2 + 5xy + y^2 = (x + y)^2 + 3xy$

$= (50)^2 + 3 = 2503$ Ans: (2503)

Solutions for questions 36 to 40:

36. From statement I, $x^y = 16$.

So if x = 2, then y = 4 and if x = 4, then y = 2.

If x = 2, then $(2x)^y = 4^4 = 256$

If x = 4, then $(2x)^y = 8^2 = 64$

So statement I alone is not sufficient.

From statement II, $2x = 8 \Rightarrow x = 4$.

Combining both the statements y = 2.

$(2x)^y = (8)^2 = 64$. Choice (C)

37. $\left(4^{\frac{1}{qr}}\right)^{p^2} \times \left(4^{\frac{1}{pq}}\right)^{r^2} \times \left(4^{\frac{1}{pr}}\right)^{q^2}$

$= 4^{\frac{p^2}{qr}} \times 4^{\frac{r^2}{pq}} \times 4^{\frac{q^2}{pr}} = 4^{\frac{p^2}{qr}+\frac{r^2}{pq}+\frac{q^2}{pr}} = 4^{\frac{p^3+r^3+q^3}{pqr}}$

From statement I, we know $\frac{p^3+r^3+q^3}{pqr} = 3$

I is sufficient.
Using Statement II, we get p + q + r = 0
When p + q + r = 0, it follows that
$\frac{p^3+r^3+q^3}{pqr} = 3$

II is sufficient.
Either of the statements is sufficient. Choice (B)

38. $a^{3b} = (a^3)^b$
$(a+1)^{2b} = ((a+1)^2)^b$
The greater of a^{3b} and $(a + 1)^{2b}$ is the greater of a^3 and $(a + 1)^2$
The first few values of a, a^3, $(a+1)^2$ are tabulated below.

a	a^3	$(a+1)^2$
1	1	4
2	8	9
3	27	16

We see that a^3 may be less or more than $(a+1)^2$
Statement I and II are not significantly different. Combining them results in statement II itself. ∴ Even the combination is not sufficient. Choice (D)

39. Using statement I, we know that
If b = 0, a = 4 and if b = 2, a = 3.
∴ a + b is not unique
I is not sufficient.
Using statement II, if a ≥ 3, 8^b would be negative which is not possible. ∴ a < 3.
If a = 0, or 1, b would not be an integer.
∴ a = 2, ∴ b = 2
II is sufficient. Choice (A)

40. As P and Q are given in different statements, we cannot answer the question using either statement.
Using both statements, we get

$P^6 = \left(\sqrt[3]{a^3+b^3}\right)^6 = (a^3 + b^3)^2 = a^6 + b^6 + 2a^3 b^3$

$Q^6 = \left(\sqrt{a^2+b^2}\right)^6 = (a^2 + b^2)^3 = a^6 + b^6 + 3 a^2 b^2(a^2 + b^2)$

$P^6 – Q^6 = 2a^3 b^3 – 3a^2 b^2 (a^2 + b^2)$
$= a^2 b^2 (2ab – 3 (a^2 + b^2)) = a^2 b^2 (–3 (a – b)^2 – 4 ab)$
$= a^2 b^2$ (A number which is negative)
∴ $P^6 – Q^6$ is negative.
Both statements together are required to answer the question. Choice (C)

Chapter – 8
(Logarithms)

Concept Review Questions

Solutions for questions 1 to 30:

1. $\text{Log}_{(48)(12)} (32)(18) = \log_{(2^4)(3)(3)(2^2)} 2^5 3^2 2$

$= \log_{2^6 3^2} 2^6 3^2 = 1$ Ans: (1)

2. (i) $\log_{9\sqrt{3}} 243 = \log_{3^2.3^{1/2}} 3^5$

$= \log_{3^{5/2}} 3^5$

$= 5 \times \frac{2}{5} \times \log_3 3$

$= 2$ Choice (A)

(ii) $\log_{\sqrt{3125}} 625 = \log_{(3125)^{1/2}} 5^4$

$= \log_{5^{5/2}} 5^4$

$= 4 \times \frac{2}{5} \times \log_5 5 = \frac{8}{5}$ Choice (B)

(iii) $\log_{3125} 5^{25} + \log_{125} 25^{60}$

$= \log_{5^5} 5^{25} + \log_{5^3} 5^{120}$

$= \frac{25}{5} \times \log_5 5 + \frac{120}{3} \times \log_5 5$

$= 5 + 40 = 45$ Choice (C)

(iv) $\log_{\sqrt{32}}(1/1024)$

$= \log_{(32)^{1/2}} (1024)^{-1}$

$= \log_{(32)^{1/2}} (32)^{-2}$

$= (–2) \times 2 \times \log_{32} 32 = –4$ Choice (D)

3. (i) $\log 7 + \log 2^3 – \log 14 – \log 4$

$= \log\left(\frac{7 \times 2^3}{14 \times 4}\right)$

$= \log_{10} 1 = 0$ Choice (A)

(ii) $\log 7^2 – \log 81 + \log 189 – \log 343$

$= \log\left(\frac{7^2 \times 189}{81 \times 343}\right)$

$= \log (1/3) = – \log 3$ Choice (D)

(iii) $5 + \log_{13}(1/2197)$
$= 5 + \log_{13}(2197)^{-1}$
$= 5 – \log_{13} 13^3 = 5 – 3\log_{13} 13$
$= 5 – 3 = 2$ Choice (B)

(iv) $\log 25 + \log 49 + \log 175 + \log 2^7 – \log 14^3$

$= \log\left(\frac{25 \times 49 \times 175 \times 2^7}{14^3}\right)$

$= \log_{10}(5^4 \times 2^4) = \log_{10} 10^4 = 4$ Choice (C)

(v) $\log(25)^2 + \log(16)^8 – \log(32)^5 + \log 5^3$

$= \log\left(\frac{(25)^2 \times (16)^8 \times 5^3}{(32)^5}\right)$

$= \log(5^7 \times 2^7) = \log_{10} 10^7 = 7$ Choice (D)

4. $\log_2(\log_2(\log_2(\log_{11}(14641)^4)))$
$= \log_2(\log_2(\log_2(\log_{11}(11)^{16})))$
$= \log_2(\log_2(\log_2 16))$
$= \log_2(\log_2(\log_2 2^4))$
$= \log_2(\log_2 4)$
$= \log_2(\log_2 2^2) = \log_2 2 = 1$ Ans: (1)

5. $\log(169)^2 – \log(143)^3 + \log(1100) – \log(1300) + \log(121)$

$= \log \frac{(169)^2 \times 1100 \times 121}{(143)^3 \times 1300} = \log(1) = 0$ Choice (A)

6. $\log_3 3^4 + \log_9 (243)^{-1} + \log_{27} 6561 + \log_{3^6} (3)^{4^{5/4}}$

$= 4\log_3 3 - \log_{3^2} 3^5 + \log_{3^3} 3^8 + \log_{3^6} 3^5$

$= 4\log_3 3 - \frac{5}{2}\log_3 3 + \frac{8}{3}\log_3 3 + \frac{5}{6}\log_3 3$

$= 4 – \frac{5}{2} + \frac{8}{3} + \frac{5}{6} = \frac{30}{6} = 5$ Choice (C)

7. $\dfrac{\log_5 5^5 \times \log_3 3^7 \times \log_4 4^5}{\log_6 6^5 \times \log_{11^3}(11^4)^5}$

$= \dfrac{5\log_5 5 \times 7\log_3 3 \times 5\log_4 4}{5\log_6 6 \times \dfrac{20}{3}\log_{11} 11}$

$= \dfrac{5\times 7\times 5\times 3}{5\times 20} = \dfrac{21}{4}$ Ans: (5.25)

8. $\log_{(343)^4}(2401)^3$

$\log_{\left(7^3\right)^4}\left(7^4\right)^3$

$= \log_{7^3} 7^{12} = \dfrac{12\log 7}{12\log 7} = 1$ Choice (C)

9. $729^{\log_{9^3}\sqrt{512}} \Rightarrow 729^{\log_{729}(29)^{1/2}} = 2^{9/2}$ Choice (A)

10. $\log_3 x^0 = \log_3 1 = 0$ Ans: (0)

11. $\log_9 27^2 = \log_9 729 = \log_9 9^3 = 3$ Choice (A)

12. $\dfrac{\log_{11} 64}{\log_{11} 81} = \dfrac{\log_{11} 8^2}{\log_{11} 9^2} = \log_{9^2} 8^2 = \dfrac{2\log 8}{2\log 9} = \log_9 8$

Choice (D)

13. $0.0000128 = \dfrac{128}{10^7} = \dfrac{2^7}{10^7} = \left(\dfrac{2}{10}\right)^7 = \left(\dfrac{1}{5}\right)^7$

$\therefore \log_{\left(\frac{1}{5}\right)} 0.0000128 = \log_{\left(\frac{1}{5}\right)}\left(\dfrac{1}{5}\right)^7 = 7$

Choice (D)

14. Give base = $\dfrac{625}{10000} = \dfrac{25}{100} = \dfrac{1}{16} = 2^{-4}$

$\therefore$ Required number = $\log_{2^{-4}} 2 = -1/4$.

Choice (A)

15. $\log_3 4 + \log_3 16 = \log_3 64$

($\log_b x + \log_b y = \log_b xy$) Ans: (64)

16. $\log_2 72 - \log_2 3 = \log_2 24$

($\log_b x - \log_b y = \log_b \dfrac{x}{y}$) Choice (C)

17. $\log_{27} 8 \times \log_x 3 = 1$

It is known that

$\log_{27} 8 \times \log_8 27 = 1$

($\log_b a \times \log_a b = 1$)

$\therefore \log_x 3 = \log_8 27$

$\log_x 3 = \log_{2^3} 3^3 = \dfrac{3\log 3}{3\log 2} = \log_2 3$

Comparing the two sides, x = 2 Choice (A)

18. $4^{\log_4 5^2} = 5^2 \left(\because a^{\log_a b} = b\right)$

$\therefore$ x = 25 Ans: (25)

19. (i) $\dfrac{\log 729}{\log 81} = \log\sqrt{x}$

$\Rightarrow \log\sqrt{x} = \log_{81} 729$

$\Rightarrow \log\sqrt{x} = \log_{9^2} 9^3 = \dfrac{3}{2}\log_9 9 = \dfrac{3}{2}$

$\log_{10}\sqrt{x} = 3/2 \Rightarrow \sqrt{x} = 10^{3/2}$

$\Rightarrow x = 10^3 = 1000$ Choice (B)

(ii) $\log x + \log 4 + \log 50 = 3$

$\Rightarrow \log_{10}(x \times 4 \times 50) = 3$

$\Rightarrow x \times 200 = 10^3$

$\Rightarrow x = 5$ Choice (A)

(iii) $\log[(x-1)(x^2+x+1)] = \log 7$

$\Rightarrow \log_{10}(x^3-1) = \log_{10} 7$

$\Rightarrow x^3 - 1 = 7 \Rightarrow x^3 = 8 \Rightarrow x = 2$ Choice (C)

(iv) $\log\left(\dfrac{(x^2-4)}{(x+2)}\right) = \log 3$

$= \log_{10}(x-2) = \log_{10} 3$

$\Rightarrow (x-2) = 3 \Rightarrow x = 5$ Choice (B)

(v) $\log_{\sqrt{3}}(x^3-18) = 4$

$\Rightarrow x^3 - 18 = (\sqrt{3})^4$

$\Rightarrow x^3 - 18 = 9 \Rightarrow x^3 = 27 \Rightarrow x = 3$ Choice (C)

(vi) $\log\left(\dfrac{4(3x+4)}{7}\right) = \log(x+3)$

$\Rightarrow \dfrac{4(3x+4)}{7} = x + 3$

$\Rightarrow 12x + 16 = 7x + 21$

$\Rightarrow 5x = 5 \Rightarrow x = 1$ Choice (D)

(vii) $\log_{10}50 + \log_{10}(5x+1) = \log_{10}(5x-7) + 2\log_{10}10$

$\Rightarrow \log_{10}[50(5x+1)] = \log_{10}(5x-7) + \log_{10}100$

$\Rightarrow \log_{10}[50(5x+1)] = \log_{10}[100(5x-7)]$

$\Rightarrow 50(5x+1) = 100(5x-7)$

$\Rightarrow 5x + 1 = 10x - 14$

$\Rightarrow x = 3$ Choice (D)

(viii) $\dfrac{\log 81}{\log 3} = \log_7(7x)$

$\Rightarrow \dfrac{\log 3^4}{\log 3} = \log_7(7x)$

$\Rightarrow \dfrac{4\log 3}{\log 3} = \log_7(7x)$

$\Rightarrow \log_7(7x) = 4$

$\Rightarrow 7x = 7^4$

$\Rightarrow x = 343$ Choice (B)

20. $\log_{10}(7x+8) - 2\log_{10}10$

$= \log_{10}(x+5) - \log_{10}25$

$\log_{10}\left(\dfrac{7x+8}{100}\right) = \log_{10}\left(\dfrac{x+5}{25}\right)$

$\Rightarrow \dfrac{(7x+8)}{100} = \dfrac{(x+5)}{25}$

$\Rightarrow 7x + 8 = 4(x+5)$

$\Rightarrow 3x = 12$

$\Rightarrow x = 4$ Ans: (4)

21. $\log_7(x-7) + \log_7(x^2+7x+49) = 4$

$\Rightarrow \log_7[(x-7)(x^2+7x+49)] = 4$

$\Rightarrow \log_7(x^3 - 7^3) = 4$

$\Rightarrow (x^3 - 7^3) = 2401$

$\Rightarrow x^3 = 2744 = (14)^3 \Rightarrow x = 14$ Choice (D)

22. $\log_y[(x-1)(x+1)] = 2$

$\Rightarrow \log_y(x^2-1) = 2$

$\Rightarrow x^2 - 1 = y^2 \Rightarrow x^2 - y^2 = 1$ Choice (C)

23. $2\dfrac{\log 6561}{\log 243} = \log_{32}x + 2$

$\Rightarrow 2x\dfrac{\log 3^8}{\log 3^5} = \log_{32}x + 2$

$\Rightarrow \dfrac{16}{5} - 2 = \log_{32}x$

$\Rightarrow \log_{32}x = 6/5$

$\Rightarrow x = (32)^{6/5} = 2^6 = 64$ Ans: (64)

24. $\log_{3125}p \times \log_9 25 \times \log_{343}243 \times \log_2 49 = 4$

$\Rightarrow \log_{5^5} p \times \log_{3^2} 5^2 \times \log_{7^3} 3^5 \times \log_2 7^2 = 4$

$\Rightarrow \frac{1}{5}\log_5 p \times \frac{2}{2}.\log_3 5 \times \frac{5}{3}.\log_7 3 \times 2.\log_2 7 = 4$

$\Rightarrow \log_5 p \times \log_3 5 \times \log_7 3 \times \log_2 7 = 6$

$\Rightarrow \log_2 p = 6 \Rightarrow x = 2^6 = 64$ Choice (B)

25. $\log_a a + \log_{a^{1/2}} a + \log_{a^{1/3}} a + \ldots + \log_{a^{1/20}} a$

$= \frac{\log a}{\log a} + \frac{\log a}{\log a^{\frac{1}{2}}} + \frac{\log a}{\log a^{\frac{1}{3}}} + \ldots + \frac{\log a}{\log a^{\frac{1}{20}}}$

$= 1 + 2 + 3 + \ldots + 20 = \frac{20 \times 21}{2} = 210$ Ans: (210)

26. Let $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7} = k$

$\Rightarrow \log a = 5k$
$\Rightarrow a = 10^{5k}$
$\log b = 6k$
$\Rightarrow b = 10^{6k}$
$\log c = 7k$
$\Rightarrow c = 10^{7k}$
$b^2 = (10^{6k})^2 = 10^{12k}$
$= 10^{7k} \times 10^{5k} = ac$ Choice (A)

27. If a is a natural number and b > a and c > a and $\log_b a = \log_c a$, it follows that $\frac{\log a}{\log b} = \frac{\log a}{\log c}$

$\therefore \log b = \log c$ or $\log a = 0$
$\therefore b = c$ or $a = 1$ Choice (C)

28. $\therefore (\log 1)(\log 2)(\log 3) \ldots (\log 10) = 0$ since $\log 1 = 0$ Choice (C)

29. $\log_2 20000 = \log_2 (625)(32) = \log_2 625 + \log_2 32$
$= \log_2 625 + 5$
$2^9 = 512$ and $2^{10} = 1024$
$\therefore \log_2 625$ lies between 9 and 10.
$\therefore$ The $\log_2 20000$ lies between 14 and 15 and its integral part is 14. Choice (C)

30. The integral part of the logarithm (base 10) (of a natural number is always 1 less than the number of digits in it
$\therefore$ The integral part of $\log_{10} N$ is 17. Ans: (17)

Exercise – 8(a)

Solutions for questions 1 to 30:

1. $5^{\frac{\log_7(\log_7 x)}{\log_7 5}} = 5^{\log_5(\log_7 x)}$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$ Ans: (49)

2. $\log_a b = \log_x b / \log_x a \Rightarrow \frac{\log_a x}{\log_b x} = 1$ Choice (C)

3. $\log_{10} x = y$, $x = 10^y$, $\log_e 10 = 1/m$
$10 = e^{1/m}$ $\therefore x = (e^{1/m})^y = e^{y/m}$ Choice (C)

4. $\log_{\left(\sqrt{b\sqrt{b\sqrt{b\sqrt{b\ldots\infty}}}}\right)}\left(\sqrt{a\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}}}\right)$

Let $\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}} = x$

$ax = x^2$, $x = a$
Similarly $y = b$ $\therefore \log_b a = (\log a)/(\log b)$ Choice (A)

5. $\tan 45° = 1$
$\log(\tan 45°) = 0$
$\therefore$ Product = 0
$\therefore x = 0$ Choice (C)

6. $\log_2 \log_2 \log_x 6561 = 2$
$\log_2 \log_x 6561 = 2^2 \Rightarrow \log_x 3^8 = 2^4$
$3^8 = x^{16}$
$\left(\sqrt{3}\right)^{16} = x^{16}$
$\therefore x = \pm\sqrt{3}$; and as $x > 0$, $x = +\sqrt{3}$ Choice (D)

7. (1) $2^3 < 10 < 2^4$
$3\log_2 2 < \log_2 10 < 4\log_2 2$
Taking the reciprocal we get
$1/3 > \log_{10} 2 > 1/4$

(2) $3^2 < 20 < 3^3$
$2\log_3 3 < \log_3 20 < 3\log_3 3$
Taking the reciprocal
$1/2 > \log_{20} 3 > 1/3$

(3) $3^2 < 10 < 3^3$
$2\log_3 3 < \log_3 10 < 3\log_3 3$
Taking the reciprocal
$1/2 > \log_{10} 3 > 1/3$

(4) $4 < 10 < 4^2$
$\log_4 4 < \log_4 10 < 2\log_4 4$
Taking the reciprocal
$1/4 > \log_{10} 4 > 1/2$

Alternate method:
Option (A) is : $(1/4) > \log_{10} 2 > (1/8)$
Taking the reciprocals, $4 < \log_2 10 < 8$
$\Rightarrow 2^4 < 10 < 2^8$, $\Rightarrow 16 < 10 < 256$ which is false.
Option (B) is : $(1/2) > \log_{20} 3 > (1/3)$
$\Rightarrow 2 < \log_3 20 < 3 \Rightarrow 3^2 < 20 < 3^3$
$\Rightarrow 9 < 20 < 27$; and this is true.
(Note : Other options, when similarly transformed lead to false statements)
Note : The remaining options also can be solved as follows:

Option (C) is : $\frac{1}{9} < \log_{10} 3 < 1/3$

$\Rightarrow 9 > \log_3 10 > 3 \Rightarrow 3^9 > 10 > 3^3$
As 10 is not greater than 3^3, the above is false.
Option (D) is : $1/2 > \log_{10^4} > 1/4$
$\Rightarrow 2 < \log_4 10 < 4; \Rightarrow 4^2 < 10 < 4^4$
As $4^2 < 10$, the above is false.
Option (B) alone is true. Choice (B)

8. By considering $1 = \log_a a^a = \log_b b^b = \log_c c^c$, the given data becomes
$x = \log_a abc$, $y = \log_b abc$, $z = \log_c abc$
$1/x + 1/y + 1/z = \log_{abc} abc = 1$
$xy + yz + zx = xyz$ Choice (B)

9. $x \log a = y \log b = z \log c = k$

$\frac{\log_b a}{\log_c b} = \frac{(\log a)(\log c)}{(\log b)(\log b)}$ [($\therefore \log_n m = (\log m)/(\log n)$)]

$= \frac{\left(\frac{k}{x}\right)\left(\frac{k}{z}\right)}{\left(\frac{k}{y}\right)\left(\frac{k}{y}\right)} = y^2/xz = 1$ Choice (A)

10. $a^{\frac{3}{2}[(\log_2 a)-3]} = \frac{1}{8}$

Applying log to base 2 in both sides, we get

$\frac{3}{2}\left[(\log_2 a) - 3\right]\log_2 a = \log_2(1/8) = -3$

Let $\log_2 a = x$; $\frac{3}{2}(x - 3)x = -3$; $x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$ $\log_2 a = 1$ or $\log_2 a = 2$
$\Rightarrow a = 2$ or $a = 4$; since a is a perfect square, $a = 4$
Ans: (4)

11. $f(x) = \frac{e^{2x} + e^{-2x}}{2}$

$f(\log_e 3) = \dfrac{9+\frac{1}{9}}{2} = \dfrac{82}{9 \times 2} = \dfrac{41}{9}$ Choice (C)

12. The given equation is :
$\log_b^a \cdot \log_c^a + \log_c^b \log_a^b + \log_a^c \cdot \log_b^c - 3 = 0$

$\Rightarrow \dfrac{(\log a)^2}{\log b \cdot \log c} + \dfrac{(\log b)^2}{\log c \cdot \log a} + \dfrac{(\log c)^2}{\log a \cdot \log b} - 3 = 0$

[(as $\log_b^a$ = (log a)/log (b)]
$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 - 3 \log a \cdot \log b \cdot \log c = 0$
when $p^3 + q^3 + r^3 - 3pqr = 0$, either p = q = r
or p + q + r = 0
If p = q = r ; log a = log b = log c; $\Rightarrow$ a = b = c
which is contrary to the data. Hence p = q = r is not acceptable.
$\therefore$ p + q + r = 0; $\Rightarrow$ log a + log b + log c = 0
$\Rightarrow$ log abc = 0; $\Rightarrow$ abc = 1 Ans: (1)

13. $\log_{12} 27 = a$

$\dfrac{3\log 3}{\log 3 + 2\log 2} = a$ [$\because \log_b a = (\log a)/(\log b)$]

$\log 3 = \dfrac{2a\log 2}{3-a}$ ----------- (1)

$\log_6 16 = \dfrac{4\log 2}{\log 2 + \log 3} = \dfrac{4\log 2}{\log 2 + \dfrac{2a\log 2}{3-a}}$

$\log_6 16 = \dfrac{4(3-a)}{3+a}$ Choice (A)

14. $\log\left\{\dfrac{\sqrt[3]{a^4}}{\sqrt{b^3 c}} \Big/ \left(\dfrac{a^2 b^3}{c^3}\right)^{1/6}\right\}$

$= \log\left\{\dfrac{a^{4/3}}{b^{3/2} \,.\, c^{1/2}} \Big/ \left(\dfrac{a^{1/3} \,.\, b^{1/2}}{c^{1/2}}\right)\right\}$

$= \log\left\{\dfrac{a^{4/3 - 1/3}}{b^{3/2 - 1/2}}\right\} = \log\left\{\dfrac{a}{b^2}\right\} = \log a - 2\log b$

Choice (D)

15. It is given that z log y + log(log x)
= log[log x + log y + log z]
$\Rightarrow \log y^z + \log(\log x) = \log[\log xyz]$
$\Rightarrow \log y^z \log x = \log[\log (xyz)]$,
$\Rightarrow y^z \cdot \log x = \log (xyz)$

$\Rightarrow \log x^{y^z} = \log xyz$

$\Rightarrow x^{y^z} = xyz$ Choice (B)

16. (a) Given a = $\log_6 161 = \log_6 (23 \times 7)$
$= \log_6 23 + \log_6 7$
$a = b + \log_6 7$
$a - b = \log_6 7$
$\log_7 6 = 1/(a-b)$ Choice (C)

(b) $\log_{\sqrt{3}} 36 + \log_{\sqrt{3}} 36 + \ldots 20$ times

$20\log_{\sqrt{3}} 36 = 20\log_{3^{1/2}}(3^2 \times 4)$

$= \dfrac{20}{1/2}\left[\log_3 3^2 + \log_3 4\right] = 40\left[2\log_3 3 + \dfrac{1}{\log_4 3}\right]$

= 40 [2 + a] = 80 + 40a Choice (B)

17. As (log2) / (logx) = $\log_x 2$, the give equation can be written as:

$\dfrac{1}{\log_x 64} = \log_2 y$; $\log_{64} x = \log_2 y$

$1/6 \log_2 x = \log_2 y$; $\log_2 x = 6 \log_2 y = \log_2 y^6$
$x = y^6$ Choice (A)

18. If $\log_2 (1 - 1/2^x) = x - 2$

$\Rightarrow \left(1 - \dfrac{1}{2^x}\right) = 2^{x-2} = 2^x/4$

if $2^x = a$, then $\dfrac{a-1}{a} = \dfrac{a}{4}$

$a^2 - 4a + 4 = (a-2)^2 = 0 \Rightarrow a = 2, 2^x = 2$
$\therefore$ x = 1 Ans: (1)

19. It is given that, $\log_x 2 \cdot \log_{(x/16)} 2 = \log_{(x/64)} 2$

As $\log_b a = \dfrac{1}{\log_a b}$, the above equation can be written

as: $[1/\log_2 x][1/\log_2(x/16)] = [1/\log_2(x/64)]$

$\Rightarrow [\log_2 x][\log_2(x/16)] = [\log_2(x/64)]$

As log (a/b) = log a – log b, the above can be written as

$\log_2 x[\log_2 x - \log_2 16] = [\log_2 x - \log_2 64]$

Substituting p for ($\log_2 x$) and simplifying the remaining log functions, the above can be written as
p(p – 4) = p – 6,
$\Rightarrow p^2 - 4p - p + 6 = 0$
$\Rightarrow$ (p – 3) (p – 2) = 0
$\Rightarrow$ p = 3 or 2
Substituting the function $\log_2 x$ for p, we have,
$\log_2 x$ = 3 or 2,
$\Rightarrow x = 2^3$ or 2^2; i.e. x = 8 or 4. Hence correct answer is 4 or 8 Ans: (4 or 8)

20. $2\log_4 (2^{1-x} + 1) = \log_2 [5 (2^x) + 1] + 1$;
$\Rightarrow \log_2(2^{1-x} + 1) = \log_2[5(2^x) + 1] + \log_2 2$
$\Rightarrow (2^{1-x} + 1) = 2[5 (2^x) + 1]$
$(2/2^x) + 1 = 10.2^x + 2$; substituting $a = 2^x$,

$\dfrac{2+a}{a} = 10a + 2$

$2 + a = 10a^2 + 2a, \Rightarrow 10a^2 + a - 2 = 0$
$\Rightarrow$ (5a – 2) (2a + 1) = 0
$\Rightarrow$ a = 2/5 or (–1/2)
$\Rightarrow 2^x$ = 2/5 or (–1/2)
$x = \log_2(2/5)$ · [(–1/2) is ignored] Choice (C)

21. Given expression equals

$\log_a(x) + \dfrac{1}{\frac{1/4}{1/2}\log_a x} + \ldots\ldots \dfrac{1}{\frac{1/400}{1/20}\log_a x}$

$= \log_x a + 2 \log_x a + \ldots\ldots. + 20 \log_x a$
$= \log_x a^{1+2+\ldots\ldots+20} = \log_x a^{210}$ Choice (A)

22. $49^{\log_7 5} = (7^2)^{\log_7 5} = 7^{2\log_7 5} = 7^{\log_7 25} = 25$

$\log_2 (2^{2^{2^2}}) = 2^{2^2} = 16$

$\therefore 25 + 16 = 31 + \log_{10} x \Rightarrow x = 10^{10}$ Choice (D)

23. Let $\sqrt[8]{0.000000001234} = x$

$\therefore \log_{10} x = (1/8)[\log_{10} 1234 - 12] = -1.11358$

$\log_{10} x = \bar{2}.88642 \rightarrow (1)$

$\log_{10} 769874 = 5.88642 \rightarrow (2)$

$\therefore$Subtracting 7 form both sides,

$(\log_{10}769874) - 7 = \bar{2}.88642$

As $7 = \log_7 10^7$ and log p – log q = log (p/q), the equation becomes:

$\log_{10} 0.0769874 = \bar{2}.88642$

$\therefore$ x = 0.0769874 Choice (A)

24. Given log7623 = 3.8821
Let logx = –0.1179, converting it into bar form,

$\log x = \bar{1}.8821$ $\therefore x = 0.7623$ Choice (B)

25. **(a)** Let $x = (441)^{50}$

log x = 50 log 441 = 50 log (49 x 9)
= 50[log 49 + log 9] = 132.22
$\therefore$ The number of digits is 132 + 1 = 133

Ans: (133)

(b) Suppose N = 0.000001
log N = log 10^{-6} = –6
So, the least integer greater than or equal to $\log_{10}$N is -6.
Suppose N = 0.000002
log N = log (2) (10^{-6}) = log 2 + log 10^{-6}
= (A number between 0 and 1) + (–6)
$\therefore$ The Least integer greater than or equal to $\log_{10}$N is –5
$\therefore$ We cannot determine the integral part of log N uniquely. Choice (D)

26. Let E = $\log_a \frac{a^3}{b^2} + \log_b \frac{b^3}{a^2} = \left(\log_a a^3 - \log_a b^2\right) + \left(\log_b b^3 - \log_b a^2\right) = 6 - 2\ \left(\log_a b + \log_b a\right)$

$\therefore$ $\log_b a + \log_a b$ is the sum of a positive number and its reciprocal. (If both a, b are greater than 1 or both are positive and less than 1, there $\log_b a$ is positive) Here both are positive and less than 1) Any such sum must be at least 2.
$\therefore$ E must be at most 2. $\therefore$ It can be 1 but not 3.

Choice (A)

27. $\log_5 \log_2 \log_3 \left(\sqrt{x+14} + \sqrt{x-13}\right) = 0$

$\log_2 \log_3 \left(\sqrt{x+14} + \sqrt{x-13}\right) = 1$

$\log_3 \left(\sqrt{x+14} + \sqrt{x-13}\right) = 2^1 \Rightarrow \sqrt{x+14} + \sqrt{x-13} = 3^2$

$\sqrt{x+14} = 9 - \sqrt{x-13}$

Squaring both sides

$x + 14 = 81 + x - 13 - 18\sqrt{x-13}$

$-54 = -18\sqrt{x-13}$

$\Rightarrow 3 = \sqrt{x-13}$

$\Rightarrow$ x – 13 = 9 $\Rightarrow$ x = 22 Ans: (22)

28. Given
log $|x^2 + y^3|$ – log $|x^2 - xy + y^2|$ + log $|x^3 - y^3|$ – log $|x^2 + xy + y^2|$ = log 247

$\Rightarrow \log \left|\frac{x^3+y^3}{x^2-xy+y^2}\right| + \log\left|\frac{x^3-y^3}{x^2+xy+y^2}\right| = \log 247$

$\Rightarrow$ log |x + y| + log |x – y| = log 247
$\Rightarrow$ |x + y| |x – y| = 247
$\Rightarrow$ (x + y) (x – y) = $\pm$ 247
Let (x + y) (x – y) = 247
(x + y) (x – y) = (1) (247)
(x + y) (x – y) = (247) (1)
(x + y) (x – y) = (–1) (–247)
(x + y) (x – y) = (–247) (–1)
From the four possibilities above, it is clear that (x, y) will have 4 values. Similarly when we consider that 247 = (13) (19) and (–13) (–19) (x, y) will have 4 more values.
Hence if (x + y) (x – y) = 247, (x, y) will have 8 more values.
Similarly if (x + y) (x – y) = – 247
(x, y) will have 8 values
$\therefore$ Totally (x , y) can assume 16 integral values.

Ans: (16)

29. $\log_6 {}^{(x+18)} > \log_6 x + \log_6{}^{1.06}$
$\log_6 {}^{(x+18)} > \log_6 x(1.06)$
x + 18 > x (1.06)
18 > x (1.06) – x
18 > x (1.06)
0.06 (x) <18

$\therefore x < \frac{18}{0.06}$

x < 300 Choice (C)

30. $\log_7 \left(\frac{9}{4}\right) + 3\log_{343}\left(\frac{16}{x}\right) \geq 2$

$\log_7 \frac{9}{4} + 3\log_{373}\left(\frac{16}{x}\right) \geq 2$

$\log_7 \left(\frac{9}{4}\right) + \frac{3}{3}\ \log_7 \left(\frac{16}{x}\right) \geq 2$

$\Rightarrow \log_7 \left(\frac{9}{4}\right)\left(\frac{16}{x}\right) \geq \log_7 7^2$

$\Rightarrow \frac{9}{4}\ \frac{16}{x} \geq 49$

$\Rightarrow 36 \geq 49x$

$\Rightarrow x \leq \frac{36}{49}$ Choice (A)

Exercise – 8(b)

Solutions for questions 1 to 35:

1. $\log_{25}125 - \log_{125}25$ = (3/2) $\log_5 5$ – (2/3) $\log_5 5$
= 3/2 – 2/3 = 5/6 Choice (D)

2. a $\log_{0.2}$ x = (b/3) $\log_{0.2}$ x
a = b/3 Choice (B)

3. x = y – 1
y = y
z = y + 1
log (xz + 1) = log $[y^2 - 1 + 1]$ = 2log y

Alternate method:

Such questions can be solved by numerical method. Assume the smallest numerical values satisfy the given conditions and substitute in the function.
1, 2 and 3 are three consecutive positive integers.
log(xz + 1) = log [(1 x 3) + 1] = log4
For the values x = 1, y = 2, z = 3, first option log (x + y + z) = log 6;
Second option is zero; third option is (1 – log2); and fifth option is 2log2, which is $\log 2^2$ = log 4. This is equal to log (xz + 1).
Hence the answer is Choice (D). Choice (D)

4. $\log_{\sqrt{y}} x = 2$

$(\sqrt{y})^2 = x \Rightarrow x = y$

$\therefore \log_{\sqrt[3]{y}} x^3 = \frac{3}{(1/3)} \log_y x = 9$ Ans: (9)

5. Taking logarithms to the base 5, the given equation becomes, $\log_5 1000 = \log_{10}1000$, $\log_5 10 = 3\log_5 10 = y$
$\log_{0.5} 1000 = 3\log_{0.5} 10 = x$

$-\frac{1}{x} + \frac{1}{y} = \frac{-\log_{10} 0.5}{3} + \frac{\log_{10} 5}{3} = \frac{\log_{10} 10}{3} = 1/3$

Alternate method:

Given that $5^y = 1000$,
$\Rightarrow 5 = 10^{(3/y)}$ --------- (1)
$(0.5)^x = 1000$, $\Rightarrow 10^{(3/x)} = 0.5$,

$\Rightarrow \frac{5}{10} = 10^{(3/x)}$

$\Rightarrow$ 5 = 10(3/x + 1) --------- (2)
(1) and (2) are equal; $10^{(3/y)} = 10^{(3/x+1)}$;

$\Rightarrow \frac{3}{y} = \frac{3}{x} + 1, \Rightarrow \left(\frac{1}{y} - \frac{1}{x}\right) = \frac{1}{3}$ Choice (A)

6. $15\log\left(\frac{48}{35}\right) + 9\log\left(\frac{80}{243}\right) - 15\log\left(\frac{64}{63}\right) + 6\log\left(\frac{5}{2}\right)$

$= \log\left[\frac{48^{15}}{35^{15}} \times \frac{80^9}{243^9} \times \frac{63^{15}}{64^{15}} \times \frac{5^6}{2^6}\right]$

$= \log\left[\frac{(2^4)^{15} \times 3^{15}}{5^{15} \times 7^{15}} \times \frac{(2^4)^9 \times 5^9}{(3^5)^9} \times \frac{(3^2)^{15} \times 7^{15}}{(2^6)^{15}} \times \frac{5^6}{2^6}\right]$

$= \log 1 = 0$ Choice (B)

7. $\log_{x^2-y^2}\left(x^2+2xy+y^2\right)$

$\frac{2\log(x+y)}{\log(x+y)+\log(x-y)}$ [($\because$ $\log_b a = (\log a)/(\log b)$]

Considering the logarithm to the base (x + y) the given function is $\frac{2}{1+\log_{(x+y)} x-y} = \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Choice (D)

8. $\log_r (r/2) = 1 - \log_r 2$

$1 - \log_r 6 + \log_r 3 = 1 - a + b$ Choice (A)

9. $a^2 + 4b^2 = 12ab$; adding 4ab to both sides of the equation, we get $(a + 2b)^2 = 16ab$

$2 \log (a + 2b) = 4 \log 2 + \log a + \log b$

$\log (a + 2b) = 1/2 [\log a + \log b + 4 \log 2]$

Choice (C)

10. $\log_2 x = \frac{3\log_{10} 8}{2\log_{10} 8}$

$\log_2 x = \frac{3}{2}$

$x = (2)^{3/2} = \sqrt{8} = 2\sqrt{2}$ Choice (A)

11. $\log (2x + 3) - 1 = \log x$

$\log (2x + 3) - \log 10 = \log x$

$\Rightarrow \log\left(\frac{2x+3}{10}\right) = \log x$

$\Rightarrow \frac{2x+3}{10} = x$

$\Rightarrow 2x + 3 = 10x$

$\Rightarrow x = \frac{3}{8}$ Choice (D)

12. $\log_{2x}\frac{2x}{3y} + \log_{3y}\frac{3y}{2x}$

$= 1 - \log_{2x}3y + 1 - \log_{3y}2x$

$= 2 - (\log_{2x}3y + \log_{3y}2x)$

let $\log_{3y} 2x = p$

It is given that $y > \frac{1}{3}$

$\Rightarrow$ the base $3y > 1$

Also $2x \ge 3y$

$\Rightarrow \log_{3y}2x \ge 1$

$\Rightarrow p \ge 1$

Consider $2 - (\log_{2x} 3y + \log_{3y}2x)$

$= 2 - \left(\frac{1}{p}+p\right)$

since $p \ge 1$, the value of $\left(p+\frac{1}{p}\right) \ge 2$

Hence the maximum value of $2 - \left(p+\frac{1}{p}\right) = 0$

From the given options 1 cannot be the value of the given expression. Choice (C)

13. Given $\frac{1}{x-1} = \log_{bc} a$

$\Rightarrow x - 1 = \log_a bc$

$\Rightarrow x = 1 + \log_a bc = \log_a a + \log_a bc$

$\Rightarrow x = \log_a abc$

Similarly $y = \log_b abc$ and

$z = \log_c abc$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \log_{abc} abc = 1$ Choice (B)

14. Given $a = \sqrt{b} = 3\sqrt{c} = 4\sqrt{d} = 5\sqrt{e}$

$\Rightarrow b = a^2$, $c = a^3$ $d = a^4$ and $e = a^5$

$\therefore \log_a (abcde)$

$= \log_a (a) (a^2) (a^3) (a^4) (a^5)$

$= \log_a a^{15} = 15$ Ans: (15)

15. Given $\frac{1}{3}\log_7 x - 3\log_7 y = 1 + \log_{0.125} 2$

$\Rightarrow \log_7 \frac{x^{\frac{1}{3}}}{y^3} = 1 + \log_{\frac{1}{8}} 2$

$\Rightarrow \log_7 \frac{x^{\frac{1}{3}}}{y^3} = 1 + \log_{2^{-3}} 2$

$\Rightarrow \log_7 \frac{x^{\frac{1}{3}}}{y^3} = 1 - \frac{1}{3} \Rightarrow \log_7\left(\frac{x^{\frac{1}{3}}}{y^3}\right) = \frac{2}{3}$

$\Rightarrow \frac{x^{\frac{1}{3}}}{y^3} = 7^{\frac{2}{3}}$

$\Rightarrow \frac{x}{y^9} = 49$

$\Rightarrow x = 49\, y^9$ Choice (C)

16. Given $\log_4 3 + \log_4\left(3^m - \frac{8}{3}\right) = 2\log_4\left(3^m - 2\right)$

$\Rightarrow 3\left(3^m - \frac{8}{3}\right) = \left(3^m - 2\right)^2$

$\Rightarrow 3^{m+1} - 8 = 3^{2m} + 4 - 4 (3^m)$

$\Rightarrow 3^{2m} - 7 (3^m) + 12 = 0$

Let $3^m = x$

$\Rightarrow x^2 - 7x + 12 = 0$

$x^2 - 4x - 3x + 12 = 0$

$x (x - 4) - 3 (x - 4) = 0$

$\Rightarrow (x - 4) (x - 3) = 0$

$\Rightarrow x = 4$ or 3

$\Rightarrow 3^m = 4$ or $3^m = 3$

$\Rightarrow m = \log_3 4$ (or) $m = 1$

Hence m can take two values. Ans: (2)

17. $\log 6250$

$= \log 5^4 (10)$

$= 4 \log 5 + \log 10$

$= 4 \log \frac{10}{2} + \log 10$

$= 4 (\log 10 - \log 2) + \log 10$

$= 4 (1 - 0{\cdot}301) + 1$

$= 4 (0{\cdot}699) + 1$

$= 2{\cdot}796 + 1 = 3.796$ Ans: (3.796)

18. Given $\log_x 3 \; \log_{\frac{x}{81}} 3 = \log_{\frac{x}{729}} 3$

$\Rightarrow \log_3 x \log_3 \frac{x}{81} = \log_3 \frac{x}{729}$

$\Rightarrow$ $\log_3 x \left[\log_3 x - \log_3 81\right] = \log_3 x - \log_3 729$

Let $\log_3 x = k$

$\Rightarrow k [k - 4] = k - 6$

$\Rightarrow k^2 - 5k + 6 = 0$

$\Rightarrow k = 3$ or $k = 2$

If k = 3, x = 27 and if k = 2, x = 9 Choice (D)

19. Given

$\log_k x + \log_{kx} x^2 + \log_{kx^2} x^3 = 0$

Consider $\log_{kx} x^2$

$= \frac{1}{\log_{x^2} kx} = \frac{2}{\log_x kx} = \frac{2}{\log_x k + 1}$

consider $\log_{kx^2} x^3$

$= \frac{1}{\log_{x^3} kx^2} = \frac{3}{\log_x kx^2} = \frac{3}{\log_x k + 2}$

Let $\log_x k = m$

The given equation becomes

$\frac{1}{m} + \frac{2}{1+m} + \frac{3}{2+m} = 0$

$\Rightarrow 6m^2 + 10m + 2 = 0$

As the discriminant is positive there are two real values of m and hence x has two values. Choice (B)

20. Let $x^y = y^z = z^x = k$

$\Rightarrow x = k^{\frac{1}{y}}, y = k^{\frac{1}{z}}, z = k^{\frac{1}{x}}$

consider $\frac{1}{x} \log_z xyz$

$= \frac{1}{x} \log_{k^{\frac{1}{x}}} \left(k^{\frac{1}{y}} . k^{\frac{1}{z}} . k^{\frac{1}{x}}\right)$

$= \frac{1}{x} \log_{k^{\frac{1}{x}}} \left(k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

Similarly

$\frac{1}{y} \log_x xyz = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ and $\frac{1}{z} \log_y xyz = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

Hence the given expression is equal to

$3\left[\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right] = 3\left(\frac{xy + yz + zx}{xyz}\right)$ Choice (A)

21. $\log_5 (\log_3 x) = 8^0 = 1$

$\Rightarrow \log_3 x = 5^1 = 5$

$\Rightarrow x = 3^5 = 243.$ Ans: (243)

22. $\log_3 x^2 - \log_3 x\sqrt{x} = 8 \log_x 3$

$\Rightarrow \log_3 \frac{x^2}{x\sqrt{x}} = 8 \log_x 3$

$\Rightarrow \log_3 \sqrt{x} = 8 \log_x 3$

$\Rightarrow \frac{1}{2} \log_3 x = \frac{8}{\log_3 x}$

$\Rightarrow (\log_3 x)^2 = 16$

$\Rightarrow \log_3 x = 4$

$\Rightarrow x = 3^4 = 81$

or

$x = 3^{-4} = \frac{1}{81}$ Choice (A)

23. Given $\log_x 162 = m$

$\Rightarrow \log_x 3^4 \ (2) = m$

$\therefore m = 4 \log_x 3 + \log_x 2$

Given $\log_x 72 = n$

$\Rightarrow \log_x 3^2 2^3 = n$

$\therefore n = 2\log_x 3 + 3\log_x 2$

Let $\log_x 3 = 1$ and $\log_x 2 = b$

$\Rightarrow m = 4a + b$ --- (1)

$n = 2a + 3b$ --- (2)

2 (2) – (1) gives

$5b = 2n - m \Rightarrow b = \frac{2n - m}{5}$

similarly $a = \frac{3m - n}{10}$

Now consider $\log_x 7776$

$= \log_x 3^5.2^5$

$= 5[\log_x 3 + \log_x 2]$

$= 5\left[\frac{3m - n}{10} + \frac{2n - m}{5}\right]$

$= 5\left[\frac{m + 3n}{10}\right] = \frac{m + 3n}{2}$ Choice (C)

24. As x = 1, 2, ... 99, $5x \geq 5$.

Given $\log_{5x} (4x - 15) > \frac{1}{2}$

$\Rightarrow (4x - 15) > \left(5x^{\frac{1}{2}}\right)$

$\because$ For a strong base (> 1) the log increases with number

$\Rightarrow (4x - 15)^2 > 5x$

$\Rightarrow 16x^2 - 120x + 225 > 5x$

$\Rightarrow 16x^2 - 125x + 225 > 0$

$\Rightarrow 16x^2 - 80x - 45x + 225 > 0$

$\Rightarrow 16x (x - 5) - 45 (x - 5) > 0$

$\Rightarrow (16x - 45) (x - 5) > 0$

$\Rightarrow x \in \left(-\infty, \frac{45}{16}\right) \cup (5, \infty)$

Since $4x - 15 > 0$

$\Rightarrow x > 3 \therefore x \in (5, \infty)$

Hence x can take values from 6 to 99 i.e. a total of 94 values. Ans: (94)

25. Given $49^{\left(\log_7 \frac{1}{3} + 2\log_x \sqrt{3}\right)} = \frac{1}{3}$

$\Rightarrow \log_7 \frac{1}{3} + 2\log_x \sqrt{3} = \log_{49} \frac{1}{3}$

$\Rightarrow \log_7 \frac{1}{3} + \log_x 3 = \frac{1}{2} \log_7 \frac{1}{3}$

$\Rightarrow \log_x 3 = -\frac{1}{2} \log_7 \frac{1}{3}$

$\Rightarrow \log_x 3 = \frac{1}{2} \log_7 3$

$\Rightarrow \log_x 3 = \log_{49} 3$

$\therefore x = 49$ Choice (C)

26. $\frac{\log_m p.\log_n p}{\log_m p + \log_n p}$

$= \dfrac{1}{\dfrac{\log_m p + \log_n p}{\log_m p . \log_n p}}$

$= \dfrac{1}{\dfrac{1}{\log_n p} + \dfrac{1}{\log_m p}}$

$= \dfrac{1}{\log_p n + \log_p m} = \dfrac{1}{\log_p mn}$

$= \log_{mn}p$ Choice (D)

27. Given $m^n = \dfrac{1}{5^6}$ and $n = \dfrac{6}{5}[(\log_5 m) - 6]$

$\Rightarrow n = \log_m\left(\dfrac{1}{5^6}\right) = -6\log_m 5$

$-6\dfrac{\log 5}{\log m} = \dfrac{6}{5}\left[\dfrac{\log m}{\log 5} - 6\right]$

Let logm = x

$-6\dfrac{\log 5}{x} = \dfrac{6}{5}\left[\dfrac{x}{\log 5} - 6\right]$

$\dfrac{\log 5}{x} = \dfrac{1}{5}\left[6 - \dfrac{x}{\log 5}\right]$

$5(\log 5)^2 = 6x\log 5 - x^2$

$(x - \log 5)(x - 5\log 5) = 0$

x= log5; 5log5

m= 5; m = 5^5

$m_1 + m_2 = 5 + 5^5 = 3130$. Ans: (3130)

28. $3\log_{20}x - \log_x 0{\cdot}125 + 3\log_x 10$

$= 3\log_{20}x + \log_x\left(\dfrac{1000}{0.125}\right)$

$= 3\log_{20}x + \log_x 8000$

$= 3\log_{20}x + 3\log_x 20$

$= 3[\log_{20}x + \log_x 20]$

Since x > 1, $\log_{20}x > 0$

$\Rightarrow \log_{20}x + \log_x 20 \geq 2$

$\Rightarrow 3[\log_{20}x + \log_x 20] \geq 6$

minimum value is 6 Choice (A)

29. $\log 10 + \log(x^2 + 5x) = 2\log\sqrt{60}$

$\Rightarrow \log 10 + \log(x^2 + 5x) = \log 60$

$\Rightarrow \log\dfrac{10(x^2+5x)}{60} = 0$

$\Rightarrow \dfrac{x^2+5x}{6} = 1$

$\Rightarrow x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$\Rightarrow x = -6$ or $x = 1$

$\therefore$ sum of the possible values of x = –5 Ans: (–5)

30. $N = 1764^{50}$

$N = (42)^{100}$

$N = [(2 \times 3 \times 7)]^{100}$

logN = 100 [log2 + log3+ log7)

logN = 100 [1·6231]

logN = 162·31

Hence the number of digits in N is 163. Choice (B)

31. $(5/6)^{400} = x$

log x = 400[log 5 – log 6]

= 400 [log 10/2 – log (3 x 2)]

= 400 [1 – log 2 – log 3 – log 2]

= 400[1 – log3 – 2 log 2] = –31.64

$= -32 + 32 - 31.64 = \overline{32}.36$

$\therefore$Number of zeros after the decimal point in $(5/6)^{400}$ is 32 – 1 = 31 Ans: (31)

32. Given log 3 = 0.4771, log 2 = 0.3010,

$5^x . 27^{1-x} = 0.1$

$x(1 - 0.3010) + 3(0.4771)(1 - x) = -1$

$\Rightarrow 0.699x + 1.4313 - 1.4313x = -1$

$\Rightarrow 2.4313 = 0.7323x \Rightarrow x = 3.32$ Choice (D)

33. $\sqrt{x+5} = 5 - \sqrt{x}$

Squaring both sides

$\Rightarrow x + 5 = 25 - 10\sqrt{x} + x$

$\sqrt{x} = \dfrac{20}{10} = 2 \Rightarrow x = 4$ Ans: (4)

34. $\log_b a = \dfrac{2}{3}$ and $\log_d c = \dfrac{4}{5}$

$\therefore a = b^{\frac{2}{3}}$ and $c = d^{\frac{4}{5}}$

$a^3 = b^2$. Let each of these be k. k is a perfect cube as well as a perfect square. $\therefore$k must have the form i^6 where i is an integer

$c^5 = d^4$. Each of these must have the form k^{20} where k is an integer.

$\therefore a^3 = b^2 = i^6$ and $c^5 = d^4 = k^{20}$

$a = i^2$, $b = i^3$, $c = k^4$, $d = k^5$

$c - a = k^4 - i^2 = 7$ (given)

$\therefore (k^2 - i)(k^2 + i) = 7$

As $k^2 > 0$, the only possibility is $k^2 - i = 1$, $k^2 + i = 7$ or

$k^2 = 4$, i = 3

$b - d = i^3 - k^5 = 3^3 - 2^5 = -5$ Choice (A)

35. $\log_4 31 = \log_{2^2} 31 = \dfrac{1}{2}\log_2 31$

$2^4 < 31 < 2^5 \Rightarrow \log_2 2^4 < \log_2 31 < \log_2 2^5$

$\Rightarrow 4\log_2 2 < \log_2 31 < 5\log_2 2$

$\Rightarrow \dfrac{4}{2} < \dfrac{1}{2}\log_2 31 < \dfrac{5}{2}$

$\Rightarrow 2 < \dfrac{1}{2}\log_2 31 < 2.5.$ Choice (B)

Chapter – 9
(Permutations and Combinations)

Concept Review Questions

Solutions for questions 1 to 7:

1. **(a)** ${}^8P_2 = 8(7) = 56$ Choice (B)

(b) ${}^{10}C_2 = \dfrac{10(9)}{1(2)} = 45$ Choice (C)

(c). ${}^{45}C_{42} = {}^{45}C_3$ as ${}^nC_r = {}^nC_{n-r}$ Choice (D)

(d) ${}^{2009}C_0 = 1$ ($\because {}^nC_0 = 1$) Choice (B)

(e) ${}^{2009}C_1 = 2009$ ($\because {}^nC_1 = 1$) Choice (C)

(f) ${}^{2009}C_{2008} = {}^{2009}C_1 = 2009$ Choice (A)

2. ${}^nC_2 = {}^nC_{10} \Rightarrow n = 10 + 2 = 12$ as ${}^nC_r = {}^nC_s \Rightarrow r + s = n$ Ans: (12)

3. We know ${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

$\therefore {}^8C_3 + {}^8C_4 = {}^9C_4$

$\therefore$n = 9 Ans: (9)

4. ${}^nP_r = r!\ {}^nC_r$ Choice (C)

5. ${}^nP_4 = n(n-1)(n-2)(n-3) = 10(4)(198) = 8(9)(10)(11)$
$\therefore n = 11$
$\therefore {}^nC_4 = {}^{11}C_4 = {}^nP_4\left(\frac{1}{4!}\right) = \frac{1}{4!}\,7920 = 330$ Choice (B)

6. The total number of ways that 12 blazers, 10 shirts and 5 ties can be worn is 12(10)(5) = 600 (by fundamental rule).
Ans : 600

7. We know that n persons can be arranged in a row in n! ways
$\therefore$ 6 persons can be arranged in 6 ! = 6 (5) (4) (3) (2) (1) = 720 ways Choice (D)

Solutions for questions 8 to 10:

8. There are 7 letters in the word RAINBOW
$\therefore$ Number of 7 letter words possible is 7! Choice (D)

9.

$$\boxed{R}\underbrace{\boxed{\qquad\qquad}}_{6!}$$

$\therefore$ Number of words that begin with R = 6! Choice (C)

10. Fixing R and W as required, the remaining 5 letters can be arranged in 5! ways Choice (B)

Solutions for questions 11 to 45:

11. For each letter there are 5 ways of posting.
$\therefore$ Required number of ways = 5^4 Choice (A)

12. The 5 vowels are a, e, i, o, u. For writing pass word repetition of letters is allowed.
$\therefore$ Number of passwords possible is 5^5 Ans: (3125)

13. A palindrome is a word which when read from left to right or right to left, remains the same.
In a palindrome, only in the first half the letters are different. The same letters that appear in the first half are repeated in the second. i.e. in a five letter palindrome word, first three letters will be different.
First place can be filled in 7 ways,
Similarly second and 3rd places can be filled in 7 ways
$\therefore$ required number of palindrome words formed is $= 7 \times 7 \times 7 = 343$ Choice (C)

14. There are 6 letters in the word MOBILE.
If we fix M in the first place and E in the last place then the remaining 4 letters can be arranged in 4 places in 4! ways.
The number of 6 letter words formed is 24.
Choice (B)

15. Given word is RELATION. Total number of letters in the word is 8. The number of three letter words that can be formed using these 8 letters is ${}^8P_3 = 8 \times 7 \times 6 = 336$
Ans: (336)

16. 1st book can be distributed to any one of the 6 students in 6 ways
2nd book can also be distributed in 6 ways
similarly 9th book can be distributed in 6 ways
$\therefore$ required number of ways
$= 6.6.6.6.6.6.6.6.6 = 6^9$. Choice (A)

17. The given word INSTITUTE contains a total of 9 occurrences – 3Ts, 2Is and the rest four are distinct.
The total number of different words that can be formed, is $\frac{9!}{3!2!}$ Choice (D)

18. Since one player is in the team and one player is not in the team, we have to select 10 players from 13. This can be done in ${}^{13}C_{10}$ or ${}^{13}C_3$ or 286 ways. Choice (C)

19. Consider the letters $L_1, L_2, \ldots, L_8$ and the covers $C_1, C_2, \ldots, C_8$.
Suppose letter L_1, is placed into the wrong envelope say E_2, then L_2 letter is also placed in the wrong envelope. At least two letters must be placed in wrong envelopes. It is not possible to place exactly 1 letter into a wrong envelope.
$\therefore$ Number of ways = '0' Choice (B)

20. The number of ways of selecting 6 students from n students is ${}^nC_6$. The number of ways of selecting 9 students from n students is ${}^nC_9$.
Given ${}^nC_6 = {}^nC_9 \Rightarrow n = 15$
The number of ways of selecting 4 students from 15 students is ${}^{15}C_4 = \frac{15(14)(13)(12)}{4(3)(2)(1)} = 1365$
Ans: (1365)

21. When two books are to be together, we assume those two books as 1 unit. With this one unit, there are 9 books which can be arranged in 9! ways. But these two books can be arranged internally in 2! ways
$\therefore$ Required number of arrangements = 9! 2!
Choice (A)

22. The hundred digit can be selected in 6 ways. For each way, the tens digit can be selected in 5 ways. For each of these 6(5) ways the units digit can be selected in 4 ways. $\therefore$ The total number of ways is 6 (5) (4) or ${}^6P_3$ Ans: (120)

23. We have to fill four blanks. Each blank can be filled by any of the six digits.

$$\underset{6}{\boxed{\quad}}\ \underset{6}{\boxed{\quad}}\ \underset{6}{\boxed{\quad}}\ \underset{6}{\boxed{\quad}}$$

$\therefore$ Required number of ways = 6^4 Choice (D)

24. A number is even if its units place is divisible by 2
$\therefore$ units place can be filled by either 2 or 4 i.e. in 2 ways. The remaining 4 places can be filled by the remaining 4 digits in 4! ways
Total number of even numbers that can be formed
= 2 (4!) = 48 Choice (B)

25. n persons can be arranged around a circle in (n – 1)! ways.
As n = 6, the required number of arrangements
= (6 – 1)! = 5! Choice (C)

26. Number of selections required = ${}^8C_5 = {}^8C_3$
Choice (A)

27. **(a)** Since a particular person has to be selected, we have to select only 5 persons from 9 persons which is possible in ${}^9C_5$ ways. Choice (A)

(b) Since particular person should not be included in the team, the selection must be made excluding that person, i.e. The selection is to be made from 6 persons. This can be done in ${}^6C_4$ ways.
Choice (C)

28. **(a)** n beads can be strung in a necklace in $\frac{(n-1)!}{2}$ ways.
Here n = 10
$\Rightarrow$ Required number of ways = $\frac{9!}{2}$ Choice (A)

(b) Number of ways of inviting at least one of n people is given by $2^n - 1$ Here n = 6
$\therefore$ Required number of ways $2^6 - 1$ Choice (B)

29. The total number of members is 4 + 6 or 10. A committee of six can be formed from 10 in ${}^{10}C_6$ ways
= (${}^{10}C_4$) or 210 ways Ans : 210

30. The number of lines that can be drawn joining n points on a plane = ${}^nC_2$ (when no three points are collinear). Here n = 20
$\therefore$ Required number of lines = ${}^{20}C_2 = 190$
Choice (A)

31. The number of triangles that can be formed with n points on a plane, when no three of them are collinear is ${}^nC_3$
As n = 24, the number of triangles is ${}^{24}C_3$ or 2024.
Choice (A)

32. **(a)** The number of rectangles that can be formed on an $n \times n$ chess board (R) = $\sum_1^n i^3 = \left[\frac{n(n+1)}{2}\right]^2$
As n = 8, R = 36^2 = 1296 Ans: (1296)

(b) Number of squares (S) that can be formed in an $n \times n$ chess board = $\sum_1^n i^2 = \frac{n(n+1)\,(2n+1)}{6}$
As n = 8, S = $\frac{8(9)\,(17)}{6}$ = 204 Ans: (204)

33. **(a)** The number of diagonals of an n sided regular polygon is $\frac{n(n-3)}{2}$. $\therefore$The number of diagonals of a 10-sided (decagon) polygon is $\frac{10(10-3)}{2}$ = 35.
Choice (B)

(b) The number of diagonals of a regular n sided polygon is $\frac{n(n-3)}{2}$
$\frac{n(n-3)}{2} = 77$
n(n –3) = 154
n = 14 satisfies the above equation. Choice (B)

34. Total number of persons = 6 + 2 + 3 + 4 = 15
The number of ways of selecting 7 persons from 15 persons is ${}^{15}C_7$ = 6435 ways Ans: (6435)

35. Given word is VALEDICTORY
Consonants are V, L, D, C, T, R, Y and vowels are A, E, I, O
4 consonants can be selected from 7 in ${}^7C_4$ ways
3 vowels can be selected from 4 in ${}^4C_3$ ways
The required number of ways of selecting 4 consonants and 3 vowels = ${}^7C_4 \times {}^4C_3 = 35 \times 4 = 140$. Choice (A)

36. A pack of 52 cards contains 4 different suits. Number of ways of drawing four cards each from a different suit is
= ${}^{13}C_1.{}^{13}C_1.{}^{13}C_1.{}^{13}C_1 = 13^4$ Choice (C)

37. The word EQUATION contains 5 vowels and 3 consonants.
Since the words begin with a consonant, the first place can be filled in 3 ways and last place can be filled in 5 ways. The remaining 6 places can be filled with the remaining 6 letters in 6! ways.
$\therefore$The total number of words that can be formed is
3(5)(6!) = 10800. Ans: (10800)

38. Since the number of boys is greater than the number of girls, the first and the last place has to be occupied by boys. 6 boys can be arranged in a row in 6! ways. There are five gaps in between them. The 5 girls can be arranged in these gaps in 5! ways.
$\therefore$Total number of arrangements = 6! 5!. Choice (C)

39. The possible number of men and women and the corresponding number of ways in which the committee can be selected are tabulated below.

Men 6	Women 4	Number of selections
3	2	${}^6C_3{}^4C_2$
4	1	${}^6C_4{}^4C_1$
5	0	${}^6C_5{}^4C_0$

$\therefore$total number of ways that the committee formed is ${}^6C_3.\,{}^4C_2 + {}^6C_4.\,{}^4C_1 + {}^6C_5.\,{}^4C_0$
= 20(6) + 15(4) + 6(1) = 120 + 60 + 6 = 186
Choice (B)

40. The word PREVIOUS contains 8 letters.
$\therefore$The number of 4-letter words that can be formed using 8 letters is ${}^8P_4$= 1680 Ans: (1680)

41. Assume that the three students have to sit together as one unit. Now there are 8 units (7 students + 1 unit of three students) and they can be arranged at a circular table in 7! ways. Again the three students can be arranged among themselves in 3! ways.
$\therefore$The total number of ways = 7! 3!. Choice (B)

42. First arrange the 7 boys.
$\times B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6 \times B_7 \times$
Now there are 8 gaps between boys, marked by $\times$.
7 boys can be arranged in 7 places in 7! ways
The 6 girls can be arranged in these gaps in ${}^8P_6$ ways
$\therefore$Required number of ways = ${}^8P_6 \times 7!$ Choice (B)

43. The number of ways (or combinations) of selecting atleast one of n different things is
${}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n - 1$
$\therefore$ The number of ways that the man can invite at least one of his friends for dinner
= $2^7 - 1 = 128 - 1 = 127$ Ans: (127)

44. 13 different beads can be arranged in a circular order in (13 – 1)! = 12! ways
now in this case it is a necklace, and so there is no distinction between clockwise and anticlockwise arrangements. So the required number of arrangements is
= $\frac{1}{2}$(12!) Choice (D)

45. We treat 10 girls as 1 unit.
Then total number of students = 11
$\therefore$ 11 students can be arranged in a row in 11! ways
Again the 10 girls can be arranged among themselves in 10! ways.
$\therefore$required number of ways = 10! 11! Choice (D)

Exercise – 9(a)

Solutions for questions 1 to 20:

1. The word QUESTION has 8 letters of which 4 are vowels and 4 consonants. There are 4 even places and the vowels can be arranged in these 4 places in 4! ways while the consonants can be arranged in the remaining 4 places in 4! ways.
CVCVCVCV
Hence total arrangements are $4! \times 4! = 24 \times 24 = 576$.
Ans: (576)

2. The word HEPTAGON has 8 letters of which 3 are vowels. As the vowels have to be together, we treat them as 1 unit. Now there are 5 other letters. These 5 letters and the unit of vowels can be permuted in 6! ways, while the vowels can be permuted among themselves in 3! ways.
Hence the required permutations are $6! \times 3!$ = 4320
Choice (D)

3. There are 11 letters in the word, of which the letters O, I and N are each repeated twice. Hence of the 11 items there are 2 alike of one kind 2 alike of the second, and 2 alike of the third while the remaining are distinct.

Hence the number of arrangements is $\frac{11!}{2!\,2!\,2!}$

Choice (B)

4. The digits can be any of 0 to 9 i.e., 10 digits

$\underset{9}{\downarrow}\quad \underset{9}{\downarrow}\quad \underset{8}{\downarrow}\quad \underset{7}{\downarrow}$

The thousands position can be filled with any of the digits 1 to 9. Having filled up the thousands position, we are left with 9 other digits. Hence the hundreds position can now be filled in 9 ways and likewise the ten's position in 8 ways and units in 7 ways. Hence the required four-digit numbers are $9 \times 9 \times 8 \times 7 = 4536$ in number. Choice (D)

5. As the numbers have to be divisible by 5, they have to end in either 0 or 5. The number of ways in each case is

$\underset{9}{\downarrow} \times \underset{8}{\downarrow} \times \underset{7}{\downarrow}\ \ \underline{0} = 504$

$\underset{8}{\downarrow} \times \underset{8}{\downarrow} \times \underset{7}{\downarrow}\ \ \underline{5} = 448$

Hence the total number is = 504 + 448 = 952

Choice (D)

6. From the given digits 0, 2, 3, 5, 8 we need to select 4 digits which add on to a multiple of 3. The combination 0, 2, 5, 8 is one possible combination while 2, 3, 5, 8 is the only other combination. As repetition is not allowed, with 0, 2, 5, 8 we can have 18 numbers and with 2, 3, 5, 8 we can have 24 numbers. Hence total of such numbers are 18 + 24 = 42. Ans: (42)

7. As the numbers have to be between 20,000 and 40,000 they have to be 5 digit numbers beginning with 2 or with 3. Further the numbers have to be even. Hence, they have to end in any of 0, 2, 4, 6, 8. Hence while the first position can be filled in 2 ways (with 2 or 3), the last position can be filled in 5 ways (as repetition is allowed). Now each of the other positions can be filled in 6 ways

$\underset{2}{\downarrow} \times \underset{6}{\downarrow} \times \underset{6}{\downarrow} \times \underset{6}{\downarrow} \times \underset{5}{\downarrow}$

Hence the required numbers
$= (2 \times 6^3 \times 5) - 1 = 2160 - 1 = 2159$

Note: We exclude the case of getting 20,000 in the above calculation as the extremes are not included.

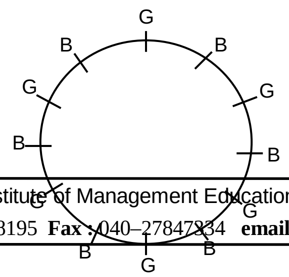
Choice (D)

8. The boat requires 4 on the bow side and 4 on the stroke side. As one of the 8 persons available cannot row on the bow side, we shall get him on to the stroke side and the two who cannot row on the stroke side should be sent to the bow side. Having fixed one on the stroke side and 2 on the bow side, we need to get 3 and 2 on each of the respective sides from the 5 remaining persons which can be done in $\frac{5!}{3!2!}$ ways.

Now the 4 persons on each side can be arranged among themselves in 4! ways.
Hence the total ways of arranging the crew is

$\frac{5!}{3!2!} \times 4! \times 4! = 48 \times 5! = 5760$ Ans: (5760)

9. Since no two boys sit next to each other, we first take care of the girls. The six girls can be arranged in 5! ways.



Now the 6 boys can be arranged in 6! ways.
Hence the total arrangements are 5! 6! Choice (C)

10. Sheetal has to initially divide her 10 friends into groups of 5 each which she can do in $\frac{10!}{5!\,5!}$ ways. Now each group can be arranged in a circle in 4! ways. Hence total ways in which she can arrange her friends around two circular tables is $\frac{10!(4!)^2}{(5!)^2}$ Choice (B)

11. Of the 100 passengers 15 have to be accommodated in the lower deck and 10 in the upper deck. As the lower deck can take 60 and upper deck 40, of the remaining 75 passengers we need to accommodate 45 in the lower deck and 30 in the upper deck, which can be done in $\frac{75!}{45!\,30!}$ ways.

Choice (B)

12. As more surgeons have to be selected than physicians, we can select 6 doctors from 5 surgeons and 6 physicians in the following ways.
Case (i) 4 surgeons and 2 physicians
Case (ii) 5 surgeons and 1 physician
The number of ways is
Case (i) ${}^5C_4 \times {}^6C_2 = 75$
Case (ii) ${}^5C_5 \times {}^6C_1 = 6$
$\therefore$ required ways are 81 Choice (B)

13. The total ways of selecting 4 professors and 3 students from 8 professors and 5 students is ${}^8C_4 \times {}^5C_3$.
The number of ways where Mr. Balamurli and Mr. Siddharth both happen to be on the delegation is ${}^7C_3 \times {}^4C_2$ which we exclude from the total possibilities, as they do not serve together on the delegation.
Hence required ways are
${}^8C_4 \times {}^5C_3 - {}^7C_3 \times {}^4C_2 = 700 - 210 = 490$ Ans: (490)

14. Prahaas can select the questions in the following combinations

(i) 3, 3, 2	(ii) 3, 2, 3	(iii) 2, 3, 3
(iv) 4, 2, 2	(v) 2, 4, 2	(vi) 2, 2, 4

The number of selections in each of the cases (i), (ii) and (iii) is ${}^6C_3 \times {}^6C_3 \times {}^6C_2$ while
The number of selections in each of the cases (iv), (v), (vi) is ${}^6C_4 \times {}^6C_2 \times {}^6C_2$
Hence total selections are
$3 \times {}^6C_3 \times {}^6C_3 \times {}^6C_2 + 3 \times {}^6C_4 \times {}^6C_2 \times {}^6C_2$
$= 3\,[6000 + 3375] = 28125$ Choice (C)

15. A group of 4n distinct items can be divided equally

(i) among 4 boys in $\frac{(4n)!}{(n!)^4}$ ways

(ii) into 4 parcels in $\frac{(4n)!}{4!\,(n!)^4}$ ways

Hence 20 items can be divided equally

(i) among 4 boys in $\frac{20!}{(5!)^4}$ ways Choice (D)

(ii) into 4 parcels in $\frac{20!}{4!\,(5!)^4}$ ways Choice (B)

16. Let the number of persons in the group be n. As there is a handshake being exchanged between any two persons, there are ${}^nC_2$ distinct handshakes which are given to be 66

Hence ${}^nC_2 = 66$ i.e., $\frac{n(n-1)}{2} = 66$

$\Rightarrow$ n= 12.
Further, these 12 persons exchange greeting cards which are ${}^{12}P_2 = 132$ Ans: (132)

Note: It may be noted that in case of handshakes the order of the persons shaking hands does not play a role, so we consider Combinations. While, in case of greeting cards, since a card sent from A to B is different from that sent from B to A, we consider Permutations.

17. There are 12 persons in the group including Kapil. Now Kapil wants to invite one or more of his 11 friends for dinner. He can deal with each of his friends in two ways - either invite him or not. Hence he can deal with his 11 friends in 2^{11} ways, of which the case of not inviting any of the friends has to be ruled out. Hence Kapil can invite one or more of his friends in $2^{11} - 1 = 2047$ ways. Choice (D)

18. Neha can deal with 4 Kit Kats in 5 ways i.e., give either 0 or 1 or 2 or 3 or 4 (since Neha wants to give one or more chocolates, it is possible that she does not give a Kit Kat at all). Like wise she can deal with 5 Perks in 6 ways, 3 Milky Bars in 4 ways. Hence total ways are $5 \times 6 \times 4$ which include a possibility of not giving any of the chocolates which has to be ruled out.
Hence required ways are $5 \times 6 \times 4 - 1 = 119$
Choice (B)

19. Number of ways of choosing at least one green dye is $2^3 - 1$.
Number of ways of choosing at least one yellow dye is $2^2 - 1$.
Number of ways of choosing a red dye is 2^4.
$\therefore$ Required number of ways = $(2^3 - 1)(2^2 - 1)(2^4)$
= (7) (3) (16) = 48 (7) = 336 Choice (A)

20. **Case 1:** The number is a 6 digit number.
$\underline{1}$ _ _ _ _ _
First place fixed with 1 other places can be filled by three ones and two zeros

$\therefore$ Required number of numbers is $\dfrac{5!}{3!\,2!} = \dfrac{120}{12} = 10$

Case 2: The number is a seven digit number.
$\underline{1}$ _ _ _ _ _ _
First place fixed with 1. Other places can be filled by three ones and three zeroes.

$\therefore$ Required number of numbers = $\dfrac{6!}{3!\,3!} = \dfrac{720}{36} = 20$

$\therefore$ Total required numbers = 20 + 10 = 30 Ans: (30)

Solutions for question 21:

The word has 7 letters I, I, N, N, K, L, G. The following are the possibilities while selecting 4 letters.

Case (i) All 4 are distinct.
Case (ii) Two are alike and two are distinct.
Case (iii) Two are alike of one kind and two alike of the other.
Now the number of selections and arrangements in each case is given below.

	Combinations	Permutations
Case (i)	${}^5C_4 = 5$	$5 \times 4! = 120$
Case (ii)	${}^2C_1 \times {}^4C_2 = 12$	$12 \times \dfrac{4!}{2!} = 144$
Case (iii)	${}^2C_2 = 1$	$1 \times \dfrac{4!}{2!2!} = 6$

21. **(a)** Hence total Combinations are 18. Ans: (18)

(b) Total Permutations are 270. Ans: (270)

Solutions for questions 22 to 35:

22. The letters in alphabetical order are E, N, O, S, T. The words that begin with E, N, O, SE, SN, SO, STE, STN and STOE will preceede the word STONE
The number of words that begin with each of E, N, O are 4!
Number of words that begin with each of SE, SN, SO are 3!
While those that begin with STE, STN are each 2!
Finally there is 1 word that begins with STOE before we reach STONE.
Hence $3 \times 4! + 3 \times 3! + 2 \times 2! + 1 = 95$ words precede STONE. Hence the rank of STONE is 96. Choice (B)

23. The sum of all n-digit numbers that can be formed using n distinct positive digits is
$(n-1)! \times \underbrace{111\,.\,.\,1}_{\text{n times}}(\text{sum of all digits})$.

Hence the required sum is
$3! \times 1111 \times [2 + 4 + 6 + 8] = 133320$ Choice (A)

24. Using 'n' points of which 'm' are on a straight line and no other three points are on a straight line, we can form
(i) ${}^nC_3 - {}^mC_3$ triangles and
(ii) ${}^nC_2 - {}^mC_2 + 1$ straight lines
Hence we can form

(i) ${}^{12}C_3 - {}^4C_3 = 220 - 4 = 216$ triangles
(ii) ${}^{12}C_2 - {}^4C_2 + 1 = 66 - 6 + 1 = 61$ straight lines
Hence the difference between the triangles and the straight lines is 216 – 61 = 155 Ans: (155)

25. In a n sided convex polygon the number of points of intersection of diagonals inside the polygon is ${}^nC_4$.
Given n = 8
Required number of points of intersection = ${}^8C_4 = 70$.
Choice (C)

26. The number of derangements of n objects is

$D_n = n!\left[\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!} + \dots\dots(-1)^n\dfrac{1}{n!}\right]$

Out of seven letters any two letters are placed into its corresponding envelopes and remaining 5 letters, no letter is placed into its corresponding envelope.
i.e. ${}^7C_2 \times D_5$

$= 21\ (5!)\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!}\right) = 21 \times (44) = 924$

Ans: (924)

27. Let the prizes be P_1, P_2, P_3, P_4 and P_5. P_1 can be dealt in 3 ways i.e., it can be given away to any of the 3 boys as each boy is eligible for one or more prizes. P_2 and infact each of P_3, P_4, P_5 can be given away in 3 ways. Now using the fundamental theorem of counting, the 5 prizes can be given away in $3 \times 3 \times 3 \times 3 \times 3 = 3^5$ ways.
Choice (B)

28. We know that, the number of non negative integral solutions of the equation $x_1 + x_2 + \dots + x_r = n$ is ${}^{n+r-1}C_{r-1}$
$\therefore$ Here, n = 15, r = 4
$\therefore$ Required answer is ${}^{15+4-1}C_{4-1} = {}^{18}C_3 = 816$.
Choice (D)

29. We know that,
$\therefore$ The number of positive integral solutions of
$x_1\ x_2 + \text{------} + x_r = n$ is ${}^{n-1}C_{r-1}$

$\therefore$ Required answer is ${}^{19}C_2 = \dfrac{19 \times 18}{2} = 171$

Choice (D)

30. The number of ways that the man can invite at least n + 1 friends for a dinner is
${}^{(2n+1)}C_{n+1} + \dots + {}^{2n+1}C_{2n+1} = 4096$--------(1)
$\therefore$ ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n = 4096$ -----------(2)
(1) + (2) $\Rightarrow$
${}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_{2n-1} = 8192$
$\Rightarrow 2^{2n+1} = 8192$ ($\because$ ${}^nC_0 + {}^nC_1 + \dots\dots {}^nC_n = 2^n$)
$\Rightarrow 2^{2n+1} = 2^{13}$
$\Rightarrow 2n + 1 = 13$

$\therefore$ the number of friends = 13 Ans: (13)

31. A has 6 elements
The 6 elements have to be split into two groups.
The number of elements in the groups could be (1, 5), (2, 4) or (3, 3). The group can be formed in

$\frac{6!}{1!\,5!} + \frac{6!}{2!\,4!} + \frac{6!}{2!\,3!\,3!}$, ways

6 + 15 + 10 = 31 Choice (B)

32. There are 4 vowels and 3 consonants in the given word. From these 3 vowels and 2 consonants can be selected in $^4C_3\ ^3C_2$ = 12 ways and using these 5 letters (3 vowels, 2 consonants) we can form 5! different words.
$\therefore$ The number of required words = 12 (5!) = 1440
Choice (B)

33. First door can be painted with any of the four colors. Second door is painted with three colors. Similarly remaining doors also painted with three colors each.
$\therefore$ total possibilities = 4 × 3 × 3 × 3 × 3 × 3 = 972
Ans: (972)

34. We know that if there are m horizontal blocks n vertical blocks then the number of ways travelling from one corner to diagonal opposite corner is $^{m+n}C_n$
$\therefore$ here m = 6; n = 4.
$\therefore$ required possible ways
= $^{10}C_4$ = 210 Ans: (210)

35. We know that if there are n lines such that no two are parallel and no three are concurrent then the number of regions formed with these n lines is $\sum n + 1$.
here n = 10
required regions = $\sum 10 + 1$
= $\frac{10.11}{2}$ + 1 = 56 Choice (D)

Exercise – 9(b)

Solutions for questions 1 to 40:

1. Set A has 8 elements. We want the number of subsets with 6, 7 or 8 elements that contain c and e. Therefore we have to select 4 or 5 or 6 elements from remaining 6, elements. [c, e are excluded from 8 elements]
The number of subsets which contain exactly 6 elements = $^6C_4$ = 15.
The number of subsets which contain exactly 7 elements = $^6C_5$ = 6
The number of subsets which contain exactly 8 elements = $^6C_6$ = 1.
$\therefore$ Required number of subsets = 15 + 6 + 1 = 22
Ans: (22)

2. C A L E N D AR
1 2 3 4 5 6 7 8
There are 5 positions to fix the L and D i.e. (1, 4), (2, 5), (3, 6), (4, 7) and (5, 8) and L and D can be intercharged.
The remaining 6 letters can be arranged in $\frac{6\,!}{2\,!}$ ways.

$\therefore$ Required number of ways = $\frac{6\,!}{2\,!} \times 5 \times 2 = 360 \times 10$

= 3600. Choice (B)

3. In the given word, there are 4 vowels and 2 consonants.
$\therefore$ It is not possible to arrange the letters as required.
Choice (D)

4. The six-digit number may start with 15 or 26 and also it is an even number. We can have the following possible cases.

Case 1:
1 5 – – – 6
If the last digit is 6, then in the remaining 3 places, one place can be filled by 6 and the other two places can be filled in 9 (9) ways.
$\therefore$ Hence, the number of trials = 9 (9) (3) = 243

Case 2:
1 5 – – – –
If last position is filled by one of the digits 0, 2, 4, 8, then in the remaining 3 places, two of the place can be filled by 6 and third place can be filled by 9 ways.
$\therefore$ Hence, required number of trials = 4 (9) (3) = 108

Case 3:
2 6 _ _ _ 6
If the last position is filled by 6, then the remaining 3 positions can be filled in 9 (9) (9) ways.
$\therefore$ Hence, required number of trials = 9 × 9 × 9 = 729

Case 4:
0, 2, 4, 6, 8
2 6 6 $\downarrow^9$ $\downarrow^9$ $\downarrow$

If the last position is filled with 0, 2, 4, 8, then in remaining 3 positions one position has to be filled by 6 and the other two positions can be filled in 9 (9) ways.
$\therefore$ Hence, the required number of trials = 9 (9) (3) (4) = 972
$\therefore$ At the most, Raju has to make 243 + 108 + 729 + 972 or 2052 trials to succeed. Choice (B)

5. The first digit (thousands) can be selected in 4 ways.
The other 3 places can be filled in $^4P_3$ ways.
We can form a total of 4 $^4P_3$ four-digited numbers by using all the even digits. We have to add all these numbers. Let us look at the contribution of each of the digit.

3 (3!) numbers contain 2 in the units place
3 (3!) numbers contain 2 in the tens place
3 (3!) numbers contain 2 in the hundreds place
Similarly, we can work out the value contributed by the other digits. The digits and the total contribution of the digits is tabulated below.

Digit	Contribution
2	24 (2000) + 18 (222)
4	24 (4000) + 18 (444)
6	24 (6000) + 18 (666)
8	24 (8000) + 18 (888)

The total sum is 24 (20000) + 18 (20) (111)
= 480000 + 39960 = 519960 Ans: (519960)

6. The possible digits in the different places and the corresponding number of numbers are tabulated below. The units digit or the tens digit cannot be 3.

	Possible Digits				No. of Number	
A.	3	6	__	odd	9 (4) =	36
B.	3	__	6	odd	10 (4) =	40
C.	__	3	6	odd	9 (4) =	36
D	__	__	__	odd	8 (9) (9) (4) =	2592

						2704

The numbers 3361, 3365, 3367 and 3369 have been counted in B as well as C.
The numbers 3661, 3665, 3667, 3669 has been counted in A as well as B.
$\therefore$ The required number of numbers is 2696.
Choice (C)

7. The digits in the thousands place and the possible number of digits in the hundreds, tens and units places (enclosed in brackets) are tabulated below.

Th	H	T	U	
3	(5)	(7)	(7)	245
4	(7)	(7)	(7)	343
5	(7)	(7)	(7)	343
7	(3)	(7)	(7)	147

				1078

These 1078 numbers include 3200, but not 7300.
$\therefore$ The number of numbers between 3200 and 7300 is 1077. Choice (D)

8. **Case 1:**
The number of 2-digit and 3-digit numbers having only one 5 is 3 (9) (9) = 243

Case 2:
$\underline{5}\ \underline{5}\ \overline{\uparrow}_{9}$
The number of 2-digit and 3-digit numbers having exactly two 5's is 3 (9) = 27

Case 3:
$\underline{5}\ \underline{5}\ \underline{5}$
The number of numbers having exactly three 5s is 1.
$\therefore$ The total number of times 5 occurs, in all possible natural numbers less than 1000 is 243 + 2 (27) + 3(1) = 300
The number of times 5 occurs in between 9 to 1000 is 300 – 1 = 299 Ans: (299)

9. A 4×3 matrix has 12 elements. Each element can be 0 or 1 or 2.
The total number of matrices 0, 1, 2 as the elements is 3^{12}
Choice (C)

10. The number of black and green balls and the number of ways they can be arranged in the 5 bowls, so that no two adjacent bowls have green balls are tabulated below.

b (6)	g (5)	No. of arrangements	Position of g balls
5	0	1	
4	1	5	1, 2, 3, 4, 5
3	2	6	1, 3 or 1,4 or 1,5 or 2,4 or 2, 5 or 3, 5
2	3	1	1, 3, 5

$\therefore$ The total number of arrangements is 13. Choice (C)

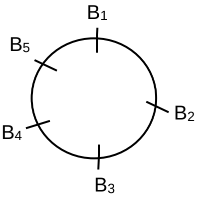
11. $\underline{\downarrow}^{9}\ \_\ \_\ \underline{\downarrow}^{10}$
As there is no restriction on the units place, this place can be filled by any of the 10 digits. The, thousands place can be filled by any of the 9 digits. (all except 0)
To fill the other two places, we have to select two distinct digits. This can be done in ${}^{10}C_2$ ways.
Required number of ways = ${}^{10}C_2$ (10) (9) = 4050
Ans: (4050)

12. One postcard can be dropped into 8 letter boxes, in 8 ways, 4 postcards can be dropped in 8^4 ways. Choice (C)

13. a + b + c + d = 20
Items can be divided into r parts in ${}^{n-1}C_{r-1}$ ways.
${}^{20-1}C_3$ or ${}^{19}C_3$ or 969 ways. Ans: (969)

14. We know that, if p things are alike of one kind, q things are alike of second kind and r things are alike of third kind, then one or more things can be selected in (p + 1) (q + 1) (r + 1) – 1 ways.
$\therefore$ Required number of ways = (4 + 1) (3 + 1) (2 + 1) – 1
= (5) (4) (3) – 1 = 59 Choice (B)

15. The 5 boys can be seated around a table in 4! ways. In between them, there are 5 places.
The 3 girls can be placed in the 5 places in ${}^5p_3$ ways.



$\therefore$ Required number of ways = 4! ${}^5P_3$ =24 $\times$ 60
= 1440 Choice (D)

16. In the given word, there are 2 Ms, 2 Ts, 2As and 5 single letters.
Taking the 2 Ms as one unit and 2 Ts as one unit, with remaining 7 letters can be arranged $\frac{9!}{2!}$ ways. (There are 2 As)
$\therefore$ Required number. of ways = $\frac{9!}{2!}$ Choice (D)

17. The total number of words that can be formed is $\frac{11!}{2!2!2!}$
Number of arrangements in which the 2 As are together = $\frac{10!}{2!2!}$
Total number of arrangements in which the As are separated = Total number of words – number of words, in which the two A's together.
= $\frac{10!}{2!2!}\left(\frac{11}{2}-1\right) = \frac{9(10!)}{2!2!2!}$ Choice (D)

18. The word is INSTITUTE

Letter	N	S	U	E	I	T
Number of times repeated	1	1	1	1	2	3

The distribution of the 5 letters, combination and permutations are tabulated below.

Distribution	Combinations	Permutations
1, 1, 1, 1, 1	${}^6C_5$ = 6	6 (5!) = 720
1, 1, 1, 2	2 (${}^5C_3$) = 20	$20\left(\frac{5!}{2!}\right)$ = 1200
1, 1, 3	${}^5C_2$ = 10	$10\left(\frac{5!}{3!}\right)$ = 200
1, 2, 2	${}^4C_1$ = 4	$4\left(\frac{5!}{2!2!}\right)$ = 120
2, 3	1	$\frac{5!}{2!3!}=10$
	41	2250

(i) 5 letters can be selected in 41 ways.
Choice (A)
(ii) The total number of arrangements that can be made is 2250. Choice (B)

19. Arranging the letters of the word 'AGAIN' in dictionary order is A, A, G, I, N.
The letters and the number of words are tabulated below.

Initial Letters	Number. of words
A	24
GAA	2
GAIAN	1
GAINA	1

$\therefore$ The 28th word GAINA Choice (B)

20. The initial digits and the number of numbers are tabulated below.

Initial Digits	Number of numbers
3	24
4	24
6	24
8	24

The 96th number is 89643
The 95th number is 89634 Ans: (89634)

21. The number of diagonals of a polygon of n sides is $\frac{n(n-3)}{2}$

$\frac{n(n-3)}{2} = \frac{5n}{2} \Rightarrow n = 8$ Choice (D)

22. (i) 12 pens can be distributed among 3 children and each one gets 4 pens.
$\therefore$ Hence, required number. of ways = ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4$
$= \frac{12!}{8!\,4!} \times \frac{8!}{4!\,4!} = \frac{12!}{(4!)^3}$ Choice (D)

(ii) 12 pens can be distributed among 3 parcels
$= \frac{12!}{(4!)^3\,3!}$ Choice (D)

23.

Section (1) (4 Qns)	Section (2) 4 (Qns)	required combinations
2 3	3 2	${}^4C_2\,{}^4C_3$ ${}^4C_3\,{}^4C_2$

Required number of ways = 2 (${}^4C_3$) (${}^4C_2$)
= 2 (4) (6) = 48 Choice (C)

24. We know that,
The number of triangles formed with 'n' non collinear points is ${}^nC_3$.
$\therefore$ Here, number of triangles = ${}^{15}C_3 - {}^4C_3 - {}^5C_3 - {}^6C_3$
= 455 – 4 – 10 – 20
= 455 – 34 = 421 Ans: (421)

25. For each book, 0 or 1 or 2 copies can be selected. Hence, the required number of ways = $3^8 - 1$
(At least 1 book has to be selected) Choice (B)

26. $\therefore$ Required number of ways = (total 4 digited telephone numbers) – (The number of 4 digited numbers without repetition)
= (9) (10^3) – 9 (${}^9P_3$) = 9 (1000 – (9) (8) (7))
= 9 (1000 –504) = 9 (496) = 4464 Ans: (4464)

27. $\therefore$ Required number of ways

$= n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+-----+(-1)^n\frac{1}{n!}\right)$

$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 5!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$

= 60 – 20 + 5 – 1 = 44 Choice (B)

28. Given that, the question paper consists of 5 problems. For each problem, one or two or three or none of the choices can be attempted.
$\therefore$ Hence, the required number of ways = $4^5 - 1$.
= $2^{10} - 1$= 1024 – 1 = 1023 Choice (C)

29. We know that, the number of straight lines that can be formed by the 'n' points in which r points are collinear and no other set of three points, except those that can be selected out of these r points are collinear) is ${}^nC_2 - {}^rC_2 + 1$.
$\therefore$ Hence, the required number of straight lines
= ${}^{11}C_2 - {}^6C_2 - {}^5C_2 + 1 + 1$
= 55 – 15 – 10 + 2 = 32 Choice (D)

30. Not younger player

_ _ _ _ _ _ $\underline{\uparrow}$

The last ball can be thrown by any of the remaining 6 players. The first 6 players can throw the ball in ${}^6p_6$ ways.
$\therefore$ The required number of ways = 6 (6!) = 4320
Choice (C)

31. After arranging 3 and 4 particular guests, the remaining number of people is 9.
To arrange on first table we require 5 members. They can be selected in ${}^9C_5$ ways.
To arrange on the second table, we require 4 members. They can be selected in ${}^4C_4$ ways.
$\therefore$ Hence, required arrangements is = ${}^9C_5$ (7!) (7!)
= ${}^9C_5$ $(7!)^2$ Choice (C)

32. In an 8 $\times$ 8 chess board there are 8 rows and 8 columns. In every row and column there are four white squares each. Number of ways of selecting two white squares which are in same row or column = 8 $\times$ ${}^4C_2$ + 8 $\times$ ${}^4C_2$ = 8 [6 + 6] = 96
Ans: (96)

33. We know that, the number of non negative integral solutions of $a_1 + a_2 + a_3$+-------+ a_r = n is ${}^{n+r-1}C_{r-1}$ here n = 14 r = 3
$\therefore$ Required answer is (14 + 3 – 1) C_{3-1} = ${}^{16}C_2$ = 120
Choice (C)

34. The word RESULT has 2 vowels while there are 3 even places (e) available.

__ $\underline{e}$ __ $\underline{e}$ __ $\underline{e}$

The two vowels can be arranged in any of the 3 even places in ${}^3P_2$ = 3 $\times$ 2 = 6 ways. Having taken care of the two of the even places, now there are 4 places available and 4 letters left and these letters can be arranged in 4! = 24 ways.
Hence, required number of arrangements is 6 $\times$ 24
= 144. Ans: (144)

35. Let the letters be L_1, L_2, L_3, L_4 and L_5 and the mail boxes be B_1, B_2, B_3, B_4. Now L_1 can be dealt in 5 ways i.e., either post it into B_1 or B_2 or B_3 or B_4 or do not post it at all (since one or more letters have to be posted, there is a possibility of not posting L_1 at all). Similarly each of L_2, L_3, L_4 and L_5 can be dealt in 5 ways, giving us a total of 5^5 possibilities which includes the case of not posting any of the letters, which has to be ruled out. Hence the required ways are 5^5 –1
Choice (C)

36. Let the number of persons be n.
Given ${}^nC_8 = {}^nC_{12}$
$\Rightarrow$ n = 20.
Now ${}^nC_{18} = {}^{20}C_{18} = {}^{20}C_2 = \frac{20\times19}{2}$ = 190
Ans: (190)

37. n distinct items can be arranged in a circle in $\frac{(n-1)!}{2}$ ways, if there is no difference between the clockwise arrangements and anti-clockwise direction. Hence the 12 beads can be arranged in a necklace in $\frac{11!}{2}$ ways Choice (D)

38. The word INCLUDE has 7 letters, of which 3 are vowels. As no two vowels are together, we need to have a consonant present in between any two vowels which act as separator for the vowels. As there is no condition on consonants, we first arrange them and this can be done in 4! ways. Now there are 5 possible positions for the 3 vowels as indicated below
VCVCVCVCV
Hence the vowels can be arranged in ${}^5P_3$ ways.
$\therefore$The required number of words is $4!\times{}^5P_3 = \frac{4!\times5!}{2!}$
Choice (D)

39. The number of ways person travel from A to B is ${}^4C_3$ i.e 35
The number of ways a person can travel from B to c is ${}^5C_2$, i.e 10.
$\therefore$ The number of ways a person can travel from A to C via B is 350 Ans: (350)

40. We need to divide a group of 50 into groups of 25, 15 and 10 and this can be done in $\frac{50!}{25!15!10!}$. Choice (B)

Note: A group of (p + q + r) items can be divided into groups of p, q, r items in $\frac{(p+q+r)!}{p!q!r!}$

Chapter – 10
(Probability)

Concept Review Questions

Solutions for questions 1 to 40:

1. **(a)** The probability of any event is always between zero and one (both inclusive). Choice (A)

(b) The probability of an impossible event is zero. Choice (B)

(c) The probability of a sure or a certain event is equal to one. Choice (A)

2. **(a)** $P(\overline{E}) = 1 - P(E) = 1 - 0.2 = 0.8$ Ans: (0.8)

(b) $P(E) + P(\overline{E}) = 1$ Choice (B)

3. If two events E_1 and E_2 are mutually exclusive, then $P(E_1 \cap E_2) = 0$ Choice (D)

4. **(a)** When a coin is tossed for n times, the total number of possible outcomes is 2^n Choice (C)

(b) When a coin is tossed n times the number of ways in which heads appears is ${}^nC_r$
Here n = 3, r = 2
i.e., ${}^3C_2 = 3$ ways Choice (B)

5. Let E be the event of heads occurring at least once and $\overline{E}$ be the event of heads not occurring at all i.e., all tails.
$P(E) = 1 - P(\overline{E})$
$= 1 - \frac{1}{2^{10}} = \frac{2^{10}-1}{2^{10}}$ Choice (D)

6. **(a)** When n coins are tossed, the probability of getting 'r' heads is $\frac{{}^nC_r}{2^n}$.
Probability of getting at least 4 tails = P(4 tails) + P (5 tails) + P (6 tails) = $\frac{{}^6C_4}{2^6} + \frac{{}^6C_5}{2^6} + \frac{{}^6C_6}{2^6}$
$= \frac{15+6+1}{64} = \frac{22}{64} = \frac{11}{32}$. Choice (D)

(b) When n coins are tossed, the probability of getting 'r' heads is $\frac{{}^nCr}{2^n}$.
The probability of getting no tail when 4 coins are tossed, is
$= \frac{{}^4C_0}{2^4} = \frac{1}{16}$. Choice (A)

7. When a dice is rolled n times, the total number of possible outcomes is 6^n
Here $n = 3 \Rightarrow 6^3$ Ans: (216)

8. When two dice are rolled, the ways in which the sum of the numbers is a prime number are:

1,1	2, 3	3, 4	5, 6
1, 2			
1,4	2, 5		
1, 6			

Each of the above outcomes can be reversed. (1, 1) doesn't give a different outcome. There are 15 outcomes in which the sum is prime. Ans: (15)

9. The outcomes in which the sum of the numbers that turn the 3 occasions is 17 or 18 are shown below.
5, 6, 6 ; 6, 5, 6 ; 6, 6, 5 and 6, 6, 6
There are 4 such outcomes. Ans: (4)

10. If two dice are rolled, the total number of possibilities is n(s) = 36.
The possible cases for the sum to be 9 are
{(3, 6), (4, 5), (6, 3), (5, 4)}. i.e. 4.
$\therefore$ Required probability = $\frac{4}{36} = \frac{1}{9}$ Choice (D)

11. When three dice are rolled, the total number of possibilities are $6^3 = 216$

the possible cases for the sum to be 10 in given in the following table

arrangement	Possibility	
(1, 3, 6)	3!	6
(1, 4, 5)	3!	6
(2, 2, 6)	$\frac{3!}{2!}$	3
(2, 3, 5)	3!	6
(2, 4, 4)	$\frac{3!}{2!}$	3
(3, 4, 3)	$\frac{3!}{2!}$	3
	Total	27

favourable cases = 27
$\therefore$ The probability that the sum 10 = $\frac{27}{216} = \frac{1}{8}$
Choice (C)

12. The number of possibilities of getting a composite number when a dice is rolled is 2.
The probability of getting a composite number when one dice is rolled is = $\frac{2}{6}$.
The probability of getting a composite number when three dice are rolled in all the three dice is $\frac{2}{6} \times \frac{2}{6} \times \frac{2}{6} = \frac{1}{27}$.
Choice (B)

13. If three dice are rolled, the total number of possibilities are $6^3 = 216$
P(at least one 6 is obtained) = 1 – P(no six is obtained on any face of the dice) = $1 - \frac{5\times5\times5}{6^3} = 1 - \frac{125}{216} = \frac{91}{216}$.
Choice (B)

14. There are four kings in a pack, one of them can be drawn in ${}^4C_1 = 4$ ways. Ans: (4)

15. In a pack of cards Ace, King, Queen and Jack are called honours. There are 16 honours in a pack, ie, an honour can be drawn in ${}^{16}C_1$ ways i.e., n(E) = 16, One card can be drawn from a pack in ${}^{52}C_1$ ways.
Required probability = $\frac{16}{52} = \frac{4}{13}$ Choice (A)

16. A card can be drawn from a pack of cards in $^{52}C_1$ ways i.e., n(S) = 52. There are 36 numbered cards, of which one card can be drawn in $^{36}C_1$ ways, i.e., n(E) = 36

Required probability $= \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$ Choice (D)

17. Let E_1 be the event of drawing a spade and E_2 be the event of drawing an Honour.
$n(E_1) = {}^{13}C_1$ and $n(E_2) = {}^{16}C_1$
by addition theorem of probability we have
$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{^{13}C_1}{^{52}C_1} + \frac{^{16}C_1}{^{52}C_1} - \frac{^{4}C_1}{^{52}C_1} = \frac{25}{52}$ Choice (A)

18. Three cards can be selected from 52 cards in $^{52}C_3$ ways.
3 queens can be selected from 4 queens in $^{4}C_3$ ways.

$\therefore$ required probability $= \frac{^{4}C_3}{^{52}C_3} = \frac{4\times3\times2}{52\times51\times50} = \frac{1}{5525}$

Ans: (5525)

19. Two cards can be selected from 52 cards in $^{52}C_2$ ways.

Probability of drawing queen cards is $\frac{^{4}C_2}{^{52}C_2}$.

Probability of drawing diamond cards is $\frac{^{13}C_2}{^{52}C_2}$.

$\therefore$ probability that both are queens or both are diamonds

$= \frac{^{4}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} = \frac{4\times3}{52\times51} + \frac{13\times12}{52\times51} = \frac{12\times14}{52\times51} = \frac{14}{221}$

Choice (B)

20. For two mutually exclusive events E_1 and E_2
$P(E_1 \cup E_2) = P(E_1) + P(E_2)$
= 0.75 + 0.15 = 0.9 Ans: (0.9)

21. There are 25 natural number in which 9 are primes.

$\therefore$ The required probability $= \frac{9}{25}$. Ans: (0.36)

22. There are 56 natural numbers present in the given set {56, 55, 54, -------, 1}
The set of numbers which are multiples of 9 from the given set is {54, 45, 36, 27, 18, 9} i.e. = 6

$\therefore$ The required probability $= \frac{6}{56} = \frac{3}{28}$. Choice (B)

23. In the given set B, there are 8 elements of which 3, 9, 15 are the multiples of.3.
Total out comes = 8
Favourable outcomes = 3

$\therefore$ Required probability $= \frac{3}{8}$. Choice (D)

24. Two distinct number can be selected from the set {1, 3, 6, 8, 9, 10} in $^{6}C_2$ ways i.e., 15 ways
When one number is odd and the other is even then the sum is odd.

The number of ways of selecting one odd number and one even number is $^{3}C_1 \cdot {}^{3}C_1 = 9$.

$\therefore$ Required probability $= \frac{9}{15} = \frac{3}{5}$ Ans: (0.6)

25. Twenty boys can be arranged in a row in 20! ways. n(S) = 20!
Let E be the event that two boys are always sit together.
Treat the two boys as one unit.
Now there are 19 boys and they can be arranged in 19! ways. Again the two boys can be arranged among them selves in 2! ways.
Hence n(E) = 19! 2!

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{19!\,2!}{20!} = \frac{19!\,2!}{20.19!} = \frac{1}{10}$. Ans: (0.1)

26. Given set is {1, 3, 5, ……… 47}
Total numbers in the given set are 24. n(S) = 24.
The values that satisfies the equation (x – 2) (x – 5) (x – 7) (x – 47) = 0 are 2, 5, 7, 47
favourable values of x taken from the given set are 5, 7, 47

Required probability $= \frac{3}{24} = \frac{1}{8}$. Choice (A)

27. Given that, A and B are independent events,
$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$
0.71 = P(A) + 0.19 – (0.19) P(A)
0.71 – 0.19 = P(A) (1 – 0.19)
0.52 = P(A) (0.81)

$P(A) = \frac{52}{81}$ Choice (B)

28. If A, B are independent events then

$P(A \cap B) = P(A)\,P(B) = \frac{1}{3}\left(\frac{3}{5}\right) = \frac{1}{5}$ Choice (B)

29. Given odds against an event E are 3 : 4

$\Rightarrow P(\overline{E}) : P(E) = 3 : 4$

$\Rightarrow P(E) = \frac{4}{7}$ Choice (B)

30. Given A and B are mutually exclusive and exhaustive events
$\Rightarrow$ P(A) + P(B) = 1 . . . (1)
Also odds in favour of A are 2 : 3

$\Rightarrow P(A) = \frac{2}{5}$

From (1), P(B) = 1 – P(A)

$= 1 - \frac{2}{5} = \frac{3}{5}$ Ans: (0.6)

31. Given $P(A) = \frac{2}{3}$, $P(B) = \frac{5}{7}$ and $P(C) = \frac{4}{5}$

Probability of problem being solved
$P(A \cup B \cup C) = 1 - P(A \cap B \cap C)$
=1 – P(A). P(B) P(C)

$= 1\ P(\overline{A})\,P(\overline{B})\,P(\overline{C})$

$= 1 - \frac{1}{3}\left(\frac{2}{7}\right)\left(\frac{1}{5}\right) = 1 - \frac{2}{105}$

$P(A \cup B \cup C) = \frac{103}{105}$ Choice (C)

32. Since A and B are equally likely events P(A) = P(B)

$\therefore P(\overline{A}) = 1 - P(A) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

Choice (B)

33. The basket contains 6 good and 9 rotten fruits.

P(drawing 3 good fruits) $= \frac{^{6}C_3}{^{10}C_3}$ Choice (B)

34. Expected value $= \Sigma X_i P(X_i)$

$= 0\left(\frac{1}{17}\right) + 1\left(\frac{5}{17}\right) + 3\left(\frac{4}{17}\right) + 5\left(\frac{4}{17}\right) + 6\left(\frac{3}{17}\right)$

$= \frac{55}{17} = 3\frac{4}{17}$ Choice (B)

35. Expected value $= \Sigma P(X_i) \times X_i$

Outcome	2, 4, 6	1, 3, 5
Amount	50	–30

p(getting even number) $= \frac{3}{6} = \frac{1}{2}$

p(getting an odd number) $= \frac{1}{2}$

Expected amount (in ₹)= $\frac{1}{2}(50) - \frac{1}{2}(30)$ = 10

In a long run he will get per turn of ₹10 Ans: (10)

Exercise – 10(a)

Solutions for questions 1 to 31:

1. Total books – 10
Biographies – 6,
Autobiographies – 4

(i) Probability of one being a biography and the other an autobiography is

$$\frac{{}^6C_1 \times {}^4C_1}{{}^{10}C_2} = \frac{6 \times 4}{45} = \frac{8}{15} \quad \left[{}^{10}C_2 = \frac{10 \times 9}{2} = 45\right]$$

Choice (A)

(ii) Probability of both being autobiographies

$= \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45} = \frac{2}{15}$. Choice (B)

2. The cards picked up should contain the letters **I**, **I** and **M** in that order.
As there are 7 cards bearing **I** and 3 bearing **M** and the cards picked up are not being replaced, the required probability is $\frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} = \frac{7}{40}$ Ans: (0.175)

3. Probability of getting head at least once
= 1 – probability of not getting a head at all
= 1 – probability of getting a tail in each toss
= 1 – $(1/2)^9$ = 1 – 1/512 = 511/512 Choice (B)

4. The number of tosses may be 2 or 3 or 4.
The possible cases and their corresponding probabilities:

Case 1 : HH OR TT → $2(1/2)^2$
Case 2 : HTT OR THH → $2(1/2)^3$
Case 3 : HTHH OR THTT → $2(1/2)^4$

Hence, the required probability is
2[1/4 + 1/8 + 1/16] = 7/8 Ans: (0.875)

5. We have 4 five rupee coins, 3 two rupee coins and 3 one rupee coins.
For the draw to yield a maximum amount, of the 6 coins drawn 4 should be five rupee coins and 2 should be two rupee coins. The required probability is

$$\frac{{}^4C_4 \times {}^3C_2}{{}^{10}C_6} = \frac{3}{210} = \frac{1}{70}$$

Hence, odds in favour are
favourable ways : unfavourable ways = 1 : 69.
Choice (B)

6. Considering different values of a, b and c from the set {1, 2, 3, 4, 6, 8, 9}, we get different quadratic equations.
As a, b and c are distinct, ${}^7P_3$ = 210 different quadratic equations can be formed.
∴ Total ways are 210
For the quadratic equation $ax^2 + bx + c = 0$ to have equal roots, $b^2 = 4ac$.
The possible combinations of a, b and c respectively are 1, 6, 9 and 9, 6, 1.
Hence favourable cases are 2
∴ Required probability = 2/210 = 1/105 Choice (C)

7. The sum has to be less than 7.
The possibilities are (1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)
Hence, the favourable cases are 15 while total cases are 6 × 6 = 36
Hence, the required probability is 15/36 = 5/12
Choice (D)

8. Probability of throwing 6 at least once = 1 – probability of throwing a number other than 6 in each trial.
= 1 – (5/6 × 5/6 × 5/6) = 1 – 125/216 = 91/216
Hence, the odds against the event are
unfavourable ways: favourable ways = 125 : 91
Choice (C)

9. Total number of cases are 6^4 = 1296
The various combinations for the sum to be 20 and 21 and the corresponding number of arrangements in each case are

Sum = 20	**Sum = 21**
6, 6, 6, 2 → 4!/3! = 4	6, 6, 6, 3 → 4!/3! = 4
6, 6, 5, 3 → 4!/2! = 12	6, 6, 5, 4 → 4!/2! = 12
6, 6, 4, 4 → 4!/2!2! = 6	6, 5, 5, 5 → 4!/3! = 4
6, 5, 5, 4 → 4!/2! = 12	
5, 5, 5, 5 → 4!/4! = 1	

favourable cases for the sum to be 20 are 35 and for the sum to be 21 are 20.
∴ p = $35/6^4$ and q = $20/6^4$
∴ p : q = 35 : 20 = 7 : 4 Choice (D)

10. The cube has four faces blank and 2 faces numbered.
P(A has a success) = P(A throws a numbered face at least once)
= 1 – P(A throws a blank face in each trial)
= 1 – $(4/6)^3$ = 1 – 8/27 = 19/27
P(B has a success)
= P(B throws a numbered face) = 2/6 = 1/3
∴ The ratio of A's chance of winning to that of B is 19/27 : 1/3 = 19 : 9 Choice (A)

11. There are 5 boxes and 5 labels. Hence the boxes can be labelled in 5! i.e. 120 different ways

(i) There is exactly one way in which all the boxes are labelled correctly (i.e., with their corresponding colours)
∴ The required probability is 1/5! = 1/120
Ans: (1)

(ii) P(at least one box is labelled incorrectly)
= 1 – P(none labelled incorrectly)
= 1 – P(all labelled correctly)
= 1 – 1/5! = 1 – 1/120 = 119/120 Choice (D)

(iii) There is no way of exactly one box being labelled incorrectly. For example if the box of yellow balls has been labelled red, then the box of red balls would also have been incorrectly labelled. Hence the required probability is 0. Choice (B)

(iv) P (all labelled incorrectly) = 1/2! – 1/3! + 1/4! – 1/5!
= 44/120 = 11/30 Ans: (11)

12. There are 4 aces, 4 kings, and 4 jacks in a pack of 52 cards. The probability that the first card is an ace = 4/52. Since, the card drawn is not replaced, the probability that the second card is a king is 4/51 and similarly, the probability of the third card being a jack is 4/50.
∴ The required probability is 4/52 × 4/51 × 4/50
= 8/16575 Choice (D)

13. There are 36 numbered cards and 16 honours.

(i) P(both are numbered cards or both honours)

$$= \frac{{}^{36}C_2 + {}^{16}C_2}{{}^{52}C_2} = \frac{630 + 120}{1326} = \frac{750}{1326} = \frac{125}{221}$$

Choice (A)

(ii) There are exactly 4 cards which are common to the 16 honours and 13 hearts.
∴ P(both are red cards or both honours)

$$= \frac{{}^{16}C_2 + {}^{13}C_2 - {}^4C_2}{{}^{52}C_2} = \frac{120 + 78 - 6}{1326} = \frac{32}{221}$$

Choice (D)

14. There are 9 numbered cards in each suit.

(i) P(all the 4 cards are numbered cards of same suit)

$$= \frac{{}^{9}C_4 + {}^{9}C_4 + {}^{9}C_4 + {}^{9}C_4}{{}^{52}C_4} = \frac{4 \times {}^{9}C_4}{{}^{52}C_4}$$

Choice (A)

(ii) P(all the 4 cards are numbered cards belonging to different suits)

$$= \frac{{}^{9}C_1 \times {}^{9}C_1 \times {}^{9}C_1 \times {}^{9}C_1}{{}^{52}C_4} = \frac{({}^{9}C_1)^4}{{}^{52}C_4}$$ Choice (D)

15. The bag contains 6 blue and 8 yellow balls.
Let the probability that the balls drawn in succession are both yellow be P(YY)

(i) when the first ball is replaced
P(YY) = $8/14 \times 8/14 = 16/49$ Choice (B)

(ii) When the first ball is not replaced
P(YY) = $8/14 \times 7/13 = 4/13$ Choice (B)

16. Let the bags be B_1 and B_2. B_1 contains 6 red and 4 white balls while B_2 contains 5 red and 5 white balls. The possibility is that either of B_1 or B_2 is selected with a probability of 1/2 in each case. Having selected a bag, two balls of different colours have to be selected.

The probability is $\frac{1}{2} \times \frac{{}^{4}C_2}{{}^{10}C_2} + \frac{1}{2} \times \frac{{}^{5}C_2}{{}^{10}C_2}$

$$= \frac{1}{2}\left[\frac{6+10}{45}\right] = \frac{8}{45}$$ Choice (B)

17. The bag contains 2 Pears, 3 Peaches, and 4 Figs.

(i) Since all the three fruits should be of same variety, they have to be all Peaches or all Figs.

Required probability = $\frac{{}^{3}C_3 + {}^{4}C_3}{{}^{9}C_3} = \frac{1+4}{84} = \frac{5}{84}$

Choice (A)

(ii) Two of them have to be of same variety. The possibilities are 2 Pears and the other any one of the remaining 7 fruits or 2 Peaches and the other any one of the remaining 6 fruits (or) 2 Figs and the other any one of the remaining 5 fruits.

$\therefore$ Required probability

$$= \frac{{}^{2}C_2 \times {}^{7}C_1 + {}^{3}C_2 \times {}^{6}C_1 + {}^{4}C_2 \times {}^{5}C_1}{{}^{9}C_3} = 55/84$$

Choice (B)

(iii) As the three fruits should be of different variety, we must have one each of Pear, Peach and Fig.

The required probability = $\frac{{}^{2}C_1 \times {}^{3}C_1 \times {}^{4}C_1}{{}^{9}C_3}$

$$= \frac{2 \times 3 \times 4}{84} = \frac{2}{7}$$ Choice (C)

18. Let E_1 be the event of Saurabh getting selected and E_2 be the event of Sweth getting selected.
Odds in favour of E_1 are 5 to 7 while odds against E_2 are 4 to 3.
$\therefore$ P(E_1) = 5/12 and P(E_2) = 3/7

(i) P(at least one of E_1, E_2 occurs)
= 1– P(none of E_1, E_2 occurs)
= $1 - 7/12 \times 4/7 = 2/3$ Choice (A)

(ii) P(exactly one of E_1, E_2 occur)
= P($E_1\ \overline{E_2}$ or $\overline{E_1}\ E_2$)

($\overline{E}$ denotes non-happening of E)

= P(E_1) P($\overline{E_2}$) + P($\overline{E_1}$) P(E_2)
(events being independent)
= $5/12 \times 4/7 + 7/12 \times 3/7 = 41/84$ Choice (A)

19. In a 8×8 chess board there are 1×1, 2×2, 3×3, ……, 8×8 squares.
In any given row, 1×1 squares are 8 and the same number in any given column. Hence, there are a total of 8^2 of them on the chess board. 2×2 squares are 7 in a row and 7 in a column. Hence, 7^2 of them. Similarly 3×3 squares are 6^2, 4×4 squares are 5^2….. and 7×7 squares are 2^2 and finally 8×8 is 1^2
Hence, the total number of squares on a chess board is $1^2 + 2^2 + 3^2 + …. + 8^2 = 204$.
Hence, the probability of a square selected at random to be a 3×3 square is $6^2/204 = 3/17$ Choice (B)

20. The probability of picking up an orange ball is 3/10 while not picking up an orange ball is 7/10.
We compute the probability of Arpit (the beginner) winning the game.
Let A and B be the events of Arpit and Bipin picking up an orange ball respectively
The winning sequence of Arpit can be

A, $\overline{A}\ \overline{B}\ A$, $\overline{A}\ \overline{B}\ \overline{A}\ \overline{B}\ A$, ...

As the above sequence indicates, Arpit may pick an orange ball right in the 1st trial with a probability of 3/10 (or) in the third trial (as the 2nd trial is made by Bipin, and for Arpit to win, Bipin should not be getting an orange ball). The probability here being $(7/10)^2 \times 3/10$ (or) in the fifth trial with a probability of $(7/10)^4 \times 3/10$ and so on.

$$\therefore P(A) = \frac{3}{10} + \left(\frac{7}{10}\right)^2 \times \frac{3}{10} + \left(\frac{7}{10}\right)^4 \times \frac{3}{10} + ...$$

$$= \frac{3/10}{1-\left(\frac{7}{10}\right)^2} = \frac{30}{51} = \frac{10}{17}$$

Probability of Bipin winning is the same as probability of Arpit losing i.e.,

$\therefore$ P(B) = P($\overline{A}$) = 1 – 10/17 = 7/17 Choice (B)

Note: If 'p' is the probability of success (in this case picking up an orange ball), the probability that the beginner wins the game = $\frac{1}{2-p}$

21. Consider the die D_1 on which 6 appears twice as often as any other number.

Out comes	1	2	3	4	5	6
Probability	x	x	x	X	x	2x

As the outcomes are mutually exclusive and collectively exhaustive, we have x + x + x + x + x + 2x = 1 i.e., x = 1/7
$\therefore$ P(6 appears) = 2/7
P(other than 6 appears) = 1/7
Consider the die D_2 on which an odd number appears thrice as frequent as an even number.

Out comes	1	2	3	4	5	6
Probability	3y	y	3y	y	3y	Y

Here 12y = 1 i.e., y = 1/12
P(an even number appears) = 1/12.

P(an odd number appears) = 3/12.
For the sum to be 11, the possibilities are 5 on D_1 and 6 on D_2 or 6 on D_1 and 5 on D_2 while for 12, it has to be 6 on D_1 and 6 on D_2.
Hence the required probability is
$(1/7 \times 1/12) + (2/7 \times 3/12) + (2/7 \times 1/12)$
= $9/7 \times 12 = 3/28$ Choice (C)

22. The probability of Hrithik drawing a red king (p_1) is 2/52, black honour (p_2) is 8/52 and card other than the above cards (p_3) is 42/52.

The expected value (E.V) is $p_1M_1 + p_2M_2 + p_3M_3$ where M_1, M_2, M_3 are the corresponding monetary values associated with p_1, p_2 and p_3

$\therefore$ E.V. = $2/52 \times 39 + 8/52 \times 26 - 42/52 \times 13$
$= 1.50 + 4 - 10.50 = -5.00$

$\therefore$ In the long run, Hrithik incurs an average loss of ₹5 per draw. Choice (B)

23. On the first coin P(H) = 2P(T)
As P(H) + P(T) = 1,
We have 2P(T) + P(T) = 1
i.e., P(T) = 1/3; P(H) = 2/3
On the second coin, P(T) = 3/2 P(H)
$\therefore$ P(H) + 2/3 P(H) = 1
$\Rightarrow$ P(H) = 2/5 and P(T) = 3/5

	P(H)	P(T)
1st Coin	2/3	1/3
2nd Coin	2/5	3/5

Let E_1 be the event of Shreya getting the same face value on both the dice and E_2 be event of E_1 not happening.
Now, $P(E_1)$ = P(HH or TT) = P(H) P(H) + P(T) P(T)
$= 2/3 \times 2/5 + 1/3 \times 3/5 = 7/15$
$P(E_2) = 1 - P(E_1) = 8/15$
E.V. = $7/15 \times 35 + 8/15 \times 25$ = ₹29.66
For Shreya to make an average profit of ₹15, she would be willing to pay ₹14.66 as an entry fee. Ans: (44)

24. Number of multiples of 7 between 201 and 300 is 14.
Number of multiples of 13 between 201 and 300 is 8.
Number of multiples of both 7 and 13 i.e., 91 between 201 and 300 is 1.
For every multiple of 7 (except that of 91), Kiran wins ₹7,000. Thus he wins ₹7000 in 13 cases.
For every multiple of 13 (except that of 91) Kiran wins ₹13,000. Thus he wins ₹13,000 in 7 cases, Kiran wins ₹91,000 in 1 case.
Hence the expected value is
$(13/100 \times 7000) + (7/100 \times 13000) + (1/100 \times 91000) = 2730$
As Kiran pays a participation fee of ₹2700, he makes a profit of ₹30 per game on an average. Ans: (30)

25. Total bulbs are 20 of which fused bulbs are 5.
P(room is lighted)
= P(at least one good bulb is selected)
= 1 – P(no good bulb is selected)
= 1 – P(all bulbs chosen are bad)

$= 1-\frac{{}^5C_3}{{}^{20}C_3} = 1-\frac{10}{1140} = \frac{113}{114}$ Choice (B)

26. Probability that either A or B occurs is $P(A \cup B)$
From addition theorem in probability,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(i) When A and B are independent,
$P(A \cap B) = P(A) \cdot P(B)$
$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) . P(B)$
$= 0.6 + 0.25 - 0.6 \times 0.25 = 0.7$ Ans: (0.7)
(ii) When A and B are mutually exclusive,
$P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$
$= 0.6 + 0.25 = 0.85$ Ans: (0.85)

27. Let us look at the number of four-digit even numbers that can be formed using 0, 2, 5, 7.
Since all such possible numbers are being considered, repetition of digits is allowed.

__ __ __ 0   __ __ __ 2
↓ ↓ ↓ fix   ↓ ↓ ↓ fix
$3 \times 4 \times 4$ $3 \times 4 \times 4$

Hence a total of $2(3 \times 4 \times 4) = 96$ four digit even numbers can be formed. Of these 96 even numbers, all those which end in 0 are divisible by 5 which are 48 in number.
Hence the required probability is 48/96 = 1/2
Note: When repetition is allowed, the number of numbers ending in 0, will be the same as ending in any non-zero number. In the above situation if numbers ending in 2 are x, then ending in 0 are also x. Hence required probability is x/2x = 1/2.
Actual calculations can be avoided in such problems.
Ans: (0.5)

28. From 1 to 25 there are 12 even numbers. There is only one favorable case that is 12.
$\therefore$ The probability that the number is 12 is $\frac{1}{12}$
Choice (C)

29. Probability of selecting urn A is $P(A)=\frac{1}{2}$.
and that of selecting urn B is $P(B) = \frac{1}{2}$
Probability of drawing a black ball (event E) when urn A is selected $P\left(\frac{E}{A}\right)=\frac{{}^7C_1}{{}^{13}C_1}$ and probability of E when urn B is selected $P\left(\frac{E}{B}\right) = \frac{{}^6C_1}{{}^{14}C_1}$
Probability of selecting black ball
$= P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right)$

$\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}+\frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}$

Required Probability = $\dfrac{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}}{\frac{1}{2}.\frac{{}^7C_1}{{}^{13}C_1}+\frac{1}{2}.\frac{{}^6C_1}{{}^{14}C_1}}$

$= \dfrac{\frac{7}{13}}{\frac{7}{13}+\frac{6}{14}} = \dfrac{\frac{7}{13}}{\frac{98+75}{13\times14}} = \frac{7\times14}{176} = \frac{49}{88}$ Choice (B)

30. Given that, the group contains 3 boys and 4 girls out of which 4 members are to be selected.
$n(s) = {}^7C_4 = 35$.
The team contains two girls or 3 girls or 4 girls.
$\therefore$ Required number of ways of forming the team
$= {}^4C_2.{}^3C_2 + {}^4C_3 \times {}^3C_1 + {}^4C_4 \times {}^3C_0$.
$= 6 \times 3 + 4 \times 3 + 1 = 18 + 13 = 31$.
$\therefore$ The required probability = $\frac{31}{35}$ Choice (C)

31. The data is tabulated below

	Route 1	Route 2
Prob	0.6	0.4
Time	20 min	30 min

$\therefore$Expected time = $\frac{0.6(20)+0.4(30)}{0.6+0.4}$ min
= (12 + 12) min = 24 min Ans: (24)

Solutions for questions 32 to 35:

Given that, there are 11 fruits in which 4 fruits are chosen.
$\therefore n(S) = {}^{11}C_4$.

32. There are only 3 fruits that are spoiled. But, we have to draw 4 spoiled fruits which is not possible.
$\therefore$ Required probability = '0' Choice (A)

33. There are 8 good fruits and 3 spoiled fruits of which 2 good fruits and 2 spoiled fruits are selected.
It can be done in ${}^8C_2.{}^3C_2$.ways.
$\therefore$ The required probability = $\frac{{}^8C_2 \times {}^3C_2}{{}^{11}C_4} = \frac{14}{55}$
Choice (D)

34. There are 8 good fruits and 3 spoiled fruits of which one good and 3 spoiled fruits are selected.
It can be done in ${}^8C_1 . {}^3C_3$.ways.

$\therefore$ The required probability = $\dfrac{{}^8C_1 \times {}^3C_3}{{}^{11}C_4}$

$= \dfrac{8\times1}{\dfrac{11\times10\times9\times8}{24}} = \dfrac{4}{165}$ Choice (B)

35. P(at least one fruit is good) = 1 – P(no fruit is good)
= 1 – 0 = 1. Choice (B)

Exercise – 10(b)

Solutions for questions 1 to 31:

1. There are 23(26 – 3) sets of 4 consecutive letters in the alphabet
$\therefore$ Total number of favourable possibilities = 17

$\therefore$ Required probability = $\dfrac{17}{23}$ Choice (D)

2.

Box A	Box B
4 Red, 6 Green	7 Red, 3 Green

Case1:
From box A, a red ball is drawn

Probability of drawing a red ball from box A = $\dfrac{4}{10}$

If red ball is drawn from box A and placed in box B; then the probability of drawing a green ball from box B is 3/11.

Case 2:
From box A, a green ball is drawn

Probability of drawing a green ball from Box A = $\dfrac{6}{10}$

If a green ball is drawn from box A and placed in box B, then the probability of drawing a green ball from box B is 4/11.

$\therefore$ Required probability = $\dfrac{4}{10}\left(\dfrac{3}{11}\right) + \dfrac{6}{10}\left(\dfrac{4}{11}\right)$

$= \dfrac{12+24}{110} = \dfrac{36}{110} = \dfrac{18}{55}$ Choice (C)

3. Given word is 'ANSWER'
Total number of arrangements = n (S) = 6! = 720
The possible position of A, E are 1, 4; 2, 5 or 3, 6. For each case the number of possible words 2(4!).
$\therefore$ The total number of words is 6(4!) = 144.

$\therefore$ Required probability = $\dfrac{144}{720} = \dfrac{1}{5}$ Ans: (0.2)

4. The total number of three digit numbers = 400
The possible digits and the number of numbers from 000 to 399 only with those digits, are tabulated below.

Possible digits	Number of numbers
006	2
015	4
024	4
033	3
114	2
123	6
222	1
	22

Required probability = $\dfrac{22}{400} = \dfrac{11}{200}$ Ans: (0.055)

5. Given, the least number is 4 or 5 and the greatest is 5 or 4
The numbers that come up and the number of ways in which they can come up are tabulated below.

Numbers	Number of ways
4, 4, 4, 4	1
4, 4, 4, 5	4
4, 4, 5, 5	6
4, 5, 5,5	4
5, 5, 5,5	1
	16

$\therefore$ Required probability = $\dfrac{16}{6\times6\times6\times6} = \dfrac{1}{81}$

Choice (C)

6. The number of ways (M_n) which n balls numbered 1 to n (n > 2) can be placed in n boxes also numbered from 1 to n, such that no ball goes into its corresponding box is given by $n!\left[\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} + (-1)^n\dfrac{1}{n!}\right]$
For n =1, then number is 0.
The values of M_n are tabulated below for n = 1 to 6

n	1	2	3	4	5	6
M_n	0	1	2	9	44	265

The number of balls that are placed in a wrong box (i), the corresponding number of balls, which are placed in the right box (6 –i), the number of ways in which the (i) balls can be selected out of 6 (${}^6C_i$) and the corresponding number of ways in which the balls can be placed in the boxes $M_i = {}^6C_i\ m_i$ are tabulated below.

Wrong	Right			
i	6 – i	${}^6C_i$	m_i	M_i
0	6	1	1	1
1	5	6	0	0
2	4	15	1	15
3	3	20	2	40
4	2	15	9	135
5	1	6	44	264
6	0	1	265	265
				720

(i) The probability that all the balls are in the right boxes (i.e I = 0) is 1/720. Choice (C)

(ii) The probability that at least 2 are in the right box 1 – P(0 right or 1 right)
$= 1 - \dfrac{265+264}{720} = \dfrac{191}{720}$ Choice (A)

(iii) The probability that none of the balls are in the right box (i.e i = 6) in $\dfrac{265}{720} = \dfrac{53}{144}$ Choice (D)

7. The following are the possibilities for exactly two consecutive falls showing up heads.
(HHTHTHT), (THHTHTH), (HTHHTHT), (THTHHTH), (HTHTHHT), (THTHTHH)
Similarly, there are 6 ways in which exactly two consecutive tails occur.

$\therefore$ Required probability = $\dfrac{12}{2^7} = \dfrac{3}{32}$ Ans: (3)

8. Total number of squares on an 8 $\times$ 8 chess board
$= \sum_1^8 n^2 = \dfrac{8(8+1)(16+1)}{6} = \dfrac{8(9)(17)}{6}$
The number of squares of 4 $\times$ 4 size is = 5(5) = 25

$\therefore$ Required probability = $\dfrac{25\times6}{8\times9\times17} = \dfrac{25}{204}$

Choice (C)

9. When five dice are rolled together, the 5 numbers and the number of ways in which these numbers can come up are shown below.

(5, 5, 6, 6, 6) $\rightarrow \dfrac{5!}{2!3!} = \dfrac{120}{12} = 10$

(4, 6, 6, 6, 6) $\rightarrow \dfrac{5!}{4!} = 5$

$\therefore$ There are 15 ways in which the total can be 28.

$\therefore$ Required probability = $\dfrac{15}{6^5} = \dfrac{5}{2\left(6^4\right)}$ Choice (D)

10. All the letters of word 'GRAPHICS' can be arranged in 8! ways. The number of words in which vowels are together is 7! 2!.

$\therefore$ Required probability = $1 - \dfrac{7!2!}{8!} = 1 - \dfrac{2}{8} = \dfrac{3}{4}$

Ans: (0.75)

11. A chess board has 64 unit squares. The total number of ways of selecting four squares = ${}^{64}C_4$
Number of squares along the positive diagonals is $2({}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4) + {}^8C_4$
= 2 (1 + 5 + 15+ 35) + 70
= 112 + 70 = 182
The positive diagonals are the diagonals with positive slope, i.e. going from lower left side to upper right side. Similarly, there are 182 ways of selecting 4 squares, such that all 4 are in a negative diagonal.

$\therefore$ Required probability = $\dfrac{2(182)}{{}^{64}C_4} = \dfrac{364}{{}^{64}C_4}$

Choice (D)

12. In the last four throws there can be 0, 1, 2, 3, 4 tails. The same number of tails should be in the first six throws
Hence, the number of favourable cases
$= {}^4C_0{}^6C_0 + {}^4C_1{}^6C_1 + {}^4C_2{}^6C_2 + {}^4C_3\,{}^6C_3 + {}^4C_4{}^6C_4$
= 1 + 24 + 6 (15) + 4 × 20 + 15
= 1 + 24 +90 + 80 + 15 = 210

$\therefore$ Required probability = $\dfrac{210}{1024} = \dfrac{105}{512}$ Choice (A)

13. We first find the probability that no two persons have the same birthday and subtract the result from 1. As leap years are excluded, there can be 365 different birthdays in a year. Second person also can have 365 birthdays and so on for the remaining persons also.
Hence, the total number of cases = 365^{10}.
And the number of possible ways for none of the 10 birthdays to coincide is ${}^{365}P_{10}$.

$\therefore$ Required probability = $1 - \dfrac{{}^{365}P_{10}}{365^{10}} = 1 - \dfrac{{}^{364}P_9}{365^9}$

$= \dfrac{365^9 - {}^{364}P_9}{365^9}$ Choice (C)

14. Rem $(4^m/5)$ = 1 if m is even
= 4 if m is odd

$\therefore$ Rem $\left(\dfrac{4^m + 4^n}{5}\right)$ = 2 if both m, n are even

= 0 if one of m, n is even and the other is odd = 3 if both are odd.
As there are 24 even numbers and 24 odd numbers between 1 and 50 we get the following results for the number of ways in which the different remainders can be obtained.

$R(4^m + 4^n / 5)$	Number of ways
2	24(24)
0	2(24)(24)
3	(24)(24)

$\therefore$ Probability that $R(4^m + 4^n / 5) = 0$

is $\dfrac{2(24)(24)}{4(24)(24)} = \dfrac{1}{2}$ Choice (C)

15. Let P, Q, R be the three men and the probabilities of their hitting the target are
P(A) = 0.3, P(B) = 0.5 and P(C) = 0.6
$P(\overline{A}) = 0.7$, $P(\overline{B}) = 0.5$ and $P(\overline{C}) = 0.4$
Exactly one of them hits the target = $P(A)P(\overline{B})P(\overline{C})$ + $P(\overline{A})\,P(B)\,P(\overline{C}) + P(\overline{A})\,P(\overline{B})\,P(C)$
= 0.3 (0.5)(0.4) + (0.5)(0.7)(0.4) + 0.6 (0.7)(0.5)
$= \dfrac{60}{1000} + \dfrac{140}{1000} + \dfrac{210}{1000} = \dfrac{410}{1000} = 0.41$ Ans: (0.41)

16. (i) Given that,
$P(\overline{A}) = 0.7$
P(A) = 0.3
$P\left(\overline{A} \cap \overline{B}\right) = 0.2$
$P(A \cup B) = 1 - P\left(\overline{A} \cap \overline{B}\right) = 1 - 0.2 = 0.8$
given
A and B are mutually exclusive events
$\Rightarrow P(A\cup B) = P(A) + P(B)$
0.8 = 0.3 + P(B)
P(B) = 0.5 Ans: (0.5)

(ii) Given, A and B are independent
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$
0.8 = 0.3 + P(B) – 0.3P(B)
0.8 – 0.3 = P(B) (0.7)
$\dfrac{5}{7}$ = P (B) Choice (A)

17. (i) In a pack of 52 cards there are 4 aces and 26 red cards

The probability of drawing a red card = $\dfrac{26}{52}$

The probability of drawing an ace = $\dfrac{4}{52}$

$\therefore$ Required probability = $\dfrac{26}{52}\left(\dfrac{4}{52}\right) = \dfrac{1}{26}$

Choice (C)

(ii) **Case 1:**
If first drawn card could be a red ace

$\therefore$ Probability of drawing a Red ace = $\dfrac{2}{52}$

Probability of drawing an ace card = $\dfrac{3}{51}$

$\therefore$ Required probability = $\dfrac{2}{52}\left(\dfrac{3}{51}\right)$

$= \dfrac{1}{(26)\,(17)} = \dfrac{1}{442}$

Case 2:
The first card could be red but not an ace

Probability of drawing a red non - ace = $\dfrac{24}{52}$

Probability of drawing an ace = $\dfrac{4}{51}$

$\therefore$ Required probability = $\dfrac{24}{52}\left(\dfrac{4}{51}\right) = \dfrac{8}{221}$

$\therefore$ Hence, required probability = $\dfrac{1}{442} + \dfrac{8}{221} = \dfrac{17}{442}$

Choice (D)

18. Given,

$P(A) : P(\bar{A}) = 4 : 3$

$P(A) = \frac{4}{7}$ and $P(\bar{A}) = \frac{3}{7}$

$P(B) : P(\bar{B}) = 2 : 1$

$P(B) = \frac{2}{3}$ and $P(\bar{B}) = \frac{1}{3}$

$P(C) : P(\bar{C}) = 1 : 4$

$P(C) = \frac{1}{5}$ and $P(\bar{C}) = \frac{4}{5}$

Now, the majority of the selectors are favorable if any two are favorable and the third is unfavorable or all the three are favorable.

Hence, required probability = $\frac{4}{7}\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) + \frac{3}{7}\left(\frac{1}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{4}{7}\right)\left(\frac{2}{3}\right)\left(\frac{1}{5}\right)$

$= \frac{32+6+4+8}{105} = \frac{50}{105} = \frac{10}{21}$ Choice (B)

19. Probability of Sadikh winning the game is $P(S) = \frac{1}{2}$

Probability of Akheel winning the game is $P(A) = \frac{3}{6} = \frac{1}{2}$

Probability of Afroz winning the game is $P(Z) = \frac{1}{2}$

And also

$P(\bar{S}) = \frac{1}{2}, P(\bar{A}) = \frac{1}{2}, P(\bar{Z}) = \frac{1}{2}$

Required probability = $P(S) + P(\bar{S}).\ P(\bar{A}).\ P(\bar{Z}).\ P(S) + P(\bar{S}).\ P(\bar{A}).\ P(\bar{Z}).P(\bar{S}).\ P(\bar{A}).\ P(\bar{Z}).\ P(S) + \_\_\_$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^{10} + \_\_\_\_$

$= \frac{\frac{1}{2}}{1-\frac{1}{8}} = \frac{\frac{1}{2}}{\frac{7}{8}} = \frac{1}{2}\left(\frac{8}{7}\right) = \frac{4}{7}$ Ans: (4)

20. Given,
P(getting composite number) = 3 × P(getting prime number) = 2 × P(getting number 1)

Number	1	2	3	4	5	6
probability	x	$\frac{2x}{3}$	$\frac{2x}{3}$	2x	$\frac{2x}{3}$	2x

Total probability = 1

$x + \frac{2x}{3} + \frac{2x}{3} + 2x + \frac{2x}{3} + 2x = 1$

$7x = 1, x = \frac{1}{7}$

When one dice is rolled probability of getting 1 is 1/7.

∴ Required probability = $\frac{1}{7}\left(\frac{1}{7}\right) = \frac{1}{49}$

Choice (A)

21. P(Choosing urn B_1) = P (Choosing urn B_2) = P(Choosing urn B_3) = $\frac{1}{3}$

$P(G/B_1) = \frac{4}{7}$; $P(G/B_2) = \frac{4}{6}$; $P(G/B_3) = \frac{1}{2}$

Hence, P(Choosing green ball)

$= \frac{1}{3}\left(\frac{4}{7}\right) + \frac{1}{3}\left(\frac{4}{6}\right) + \frac{1}{3}\left(\frac{1}{2}\right)$

$= \frac{1}{3}\left[\frac{24+28+21}{42}\right] = \frac{1}{3}\left(\frac{73}{42}\right) = \frac{73}{126}$ Choice (A)

22. The probability of the selected bag is defective $= \frac{100}{1000} = \frac{1}{10}$

The probability of the selected bag is non defective $= \frac{900}{1000} = \frac{9}{10}$

The required probability = ${}^8C_0\left(\frac{9}{10}\right)^8 = (0.9)^8$

Choice (B)

23. P(getting same color balls) = P(getting 2 white balls) + P(getting 2 blue balls) + P(getting 2 green balls)

$= \frac{{}^3C_2}{{}^{12}C_2} + \frac{{}^5C_2}{{}^{12}C_2} + \frac{{}^4C_2}{{}^{12}C_2} = \frac{3+10+6}{{}^{12}C_2} = \frac{19}{66}$

Ans: (19)

24. When a dice is rolled, the possible outcomes are {1,2,3,4,5, 6}
Odd number appears in three cases ie {1, 3, 5}
Even number appears in three cases ie {2, 4, 6}
Expected value $\Sigma P_i \times$ Monitary value (M.V)

Number	P_i	M.V	$P_i \times$ M.V
1	1/6	3	1/6 (3)
2	1/6	8	1/6 (8)
3	1/6	9	1/6 (9)
4	1/6	16	1/6 (16)
5	1/6	15	1/6 (15)
6	1/6	24	1/6 (24)

$\Sigma P_i \times M.V = 1/6[3 + 8 + 9 + 16 + 15 + 24] = \frac{76}{6}$ = ₹12.50

∴ Amount to be paid = expected value – profit
= ₹12.50 – ₹10.00 = ₹2.50. Ans: (2.50)

25. The number of multiples of 8 in between 101 to 200 is 13
The number of multiples of 12 in between 101 to 200 is 8
The number of multiples of 8 and 12 in between 101 to 200 is 4
The number of multiples of 8 that fetches ₹40 are 13 – 4 = 9
The number of multiples of 12 that fetches ₹65 are 8 – 4 = 4
The number of multiples of both 8 and 12 that fetches ₹80 are 4
Expected value = P(multiples of 8) × 40 + P(multiples of 12) × 65 + P(multiples of both 8 and 12) × 80

$= \frac{9}{100}(40) + \frac{4}{100}(65) + \frac{4}{100}(80)$

$= \frac{360+260+320}{100} = \frac{940}{100}$ = ₹9.40

Expected average gain in long run = 9.40 – 3.60 = ₹5.80.
Ans: (5.80)

26. The probability of getting heads $= \frac{80}{100} = \frac{4}{5}$

The probability of getting tails $= \frac{20}{100} = \frac{1}{5}$

Expected amount $= \frac{4}{5}(25) - \frac{1}{5} \times 30$

= 20 – 6 = ₹14 Choice (D)

27. Probability that the card is a red honour $= \frac{8}{52}$

Probability that the card is a black Jack = $\frac{2}{52}$

Probability that the card is neither red honour nor black Jack = $\frac{42}{52}$

Expected value = $\frac{8}{52}$ (65) + $\frac{2}{52}$ (52) + $\frac{42}{52}$ (26)

= 10 + 2 – 21 = 12 – 21 = 9 ₹ loss Choice (C)

28. We know that, there are 21 consonants in the English alphabet. The probability that the consonant is 'c' = $\frac{1}{21}$

Choice (A)

29. The probability that the envelope is from Hyderabad (H) or Ahmedabad (A) is ½ in each case. But we need the probability of A, given a particular condition. Let E be the event that the two consecutive letters are AD.

$P\left(\frac{E}{A}\right) = \frac{1}{8}, P\left(\frac{E}{B}\right) = \frac{2}{7}$

$\therefore$ P(E) = P(A) $P\left(\frac{E}{A}\right)$ + P(B) $P\left(\frac{E}{B}\right)$

$= \frac{1}{2}\left(\frac{1}{8}\right)+\frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{2}\left(\frac{23}{56}\right)$

By Bayes' Theorem

$$P\left(\frac{B}{E}\right) = \frac{P(B)\ P\left(\frac{E}{B}\right)}{P(E)}$$

$$= \frac{\frac{1}{2}\left(\frac{2}{7}\right)}{\frac{1}{2}\left(\frac{23}{56}\right)} = \frac{16}{23}$$ Choice (C)

30. According to the conditions P the committee can be formed in two ways i.e., 1 man and 2 women or 3 women.

Total number of persons = 4 + 5 = 9. 3 members can be chosen in $^9C_3$ ways i.e. 84 ways.

The number of ways of forming the committee.

= $^5C_2 \times {}^4C_1 + {}^5C_3 \times {}^4C_0$ = 40 + 10 = 50.

$\therefore$ Required probability = $\frac{50}{84} = \frac{25}{42}$ Ans: (25)

31. E = 1 (0.3) + 2 (0.2) + 3 (0.2) + 4 (0.2) + 5 (0.1)

= 0.3 + 0.4 + 0.6 + 0.8 + 0.5 = 2.6 Ans: (2.6)

Solutions for questions 32 to 35:

Given that, bag contains 10 mobiles of which 4 are damaged. The number of ways of selecting 3 mobiles from 10 mobiles is $^{10}C_3$ i.e., = 120 ways

32. As all the mobiles chosen are damaged, from 4 mobiles 3 can be selected in $^4C_3$ ways.

$\therefore$The probability that all mobiles are damaged is

$= \frac{^4C_3}{^{10}C_3} = \frac{4\times3\times2}{10\times9\times8} = \frac{1}{30}$. Choice (C)

33. Two good mobiles can be selected from 6 good mobiles in $^6C_2$ ways and one defective mobile can be selected from 4 mobiles in $^4C_1$ ways.

$\therefore$ The number of ways of selecting two good mobiles and one defective mobile is $^6C_2 \times {}^4C_1$ ways = 60 ways.

The required probability = $\frac{60}{120} = \frac{1}{2}$ Choice (D)

34. One good mobile can be selected in $^6C_1$ ways and two defective mobiles can be selected in $^4C_2$ ways.

Number of ways of selecting one good mobile and two defective mobiles is $^6C_1 . {}^4C_2$ = 36 ways

$\therefore$ Required probability = $\frac{36}{120} = \frac{3}{10}$ Choice (B)

35. P(at least one mobile is damaged) = 1 – P(no mobile is damaged) = $1-\frac{^6C_3}{^{10}C_3} = 1-\frac{1}{6} = \frac{5}{6}$. Choice (C)