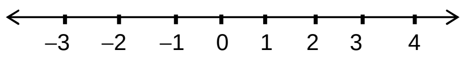
Chapter – 6

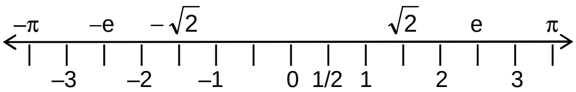
COORDINATE GEOMETRY

Real numbers can be represented geometrically on a horizontal line. We begin by selecting an arbitrary point O, called the origin, and associating it with the real number 0. By convention, we take all positive real numbers to the right of O and the negative real numbers to the left of O.



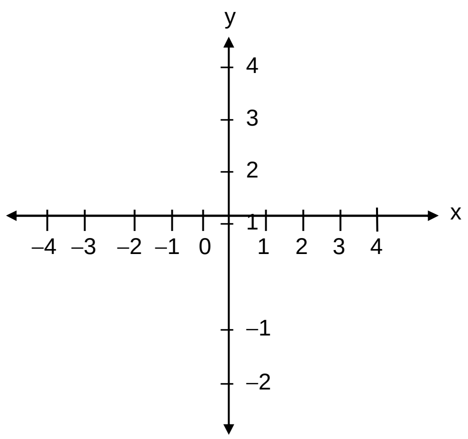
Integer line

Primarily, we plot the integers. On subdividing of these segments, it is possible to locate rational and irrational numbers.



Rectangular coordinates

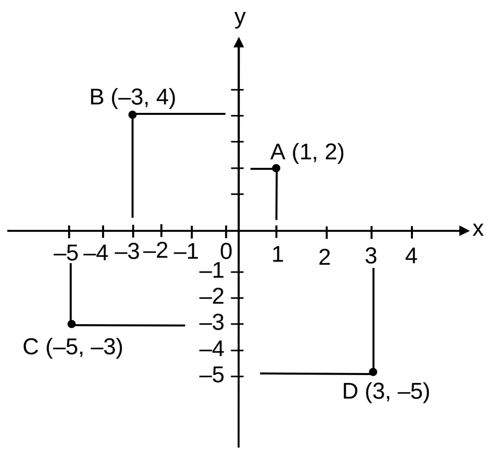
Consider two lines, one vertical and the other horizontal. Let the horizontal line be named as the "x-axis" and the vertical line the "y-axis".



This time, we take the point of intersection of the axes as the origin (O). Once again on the x-axis we follow the convention of associating the positive real numbers to the right of O and the negative real numbers to the left of O. On the y-axis, the positive real numbers are associated above O and the negative real numbers below O.

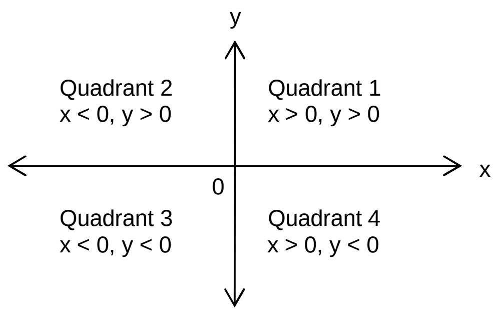
Ordered Pair

Any point P in the plane formed by the x-axis and y-axis can be located by using an ordered pair of real numbers. Let x denote the signed distance of P from the y-axis (by signed distance we mean, if P is to the right of the y-axis, then x > 0 and if P is to the left of the y-axis, then x < 0); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y) denotes the coordinates of P. This gives us the information to locate the point P. The points A, B, C, D located in the figure can be observed by the reader.



x-coordinate and y-coordinate

If (x, y) are the coordinates of a point P, then x is called the x-coordinate of P and y is called the y-coordinate of P. For instance, the coordinates of the origin are (0, 0). The x-coordinate of any point on the y-axis is 0, the y-coordinate of any point on the x-axis is 0. The coordinate system described here is also termed as the Cartesian Coordinate System. The plane is divided into 4 sections termed as quadrants.



Examples:

The point (3, 5) lies in the 1^{st} quadrant.
The point (–4, 6) lies in the 2^{nd} quadrant.
The point (–5, –6) lies in the 3^{rd} quadrant.
The point (7, –2) lies in the 4^{th} quadrant.

Some basic rules and formulae, which have to be remembered, are given below. Each formula is followed by one or more examples, which clearly explain its application.

1. Distance formula

(i) The distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(ii) The distance between the origin (0, 0) and the point (x, y) is $\sqrt{x^2 + y^2}$.

Examples

6.01. Find the distance between the points (4, 3) and (3, –2).

Sol: Distance = $\sqrt{(4-3)^2+[3-(-2)]^2} = \sqrt{26}$.

6.02. Find the distance between the points (0, 0) and (9, 12).

Sol: Distance = $\sqrt{(9-0)^2+(12-0)^2}$ = 15.

6.03. Prove that the lines formed by joining the points (2, –3), (–3, –7) and (–8, –11) do not form a triangle.

Sol: Let X = (2, –3), Y = (–3, –7) and Z = (–8, –11)

XY = $\sqrt{(2-(-3))^2+(-3-(-7))^2} = \sqrt{41}$ units.

YZ = $\sqrt{(-3-(-8))^2+(-7-(-11))^2} = \sqrt{41}$ units.

XZ = $\sqrt{(2-(-8))^2+(-3-(-11))^2} = 2\sqrt{41}$ units.

XY + YZ = XZ

$\therefore$ X, Y and Z are three collinear points and do not form a triangle.

2. Area of triangle

(i) The area of the triangle formed by the vertices A(x_1, y_1), B(x_2, y_2) and C(x_3 , y_3) is equal to value of the determinant

$\frac{1}{2}\begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}$; i.e.,

$1/2\{(x_1 - x_2)(y_2 - y_3) - (y_1 - y_2)(x_2 - x_3)\}$.

Alternately, the area can be found using

$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

$= \frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right|$

(ii) The area of the triangle formed by the vertices (0, 0), (x_1, y_1) and (x_2, y_2) is $1/2\ |x_1y_2 - x_2y_1|$.

6.04. Find the area of the triangle formed by joining the points (–1, –5), (2, –2) and (4, –1).

Sol: Given (x_1, y_1) = (–1, –5), (x_2, y_2) = (2, –2) and (x_3, y_3) = (4, –1).

Area = Mod of $\frac{1}{2}\begin{vmatrix} -1-2 & -5-(-2) \\ 2-4 & -2-(-1) \end{vmatrix}$

$= \frac{1}{2}\ |(-3)(-1) - (-3)(-2)| = \frac{3}{2}$ sq. units.

6.05. Find the area of the triangle formed by joining the points (0, 0), (2, 0) and (2, 4).

Sol: Given (x_1, y_1) = (0, 0), (x_2, y_2) = (2, 0) and (x_3, y_3) = (2, 4).

Area = $\frac{1}{2}\begin{vmatrix} 0-2 & 0-0 \\ 2-2 & 0-4 \end{vmatrix}$

$\frac{1}{2}|(-2)(-4)|$ = 4 sq. units.

6.06. Find the area of the triangle formed by joining the points (–1, –4), (–3, –5) and (–5, –6).

Sol: Given (x_1, y_1) = (–1, –4), (x_2, y_2) = (–3, –5) and (x_3, y_3) = (–5, –6).

Area = Mod of $\frac{1}{2}\begin{vmatrix} -1-(-3) & -4-(-5) \\ -3-(-5) & -5-(-6) \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$

Since the area is 0, the points do not form a triangle but form a straight line.

Note:

The area of a quadrilateral formed by the points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) describing the consecutive vertices is given by $\frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_4 & y_2-y_4 \end{vmatrix}$.

Alternately, the area can be found using

$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ i.e.,

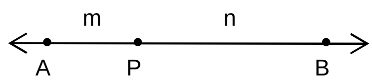
$\frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\right|$

3. Section formulae

(i) Internal Division

If A (x_1, y_1) and B (x_2, y_2) are two points given, then the coordinates of a point P, which divides the line joining AB internally in the ratio m : n is given by

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$



Note: The point P is between A and B for internal division.

6.07. Find the co-ordinates of the point P which divides the line segment joining the points A(3, 5) and B(1, –1) internally in the ratio 2 : 3.

Sol: From the section formula,

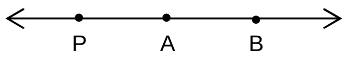
P = $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{2(1)+(3)(3)}{2+3}, \frac{2(-1)+(3)(5)}{2+3}\right) = \left(\frac{11}{5}, \frac{13}{5}\right)$

(ii) External Division:

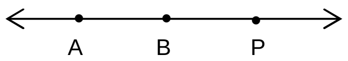
If A(x_1, y_1) and B(x_2, y_2) are two points then the coordinates of a point P, which divides the line segment joining AB in the ratio m : n externally are given by

$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Fig 1. 

(if m < n)

OR

Fig 2. 

(if m > n)

Note: The point P is beyond A and B for external division. It can be either beyond A (Fig 1) or beyond B (fig 2).

6.08. Find the co-ordinates of the point P which divides the line segment joining the points A(3, 5) and B(1, –1) in the ratio 2 : 3 externally.

Sol: From the section formula,

$$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

$$= \left(\frac{(2)\,(1) - (3)(3)}{2-3}, \frac{2(-1)-(3)(5)}{2-3}\right) = (7, 17)$$

Note: The midpoint of the line segment joining the two points is a special case of section formula, in which the ratio is 1 : 1.

6.09. Find the centre of the circle which has (7, 20) and (1, –4) as the extremities of one of its diameters.

Sol: Centre = Midpoints of its diameter

$$= \left(\frac{7+1}{2}, \frac{20-4}{2}\right) = (4, 8)$$

Centroid: If A(x_1, y_1) , B(x_2, y_2) and C(x_3, y_3) are the vertices of a triangle, then the centroid G of the triangle ABC is given by

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Note: The centroid of a triangle is the point of concurrence of the medians of a triangle.

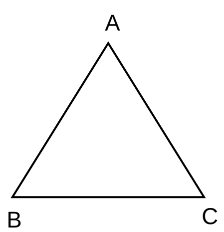
6.10. Find the centroid of the triangle formed by the vertices P(–3, 5), Q(–5, 7) and R(–1, 3).

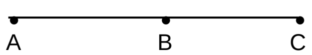
Sol: G = Centroid of the triangle

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{-3+-5+-1}{3}, \frac{5+7+3}{3}\right) = (-3, 5)$$

Collinearity: Given three distinct points in a plane A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3), there are two possibilities. They may form a triangle or a straight line.





In case the three points A, B and C form a straight line, we say that they are "collinear".

Any of the following conditions are enough to show collinearity of the given three points.

1. AB + BC = CA or AC + CB = AB or AB + AC = BC.
2. The area of the triangle formed by A, B and C equals zero.
3. Slope of AB = Slope of BC.

6.11. Show that the points P(4, –5), Q(6, –7) and R(8, –9) are collinear.

Sol: Area of the triangle formed by the points P, Q and R = $\frac{1}{2}\begin{vmatrix} 4-6 & -5-(-7) \\ 6-8 & -7-(-9) \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 2 \\ -2 & 2 \end{vmatrix}$

$\frac{1}{2}$[(–2) (B) – (B) (–2)] = 0.

∴ P, Q and R are collinear.

Alternate method:

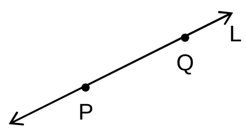
If PQ, QR and PR are computed, they would be $2\sqrt{2}$, $2\sqrt{2}$ and $4\sqrt{2}$ respectively.

PQ + QR = PR. This is possible only when P, Q, R are collinear and Q lies between P and R. Hence P, Q, R do not form a triangle.

∴ P, Q and R are collinear.

The Straight Line

We now deal with a case where a specified relationship (equation) between x and y is given for various points P(x, y). One such relationship is the linear equation. Its graph is called a straight line. We know that there is one and only one line containing two distinct points P and Q, from plane geometry.



If P and Q are each represented by ordered pairs of real numbers, the following definition can be given:

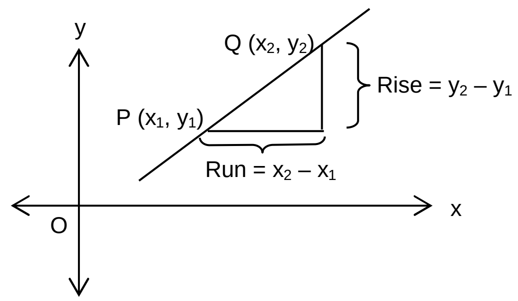
Slope of a line: The slope of a line is a number that describes both the direction and steepness of a line.

Let P and Q be two distinct points with coordinates (x_1, y_1) and (x_2, y_2) respectively. The slope m of the line L containing P and Q is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ if } x_1 \neq x_2.$$

If $x_1 = x_2$, the slope m of line L is undefined (since this results in division by 0) and L in which case, is a vertical line.

For a non vertical line, slope = $\frac{\text{Change in y}}{\text{Change in x}}$ or "rise over run"



Note: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$

So the slope is same whether changes are computed from P to Q or Q to P.

Alternately, the slope of a line is the tangent value of the angle (θ) made by the line with the positive direction of the x-axis in the anticlockwise direction.
$m = \tan\theta$.

6.12. Compute the slope of the line passing through the points (–3, 5) and (5, –1).

Sol: Slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-1)}{-3-(5)} = \dfrac{-3}{4}$

6.13. The lines L_1, L_2, L_3 and L_4 pass through the following pairs of points. Find the slopes of the lines.
L_1: (2, –3), (–1, 2)
L_2: (3, –4), (–4, 6)
L_3 : (–4, 5), (–5, 8)
L_4 : (5, –6), (3, –10)

Sol: Let m_1, m_2, m_3 and m_4 denote the slopes of the lines L_1, L_2, L_3 and L_4 respectively.

$m_1 = \dfrac{2-(-3)}{-1-2} = -\dfrac{5}{3}$

$m_2 = \dfrac{6-(-4)}{-4-3} = -\dfrac{10}{7}$

$m_3 = \dfrac{8-5}{-5-(-4)} = -3$

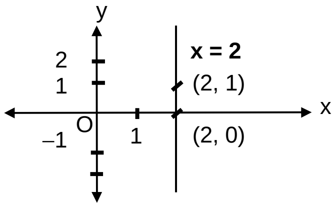
$m_4 = \dfrac{-10-(-6)}{3-5} = 2$

Equations of lines

Vertical lines

The equation of a vertical line passing through a point (a, 0) is given by the equation x = a where a is a given real number.

Example:



Non-vertical lines: Let L be a non-vertical line with slope m containing (x_1, y_1). For any other point (x, y) on L, we have

$m = \dfrac{y - y_1}{x - x_1}$ or $y - y_1 = m(x - x_1)$

Point-Slope Form: The equation of a non-vertical line of slope m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

6.14. Find the equation of the line which passes through (2, –2) and has a slope 3.

Sol: $y - y_1 = m(x - x_1)$
$m = 3$, $(x_1, y_1) = (2, -2)$
$y - (-2) = 3(x - 2)$
$y = 3x - 8$

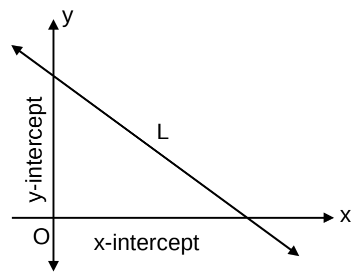
Two-point Form: **The equation of a non-vertical line passing through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is** $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

6.15. Find the equation of the line which passes through (3, 2) and (6, 7).

Sol: $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (6, 7)$
$y - 2 = \dfrac{7-2}{6-3}(x - 3)$
$3y - 6 = 5x - 15$
$5x - 3y = 9$

Note:
(1) In algebra ax + by + c = 0 is termed as a first-degree equation in x and y.
(2) If a = 0 and b ≠ 0, then L will be a horizontal line.
(3) If b = 0 and a ≠ 0, then L will be a vertical line.
(4) If c = 0, then L passes through the origin.

Intercepts: The portions cut off by a line on the coordinate axes are called intercepts.
The x-intercept is the portion on the x-axis and the y-intercept is the portion on the y-axis.



6.16. Find the x and y intercepts of the line 14x + 8y = 112

Sol: 14x + 8y = 112
∴ when x = 0, y = 14
and when y = 0, x = 8
∴ x – intercept = 8 and y – intercept = 14.

Intercept Form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where x-intercept is ‘a’ and y-intercept is ‘b’.

6.17. Write the intercept form of the line whose general form is 4x + 5y – 20 = 0.

Sol: 4x + 5y = 20
We get the intercept form by dividing by 20.
∴ The intercept form of the line is $\dfrac{x}{5} + \dfrac{y}{4} = 1$.

> Slope Intercept Form: **The equation of a line with slope m and y-intercept b is y = mx + b**

Note:

(A) When the equation is written in this form, the coefficient of x is the slope and the constant term gives the y-intercept of the line.

(B) y is explicitly written in terms of x. So this form is also termed as the explicit form of the line.

6.18. Find the slope and the y – intercept of the line $y = 3x + 4$.

Sol: The given line has the form $y = 3x + 4$.
Slope = 3 and y – intercept = 4.

> General Form: **The equation of a line L is in the general form when it is written as ax + by + c = 0 where a, b and c are real numbers with either a ≠ 0 or b ≠ 0.**

Note:

(1) When the equation is written in this form, the coefficient of x is the slope and the constant term gives the y-intercept of the line.

(2) y is explicitly written in terms of x. So this form is also termed as the explicit form of the line.

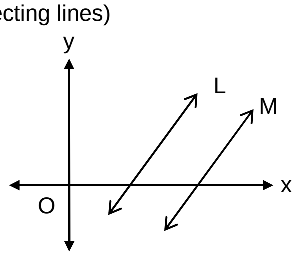
The following table summarises the various forms of equations of straight lines.

	You are Given	You Use	Equation
1.	Point (x_1, y_1) and slope m	Point - slope form	$y - y_1 = m(x - x_1)$
2.	Two points (x_1, y_1), (x_2, y_2)	If $x_1 = x_2$, use vertical line equation	$x = x_1$
		If $x_1 \neq x_2$, Two Point Form	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
3.	x and y intercepts a and b	Intercept-Form	x/a + y/b = 1
4.	Slope m, y-intercept b	Slope - Intercept Form	y = mx + b

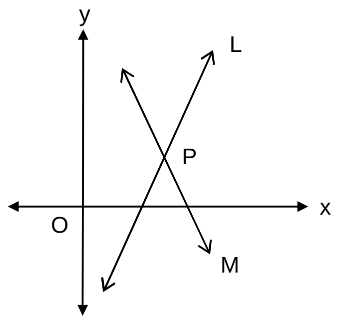
Parallel and intersecting lines

Let L and M be two lines. Exactly one of the following three relationships must hold for the lines L and M.

1. All the points on L are the same as the points on M. (Identical lines)
2. L and M have no points in common. (Parallel lines)
3. L and M have exactly one point in common. (Intersecting lines)



Parallel lines



Intersecting lines

Note: To find the coordinates of the point of intersection of the lines
L : $a_1x + b_1y + c_1 = 0$ and
M : $a_2x + b_2y + c_2 = 0$,

we solve the two equations to get the point of intersection as

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

6.19. Find the point of intersection of the lines $4x - 3y - 33 = 0$ and $3x - 2y - 25 = 0$ and also find the number of regions into which the xy plane is divided by these lines.

Sol: L_1 is $4x - 3y - 33 = 0$
L_2 is $3x - 2y - 25 = 0$
On solving these equations,
x = 9 and y = 1
∴ (9, 1), is the point of intersection.
As the lines intersect, the plane gets divided into 4 infinite regions.

Angle between two lines

If m_1 and m_2 are the slopes of two lines, the angle 'θ' between them is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Note:

(1) Condition for parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ or $m_1 = m_2$

(2) Condition for perpendicular lines:
$a_1 a_2 + b_1 b_2 = 0$ or $m_1 m_2 = -1$.

6.20. Show that the lines $4x - 3y - 6 = 0$ and $-12x + 9y + 10 = 0$ are parallel.

Sol: $a_1 = 4$, $a_2 = -12$, $b_1 = -3$ and $b_2 = 9$

$$\frac{a_1}{a_2} = \frac{4}{-12} = -\frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{-3}{9} = -\frac{1}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

∴ the given lines are parallel.

6.21. Show that the lines $4x - 3y - 6 = 0$ and $3x + 4y + 10 = 0$ are perpendicular.

Sol: $a_1 = 4$, $b_1 = -3$, $a_2 = 3$ and $b_2 = 4$
$a_1 a_2 + b_1 b_2 = (4)(3) + (-3)(4) = 0$
$\therefore$ the given lines are perpendicular.

Note: The equation of a line through a point $P(x_1, y_1)$ and
(i) parallel to $ax + by + c = 0$ is given by $a(x - x_1) + b(y - y_1) = 0$.
(ii) perpendicular to $ax + by + c = 0$ is given by $b(x - x_1) - a(y - y_1) = 0$.

6.22. Find the equation of the line passing through (4, –4) and
(i) parallel to $2x + 3y + 6 = 0$;
(ii) perpendicular to $2x + 3y + 6 = 0$

Sol: (i) As the required line is parallel to $2x + 3y = -6$, it is of the form $2x + 3y = k$. As it passes through (4, –4), it is $2x + 3y = 2(4) + 3(-4) = -4$.
(ii) As the required line is perpendicular to $2x + 3y = -6$, it is of the form $3x - 2y = k$. As it passes through (4, –4) it is $3x - 2y = 3(4) - 2(-4) = 20$

Some formulae to remember

(1) The general form of the equation of a straight line is $ax + by + c = 0$. Here, the y-intercept is $-c/b$, the x-intercept is $-c/a$ and the slope is $-a/b$.
(2) If $ax + by + c = 0$ is the equation of a line, the perpendicular distance from a point (x_1, y_1) to this line is given by: $\left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$
(3) The distance between two parallel straight lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
(4) The equation of a circle centred at (h, k) with radius r units is $(x - h)^2 + (y - k)^2 = r^2$.
(5) The equation of a circle centred at the origin with radius r units is $x^2 + y^2 = r^2$.
(6) The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents of a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

Some more worked examples

6.23. Find the equation of the line whose x and y intercepts are 4 and 5 respectively.

Sol: $\dfrac{x}{4} + \dfrac{y}{5} = 1$
$\Rightarrow 5x + 4y = 20$.

6.24. Find the equation of the line passing through (3, –4) and parallel to the y axis.

Sol: The equation of a line parallel to the y axis is of the form x = a constant.
As the line required passes through (3, –4) the required equation is $x = 3$.

6.25. Find the perpendicular distance of the point (1, 1) from the line $12x + 5y + 8 = 0$.

Sol: The length of the perpendicular
$= \left|\dfrac{12(1) + 5(1) + 8}{\sqrt{12^2 + 5^2}}\right| = \dfrac{25}{13}$ units.

6.26. Find the equation of the circle with centre as (2, 3) and radius 5 units.

Sol: Let (x_1, y_1) be any point on the circle.
$\sqrt{(x_1 - 2)^2 + (y_1 - 3)^2} = 5$
On squaring both sides, we get
$x_1^2 - 4x_1 + 4 + y_1^2 - 6y_1 + 9 = 25$
$x_1^2 + y_1^2 - 4x_1 - 6y_1 - 12 = 0$
$\therefore$ The required equation is $x^2 + y^2 - 4x - 6y - 12 = 0$

Note: The general form of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ with $g^2 + f^2 - c \geq 0$.

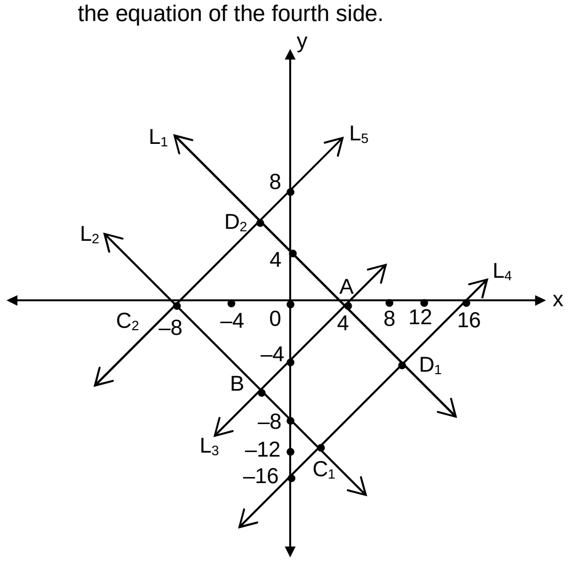
6.27. A line drawn through P(3, –4) makes an angle of 45° with the x-axis and cuts it at Q. Find PQ.

Sol: Given: $\theta = 45$; Slope of the line is $\tan\theta = \tan 45° = 1$.
Equation of the line is $\dfrac{y - (-4)}{x - 3} = 1$
$\Rightarrow y - x = -7$
At Q, $y = 0$; $\therefore x = 7$, $\therefore$ Q(7, 0).
$\therefore PQ = \sqrt{(7 - 3)^2 + (0 - (-4))^2} = 4\sqrt{2}$ units.

6.28. Find the value of a if the line $(x + y + 2) + a(4x + 6y + 18) = 0$ is parallel to the x-axis.

Sol: $x(1 + 4a) + y(1 + 6a) + 2 + 18a = 0$
Slope $= \dfrac{-(1 + 4a)}{1 + 6a}$
Since the given equation is parallel to the x-axis, slope = 0.
$\therefore \dfrac{-(1 + 4a)}{1 + 6a} = 0$, $\therefore a = \dfrac{-1}{4}$.

6.29. The lines $x + y - 4 = 0$, $x + y + 8 = 0$ and $-x + y + 4 = 0$ form three sides of a square. Find the equation of the fourth side.



$L_1 = x + y - 4 = 0$ $\quad$ $L_3 = x - y - 4 = 0$
$L_2 = x + y + 8 = 0$

Let the point of intersection of L_1 and L_3 be A and the point of intersection of L_2 and L_3 be B. There are two possible squares determined by these three lines. The fourth side of one square lies on L_4 which lies on one side of L_3 and the fourth side of the other lies on L_5 which lies on the opposite side of L_3.

The equation of L_4 and L_5 are of the form $x - y + k = 0$

Since the lines form a square, the distance between L_1 and L_2 is equal to the distance between L_3 and L_4 or L_3 and L_5.

$$\therefore \frac{|8-(-4)|}{\sqrt{1^2+1^2}} = \frac{|k+4|}{\sqrt{1^2+1^2}}$$

i.e. $k + 4 = 12$ or $k + 4 = -12$
i.e. $k = -16$ or $k = 8$
$\therefore$ the required equation is $x - y - 16 = 0$ or $x - y + 8 = 0$.

Locus

Locus is a set of points that satisfy a geometrical condition. For example

(1) If a point moves so that its distances from two fixed points are always equal, its locus is the perpendicular bisector of the line segment joining the two points.

(2) If a point moves so that its distance from a fixed point is constant then its locus is a circle with centre as the fixed point and the constant distance as radius. We can also say that the locus of a point equidistant from two fixed points, say A and B is the perpendicular bisector of the segment AB. Similarly, the locus of a point which is at a fixed distance (say r) from a fixed point, say O, is the circle with centre O and radius r., i.e., the idea of motion is not a necessary part of the concept of locus. It is merely one way of expressing the idea.

6.30. Find the equation of locus of the point P such that $PA^2 + PB^2 = 36$, where A = (2, 3) and B = (4, –1).

Sol: P(x, y) is any point on the locus and A = (2, 3) and B = (4, –1)
Given $PA^2 + PB^2 = 36$
$\Rightarrow (x-2)^2 + (y-3)^2 + (x-4)^2 + (y+1)^2 = 36$
$\Rightarrow 2(x^2 + y^2 - 6x - 2y - 3) = 0$
$\Rightarrow x^2 + y^2 - 6x - 2y - 3 = 0$. This is the equation of the locus. It represents a circle.

Pair of straight lines

Consider a general second degree equation of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ . . . (A). If it can be split into two linear factors of the form $(a_1x + b_1 y + c_1)(a_2x + b_2y + c_2)$,
i.e., if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$
then the equation (A) represents a pair of straight lines represented by $a_1x + b_1y + c_1 = 0$ . . . . (1) and $a_2x + b_2y + c_2 = 0$ . . . . (2)

6.31. Does the equation $x^2 + 2xy + y^2 - 5x - 5y + 6 = 0$ represents a pair of parallel lines?

Sol: $x^2 + 2xy + y^2 - 5x - 5y + 6 = 0$ can be expressed as $(x + y - 2)(x + y - 3) = 0$
i.e., $(x^2 + 2xy + y^2 - 5x - 5y + 6) = (x + y - 2)(x + y - 3) = 0$

$\therefore$ The two straight lines represented by the given equations are $x + y - 2 = 0$ and $x + y - 3 = 0$ or $x + y = 2$ and $x + y = 3$. These are two parallel lines.

Change of Axes

Sometimes, to be able to express an equation in a simpler form, it may be required to change the original co-ordinate axes, either by way of shifting the origin or by changing the direction of the axes.

Translation of Axes

In this case, the origin is shifted to a new point, keeping the direction of the axes intact. The new axes are parallel to the original axes.

Suppose the origin is shifted to (h, k). If the original coordinates of a point P are (x, y) and (X, Y) denote the coordinates of P with reference to the new axes, then we have $x = X + h$ and $y = Y + k$.

Rotation of Axes

In this process, the origin is kept intact and the axes are rotated about the origin, through a required angle.

If θ is the angle of rotation and (x, y) are the coordinates of a point P with reference to the original axes and (X, Y) with reference to the new axes, then the relation between them is given below:

$x = X\cos\theta - Y\sin\theta$	$y = X\sin\theta + Y\cos\theta$
$X = x\cos\theta + y\sin\theta$	$Y = -x\sin\theta + y\cos\theta$

These equations are called transformation equations.

Note:

(1) If the axes are rotated at an angle θ in the anticlockwise direction, θ is considered positive.
(2) Sometimes, we may have to translate and rotate or rotate and translate. The order is unimportant.

6.32. Find the coordinates of the point (–3, 4), when the origin is translated to (1, –4).

Sol: The equations relating the coordinates are
$x = X + h$,
$y = Y + k$
Given (x, y) = (–3, 4) and (h, k) = (1 , –4)
$X = x - h$ and $Y = y - k$
$X = -3 - 1$ and $Y = 4 - (-4)$
$\Rightarrow$ (X, Y) = (–4, 8)

6.33. The point to which the origin has to be shifted, so that the point (–3, 5) changes to (1, –2) after translation, is

Sol: The equations relating the coordinates are
x = X + h, y = Y + k
Here (x, y) = (–3, 5), and (X, Y) = (1, –2)
∴ h = x – X and k = y – Y
h = (–3, –1) , k = 5 – (–2)
∴ (h, k) = (–4, 7)

6.34. The coordinates of the point $\left(-3\sqrt{2}, 5\sqrt{2}\right)$ in the new system, when the axes are rotated by an angle of 45° in the anticlockwise direction, are ______.

Sol: The transformation equations are
X = x cosθ + y sinθ , Y = –x sinθ + y cos θ
Here (x, y) = $\left(-3\sqrt{2},\ 5\sqrt{2}\right)$ and θ= 45°
∴ X = $-3\sqrt{2}\cos 45° + 5\sqrt{2}\sin 45°$ and
Y = $+3\sqrt{2}\sin 45 + 5\sqrt{2}\cos 45°$

X = $-3\sqrt{2}.\dfrac{1}{\sqrt{2}} + 5\sqrt{2}.\dfrac{1}{\sqrt{2}}$ and

Y = $3\sqrt{2}.\dfrac{1}{\sqrt{2}} + 5\sqrt{2}.\dfrac{1}{\sqrt{2}}$

X = –3 + 5 and Y = 3 + 5
∴ (X, Y) = (2, 8)

6.35. If the coordinates of a point P change to (4, 0) when the axes are rotated by an angle 30° in the clockwise direction, then the coordinates of P are

Sol: Given (X, Y) = (4, 0) let P (x, y) and θ = –30°
∴ x = X cosθ – Ysinθ , Y = xsinθ + ycosθ
x = 4cos(–30°) – 0sin(–30)°, y = 4sin(–30°) + 0 . cos(–30°)
x = $4.\dfrac{\sqrt{3}}{2}$, y = $-4.\dfrac{1}{2}$
∴ (x, y) = $(2\sqrt{3}, -2)$

Concept Review Questions

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. (a) What is the equation of the X-axis?
(A) x = 0 (B) y = 0
(C) x + y = 0 (D) x – y = 0

(b) What is the equation of the Y-axis?
(A) x = 0 (B) y = 0
(C) x + y = 0 (D) x – y = 0

2. What is the slope of a line parallel to the X-axis?

$\boxed{}$

3. (a) What is the equation of the line parallel to the X-axis and passing through (5, 9)?
(A) x = 5 (B) y = 9
(C) x + y = 14 (D) x – y = –4

(b) What is the equation of the line parallel to the Y-axis and passing through (5, 9)?
(A) x = 5 (B) y = 9
(C) x + y = 14 (D) x – y = –4

4. What is the point of intersection of the X-axis and Y-axis?
(A) (1, 1) (B) (1, 0)
(C) (0, 1) (D) None of these

5. What is the point of intersection of the lines x = 2 and y = 3?
(A) (2, 3) (B) (3, 2) (C) (–2, –3) (D) (–3, –2)

6. If the line 2x + 3y + k = 0 passes through the point (3, 2), what is the value of k?

$\boxed{}$

7. If the first, second, third and fourth quadrants are denoted as Q_1, Q_2, Q_3 and Q_4 respectively, the point (–3, –5) belongs to
(A) Q_1 (B) Q_2 (C) Q_3 (D) Q_4

8. The coordinates of the point that is at a distance of 3 units from the X-axis and 2 units from the Y-axis and lies in Q_2 is
(A) (–3, 2) (B) (–2, 3) (C) (–3, –2) (D) (–2, –3)

9. The distance from the point (4, 3) to X-axis is $\boxed{}$ units

10. The distance between the points A (2, –3) and B (–1, 1) in units is
(A) 5 (B) $\sqrt{5}$
(C) 7 (D) None of these

11. The distance from origin to the point (–5, –12) is
(A) 5 (B) 12 (C) 13 (D) 17

12. PQ is the perpendicular bisector of AB, where A = (–3, 4) and B = (–7, 2). If Q lies on AB, then find the coordinates of Q.
(A) (–10, 6) (B) (– 5, –11)
(C) (–5, 3) (D) (– 1, –3)

13. The slope of the line that makes an angle of 60° with the positive X-axis is
(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\sqrt{3}$ (D) $\sqrt{3}$

14. (a) The equation of the line with slope –1 and y-intercept 3 is
(A) x – y = 3 (B) x + y = 3
(C) x – y + 3 = 0 (D) x + y + 3 = 0

(b) Find the equation of the line whose slope is 4/3 and x-intercept is 6.
(A) 3y = 4x – 24 (B) 3y = – 4x + 24
(C) 3y = 4x + 24 (D) 4y = – 3x + 18

15. The intercepts made by the line 3x – 4y – 12 = 0 on the X-axis and on the Y-axis respectively are

$\boxed{}$, $\boxed{}$.

16. The slope of the line 3x – 4y + 7 = 0 is
(A) $\frac{3}{4}$ (B) $\frac{-3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{-4}{3}$

17. The equation of the line making intercepts of 3 and –2 on the X-axis and Y-axis respectively is
(A) 2x + 3y = 6 (B) 2x – 3y – 6 = 0
(C) 3x + 2y = –6 (D) 3x – 2y – 6 = 0

18. The equation of the line passing through the point (–1, 4) with slope $\frac{-2}{3}$ is
(A) 2x – 3y + 10 = 0 (B) 2x + 3y + 10 = 0
(C) 2x + 3y – 10 = 0 (D) 3x + 2y – 10 = 0

19. If the slope of a line is – 2, then the slope of the line perpendicular to it is $\boxed{}$

20. The slope of the line parallel to 3x – 4y + 8 = 0 is
(A) $\frac{-3}{4}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{-4}{3}$

21. The angle made by the line 2x –2y + 7 = 0 with the X-axis is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

22. The distance from the origin to the line 3x – 4y + 10 = 0 is (in units)
(A) 0 (B) 1 (C) 2 (D) 5

23. X = (2, 4) and Y = (6, 10). Find the point on the X-axis which is equidistant from X and Y.
(A) $(\frac{25}{2}, 0)$ (B) $(\frac{27}{2}, 0)$ (C) $(\frac{23}{2}, 0)$ (D) $(\frac{29}{2}, 0)$

24. The midpoints of the sides of triangle PQR are (–4, 0), (–2, 2) and (2, 4). Find the centroid of ΔPQR.

(A) $\left(\frac{-4}{3}, 1\right)$ (B) $\left(\frac{-2}{3}, 1\right)$

(C) $\left(\frac{-4}{3}, 2\right)$ (D) $\left(\frac{-2}{3}, 2\right)$

25. (–7, 8), (–3, 9) and (–5, 6) are the vertices of a parallelogram taken in that order. Find the coordinates of the fourth vertex.

(A) (–9, 5) (B) (–8, 4)
(C) (–8, 5) (D) (–9, 4)

26. L_1 is $3x - 4y + 7 = 0$ and L_2 is $ax + 8y - 6 = 0$. L_1 and L_2 do not intersect. Find a.

$\boxed{}$

27. L_1 is $2x - 3y + 8 = 0$ and L_2 is $3x + by + 9 = 0$. $L_1 \perp L_2$. Find b.

$\boxed{}$

28. Find the point of intersection of the lines $8x - 3y = 13$ and $2x + y = 5$.

(A) (–2, –1) (B) (2, 1)
(C) (2, –1) (D) (–2, 1)

29. If $6x + py + 18 = 0$ and $x + y + q = 0$ represent the same line, then find p + q.

$\boxed{}$

30. Find the orthocentre of the triangle whose vertices are (4, 3), (0, 7) and (–4, 3).

(A) (0, 7) (B) (4, 3)
(C) (–4, 3) (D) (0, 0)

31. ABCD is a square. A is (3, 5) and C is (7, 13). Find its area (in sq.units).

$\boxed{}$

32. The angle between the lines x = 10 and y = 5 is $\boxed{}$ degrees

33. Find the distance from (2, 3) to the line $3x + 4y + 10 = 0$.

(A) 18/5 (B) 28/5
(C) 27/5 (D) 33/5

34. The coordinates of point A, when origin is translated to (–2, –3) are (2, 4). The original coordinates of A are

(A) (1, 0) (B) (0, 1)
(C) (–1, 0) (D) (0, –1)

35. The coordinates of the point (4, –2) in the new system, when the origin is translated to the point (–7, 5), are

(A) (–11, 7) (B) (11, 7)
(C) (–11, –7) (D) (11, –7)

36. The circumference of the circle $x^2 + y^2 + 8x - 6y - 11 = 0$ is $\boxed{}$ π

37. Find the centre and radius of the circle $(x-3)^2 + (y+2)^2 = 36$

(A) (–3, 2), 4 (B) (3, 2), 5
(C) (3, –2), 6 (D) (–3, –2), 6

38. The centre of the circle $x^2 + y^2 + 8x - 6y + 11 = 0$ is

(A) (–4, 3) (B) (4, –3)
(C) (–4, –3) (D) (4, 3)

39. The area of the circle $x^2 + y^2 - 6x + 4y - 23 = 0$ is

(A) 12π (B) 6π
(C) 24π (D) 36π

40. The diameter of the circle $x^2 + y^2 + 6x - 8y - 56 = 0$ is $\boxed{}$.

Exercise – 6(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The distance between the point (24, 7) and the centre of the circle $x^2 + y^2 = a^2$ is $\boxed{}$ units.

2. The slope of the line joining the points (a – b, a + b) and (–b, a) is _______.

(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{-b}{a}$ (D) $\frac{-a}{b}$

3. The equation of the line parallel to y-axis and passing through the point $\left(\frac{7}{3}, -2\right)$ is _______.

(A) y + 2 = 0 (B) 7x + 3 = 0
(C) 7x – 3 = 0 (D) 3x – 7 = 0

4. The lines 5x – y + 6 = 0 and 4x + 3y + 1 = 0 intersect in the _______.

(A) 1st quadrant (B) 2nd quadrant
(C) 3rd quadrant (D) 4th quadrant

5. The centroid of the triangle formed by joining the points A (2, 8), B (4, –2) and C (0, 6) is _______.

(A) (6, 6) (B) (4, 4)
(C) (1, 1) (D) (2, 4)

6. The ratio in which the x-axis divides the line joining the points (4, 7) and (1, 1) is _______.

(A) 4 : 1 externally (B) 4 : 1 internally
(C) 7 : 1 externally (D) 7: 1 internally

7. If the points (1, 7), (3, 3) and (7, k) are collinear, then k = $\boxed{}$

8. Which of the following points is collinear with the points (1, 3) and (3,7)?

(A) (0, 0) (B) (–1, 1)
(C) (–2, –6) (D) (2, 5)

9. The area of the triangle formed by joining the points (1, 1), (3, 5) and (–1, 2) is $\boxed{}$ sq.units.

10. The area of the triangle formed by the line 4x – 3y = 24 with the co-ordinate axes is $\boxed{}$ sq.units.

11. The points (–1, –1) $\left(\sqrt{3}, -\sqrt{3}\right)$ and (1, 1) form _______.

(A) a straight line.
(B) a scalene triangle.
(C) an isosceles triangle.
(D) an equilateral triangle.

12. If the lines 3x + y – 4 = 0, x – 3y + 2 = 0, 2x + 5y – 1 = 0 form a right-angled triangle, then the vertex containing the right angle is _______.

(A) (–1, 1) (B) (1, 1) (C) (1,–1) (D) (–1, –1)

13. If the roots of the quadratic equation $x^2 - 7x + 6 = 0$ are intercepts of a line L, the equation of the line L can be _______.

(A) x + 6y = 6
(B) 6x + y = 6
(C) 6x + y –3= 0
(D) either (A) or (B)

14. If the points A(2, 4), B(–1, 3) and C(3, –2) are the three consecutive vertices of a parallelogram, then, the fourth vertex is _______.

(A) (6, 1) (B) (3, –1)
(C) (–6, 1) (D) (6, –1)

15. Which of the following lines makes equal intercepts on both the axes?

(A) x + y – 5 = 0
(B) 2x + y + 5 = 0
(C) 2x – y + 2 = 0
(D) y = 4x + 5

16. The distance between the parallel lines x – 2y = 4 and –3x + 6y + 2 = 0 is _______.

(A) $\frac{1}{\sqrt{5}}$ units (B) $\sqrt{5}$ units

(C) $\frac{2\sqrt{5}}{3}$ units (D) $\frac{5}{12}$ units

17. The distance of the point (1, 4) from the line 2x – y + 7 = 0 is _______.

(A) $2\sqrt{5}$ units (B) $\sqrt{5}$ units
(C) 5 units (D) 10 units

18. If the lines $\sqrt{7}x - \sqrt{3}y = 3$ and $\sqrt{3}x + ky = 4$ are parallel, then k = _______.

(A) $\sqrt{6}$ (B) $\frac{-3\sqrt{7}}{7}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

19. If the lines 4x – y – 1 = 0 and 12x – 3y + 3 = 0 represent a pair of opposite sides of a square, then the area of the square is _______.

(A) $\frac{2}{5}$ sq.units

(B) $\frac{36}{\sqrt{153}}$ sq.units

(C) $\frac{18}{153}$ sq.units

(D) $\frac{4}{17}$ sq.units

Directions for question 20: Match the lines with their equations.

20.

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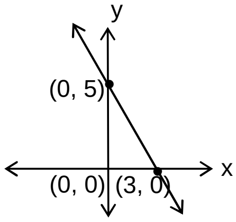
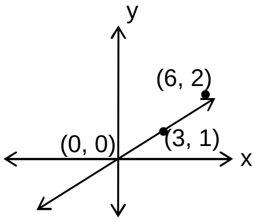
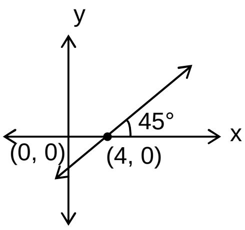
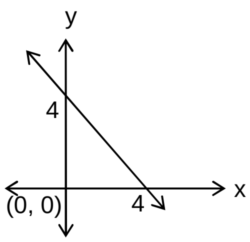
<tr><td>i</td><td>
</td><td>(A) x – 3y = 0</td></tr>
<tr><td>ii.</td><td>
</td><td>(B) x – y = 4</td></tr>
<tr><td>iii.</td><td>
</td><td>(C) 5x + 3y = 15</td></tr>
<tr><td>iv.</td><td>
</td><td>(D) x + y = 4</td></tr>
</table>

21. The acute angle between the lines $x+\sqrt{3}y+6\sqrt{3}=0$ and $y+\sqrt{3}x+2=0$ is _______.
(A) 15° (B) 30° (C) 60° (D) 75°

22. If the line 2x + y – k = 0 passes through the point of intersection of the lines 4x + y – 13 = 0 and x – 3y – 13 = 0, then k = ☐

23. Find the area of the quadrilateral formed by joining the points (–2, –3), (–2, 3), (2, 3) and (2, –3) (in sq.units).
☐

24. L_1 is 2x + 3y – 8 = 0 and L_2 is kx – 9y + 24 = 0. L_1 and L_2 do not intersect. Find k.
(A) 6 (B) 1/6 (C) –6 (D) –1/6

25. Find the equation of the line passing through the point (18, 6) and perpendicular to the line passing through the points (2, 7) and (3, 10).
(A) 3x + y – 60 = 0
(B) 3x – y – 48 = 0
(C) 3y = x
(D) 3y + x – 36 = 0

26. A is (3, 5). The equation of perpendicular bisector of AB is 2y + x – 23 = 0. Find B.
(A) (7, 13) (B) (6, 11)
(C) (8, 15) (D) (9, 17)

27. Find the transformed equation of 2x – 3y + 7 = 0, when the origin is translated to (1, –1).
(A) 2X – 3Y + 12 = 0
(B) 2X – 3Y – 12 = 0
(C) 2X – 3Y –12 = 0
(D) 2X + 3Y– 12 = 0

28. The transformed equation of x – 2y + 5 = 0, when the axes are rotated by an angle of 45° in the anticlockwise direction is

(A) $3X + Y - 5 = 0$

(B) $3X + Y + 5\sqrt{2} = 0$

(C) $X - 3Y + 5\sqrt{2} = 0$

(D) $X + 3Y - 5\sqrt{2} = 0$

29. A circle $(x - 3)^2 + (y - 3)^2 = 9$ is drawn such that both the positive coordinate axes are tangents to it. Find the greatest possible distance from a point P (2, 2) to a point on the circle.

(A) $3 + \sqrt{2}$ (B) $3 - \sqrt{2}$

(C) $\sqrt{3} - 2$ (D) $\sqrt{3} + 2$

30. The maximum distance between any point on $(x + 8)^2 + y^2 = 36$ and any point on $x^2 + (y + 15)^2 = 64$ is ☐.

31. The number of points lying strictly inside the circle $x^2 + y^2 = 9$, which have integer coordinates is _______.

(A) 30 (B) 25 (C) 13 (D) 15

32. R_1 is a road whose alignment is described by the equation is $3x + 2y = 30$. R_2 is a road parallel to R_1. R_3 is a road exactly midway between the roads R_1 and R_2 and has the equation $3x + 2y = 45$. The distance between the origin and the closer of R_1 R_2 is ☐ times the distance between the origin and the farther of R_1 and R_2.

33. The point of intersection of $4x + 5y = 26$ and $y = kx + 2$ has integral coordinates. What is the number of integral values that k can take?

☐

34. If the area of the convex quadrilateral formed by the lines $x + 3y = 22$, $3x + y = 22$ and the coordinate axes is S, find the value of 3S.

☐

35. At how many points does the line $8x - 15y + 140 = 0$ meet the curve $x^2 + y^2 = 64$?

☐

Exercise – 6(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the distance between (8, 8) and the centre of the circle $(x - 5)^2 + (y - 4)^2 = 40$ (in units).

☐

2. Find the angle made by the line $\sqrt{2}\,x - y + 18 = 0$ with the Y – axis.

(A) $\text{Tan}^{-1}\left(\frac{3}{\sqrt{2}}\right)$ (B) $\text{Tan}^{-1}\left(\frac{5}{\sqrt{2}}\right)$

(C) $\text{Tan}^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (D) $\text{Tan}^{-1}\left(\frac{1}{3\sqrt{2}}\right)$

3. Find the distance between the parallel lines $2x + 3y + 7 = 0$ and $4x + 6y + 15 = 0$ (in units).

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2\sqrt{13}}$ (C) $\frac{2}{\sqrt{13}}$ (D) $\frac{3}{\sqrt{13}}$

4. Find the acute angle between the lines $3x + 4y + 8 = 0$ and $12x - 5y + 9 = 0$.

(A) $\text{Cos}^{-1}\left(\frac{12}{65}\right)$

(B) $\text{Cos}^{-1}\left(\frac{18}{65}\right)$

(C) $\text{Cos}^{-1}\left(\frac{14}{65}\right)$

(D) $\text{Cos}^{-1}\left(\frac{16}{65}\right)$

5. The points of trisection of AB are C and D. If A = (6, 13) and B = (3, 7), then find $\overline{CD}$ (in units).

(A) $\sqrt{5}$ (B) $2\sqrt{5}$

(C) $\frac{3}{2}\sqrt{5}$ (D) $\frac{5}{2}\sqrt{5}$

6. (3, –4) and (–3, 4) are the vertices of an equilateral triangle. Which of the following can be the third vertex?

(A) $\left(4\sqrt{3}, 3\sqrt{3}\right)$ (B) (–4, –3)

(C) (3, 3) (D) (–4, 4)

7. ABC is a triangle. A = (8, 4). AD is the median drawn to BC. D = (12, 8). Find the centroid of the triangle.

(A) $\left(\frac{26}{3}, \frac{14}{3}\right)$ (B) $\left(\frac{32}{3}, \frac{20}{3}\right)$

(C) $\left(\frac{28}{3}, \frac{16}{3}\right)$ (D) $\left(\frac{28}{3}, \frac{14}{3}\right)$

8. If (5, 6) and (7, 9) are two vertices of triangle ABC and origin is centroid, find the third vertex.

(A) (–15, –12) (B) (15, 12)

(C) (12, 15) (D) (–12, –15)

9. Find the equation of the line parallel to $3x + 4y + 11 = 0$ which passes through (3, 4).

(A) $3y + 4x - 24 = 0$

(B) $3y = x$

(C) $4y + 3x - 25 = 0$

(D) $4y - 3x - 7 = 0$

10. A line passes through (5, 12) and makes equal intercepts on the coordinate axes. Find the intercepts.

$\boxed{}$ and $\boxed{}$

11. Find the area of the triangle formed by the X-axis, Y-axis and x + 4y – 16 = 0 (in sq.units).
(A) 24 (B) 36 (C) 32 (D) 40

12. Find the equation of the line whose slope and y –intercepts are the roots of $y^2 - 9y + 18 = 0$.
(A) y = 6x + 3
(B) y = 3x + 6
(C) Neither (A) nor (B)
(D) Either (A) or (B)

13. Find the value of k for which the lines 3x + 4y = 14, 2x + 3y = 10 and 5x + ky = 16 are concurrent.

$\boxed{}$

14. If the lines 2x + 3y + 4 = 0, 5x – 7y – 19 = 0 and 4x + ky + 6 = 0 are concurrent, then k =
(A) 5 (B) 4 (C) 3 (D) –2

15. Find the area of the triangle whose midpoints of the sides are given by (1, 5), (3, 6) and (4, 8) (in sq. units).

$\boxed{}$

16. The area of the quadrilateral formed by the points (1, 3), (3, 1), (5, 3) and (3, 5) is (sq. units)
(A) 16 (B) 6 (C) 8 (D) 4

17. If the line 2x + 3y + 6 + k(x – 4y + 8) = 0 is parallel to the Y – axis, then find k.

$\boxed{}$

18. If the points (3, 6), (4, 9) and (5, k) are collinear, then find k.
(A) 12 (B) 11 (C) 14 (D) 15

19. The roots of the equation $p^2 - p - 12 = 0$ represent the slope and the x-intercept of a line. Find the equation of the line.
(A) y = 4x + 12
(B) y = –3x + 12
(C) Either (A) or (B)
(D) y = –4x + 12

20. Find the equation of the line passing through (4, 7) and (6, 11).
(A) y = x + 3 (B) y = 2x + 1
(C) y = x + 5 (D) y = 2x – 1

21. The points (4, –5), (0, 0) and (5, 4) form a/an ______.
(A) scalene triangle
(B) right angled isosceles triangle
(C) equilateral triangle
(D) straight line

22. Two opposite sides of a square are given by the lines 2x + y + 4 = 0 and 8x + 4y – 12 = 0. Find the perimeter of the square.

$\dfrac{\boxed{}}{\sqrt{5}}$

23. If the lines 2x + 3y – 12 = 0, 3x + 4y – 17 = 0 and 4x + ay – 22 = 0 are concurrent, then find a.

$\boxed{}$

24. A line passing through A(3, 4) makes an angle of 30° with the X-axis and cuts the Y-axis at B. Find AB (in units).
(A) $3\sqrt{3}$ (B) $2\sqrt{3}$
(C) $4\sqrt{3}$ (D) $5\sqrt{3}$

25. Find the ratio in which the line joining (1, –1) and (4, 3) is divided by the X – axis.
(A) 1 : 3 externally (B) 3 : 1 externally
(C) 1 : 3 internally (D) 3 : 1 internally

26. A line passes through (3, 4) and the sum of its intercepts on the coordinate axes is 14. Find its equation.
(A) x + y = 7 (B) 4x + 3y = 24
(C) y = x + 7 (D) Either (A) or (B)

27. Which of the following never holds true?
(A) Parallel lines have equal gradients.
(B) Perpendicular lines have the product of their gradients as –1
(C) Lines parallel to the X – axis have a gradient of 0
(D) All statements are true

28. The lines 4x – 3ky + 4 = 0 and 2x – 5y + 1 = 0 intersect in the first quadrant. The x – coordinate of the point of intersection is twice the y – coordinate. Find the value of k.

$\boxed{}$

29. Find the area of the triangle formed by the line 10x + 3y = 30 and the coordinate axes (in square units).
(A) 10 (B) 20
(C) 15 (D) 25

30. Find the coordinates of the point which divides the line segment joining the points (–1, 4) and (2, 6) externally in the ratio 2 : 3.
(A) (–3, 0) (B) (–4, 0)
(C) (–6, 0) (D) (–7, 0)

31. Find the area of the triangle formed by (a + 1, a + 2), (a – 1, a + 1) and (a + 3, a – 3). (in square units)

$\boxed{}$

32. A line passing through (2, 12) has the difference of its intercepts on the coordinate axes as 12. Its gradient is a positive integer. Find its gradient.

$\boxed{}$

33. A line makes an angle of 150° with the X – axis. The sum of its intercepts on the coordinate axes is 3. Find its x – intercept.

(A) $\frac{3\sqrt{3}\left(\sqrt{3}-1\right)}{2}$

(B) $\frac{3\sqrt{3}\left(\sqrt{3}-1\right)}{4}$

(C) $\frac{3\sqrt{3}\left(\sqrt{3}+1\right)}{2}$

(D) $\frac{3\sqrt{3}\left(\sqrt{3}+1\right)}{4}$

34. Find the equation of the line passing through (1, 1) and perpendicular to $6x – 3y + 5 = 0$.

(A) $2x + y = 3$ (B) $2y + x = 3$
(C) $2x – y = 1$ (D) $2y – x = 1$

35. The line $2x + 6y + k = 0$ passes through the point of intersection of the lines $2x + 3y – 13 = 0$ and $3x + 2y – 12 = 0$. Find k.

(A) –20 (B) –18
(C) –16 (D) –22

36. Find the equation of the line which passes through (3, 4) and the point of intersection of $2x + 3y – 8 = 0$ and $2y – 3x – 1 = 0$.

(A) $y = x + 1$ (B) $y = 2x – 2$
(C) $y = 3x – 5$ (D) $y = 7 – x$

37. The equation of the line passing through the points (1, 4) and (4, 1) is ________.

(A) $x + y – 5 = 0$ (B) $x – y –1 = 0$
(C) $x + y + 1 = 0$ (D) $x – y + 1 = 0$

38. The points (0,0), (–2,3) and (6,–9) when joined form a/an ________.

(A) scalene triangle
(B) right-angled triangle
(C) equilateral triangle
(D) straight line

39. Find the equation in the original system, if the transformed equation of a curve when the origin is translated to (–1, 2) is $aX + bY + c = 0$.

(A) $ax + by – a – 2b + c = 0$
(B) $ax + by + a – 2b + c = 0$
(C) $ax – by + a – 2b + c = 0$
(D) $ax – by + a – 2b + c = 0$

40. The transformed equation, when the axes are rotated through an angle 60° is $X^2 + Y^2 = 2$, then the original equation is

(A) $x^2 – y^2 = 2$ (B) $x^2 + y^2 = 2$
(C) $x^2 + y^2 = 1$ (D) $x^2 – y^2 = 1$

41. A circle in the first quadrant, with radius 2 units, is drawn such that it touches both the coordinate axes. The shortest distance from the origin to the circle is

(A) $2\sqrt{2}$ (B) $\sqrt{2} – 1$
(C) $2\sqrt{2} – 1$ (D) $2\left(\sqrt{2} – 1\right)$

42. A circle has the equation $x^2 + y^2 = r^2$. One of the lines $5x – 4y – 20 = 0$ and $5x – 4y + 40 = 0$ is a secant of the circle and the other is a tangent to the circle. Find the radius of the circle.

(A) $\frac{40}{\sqrt{41}}$ (B) $\frac{60}{\sqrt{41}}$ (C) $\frac{30}{\sqrt{41}}$ (D) $\frac{20}{\sqrt{41}}$

43. The minimum possible distance between the curves $x^2 + y^2 – 8y + 12 = 0$ and $x^2 + y^2 + 6y = 0$ is $\boxed{}$.

44. If the lines $3x + 4y = 10$ and $my – x + 4 = 0$ intersect, then for how many integer values of m, does the point of intersection have integer coordinates?

$\boxed{}$

45. The number of points lying strictly inside the curve whose equation is $x^2 + 2y^2 = 24$, which have integer coordinates is

$\boxed{}$

Directions for questions 46 to 50: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using I and II together.

46. Find the centroid of the triangle ABC.

I. The coordinates of B are (1, 2).
II. The mid point of AB is (4, 7).

47. Does the point (7, 2) lie on the line L?

I. L passes through (5, 6).
II. The slope of the line L is 3/5.

48. Is point P in the first quadrant?

I. P lies within the circle with centre at the origin and radius 3.
II. P lies on the straight line $x + 2y = 4$.

49. What is the area of the triangle?

I. One of the sides of the triangle is on the y-axis.
II. Two of the sides lie on $x + y = 1$ and $x – y = 1$.

50. What is the area of the square ABCD?

I. The midpoints of AB and CD are (1, 4) and (1, 8) respectively.
II. The point of intersection of the diagonals is (2, 10) and one of the verices of the square is (4, 6).

Key

Concept Review Questions

1. (a) B
   (b) A
2. 0
3. (a) B
   (b) A
4. D
5. A
6. –12
7. C
8. B
9. 3
10. A
11. C
12. C
13. D
14. (a) B
    (b) A
15. 4, –3
16. A
17. B
18. C
19. 0.5
20. B
21. A
22. C
23. D
24. C
25. A
26. –6
27. 2
28. B
29. 9
30. A
31. 40
32. 90
33. B
34. B
35. D
36. 12
37. C
38. A
39. D
40. 18

Exercise – 6(a)

1. 25
2. A
3. D
4. B
5. D
6. C
7. –5
8. D
9. 5
10. 24
11. D
12. B
13. D
14. D
15. A
16. C
17. B
18. B
19. D
20. (i) C
    (ii) A
    (iii) B
    (iv) D
21. B
22. 5
23. 24
24. C
25. D
26. A
27. A
28. D
29. A
30. 31
31. B
32. 0.5
33. 1
34. 121
35. 0

Exercise – 6(b)

1. 5
2. C
3. B
4. D
5. A
6. A
7. B
8. D
9. C
10. 17, 17
11. C
12. D
13. 3
14. A
15. 6
16. C
17. 0.75
18. A
19. C
20. D
21. B
22. 28
23. 5
24. B
25. C
26. D
27. D
28. 4
29. C
30. D
31. 6
32. 2
33. A
34. B
35. D
36. A
37. A
38. D
39. B
40. B
41. D
42. A
43. 2
44. 2
45. 51
46. D
47. C
48. D
49. C
50. B