Chapter – 9

STATISTICS

Statistics deals with the collection, classification, presentation, analysis and interpretation of numeric data (quantitative data).

The simplest form of numerical data is a series of values of a certain variable. Such values can be given as individual numbers. Therefore, such data is called an individual series of raw data.

In such a series, some numbers could occur repeatedly. In such cases, it would be convenient to group these numbers. This grouping can be done in two ways. The numbers can be arranged in ascending (or descending) order and the number of times each number occurs (its frequency) can be specified. This results in a discrete series. Alternately, we may choose to ignore the actual number and record only the fact that it lies between certain limits. Such grouping results in a continuous series. The values between the limits are called intervals. For example, consider the marks obtained by the students of a class. This data has been represented in each of the ways below.

Individual series
60, 55, 70, 60, 70
65, 75, 65, 70, 55
58, 74, 67, 68, 73
Discrete series

Frequency	2	1	2	2	1	1	3	1	1	1
Number	55	58	60	65	67	68	70	73	74	75

Continuous series
We denote the marks obtained by the students as

Intervals	Frequency
$x \leq 55$	2
$55 < x \leq 60$	3
$60 < x \leq 65$	2
$65 < x \leq 70$	5
$70 < x \leq 75$	3

Collection of data, especially on a large scale, is typically an expensive process and requires big teams of field workers. Therefore it needs careful planning. The primary consideration is the purpose for which the collected data will be used

After data has been collected, classified and presented, it has to be analysed and interpreted. To analyse data, the important features of the data have to identified and relations between quantities (variables) have to be studied. Two important features of any data are the central tendency and dispersion. Instead of thinking of all the numbers in a set, we would like to think of a typical number, the 'central' value. Two sets of numbers, which may have the same 'central' value, may differ in terms of the variation of all the numbers in the set, i.e. the extent to which the numbers 'disperse' from this central value.

Various quantitative measures can be defined to measure these two properties, ie central tendency and dispersion.

In this chapter, we shall limit ourselves to an individual series and study such properties.

I. Measures of Central Tendencies

The measures we discuss here are
(A) Arithmetic Mean,
(B) Geometric Mean,
(C) Harmonic Mean,
(D) Median and
(5) Mode.

1. Arithmetic Mean (A.M.) ($\overline{x}$)

Given $x_1, x_2, \ldots\ldots\ldots, x_n$ (n individual items)

$$\boxed{\text{A.M.} = \overline{x} = \frac{x_1 + x_2 + \ldots. + x_n}{n}}$$

or $\overline{x} = \dfrac{\text{Sum of the observations}}{\text{The number of observations}}$

Examples

(i) The arithmetic mean of (5, 8, 9, 14, –16) is
$= \dfrac{5 + 8 + 9 + 14 - 16}{5} = \dfrac{20}{5} = 4$

(ii) The arithmetic mean of (3, 4, 5, 6, 7, 8, 9, 10) is
$= \dfrac{3+4+5+6+7+8+9+10}{8} = \dfrac{52}{10} = 5.2$

Note:
(a) The algebraic sum of deviations about the mean is 0 or $\Sigma(x - \overline{x}) = 0$.
(b) The arithmetic mean of two numbers a, b is $\dfrac{a+b}{2}$.
(c) If b = AM of (a, c), then a, b and c are in arithmetic progression.

2. Geometric Mean (G.M.)

Given $x_1, x_2, \ldots\ldots\ldots, x_n$ (n individual items all being positive)

$$\boxed{\text{G.M.} = (x_1 \cdot x_2 \cdot \ldots\ldots, x_n)^{1/n}}$$

or G.M. = n^{th} root of the product of the numbers.

Examples

(i) The geometric mean of (10, 30, 90) is
$= (10 \times 30 \times 90)^{1/3}$
$= (10 \times 30 \times 3 \times 30)^{1/3}$
$= (30 \times 30 \times 30)^{1/3} = 30$

(ii) The geometric mean of (24, 20, 15, 45, 75) is
$= (8 \times 3 \times 5 \times 4 \times 3 \times 5 \times 3^2 \times 5 \times 3 \times 5^2)^{1/5}$
$= (2^5 \times 3^5 \times 5^5)^{1/5}$
$= 2 \times 3 \times 5 = 30$

Note:

(a) Geometric mean is not very commonly used as it involves finding the n^{th} root, and hence requires complex calculations for higher values of n.

(b) The geometric mean of two positive numbers a, b is $\sqrt{ab}$.

(c) If b = GM of (a, c), then a, b and c are in geometric progression.

3. Harmonic Mean (H.M.)

Given x_1, x_2, …, x_n (n individual observations such that none of them is equal to 0),

$$\boxed{H.M. = \frac{n}{1/x_1 + 1/x_2 + ... + 1/x_n}}$$

Examples:

(i) The harmonic mean of (1, 2, 3, 4, 5) is

$$= \frac{5}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = \frac{5\times 60}{60+30+20+15+12}$$

$$= \frac{300}{137}$$

(ii) The harmonic mean of $\left(\frac{1}{1},\frac{1}{3},\frac{1}{9},\frac{1}{27}\right)$

$$= \frac{4}{1+3+9+27} = \frac{4}{40} = \frac{1}{10}$$

Note:

(a) HM of two numbers a, b is $\frac{2ab}{a+b}$.

(b) If b = HM of (a, c), then a, b and c are in harmonic progression.

(c) For any two positive numbers a, b
  (i) AM $\geq$GM $\geq$ HM.
  (ii) $(GM)^2$ = (AM) (HM).

4. Median

Median is the magnitude of the "middle-most" item in a series of value of variables, when the values have been arranged in order of their magnitude. When there are odd number of observations, the middle number, when the values are arranged in ascending or descending order, is the median. When there are even number of observations, the average of the two numbers, at the middle when the values are arranged in ascending or in descending order is the median.

Examples

(i) The median of 3, 7, 12, 16, 22 is 12.

(ii) The median of 4, 5, 9, 11, 14, 15 is $\frac{9+11}{2}$ =10.

Note:

(a) The median divides the distribution into two equal parts.

(b) Median is suitable for qualitative data as well.

5. Mode

It is the item which is "most often" found in the given set of observations, i.e, the value occurring the highest number of times.

Examples

(i) For the observations; 2, 1, 4, 2, 6, 4, 3, 2, 8, 2, 2, 1, 4, 6, 7.
Mode = 2

(ii) For the observations; 1, 2, 2, 3, 3, 3, 3 4, 4, 4, 4, 5, 5, 6
Modes are 3, 4

(iii) Consider the observations 4, 7, 11, 35, 36, 26.

Here no item occurs more than once. So, mode is ill-defined.

Empirical Formula:

Mode = 3Median – 2Mean.

This formula is valid for the distribution which are moderately symmetric. (symmetry being coincidence of mean, median and mode)

II. Measures of Dispersion:

The measures we discuss here are
(A) Range,
(B) Quartile Deviation,
(C) Mean Deviation and
(D) Standard Deviation/Variance.

1. Range

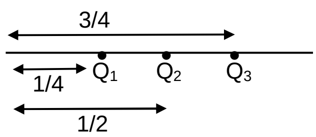
Given, x_1, x_2, ……, x_n (n individual observations).
Range = maximum value – minimum value

Example

(i) Range (2, 4, 5, 7, 8, 12, 13) = 13 – 2 = 11.
(ii) Range (5, 4, 9, 1, 6, 12, 15) = 15 – 1 = 14.

2. Quartile Deviation (Q.D.) or Semi Inter Quartile Range

Quartiles are those values, which divide the distribution into four equal parts, when the values are arranged in ascending or descending order of magnitude.



Q_1 is called the first quartile, Q_2 is the middle quartile and Q_3 is the third quartile. The second quartile is also referred to as the median.
As the name semi-inter-quartile range itself suggests

Q.D. = $\frac{Q_3 - Q_1}{2}$ (one-half the range of quartiles)

For calculation

Q_1 = size of $\left(\frac{n+1}{4}\right)^{th}$ item

Q_3 = size of $3\left(\frac{n+1}{4}\right)^{th}$ item

Example

(i) Find the Q.D. of the observations 4, 9, 12, 15, 20, 23, 25.

Sol: Since there are 7 terms; n = 7

$Q_1 = \left(\frac{7+1}{4}\right)^{th}$ item = 2^{nd} item = 9.

$Q_3 = 3\left(\frac{7+1}{4}\right)^{th}$ item = 6^{th} item = 23.

$\therefore$ Q.D = $\frac{Q_3 - Q_1}{2}$

$= \frac{23-9}{2} = \frac{14}{2} = 7.$

(ii) Find the Q.D. of 3, 7, 18, 24, 27, 31.

Sol: The observations 3, 7, 18, 24, 27, 31.

$Q_1 = \left(\frac{6+1}{4}\right)^{th}$ item = $1^3/_4{}^{th}$ item

= 1^{st} item + $\frac{3}{4}$ (2^{nd} item – 1^{st} item)

$Q_1 = 3 + \frac{3}{4}(7-3) = 6.$

and $Q_3 = 3\left(\frac{6+1}{4}\right)^{th}$ item = $5^1/_4{}^{th}$ item

= 5^{th} item + $\frac{1}{4}$ (6^{th} item – 5^{th} item)

$Q_3 = 27 + \frac{1}{4}(31-27) = 28.$

$\therefore$ Q.D = $\frac{Q_3 - Q_1}{2} = \frac{28-6}{2} = 11.$

3. Mean Deviation (M.D.)

The mean deviation is calculated about mean or median or mode. But by default mean deviation is about mean. Mean deviation is the average of deviations of each item in the data set from the mean.

$$\boxed{\text{M.D.} = \frac{\sum_{i=1}^{n} |x_i - A|}{n}}$$

A = mean / median / mode; n = number of items.

Example

Find the mean deviation of 2, 3, 9, 11, 15.

Sol: Mean of the observations $\bar{x}$ = 40/5 = 8
So the mean deviation

$$\text{M.D.} = \frac{|2-8|+|3-8|+|9-8|+|11-8|+|15-8|}{5}$$

$= \frac{6+5+1+3+7}{5} = \frac{22}{5} = 4.4$

Note:

(A) Mean Deviation of two numbers a, b = $\frac{|a-b|}{2}$.

(B) Mean deviation is based on each and every observation.

4. Standard Deviation (S.D.):

It is the root mean squared deviation taken about the mean.

S.D. = $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$, where $x_1, x_2, x_3, \ldots\ldots x_n$ are the n items given.

The expression $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ also equals to $\sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

Example

Find the S.D. of (2, 5, 7, 10, 13, 17).

Sol: Mean of the observations = 54/6 = 9

S.D. = $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$

$= \sqrt{\frac{(-7)^2 + (-4)^2 + (-2)^2 + (1)^2 + (4)^2 + (8)^2}{6}}$

$= \sqrt{\frac{49+16+4+1+16+64}{6}} = \sqrt{\frac{150}{6}} = \sqrt{25} = 5$

Note:
(A) The square of the standard deviation is variance.
(B) The standard deviation is non-negative.

Examples

9.01. Find the mean, median and mode of the individual series 4, 6, 10, 12, 14, 14, 20 and 22.

Sol: Mean = $\frac{4+6+10+12+2(14)+20+22}{8}$ = 12.75

Median: The given numbers are in a non-decreasing order.
The two middle numbers are 12 and 14.

$\therefore$ Median = $\frac{12+14}{2}$ = 13.

Mode: The observation 14 has the greatest frequency.
$\therefore$ 14 is the mode.

9.02. If the arithmetic mean of 25, 27, 31, 35, 39 and x is 38, find x.

Sol: $\bar{x} = \frac{25+27+31+35+39+x}{6} = 38$

$\Rightarrow$ x = 71

9.03. Find the arithmetic mean and the median of the first 9 consecutive natural numbers.

Sol: The numbers are 1, 2, 3, 4, 5, 6, 7, 8 and 9.
The numbers are in A.P.
$\therefore$ their mean is the mean of the first and last terms i.e.,

$\frac{1+9}{2}$ = 5, Which is the middle term.

Since the middle term is 5, the median is also 5.

9.04. Find the geometric mean of 2, 6, 18, 54 and 162.

Sol: Geometric mean = $[(2)\,(6)\,(18)\,(54)\,(162)]^{\frac{1}{5}}$

$= \left[(2)\,(2)\,(3)\,(2)(3)^2\;(2)\,(3)^3\;(2)(3^4)\right]^{\frac{1}{5}}$

$= \left[(2)^5\;(3^{10})\right]^{\frac{1}{5}}$ = (2) (3^2) = 18

9.05. Find the geometric mean of the observations 8, 12, 18 and 27.

Sol: Geometric mean = $[(8)\,(12)\,(18)\,(27)]^{\frac{1}{4}}$

$= \left[(8)\,(8)\left(\frac{3}{2}\right)(8)\left(\frac{3}{2}\right)^2(8)\left(\frac{3}{2}\right)^3\right]^{\frac{1}{4}}$

$= \left[(8)^4\left(\frac{3}{2}\right)^6\right]^{\frac{1}{4}} = 12\sqrt{\frac{3}{2}} = 6\sqrt{6}$

9.06. Find the harmonic mean of the observations 4, 8, 12, 16 and 20.

Sol: H.M. (4, 8, 12, 16, 20)

$= \dfrac{5}{\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}} = \dfrac{1200}{137}$.

9.07. The H.M., G.M., and the A.M., of the positive numbers p and q must be in – (AP/GP/HP)

Sol: H.M. (p, q) = $\dfrac{2pq}{p+q}$

G.M(p, q) = $\sqrt{pq}$

A.M(p, q) = $\dfrac{p+q}{2}$

[G.M. (p, q)]^2 = pq

[A.M. (p, q)] [H.M (p, q)] = pq = [G.M (p, q)]^2

∴ H.M. (p, q), G.M. (p, q) and A.M. (p, q) are in geometric progression.

9.08. Find the range, quartile deviation and mean deviation for the data 5, 7, 12, 14, 20, 26 and 28.

Sol: Range = maximum value – minimum value

= 28 – 5 = 23.

Quartile deviation:

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ item = 2nd item = 7.

$Q_3 = \left(\frac{n+1}{4}\right)^{th}$ item = 6th item = 26.

$\therefore$ Q.D. = $\dfrac{Q_3 - Q_1}{2}$ = 9.5

Mean deviation:

Mean ($\bar{x}$) of the observations = $\dfrac{112}{7}$ = 16

M.D. = $\dfrac{\sum\left|xi - \bar{x}\right|}{n}$

= $\frac{1}{7}$ [| 5 – 16 | + | 7 – 16 | + |12 – 16 | + |14 – 16

| + |20 – 16| + |26 – 16 | + | 28 – 16|] = $\frac{52}{7}$.

9.09. Find the standard deviation and variance of the observations 6, 10, 16, 20 and 24.

Sol: Mean of the observations

$= \dfrac{\sum x_i}{n} = \dfrac{6+10+16+20+24}{5} = 15.2$

$\dfrac{\sum x_i^2}{n} = \dfrac{36+100+256+400+576}{5} = \dfrac{1368}{5}$

S.D. = $\sqrt{\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2} = \sqrt{\dfrac{1368}{5} - 15.2^2}$

= $\sqrt{42.56}$ Variance = (S.D)^2 = 42.56.

9.10. Find the range and the quartile deviation of the following prices of 15 items in a general store: 7, 9, 15, 21, 27, 17, 33, 34, 39, 25, 11, 29, 23, 43 and 41.

Sol: Arranging the prices in ascending order, we have 7, 9, 11, 15, 17, 21, 23, 25, 27, 29, 33, 34, 39, 41 and 43.

Range = Maximum price – Minimum price

= 43 – 7 = 36.

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ item = $\left(\frac{15+1}{4}\right)^{th}$ item = 15.

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ item = 12th item = 34.

Q.D. = $\dfrac{34-15}{2}$ = 9.5

9.11. The median of a set of observations is 12. Their mean is 13. Find their mode.

Sol: Mode = 3Median – 2Mean

= 3(12) – 2(13) = 10.

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Data that consists of only observations is called ______ data.
(A) Discrete
(B) Continuous
(C) raw
(D) None of these

2. Data that consists of class intervals and frequencies is called ________ data.
(A) raw
(B) grouped
(C) either (A) or (B)
(D) None of these

3. In grouped data, if two successive classes are 0 – 9 and 10 – 19 respectively, then the size of each class is _______.
(A) 10 (B) 9
(C) 11 (D) None

4. The midvalue of the class 14.5 – 25.5 is $\boxed{}$

5. The arithmetic mean of the data 12, 17, 15, 21, 36, 40 is _____
(A) 23 (B) 22.5
(C) 21.5 (D) 23.5

6. The arithmetic mean of the series 8, 15, 22, 29, 36, 43, 50 is _____
(A) 26 (B) 29 (C) 36 (D) 22

7. If the arithmetic mean of the observations $x_1, x_2, \ldots, x_n$ is 50, then the arithmetic mean of $x_1 - 10, x_2 - 10, x_3 - 10, \ldots x_n - 10$ is $\boxed{}$

8. The arithmetic mean of $x_1, x_2, \ldots, x_{50}$ is k. Therefore, the arithmetic mean of $cx_1 + 8, cx_2 + 8, cx_3 + 8, \ldots, cx_{50} + 8$ is ______.
(A) ck (B) k + 8
(C) ck + 8 (D) k

9. The sum of the deviations about the mean is equal to ______.
(A) 1 (B) 8 (C) 7 (D) 0

10. The median of the first 49 natural numbers is $\boxed{}$
(A) 24 (B) 23 (C) 26 (D) 25

11. If x_1, x_2, x_3 are in arithmetic progression, then the arithmetic mean of x_1 and x_3 is ______.
(A) x_2 (B) x_1
(C) $x_1 + x_3$ (D) x_3

12. The mode of the data 3, 4, 5, 3, 6, 5, x, 3, 4, 3 is ______.
(A) 5
(B) 3
(C) 4
(D) Cannot be determined

13. If y is the median of the data 8, 12, 16, 14, 10, 6 and x, then which of the following is necessarily true?
(A) $10 < y < 12$ (B) $10 \le y \le 12$
(C) $10 \le y < 12$ (D) None of these

14. The mode (s) of the data 3, 5, 7, 3, 6, 5, 3, 9, 5, 6, 7 is/are ______.
(A) 3 alone (B) 5 alone
(C) 7 (D) Both 3 and 5

15. A data that consists of two modes is called _______.
(A) unimodel (B) bimodel
(C) trimodel (D) ill-defined

16. The geometric mean of 12 and 3 is $\boxed{}$

17. The harmonic mean of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ is $\boxed{}$

18. The harmonic mean of 3 and 5 is ______.
(A) 4 (B) 3.75
(C) $\frac{8}{15}$ (D) None of these

19. If A, G, H are the arithmetic mean, the geometric mean and the harmonic mean of any two positive numbers, then the inequality that holds between them is _______.
(A) $A \ge G \ge H$ (B) $A \ge G$ and $G \le H$
(C) $G \ge H \ge A$ (D) $G \ge H \ge A$

20. If the arithmetic mean and the geometric mean of two positive numbers are 12 and 6 respectively, then the harmonic mean of those numbers is _______.
(A) 6 (B) $\sqrt{3}$ (C) 3 (D) 12

21. For a distribution, the arithmetic mean and the mode are 26 and 24 respectively. Find the median.
(A) 50 (B) 26 (C) 24 (D) $\frac{76}{3}$

22. For a symmetric distribution the relation between mean, median and mode is _______.
(A) Mean < Median < Mode
(B) Mean > Median > Mode
(C) Median < Mean < Mode
(D) None of these

23. For a moderately symmetric distribution the difference between mode and median is 24. Find the difference between median and the mean of the distribution.
$\boxed{}$

24. Range of the data 62, 41, 83, 72, 24, 19 is ______.
(A) 72 (B) 64 (C) 59 (D) 62

25. If the range of the observations $x_1, x_2, - - - -, x_n$ is 28, then the range of $x_1 - 10, x_2 - 10, - - -, x_n - 10$ is _______.

$\boxed{}$

26. The first quartile Q_1 of the data 8, 12, 6, 17, 21, 14 and 26 is _______.
(A) 12 (B) 8 (C) 10 (D) 21

27. The third quartile Q_3 of the data 42, 38, 21, 56, 30, 29, 26, 14, 23, 72, 47 is $\boxed{}$

28. The mean deviation of 24 and 36 is $\boxed{}$

29. If σ is the standard deviation of the observations $x_1, x_2 . . . . ., x_{25,}$ then the standard deviation of $3x_1 + 5, 3x_2 + 5 . . . . . . , 3x_{25} + 5$ is _______.
(A) σ (B) 3σ + 5 (C) 3σ – 5 (D) 3σ

30. The standard deviation of 8, 12, 4, 16, 24, 20, 28 is _______.
(A) 4 (B) 6 (C) 8 (D) $\sqrt{8}$

Exercise – 9(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the arithmetic mean of $4^2, 5^2, 6^2, 7^2, 8^2$ and 9^2.
(A) 31.66 (B) 35.6
(C) 44.5 (D) 45.16

2. Find the arithmetic mean of 4, 16, 64,….., 4096.

$\boxed{}$

3. Find the arithmetic mean of 9, 17, 25, 33, .., 105.
(A) 57.5 (B) 58 (C) 57 (D) 58.5

4. The arithmetic mean of a set of 50 numbers is 42. If two numbers 75 and 105 are discarded, the arithmetic mean of the remaining set is _______.
(A) 40 (B) 52 (C) 48 (D) 38.4

5. The arithmetic mean of a set of 7 observations is 36 and that of another set of 13 observations is 46. The mean of the combined set is _______.
(A) 41 (B) 42.5 (C) 41.5 (D) 43.5

6. The harmonic mean of $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{8}, \frac{1}{10}$ is _______.
(A) $\frac{5}{29}$ (B) $\frac{1}{5}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$

7. The geometric mean of $3^1, 3^2, 3^3, ..., 3^{99}$ is _______.
(A) 27^{10} (B) 3^{40}
(C) 9^{25} (D) 9^{15}

8. The median of 15.5213, 15.3125, 15.4132, 15.3215, 15.3142, 15.2413 and 15.3143 is $\boxed{}$

9. The median of 15, 22, 8, 6, 16, 20, 14, 4 is _______.
(A) 15 (B) 15.5
(C) 14.5 (D) 11

10. Find the median of the first 17 prime numbers.
(A) 31 (B) 29 (C) 19 (D) 23

11. The median of a series of 17 numbers is 23, two more numbers 20 and 27 are included to this set. Find the median of the extended set.
(A) 27
(B) 23
(C) 23.5
(D) Cannot be determined

12. Find the range of values that median can take for the series 14, 18, 16, 5, 12, x, 7
(A) [5, 16] (B) [12, 14]
(C) [16, 18] (D) [12, 16]

13. Find the mode of the following set of observations 4, 8, 10, 10, 10, 8, 10, x, 5.
(A) x
(B) 10
(C) 8
(D) Cannot be determined

14. If the mean and the median of a distribution are 1.25 and 4.5 respectively, then the mode is $\boxed{}$

15. The range of the set $S = \{x_1, x_2, x_3, ……, x_n\}$ is r. Find the range of the set $S^+ = \{x_1 + 4, x_2 + 4, x_3 + 4, ……, x_n + 4\}$.
(A) $\frac{r}{4}$ (B) r + 4 (C) r – 4 (D) r

16. The quartile deviation of 15, 9, 10, 13, 16, 5, 20 is

$\boxed{}$

17. The mean deviation of 1, 11, 13, 5, 7, 2, 16, 17 is _______.
(A) 4.75 (B) 5 (C) 4.25 (D) 5.25

18. Find the standard deviation of a, b where a > b.
(A) a – b (B) $\frac{a^2 + b^2}{2}$
(C) $\sqrt{\frac{a^2 - b^2}{2}}$ (D) $\frac{a - b}{2}$

19. The variance of seven consecutive integers is _______.
(A) 4 (B) 1 (C) 2 (D) 13

20. The standard deviation of 18, 18, 18, 18, 18, 18, 18, 18, 18, 23 is _______.
(A) 50.6 (B) 15 (C) 2.25 (D) 1.5

21. The standard deviation of x_1, x_2, x_3,…….x_n is 5. The standard deviation of $3x_1+2$, $3x_2+2$, $3x_3+2$,…….$3x_n+2$ is $\boxed{}$

22. The sum of the cubes of the first n positive integers is 3025. The arithmetic mean of these cubes is _______.
(A) 15.25 (B) 130.25
(C) 306.5 (D) 302.5

23. The algebraic sum of the deviations of 10 numbers taken about 7 is 40. The arithmetic mean of these numbers is $\boxed{}$

24. If the geometric mean of two positive numbers x, y is defined as G = $\sqrt{xy}$ and their harmonic mean is defined as H = 2xy / x + y. If A is the arithmetic mean of x and y, then
(A) $G^2 = AH$ (B) $A^2 = GH$
(C) $H^2 = AG$ (D) $G = A^2H^2$

25. If a = max (x, –x) and b = min (x, –x); x is a real number, then the arithmetic mean of a and b is

$\boxed{}$

Exercise – 9(b)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the arithmetic mean of the first 80 even natural numbers.

$\boxed{}$

2. The arithmetic mean of the first x odd natural numbers is y. Find the arithmetic mean of the first x even natural numbers in terms of y.
(A) y (B) y + 1 (C) y + 2 (D) y + 3

3. Find the harmonic mean of 1, 3, 6, 8, 16 and 32.
(A) $\frac{182}{55}$ (B) $\frac{172}{55}$ (C) $\frac{192}{55}$ (D) $\frac{202}{55}$

4. Find the geometric mean of 1, 6, 6^2, ... and 6^{51}.
(A) $6^{\frac{49}{2}}$ (B) $6^{\frac{51}{2}}$ (C) $6^{\frac{53}{2}}$ (D) 6^{26}

5. Find the mean deviation of 16, 4, 10, 18, 30, 14, 2, 24 and 26.
(A) $\frac{68}{9}$ (B) $\frac{65}{9}$ (C) $\frac{62}{9}$ (D) $\frac{71}{9}$

6. The arithmetic mean of a set of 10 numbers was calculated wrongly as 20 since three observations 14, 12 and 16 were misread as 6, 8 and 10. Find the actual mean.

$\boxed{}$

7. In an office, there are 80 employees. 60 of them have an average wage of ₹4000 per month. The remaining employees have an average wage of ₹5000 per month. Find the average wage of all the employees.
(A) ₹4100 (B) ₹4150 (C) ₹4200 (D) ₹4250

8. Find the standard deviation of 6, 6, 6, 6, 10, 10, 8, 8 and 12.
(A) $\sqrt{\frac{40}{3}}$ (B) $\frac{\sqrt{42}}{3}$ (C) $\frac{\sqrt{46}}{3}$ (D) $\frac{\sqrt{40}}{3}$

9. Find the median of the first 15 prime numbers.

$\boxed{}$

10. The arithmetic mean of the series y_1, y_2, y_3, . . . y_n is A. Find the arithmetic mean of the series $\frac{3y_1+2}{4}, \frac{3y_2+2}{4}, \ldots\ldots \frac{3y_n+2}{4}$
(A) $\frac{3A+2}{4}$ (B) $\frac{3A-2}{4}$
(C) $A-\frac{1}{2}$ (D) $A+\frac{1}{2}$

11. The arithmetic mean of a set of 25 observations is 24. Two of the observations whose values were 35 and 36 were discarded. Find the arithmetic mean of the remaining numbers.
(A) 22 (B) 23 (C) 24 (D) 25

12. Thirty observations have their range as 60. Each of them is tripled. Find the range of the new set of observations.

$\boxed{}$

13. Twenty three observations had their median as 60. Three was subtracted from each. Find the median of the new set of observations.
(A) 57 (B) 60 (C) 63 (D) 30

14. Find the quartile deviation of 2, 5, 7, 10, 12, 16, 20, 24, 29, 33, 37.

$\boxed{}$

15. If 5, 4 and 6 occur with frequencies of 2, 3 and 4 respectively, find their arithmetic mean.
(A) $\frac{44}{9}$ (B) $\frac{50}{9}$ (C) $\frac{52}{9}$ (D) $\frac{46}{9}$

16. Find the mode of the data 3, 4, 5, 6, 7, 5, 7, 8, 8, 9, 8, 9, 4 and 3.

$\boxed{}$

17. Find the arithmetic mean of the multiples of 8 between 100 and 200.
(A) 140 (B) 132 (C) 148 (D) 156

18. The standard deviation of a series of n-observations is σ. If each observation is divided by 3, the standard deviation of the new series is _______.
(A) σ (B) $\frac{\sigma}{3}$ (C) 3σ (D) 9σ

19. Raju ran once around a square plot ABCD. He ran along AB at an average speed of S_1 kmph and ran along BC at an average speed of S_2 kmph. His average speed of running from A to C is _______.
(A) the arithmetic mean of S_1 and S_2.
(B) the geometric mean of S_1 and S_2.
(C) the harmonic mean of S_1 and S_2.
(D) the arithmetic mean of $\frac{1}{S_1}$ and $\frac{1}{S_2}$.

20. The terms $Y_1, Y_2, \ldots, Y_n$ have an arithmetic mean of A.
Let Z_i = AM(Y_i, Y_{i+1}, Y_{i+2}, Y_{i+3}) for1 ≤ i ≤ n – 3.
Z_{n-2} =AM(Y_{n-2} Y_{n-1}, Y_n, Y_1)
Z_{n-1} =AM(Y_{n-1} Y_n, Y_1, Y_2)
Z_n = A.M(Y_n, Y_1, Y_2, Y_3).

Find the arithmetic mean of $Z_1, Z_2, \ldots Z_n$. (Assume n > 10) $\boxed{}$ A

21. Find the arithmetic mean of the cubes of the first n even natural numbers.
(A) $2n^2(n + 1)^2$ (B) $2n(n + 1)^2$
(C) $2n^2(n + 1)^2$ (D) $n^2(n + 1)^2$

22. The arithmetic mean of the series $Y_1, Y_2, \ldots Y_n$ is M. If Y_{n-1} is replaced by Y_r, the arithmetic mean of the new series will be _______.
(A) $\frac{Mn - Y_r + Y_n - 1}{n}$ (B) $\frac{Mn - Y_r - Y_n - 1}{}$
(C) $\frac{nM + Y_r + Y_{n-1}}{n}$ (D) $\frac{Mn + Y_r - Y_{n-1}}{n}$

23. The arithmetic mean of the series $x_1, x_2, \ldots x_n$ is x. The arithmetic mean of the series $y_1, y_2, \ldots y_n$ is y. Find the arithmetic mean of the series $x_1 - y_1, x_2 - y_2, \ldots\ldots x_n - y_n$.
(A) x – y (B) $\frac{x-y}{n}$ (C) n(x – y) (D) $\frac{x-y}{2}$

24. The arithmetic mean of a set of 150 observations is 30.The arithmetic mean of 100 of them is also 30. Find the arithmetic mean of the remaining observations.

$\boxed{}$

25. The sum of the squares of the deviations of eleven observations about their arithmetic mean is 110. Find the standard deviation of the observations.
(A) $\sqrt{11}$ (B) $\sqrt{10}$ (C) $\sqrt{5}$ (D) $\sqrt{22}$

Key

Concept Review Questions

1. C	6. B	11. A	16. 6	21. D	26. B
2. B	7. 40	12. B	17. 0.2	22. D	27. 47
3. A	8. C	13. B	18. B	23. 12	28. 6
4. 20	9. D	14. D	19. A	24. B	29. D
5. D	10. 25	15. B	20. C	25. 28	30. C

Exercise – 9(a)

1. D	6. C	11. B	16. 3.5	21. 15
2. 910	7. C	12. B	17. D	22. D
3. C	8. 15.3143	13. B	18. D	23. 11
4. A	9. C	14. 11	19. A	24. A
5. B	10. D	15. D	20. D	25. 0

Exercise – 9(b)

1. 81	6. 21.8	11. B	16. 8	21. B
2. B	7. D	12. 180	17. C	22. D
3. C	8. D	13. A	18. B	23. A
4. B	9. 19	14. 11	19. C	24. 30
5. A	10. A	15. D	20. 1	25. B