CHAPTER – 2

Squares and Cubes

In competitive examinations, there can be questions on direct application of squares, cubes, square-roots and cube-roots. For example, there can be a question which asks you to find the tens-digit of a four-digit perfect square. Also, an understanding of Squares and Cubes of useful while performing calculations.

Remembering squares (upto first 25 natural numbers), cubes (upto first 12 natural numbers) is very important in calculations. By remembering these (squares upto 25), one can calculate squares of any natural number from 26 to 125 in no time, which in turn will help in solving some other questions too. Similarly, by remembering cubes (upto 12) one can calculate cubes of any two-digit number with greater speed. Given below are some methods for finding squares and cubes of numbers.

How to find the square of a number ending in 5:

Getting the square of a number ending in 5 is very simple. If the last digit of the number is 5, the last two digits of the square will be 25. Consider the earlier part of the number and multiply it with one more than itself and that product will be the first part of the answer. (The second part of the answer will be 25 itself).

$35^2 = 1225$ (Here, $3 \times 4 = 12$, so the answer is 1225)
$45^2 = 2025$
$55^2 = 3025$
$75^2 = 5625$
$95^2 = 9025$
$125^2 = 15625$
$175^2 = 30625$
$235^2 = 55225$
$195^2 = 38025$
$245^2 = 60025$

So, now we know the squares of numbers 25, 30, 35, 40, 45, 50, 55, 60, 65, etc. If we want to find the squares of any other number, we can find it using these squares which we already know.

To find the square of a number which is one more than the number whose square we already know:

For 26^2, we will go from 25^2; for 31^2 we go from 30^2 and so on.

One way is by writing $26^2 = (25+1)^2$. But we need not even calculate $(a + b)^2$ by adopting the following method;

$26^2 = 25^2 + 26^{th}$ odd number, i.e., $625 + 51 = 676$
$(a + b)^2 = a^2 + 2ab + b^2$
$26^2 = (25 + 1)^2$
$(25)^2 + 2(25 \times 1) + (1)^2$
$625 + 50 + 1 = 625 + 51 = 676$

But we will look at a different method which will enable the student perform the calculations for squares mentally.

$1^2 = 1 = 1$
$2^2 = 4 = 1 + 3$
$3^2 = 9 = 1 + 3 + 5$
$4^2 = 16 = 1 + 3 + 5 + 7$
$5^2 = 25 = 1 + 3 + 5 + 7 + 9$

i.e., to get n^2, we add up the FIRST n odd numbers. If we want 13^2, it will be the sum of the FIRST 13 odd numbers.

n^{th} odd number is equal to $(2n - 1)$.

Suppose we want to find out 6^2, knowing what 5^2 is, we can move from 5^2 to 6^2.

6^2 will be the sum of 1^{st} 6 odd numbers. But the sum of the first 6 odd numbers can be written as "sum of the first 5 odd numbers" + "sixth odd number". Since we already know that the sum of the FIRST 5 odd numbers is 5^2, i.e., 25, we need to add the sixth odd number i.e., $(2 \times 6 – 1 =)$ 11 to 25 to give us $6^2 = 36$.

Similarly

$31^2 = 900 + 31$st odd number $= 900 + 61 = 961$

$36^2 = 1225 + 36$th odd number $= 1225 + 71$

$= 1296$ (Since $35^2 = 1225$)

$41^2 = 1600 + 81 = 1681$

$46^2 = 2025 + 91 = 2116$

$126^2 = 15625 + 251 = 15876$

$196^2 = 38025 + 391 = 38416$

$216^2 = 46225 + 431 = 46656$

We have now seen how to find the squares of numbers which are one more than those numbers whose squares we already know (eg. 25, 30, 35, etc.)

To find the square of a number which is one less than the number whose squares we already know

Similarly, we can find the squares of numbers which are one less than the numbers whose squares are known. For example,

$29^2 = 30^2 – 30^{th}$ odd number
$= 900 – 59 = 841$
$39^2 = 40^2 – 40^{th}$ odd number $= 1600 – 79 = 1521$
$34^2 = 1225 – 69 = 1156$
$54^2 = 3025 – 109 = 2916$
$74^2 = 5625 – 149 = 5476$
$94^2 = 9025 – 189 = 8836$
$214^2 = 46225 – 429 = 45796$

Thus, we have seen how to arrive at the squares of numbers which are one more or one less than the numbers whose squares we already know (i.e., 25, 30, 35, 40, 45, 50, 55, etc.).

To find the square of a number which is 2 more than the number whose squares we already know:

Now, we will see how to get the squares of numbers which are 2 more (or less) than the numbers whose squares we already know.

$27^2 = 26^2 + 27^{th}$ odd number $= 25^2 + 26^{th}$ odd number + 27^{th} odd number.

The sum of the 26^{th} odd number and 27^{th} odd number is the same as 4 times 26. Hence

$27^2 = 25^2 + 4 \times 26 = 625 + 104 = 729$

$57^2 = 3025 + 224$ (4 times 56) $= 3249$

$77^2 = 5625 + 304$ (4 times 76) $= 5929$

$97^2 = 9025 + 384$ (4 times 96) $= 9409$

To find the square of a number which is 2 less than the number whose squares we already know

Similarly, we can find out the squares of numbers which are 2 less than the numbers whose squares we know.

28^2 = (30^2 – 4 times 29) = 900 – 116 = 784
53^2 = (55^2 – 4 times 54) = 3025 – 216 = 2809
93^2 = 9025 – 376 = 8649
243^2 = 60025 – 976 = 59049
143^2 =(145^2– 4 x 144) = 21025 – 576 = 20449

To find the square of a number from 26 to 50

The squares of numbers from 26 to 50 can be calculated by writing down and adding two parts as explained below:

The first part is as many times 100 as the number is more than 25, for example in finding 31^2, as 31 is 6 more than 25, the first part is 100 x 6 = 600

The second part is the square of the number that is as much less than 25 as the number is more than 25, i.e., in finding 31^2, the second part is the square of 6 less than 25, i.e., $(25 – 6)^2 = 19^2$ = 361
Hence, 31^2 = First part + Second part = 600 + 361 = 961

The above method can be summarized as

1. Finding 31^2
   i. 31 = 25 + <u>6</u>
   ii. 25 – 6 $\rightarrow$ 19^2 $\rightarrow$ 361
   iii. 31^2 = 6 x 100 + 361 = 961

2. Finding 33^2
   i. 33 = 25 + <u>8</u>
   ii. 25 – 8 $\rightarrow$ 17^2 $\rightarrow$ 289
   iii. 31^2 = 8 x 100 + 289 = 1089

To find the square of a number from 51 to 75:

The squares of numbers from 51 to 75 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits which are to the extreme right of the square of the number by which the given number is more than 50.

For example, in finding 63^2, as 63 is 13 more than 50, the second part will be the two digits to the extreme right of 13^2 (= 1<u>69</u>), i.e. 69. Since there are more than two digits in 13^2, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any, from the second part and (ii) the sum of 25 (for this range (i.e., 51 to 75) 25 is taken as the base) and the number by which the given number is more than 50.
As 63 is 13 more than 50, the first part will be 25 + 13 + 1 (carry forward from the second part) = 39
Therefore 63^2 = <u>39</u> <u>69</u>

The above can be summarized as

First Part Second Part

1. 63^2 = (25 + 13) / 13^2 = 38 / <u>69</u> = 38 + 1 / 69 = <u>39</u> <u>69</u>
   First Part Second Part
   C.F

2. 61^2 = (25 + 11) / 11^2 = 36 / <u>21</u> = <u>37</u> <u>21</u>
   C.F
   First Part Second Part

3. 56^2 = (25 + 6) / 6^2 = 31/36 = <u>31</u> <u>36</u>

Note: This process is faster for Squares in the range of 50 to 60, as there is no carry forward.

To find the square of a number from 76 to 100:

The squares of numbers from 76 to 100 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits to the extreme right of the square of the number by which the given number is less than 100.
For example in 88^2, as 88 is 12 less than 100, the second part will be the two digits to the extreme right of 12^2(= 1<u>44</u>), i.e., 44. Since there are more than two digits in 12^2, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any, from the second part and (ii) the difference between the given number and the number by which the given number is less than 100. As 88 is 12 less than 100, the first part will be 88 – 12 + 1(carry forward from the second part) = 77.
Therefore, 88^2 = <u>77</u> <u>44</u>

The above can be summarized as

First Part Second Part

1. 88^2 = (88 - 12) / 12^2 = 76 / 44 = (76 + 1) / 44 = <u>77</u> <u>44</u>
   C.F
   First Part Second Part

2. 89^2 = (89 - 11) / 11^2 = 78 / 21 = <u>79</u> <u>21</u>
   C.F
   First Part Second Part

3. 96^2 = (96 - 4) / 4^2 = 92/16 = <u>92</u> <u>16</u>

Note: This process is faster for Squares in the range of 90 to 100, as there is no carry forward.

To find the square of a number from 101 to 125:

The squares of numbers from 101 to 125 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits to the extreme right of the square of the number by which the given number is more than 100.

For example, finding 112^2, as 112 is 12 more than 100, the second part will be the two digits of 12^2(= 144), i.e., 44. Since there are more than two digits in 12^2, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any from the second part and (ii) the sum of the given number and the number by which the given number is more than 100.
As 112 is 12 more than 100, the first part will be 112 + 12 + 1 (carry forward from the first part) = 125

Therefore, 88^2 = <u>125</u> <u>44</u>
The above can be summarized as

First Part Second Part

1. $112^2 = (112 + 12) / 12^2 = 124 / 44 = (124 + 1) / 44 = \underline{125}\ \underline{44}$

C.F

First Part Second Part

2. $113^2 = (113 + 13) / 13^2 = 126 / 169 = \underline{127}\ \underline{69}$

C.F

3. $106^2 = (106 + 6) / 6^2 = 112/36 = \underline{112}\ \underline{36}$

By observing and remembering a few properties regarding the behaviour of the last digits of numbers and of their squares and cubes, it is sometimes possible to solve certain kinds of questions. Hence, some of the important properties of the last digits of numbers are given below:

Last digit of any number	0	1	2	3	4	5	6	7	8	9
Last digit of its square	0	1	4	9	6	5	6	9	4	1
Last digit of its cube	0	1	8	7	4	5	6	3	2	9

Some important observations:

1. The square of a number can never end with 2, 3, 7 or 8.
2. Any power of any number ending in 0, 1, 5 or 6 ends with 0, 1, 5, 6 respectively.
3. If the last digits of two numbers are 10's complements, then the last digits of their squares will be equal. Hence, if the last digit of the square of a number is given, it is not possible to determine the last digit of that number uniquely. For example if n^2 ends with 9, n may end with 3 or 7.
4. We can uniquely determine the last digit of a number given the cube of that number, for example $(\ldots)^3$ = __3, the number can end only in 7.
5. If the last digits of two numbers are 10's complements then last digits of their cubes will be also 10's complements.
6. The square of a number with only 1's will always be a palindrome. For example :
   $11^2 = 121$; $111^2 = 12321$; $1111^2 = 1234321$.
7. The last two digits of any power of a number ending in 25 or 76 always end in 25 and 76 respectively.
8. a. If the square of any number is ending in 1, then the ten's digit of that square should be an even number.
   b. If the square of any number is ending in 4, then the ten's digit of that square should be an even number.
   c. If the square of any number is ending in 5, then the ten's digit of that square should be 2.
   d. If the square of any number is ending in 6, then the ten's digit of that square should be an odd number.
   e. If the square of any number is ending in 9, then the ten's digit of that square should be an even number.

Finding the cube of a number:

The following method is useful for finding the cube of a two-digit number.

2.01. Find the cube of 12.

Step 1: Cube the left most digit, i.e., 1 in this case, and write it down on the extreme left.

Step 2: Write three more numbers to its right such that the ratio of successive pairs of numbers is same as the ratio of the digits (1 : 2) in the original number. We get the following <u>1</u> <u>2</u> <u>4</u> <u>8</u>. (1 : 2 = 2 : 4 = 4 : 8)

Step 3: Double the second number (i.e., 2) and the third number (i.e., 4) of the above four numbers and write the result (i.e., $2 \times 2 = 4$ and $2 \times 4 = 8$) under the respective numbers.

Step 4: Add the two rows – one column at a time – such that each column contributes only one digit to the total. (If any column gives more than one digit, the additional digits are carried forward)

```
        1  <-----  carry forward
        1   2   4   8
            4   8
        -------------------
12³ =   1   7   2   8
        -------------------
```

2.02. Find the cube of 23.

Step 1 : Cube the left most digit (i.e., $2^3 = 8$) and write it down on the extreme left.

Step 2 : Write three numbers next to the above, such that the ratio between any two successive numbers is the same as the ratio of the digits of the given number. (Therefore in the number 23, the ratio of the digits is 2 : 3). We get <u>8</u> <u>12</u> <u>18</u> <u>27</u>

Note: It may sometimes be difficult to find the numbers, i.e.. 12, 18 and 27. Note that these numbers are obtained, as $12 = 8 \times 3/2$; $18 = 12 \times 3/2$; $27 = 18 \times 3/2$. i.e., to get any number, multiply the previous number by the units digit value (i.e., 3) and divide by the ten's digit value (i.e., 2).

Step 3 : Double the 2^{nd} number (i.e., 12) and the 3^{rd} number (i.e., 18) and write them down below the respective numbers.

```
8   12   18   27
    24   36
```

Step 4 : Add all the numbers, column wise, as shown below, each time carrying forward all digits except the units digit.

```
4    5    2  <-----  carry forward
8   12   18   27
         24   36
-------------------
12   1    6    7
-------------------
```

$\therefore 23^3 = 12167$

2.03. Find the cube of 37.

23	47	34	⟵ carry forward	
<u>27</u>	<u>63</u>	<u>147</u>	<u>343</u>	
	<u>126</u>	<u>294</u>		
50	6	5	3	

$\therefore 37^3 = 50653$

Square Root of a Perfect Square

We will see how to find the square root of a perfect square by "square root division method." Let us find the square root of 95824521.

In normal division, we take one digit at a time from the dividend for the purpose of division. In square root division, we take two digits at a time from the given number. For this purpose, we first mark blocks of two digits in the given number starting from the units digit.
The number 95824521 will then look like

$\overline{95}\;\overline{82}\;\overline{45}\;\overline{21}$

(If the number of digits in the number is odd, then the last block, i.e., the extreme left block, will have only one digit)

$\big)\overline{\overline{95}\;\overline{82}\;\overline{45}\;\overline{21}}$

In normal division, the divisor is given and it is the same in each step. However, in square root division, the divisor in all the steps is not the same and it will have to be built separately in each step. We will first build the divisor in the first step in this case. The divisor in the first step is the integral part of the square root of the first block of two digits. Since the square root of 95 will be between 9 and 10, the divisor in the first step is 9. We write this as the divisor in the divison shown above and also on the top of the horizontal line as shown below:

```
     9
9)95 82 45 21
```

The figure that we will finally get on the top of the line above (shown by arrow mark) will be the square root of the given number.
The 9 written as divisor and the digit 9 written in the quotient are multiplied to give 81 which is then subtracted from the block of two digits 95 giving a remainder of 14 (= 95 – 81)

```
     9
9)95 82 45 21
  81
  ––––
  1482
```

Now we bring down the next block of two digits 82 thus giving us 1482 as the dividend in the next step.
Now we will have to build the divisor for this step. The divisor in any step during the square root division has two parts and is built as follows:

Take the divisor in the previous step and add its last digit to itself. This gives the first part of the divisor. In this case, take the divisor 9 and add 9 to itself getting 18 as the first part of the divisor.

```
         9k
   9     95 82 45 21
         81
         --------
18k      1482
```

Now we need the second part of the divisor which will come in the place of the letter k (shown by arrow mark) in this step.

To get the second part of the divisor, we need to find a digit so that it can come in place of k in the divisor as well as the quotient such that the product of the entire divisor (including the digit coming in place of k) and the digit k is the greatest possible number which is less than or equal to 1482 (which is the number being divided in this step). In this case we find that if we take the digit 7 in place of k, 187 multiplied by 7 gives 1309 which is less than 1482. However if we take 8 in place of k, we have 188 multiplied by 8 giving us 1504 which is greater than 1482. So, we need to take the digit 7 in place of k and the product 1309 subtracted from 1482 gives a remainder of 173 as shown below.

```
       97
    9)95 82 45 21
      81
      –––––
 187) 1482
      1309
      –––––
      17345
```

Now we bring down the next block of two digits 45 next to the remainder of 173 giving us the new number 17345 as the dividend.

This process of building the divisor in the step from the divisor of the previous step and finding a new digit as the second part of the divisor should be continued in the same manner as explained above. In this case, the first part of the divisor is 187 + 7 (which is the last digit of the divisor) giving us 194. Now a digit (k) has to be selected such that the number having 194 followed by that digit (i.e., the number 194k) multiplied by that digit itself i.e., k, will give the largest product less than or equal to 17345.

```
        97k
     9)95 82 45 21
       81
       –––––
  187) 1482
       1309
       –––––
 194k) 17345
```

We find that k = 8 will give 1948 x 8 = 15584 where as for k = 9, we get 1949 x 9 = 17541 which is greater than 17345.

Hence the value of k is taken as 8 and the product 15584 is subtracted from 17345 giving a remainder of 1761. We now bring down the next block of two digits 21 giving us the number to be divided in the next step as 176121.

The first part of the divisor in this step is 1956
(= 1948 + the last digit 8).

```
           978k
       9)95 82 45 21
         81
         -----
    187) 1482
         1309
         -----
   1948) 17345
         15584
         -----
  1956k) 176121
```

Now we need to find a value for k such that 1956k multiplied by k should be less than or equal to 176121. We find that if we take k equal to 9, 19569 x 9 = 176121 giving us a remainder 0.

```
           9789
       9)95 82 45 21
         81
         -----
    187) 1482
         1309
         -----
   1948) 17345
         15584
         -----
  19569) 176121
         176121
         -----
              0
```

At this stage, there are no more digits to bring down. At the stage where there are no more digits to bring down, if the remainder is 0, then the given number is a perfect square and the quotient which we wrote at the top is the square root. (When there are no more digits to be brought down, if the remainder is not 0, the given number is not a perfect square).
In this case, the given number is a perfect square and the square root is 9789

Square Root of any Number

If the given number is not a perfect square, then the square root can be found to any desired decimal place when the divisior does not terminate. The procedure for finding out the square root is the same as for a perfect square till all the digits in the given number have been brought down. At this stage where there are no more digits to bring down and the remainder is not zero, we **put a decimal point in the quotient** and take TWO zeroes in number to be divided in that particular step (i.e., it is treated as if we have blocks of two zeroes at the end of the given number) and we just continue the square root division as usual till the required number of decimal places are obtained in the quotient. (Compare this process with normal division where, if all the digits are used up and we still have some remainder, we put a decimal point in the quotient and then keep bringing down a zero in each step and continue the division process. The only difference is that we take blocks of two zeroes in square root division. The process of building the divisor in each step does not change at any stage of the square root division).

If the given number itself has some decimal places, then the process of making the blocks of two digits will start at the decimal point and proceed with two digits at a time to the left of the decimal **as well as** to the right of the decimal point. For example, if we have to find the square root of the number 2738474.235, then making the blocks of two digits will be as shown below:

$$\overline{2}\ \overline{73}\ \overline{84}\ \overline{74}\ .\ \overline{23}\ \overline{5}$$

On the left side of the decimal place, the extreme left block has only one digit and it will be dealt with as it is (as discussed in case of perfect squares). On the right side of the decimal place, on the extreme right, the block has only one digit 5. In the square root division, this will be treated as "50" which will not change the value of the number, but gives a block of two digits.

Powers of 2 and 3

Remembering powers of 2 upto 12 and powers of 3 upto 8 will be of great help. It has been observed that various competitive examinations have direct questions on the application of these.

Property for the powers of 2:

$2^0 = 1$,	$2^1 = 2$,	$2^2 = 4$,	$2^3 = 8$
$2^4 = 16$,	25 = 32,	$2^6 = 64$,	$2^7 = 128$
$2^8 = 256$,	$2^9 = 512$,	$2^{10} = 1024$,	$2^{11} = 2048$

By observing the following, we can see that

$2^0 + 2^1 = 3 = 2^2 – 1$
$2^0 + 2^1 + 2^2 = 7 = 2^3 – 1$
$2^0 + 2^1 + 2^2 + 2^3 = 15 = 2^4 – 1$
$2^0 + 2^1 + 2^2 + + 2^3 + 2^4 = 31 = 2^5 – 1$

Similarly,

$2^0 + 2^1 + 2^2 + + 2^3 + 2^4 = 31 = 2^5 – 1$
That is, the sum of powers of 2 from 0 to any number k will be equal to $2^{k+1} – 1$.
The above concept can be used in the following example:

For example the sum $2^0 + 2^1 + \ldots\ldots 2^n$ is equal to $(2^{n+1} – 1)$. This can help us arrive at the answer to a question like “If ten brothers have some marbles each, such that every brother, except the youngest, has twice the number of marbles than those that the brother immediately younger to him has, then find the least possible total number of marbles with the ten brothers”.

To have the least total, the youngest should have the least number of marbles i.e. only one marble.

The second youngest will have 2 (i.e., 2^1) the next brother will have 4 (i.e., 2^2) and so on.

The eldest will have 2^{10}. The sum of all the marbles with them will be $2^0 + 2^1 + 2^2 + \ldots\ldots 2^{10} = 2^{10+1} – 1 = 2^{11} – 1 = 2048 – 1 = 2047$.

2.04. A trader uses only five weights which together weigh 31 kg. With these five weights he can measure all integer weights from 1 kg to 31 kg, with the weight kept only in one pan of the weighing scale. Find the individual weights of the five pieces?

Sol. For measuring all integer weights upto 31 kg, the individual weights needed are the powers of 2, i.e. 2^0, 2^1, 2^2, 2^3 and 2^4

$\therefore$ if we have weights of 1 kg, 2 kg, 4 kg,, 8 kg and 16 kg, we can measure all integer weights upto 31 kg.

For example if we have to measure 23 kg, we have to use the weights 16 kg, 4 kg, 2 kg and 1 kg on one pan.

If you want to write any number from 1 to M as a sum of one or more of the integers of a given set of integers (each integer being used at the most once), it can be done by using the powers of 2. The set of integers we can use consists of all the powers of 2 starting from 1 (i.e. 2^0) to the largest power of 2 less than or equal to M. For example, if you want to build all the integers upto 255, the numbers 1, 2, 4, 8, 16, 32, 64, 128 are sufficient.

Property for the powers of 3:

$3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$
$3^4 = 81$, $3^5 = 243$, $3^6 = 729$, $3^7 = 2187$

$3^0 + 3^1 + 3^2 + 3^3 = 40$

Using a combination of these numbers, each occurring at the most once, we can obtain all the numbers from 1 to 40 by using the operation of only addition and/or subtraction.

The above concept can be used in the following example

2.05. A trader uses only four weights, which together weigh 40 kg. With the four weights he could measure all integer weights from 1 kg to 40 kg, placing weights in both the pans. Find the weights of the four pieces?

Sol. For measuring all weights upto 40 kg, the weights needed are the powers of 3 whose sum adds upto 40, i.e., 3^0, 3^1, 3^2 and 3^3 . . . if we have weights of 1 kg, 3 kg, 9 kg and 27 kg, we can measure all weights from 1 to 40 kg

For example, if we have to measure 33 kg, we have to keep the 27 kg and 9 kg weights on one pan and 3 kg weight on the other i.e., 27 + 9 – 3 = 33 kg.

Exercise – 2(a)

Questions 1 to 25: Squares/Cubes

1. $(503)^2 = ?$

2. $(132)^2 = ?$

3. $(484)^2 - (316)^2 = ?$

4. $(398)^2 = ?$

5. $(1008)^2 = ?$

6. $(10.12)^2 = ?$

7. $(392)^2 = ?$

8. $(14.85)^2 = ?$

9. $(264)^2 = ?$

10. $(596)^2 = ?$

11. $(117)^2 = ?$

12. $(69)^2 = ?$

13. $(414)^2 = ?$

14. $(235)^2 = ?$

15. $473^2 = ?$

16. $126^2 = ?$

17. $4444^2 = ?$

18. $576^2 = ?$

19. $891^2 + 819^2 = ?$

20. $(1531)^2 = ?$

21. $2652 \times 2652 = ?$

22. $275^2 = ?$

23. $146^2 = ?$

24. $24^3 + 18^3 = ?$

25. $19^3 - 18^3 = ?$

Directions for questions 26 to 45: Select the correct alternative from the given choices.

26. $\dfrac{(5.672)^3 + (4.278)^3}{(5.672)^2 - (5.672)\times(4.278) + (4.278)^2}$ is equal to ___________.

(A) 1.39 (B) 2.43 (C) 5.65 (D) 9.95

27. The value of $(4.95)^3 - (4.00)^3 - (0.95)^3$ is _________.

(A) 56.43 (B) 363.86 (C) 192 (D) 13.4

28. $\sqrt{4762} \times 79.23$ is approximately equal to

(A) 4515 (B) 5514 (C) 5451 (D) 5530

29. If $\sqrt[3]{8100} + 375 = x \div 24.93$, approximate value of x is

(A) 8785 (B) 9875 (C) 9785 (D) 9900

30. $\sqrt[3]{512680} + \sqrt{379} + \sqrt{580}$ is approximately

(A) 104 (B) 100 (C) 99 (D) 124

31. $(445)^2 \times 4^3 + 35625 = (?)^2$

(A) 3500 (B) 3600 (C) 3650 (D) 3565

32. $\sqrt[3]{\dfrac{?}{32768}} = \dfrac{15}{32}$

(A) 3375 (B) 3500 (C) 4560 (D) 4975

33. $\dfrac{64^2 \times (29^2 - 20^2)}{441} = ?$

(A) 64 (B) 46 (C) 64^2 (D) 86

34. $\sqrt[3]{343500} + \sqrt[3]{216600} + \sqrt[3]{125400}$ is approximately

(A) 70 (B) 120 (C) 150 (D) 180

35. $\dfrac{(3.7)^3 + (1.3)^3}{13.69 - 4.81 + 1.69} = ?$

(A) 5 (B) 3 (C) 2 (D) 6

36. $(34.41)^2 + (26.64)^2 - (17.98)^2$ is approximately
(A) 1460 (B) 1560 (C) 1660 (D) 1760

37. $\sqrt{484} \times \sqrt{1681} \div \sqrt{121} = ?$
(A) 28 (B) 62 (C) 82 (D) 86

38. $\sqrt{?} + 28 = \sqrt{4096}$
(A) 1681 (B) 41 (C) 1861 (D) 1296

39. $\sqrt{4096} + \sqrt{2916} = \sqrt{9801} + \sqrt{?}$
(A) 234 (B) 361 (C) 432 (D) 564

40. $\sqrt[3]{205379} = ?$
(A) 39 (B) 49 (C) 53 (D) 59

41. $(81)^2 \div \sqrt{?} = 3^4 \times 9$
(A) 81 (B) 27 (C) 9 (D) 243

42. $(?)^3 + 7^3 \times 15 = (96)^2 + 5^2$
(A) 14 (B) 16 (C) 18 (D) 12

43. $\sqrt{42025} \times \sqrt{3481} - (83)^2 = \sqrt{?} + (72)^2$
(A) 488 (B) 484 (C) 464 (D) 488

44. $\sqrt{23716} + \sqrt{4096} = ? \div 32$
(A) 6796 (B) 6676 (C) 6900 (D) 6976

45. $\sqrt{46656} + \sqrt{4096} \times 52 = (?)^2 + 115$
(A) 48 (B) 52 (C) 57 (D) 68

Exercise – 2(b)

Questions 1 to 22: Squares

1. $308^2 = ?$

2. $(1962)^2 = ?$

3. $131^2 + 169^2 = ?$

4. $1024^2 - 576^2 = ?$

5. $(1667)^2 = ?$

6. $83^2 = ?$

7. $127^2 = ?$

8. $39^2 - 27^2 = ?$

9. $(491)^2 = ?$

10. $45^2 - 30^2 = ?$

11. $(152)^2 = ?$

12. $72^2 - 48^2 = ?$

13. $75^2 - 65^2 = ?$

14. $(212)^2 = ?$

15. $(153)^2 = ?$

16. $(412)^2 = ?$

17. $(788)^2 = ?$

18. The square of 651 is

19. The square of 738 is

20. $(128)^2 = ?$

21. $(632)^2 = ?$

22. $(239)^2 = ?$

Directions for questions 23 to 45: Select the correct alternative from the given choices.

23. $40^2 + 42^2 = (?)^2$
(A) 43 (B) 44 (C) 58 (D) 81

24. $32^2 \times 7^2 - 126^2 = (?) \times 5^2$
(A) 1372 (B) 1271 (C) 1475 (D) 1732

25. $(?)^2 \times 36 = (294)^2$
(A) 47 (B) 49 (C) 64 (D) 81

26. $\sqrt[3]{140608} \times ? = 1820$
(A) 25 (B) 28 (C) 35 (D) 45

27. $(49)^2 + ?^2 = 3564 + 2933$
(A) 52 (B) 54 (C) 58 (D) 64

28. $\dfrac{35^2 - 15^2}{6^2 + 8^2} = ?$
(A) 10 (B) 50 (C) 100 (D) 1000

29. $[(14)^3 \times (?)^2] \div 56 = 2401$
(A) 5 (B) 6 (C) 7 (D) 9

30. $(39)^2 - (60)^2 + (48)^2 = ?$
(A) 105 (B) 225 (C) 250 (D) 275

31. $\sqrt{6400} \times \sqrt{1296} \div \sqrt{8100} = ?$
(A) 32 (B) 23 (C) 34 (D) 48

32. $(225)^2 \div \sqrt[3]{15625} = ?^2$
(A) 25 (B) 35 (C) 45 (D) 55

33. $\dfrac{\sqrt{0.36 \times 0.000081}}{0.0018} \times 100 = ?$
(A) 0.3 (B) 0.1 (C) 3 (D) 300

34. $\sqrt{\dfrac{2916}{256}} = \dfrac{?}{\sqrt{2304}}$
(A) 126 (B) 162 (C) 262 (D) 178

35. $\sqrt{(20.16 \times 100) \div 63 \div 8} = ?$
(A) 1 (B) 2 (C) 3 (D) 4

36. $68^3 \times 4^3 - (3578)^2 = ?$
(A) 3721564 (B) 217364
(C) 7321564 (D) 78321564

37. $\sqrt{725200 \div 45325} = ? \times 4$

(A) 0 (B) 1 (C) 2 (D) 3

38. $\sqrt{5.0625} \times \sqrt{10.24} \times ? = 468$

(A) 65 (B) 56 (C) 75 (D) 57

39. $\dfrac{35^2 - 15^2 + 8^2}{28} = ?$

(A) 18 (B) 81 (C) 83 (D) 38

40. $\sqrt{729.03} - \sqrt{523.98} = ?$

(A) 1 (B) 2 (C) 3 (D) 4

41. $\sqrt{121.32} + \sqrt[3]{728.98} = ?$

(A) 2 (B) 4 (C) 6 (D) 20

42. $\sqrt[3]{1330.988} - \sqrt{81.032} = ?$

(A) 1 (B) 2 (C) 4 (D) 5

43. $\sqrt{9218} \times \sqrt{2210} \div \sqrt{1028} = ?$

(A) 104 (B) 114 (C) 141 (D) 144

44. $(63)^2 - (45)^2 = (?)^2 - 81$

(A) 5 (B) 35 (C) 45 (D) 65

45. $(84)^3 - (84)^2 = ?$

(A) 585468 (B) 558468 (C) 558648 (D) 585648

Key

Exercise – 2(a)

1. 253009	9. 69696	17. 19749136	25. 1027	33. C	41. A
2. 17424	10. 355216	18. 331776	26. D	34. D	42. B
3. 134400	11. 13689	19. 1464642	27. A	35. A	43. B
4. 158404	12. 4761	20. 2343961	28. C	36. B	44. D
5. 1016064	13. 171396	21. 7033104	29. B	37. C	45. C
6. 102.4144	14. 55225	22. 75625	30. D	38. D	
7. 153664	15. 223729	23. 21316	31. D	39. B	
8. 220.5225	16. 15876	24. 19656	32. A	40. D	

Exercise – 2(b)

1. 94864	9. 241081	17. 620944	25. B	33. D	41. D
2. 3849444	10. 1125	18. 423801	26. C	34. B	42. B
3. 45722	11. 23104	19. 544644	27. D	35. B	43. C
4. 716800	12. 2880	20. 16384	28. A	36. C	44. C
5. 2778889	13. 1400	21. 399424	29. C	37. B	45. D
6. 6889	14. 44944	22. 57121	30. B	38. A	
7. 16129	15. 23409	23. C	31. A	39. D	
8. 792	16. 169744	24. A	32. C	40. D	