CHAPTER – 5

Approximations

'Approximate calculation' is one of the approaches in solving a problem / arriving at the answer to a question at a faster rate. With the help of approximate calculations, one can save a lot of time and this can be utilized in other areas.

In most cases in various exams, the approach towards a question depends on the answer choices. From the answer choices, one should decide which method to follow – actual calculations or approximate calculations.

In most exams, for solving questions based on simplifications, data interpretation, ratios, percentages, etc., the use of approximations is very handy for solving the question at a faster pace. Approximation in any calculation depends on the degree of accuracy required. The closer the given answer choices, the greater the need for closer approximation.

Ex.1: For which of the following values is the increase the highest

a. 3164 to 4072
b. 2422 to 3218
c. 4234 to 5866
d. 1876 to 2761

In order to solve the above question, if we calculate accurately, we will get 4072 – 3164 = 908; 3218 – 2422 = 354; 5866 – 5234 = 632; 2761 – 1876 = 885.

If we try approximate calculations,

4072 → (Rounded off to) 4100

3164 → (Rounded off to) 3200

Here the subtraction is very simple.
4100 – 3200 = 900.

During the process of rounding off, if the last two digits are 50 or greater than 50, then the figure must be rounded off to the next highest hundred. Otherwise, it should be rounded off to the next lowest hundred. Thus, 3164 would be rounded off to 3200, while 4072 would be rounded off to 4100. In fact, in such calculations, even the hundreds (i.e., the last two zeroes) need not be considered since the two zeroes are present in every case. This means that the above calculation would be further simplified by mentally treating it as 41 – 32 = 9. Other calculations can be done in a similar manner.

In case of multiplications like 389 x 1456, suppose the answer choices are given as below:

(A) 564322 (B) 565400
(C) 566384 (D) 572356

We can go for 390 x 1450 which is 565500 whereas the actual answer here is 566384. This is far from the actual answer. But as none of the answer choices lie between these values, this is the required answer. In the above calculation, only one answer ends with 4; hence without actually calculating, we can say that Choice (C) is the answer.

Suppose the answer choices are closer, like

(A) 565424 (B) 566644
(C) 566384 (D) 572354

then the above approximation will not be useful. Then the approach should be 1455 x 389 which gives us 565995. Hence the answer should be very much close to 565995 but should be more than that which is 566384.

If we have to calculate 37.22 % of 1384

The actual calculation takes around 45 seconds to 75 seconds, depending on the speed of the person.

Suppose the answer choices for the above calculation are mentioned as

(A) 564 (B) 515 (C) 529 () 542

As the answer choices are not very close, calculating for 40% and reducing that by 3% may be sufficient for answering the question.
10% of 1384 = 138.4 . Four times that is 554 and 3% is approximately equivalent to 42. Hence the answer is 552 – 42 i.e., 510.
Therefore, the closest answer is 510.

Suppose the answer choices for the above calculation are mentioned as:

(A) 510.264 (B) 515.124
(C) 519.316 (D) 522.356

It is clear that the answers are very close. But if you approximate 37.22% equivalent to 37.5%, you can simply convert the calculation into 3/8 of 1384. As 1384 goes 173 times, the answer is 519. Hence, 519.316 cannot be the answer, as it should be less than 519 but very close to 519, which is 515.124.

If the answer choices are even closer than the above example, we go for subtraction of 28%, which is approximately 1/400th part of 1384. Hence, by using approximate calculations we can answer questions at a faster rate.

Now, let us consider another question.

Find the value of $\frac{5843}{31200}$ x 100

(A) 17.56 (B) 18.38 (C) 18.72 (D) 16.96

This ratio can be calculated faster by two very useful methods than by conventional division.

The first approach can be called the "*ten percent method*". In this method, 10% of the denominator is first obtained by simply shifting the decimal point in the denominator it by one place to the left. Similarly, 1% of the denominator, 0.1% etc., can also be successively obtained by shifting the decimal point to the left by one more place in each successive step. Then the numerator is expressed as the nearest possible multiple of 10% of the denominator along with some excess or shortfall.

For example,
10% of 31200 = 3120
and
5843 = 1 × 3120 + 2723 (excess)
Again, the excess of 2723 can be expressed as a multiple of say,
5% of denominator plus some excess.
Half of 10% of 31200 = 5% of 31200 = 1560
Hence, 2723 = 5% of 31200 + 1163
Further, 1163 is slightly less than 4 times (1% of 31200) i.e., 1248.

Thus,
5843 ≅ (10% + 5% + 4%) of 31200
≅ 19% of 31200.

The correct figure must be slightly less than 19% of 31200. Hence from the choices, the answer can be Choice (C).

The second approach to quickly calculate the ratio $\frac{5843}{31200}$ is by using of the decimal equivalent values of the reciprocals of the first few natural numbers. In this approach, the numerator and denominator are first approximated as $\frac{5800}{31200}$, which is further approximated to $\frac{5.8}{31.2}$ which is close to $\frac{5.8}{3\times1.04}$
(i.e., 4% less than 19.3),
i.e., 18.54. But since in the first approximation we had taken 5800 instead of 5843, the answer has to be slightly more than 18.54. Thus, Choice (C) is the answer.

Find the value of $\frac{6164}{26879}$ x 100

(A) 21.68 (B) 22.16 (C) 22.93 (D) 23.37

By observing the given choices, we understand that the answer should be close to 22.22% (i.e., 2/9) (Here, one should remember reciprocals and their multiples.). The calculation is:

$\frac{2}{9}\times 26879 = 5973$. As 5973 is about 190 less than 6164, we need to add about 190 to 5973. But as we are interested in percentages, 190 forms slightly more than 0.5% but less than 1%. The answer should be more than 22.7% but less than 23.22%. From the choices, only choice (C) is satisfied.

Find the value of (2911 / (3784 x 4)) x 100.
(A) 17.86 (B) 18.15 (C) 21. 76 (D) 19.23

$$= \frac{\left(\frac{6695-3784}{3784}\right)\times 100}{4}$$

This is approximated as:

$$\frac{6700-3800}{3800\times 4}\times 100$$

$$= \frac{2900}{38\times 4} = \frac{2900}{152}$$

$\frac{2900}{152}$ is slightly less than $\frac{3000}{150} = 20$

Thus, 19.23% is close to 20%. Therefore Choice (D) is correct.

Approximations for divisions can be done in two ways. The first one is cross multiplication. The following example shows how to solve an approximation problem using cross multiplication.

5.01. Find the value of x.

$$\frac{38}{154} = \frac{x}{190}$$

Sol. $x = \frac{38\times 190}{154} = \frac{19}{77}\times 190 \simeq \frac{1}{4}\times 190 \simeq 47.5$

The second method is to find the approximate ratio of the numerators or denominators and arrive at the solution. This is illustrated in the following two examples.

5.02. Find the value of x.

$$\frac{54}{238} = \frac{11}{x}$$

Sol. 5.4 x 2 = 10.8

$11 - 10.8 = 0.2 \cong \frac{1}{3}$ x 0.54

So, $11 = 5.4 \times 2 + \frac{1}{3}\times 0.54$

$\therefore x = 23.8 \times 2 + \frac{1}{3}\times 2.38$

= 47.6 + 0.8 = 48.4

5.03. Find the value of x

$$\frac{125}{220} = \frac{175}{x}$$

175 is 40% more than 125
∴ x is 40% more than 220, i.e. 308.

Therefore we understand that approximations are very useful in additions, subtractions, multiplications, divisions, percentage calculations, etc.

Exercise – 5(a)

Questions 1 to 6: Find the value of x.

1. $\frac{x}{540} = \frac{237}{681}$

2. $\frac{538}{x} = \frac{173}{71}$

3. $\frac{961}{384} = \frac{x}{283}$

4. $\frac{x}{174} = \frac{1864}{523}$.

5. $\frac{x}{416} = \frac{1}{585}$.

6. $\frac{x}{1334} = \frac{17}{667}$.

Questions 7 to 14: Find the value of x to the nearest integer.

7. $\frac{39}{29} = \frac{x}{41}$

8. $\frac{91}{36} = \frac{x}{476}$

9. $\frac{32}{209} = \frac{x}{638}$

10. Find x given that $\frac{4x}{35} = \frac{440}{19}$.

11. $\frac{18}{29} = \frac{41}{x}$.

12. Find x given that $\frac{16}{59} = \frac{73}{x}$.

13. Find x given that $\frac{x}{294} = \frac{864}{63}$.

14. Find the approximate value of x, if $\frac{x}{12} = \frac{16}{215}$.

Directions for questions 15 to 45: From the given choices, select the correct alternative which gives an approximate value to replace the question mark (?).

15. Find the fractional value of x satisfying the equation $\frac{187x}{221} = \frac{247}{2197}$.

(A) $\frac{13}{137}$ (B) $\frac{21}{185}$ (C) $\frac{19}{143}$ (D) $\frac{17}{132}$

16. $120.01 \times 4.99 + 5.99 \times 80.01 = ?$
(A) 1020 (B) 1040 (C) 1060 (D) 1080

17. 28% of 200.04 + 18% of 120 + 25.9% of 300 = ?
(A) 140 (B) 156 (C) 164 (D) 136

18. $\sqrt{728.99} \div \sqrt{8.99} \times 6.01 = ?$
(A) 54 (B) 60 (C) 50 (D) 52

19. $320.98 \div 107.1 + 15.98 \div 4.01 = ?$
(A) 6 (B) 8 (C) 7 (D) 9

20. 484.71 + 285.33 – 827.38 + 73.9 = ?
(A) 138 (B) –25 (C) –75 (D) 17

21. $111.9 \times 5.01 + 4.01 \times 89.9 = ?$
(A) 945 (B) 930 (C) 950 (D) 920

22. $\sqrt{225.01} \times \sqrt{8.98} + 26.9 \times 5.01 = ?$
(A) 180 (B) 200 (C) 190 (D) 195

23. $19.98 \times 39.01 \div 20.01 + 18.03 = ?$
(A) 54 (B) 60 (C) 75 (D) 57

24. 24% of 300.13 + 17.9% of 90 + 23.9% of 200 = ?
(A) 120 (B) 108 (C) 136 (D) 156

25. $\frac{(9.99)(7.01)(2.05)+(17.05)(2.95)(15.01)}{13^2+12} = ?$
(A) 14 (B) 10 (C) 4 (D) 5

26. 34% of 400.17 + 27.9% of 90 – 23.9% of 200 = ?
(A) 88 (B) 67 (C) 76 (D) 79

27. $\frac{28.1\%\text{ of }300+57.1\%\text{ of }500}{37.9\%\text{ of }200} = ?$
(A) 6 (B) 7 (C) 5 (D) 4

28. $\frac{16\%\text{ of }700+20\%\text{ of }500.01}{8\%\text{ of }690+5.1\%\text{ of }200.09} = ?$
(A) 8 (B) 5 (C) 3 (D) 7

29. $\frac{(8.99)(7.01)(2.05)+(17.05)(29.88)(15.01)}{(2.01)(16.01)(19.9)} = ?$
(A) 14 (B) 10 (C) 11 (D) 12

30. $\sqrt[3]{343.01} \times \sqrt[3]{124.99} \times \sqrt[3]{64.02} = ?$
(A) 136 (B) 138 (C) 140 (D) 145

31. $\sqrt{146} \times 18 = ?^2 + 3\%$ of 5973
(A) 6 (B) 9 (C) 12 (D) 3

32. $6^4/_7 + 8^1/_2 + 11^7/_8 = ?$
(A) 33 (B) 27 (C) 41 (D) 29

33. 12468 – 6490 + 11811 = ? + 7829 (to the nearest tens).
(A) 9960 (B) 9000
(C) 11500 (D) 12000

34. If a = $\frac{6}{7}$, b = $\frac{13}{16}$ and c = $\frac{9}{11}$, then ab ÷ c = ?
(A) 2 (B) 0.85 (C) 2.5 (D) 1.5

35. $\sqrt{36.1} \times 34 + 15\sqrt{8.92} = ? \times (14.28\%\text{ of }217)$
(A) 10 (B) 6 (C) 8 (D) 11

36. 1256 ÷ 45896 + 205 – 15 = ?
(A) 170 (B) 180 (C) 190 (D) 200

37. 8569.4 – 5698.3 + 5263.2 – 1256 = ?
(A) 6900 (B) 7200
(C) 6800 (D) 6650

38. 4569.3 – 7895.8 + ? + 125 = 5368
(A) 7585 (B) 8100
(C) 8500 (D) 8200

39. $\frac{42}{15} = \frac{?}{122}$
(A) 400 (B) 349 (C) 345 (D) 342

40. 196% of 73 = $?^2$
(A) 15 (B) 13 (C) 12 (D) 14

41. $534.95 – 15.23 + 35 \times 6.78 + 40\%$ of 478 = ?
(A) 256 (B) 354
(C) 478 (D) 956

42. $\frac{60\%\text{of}420.44+\frac{368}{4}}{\frac{7.2}{13.2}\text{of}88+48\times\frac{3}{9}} = ?$
(A) 2 (B) 5 (C) 8 (D) 9

43. $1911 \div 13$ of $9 \div \frac{1}{7} \times 3 – 8 + 12 = ?$
(A) 26587 (B) 26887
(C) 27187 (D) 27787

44. $\sqrt[3]{91} \times 162\%$ of $48 = ?^3$
(A) 4 (B) 5 (C) 7 (D) 9

45. $356.45 \times 13.95 – 230.15 \div 16.52 = ?$
(A) 2648 (B) 3969
(C) 4958 (D) 8594

Exercise – 5(b)

Questions 1 to 15: Find the approximate value of 'x'.

1. $\frac{64}{x}=\frac{17}{37}$

2. $\frac{23}{63}=\frac{x}{84}$

3. $\frac{92}{47}=\frac{113}{x}$

4. $\frac{x+14}{162}=\frac{19}{23}$

5. $\frac{63}{28}=\frac{x}{52}$

6. $\frac{69}{x}=\frac{112}{504}$

7. $\frac{185}{246}=\frac{93}{x}$

8. Find the value of x to the nearest integer given that $\frac{x}{749}=\frac{3748}{1249}$

9. Find the value of x to two decimal places given that $\frac{x}{159}=\frac{53}{83}$

10. Find the value of x to the nearest integer given that $\frac{19}{297}=\frac{x}{142}$.

11. Find the value of x to one decimal place given that $\frac{30}{97}=\frac{x}{107}$.

12. $\frac{24}{75}=\frac{x}{88}$

13. $\frac{126}{x}=\frac{375}{648}$ (to the nearest integer)

14. $\frac{33}{63}=\frac{39}{x}$

15. $\frac{72}{x}=\frac{x}{338}$

Directions for questions 16 to 45: Select the correct alternative from the given choices which comes in the place of (?).

16. 49.5% of 2304 =?
(A) 1120.28 (B) 1140.48
(C) 1165.38 (D) 1195.78

17. $\frac{123}{279}=\frac{267}{?}$
(A) 605.6 (B) 621.2
(C) 647.1 (D) 675.9

18. 18.17% of 229 =?
(A) 32.413 (B) 34.823
(C) 41.613 (D) 43.863

19. $31^2 - 29^2$ =?
(A) 102 (B) 120
(C) 126 (D) 138

20. 345×109 =?
(A) 28150 (B) 30245
(C) 31850 (D) 37605

21. $105.126 \times 35.201 - 90.23 \times 3 + 55.11 \times 27.01$ = ?
(A) 4890 (B) 40000
(C) 271 (D) 5996

22. $105 \times 99.9 \times 299.8$ = ?
(A) 7856000 (B) 3150000
(C) 8450000 (D) 4420000

23. $\sqrt[3]{216400} + \sqrt{280} + \sqrt{322}$ = ?
(A) 651 (B) 361 (C) 85 (D) 95

24. 15% of 199 + 14% of 202 = ?
(A) 52 (B) 62 (C) 58 (D) 78

25. $\frac{19}{52}=\frac{?}{71}$
(A) 26 (B) 28 (C) 24 (D) 22

26. $729 \times ? \div 0.125 = 920$
(A) 0.5 (B) 0.15 (C) 0.4 (D) 0.8

27. $659 \div 13 + ?^2 \times 15.6 = 2051$
(A) 11 (B) 13 (C) 15 (D) 18

28. $7874 \div 14 \times 12 - 3.94$ = ?
(A) 6200 (B) 6420 (C) 6740 (D) 6590

29. $17.43 \div 32 \times 0.01 = ? \div 1000$
(A) 1 (B) 3 (C) 5 (D) 8

30. $(?^3 + 23456 - 21246) \div 31421 = 25$
(A) 65 (B) 80 (C) 69 (D) 92

31. $(430)^2 \div 24262 \times 0.44$ = ?
(A) 3 (B) 5 (C) 8 (D) 10

32. $7 \div 9 \times 3.5 \times 3.5 \div 5$ = ?
(A) 0 (B) 4 (C) 2 (D) 6

33. $?^2 \times 3.5 - 63 = 245$
(A) 2 (B) 4
(C) 5 (D) 9

34. 64.85% of 4200 = $?^3$
(A) 11 (B) 12
(C) 14 (D) 16

35. $8\frac{1}{5}+7\frac{5}{6}-9\frac{1}{5}-3\frac{1}{3}$ = ?

(A) 1 (B) 2 (C) 3 (D) 5

36. $\left(\frac{25.68+19.23}{32.38+55.29}\right)\times 32$ = ?

(A) 2 (B) 4
(C) 8 (D) 16

37. 625.48 – 9 $\sqrt[3]{343500}$ + 199.96 = ?

(A) 85 (B) 195
(C) 525 (D) 630

38. $\frac{\sqrt{532.69}+\sqrt{230.15}}{\sqrt{290.96}+\sqrt{364.56}}$ = ?

(A) 1 (B) 3
(C) 5 (D) 6

39. 22% of 4588 + 58% of 6814 = ?

(A) 1649 (B) 2649
(C) 4961 (D) 9641

40. 78927.95 ÷ 448.29 + 3425.6925 = ?

(A) 1600 (B) 2600
(C) 4600 (D) 3600

41. 256 ÷ 45 – 205 + 45 = ?

(A) –170 (B) –180
(C) –154 (D) 200

42. 569.4 + 5698.3 – 5263.2 + 1256 = ?

(A) 2700 (B) 2260
(C) 2800 (D) 2350

43. 459.3 + 7895.8 – ? + 125 = 536

(A) 7950 (B) 8100
(C) 7800 (D) 8200

44. $\frac{92}{15}=\frac{?}{314}$

(A) 4005 (B) 3490
(C) 2045 (D) 1926

45. 96% of 83 = $?^2$

(A) 8.1 (B) 8.3 (C) 8.9 (D) 8.4

Key

Exercise – 5(a)

1. 188
2. 220.8
3. 708
4. 621.3
5. $\frac{32}{45}$
6. 34
7. 55
8. 1203
9. 98
10. 203
11. 66
12. 269
13. 4032
14. $0.\overline{8}$
15. C
16. D
17. B
18. A
19. C
20. D
21. D
22. A
23. D
24. C
25. D
26. D
27. C
28. C
29. D
30. C
31. A
32. B
33. A
34. B
35. C
36. C
37. A
38. C
39. D
40. C
41. D
42. B
43. D
44. C
45. C

Exercise – 5(b)

1. 139.29
2. 30.67
3. 57.73
4. 119.83
5. 117
6. 310.5
7. 123
8. 2248
9. 101.53
10. 9
11. 33.1
12. 28.16
13. 218
14. 74.45
15. 156
16. B
17. A
18. C
19. B
20. D
21. A
22. B
23. D
24. C
25. A
26. B
27. A
28. C
29. C
30. D
31. A
32. C
33. D
34. C
35. C
36. D
37. B
38. A
39. C
40. D
41. C
42. B
43. A
44. D
45. C