Chapter – 7

TRIGONOMETRY

This is a branch of Mathematics that deals with the relation between the sides and angles in any triangle. Trigonometry literally means three angle measure.

Measurement of angles

The angle between two lines can be measured in various systems. We consider two of these systems,

(a) Sexagesimal System and

(b) Circular System.

(a) Sexagesimal System: A right angle is taken as the unit of measurement.

$^1/_{90}{}^{th}$ of a right angle is termed as a degree (1°).

$^1/_{60}{}^{th}$ of a degree is termed as a minute (1').

$^1/_{60}{}^{th}$ of a minute is termed as a second (1'').

Basic Conversions

60'' = 1'.

60' = 1°.

and 90° = 1 right angle.

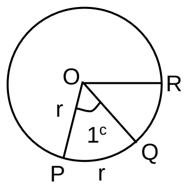
(b) Circular System: A radian (1^c) is taken as the basic unit of this measure.

A radian (1^c) is defined as the angle subtended at the centre of a circle by an arc whose length (ℓ) equals the radius (r).

i.e., $\ell = r$

$\therefore$ If arc PQ = r,

then $\angle POQ$ = 1 radian.



Note:

(i) Radian is a real constant.

(ii) The circular and sexagesimal measures are related by π^c or π radians = 180°.

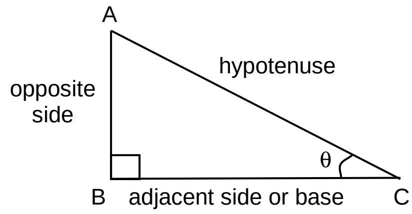
2π radians = 360°.

(iii) 1 revolution = 2π radians.

Important Conversions

Sexagesimal Measure	Circular Measure
30°	$\pi/6$
45°	$\pi/4$
60°	$\pi/3$
90°	$\pi/2$
270°	$3\pi/2$

Given a right-angled triangle ABC, and $\angle ACB = \theta$ the trigonometric ratios are defined as follows:



$$\sin\theta = \frac{opp}{hyp} = \frac{AB}{AC}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{BC}{AC}$$

$$\tan\theta = \frac{opp}{adj} = \frac{AB}{BC}$$

$$\cot\theta = \frac{adj}{opp} = \frac{BC}{AB}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{AC}{BC}$$

$$\text{cosec}\theta = \frac{hyp}{opp} = \frac{AC}{AB}$$

From the ratios, we can easily observe the following relations:

(a) Reciprocal Relations:

$$\text{cosec}\theta = \frac{1}{\sin\theta}; \qquad \sin\theta = \frac{1}{\text{cosec}\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}; \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}; \qquad \cot\theta = \frac{1}{\tan\theta}$$

(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

Signs of Trigonometric Ratios

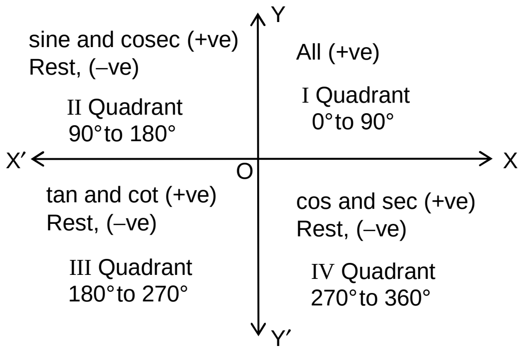
(i) If θ lies in the first quadrant ($0 < \theta < \pi/2$), all the trigonometric ratios are positive.

(ii) If θ lies in the second quadrant ($\pi/2 < \theta < \pi$), only sin θ and cosec θ are positive and the rest of the ratios are negative.

(iii) In the third quadrant ($\pi < \theta < 3\pi/2$), only tan θ and cot θ are positive and the rest of the ratios are negative.

(iv) In the fourth quadrant ($3\pi/2 < \theta < 2\pi$), only cos θ and sec θ are positive and the rest of the ratios are negative.

The signs of the trigonometric ratios can be remembered with the help of the diagram given below.



(i) The absolute value of the ratio is to be arrived at as per the following table

Ratio	**when n is even**	**when n is odd**
$\sin(n\pi/2 \pm \theta)$	$\sin\theta$	$\cos\theta$
$\cos(n\pi/2 \pm \theta)$	$\cos\theta$	$\sin\theta$
$\tan(n\pi/2 \pm \theta)$	$\tan\theta$	$\cot\theta$

(ii) Values of Trigonometric functions of some standard angles

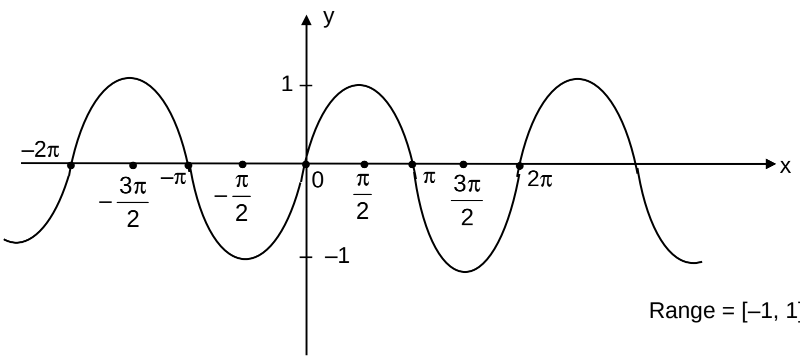
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1/2	0	–1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

Maximum and Minimum values of Trigonometric Ratios

(i) The sine and cosine of an angle can never be less than –1 and cannot be greater than +1.
(ii) The secant and cosecant of an angle will not have any value between –1 and +1.
(iii) The tangent and co-tangent of an angle can take any real value.

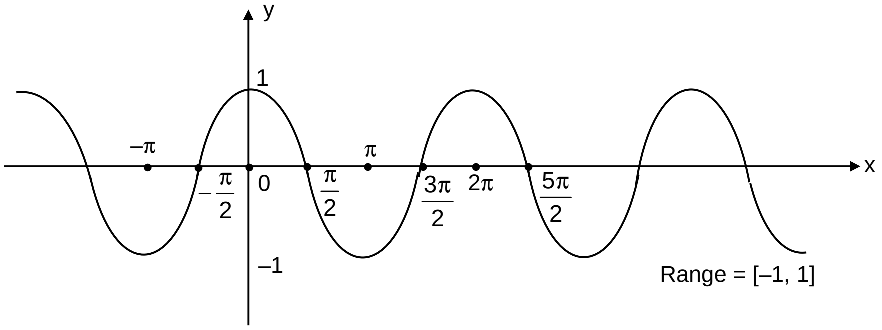
Graphs of sine, cosine and tangent functions

sine function: $y = \sin x$



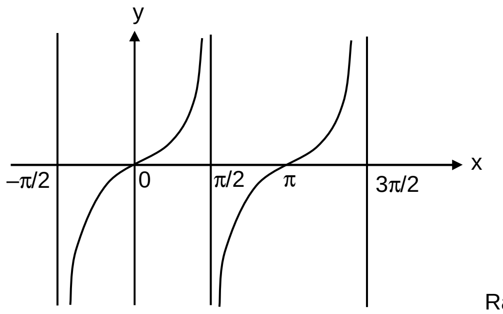
Range = [–1, 1]

cosine function: y = cos x



Range = [–1, 1]

tangent function: y = tan x



Range = R or (–∞, ∞)

Some important trigonometric identities

1. (a) $\sin^2\theta + \cos^2\theta = 1$
   (b) $1 + \tan^2\theta = \sec^2\theta$
   (c) $1 + \cot^2\theta = \text{cosec}^2\theta$

2. (a) $\sin(-\theta) = -\sin\theta$
   (b) $\cos(-\theta) = \cos\theta$
   (c) $\tan(-\theta) = -\tan\theta$

Note : $(\sin\theta)^2$ is represented as $\sin^2\theta$ and so on.

Compound Angle: An angle made by the sum or difference of two or more angles is called a compound angle.

1. Addition and Subtraction formulae

(i) $\sin(A + B) = \sin A\cos B + \cos A\sin B$

(ii) $\sin(A - B) = \sin A\cos B - \cos A\sin B$

(iii) $\cos(A + B) = \cos A\cos B - \sin A\sin B$

(iv) $\cos(A - B) = \cos A\cos B + \sin A\sin B$

(v) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

(vi) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(vii) $\cot(A + B) = \dfrac{\cot A\cot B - 1}{\cot A + \cot B}$

(viii) $\cot(A - B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

(ix) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(x) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

(xi) $\tan(A + B + C) = \dfrac{\sum\tan A - \pi\tan A}{1 - \sum\tan A\tan B}$

2. Double angle formulae

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

(iii) From (ii) above,

$\cos\theta = \sqrt{\dfrac{1 + \cos 2\theta}{2}}$; $\sin\theta = \sqrt{\dfrac{1 - \cos 2\theta}{2}}$

(iv) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

(v) $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

3. Half - angle formulae

Replacing θ by θ/2 in the formulae given in (2), we get the following formulae.

(i) $\sin\theta = 2\sin\theta/2\cos\theta/2$

$= \dfrac{2\tan\theta/2}{1 + \tan^2\theta/2}$

(ii) $\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

$= 2\cos^2\theta/2 - 1$

$= 1 - 2\sin^2\theta/2 = \dfrac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2}$

(iii) $\tan\theta = \dfrac{2\tan\theta/2}{1 - \tan^2\theta/2}$

(iv) $\cot\theta = \dfrac{\cot^2\theta/2 - 1}{2\cot\theta/2}$

4. *Triple angle formulae:*

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
(iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$
(iv) $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1} = \dfrac{3\cot\theta - \cot^3\theta}{1-3\cot^2\theta}$

5. *Formulae for changing the Product into Sum or Difference:*

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$
$2\cos A\sin B = \sin(A + B) - \sin(A - B)$
$2\cos A\cos B = \cos(A + B) + \cos(A - B)$
$2\sin A\sin B = \cos(A - B) - \cos(A + B)$

6. *Formulae for changing the Sum or Difference into Products:*

$\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

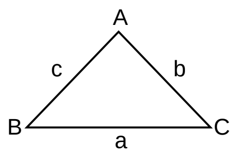
$\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

$\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

Other Standard Results

(1) Sine rule:
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$
(R is the circumradius)

(2) Area of a triangle ABC, $(\Delta) = (1/2)\,ab\sin C$ (where C is the angle included between the sides of the lengths a and b)



Alternatively $\Delta = 1/2\ bc\sin A$ and $1/2\ ca\sin B$ can also be taken.

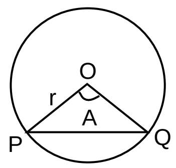
(3) Cosine Rule:

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

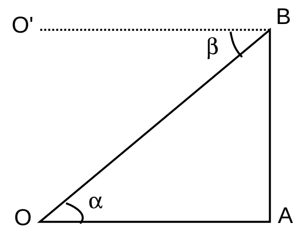
(4)



O is the centre
PQ is the chord

Chord $PQ = 2r\sin\dfrac{A}{2}$, r = radius of the circle.

Heights and Distances



Let AB be a vertical line drawn from B to A to meet the horizontal line drawn from O (which is at a lower level than B) and let BO' be the horizontal drawn through B (i.e., BO' is parallel to OA)

Then ∠AOB is called the angle of elevation of point B as seen from point O and ∠O' BO is the angle of depression of point O as seen from point B. It can be readily seen that $\alpha = \beta$ i.e.,

Angle of Elevation = Angle of Depression (since AO is parallel to BO').

Using the trigonometric ratios on angles of elevation and depression, we can find out heights and distances as seen in the examples given in the later part of this chapter.

Examples

7.01. Convert $\dfrac{5\pi}{12}$ radians into degrees.

Sol: π radians = 180°
$\dfrac{5\pi}{12}$ radians $= \dfrac{5}{12}(180°) = 75°$

7.02. An athlete ran 2000 m around a circular path of circumference 200 m. Find the angle covered by him (in radians).

Sol: The athlete makes $\dfrac{2000}{200} = 10$ rounds of the circular path.
1 round = 2π radians.
10 rounds = (10) (2π radians)
= 20π radians.

7.03. The angles of a triangle are in arithmetic progression with a common difference of 20°. Find the measures of the angles in radians.

Sol: Let the measures of the angles be P°, Q° and R°
P + Q + R = 180°
P = Q – 20 and R = Q + 20
∴ (Q – 20) + Q + (Q + 20) = 180
⇒ Q = 60
P = 40 and R = 80
The measures of the angles (in radians) are $\dfrac{2\pi}{9}$, $\dfrac{\pi}{3}$ and $\dfrac{4\pi}{9}$.

7.04. Find the angle covered by the minutes hand in 36 minutes.

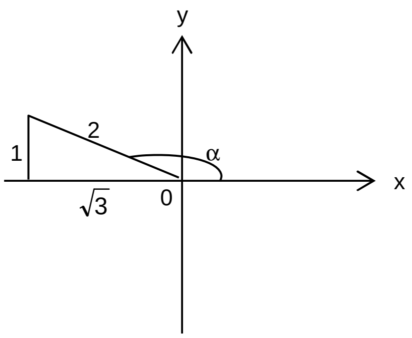
Sol: The angle covered by the minute hand in 60 minutes = 360°
Angle covered by it in 36 minutes
$= \frac{36}{60}(360)° = 216° = \frac{6\pi}{5}$ radians.

7.05. If $\tan^2\alpha = \frac{1}{3}$ and α lies in the IInd quadrant find the values of $\sin\alpha$ and $\cos\alpha$.

Sol: $\tan^2\alpha = \frac{1}{3} \Rightarrow \tan\alpha = \pm\frac{1}{\sqrt{3}}$

Since α lies in the IInd quadrant, $\sin\alpha > 0$, $\cos\alpha < 0$ and $\tan\alpha < 0$.

$\therefore \tan\alpha = -\frac{1}{\sqrt{3}}$



$\sec\alpha = \sqrt{1+\tan^2\alpha} = -\frac{2}{\sqrt{3}}$

$\therefore \cos\alpha = -\sqrt{3}/2$ and $\sin\alpha = \tan\alpha \cos\alpha$

$= \frac{-1}{\sqrt{3}}\left(\frac{-\sqrt{3}}{2}\right) = \frac{1}{2}$.

7.06. Show that $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

Sol: $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta(1+\sin\theta + 1 - \sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2\cos\theta}{1-\sin^2\theta} = \frac{2\cos\theta}{\cos^2\theta} = 2\sec\theta$

7.07. Show that $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) + \sin 2\theta - (\sin\theta + \cos\theta)^2 = 0$

Sol: $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$
$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$
$= 1 + \sin 2\theta$
$\therefore \sin 2\theta - (\sin\theta + \cos\theta)^2 = -1$
$\therefore (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) + \sin 2\theta - (\sin\theta + \cos\theta)^2 = 1 + (-1) = 0$

7.08. If $\cos\theta = -\frac{\sqrt{7}}{2\sqrt{2}}$ and θ is not in second quadrant, then find the value of $\frac{\operatorname{cosec}^2\theta + \sec^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$.

Sol: Given $\cos\theta = -\frac{\sqrt{7}}{2\sqrt{2}}$ and $\theta \notin Q_2 \Rightarrow \theta \in Q_3$

$\therefore \operatorname{cosec}\theta = -2\sqrt{2}$

and $\sec\theta = \frac{-2\sqrt{2}}{\sqrt{7}}$

$\therefore \frac{\operatorname{cosec}^2\theta + \sec^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta} = \frac{8+\frac{8}{7}}{8-8/7} = \frac{8\left(1+\frac{1}{7}\right)}{8\left(1-\frac{1}{7}\right)}$

$= \frac{8}{7}\times\frac{7}{6} = \frac{4}{3}$

7.09. If $\theta = \pi/6$ then find the value of $\sin^6\theta + \cos^6\theta$.

Sol: $\left(\sin\frac{\pi}{6}\right)^6 + \left(\cos\frac{\pi}{6}\right)^6 = \left(\frac{1}{2}\right)^6 + \left(\frac{\sqrt{3}}{2}\right)^6$

$= \frac{1+27}{64} = \frac{7}{16}$

7.10. Find the value of cos (–1200°).

Sol: We know that $\cos(-\theta) = \cos\theta$.
$\Rightarrow \cos(-1200°) = \cos 1200°$
$= \cos(3\times 360° + 120°)$
$= \cos 120°$
$= \cos(180° - 60°)$
$= -\cos 60° = -1/2$

7.11. If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, then find the value of $\sin\frac{\alpha-\beta}{2}$.

Sol: Consider $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{3}{5}.\frac{5}{13} + \frac{4}{5}.\frac{12}{13}$

$\Rightarrow \cos(\alpha - \beta) = \frac{63}{65}$

We know that

$\sin\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1-\cos(\alpha-\beta)}{2}}$

$= \sqrt{\frac{1-\frac{63}{65}}{2}} = \sqrt{\frac{2}{2(65)}} = \frac{1}{\sqrt{65}}$

7.12. Find the value of cos 20°cos 40° – sin 5°sin 25°.

Sol: Required value = $\frac{1}{2}$[2cos 20° cos 40° – 2 sin 5° sin 25°]

$= \frac{1}{2}$ [cos 60° + cos 20° + cos 30° – cos 20°]

$= \frac{1}{2}\left[\frac{1}{2}+\frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}+1}{4}$

7.13. Find the value of $\frac{\sqrt{3}}{\sin 20°}-\frac{1}{\cos 20°}$.

Sol: $\frac{\sqrt{3}\cos 20°-\sin 20°}{\sin 20°\cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20°-\frac{1}{2}\sin 20°\right]}{\frac{1}{2}(2\sin 20°\cos 20°)}$

$= \frac{4(\sin 60°\cos 20°-\cos 60°\sin 20°)}{\sin 40°}$

$= 4\left(\frac{\sin 40°}{\sin 40°}\right) = 4$

7.14. If the lengths of the sides of a triangle are 8, 9 and 10, find the cosine of the least angle in it.

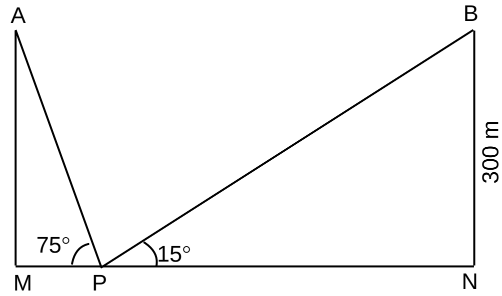
Sol: Let a = 8, b = 9 and c = 10.
The least angle of the triangle will be opposite to the least side in it.
∴ ∠A is the least angle.

$\cos \angle A = \frac{b^2+c^2-a^2}{2bc} = \frac{9^2+10^2-8^2}{(2)(9)(10)} = 0.65$

$\angle A = \cos^{-1}(0.65)$

7.15. From a point P on the ground the angles of elevations of the tops of two buildings (B and A) are 15° and 75° respectively. Each building is 300 m high. Find the distance between the tops of both the buildings. (tan 15° = 2 – $\sqrt{3}$ and tan 75° = 2 + $\sqrt{3}$)

Sol:



Distance between the tops of the two buildings
= AB = MP + PN
= 300 (cot 75° + cot 15°)
= 300 (tan 15° + tan 75°)
= 300 $\left(2-\sqrt{3} + 2 + \sqrt{3}\right)$
= 1200 m.

Concept Review Questions

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Angle $\frac{2\pi^c}{3}$ in sexagesimal measure is $\boxed{}$ degrees.

2. Angle 300° in radians (circular measure) is _______.
(A) $\frac{4\pi^c}{3}$ (B) $\frac{5\pi^c}{3}$ (C) $\frac{5\pi^c}{6}$ (D) $\frac{3\pi}{2}$

3. The angle covered by a wheel in making one complete revolution at its centre is _______.
(A) 180° (B) 90° (C) 360° (D) 300°

4. The angle covered by the minute hand in 12 minutes is _______.
(A) $\frac{\pi}{5}$ (B) $\frac{3\pi}{5}$ (C) $\frac{2\pi}{5}$ (D) $\frac{4\pi}{5}$

5. The angle covered by the hour hand in 30 minutes is ______.
(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{10}$ (C) $\frac{\pi}{30}$ (D) π

6. In a right-angled triangle, if hypotenuse and one of the sides are 17 and 8 respectively and the angle between them is θ then sinθ and tanθ are _______ respectively.
(A) $\frac{15}{12}, \frac{15}{17}$ (B) $\frac{8}{17}, \frac{8}{15}$
(C) $\frac{15}{17}, \frac{15}{8}$ (D) $\frac{8}{17}, \frac{17}{15}$

7. Given that cosθ is positive and tanθ is negative then θ is in ______.
(A) Q_1 (B) Q_2 (C) Q_3 (D) Q_4

8. Which of the following statements is false?
(A) tan 500 °< 0 (B) cosec (– 200°) > 0
(C) cot (– 500°) > 0 (D) sec (–400°) < 0

9. The number of points in which the graph of cosx meets the x-axis is _______.
(A) 0 (B) 1 (C) 2 (D) Infinite

10. sinθ expressed in terms of tanθ is ________.
(A) $\frac{\tan\theta}{\sqrt{1-\tan\theta}}$ (B) $\frac{\tan\theta}{\sqrt{1+\tan\theta}}$
(C) $\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$ (D) $\frac{\tan^2\theta}{\sqrt{1+\tan\theta}}$

11. If secθ = $\frac{17}{8}$; θ ∉ Q_1, then cotθ = _______.
(A) $\frac{15}{8}$ (B) $\frac{-8}{15}$ (C) $\frac{8}{15}$ (D) $\frac{-15}{8}$

12. If a = cos10° – sin10° and b = cos70° – sin70°, which of the following is true?
(A) a < 0, b < 0 (B) a < 0, b > 0
(C) a > 0, b < 0 (D) a > 0, b > 0

13. cot (180° + θ) = _______.
(A) tanθ (B) cotθ (C) –cotθ (D) –tanθ

14. (1 + sinθ) ($\sec^2\theta$) (1 – sinθ) = _______.
(A) 0 (B) secθ (C) sinθ (D) 1

15. (a) cosec(330°) = $\boxed{}$

(b) sec(1020°) = $\boxed{}$

16. If the measures of the angles of a triangle are in the ratio 1 : 2 : 3, then the ratio of the sides opposite to the angles (in the same order) is ________.
(A) 1 : 2 : 3 (B) 1 : $\sqrt{2}$: 3
(C) 1 : 2 : $\sqrt{3}$ (D) 1 : $\sqrt{3}$: 2

17. Which of the following statements cannot be true?
(A) sinθ = 0.5 (B) cosθ = 0.75
(C) tanθ = 100 (D) cosecθ = $\frac{1}{2}$

18. If tanθ + secθ = a and secθ – tanθ = b then the relation between a and b is _______.
(A) $\frac{a}{b} = 1$ (B) ab = 1
(C) $a^2 + b^2 = 1$ (D) $a^2 - b^2 = 1$

19. $\text{cosec}^4\theta + \cot^4\theta - 2\text{cosec}^2\theta \cot^2\theta$ = $\boxed{}$

20. If secθ = –2 and cotθ = $-\frac{1}{\sqrt{3}}$, then sinθ = _______.
(A) $-\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

21. Find the value of cosec $\frac{3\pi}{4}$.
(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) –2 (D) 2

22. If cosecθ = $\frac{1}{2}$ + cotθ, then find sinθ.

$\boxed{}$

23. If $\tan^2\theta + 2\sec^2\theta = \frac{59}{16}$, then find tanθ.

± $\boxed{}$

24. Which of the following is never possible?
(A) tanθ = 4/3
(B) cosecθ = 5/2
(C) cotθ = 3/4
(D) secθ = 2/5

25. $\sin 44°\sec 46° + \cos 44°\operatorname{cosec} 46° =$ $\boxed{}$

26. The value of tan1°tan2°tan3°……. tan89°is $\boxed{}$

27. If $180° < \beta < 270°$ and $\cos\beta = -4/5$, then find $\operatorname{cosec}\beta + \cot\beta$.

(A) –1/3 (B) –1/2 (C) –2 (D) –3

28. An angle 'θ' in sexagesimal measure is such that its complement is equal to $^2/_5$ th of its supplement.

θ = $\boxed{}$

29. If $\sin\alpha + \sin\beta + \sin\gamma = 3$, then the value of $\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2}$ is $\boxed{}$

30. If the two sides of a triangle are 5 and $3\sqrt{2}$ and the angle included by them is 45° then the area of the triangle is ________.

(A) 15 sq units. (B) $15\sqrt{2}$ sq units.

(C) $7.5\sqrt{2}$ sq units. (D) 7.5 sq units.

30. In the above problem, the length of the third side is _______.

(A) 13 (B) $\sqrt{13}$ (C) $2\sqrt{13}$ (D) $13\sqrt{2}$

31. A person standing 50 m away from the foot of a tower observes the top of a tower to be at an angle of elevation of 60°. Find the height of the tower in meters.

(A) $25\sqrt{3}$ (B) 25 (C) 50 (D) $50\sqrt{3}$

32. At an instant of time if the length of the shadow of a pole is $\frac{1}{\sqrt{3}}$ times its height then find the angle of elevation of the sun. (in degrees)

$\boxed{}$

33. At a point on the ground, the top of a flag pole subtends an angle of 60°. If the height of the pole is 300 m, find the distance (in meters) between the point of observation and the foot of the pole.

(A) $75\sqrt{3}$ (B) $90\sqrt{3}$ (C) $100\sqrt{3}$ (D) $50\sqrt{3}$

34. A ladder is placed against a wall of height 18 m. If the top of the ladder makes an angle of 60° with the wall, then the length of the ladder is ________.

(A) 36 m (B) $12\sqrt{3}$ m

(C) $18\sqrt{3}$ m (D) 48 m

35. Mohit and Rohit stand on either side of a pole and observe the top of the pole at an angle of elevation of 60° and 45° respectively. If the distance between Mohit and Rohit is 100 m, then the height of the pole is ________.

(A) $100(\sqrt{3})(\sqrt{3}-1)$ m

(B) $75(\sqrt{3})(\sqrt{3}-1)$ m

(C) $50(\sqrt{3})(\sqrt{3}-1)$ m

(D) $125(\sqrt{3})(\sqrt{3}-1)$ m

Exercise – 7(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If the angles of a triangle are in an arithmetic progression and the measure of the least angle is 20°, then find the measure of the greatest angle in radians.

$\dfrac{\boxed{}}{9}\pi^c$

2. If sinθ and cosθ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then which of the following relations hold?

(A) $b^2 = a^2 + 2ac$ (B) $b^2 = a^2 - 4ac$

(C) $a^2 + b\sqrt{b^2 - 4ac} = 0$ (D) $a^2 = b^2 (b^2 - 4ac)$

3. If $\tan\theta = \sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}$, then $\tan\left(22\dfrac{1}{2}\right)^\circ$ = _______.

(A) $2 - \sqrt{3}$ (B) $2 + \sqrt{3}$

(C) $\sqrt{2} - 1$ (D) $\sqrt{2} + 1$

4. If $3\tan^2\theta - 1 = 0$ and θ lies in III[rd] quadrant, then cosecθ = _______.

(A) $-\sqrt{3}$ (B) –1 (C) $-\dfrac{\sqrt{3}}{2}$ (D) –2

5. If 13sinθ – 12 = 0 and 'θ' is acute, then $\dfrac{\cot\theta - \tan\theta}{\sec\theta - \operatorname{cosec}\theta}$ = ________.

(A) $\dfrac{17}{13}$ (B) $\dfrac{-17}{13}$ (C) $\dfrac{-131}{91}$ (D) $\dfrac{131}{91}$

6. In a cyclic quadrilateral ABCD, the value of cosA + cosB + cosC + cosD is $\boxed{}$

7. If sinθ and cosecθ are the roots of the equation $cx^2 + ax + b = 0$, then ________.

(A) b – c = 0 (B) a + c = 0

(C) a + b = 0 (D) a – c = 0

8. If secθ + tanθ = p, then cosθ = _______.

(A) $\dfrac{p^2+1}{2p}$ (B) $\dfrac{p^2+1}{p^2-1}$ (C) $\dfrac{2p}{p^2+1}$ (D) $\dfrac{p}{p^2+1}$

9. If $\sin\theta = x + \dfrac{1}{x}$ for x > 0, then _______.

(A) $0 \le x \le 1$

(B) no such value of x exists

(C) $x \in R^+$

(D) no such value of x exists

10. Which of the following can be true for an acute angled triangle?

(A) Sine of each angle is less than $\dfrac{1}{\sqrt{2}}$.

(B) Sine of exactly two angles is less than $\dfrac{1}{\sqrt{2}}$.

(C) Sine of each angle is greater than $\dfrac{1}{\sqrt{2}}$.

(D) Cosine of exactly two angles is greater than $\dfrac{1}{\sqrt{2}}$.

11. If x = cos50° + cos55° + cos60° and y = sin20° + sin25° + sin30°, then ______.

(A) $\dfrac{x}{y} > 1$ (B) $\dfrac{x}{y} < 1$

(C) $\dfrac{x}{y} = 1$ (D) x + y = 0

12. If two sides of a triangle are 6 cm and 9 cm and the included angle is 45°, then the area of the triangl e in sq.cm is _______.

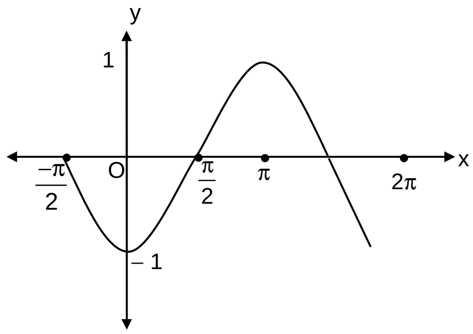
(A) $27\sqrt{2}$ (B) 27/2 (C) $27/\sqrt{2}$ (D) 27

13. In a triangle ABC, if a = 4 cm, b = 7 cm and ∠C = 90°, then the length of the longest side is _______.

(A) $\sqrt{63}$ cm (B) $\sqrt{65}$ cm

(C) 7 cm (D) 9 cm

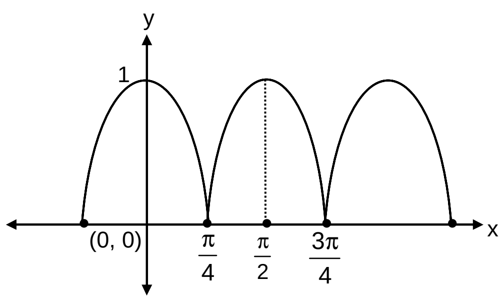
14. (i) Which of the following relations best describes the given graph?



(A) y = sinx (B) y = –cosx

(C) y = cosx (D) y = –sinx

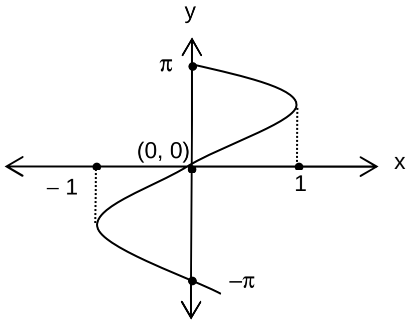
(ii) Which of the following relations best describes the given graph?



(A) y = |sin2x| (B) y = |cos2x|

(C) $y = \left|\cos\dfrac{x}{2}\right|$ (D) $y = \left|\sin\dfrac{x}{2}\right|$

(iii) Which of the following relations best describes the given graph?



(A) y = sinx (B) y = cosx
(C) x = siny (D) x = cosy

15. Value of $\dfrac{\cos 20^\circ + \sin 50^\circ}{\sin 20^\circ + \cos 50^\circ}$ is _______.

(A) 3 (B) $\sqrt{3}$ (C) 1/3 (D) $1/\sqrt{3}$

16. The maximum value of $\sin^6 x + \cos^6 x$ is $\boxed{}$

17. If h(y) = $3\left[\,|\sin y| + |\cos y|\,\right]$, then which of the following represents the range of h(y)?

(A) $3 \le h(y) \le 3\sqrt{2}$ (B) $0 \le h(y) \le 3\sqrt{2}$
(C) $3 \le h(y) \le 4$ (D) None of these

18. If the angles of elevation of the top of a tower from the top and the foot of a pole of height 20 m are 30° and 60° respectively, then the height of the tower is (in m)

$\boxed{}$

19. At a point halfway between the line joining the foot of the towers X and Y, the angles of elevations of the tops of the towers are found to be 30° and 45° respectively. If the towers are separated by a horizontal distance of 600 m, find the ratio of their heights.

(A) 3 : 1 (B) 1 : $\sqrt{3}$ (C) 2 : 1 (D) 1 : 2

20. Vinod observes an aeroplane flying exactly 1·5 km above the ground at an angle of elevation of 45°. If, after 9 seconds, the plane has moved away from him and is making an angle of elevation of 30° at the same height, then the average speed of the plane in km/sec is _______.

(A) $\dfrac{\sqrt{3}-1}{6}$ (B) $\dfrac{\sqrt{3}+1}{6}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{6}{25}$

21. From a steamer moving towards a lighthouse at a constant velocity, the angle of elevation of the top of the lighthouse is observed to be 30°. 10 minutes from that instant, the angle of elevation changes to 60°. If the steamer reaches the light house at noon, then find the time at which the first observation was made.

$\boxed{}$: $\boxed{}$ a.m

22. The angles of elevation of the tip 'P' of a pole PQ from two points, A and B, lying on either sides of the pole are found to be 30° and 60° respectively. A point 'C' lying on the line segment joining AB is such that 2∠APC = ∠APB. Find the length of AC if BC = 30 m.

(A) $30\sqrt{3}$ m (B) 60 m (C) $10\sqrt{3}$ m (D) 45 m

23. Find the angle of elevation of the sun at an instant when the length of the shadow of a pole is $\sqrt{3}$ times its height. (in degrees)

$\boxed{}$

24. A pole EP is errected vertically in the midpoint of the diagonal E of a square ABCD in such a way that the side AB subtends an angle of 60° at P. If the side of the square is 5 m, find the height of the tower in meters.

$\boxed{}\sqrt{2}$

25. A flag staff EP is situated at the midpoint E of side AB of a square ABCD. If the side DC subtends an angle of 30° at P, then find the height of the flag staff, given side of the square is 6m.

(A) $18(4 + \sqrt{3})$m (B) $9(3 + \sqrt{3})$ m
(C) $18(3 + \sqrt{3})$ m (D) $9(3 + 4\sqrt{3})$m

26. If $x^2 + y^2 \le 9$ and $|\tan^2 \pi x| + |\cos^2 \pi y| = 0$, then the number of possible values of (x, y) is $\boxed{}$

27. If θ < 45°, then find the value of $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\theta}}}$.

(A) $4\cos^2 \theta/4$ (B) $4\sin^2 \theta/4$
(C) $2\cos \theta/4$ (D) $2 \sin \theta/4$

28. ABC is a triangle.
4 sinA + 6 cosB = 8
4 cos A + 6 sin B = 6
∠C equals _______.
(A) 60° (B) 120°
(C) 90° (D) cannot be determined

29. PQR is a triangle. $\dfrac{5\sin Q + 4\sin R}{5\sin Q - 4\sin R} = \dfrac{33}{13}$. If the sides of PQR are integers, the sum of PQ and PR has a minimum value of $\boxed{}$

30. E = 10 sinxcosx(5 + sinxcosx). Which of the following cannot be the value of E?

(A) $\dfrac{55}{2}$

(B) $-\dfrac{45}{2}$

(C) $\dfrac{57}{2}$

(D) More than one of the previous choices

Exercise – 7(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If $\sin\theta + \text{cosec}\theta = \frac{5}{2}$, find the value of $\sec^2\theta + \cot^2\theta$.

$$\frac{\boxed{\quad\quad}}{3}$$

2. Which of the following is not possible?

(A) $\cot\theta = \frac{1}{3}$ (B) $\text{cosec}\theta = \frac{4}{3}$

(C) $\sec\theta = 2$ (D) $\sin\theta = \frac{3}{2}$

3. If $\text{cosec}\ \theta = \frac{5}{3}$ (θ lies in the first quadrant), then find the value of $\cot\theta$.

(A) $\frac{4}{3}$ (B) $\frac{7}{3}$ (C) 2 (D) 3

4. If $\sin\theta + \text{cosec}\theta + \sin^2\theta + \text{cosec}^2\theta = 0$, find the value of $\cot\theta$.

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{-1}{\sqrt{2}}$ (D) 0

5. If $\sec\theta$ and $\tan\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which of the following is true?

(A) $b^4 = 4ab^2c + a^4$ (B) $b^4 = 4ab^2c - a^4$

(C) $b^4 = a^4 - 4ab^4c$ (D) $b^4 + a^4 + 4ab^2c = 0$

6. If $\theta = 60°$, then find the value of $\sin2\theta\ \sec3\theta - \text{cosec}2\theta\ \cot\theta$.

(A) $\frac{-\sqrt{3}}{2} + \frac{2}{3}$ (B) $\frac{-\sqrt{3}}{2} - \frac{2}{3}$

(C) $\frac{\sqrt{3}}{2} - \frac{2}{3}$ (D) $\frac{\sqrt{3}}{2} + \frac{2}{3}$

7. If $\cot\theta = \sqrt{\frac{1+\cos2\theta}{1-\cos2\theta}}$, then find $\cot 22^1/_2{}^\circ$

(A) $\sqrt{2} + 1$ (B) $\sqrt{2} - 1$

(C) $\sqrt{2} + 3$ (D) $2\sqrt{2} + 1$

8. If PQR is a right angled isosceles triangle, then $\cos\angle P + \cos\angle Q + \cos\angle R$ = _______.

(A) $\sqrt{2}$ (B) $\sqrt{2}+1$ (C) $\sqrt{2}+2$ (D) 0

9. If $1 + \text{cosec}\theta + \cot\theta = x$, then $\cos\theta$ = _______.

(A) $\frac{x^2-2x+1}{x^2-2x+2}$ (B) $\frac{x^2+2x-1}{x^2+2x+1}$

(C) $\frac{x^2-2x}{x^2-2x+2}$ (D) $\frac{x^2-2x+2}{x^2-2x}$

10. If $13\sin\theta - 12 = 0$ and θ is acute, then find the value of $\frac{2\cos\theta + 3\tan\theta}{\text{cosec}\theta + \cot\theta}$.

$$\frac{\boxed{\quad\quad}}{195}$$

11. PQRS is a cyclic quadrilateral. $\sin P + \sin Q + \sin R + \sin S$ = _______.

(A) –2 (B) 1

(C) 2 (D) Cannot be determined

12. $\cot^6\theta - \text{cosec}^6\theta + 3\cot^2\theta\ \text{cosec}^2\theta$ = $\boxed{\quad\quad}$

13. If $\tan(\alpha - 45°) + \tan(\alpha + 45°) = 0$, then a possible value of α is ________.

(A) 0° (B) 90°

(C) 30° (D) Either (A) or (B)

14. If $270° < \alpha < 360°$ and $\sin\alpha = -3/5$, then find $\sec\alpha + \tan\alpha$.

(A) 1/2 (B) 2 (C) 3 (D) 1/3

15. If $\sin\alpha = 7/25$ and α is acute, then find $\frac{\cot\alpha - \text{cosec}\alpha}{\tan\alpha - \sec\alpha}$.

(A) $\frac{4}{21}$ (B) $\frac{2}{21}$ (C) $\frac{-4}{21}$ (D) $\frac{-2}{21}$

16. If $\cos(x + y) = \frac{\sqrt{3}-1}{2\sqrt{2}}$ and $\sin y = \frac{1}{2}$, which of the following can be the respective values of y and x?

(A) 30°, 45° (B) 30°, 135°

(C) 60°, 45° (D) 60°, 135°

17. The angle covered by the minute hand of a clock in 18 minutes is ________.

(A) 120° (B) 108° (C) 136° (D) 126°

18. $\log\sin30° + \log\cos45° + \log\cos30°$ = _______.

(A) $\log\left(\frac{3}{32}\right)$ (B) 0

(C) $\log\left(\frac{32}{3}\right)$ (D) $\frac{1}{2}\log\left(\frac{3}{32}\right)$

19. If the angles of a triangle ABC are in A.P. with a common difference of 30°, then the value of cosA cosB cosC is _______.

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

20. If $a = \text{cosec}\theta$, $b = \cot\theta$ and θ lies in the first quadrant, then $\sqrt{\frac{a+1}{a-1}} - \sqrt{\frac{a-1}{a+1}}$ = ________.

(A) $\frac{2}{b}$ (B) $\frac{-2}{b}$ (C) –2b (D) 2b

21. The minimum value of $\sin^4 x + \cos^4 x$ is $\boxed{\quad\quad}$

22. If $\text{cosec}\alpha$ and $\text{cosec}\ \beta$ are the roots of $px^2 + qx + r = 0$ where α and β are supplementary angles, which of the following is true?

(A) $p^2 = 4qr$

(B) $r^2 = 4pq$

(C) $q^2 = 4pr$

(D) More than one of the previous choices.

23. If PQRS is a cyclic quadrilateral.
cotP + cotR – (cotQ + cots) = $\boxed{\quad\quad}$

24. Two sides of a triangle are 4 units and 6 units respectively. The angle between them is 30°. Find the area of the triangle (in sq units.)

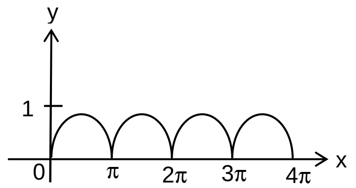
$\boxed{\quad\quad}$

25. The sides of a triangle are 4, $6\sqrt{3}$ and 8. Find the least angle of the triangle. $\cos^{-1}\left(\dfrac{\boxed{\quad\quad}}{\sqrt{3}}\right)$

26. In ΔPQR, if p = 8 cm, q = 10 cm and cosR= $\dfrac{\sqrt{5}+1}{4}$, then find r.

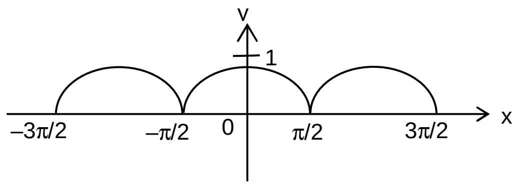
(A) $2\sqrt{41-10\sqrt{5}}$ (B) $2\sqrt{36-10\sqrt{5}}$
(C) $2\sqrt{31-10\sqrt{5}}$ (D) $2\sqrt{26-10\sqrt{5}}$

27. Which of the following relations best describes the graph?



(A) x = sin y (B) y = sin x
(C) x = $|\sin y|$ (D) y= $|\sin x|$

28. Which of the following relations best describes the graph?



(A) y = $\cos^2$x (B) y =cosx
(C) x = cos y (D) y = $|\cos x|$

29. The shadow of Ajay was equal to his height. Find the angle of elevation of the sun (in degrees)

$\boxed{\quad\quad}$

30. An aeroplane is flying at a uniform height of 2 km above the ground. Rohit at a point P observed the plane at an angle of elevation of 60°. After the plane moved for half a minute towards him, he found the angle of elevation to be 75°. Find its speed (in kmph). (tan 75°= 2 + $\sqrt{3}$)

$\dfrac{\boxed{\quad\quad}}{\sqrt{3}\left(2+\sqrt{3}\right)}$

31. A ladder was placed against a wall. It made an angle of 15°with the ground. Its top was 10 m above the foot of the wall. Find the distance (in m) between the foot of the wall and the foot of the ladder. Given that tan 15°= 2 – $\sqrt{3}$

(A) 5(2 + $\sqrt{3}$) (B) 15(2 + $\sqrt{3}$)
(C) 10(2 + $\sqrt{3}$) (D) 20(2 + $\sqrt{3}$)

32. From the top of a hill of 90 m height, the angles of depressions of the top and foot of a tower are observed as 15° and 60° respectively. Find the height of the tower. (in m)

(A) $30\left(\sqrt{3}-1\right)$ (B) $60\left(3-\sqrt{3}\right)$
(C) $90\left(\sqrt{3}-1\right)$ (D) $30\left(3-\sqrt{3}\right)$

33. The value of $\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta$ is $\boxed{\quad\quad}$

34. In a triangle if the length of one of the sides is $3\sqrt{3}$ and the angle opposite to that side is 60° then the circumradius of the triangle is ______.
(A) 1
(B) 2
(C) 3
(D) Cannot be determined

35. In triangle ABC, BC = $4\sqrt{2}$, AC = $4\sqrt{3}$, $\angle$A = 45° and C is the largest angle. Find the length of AB.

(A) 2($\sqrt{3}$ + 1) (B) 3($\sqrt{2}$ + 1)
(C) 2($\sqrt{6}$ + 1) (D) 2($\sqrt{6}$ + $\sqrt{2}$)

36. Due to the influence of wind, the upper part of a pole is broken and it made an angle of 30° with the ground. The distance from the foot of the pole to the point where the top of the pole touches the ground is 20 m. Find the length of the upper part of the pole.

$\dfrac{\boxed{\quad\quad}}{\sqrt{3}}$ m

37. Two buildings of equal heights stand on either side of a road. At a point on the road in between the buildings, the angles of elevation of their tops were observed to be 30° and 45°. If the distance between the buildings is 90 m, then find the height of each building.

$\boxed{\quad\quad}$ $(\sqrt{3}-1)$ m

38. If f(x) = $\dfrac{\sin^3 x}{\cos^2 x}-\dfrac{\cos^3 x}{\sin^2 x}$ and g(x) = $\dfrac{\cos^2 x}{\sin^3 x}-\dfrac{\sin^2 x}{\cos^3 x}$, where $\dfrac{\pi}{4}<x<\dfrac{\pi}{2}$, then which of the following best describes the relation between f(x) and g(x)?
(A) f(x) $\geq$ g(x) (B) f(x) $\leq$ g(x)
(C) f(x) > g(x) (D) f(x) < g(x)

39. If 5cosθ + 12sinθ = 13(θ$\in$$Q_1$), then find the value of tanθ.
(A) 5/12 B) 12/5 (C) 5/6 (D) 6/5

40. If α, β are complementary angles, then the maximum value of $\cos^2\alpha + \sin^2\beta$ is $\boxed{}$

Directions for questions 41 to 45: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using I and II together.

41. Does θ lie in 3rd quadrant?
I. $\cos\theta > 0$.
II. $\sin\theta < 0$.

42. Find the value of $(\cos A + \sin A)^2 - (\cos A - \sin A)^2$.
I. $A = 0°$.
II. $\cos^2 A + \sin^2 A = 1$.

43. Find the value of tanA + tanB + tanC.
I. A, B, C are angles of a triangle.
II. ΔABC is equilateral.

44. Which is taller, The Eiffel Tower or The Statue of Liberty?
I. The angle of elevation of the head of The Statue of Liberty from a distance of 15 m is 60°.
II. The angle of elevation of the middle of The Eiffel Tower from a distance of 15 m is 45°.

45. What is the height of the tower PQ?
I. A man standing on the ground at a point A, 30 m from the foot of the tower observes the top of the tower at an angle of elevation of 60°.
II. A man standing at a point B, 20 m from the fat of the tower observes the midpoint of the tower at an angle of elevation of 30°.

Key

Concept Review Questions

1. 120	7. D	13. B	18. B	24. D	30. (i) D	35. C
2. B	8. D	14. D	19. 1	25. 2	(ii) B	
3. C	9. D	15. (i) –2	20. B	26. 1	31. D	
4. C	10. C	(ii) 2	21. A	27. A	32. 60	
5. A	11. B	16. D	22. 0.8	28. 30	33. C	
6. C	12. C	17. D	23. 0.75	29. 3	34. A	

Exercise – 7(a)

1. 5	6. 0	11. A	(iii) C	19. B	24. 2.5	29. 71
2. A	7. A	12. C	15. B	20. A	25. D	30. C
3. C	8. C	13. B	16. 1	21. 11, 45	26. 26	
4. D	9. B	14. (i) B	17. A	22. A	27. C	
5. B	10. C	(ii) B	18. 30	23. 30	28. C	

Exercise – 7(b)

1. 13	9. C	17. B	25. 1.625	33. 1	41. A
2. D	10. 1036	18. D	26. C	34. C	42. A
3. A	11. D	19. A	27. D	35. D	43. A
4. D	12. –1	20. A	28. D	36. 40	44. C
5. A	13. D	21. 0.5	29. 45	37. 45	45. B
6. B	14. A	22. C	30. 480	38. C	
7. A	15. A	23. 0	31. C	39. B	
8. A	16. A	24. 6	32. B	40. 2	