CHAPTER – 4

SEQUENCES AND SERIES

PROGRESSIONS

In this chapter, we will look at the problems on sequences or progressions of numbers, where the terms of the sequence follow a particular pattern - either addition of a constant (Arithmetic Sequence or Arithmetic Progression) or multiplication by a constant (Geometric Sequence or Geometric Progression). A third type of progression–Harmonic Progression–has also been defined later.

ARITHMETIC PROGRESSION (A.P.)

An arithmetic progression is a sequence of numbers in which any number (other than the first) is more (or less) than the immediately preceding number by a constant value. This constant value is called the common difference. In other words, any term of an arithmetic progression can be obtained by adding the common difference to the preceding term.

Let a be the first term of an arithmetic progression, d the common difference and n the number of terms in the progression.
The n^{th} term is normally represented by T_n and the sum to n terms is denoted by S_n

T_n = n^{th} term = a + (n – 1) d

S_n = Sum of n terms $= \frac{n}{2} \times [2a + (n-1)d]$.

The progression can be represented as a, a + d, a + 2d, …….., [a + (n – 1)d]. Here, quantity d is to be added to any chosen term to get the next term of the progression.

The sum to n terms of an arithmetic progression can also be written in a different manner.

Sum of first n terms $= \frac{n}{2} \times [2a + (n-1)d]$

$= \frac{n}{2} \times [a + \{a + (n-1)d\}]$

But, when there are n terms in an arithmetic progression, a is the first term and {a + (n – 1)d} is the last term.

Hence, $S_n = \frac{n}{2} \times$ [First Term + Last Term]

Arithmetic Mean (A.M) is the average of a set of numerical values. Since average is equal to {sum of all the quantities/number of quantities}, arithmetic mean of an A.P is equal to the sum of the terms of the arithmetic progression divided by the number of terms in the arithmetic progression.
Arithmetic Mean of n terms in arithmetic progression

$= \frac{Sn}{n} = \frac{1}{2}\{2a + (n-1)d\}$

$= \frac{1}{2} \times$ (First Term + Last Term)

$= \frac{(\text{First Term} + \text{Last Term})}{2}$

i.e., A.M. of a A.P is the average of the first and the last terms of the A.P.
Arithmetic Mean can also be obtained by taking the average of any two terms which are EQUIDISTANT from the two ends of the A.P. i.e.,

- The average of the second term from the beginning and the second term from the end will be equal to the A.M.
- The average of the third term from the beginning and the third term from the end will also be equal to the A.M. and so on.

In general, the average of the k^{th} term from the beginning and the k^{th} term from the end will be equal to the A.M.

Conversely, if the A.M. of an A.P. is known, the sum to n terms of the series (S_n) can be expressed as
$S_n = n \times$ A.M.

For two numbers a and b their A.M. $= \frac{(a+b)}{2}$.

If three numbers are in arithmetic progression the middle number is called the Arithmetic Mean, i.e., if a, b, c are in A.P., then b is the A.M. of the three terms and $b = \frac{a+c}{2}$.

If three numbers are in A.P., we can represent the three numbers as (a – d), a and (a + d).

If four numbers are in A.P., we can represent the four numbers as (a – 3d), (a – d), (a + d) and (a + 3d); (in this case, 2d is the common difference).

If five numbers are in A.P., we can represent the five numbers as (a – 2d), (a – d), a, (a + d) and (a + 2d).

Examples

4.01. The first term and the common difference of an arithmetic progression are 4 and 3 respectively. Find the 11^{th} term.

Sol: The n^{th} term of A.P = a + (n – 1) d
11^{th} term = 4 + 10 (3) = 34

4.02. Find the number of terms in an arithmetic progression in which the first and last terms are 7 and 45 respectively and the common difference is 2.

Sol: Let the number of terms be n.
Given a = 7, d = 2 and a + (n – 1) d = 45
$\Rightarrow$ 45 = 7 + (n – 1) 2
$\therefore$n = 20

4.03. The sixth and the tenth terms of an arithmetic progression are 22 and 38 respectively. Find the first term and the common difference.

Sol: Let the first term and the common difference be a and d respectively.

$a + 5d = 22$ --- (1)
$a + 9d = 38$ --- (2)
Subtracting (1) from (2),
$4d = 16$, $d = 4$
Substituting d in (1) or (2),
we get $a = 2$

4.04. The 12^{th} term, the 14^{th} term and the last term of an arithmetic progression are 25, 31 and 37 respectively. Find the first term, common difference and the number of terms.

Sol: Let the first term, the common difference and the number of terms be a, d and n respectively.
Given that
$a + 11d = 25 \quad \rightarrow \quad (1)$
$a + 13d = 31 \quad \rightarrow \quad (2)$
Subtracting (1) from (2),
$2d = 6$
$d = 3$
Substituting $d = 3$ in (1) or (2),
$a = -8$
given, $t_n = -8 + (n - 1)\,3 = 37$
$n = 16$

4.05. Find the sum of the first 20 terms of an arithmetic progression in which the first term is 6 and the common difference is 2.

Sol: The sum of the first n terms of an A.P
$= \frac{n}{2}\,(2a + (n - 1)\,d)$
Sum of the first 20 terms
$= \frac{20}{2}\left[2(6) + 19(2)\right] = 500$

4.06. Find the arithmetic mean of the first 31 terms of an arithmetic progression in which the first term is 3 and the common difference is 5.

Sol: Arithmetic mean of the terms of an A.P

$$= \frac{\frac{n}{2}\,(2a + (n-1)d)}{n} = \frac{2a + (n-1)d}{2}$$

$$\therefore \text{ arithmetic mean } = \frac{2(3) + (30)(5)}{2} = 78$$

4.07. Divide 100 into four parts which are in arithmetic progression such that the product of the second part and third part is 72 more than the product of the other two parts.

Sol: Let the four parts be $a - 3d$, $a - d$, $a + d$ and $a + 3d$
Given, $a - 3d + a - d + a + d + a + 3d = 100$
$\therefore a = 25$
Given $(a - d)\,(a + d) = (a - 3d)\,(a + 3d) + 72$
$a^2 - d^2 = a^2 - 9d^2 + 72$
$d = \pm 3$
If $d = 3$, the numbers are 16, 22, 28 and 34. If $d = -3$, the numbers are same but in the descending order.

4.08. Three terms in arithmetic progression have a sum of 45 and a product of 3240. Find them.

Sol: Let the terms be $a - d$, a and $a + d$.
$a - d + a + a + d = 45$
$a = 15$
$(a - d)\,a\,(a + d) = 3240$
$15^2 - d^2 = 216$
$d = \pm 3$
If $d = 3$ the terms are 12, 15 and 18. If $d = -3$, the terms are same but in the descending order.

4.09. The first term and the last term of an arithmetic progression are 9 and 69 respectively. If the sum of all the terms is 468, find the number of terms and the common difference.

Sol: Let the number of terms and the common difference be n and d respectively,
$S_n = \frac{n}{2}\left[9 + 69\right] = 468 \Rightarrow 39n = 468$
$n = 12$
$t_n = 9 + 11d \Rightarrow 11d = 60$
$\Rightarrow d = \frac{60}{11}$

4.10. The 17^{th} term and the 28^{th} term of an arithmetic progression are 66 and 110 respectively. Find the 33^{rd} term.

Sol: Let the first term and the common difference be a and d respectively.
$a + 16d = 66 \quad \rightarrow \quad (1)$
$a + 27d = 110 \quad \rightarrow \quad (2)$
Subtracting (1) from (2),
$11d = 44$
$d = 4$
Substituting $d = 4$ in (1) or (2),
$a = 2$
33^{rd} term $= 2 + 32\,(4) = 130$.

4.11. The sum of three numbers which are in arithmetic progression is 24. The sum of their squares is 200. Find the numbers.

Sol: Let the numbers be $a - d$, a and $a + d$.
Given, $a - d + a + a + d = 24$
$\therefore a = 8$
$(a - d)^2 + a^2 + (a + d)^2 = 200$
$3a^2 + 2d^2 = 200$
$\Rightarrow d^2 = 4$
$\therefore d = \pm 2$
If $d = 2$, the numbers are 6, 8 and 10. If $d = -2$, the numbers are same, but in the descending order.

GEOMETRIC PROGRESSION (G.P.)

Numbers taken in a certain order, are said to be in Geometric Progression, if the ratio of any term (other than the first) to the preceding one is the same. This ratio is called the common ratio. In other words, any term of a geometric progression can be obtained by multiplying the preceding number by the common ratio.

The common ratio is normally represented by r. The first term of a geometric progression is denoted by a.

A geometric progression can be represented as a, ar, ar^2, ... where a is the first term and r is the common ratio of the geometric progression.
n^{th} term of the geometric progression is ar^{n-1}.

Sum to n terms :

$$\frac{a\left(1-r^n\right)}{1-r} \text{ or } \frac{a\left(r^n-1\right)}{r-1}$$

$$= \frac{ar^{n-1}-a}{r-1} = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

Thus the sum to n terms of a geometric progression can also be written as

$$S_n = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

If n terms a_1, a_2, a_3, ... a_n are in G.P., then the Geometric Mean (G.M.) of these n terms is given by $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots\ldots\ldots \cdot a_n}$

If three terms are in geometric progression then the middle term is a Geometric Mean of the other two terms, i.e., if a, b and c are in G.P., then b is the Geometric Mean of the three terms and $b^2 = ac$.

If there are two terms a and b, their Geometric Mean (G.M.) is given by $\sqrt{ab}$.

When there are three terms in geometric progression, we can represent the three terms to be a/r, a and ar

When there are four terms in geometric progression, we can represent the four terms as $\frac{a}{r^3}$, $\frac{a}{r}$, ar and ar^3.

(In this case r^2 is the common ratio)

INFINITE GEOMETRIC PROGRESSION

If $-1 < r < +1$ or $|r| < 1$, then the sum of the terms of a geometric progression does not increase infinitely; it "converges" to a particular value. Such a G.P. is referred to as an infinite geometric progression. The sum of an infinite geometric progression is represented by S_∞ and is given by the formula

$S_\infty = \frac{a}{1-r}$.

HARMONIC PROGRESSION (H.P)

If the reciprocals of the terms of a sequence are in arithmetic progression, the sequence is said to be a harmonic progression, For example, 1, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$,........ is a harmonic progression. In general, the sequence $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$,....... is a harmonic progression.

For two numbers a and b, their Harmonic Mean (H.M) is given by $\frac{2ab}{a+b}$.

For any two positive numbers a and b,

$$\boxed{A.M \geq G.M \geq H.M}$$

If a, b, c are in harmonic progression, b is said to be the harmonic mean of a and c. In general, if x_1, x_2, ……x_n are in harmonic progression, x_2, x_3,….x_{n-1} are the n – 2 harmonic means between x_1 and x_n.

SOME IMPORTANT RESULTS

The results of the sums to n terms of the following series are quite useful and hence should be remembered by students.

Sum of the first n natural numbers

$$= \sum n = \frac{n(n+1)}{2}$$

Sum of squares of the first n natural numbers

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \left[\sum n\right]^2$$

4.12. Find the 7^{th} term of the geometric progression whose first term is 6 and common ratio is 2.

Sol: n^{th} term of a G.P = ar^{n-1}
7^{th} term = 6 (2^6) = 384

4.13. A geometric progression has its first term as 64 and its common ratio as $\frac{1}{2}$. Find the sum of its first five terms.

Sol: Sum of the first n terms of a G.P = $\frac{a\left(1-r^n\right)}{1-r}$

Sum of its first five terms = $\frac{64\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} = 124$

4.14. Find the common ratio of the geometric progression whose first and last terms are 5 and $\frac{1}{25}$ respectively and the sum of its terms is $\frac{624}{100}$.

Sol: Sum of the terms of a geometric progression whose common ratio is r, is given by

$$\frac{r\left(\text{last term}\right)-\left(\text{first term}\right)}{r-1}$$

$$\frac{r\left(\frac{1}{25}\right)-5}{r-1} = \frac{624}{100}$$

$\Rightarrow$ 4 r – 500 = 624r – 624

$\Rightarrow$ $r = \frac{1}{5}$.

4.15. In the previous example, find the number of terms.

Sol: Let the number of terms be n.

$$\frac{1}{25} = 5\left(\frac{1}{5}\right)^{n-1}$$

$$\left(\frac{1}{5}\right)^3 = \left(\frac{1}{5}\right)^{n-1}$$

Comparing both sides, n – 1 = 3

$\therefore$ n = 4

4.16. Three numbers in geometric progression have a sum of 42 and a product of 512. Find the numbers.

Sol: Let the numbers be $\frac{a}{r}, a$ and ar.

$\frac{a}{r} + a + ar = 42$

$\left(\frac{a}{r}\right)(a)(ar) = 512$

a = 8

$\frac{8}{r} + 8 + 8r = 42$

$8r^2 - 34r + 8 = 0$

$8r^2 - 32r - 2r + 8 = 0$

(r – 4) (4r – 1) = 0

$r = 4$ or $\frac{1}{4}$

If r = 4, the numbers are 2, 8 and 32. If $r = \frac{1}{4}$, the numbers are same, but in the descending order.

4.17. The sum of the terms of an infinite geometric progression is 27. The sum of their squares is 364·5. Find the common ratio.

Sol: Let the first term and the common ratio be a and r respectively.

Given that $\frac{a}{1-r} = 27 \Rightarrow \left(\frac{a}{1-r}\right)^2 = 729$

And $\frac{a^2}{1-r^2} = 364 \cdot 5$

$\Rightarrow a^2 = 729\,(1 - r)^2 = 364{\cdot}5\,(1 - r^2)$

$729\,(1 - r)^2 - \frac{729}{2}(1-r)(1+r) = 0$

$\frac{729}{2}(1-r)\,[2(1-r)-(1+r)] = 0$

$\Rightarrow (1 - r)(1 - 3r) = 0$

$r \neq 1\ (\because |r| < 1)$

$\therefore r = \frac{1}{3}$

4.18. If |x| < 1, find the value of $3 + 6x + 9x^2 + 12x^3 + \ldots\ldots$

Sol: Let $S = 3 + 6x + 9x^2 + 12x^3 + \ldots. \quad \rightarrow \quad (1)$

$xS = 3x + 6x^2 + 9x^3 + \quad \rightarrow \quad (2)$

Subtracting (2) from (1)

$S(1 - x) = 3(1 + x + x^2 + \ldots\ldots\ldots\ldots)$

As |x| < 1, $S = \frac{3\left(\frac{1}{1-x}\right)}{1-x} = \frac{3}{(1-x)^2}$

4.19. Find the sum of the series $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots\ldots\ldots\ldots\infty$

Sol: The series is an infinite geometric progression with first term as 1 and common ratio as $\frac{2}{3}$.

Sum to infinity of an infinite geometric progression = $\frac{a}{1-r}$

$\therefore$ Sum of the series = $\frac{1}{1-\frac{2}{3}} = 3$

4.20. If $(1^3 + t_1) + (2^3 + t_2) + (3^3 + t_3) + (4^3 + t_4) + \ldots\ldots\ldots. + (n^3 + t_n) = \frac{n^2(n+6)}{4}$, find t_{10}.

Sol: $(1^3 + 2^3 + 3^3 + \ldots\ldots\ldots n^3) + (t_1 + t_2 + \ldots\ldots. t_n)$

$= \frac{n^2(n+6)}{4}$

$\Rightarrow t_1 + t_2 + \ldots\ldots\ldots\ldots.. t_n$

$= \frac{n^2(n+6)}{4} - (1^3 + 2^3 + 3^3 + \ldots.. n^3)$

$= \frac{n^2(n+6)}{4} - \frac{1}{4}\left(n(n+1)^2\right) = \frac{n^2[-n^2 - n + 5]}{4}$

Let $S_n = t_1 + t_2 + \ldots\ldots. t_n$

$t_{10} = S_{10} - S_9$

$S_{10} = \frac{10^2[-100-10+5]}{4} = -2625$

$S_9 = \frac{81[-81-9+5]}{4} = \frac{-6885}{4}$

$\therefore t_{10} = -2625 - \left(\frac{-6885}{4}\right) = \frac{-3615}{4}$

SEQUENCES AND SERIES

Any function for which the domain is the set {1, 2, ….n} is called a sequence (or a finite sequence). If the domain is N = { 1, 2, 3, ….}, it is an infinite sequence. Thus, 1, $\frac{1}{2}$, $\frac{1}{3}$, …. $\frac{1}{1000}$ is a finite sequence, while 1, $\frac{1}{2}$, $\frac{1}{3}$, …. or 1, $\frac{1}{2^2}$, $\frac{1}{3^2}$, …. are infinite sequences.

An expression of the kind $x_1 + x_2 + \ldots. + x_n$ is a series (or a finite series) while expressions of the kind $x_1 + x_2 + \ldots.$ are infinite series.

Consider the infinite sequences A and B and infinite series C and D given below.

A : 1, 2, 3,….

B : $1 + \frac{1}{1}, 1+\frac{1}{2}, 1+\frac{1}{3}, \ldots..$

A is said to be a **divergent** sequence while B is a **convergent** sequence. This is because there is no finite number to which the terms of A tend to, while in the case of B, the terms tend to 1, i.e., if t_n denotes the n^{th} term, we can make sure that t_n is as close to 1 as we please, by taking any value of n, which is sufficiently large. (The condition is stated quantitatively as follows. For any value of δ, no matter how small, we can find a number N, such that if $n \geq N$, then $|t_n - 1| < \delta$.)

C : $1 + \frac{1}{2} + \frac{1}{3} + \ldots.$

D : $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots..$

Similarly C is a **divergent** series while D is a **convergent** series. This is because there is no finite value to which C tends while D tends to 2, i.e. we can ensure that the sum upto the n^{th} term of D say S_n, is as close to 2 as we please, by taking any value of n which is sufficiently large. (This condition is stated quantitatively as follows. For any value of δ, no matter has small, we can find a number N such that if $n \geq N$, then $|s_n - 2| < \delta$.)

We shall look at some useful models on series/sequence which have appeared in management entrances. The series could include AP, GP or other patterns of summations which involve concepts of progressions. There could also be other series which appear to be related to progressions, but actually involve techniques of mathematical manipulation. These techniques are best illustrated or learned using examples. But for most questions observing the pattern proved to be a useful method for arriving at the answer.

Note: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

4.21. In a certain sequence,
$t_1 = 5$ and $t_{n+1} = 2t_n - 3n + 3$. Find t_{100}

Sol: To get any term, we need to multiply the preceding one by 2.
$\therefore$ We should try to relate the n^{th} term t_n to 2^n
$t_1 = 5 = 2 + 3$
$t_2 = 2(5) - 3(1) + 3 = 10 = 2^2 + 3(2)$
$t_3 = 2(10) - 3(2) + 3 = 17 = 2^3 + 3(3)$
$t_4 = 2(17) - 3(3) + 3 = 28 = 2^4 + 3(4)$
We can guess that $t_n = 2^n + 3n$.
$\therefore t_{100} = 2^{100} + 3\,(100)$
We shall consider this general problem in reverse, i.e., we start with the answer ($t_n = r^n + bn$) and obtain the question (the way t_{n+1} is related to t_n)
Let $t_n = ar^n + bn$ ---- (1)
$\Rightarrow rt_n = ar^{n+1} + brn$
$\therefore t_{n+1} = a\,.\,r^{n+1} + bn + b = rt_n - brn + bn + b$
$= rt_n + b\,(1 - r)\,n + b.$ ……. (2)
Let us compare this with the relation given in the example above
$t_{n+1} = 2t_n - 3n + 3$
i.e., r = 2, b = 3 and $b(1 - r) = 3(1 - 2) = -3$
We see that the relation between t_{n+1} and t_n is of the kind (2) where r = 2, b = 3
$\therefore t_n = 2^n + 3n$

In general, if a relation of the type $t_{n+1} = rt_n + cn + b$ is given, where $c = b(1 - a)$, then t_n is given by (1).

4.22. Evaluate $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{99(100)}$.

Sol: $\frac{1}{1(2)} = \frac{1}{1} - \frac{1}{2}$

$\frac{1}{2(3)} = \frac{1}{2} - \frac{1}{3}$

Finally $\frac{1}{99(100)} = \frac{1}{99} - \frac{1}{100}$

The given expression is $1 - \frac{1}{100} = \frac{99}{100}$

4.23. Find the sum of

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}, \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}, \ldots, \sqrt{1+\frac{1}{20^2}+\frac{1}{21^2}}$$

Sol: The general term is given by

$t_n^2 = 1 + \frac{1}{n^2} + \frac{1}{(n+1)^2}$

$= \frac{n^2(n+1)^2 + (n+1)^2 + n^2}{n^2(n+1)^2}$

$= \frac{[n(n+1)]^2 + 2n(n+1) + 1}{[n(n+1)]^2} = \frac{(n^2+n+1)^2}{n^2(n+1)^2}$

$\therefore t_n = \frac{n^2+n+1}{n(n+1)} = 1 + \frac{1}{n(n+1)}$

$t_1 = 1 + \frac{1}{1(2)}$

$t_2 = 1 + \frac{1}{2(3)}$

And finally $t_{20} = 1 + \frac{1}{20(21)}$

$\therefore S = 20 + 1 - \frac{1}{21} = 20\frac{20}{21}$

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the 22nd term of the arithmetic progression whose first term is 20 and common difference is 1/3.
(A) 27 (B) $27^1/_3$ (C) $27^2/_3$ (D) 28

2. Which term of the arithmetic progression 2, 6, 10, …… is 106?

3. Find the 10th term of the arithmetic progression whose 4th term is 7 and whose 17th term is 72.
(A) 32 (B) 37 (C) 42 (D) 47

4. The sixth term and the eleventh term of an arithmetic progression are 30 and 55 respectively. Find the twenty-first term of the arithmetic progression.
(A) $88^1/_3$ (B) 105 (C) 110 (D) $92^1/_2$

5. Thirteen times the thirteenth term of an arithmetic progression is equal to seven times the seventh term of the arithmetic progression. What is the twentieth term?

6. What is the 15th term of an arithmetic progression whose first term is equal to its common difference and whose 3rd term is 9?
(A) 15 (B) 30 (C) 45 (D) 60

7. If x + 4, 6x – 2 and 9x – 4 are three consecutive terms of an arithmetic progression, then find x.
(A) 2 (B) 4 (C) 6 (D) 8

8. Find the number of terms and the sum of the terms respectively of the arithmetic progression 32, 28, … 4.
(A) 8; 144 (B) 7; 126 (C) 14; 252 (D) 15; 270

9. Find the sum of the first 31 terms of the arithmetic progression whose first term is 6 and whose common difference is $\frac{8}{3}$.

10. Find the sum of the terms of the arithmetic progression whose first term, last term and common difference are 3, 101 and 7 respectively.
(A) 750 (B) 720 (C) 780 (D) 810

11. Find the sum of the terms of an arithmetic progression whose first term, last term and number of terms are –9, 51 and 21 respectively.
(A) 420 (B) 441 (C) 462 (D) 483

12. (a) Three terms are in arithmetic progression such that their sum is 36 and product is 1296. Find the three terms.
(A) 4, 12, 20 (B) 6, 12, 18
(C) 9, 12, 15 (D) 8, 12, 16

(b) The sum of five terms in arithmetic progression is 70. The product of the extreme terms is 132. Find the five terms.
(A) 4, 8, 12, 16, 20 (B) 10, 12, 14, 16, 18
(C) 6, 10, 14, 18, 22 (D) 8, 12, 16, 20, 24

13. The sum to n terms of an arithmetic progression is $5n^2 + 2n$. Find the nth term.
(A) 10n + 5 (B) 10n – 3 (C) 5n – 1 (D) 5n – 2

14. Find the sum of all the two-digit numbers which leave a remainder of 1 when divided by 4.
(A) 1201 (B) 1012 (C) 1210 (D) 1021

15. **(a)** The sum of the first 71 terms of an arithmetic progression is 0. Which of the following terms must be 0?
(A) 18th (B) 19th (C) 36th (D) 37th

(b) The sum of the first 30 terms of an arithmetic progression is 40. The sum of its first 60 terms is also 40. Find the sum of its 31st and 60th terms.
(A) 0 (B) 600 (C) 40 (D) 1200

16. Which term of the geometric progression 4, $4\sqrt{2}$, 8 ….. is $64\sqrt{2}$?

17. Find the first four terms of a geometric progression whose nth term is $4(-5)^n$.
(A) –20, 100, –500, 2500
(B) 20, –100, –500, 2500
(C) –20, –100, –500, –2500
(D) 20, 100, 500, 2500

18. Find the sixth term of the geometric progression whose first term is 2 and common ratio is 3.
(A) 96 (B) 486 (C) 1458 (D) 162

19. The sum of the first n terms of the geometric progression, whose first term is 4 and the common ratio is 3, is 4372. Find n.

20. The fourth term and the eighth term of a geometric progression are 3 and 1/27 respectively. Find the 12th term.
(A) 1/243 (B) 1/729 (C) 1/2187 (D) 1/6561

21. In a geometric progression, the 2nd term is 9 and the 6th term is 729. What is the 4th term?
(A) 81 (B) –81 (C) 27 (D) –27

22. **(a)** Find the sum of the first 4 terms of a geometric progression whose first term is 6 and whose common ratio is 2.
(A) 90 (B) 84 (C) 96 (D) 102

(b) What is the sum of the first 7 terms of a geometric progression whose first term is 1 and 4th term is 8?
(A) 129 (B) 128
(C) 127 (D) None of these

23. How many terms of the series 5, $5\sqrt{5}$, 25, ….. add up to $155 + 155\sqrt{5}$?

24. Find the common ratio of a geometric progression, whose first term is 3, last term is 2187 and the sum of the terms is 3279.

25. Three terms in geometric progression are such that their sum is 26 and their product is 216. Find the terms.
(A) 4, 8, 16 (B) 2, 6, 18 (C) 2, 8, 16 (D) 3, 6, 12

26. Find the sum to infinity of 1, $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}$...,

27. In an infinite geometric progression each term is equal to seven times the sum of the terms that follow it. What is the common ratio of the progression?
(A) 1/6 (B) 1/7 (C) 1/8 (D) 1/9

28. (a) What is the arithmetic mean of the arithmetic progression 3, 7, 11, 15, 19, 23, 27, 31?

(b) Find the geometric mean of the geometric progression 3, 9, 27, 81.
(A) 9 (B) $9\sqrt{3}$ (C) 243 (D) 18

29. (a) If x, y and z are three natural numbers in arithmetic progression, then the x^{th} term, the y^{th} term and the z^{th} term of any arithmetic progression A are in
(A) arithmetic progression
(B) geometric progression
(C) not necessarily in arithmetic progression or geometric progression.

(b) If x, y and z are three natural numbers in arithmetic progression, then the x^{th} term, the y^{th} term and the z^{th} term of any geometric progression G, are in
(A) arithmetic progression
(B) geometric progression
(C) not necessarily in arithmetic progression or geometric progression.

30. If 3 positive numbers are in geometric progression, their logarithms will be in
(A) arithmetic progression
(B) geometric progression
(C) not necessarily in arithmetic progression or geometric progression.

Exercise – 4(a)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The 67^{th} term of an arithmetic progression is 15 times the fourth term. Find the 21^{st} term, if the 11^{th} term is 23.

2. Find the smallest of the three numbers in arithmetic progression, if the product of the first and the third numbers is 252 and the sum of the three numbers is 48.
(A) 10 (B) 12 (C) 14 (D) 16

3. Find the expression for the sum of n terms of an arithmetic progression, if the tenth term is 40 and the 12^{th} term is 44.
(A) $10n + 25n^2$ (B) $20n + 20n^2$
(C) $25n + 15n^2$ (D) $n^2 + 21n$

4. If the sum to 37 terms of an arithmetic progression is 703, then find the middle term of the arithmetic progression.

5. Find the least value of the number of terms of the series 20, 18, 16, … for which the series has the maximum sum.
(A) 9 (B) 10 (C) 11 (D) 12

6. If the sum of the fifth, thirteenth and eighteenth terms of an A.P is zero, find the 12^{th} term.
(A) –2 (B) 1
(C) 0 (D) Cannot be determined

7. How many integers between 450 and 950 are divisible by both 3 and 7?
(A) 20 (B) 24 (C) 30 (D) 35

8. How many integers are there between 300 and 600 that are divisible by 9?
(A) 33 (B) 31 (C) 28 (D) 25

9. Find the sum of all two-digit numbers which leave a remainder of 3 when divided by 7.

10. The sums of n terms of two series in A.P. are in the ratio (7n – 17) : (4n + 16). Find the ratio of their 21^{st} terms.
(A) 3 : 2 (B) 4 : 3 (C) 5 : 4 (D) 4 : 5

11. The terms of an arithmetic progression are all positive. The square of fourth term equals the sum of the squares of the previous two terms. The sum of the first four terms is 14. Find the common difference.

12. Find the number of terms common to the progressions 7, 11, 15, …. 497 and 1, 6, 11, 16, ..., 501.

13. The first term of an arithmetic progression consisting of 30 terms is 10 and the common difference is 5. Find the ratio of the sum of the 30 terms of the arithmetic progression to the sum of the last 20 terms.
(A) 99 : 13 (B) 96 : 17 (C) 93 : 19 (D) 99 : 86

14. There are 30 terms in an arithmetic progression. The second and third terms are distinct integers. The ratio of the sum of first 20 terms and the sum of the first 10 terms equals twice the ratio of the second and first terms. Which of the following can be the sum of all its terms?
(A) 1120 (B) 1560 (C) 2020 (D) 3750

15. How many terms of the A.P. 2, 4, 6, . . . must be taken so that the sum is 156?
(A) 13 (B) 12 (C) 11 (D) 20

16. 73 times the 73^{rd} term of an arithmetic progression is equal to 37 times the 37^{th} term. What is the value of the 110^{th} term?
(A) 73 (B) 36 (C) 110 (D) 0

17. The sum of four positive terms in arithmetic progression is 60 and the ratio of the product of the first and the third terms to the product of the second and the fourth terms is 3 : 8. Find the four terms.
(A) 9, 12, 15, 18 (B) 6, 12, 18, 24
(C) 8, 12, 16, 24 (D) 5, 10, 15, 20

18. A person is employed in a company for a salary of ₹6000 per month. If he gets an increment of ₹300 on his monthly salary every year, what will be the total amount he receives in a period of 30 years?
(in ₹)

19. Find the sum of all the integers from 1 to 300 that are divisible either by 3 or 5.
(A) 21150 (B) 36250 (C) 35150 (D) 37350

20. If $\log_2 x + \log_2 x^2 + \log_2 x^3 + \log_2 x^4 + \ldots\ldots\ldots + \log_2 x^{10} = 220$, then find x.
(A) 4 (B) 16 (C) 8 (D) 10

21. The product of three terms in geometric progression is 1728 and the sum of the products of two of them taken at a time is 1032. Find the smallest of the three terms.

22. The sum of the first eight terms of a geometric progression is 510 and the sum of the first four terms of the geometric progression is 30. Find the first term of the geometric progression, given that it is positive.
(A) 2 (B) 4 (C) 6 (D) 8

23. What is the fifth term of an infinite geometric progression whose first term is 4 and each term is thrice the sum of all the terms following it?
(A) $\frac{1}{8}$ (B) $\frac{1}{64}$ (C) $\frac{1}{32}$ (D) $\frac{1}{256}$

24. The sum of an infinite geometric progression is 12. The sum to infinity of the squares of the terms is 48. Find the first term.

25. Find the number of terms in a G.P. whose first term is 5, sum of all the terms is 5115/512 and the common ratio is 1/2.

26. In a G.P. with a negative common ratio, the 7th and 13th terms are $5^{7.5}$ and $5^{13.5}$ respectively. What are the values of the common ratio and the first term respectively?
(A) $-5; 5^{1.5}$ (B) $-5; 5^{2.5}$
(C) $-25; 5^{0.5}$ (D) $-25; 5^{1.5}$

27. A square T-2 is formed by joining the midpoints of the sides of another square T-1 of side 16 cm. A third square T–3 is formed by joining the midpoints of the sides of T-2. Find the sum of areas of all the squares (including T-1) formed by repeating this process indefinitely. (in sq cm)

28. Find the integer value of y, if –x, 2y and 2(y + 3) are in arithmetic progression and (x + 2), 2(y + 1) and (5y – 1) are in geometric progression.
(A) 2 (B) 3 (C) 4 (D) 5

29. The first, second and third terms of a geometric progression are equal to the first, seventh and twelfth terms respectively of an arithmetic progression. If the first term and common difference have opposite signs, find the 37th term of the arithmetic progression.
(A) –1 (B) 0
(C) 1 (D) Cannot be determined

30. Three numbers 5 + x, 5x + 1 and 8x form an increasing arithmetic progression. If the third term is divided by 6, the resulting number and the first and second terms of the arithmetic progression taken in order would be in G.P. Find the common ratio.

31. The product of three numbers p, q and r which are in geometric progression is 512. If p is increased by 14, r is decreased by 8 and q remains the same, the resulting values of q, p and r are in arithmetic progression. What could be the initial value of r?
(A) 28 (B) 32 (C) 36 (D) 40

32. A is an arithmetic progression of 13 terms. G is a geometric progression of 13 terms. The product of the terms of G is 8192. The sum of the terms of A is 26. Find the sum of the seventh terms of A and G.
(A) 6 (B) 4
(C) 8 (D) Cannot be determined

33. Find the sum of the series $2 + 3x + 4x^2 + 5x^3 + \ldots$ to infinity, if $|x| < 1$.
(A) $\frac{2-x}{(1-x)^2}$ (B) $\frac{2+x}{(1+x)^2}$
(C) $\frac{2-x}{(1+x)^2}$ (D) $\frac{2+x}{(1-x)^2}$

34. Find the value of
$$1+\frac{0.9}{11}+\frac{0.99}{(11)^2}+\frac{0.999}{(11)^3}+\ldots\ldots.$$
(A) $\frac{1089}{990}$ (B) $\frac{1981}{1090}$ (C) $\frac{989}{990}$ (D) $\frac{1189}{1090}$

35. If $X=\frac{1}{80(41)}+\frac{1}{79(42)}+\frac{1}{78(43)}+\ldots+\frac{1}{42(79)}+\frac{1}{41(80)}$ and $Y = 1 - \frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots\ldots+\frac{1}{79}-\frac{1}{80}$, then find the value of $\frac{Y}{X}$.
(A) 40.5 (B) 50.5
(C) 60.5 (D) None of these

36. Find the sum of the first 10 terms of the series $3(2^2) + 4(3^2) + 5(4^2) + \ldots\ldots$

37. Find the sum of the first 10 terms of the series
$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\sqrt{1+\frac{1}{4^2}+\frac{1}{5^2}}+\ldots\ldots+\sqrt{1+\frac{1}{10^2}+\frac{1}{11^2}}$$
(A) $7\frac{7}{11}$ (B) $8\frac{8}{11}$ (C) $9\frac{9}{11}$ (D) $10\frac{10}{11}$

38. In a certain series, the nth term $T_n=2(T_{n-1}) + (n-2)$ for $n \geq 2$. If $T_1 = 2$, then find the value of T_{100}.
(A) $2(3^{99}) - 2^{99}$ (B) $3(2^{98}) - 1$
(C) $3(2^{101}) - 10^3$ (D) $3(2^{99}) - 10^2$

39. Find the sum of the first 20 terms of the series 1 + 10 + 23 + 40 + 61 + 86 + ...
(A) 6290 (B) 6940 (C) 6980 (D) 6364

40. Find the value of $(50 \times 1) + (49 \times 2) + (48 \times 3) + \ldots (1 \times 50)$.

41. $\frac{1}{2}+\frac{1}{2+4}+\frac{1}{2+4+6}+\ldots\ldots\frac{1}{2+4+6+\ldots+400}=$
(A) $\frac{399}{400}$ (B) $\frac{199}{200}$ (C) $\frac{299}{300}$ (D) $\frac{499}{500}$

42. $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots\ldots+\frac{31}{1^2+2^2+3^2+\ldots 15^2}=$
(A) $\frac{23}{4}$ (B) $\frac{45}{8}$ (C) 6 (D) $\frac{15}{2}$

43. If a, b, 2a + b, 2a – 3b –7 are the first four terms of an arithmetic progression, find the 97th term.
(A) $\frac{-193}{2}$ (B) $\frac{-195}{2}$ (C) $\frac{193}{2}$ (D) $\frac{-123}{2}$

44. There are N consecutive even integers in descending order. The first integer is N and the average of the first five is 1594 more than the last integer. Find the value of N.
(A) 700 (B) 800 (C) 600 (D) 750

45. Find the 100th term of 1, 5, 11, 19, 29, 41, ...

Exercise – 4(b)

Directions for questions 1 to 55: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The first term of an arithmetic progression is 6 and the common difference is 4. The n^{th} term is 250. Find the value of n. ☐

2. The sum of three numbers in an arithmetic progression is 39 and the sum of the squares of the three numbers is 515. Find the smallest of the three numbers.
(A) 9 (B) 10 (C) 11 (D) 12

3. Find the sum of the squares of the first 10 even natural numbers. ☐

4. $7^{1+\frac{1}{2}+\frac{1}{4}+.....\infty}$ evaluates to ☐

5. The common difference of an arithmetic progression having seven terms is 3. If the product of the first term and the last term is 595, what could be the sum of all the terms of the arithmetic progression?
(A) 108 (B) 216 (C) 432 (D) 182

6. An athlete runs a race and after every hour, his speed reduces to half the speed with which he travelled for the previous hour. Find the time taken to cover the race, if the person started the race with a speed of 16 km/hr and the length of the race was 31.5 km. (in hours) ☐

7. The first three terms of an arithmetic progression are 3x, 5x + 8 and 10x + 4. Find the sum of the first 10 terms. ☐

8. Let A(N) denote the sum of the first N natural numbers and B (N) denote the sum of the squares of the first N natural numbers. If B(N) is a multiple of A(N), then which of the following could be true?
(A) N is divisible by 6
(B) N + 1 is divisible by 6
(C) N + 2 is divisible by 6
(D) N – 3 is divisible by 6

9. Find the least number of terms of the series 2, 6, 18 …. such that their sum exceeds 500. ☐

10. If S_n is the sum of the first n terms of the series 40 + 38 + 36 + …., then find the maximum value of S_n.
(A) 450 (B) 420 (C) 390 (D) 410

11. If the sum of the first 37 terms of an arithmetic progression. is 703, then find the sum of the first 10 terms of the arithmetic progression, given the first term of the arithmetic progression is 1.
(A) 55 (B) 65 (C) 75 (D) 85

12. The sum of four numbers in an ascending arithmetic progression is 160 and the product of the extremes is 1564. Find the smallest of the numbers.
(A) 28 (B) 34 (C) 42 (D) 43

13. Find the value of $-1^2+2^2-3^2+4^2-5^2+6^2+ \ldots\ldots -19^2+20^2$ ☐

14. $\dfrac{1^2+2^2+.....+n^2}{(n+1)^2+(n+2)^2+....(2m)^2}=\dfrac{385}{2485}$ .Find the value of m.
(A) 10 (B) 20 (C) 25 (D) 30

15. $1^3+2^3+3^3+ \ldots + m^3 = 3025$. Find the value of m.
(A) 8 (B) 9 (C) 10 (D) 11

16. The sum of first n terms of an arithmetic progression. is given by $2n^2 + 6n$. Find the common difference of the A.P. ☐

17. The sum of the squares of three terms in arithmetic progression is 365. The product of the first and the third terms is 120. Find the sum of the squares of the second term and the common difference.
(A) 145 (B) 170 (C) 122 (D) 197

18. Find the number of terms common to the progressions 2, 8, 14, 20,…. 98 and 6, 10, 14, 18, …., 102.
(A) 7 (B) 6 (C) 8 (D) 9

19. The sums of the first n terms of two arithmetic progressions S_1 and S_2 are in the ratio 11n – 17 : 5n – 21.
Find the ratio of the 16^{th} terms of S_1 and S_2.
(A) 3:2 (B) 162 : 67
(C) 9:4 (D) 27:8

20. Find the sum of all the three-digit numbers which leave a remainder of 1 when divided by 8.
(A) 61472 (B) 61508 (C) 61488 (D) 61496

21. How many three-digit numbers less than 500 are divisible neither by 4 nor by 6? ☐

22. A person gets a starting salary of ₹5000 per month. During the first year of his job, he receives a monthly increment of ₹200 starting from the second month. During the second year, he receives a monthly increment of ₹400, a monthly increment of ₹600 for the third year and so on. Find the total amount received by him at the end of 4 years. (Assume that the salary he gets during the last month of a year is same as that during the first month of the next year).
(A) ₹5.864 lakhs (B) ₹5.914 lakhs
(C) ₹5.964 lakhs (D) ₹6.36 lakhs

23. If 7/12, −2m and 12/7 form a G.P., then what is the value of m?
(A) ±2 (B) ±1/2
(C) $\pm 1/\sqrt{2}$ (D) None of these

24. Find the sum of the terms of the sequence:
(1) × (20), (2) × (19), (3) × (18), …(20) × (1).
(A) 1750 (B) 1640
(C) 1540 (D) 1430

25. If $\log_3 x + \log_{\sqrt[3]{3}} x + \log_{\sqrt[5]{3}} x + \ldots\ldots + \log_{\sqrt[23]{3}} x = 432$, then find x.

26. The number of bacteria in a colony doubles every minute. If there are 1024 bacteria after 5 minutes, find the number of bacteria present initially.

27. If the sum of three numbers in a geometric progression is 38 and their product is 1728, then find the smallest number.
(A) 6 (B) 4 (C) 2 (D) 8

28. The common ratio of geometric progression is a positive number less than 1. The first term is 18. The difference of the third and second terms is 4. Find the common ratio.
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{3}$ or $\frac{2}{3}$

29. In a three digit number, the hundreds digit, the tens digit and the units digit are in ascending order and in geometric progression which has an integral common ratio. How many such numbers are there?

30. If a, c and b as well as $a^2 + b^2$, $a^2 + c^2$ and $b^2 + c^2$ are in geometric progression, then which of the following is necessarily true?
(A) a = b (B) b = c (C) a = c (D) a = b = c

31. In a geometric progression, each term is the sum of all the terms following it. The sum to infinity of the terms is 32. If all the terms are positive, then find the first term.
(A) 16 (B) $16\sqrt{2}$ (C) 64 (D) $8\sqrt{2}$

32. S_1 is a square. By joining the midpoints of sides of S_1, another square S_2 is formed. By joining the midpoints of sides of S_2, another square S_3 is formed and so on. The side of S_1 is 32 cm. Find the sum of the perimeters of all the squares (in cm).
(A) $48\sqrt{2}\left(\sqrt{2}+1\right)$ (B) $128\sqrt{2}\left(\sqrt{2}+1\right)$
(C) $72\sqrt{2}\left(\sqrt{2}+1\right)$ (D) $36\sqrt{2}\left(\sqrt{2}+1\right)$

33. If the second, third and first terms of a geometric progression form an arithmetic progression, find the first term of the geometric progression, given that the sum to infinity of the G.P. is 36.

34. In an arithmetic progression, the first term is a natural number and the common difference is a whole number. If the first term is decreased by 2 and the third term is increased by 10, the first three terms would be in geometric progression. The common difference can be
(A) 4 (B) 8 (C) 16 (D) 14

35. Between 250 and 750, how many integers are divisible by 11?

36. $S = 2 + 4x + 6x^2 + 8x^3$ …………. where $|x| < 1$. Find S.
(A) $\frac{4}{(1-x)^2}$ (B) $\frac{3}{(1-x)^2}$
(C) $\frac{2}{(1-x)^2}$ (D) $\frac{1}{(1-x)^2}$

37. If $S = \frac{1}{21}+\frac{1}{22}+\frac{1}{23}+\frac{1}{24}+\ldots\ldots+\frac{1}{40}$, then which of the following is/are equal to S?
(A) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots+\frac{1}{39}-\frac{1}{40}$
(B) $30.5\left[\frac{1}{21(40)}+\frac{1}{22(39)}+\frac{1}{23(38)}+\ldots+\frac{1}{39(22)}+\frac{1}{40(21)}\right]$
(C) $\frac{1}{31}+\frac{1}{32}+\frac{1}{33}+\ldots..+\frac{1}{60}$
(D) Both (A) and (B)

38. If $S = \frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\ldots.+\frac{1}{9999}$, then the value of S is
(A) $\frac{50}{101}$ (B) $\frac{49}{99}$ (C) $\frac{25}{51}$ (D) $\frac{50}{99}$

39. The sum of the first 2013 terms of a geometric progression is 300. The sum of the first 4026 terms of the progression is 540. The sum of the first 6039 terms of the progression is

40. Find the sum of the first nine terms of S, where S $= \frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\ldots..$.
(A) $\frac{80}{81}$ (B) $\frac{52}{51}$ (C) $\frac{119}{121}$ (D) $\frac{99}{100}$

41. How many terms of the G.P. 2, 6, 18, . . . are needed to give a sum of 6560?
(A) 4 (B) 6 (C) 7 (D) 8

42. How many terms of the A.P. 3, 9, 15, . . . must be taken so that their sum may be 2883?

43. Find the sum of the first 12 terms of S_n, where $S_n = 1 + 3 + 7 + 13 + 21 + 31 + 43 + \ldots..$.

44. Find the sum of the first 10 terms of the following series $3^2(1) + 4^2(2) + 5^2(3) + 6^2(4)+\ldots.$.
(A) 4675 (B) 4785 (C) 4875 (D) 4915

45. If $S_n = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots.,+\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}$ then find the sum of $S_1 + S_2 + S_3 + S_4 + S_5$.
(A) 12 (B) 17 (C) $18\frac{11}{20}$ (D) $21\frac{3}{10}$

46. $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+...+\frac{1}{\sqrt{119}+\sqrt{121}}=$ _____ .

(A) $2\sqrt{3}+1$ (B) 5

(C) $11-2\sqrt{3}$ (D) 10

47. What could be the value of the 19^{th} term of a G.P. whose 16^{th} and 11^{th} terms are $3^7\left(2\sqrt{18}\right)$ and $3^5\left(2\sqrt{6}\right)$ respectively?

(1) $3\sqrt[8]{2}$ (B) 9, $2\sqrt{6}$ .3

(C) $2\sqrt{2}\cdot 3^9$ (D) $2\sqrt{6}\cdot 3^9$

48. In a series T_i, each term from the second term is obtained by subtracting 2 from three times the preceding term. If $T_1 = 3$, then find the value of T_{200}.

(A) $3(2^{200}) - 2$
(B) $3(2^{199}) + 1$
(C) $2(3^{199}) + 1$
(D) $2(3^{200}) - 1$

49. Find the sum to *n* terms of the series 2 + 22 + 222 + ...

(A) $\frac{20(10^n-1)-n}{27}$

(B) $(2/9)\left(\frac{10(10^n-1)}{9}-n\right)$

(C) $(1/9)\left(\frac{20(10^n-1}{9}-n\right)$

(D) $(4/9)\left(\frac{10(10^n-1}{9}-n\right)$

50. A ball is dropped from a height of 1250 m. It rebounds to four fifths of the height from which it falls. If it continues to fall and rebound this way, how much distance does the ball cover totally before coming to rest? (in m)

51. If there are thirteen arithmetic means inserted between 10 and 80, then find the fourth arithmetic mean.

52. The number of terms in an A.P. is 20. The sum of the four middle terms is –22 and the sum of the first four terms is 74. What are the values of the first term and the common difference?

(A) 23, –3 (B) –23, 3
(C) 20, –6 (D) –20, 6

53. Find the sum of the first 20 terms of the series 1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4), (1 + 2 + 3 + 4 + 5), …

(A) 1540 (B) 1435
(C) 1450 (D) 1345

54. Find the sum of the first 100 terms of the series 1 + 2 + 3 – 4, 2 + 3 + 4 – 5, 3 + 4 + 5 – 6, 4 + 5 + 6 – 7,…

55. If $S = 1 + 4x + 9x^2 + 16x^3 + 25x^4 + 36x^5 + 49x^6 +$ …… where $|x| < 1$, then S is equal to ____.

(A) $\frac{1}{(1-x)^2}+\frac{2x}{(1-x)^3}$ (B) $\frac{1}{(1-x)^2}+\frac{x}{(1-x)^3}$

(C) $\frac{1+x}{(1-x)^2}+\frac{3x}{(1-x)^3}$ (D) $\frac{1+2x}{(1-x)^2}+\frac{x}{(1-x)^3}$

Directions for questions 56 to 65: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using I and II together.

56. What is the sum of the first 25 terms of an arithmetic progression?
I. The first term of the progression is 3.
II. The sum of the first 10 terms of the progression is equal to the sum of the first 15 terms of the progression.

57. Three numbers are in geometric progression and the least of them is 1. Find the middle number.
I. The sum of the numbers is 21.
II. The product of the numbers is 64.

58. Are x, y and z in geometric progression?
I. x + y = 20 and y + z = 60
II. y/x = – 2

59. What is the fourth term in the series?
I. First term of the series is 1.
II. Starting from the second term, each term in the series is obtained by squaring the number obtained by adding one to the previous term of the series.

60. What is the arithmetic mean of x and y?
I. The geometric mean of x and y is 4.
II. The arithmetic mean of x, y, 4 and 8 is 5.

61. G is a geometric progression whose sum of the terms is $\frac{3^8-1}{2}$ and whose product of the terms is 3^{28}. Find its common ratio.
I. If each term in G is multiplied by its common ratio, the resulting progression would have the sum of its terms as $\frac{3(3^8-1)}{2}$.
II. If each term in G is multiplied by its common ratio, the resulting progression would have the product of its terms as 3^{36}. Also G has 8 terms.

62. A geometric progression which has all positive terms has a sum to infinity of 8. Find its first term.

I. The sum of the squares of all terms of the progression is $\frac{64}{3}$.

II. Each term equals the sum of all the terms following it.

63. An arithmetic progression has the sum of its terms as 100. It has twelve terms. Find its common difference.

I. The ratio of the sum of its odd numbered terms and even numbered terms is 11 : 14.

II. The sum of its even numbered terms is 12 more than that of the odd numbered terms.

64. What is the middle term of an arithmetic progression of 11 numbers?

I. The average of 11 numbers is 63.

II. The average of the first six numbers is 60 and that of the last six numbers is 66.

65. If a, b and c are in geometric progression, is the common ratio negative?

I. b is less than a.

II. abc is greater than ac.

Key

Concept Review Questions

1. A
2. 27
3. B
4. B
5. 0
6. C
7. A
8. A
9. 1426
10. C
11. B
12. (a) B
    (b) C
13. B
14. C
15. (a) C
    (b) A
16. 10
17. A
18. B
19. 7
20. C
21. A
22. (a) A
    (b) C
23. 6
24. 3
25. B
26. 4
27. C
28. (a) 17
    (b) B
29. (a) A
    (b) B
30. A

Exercise – 4(a)

1. 43
2. C
3. D
4. 19
5. B
6. C
7. B
8. A
9. 676
10. A
11. 1
12. 25
13. D
14. D
15. B
16. D
17. B
18. 3726000
19. A
20. B
21. 2
22. A
23. B
24. 6
25. 10
26. A
27. 512
28. A
29. B
30. 2
31. B
32. B
33. A
34. D
35. C
36. 4860
37. D
38. D
39. A
40. 22100
41. B
42. B
43. A
44. B
45. 10099

Exercise – 4(b)

1. 62
2. C
3. 1540
4. 49
5. D
6. 6
7. 840
8. C
9. 6
10. B
11. A
12. B
13. 210
14. A
15. C
16. 4
17. C
18. C
19. B
20. C
21. 266
22. D
23. B
24. C
25. 27
26. 32
27. D
28. D
29. 3
30. A
31. A
32. B
33. 54
34. D
35. 46
36. C
37. D
38. A
39. 732
40. D
41. D
42. 31
43. 584
44. B
45. C
46. B
47. D
48. C
49. B
50. 11250
51. 30
52. A
53. A
54. 10100
55. A
56. A
57. B
58. C
59. C
60. A
61. B
62. B
63. B
64. B
65. C