CHAPTER – 7

INDICES AND SURDS

INDICES

If a number 'a' is added three times, then we write it as 3a.
Instead of adding, if we multiply 'a' three times, we write it as a^3. Here, 'a' is called the 'base' and 3 is called the 'power' or 'index' or 'exponent'.

Similarly, 'a' can be raised to any exponent 'n' and accordingly written as a^n. This is read as "a to the power n" or "a raised to the power n."

$a^n = a \times a \times a \times a \times$ .... n times

For example,
$2^3 = 2 \times 2 \times 2 = 8$ and $3^4 = 3 \times 3 \times 3 \times 3 = 81$

While the examples taken are for positive integer values of n, the powers can also be negative integers or positive or negative fractions or irrational numbers. In the sections that follow, we will also see how to deal with numbers where the powers are fractions or negative integers.

If a number raised to a certain power is inside brackets and quantity is then raised to a power again, {i.e., a number of the type $(a^m)^n$ - read as "a raised to the power m whole raised to the power n" or "a raised to power m whole to the power n"}, then the number inside the brackets is evaluated first and then this number is raised to the power which is outside the brackets.

For example, to evaluate $(2^3)^2$, we first find out the value of the number inside the bracket (2^3) as 8 and now raise this to the power 2. This gives 8^2 which is equal to 64. Thus $(2^3)^2$ is equal to 64.

If we have powers in the manner of "steps", then such a number is evaluated by starting at the topmost of the "steps" and coming down one "step" in each operation.

For example, 2^{4^3} is evaluated by starting at the topmost level '3'. Thus we first calculate 4^3 as equal to 64. Since 2 is raised to the power 4^3, we now have 2^{64}.

Similarly, 2^{3^2} is equal to "2 raised to the power 3^2" or "2 raised to the power 9" or 2^9 which is equal to 512.

There are certain basic rules/laws for dealing with numbers having powers. These are called Laws of Indices. The important ones are listed down and the students have to know these rules and be able to apply any of them in solving problems. Most of the problems in indices will require one or more of these rules. These rules should be internalised by the students to the extent that after some practice, application of these rules should come naturally and the student should not feel that he is applying some specific formula.

<u>Table of Rules/Laws of Indices</u>

	Rule/Law	Example
1)	$a^m \times a^n = a^{m+n}$	$5^2 \times 5^7 = 5^9$
2)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^5}{7^3} = 7^2 = 49$
3)	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
4)	$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
5)	$\sqrt[m]{a} = a^{1/m}$	$\sqrt[3]{64} = 64^{1/3} = 4$
6)	$(ab)^m = a^m \cdot b^m$	$(2 \times 3)^4 = 2^4 \cdot 3^4$
7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
8)	$a^0 = 1$ (where $a \neq 0$)	$3^0 = 1$
9)	$a^1 = a$	$4^1 = 4$

These rules/laws will help you in solving a number of problems. In addition to the above, the student should also remember the following rules:

<u>Rule 1</u>: When the bases of two equal numbers are equal, then their powers also will be equal. (If the bases are neither zero nor ± 1.)
For example : If $2^n = 2^3$, then it means n = 3

<u>Rule 2</u>: When the powers of two equal numbers are equal (integer and not equal to zero), two cases arise:
i) if the power is an odd number, then the bases are equal. For example, if $a^3 = 4^3$ then a = 4.
ii) if the powers are even numbers, then the bases are numerically equal but can have different signs. For example, if $a^4 = 3^4$ then a = + 3 or – 3.

The problems associated with indices are normally of THREE types:

<u>Simplification</u>
Here, the problem involves terms with same or different bases and powers which have to be simplified using the rules discussed in the table above.

<u>Solving for the value of an unknown</u>
Here, the problem will have an equation where an unknown (like x or y) will appear in the base or in the power and using Rule 1 and Rule 2 discussed above, values of unknown are to be determined.

<u>Comparison of numbers</u>
Here two or more quantities will be given – each being a number raised to a certain power. These numbers have to be compared in magnitude – either to find the largest or smallest of the quantities or to arrange the given quantities in ascending or descending order.

The following examples will make clear the different types of problems that you may be asked.

Examples

7.01. Simplify :

$$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

Sol: $$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

$$= \left(\frac{9^2}{12^2}\right)^{-1} \times \left(\frac{32^2}{3^2}\right)^{1/2} \times \left(\frac{324}{24}\right)$$

$$= \frac{144}{81} \times \frac{32}{3} \times \frac{324}{24} = 256$$

7.02. Simplify : $\left(\frac{a^4 b^3}{c^2}\right)^2 \times \left(\frac{b^4 c^3}{a^2}\right)^3 \times \left(\frac{c^4 a^3}{b^2}\right)^4$

Sol: $$\left(\frac{a^4 b^3}{c^2}\right)^2 \times \left(\frac{b^4 c^3}{a^2}\right)^3 \times \left(\frac{c^4 a^3}{b^2}\right)^4$$

$$= \frac{a^8 b^6}{c^4} \times \frac{b^{12} c^9}{a^6} \times \frac{c^{16} a^{12}}{b^8}$$

$= a^{8-6+12}\, b^{6+12-8}\, c^{-4+9+16} = a^{14}\, b^{10}\, c^{21}$

7.03. In the equation given below, solve for x

$$\sqrt[3]{\left(\frac{5}{7}\right)^{x+1}} = \frac{125}{343}$$

Sol: Given

$$\left(\frac{5}{7}\right)^{x+1} = \left[\left(\frac{5}{7}\right)^3\right]^3 = \left(\frac{5}{7}\right)^9.$$

By equating their indices,
$x + 1 = 9$
$x = 8.$

7.04. If $625^{x-2} = 25^{x+2}$, find x.

Sol: Given, $(5^4)^{x-2} = (5^2)^{(x+2)}$ by equating their Indices,
$4(x-2) = 2(x+2)$
$x = 6.$

7.05. If $\left(\frac{49}{2401}\right)^{4-x} = 49^{2x-6}$, find x.

Sol: $$\left(\frac{49}{2401}\right)^{4-x} = \left(49^{-1}\right)^{4-x} = 49^{x-4}$$

Given, $49^{x-4} = 49^{2x-6}$
$x - 4 = 2x - 6$
$x = 2$

7.06. Arrange the following in ascending order
625^6, 125^7 and 25^{10}

Sol: $625^6 = (5^4)^6 = 5^{24}$
$125^7 = (5^3)^7 = 5^{21}$
$25^{10} = (5^2)^{10} = 5^{20}$
$25^{10} < 125^7 < 625^6$

SURDS

Any number of the form p/q, where p and q are integers and $q \neq 0$ is called a rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Amongst surds, we will specifically be looking at 'quadratic surds' – surds of the type $a + \sqrt{b}$ and $a + \sqrt{b} + \sqrt{c}$ etc. where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds - what is required is a basic understanding of some of the operations on surds.

If there is a surd of the form $(a + \sqrt{b})$, then a surd of the form $\pm(a - \sqrt{b})$ is called the conjugate of the surd $(a + \sqrt{b})$. The product of a surd and its conjugate will always be a rational number.

RATIONALISATION OF A SURD

When there is a surd of the form $\frac{1}{a + \sqrt{b}}$, it is difficult to perform arithmetic operations on it. Hence, the denominator is converted into a rational number thereby facilitating ease of handling the surd. This process of converting the denominator into a rational number without changing the value of the surd is called rationalisation.

To convert the denominator of a surd into a rational number, multiply the denominator and the numerator with the conjugate of the surd in the denominator so that the denominator gets converted to a rational number without changing the value of the surd. That is, if there is a surd of the type $a + \sqrt{b}$ in the denominator, both the numerator and the denominator have to be multiplied with a surd of the form $a - \sqrt{b}$ or a surd of the form $(-a + \sqrt{b})$ to convert the denominator into a rational number.

If there is a surd of the form $(a + \sqrt{b} + \sqrt{c})$ in the denominator, then the process of multiplying the denominator with its conjugate surd has to be carried out TWICE to rationalise the denominator.

SQUARE ROOT OF A SURD

If there exists a square root of a surd of the type $a + \sqrt{b}$, then it will be of the form $\sqrt{x} + \sqrt{y}$. We can equate the square of $\sqrt{x} + \sqrt{y}$ to $a + \sqrt{b}$ and thus solve for x and y. Here, one point should be noted – when there is an equation with rational and irrational terms, the rational part on the left hand side is equal to the rational part on the right hand side and, the irrational part on the left hand side is equal to the irrational part on the right hand side of the equation.
However, for the problems which are expected in the entrance exams, there is no need of solving for the square root in such an elaborate manner. We will look at

finding the square root of the surd in a much simpler manner. Here, first the given surd is written in the form of $(\sqrt{x}+\sqrt{y})^2$ or $(\sqrt{x}-\sqrt{y})^2$. Then the square root of the surd will be $(\sqrt{x}+\sqrt{y})$ or $(\sqrt{x}-\sqrt{y})$.

COMPARISON OF SURDS

Sometimes we will need to compare two or more surds either to identify the largest/smallest one or to arrange the given surds in ascending/descending order. The surds given in such cases will be such that they will be close to each other and hence we will not be able to identify the largest one by taking the approximate square root of each of the terms. In such a case, the surds can both be squared and the common rational part be subtracted. At this stage, normally one will be able to make out the order of the surds. If even at this stage, it is not possible to identify the larger of the two, then the numbers should be squared once more.

Examples

7.07. Simplify : $\frac{1}{4-\sqrt{5}}-\frac{1}{4+\sqrt{5}}$

Sol: $\frac{1}{4-\sqrt{5}}-\frac{1}{4+\sqrt{5}} = \frac{4+\sqrt{5}-(4-\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} = \frac{2\sqrt{5}}{11}$

7.08. Rationalize the denominator: $\frac{1}{1+\sqrt{6}-\sqrt{7}}$

Sol: A rationalizing factor of $1+\sqrt{6}-\sqrt{7}$ is $1+\sqrt{6}+\sqrt{7}$

$$\frac{1}{1+\sqrt{6}-\sqrt{7}} = \frac{(1+\sqrt{6}+\sqrt{7})}{(1+\sqrt{6}-\sqrt{7})(1+\sqrt{6}+\sqrt{7})}$$

$$= \frac{1+\sqrt{6}+\sqrt{7}}{(1+\sqrt{6})^2-(\sqrt{7})^2} = \frac{1+\sqrt{6}+\sqrt{7}}{2\sqrt{6}}$$

The rationalizing factor of $\sqrt{6}$ is $\sqrt{6}$

$$= \frac{\sqrt{6}+6+\sqrt{42}}{12}$$

7.09. Find the value of $\sqrt{62+\sqrt{480}}$.

Sol: Let $\sqrt{62+\sqrt{480}} = \sqrt{a}+\sqrt{b}$

Squaring both sides,

$62+\sqrt{480} = a+b+2\sqrt{ab}$

$62+\sqrt{480} = a+b+\sqrt{4ab}$

Equating the corresponding rational and irrational parts on both sides, a + b = 62 and 4ab = 480 $\Rightarrow$ ab = 120

As a + b = 60 + 2 and ab = (60) (2) it follows that a = 60 and b = 2 or vice versa.

$\therefore\ \sqrt{a}+\sqrt{b} = \sqrt{60}+\sqrt{2}$

7.10. Which of the surds given below is greater?

$\sqrt{3}+\sqrt{23}$ and $\sqrt{6}+\sqrt{19}$

Sol: $(\sqrt{3}+\sqrt{23})^2 = 26+2\sqrt{69}$

$\sqrt{69}$ lies between $\sqrt{64}$ and $\sqrt{81}$

$\therefore\ 26+2\sqrt{69}$ lies between 26 + 2(8) and 26 + 2(9) i.e. 42 and 44.

Similarly $(\sqrt{6}+\sqrt{19})^2$ lies between 45 and 47.

$\therefore\ (\sqrt{3}+\sqrt{23})^2 < (\sqrt{6}+\sqrt{19})^2$

$\therefore\ \sqrt{6}+\sqrt{19} > \sqrt{3}+\sqrt{23}$

Concept Review Questions

***Directions for questions 1 to 25*:** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify the following:

(a) $\left(\frac{243}{1024}\right)^{-2/5} \times \left(\frac{144}{49}\right)^{-1/2} \div \left(\frac{8}{343}\right)^{-2/3}$

(A) $2^5 \times 3^{-1} \times 7$ (B) $2^5 \times 3^{-3} \times 7^{-2}$
(C) $2^4 \times 3^{-3} \times 7^{-1}$ (D) $2^4 \times 3^{-1} \times 7^{-2}$

(b) $\left(\frac{49^8}{6561^2}\right)^{-3/16}$

(A) $-\frac{27}{343}$ (B) $\frac{27}{343}$ (C) $-\frac{343}{27}$ (D) $\frac{343}{27}$

(c) $(3-5)^2 \div (4-7)^{-4} \times [(-3)^{-2}]^{-1}$ $\boxed{}$

(d) $\left(\frac{x^2 \cdot y^{-3}}{z^4}\right)^{-2} \times \left(\frac{x^2 \cdot y}{z^{-2}}\right)^3 \div \left(\frac{x^{-12} \cdot y^7}{z^{-8}}\right)^{-1}$

(A) $x^{10}.\ y^{16}\ .\ z^{-22}$ (B) $x^7.\ y^{-16}\ .\ z^{-22}$
(C) $x^{-7}.\ y^{16}\ .\ z^{-22}$ (D) $x^{-10}.\ y^{16}\ .\ z^{22}$

(e) $\left(\frac{3125 \cdot x^{-15}}{100000\, x\, y^{-20}}\right)^{-2/5}$

(A) $4.\ x^{-6}\ .\ y^{-8}$ (B) $4.\ x^{-6}\ .\ y^{8}$
(C) $4.\ x^{6}\ .\ y^{-8}$ (D) $4.\ x^{6}\ .\ y^{8}$

(f) $\frac{5^{2a-5} \times 25^{a/2} \times 125^{a+3}}{(3125)^{3a/5} \times 625^{a+1} \times 5^{-a}}$ $\boxed{}$

(g) $729^{1/3} - 243^{3/5} + 81^{3/2} \times (27)^{-1}$
(A) 9 (B) 3 (C) 27 (D) 81

(h) $\frac{11^{-5} \times 121^3}{1331^{-4} \times (14641)^{11/4}}$
(A) 1 (B) 11 (C) 121 (D) 11^{-1}

(i) $\frac{1-[1-\{1-(1+y)^{-1}\}]}{(1-y)}$

(A) $\frac{y}{(1-y^2)}$ (B) $\frac{y}{(1-y)^2}$ (C) $\frac{1+y}{(1-y)^2}$ (D) $\frac{1+y^2}{1-y^2}$

(j) $\left(\frac{11^{18}}{7^{-27}}\right)^{1/9}$ $\boxed{}$

2. (a) $\left(x^{a-b}\right)^{(a^2+ab+b^2)} \times \left(x^{b-c}\right)^{(b^2+bc+c^2)} \times \left(x^{c-a}\right)^{(c^2+ac+a^2)}$

(A) 0 (B) 1
(C) $x^{a^3+b^3+c^3}$ (D) $x^{3(a^2+b^2+c^2+ac+bc+ca)}$

(b) $\frac{\left(x^a\right)^{2c} \cdot \left(x^b\right)^{2a} \left(x^c\right)^{2b}}{\left(x^{a+b}\right)^c \cdot \left(x^{b+c}\right)^a \cdot \left(x^{c+a}\right)^b}$

(A) 0 (B) $x^{2a+2b+2c}$
(C) 1 (D) $x^{4ab+4ac+4bc}$

3. $\frac{50 \times 2^{x-4} + 25 \times 2^{x-5}}{10^{x+3}} =$

(A) $\frac{1}{(5^x)(2^{10})}$ (B) $\frac{1}{(5^x)(2^9)}$

(C) $\frac{1}{(5^x)(2^{11})}$ (D) $\frac{1}{(5^x)(2^8)}$

4. $343^{0.12} \times 2401^{0.08} \times 49^{0.01} \times 7^{0.1} =$
(A) 7 (B) $7^{4/5}$ (C) 7^8 (D) $7^{3/5}$

5. Simplify: $y^{\frac{p}{p+q+r}} \cdot y^{\frac{q}{p+q+r}} \cdot y^{\frac{r}{p+q+r}}$
(A) y (B) 1 (C) 1/y (D) y^2

6. **(a)** If $5^{2x} = 625$, find x.

$\boxed{}$

(b) What is the value of x if $3^{x^x} = 81$?
(A) 2 (B) 3 (C) 4 (D) $\sqrt{2}$

7. Which of the following is a rationalizing factor of $10^{\frac{1}{3}} - 9^{\frac{1}{3}}$?

(A) $10^{\frac{2}{3}} + 9^{\frac{2}{3}}$ (B) $10^{\frac{2}{3}} - 9^{\frac{2}{3}}$

(C) $10^{\frac{2}{3}} + 90^{\frac{1}{3}} + 9^{\frac{2}{3}}$ (D) $10^{\frac{2}{3}} - 9^{\frac{1}{3}} + 9^{\frac{2}{3}}$

8. If $3^x\, 7^y = 441$ where x and y are integers, what is the value of x – y?
(A) 0 (B) 1
(C) 2 (D) Cannot be determined

9. **(a)** If $5^{\frac{1}{2}}\, 5^{\frac{3}{2}}\, 5^{\frac{5}{2}}\, 5^{\frac{7}{2}}\, 5^{\frac{9}{2}} = 25^x$, find x.

$\boxed{}$

(b) Solve for x: $2^{3x+4} = 2^{4x+2}$

$\boxed{}$

(c) Solve for x: $729^{x+1} = 3^{4x-3}$

(A) $-\frac{9}{2}$ (B) $-\frac{11}{4}$

(C) –9 (D) –8

(d) Solve for x: $9^{2x+1} = 27^{5x-3}$
(A) 1 (B) 2
(C) –1 (D) –2

10. If $\frac{p}{q}=\frac{r}{s}$ and $p^a = q^b = r^c = s^d$, then $\frac{1}{a}-\frac{1}{b}=$

(A) $\frac{1}{d}-\frac{1}{c}$ (B) $\frac{1}{c}+\frac{1}{d}$

(C) $\frac{1}{c}-\frac{1}{d}$ (D) $-\left(\frac{1}{c}+\frac{1}{d}\right)$

11. Which of the following gives the relation between the pairs / triplets given below?

(a) 7^{125} ; 2^{375}
(A) $7^{125} < 2^{375}$
(B) $7^{125} > 2^{375}$
(C) $7^{125} = 2^{375}$
(D) Cannot be determined

(b) 2^{51} ; $4^{13} \times 32^4$
(A) $2^{51} < 4^{13} \times 32^4$ (B) $2^{51} > 4^{13} \times 32^4$
(C) $2^{51} = 4^{13} \times 32^4$ (D) Cannot be determined

(c) $(343)^5$, $(49)^7$, 7^{16}
(A) $(343)^5 > (49)^7 > 7^{16}$ (B) $(343)^5 < (49)^7 < 7^{16}$
(C) $7^{16} < (49)^7 > (343)^5$ (D) $(49)^7 < (343)^5 < 7^{16}$

(d) $(27)^{10}$, 5^{20}, 2^{40}
(A) $5^{20} > 2^{40} > (27)^{10}$ (B) $2^{40} < (27)^{10} < 5^{20}$
(C) $(27)^{10} < 5^{20} < 2^{40}$ (D) $(27)^{10} > 5^{20} > 2^{40}$

(e) 7^{75} ; $5^{75} \times 3^{25}$; $(200)^{25}$
(A) $7^{75} > (200)^{25} > 5^{75} \times 3^{25}$
(B) $5^{75} \times 3^{25} > 7^{75} > (200)^{25}$
(C) $(200)^{25} > 7^{75} > 5^{75} \times 3^{25}$
(D) $7^{75} > 5^{75} \times 3^{25} > (200)^{25}$

12. Which of the following is the largest in value?
(A) $6^{1/2}$ (B) $7^{1/3}$ (C) $8^{1/4}$ (D) $9^{1/5}$

13. $\dfrac{9}{6^{\frac{2}{3}} - 18^{\frac{1}{3}} + 3^{\frac{2}{3}}} =$

(A) $\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$ (B) (1/3) $\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$

(C) $\left(6^{\frac{1}{3}}-3^{\frac{1}{3}}\right)$ (D) (1/9) $\left(6^{\frac{1}{3}}+3^{\frac{1}{3}}\right)$

14. Which of the following is the conjugate of the surd $\sqrt{7} - 2$?

(A) $\sqrt{7} + 2$ (B) $-2 - \sqrt{7}$
(C) Either (A) or (B) (D) None of these

15. (a) Rationalise the denominator: $\dfrac{50}{\sqrt{15}-\sqrt{10}}$.

(A) $5\sqrt{15}-5\sqrt{10}$ (B) $10\sqrt{15}-10\sqrt{10}$
(C) $5\sqrt{15}+5\sqrt{10}$ (D) $10\sqrt{15}+10\sqrt{10}$

(b) Rationalise the denominator: $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

(A) $8-\sqrt{15}$ (B) $4+\sqrt{15}$
(C) $4-\sqrt{15}$ (D) $8+\sqrt{15}$

16. Simplify: $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{1}{\sqrt{6}+\sqrt{5}}$

(A) $\sqrt{6}-\sqrt{2}$ (B) $\sqrt{6}+\sqrt{2}$
(C) $\sqrt{5}+\sqrt{6}$ (D) $\sqrt{2}-\sqrt{6}$

17. $\dfrac{6^{\frac{3}{4}} \times \sqrt[4]{6^{10}}}{\sqrt[4]{6^9}} =$

(A) 6 (B) 36 (C) $\sqrt[4]{6}$ (D) 1

18. $2\sqrt{\frac{5}{2}} - 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000} =$

(A) $9\sqrt{10}$ (B) $8\sqrt{10}$ (C) $8\sqrt{10}$ (D) $11\sqrt{10}$

19. $\left(\dfrac{\sqrt{p}-\sqrt[4]{pq}}{\sqrt[4]{pq}-\sqrt{q}}\right)^{-4}$ **=**

(A) $\frac{p}{q}$ (B) $\sqrt{\frac{p}{q}}$ (C) $\sqrt{\frac{q}{p}}$ (D) $\frac{q}{p}$

20. $\left(\sqrt{324+2\sqrt{323}}\right) - \left(\sqrt{324-2\sqrt{323}}\right) =$

(A) 2 (B) 1 (C) 36 (D) $2\sqrt{323}$

21. If y = $12+2\sqrt{35}$, then $\sqrt{y}-\dfrac{1}{\sqrt{y}}=$

(A) $\dfrac{\sqrt{7}+\sqrt{5}}{2}$ (B) $\dfrac{3\sqrt{5}-\sqrt{7}}{2}$

(C) $\dfrac{2\sqrt{5}+\sqrt{7}}{2}$ (D) $\dfrac{3\sqrt{5}+\sqrt{7}}{2}$

22. If y = $\dfrac{9-\sqrt{77}}{2}$, then find $y^2+\dfrac{1}{y^2}$.

23. If a = $\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ and b = $\dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$, then find the value of $a^2 - ab + b^2$.

24. Find the positive square root of the following surds

(a) $14-6\sqrt{5}$
(A) $\sqrt{5}+\sqrt{3}$ (B) $3-\sqrt{5}$
(C) $3+\sqrt{5}$ (D) $\sqrt{5}-\sqrt{3}$

(b) $18+\sqrt{308}$
(A) $7+\sqrt{11}$ (B) $\sqrt{11}-\sqrt{7}$
(C) $\sqrt{7}+\sqrt{11}$ (D) $7-\sqrt{11}$

25. (a) Arrange the following in ascending order.
a = $\sqrt{2}$ + $\sqrt{11}$, b = $\sqrt{6}$ + $\sqrt{7}$, c = $\sqrt{3}$ + $\sqrt{10}$ and d = $\sqrt{5}$ + $\sqrt{8}$

(A) abcd (B) abdc (C) acdb (D) acbd

(b) Arrange the following in descending order.
$a = \sqrt{20} + \sqrt{2}$, $b = \sqrt{24} + \sqrt{6}$, $c = \sqrt{22} + 2$ and $d = \sqrt{26} + \sqrt{8}$
(A) dcba (B) dcab
(C) dbac (D) dbca

(c) Arrange the following in descending order.
$a = \sqrt{13} + \sqrt{11}$, $b = \sqrt{15} + \sqrt{9}$, $c = \sqrt{18} + \sqrt{6}$, $d = \sqrt{7} + \sqrt{17}$.
(A) abdc (B) dcab (C) adcb (D) acdb

(d) Arrange the following in ascending order.
$p = \sqrt{26} - \sqrt{23}$, $q = \sqrt{18} - \sqrt{15}$, $r = \sqrt{11} - \sqrt{8}$, $s = \sqrt{24} - \sqrt{21}$
(A) rqsp (B) psrq (C) pqrs (D) psqr

Exercise – 7(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify: $[(81)^{4a}]^a \cdot [27^{8b}]^a \cdot [(243)(9^2)]b^2$
(A) $(729)^{a+b}$ (B) $9^{3a+b} - 3^{a-b}$
(C) $(81)^{(a+b)^2}$ (D) $3^{(4a+3b)^2}$

2. If $5^x = (0.125)^y = 10^{3z}$, then what is the relation between x, y and z, given that they are non-zero real numbers?
(A) 1/x + 1/y + 1/z = 1 (B) 3/x = 1/y + 1/z
(C) 1/z – 1/y – 1/x = 1 (D) xyz = 1

3. If a is positive and not equal to 1 and $a^{-x^3} = p$, $a^{-y^3} = q$, $a^{-z^3} = r$ and x + y + z = 0, for what value of a will xyz be equal to –1/3?
(A) p + q + r (B) pqr
(C) $\sqrt[3]{pqr}$ (D) 1/pqr

4. Solve for x and y:
$3.5^x + 2^{y+2} = 107$, $5^{x+1} + 8.2^y = 189$
(A) 3, 2 (B) 5, 7 (C) 7, 5 (D) 2, 3

5. Solve for x, if $\left(5\sqrt{7}\right)^{5x-4} = (35)^3(25)^{3/2}$

$\boxed{}$

6. If $(3^{x+1}) + (4^{y-1}) = 73$ and $4(3^x) + 3(4^{y-2}) = 60$, find x + y.
(A) 5 (B) 4 (C) –1 (D) –2

7. If $5^{x+3} - 5^{x-3} = 78120$, find x.

$\boxed{}$

8. If $\dfrac{(2^a)^a \cdot (2^b)^b \cdot (2^c)^c \cdot}{(4^a)^{-b} \cdot (4^b)^{-c} \cdot (4^c)^{-a} \cdot} = 8$, which of the following is a possible value of a + b + c?
(A) $-\sqrt{3}$ (B) 1 (C) 2 (D) $-\sqrt{2}$

9. Evaluate: $\dfrac{1}{1 + m^{xy-yz} + m^{xy-zx}} + \dfrac{1}{1 + m^{yz-zx} + m^{yz-xy}} + \dfrac{1}{1 + m^{zx-xy} + m^{zx-yz}}$.
(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{2}$ (C) 1 (D) 2

10. If $x^y = y^z = z^x = k$, which of the following is equal to k?
(A) $k = (xyz)^{xyz}$
(B) $k = (xyz)^{xy + yz + zx}$
(C) $k = \dfrac{x^y + y^z + z^x}{xyz}$
(D) $k = (xyz)^{(xyz)/(xy + yz + zx)}$

11. If $a^a \cdot b^b \cdot c^c = a^b \cdot b^c \cdot c^a = a^c \cdot b^a \cdot c^b$ and a, b, c are positive integers greater than 1, then which of the following can NOT be true for any of the possible values of a, b, c?
(A) abc = 8 (B) a + b + c = 8
(C) abc = 27 (D) a + b + c = 27

12. If x and y are natural numbers such that $(y^x + y^{x+1} + y^{x+2})$ is always divisible by 14, find the minimum possible value of y.

$\boxed{}$

13. If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + 1/x^3$?

$\boxed{}$

14. If $x = \cfrac{2}{2 + \cfrac{2}{2 + \cfrac{2}{2 + \cfrac{2}{\dots\infty}}}}$, find x.
(A) $2 - \sqrt{2}$ (B) $\dfrac{1}{2} + \sqrt{3}$
(C) $-1 + \sqrt{3}$ (D) $-1 + \sqrt{2}$

15. Arrange a, b, c in ascending order, where $a = 9^{11} + 3^{12}$; $b = 81^7 + 7^{12}$ and $c = 27^8 + 5^{12}$
(A) a, c, b (B) a, b, c
(C) b, c, a (D) c, b, a

16. Arrange the following numbers in ascending order.
$3^{1/5}, 4^{1/6}, 5^{1/7}, 6^{1/8}$

(A) $3^{1/5}, 6^{1/8}, 5^{1/7}, 4^{1/6}$ (B) $3^{1/5}, 5^{1/7}, 6^{1/8}, 4^{1/6}$
(C) $3^{1/5}, 6^{1/8}, 4^{1/6}, 5^{1/7}$ (D) $4^{1/6}, 3^{1/5}, 5^{1/7}, 6^{1/8}$

17. The ascending order of $16^{\frac{7}{12}}, 81^{\frac{3}{8}}, 625^{\frac{2}{3}}$ is _____.

(A) $16^{\frac{7}{12}}, 81^{\frac{3}{8}}, 625^{\frac{2}{3}}$ (B) $16^{\frac{7}{12}}, 625^{\frac{2}{3}}, 81^{\frac{3}{8}}$,

(C) $625^{\frac{2}{3}}, 16^{\frac{7}{12}}, 81^{\frac{3}{8}}$ (D) $81^{\frac{3}{8}}, 16^{\frac{7}{12}}, 625^{\frac{2}{3}}$

18. If $p^2 + q^2 + r^2 = 1$ what is the value of $\left(\sqrt[-pq]{a^r}\right)\times\left(\sqrt[-qr]{a^p}\right)\times\left(\sqrt[-pr]{a^q}\right)$?

(A) $\frac{1}{(a)^{1/pqr}}$ (B) $(a)^{1/pqr}$

(C) $1/(a)^{pqr}$ (D) $(a)^{pqr}$

19. Simplify : $\left(\frac{a}{\sqrt{b}-\sqrt{c}}+\frac{a}{\sqrt{b}+\sqrt{c}}\right)^2$

(A) $\frac{2a^2c}{(b-c)^2}$ (B) $\frac{(b-c)^2}{4a^2b}$

(C) $\frac{4a^2b}{(b-c)^2}$ (D) $\frac{2a^2}{b^2+c^2-a^2}$

20. If $2\sqrt{2}+\sqrt{3}=x$, what is the value of $\frac{11+4\sqrt{6}}{2\sqrt{2}-\sqrt{3}}$ in terms of x?

(A) $\frac{x^2}{\sqrt{2}}$ (B) x^3

(C) $\frac{x^3}{8}$ (D) $\frac{x^3}{5}$

21. If x = $\sqrt[3]{55+12\sqrt{21}}$ then the value of $x+\frac{1}{x}$ is

$\boxed{}$

22. Which of the following is the greatest?

(A) $16\sqrt[5]{81}$ (B) $5\sqrt[4]{27}$

(C) $12\sqrt[6]{243}$ (D) $8\sqrt[3]{9}$

23. If $a = \sqrt{5} + 2$, then $a^2 - 1/a^2$ is equal to ________.

(A) 18 (B) 0

(C) $8\sqrt{5}$ (D) $-8\sqrt{5}$

24. If $x = 5-\sqrt{21}$, find the value of $\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$.

(A) $\frac{\sqrt{7}-\sqrt{3}}{2}$ (B) $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{3}}$

(C) $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$ (D) $\frac{\sqrt{7}-\sqrt{3}}{3}$

25. Arrange a, b, c and d in descending order if $a=\sqrt{13}+\sqrt{9}$, $b=\sqrt{19}+\sqrt{3}$, $c=\sqrt{17}+\sqrt{5}$, $d=\sqrt{12}+\sqrt{10}$.

(A) b, c, a, d (B) d, b, a, c
(C) d, b, c, a (D) d, a, c, b

26. Find the value of $\sqrt{16+2\sqrt{55}}$.

(A) $1+\sqrt{15}$ (B) $\sqrt{10}+\sqrt{5}+5$

(C) $1+2\sqrt{5}+\sqrt{15}$ (D) $\sqrt{11}+\sqrt{5}$

27. Simplify: $\sqrt{(a+b+c)+2\sqrt{ac+bc}}$.

(A) $\sqrt{a}+\sqrt{b}+\sqrt{c}$ (B) $\sqrt{a+b}+\sqrt{c}$

(C) $\sqrt{ab+bc}$ (D) $\sqrt{abc}$

28. Find the square root of $23+4\sqrt{10}-10\sqrt{2}-8\sqrt{5}$.

(A) $\sqrt{5}+\sqrt{10}-\sqrt{8}$
(B) $\sqrt{10}+\sqrt{8}-\sqrt{5}$
(C) $\sqrt{8}+\sqrt{5}+\sqrt{10}$
(D) $\sqrt{5}+\sqrt{8}-\sqrt{10}$

29. If $\sqrt{x\sqrt{x\sqrt{x.......\infty}}} = 11^x$, what is the value of $\left(\sqrt[x]{x}\right)$?

(A) 11 (B) $\sqrt[3]{11}$ (C) $\sqrt{11}$ (D) 1

30. If x = $\sqrt{\frac{6}{4+\frac{6}{2+\sqrt{\frac{6}{4+\frac{6}{2+...\infty}}}}}}$, find the value of x.

(A) $\frac{9+\sqrt{33}}{2}$ (B) 1

(C) $\frac{9-\sqrt{43}}{2}$ (D) $\frac{9-\sqrt{33}}{4}$

31. Simplify $\sqrt{x+\sqrt{x^2+\sqrt{x^4+\sqrt{x^8........\infty}}}}$.

(A) $\sqrt{x}\left(\frac{1+\sqrt{5}}{2}\right)$ (B) $\frac{(3+5\sqrt{2})}{\sqrt{x}}$

(C) $\frac{x}{1+\sqrt{x}}$ (D) $\frac{\sqrt{2}+\sqrt{3}}{x^{3/2}}$

32. Find the value of
$\frac{1}{\sqrt{5}+\sqrt{11-2\sqrt{30}}}-\frac{1}{\sqrt{3}-\sqrt{10+2\sqrt{21}}}$.

(A) $\frac{6\sqrt{7}-7\sqrt{6}}{42}$ (B) $\frac{7\sqrt{6}+6\sqrt{7}}{42}$

(C) $\frac{7\sqrt{7}-6\sqrt{6}}{42}$ (D) $\frac{7\sqrt{7}+6\sqrt{6}}{42}$

33. If $\frac{1}{\sqrt{x}+\sqrt{x+1}}+\frac{1}{\sqrt{x+1}+\sqrt{x+2}}+\frac{1}{\sqrt{x+2}+\sqrt{x+3}}+\ldots\ldots\frac{1}{\sqrt{x+98}+\sqrt{x+99}}=9$,

which of the following is a possible value of x?

(A) 2 (B) 1 (C) 4 (D) 3

34. Find the value of $x^2 - 3xy + y^2$, if

$x=\frac{\sqrt{19}+3\sqrt{2}}{\sqrt{19}-3\sqrt{2}}$ and $y=\frac{\sqrt{19}-3\sqrt{2}}{\sqrt{19}+3\sqrt{2}}$

(A) 5466 (B) 5471
(C) 5476 (D) 5481

35. If $\left(\sqrt{7}+\sqrt{3}+2\right)\left(a\sqrt{21}+b\sqrt{3}+c\right)=48$ and a, b and c are integers, then the value of 2a + 3b + 4c is

$\boxed{}$

Exercise – 7(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If a x 5^2 = 2020.20, what is the value of $\frac{a \times 10^{-3}}{10^4}$?

(A) 0.0000808080
(B) 0.00000808080
(C) 0.000000808080
(D) 0.0000202020

2. What is the value of z in terms of x and y if $3^x = 2^y = 6^z$?

(A) $z=\frac{x+y}{4xy}$ (B) $z=\frac{xy}{x+y}$

(C) $z=\frac{x+y}{2}$ (D) $z=\frac{x^2+y^2}{xy}$

3. If $[(a^3)^b]^3 = 8^3 \times 8^3$, which of the following could be the possible values of a and b?

(A) 2, 2 (B) 3, 2 (C) 2, 3 (D) 4, 2

4. Solve for x: $3^{2x} + 2.3^x - 99 = 0$.

$\boxed{}$

5. Solve for x, if $(a/b)^x (c/b)^x = 225/16$, given a, b, c are prime numbers.

(A) 4 (B) 3 (C) 2 (D) 1

6. If $3^{x+3} - 3^{x-3} = 6552$, then find x^2.

$\boxed{}$

7. If xyz = 1, then the expression $\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$ is equal to

(A) 1 (B) $\frac{3}{x+y+z}$

(C) $\frac{3}{x^{-1}+y^{-1}+z^{-1}}$ (D) $\frac{x+y+z}{3}$

8. If a and b are integers such that $a^b + b^a = 1025$, find the value of a + b. $\boxed{}$

9. If $\frac{\left(81^a\right)^a\left(81^b\right)^b\left(81^c\right)^c}{\left(6561^b\right)^{-c}\left(6561^c\right)^{-a}\left(6561^a\right)^{-b}}=3$, then a + b + c could be

(A) 2 (B) 1/3
(C) –1/2 (D) –1/3

10. The greatest number among $6\sqrt[3]{5}$, $9-\sqrt[3]{2}$, $15-\sqrt[4]{3}$ is

(A) $6\sqrt[3]{5}$ (B) $9-\sqrt[3]{2}$

(C) $15-\sqrt[4]{3}$ (D) Cannot say

11. If $x=\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$, find x

(A) $\sqrt{3}+\frac{3}{2}$ (B) $\sqrt{3}-\frac{3}{2}$

(C) $\sqrt{3}+\frac{1}{2}$ (D) $\sqrt{3}-\frac{1}{2}$

12. If $x = 2^{55}$, $y = 17^{14}$ and $z = 31^{11}$, then which of the following is the ascending order of the values of x, y and z?

(A) zyx (B) zxy (C) yxz (D) xyz

13. If $A = 8^{88^8}, B = 8^{8^{88}}, C = 8^{888}$ and $D = 8^{8^{8^8}}$, which of the following represents the ascending order of A, B, C, D?

(A) CDAB (B) CABD
(C) CBAD (D) ACBD

14. If $a^x = b^y = c^z$ where a > 1, b > 1, c > 1, then the value of $(abc)^{\left(\frac{xyz}{xy+yz+zx}\right)}$ is

(A) $a^x + b^y + c^z$
(B) $(abc)^{xyz}$
(C) $\frac{a^{\frac{1}{x}}+b^{\frac{1}{y}}+c^{\frac{1}{z}}}{3}$
(D) a^x

15. If $x = \sqrt{3}+\sqrt{2}$, then $x^2 - 1/x^2$ =

(A) $12\sqrt{2}$ (B) 10

(C) 50 (D) $4\sqrt{6}$

16. What is the maximum value of k, if 12345678900000 $\times 10^k$ is less than one? ☐

17. If $\frac{\sqrt[3]{x}}{392} = \frac{98}{x}$, then find $x^{2/3}$.

(A) 14 (B) 144 (C) 16 (D) 196

18. If $(x - y)\left[\frac{1}{\sqrt{x}+\sqrt{y}} + \frac{1}{\sqrt{x}-\sqrt{y}}\right] = 12$, find the value of y.

(A) 36 (B) 25

(C) 49 (D) Cannot be determined

19. If $\sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\cdot^{\cdot^{\cdot^{\infty}}}}}} = \frac{1}{4}$, find x.

(A) $\frac{1}{2^{12}}$ (B) $\frac{1}{2^{10}}$ (C) $\frac{1}{2^{16}}$ (D) $\frac{1}{2^{14}}$

20. If $\frac{3-\sqrt{a+5}}{a-4} = \frac{-1}{16}$, a = ☐

21. Find the value of $\sqrt{7-3\sqrt{5}}$.

(A) $\sqrt{7}-2\sqrt{3}$ (B) $\frac{3-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-\sqrt{7}}{2\sqrt{2}}$ (D) $\frac{\sqrt{5}+\sqrt{2}}{3}$

22. What is the mean proportional of $2-\sqrt{3}$ and $26-15\sqrt{3}$?

(A) $7-4\sqrt{3}$ (B) $12-7\sqrt{3}$

(C) $13-5\sqrt{3}$ (D) $24\sqrt{3} - 14\sqrt{3}$

23. The arithmetic mean of two surds is $5 + 9\sqrt{2}$, and one of the surds is $1 + 12\sqrt{2}$. What is the square root of the other surd?

(A) $6-21\sqrt{2}$ (B) $4-3\sqrt{2}$

(C) $\sqrt{3}\left(\sqrt{2}+1\right)$ (D) $\sqrt{2}\left(2-\sqrt{3}\right)$

24. $\frac{1}{\sqrt{6}+\sqrt{7}-\sqrt{13}} + \frac{1}{\sqrt{6}-\sqrt{7}-\sqrt{13}}$ =

(A) $\sqrt{6}$ (B) $\frac{1}{\sqrt{6}}$

(C) 6 (D) $\frac{1}{6}$

25. If $x = 3-\sqrt{5}$ then evaluate $\frac{\sqrt{x}}{\sqrt{2}+\sqrt{3x-2}}$.

(A) $\frac{1}{4-2\sqrt{5}}$ (B) $\sqrt{3}-2\sqrt{5}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{1}{\sqrt{2}-3\sqrt{5}}$

26. If $a = \sqrt{6}+\sqrt{8}$, what is $2\sqrt{2}\left(26+15\sqrt{3}\right)$ in terms of a?

(A) $a^{3/2}$ (B) $a^{5/2}$ (C) a^4 (D) a^3

27. If $a = \frac{1}{2+\sqrt{3}}$, $b = \frac{1}{2-\sqrt{3}}$, what is the value of $7b^2 + 11ab - 7a^2$?

(A) $-14 + 21\sqrt{3}$ (B) $11 + 56\sqrt{3}$

(C) $49 + 8\sqrt{3}$ (D) $\sqrt{3} + 11$

28. If $a = \sqrt{12}+2$, $b = \sqrt{3}+4$, $c = \sqrt{6} + \sqrt{8}$ and $d = \sqrt{2} + \sqrt{24}$, which of the following represents the ascending order of a, b, c and d?

(A) cbda (B) cbad

(C) cadb (D) cabd

29. Find the square root of

$$\left[1 + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots\ldots\ldots \frac{1}{\sqrt{324}+\sqrt{323}}\right].$$

(A) $3\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $2\sqrt{3}$ (D) $\frac{\sqrt{3}-1}{2}$

30. Find the sum of the given terms in the following series

$$\frac{1}{1+2^{1/3}+2^{2/3}} + \frac{1}{2^{2/3}+6^{1/3}+3^{2/3}} + \frac{1}{3^{2/3}+12^{1/3}+4^{2/3}} + \ldots + \frac{1}{26^{2/3}+702^{1/3}+27^{2/3}}$$ ☐

31. If $x = \sqrt[3]{2}+\sqrt[3]{4}$, what is the value of $x^3 - 6x$? ☐

32. If $x = 3 + \sqrt{5}$, then find the value of $x^3 - 9x^2 + 22x$. ☐

33. The value of

$\frac{1}{\sqrt{3}-\sqrt{5-2\sqrt{6}}} + \frac{1}{\sqrt{5}-\sqrt{8-2\sqrt{15}}}$ is

(A) $\frac{3\sqrt{2}+2\sqrt{3}}{6}$ (B) $\frac{2\sqrt{3}-3\sqrt{2}}{4}$

(C) $\frac{3\sqrt{2}-2\sqrt{3}}{12}$ (D) $\frac{2\sqrt{3}+4\sqrt{2}}{12}$

34. Simplify : $\dfrac{a-b}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}-\dfrac{a+b}{\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}}$.

(A) $-2b^{1/3}$ (B) $-2(ab)^{1/3}$
(C) $a^{1/3}-b^{1/3}$ (D) $(a/b)^{1/3}$

35. If $x=\dfrac{\sqrt{13}+2\sqrt{3}}{\sqrt{13}-2\sqrt{3}}$ and $y=\dfrac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}+2\sqrt{3}}$, then what is the value of $x^2+5xy+y^2$? $\boxed{}$

Directions for questions 36 to 40: Each question is followed by two statements, I and II. Answer each question using the following instructions:

Choose (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Choose (B) if the question can be answered using either statement alone.
Choose (C) if the question can be answered using I and II together but not using I or II alone
Choose (D) if the question cannot be answered even using I and II together.

36. If x and y are positive integers, then what is the value of $(2x)^y$?
I. $x^y = 16$.
II. $2x = 8$.

37. If p,q and r are non-zero real numbers, then find the value of $\left(4^{\frac{1}{qr}}\right)^{p^2}\times\left(4^{\frac{1}{pq}}\right)^{r^2}\times\left(4^{\frac{1}{pr}}\right)^{q^2}$.
I. $p^3+q^3+r^3 = 3pqr$.
II. $p = 1$, $q = 2$ and $r = -3$.

38. If b is a natural number, which is greater a^{3b} or $(a+1)^{2b}$?
I. a is a whole number.
II. a is a natural number.

39. Find a + b where a and b are non-negative integers.
I. $2^a + 3^b = 17$.
II. $9^a + 8^b = 145$.

40. If a and b are positive, which is greater between P and Q?
I. $P=\sqrt[3]{a^3+b^3}$
II. $Q=\sqrt{a^2+b^2}$

Key

Concept Review Questions

1. (a) C
   (b) B
   (c) 2916
   (d) D
   (e) C
   (f) 1
   (g) A
   (h) C
   (i) 41503
   (j) D
2. (a) B
   (b) C
3. D
4. B
5. A
6. (a) 2
   (b) A
7. C
8. A
9. (a) 6.25
   (b) 2
   (c) A
   (d) A
10. C
11. (a) A
    (b) B
    (c) D
    (d) D
    (e) B
12. A
13. A
14. B
15. (a) D
    (b) C
16. A
17. A
18. D
19. D
20. A
21. D
22. 79
23. 481
24. (a) B
    (b) C
25. (a) C
    (b) D
    (c) A
    (d) D

Exercise – 7(a)

1. D
2. B
3. B
4. D
5. 2
6. A
7. 4
8. A
9. C
10. D
11. B
12. 2
13. 52
14. C
15. A
16. A
17. A
18. A
19. C
20. D
21. 5
22. A
23. C
24. C
25. D
26. D
27. B
28. D
29. A
30. B
31. A
32. B
33. B
34. B
35. 64

Exercise – 7(b)

1. B	11. B	21. B	31. 6
2. B	12. B	22. A	32. 12
3. A	13. B	23. C	33. A
4. 2	14. D	24. B	34. A
5. C	15. D	25. C	35. 2503
6. 25	16. –14	26. D	36. C
7. A	17. D	27. B	37. B
8. 1025	18. D	28. D	38. D
9. C	19. C	29. A	39. A
10. C	20. 164	30. 2	40. C