CAT
FORMULAE BOOK

COMPLETE RECAP OF CAT MATHS IN 60 PAGES

hitbullseye.com
India's Fastest Growing Test Prep Portal

Chapter 1– Calculation Tricks___________

A. List of important fractions
B. Important Tables, squares and cubes

Chapter 2– Conversion of Units_________

Chapter 3– Number System____________

A. Types of No.
B. Operation on odd and even numbers
C. Test of Divisibility
D. Properties of Numbers
E. LCM & HCF
F. Fractions
G. Indices
H. Remainder Theory

Chapter 4– Average __________________

Chapter 5– Basic Algebra _____________

A. Algebraic Formulae
B. Linear Equations
C. Quadratic Equations
D. Inequalities
E. Arithmetic and Geometric Progression

Chapter 6– Percentage and Profit & Loss _

A. Percentage
B. Simple interest and Compound interest
C. Profit & Loss

Chapter 7– Ratio, Mixture and Partnership

A. Ratio and Proportion
B. Alligation and Mixture
C. Partnership

Chapter 8– Time and Work_____________

Chapter 9– Time Speed & Distance ______

A. Basics of Time Speed Distance
B. Boats and Steams
C. Races

Chapter 10– Geometry & Mensuration_____

A. Rectangular solids and cylinders
B. Lines and Angles
C. Triangles
D. Circles
E. Quadrilateral & Polygons

Chapter 11– Coordinate Geometry________

Chapter 12– Set Theory ________________

Chapter 13– Logarithms________________

Chapter 14– Permutation & Combination __

Chapter 15– Probability ________________

Chapter 16– Clocks and Calendars_______

Chapter 17– Mean and Basic Statistics____

Chapter 18– Trigonometry ______________

1. Calculation Tricks

A. List of important fractions

You get many questions in the exams based on Percentage, Profit, Interest etc. in which you have to calculate, say 87.5 % of 800, 58.33 % of 2400 etc. Calculating these values with the help of traditional methods is time-consuming.

If you apply the fraction approach, you can crack these easily i.e., if you know that 87.5 % is just 7/8th of the number and 58.33 % is 7/12th of the number, then it becomes easy to calculate.

Given below are the important fractions, which you should remember:

% age	Fraction	% age	Fraction
50 %	$^1/_2$	55 $^5/_9$%	$^5/_9$
33 $^1/_3$%	$^1/_3$	77 $^7/_9$%	$^7/_9$
66 $^2/_3$%	$^2/_3$	88 $^8/_9$%	$^8/_9$
25 %	$^1/_4$		
75 %	$^3/_4$	9 $^1/_{11}$%	$^1/_{11}$
20 %	$^1/_5$	18 $^2/_{11}$%	$^2/_{11}$
40 %	$^2/_5$	27 $^3/_{11}$%	$^3/_{11}$
60 %	$^3/_5$	36 $^4/_{11}$%	$^4/_{11}$
80 %	$^4/_5$	45 $^5/_{11}$%	$^5/_{11}$
		54 $^6/_{11}$%	$^6/_{11}$
16 $^2/_3$%	$^1/_6$	63 $^7/_{11}$%	$^7/_{11}$
83 $^1/_3$%	$^5/_6$	72 $^8/_{11}$%	$^8/_{11}$
14 $^2/_7$%	$^1/_7$	81 $^9/_{11}$%	$^9/_{11}$
		90 $^{10}/_{11}$%	$^{10}/_{11}$
12 $^1/_2$%	$^1/_8$		
37 $^1/_2$%	$^3/_8$	8 $^1/_3$%	$^1/_{12}$
62 $^1/_2$%	$^5/_8$	41 $^2/_3$%	$^5/_{12}$
87 $^1/_2$%	$^7/_8$	58 $^1/_3$%	$^7/_{12}$
		91 $^2/_3$%	$^{11}/_{12}$
11 $^1/_9$ %	$^1/_9$	6 $^2/_3$%	$^1/_{15}$
22 $^2/_9$%	$^2/_9$	6 $^1/_4$%	$^1/_{16}$
44 $^4/_9$%	$^4/_9$	5 %	$^1/_{20}$

Anything doubles to increase by 100 % and becomes 200%.
Anything trebles to increase by 200 % and becomes 300 %.

B. Important Tables, Squares and Cubes

In order to be good at mathematics, you have to be good at calculations. For improving calculations, you have to be good at numbers and tables. You should start with tables and make it the most important part of your preparation

Tables: Learn all these tables by heart and see how you improve your calculation speed.

Table	12	13	14	15	16	17	18	19
T×1	12	13	14	15	16	17	18	19
T×2	24	26	28	30	32	34	36	38
T×3	36	39	42	45	48	51	54	57
T×4	48	52	56	60	64	68	72	76
T×5	60	65	70	75	80	85	90	95
T×6	72	78	84	90	96	102	108	114
T×7	84	91	98	105	112	119	126	133
T×8	96	104	112	120	128	136	144	152
T×9	108	117	126	135	144	153	162	171
T×10	120	130	140	150	160	170	180	190

Squares: Learn these squares by heart.

Z	Z^2	Z	Z^2	Z	Z^2
1	1	13	169	25	625
2	4	14	196	26	676
3	9	15	225	27	729
4	16	16	256	28	784
5	25	17	289	29	841
6	36	18	324	30	900
7	49	19	361	31	961
8	64	20	400	32	1024
9	81	21	441	33	1089
10	100	22	484	34	1156
11	121	23	529	35	1225
12	144	24	576		

Cubes: Learn these cubes by heart.

Y	Y^3	Y	Y^3
1	1	12	1728
2	8	13	2197
3	27	14	2744
4	64	15	3375
5	125	16	4096
6	216	17	4913
7	343	18	5832
8	512	19	6859
9	729	20	8000
10	1000	21	9261
11	1331	22	10648

2. Conversions of Units

Number of zeroes in

1 Lakh = 5 = 100,000
1 Crore = 7 = 10,000,000
1 million = 6 = 1,000,000
1 Billion = 9 = 1,000,000,000

Linear Measures (Metric)

1. 10 millimeters (mm) = 1 centimeter (cm)
2. 10 centimeters = 1 decimeter (dm)
3. 10 decimeters = 1 meter (m or M)
4. 10 meters = 1 Decameter (Dm)
5. 10 Decameter = 1 Hectometer (Hm)
6. 10 Hectometers = 1 Kilometer (Km) = 1000 m = 100,000 cm

British

1. 12 inches (in) = 1 foot (ft)
2. 3 feet = 1 yard (yd)
3. 220 yards = 1 furlong
4. 8 furlongs = 1 mile = 1760 yards = 5280 ft

Relationship
1. 1 inch = 2.54 cm
2. 1 Km = 5 furlongs
3. 1 mile = 1600 m
4. 1 meter = 1.1 yds = 3.39 ft
5. 1 mile = 1.6 km = 1600 meters. .

Area Measures

Metric
1. 1 Hectare = 1 sq. Hm = 10,000 Sq. m

British
1. 1 acre = 4840 sq. yds = 40,000 sq ft.
2. 1 are = 120 sq. yds
3. 40 ares = 1 acre
4. 1 cent = 440 sq ft.

Relationship
1. 1 Hectare = 2.5 acres

Volume Measures
Metric
1. 1 cubic centimeter (cc) = 1 milliliter (ml).
2. 10 ml = 1 centiliter (cl).
3. 10 cl = 1 deciliter (dl).
4. 10 dl = 1 liter (L or l) = 1000 ml = 1000 cc.
5. 1000 liter = 1 Kilo liter (Kl).

British
1. 20 ounces (oz) = 1 pint
2. 8 pints = 1 gallon

Relationship
1. 1 gallon = 4.55 liters or 1 liter = 0.22 gallon.

Weight Measures

Metric
1. 10 milligrams (mg) = 1 centigram (cgm)
2. 10 centigrams = 1 decigram (dgm)
3. 10 decigrams = 1 gram (gm)
4. 10 grams = 1 Decagram (Dgm)
5. 10 Decagrams = 1 Hectogram (Hgm)
6. 10 Hectograms = 1 Kilogram (Kg) = 1000 gm
7. 100 Kg = 1 quintal
8. 10 quintals = 1 Metric Ton = 1000 Kg

British
1. 12 ounce (oz) = 1 pound (lb)
2. 14 pounds = 1 quarter (qr)
3. 4 quarters = 1 hundred weight (cwt)
4. 20 hundred weights = 1 ton
5. 1 ton = 2240 pounds

Speed Measures.

Metric

1 km = 1000 meters

1 hr = 3600 seconds

1 km/hr = $\left(\frac{1\times 1000}{1\times 3600}=\frac{5}{18}\right)$ m/sec

British

1 mile = 1760 yards

1 yard = 3 feet

1 mile = 5280 feet

1 mph = $\left(\frac{1\times 1760}{1\times 3600}=\frac{22}{45}\right)$ yards/sec

1 mph = $\left(\frac{1\times 5280}{1\times 3600}=\frac{22}{15}\right)$ ft/sec

3. Number System

A. Types of Numbers

Following are different types of numbers which we generally use in our calculations.

Natural Numbers: The numbers 1,2,3,4.... are called natural numbers or positive integers.

Whole Numbers: The numbers 0,1,2,3.... are called whole numbers. Whole numbers include "0". Every Natural number is a whole number but the converse is not true.

Integers: The numbers .... -3, -2, -1, 0, 1, 2, 3,....are called integers. Every Whole number is an integer but the converse is not true.

Negative Integers: The numbers -1, -2, -3, .. are called negative integers.

Rational Numbers: Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format ⇒ $\frac{a}{b}$, where b $\neq$ 0 and a & b are positive or negative integers. Every Integer is a rational number but the converse is not true.

Positive Fractions: The numbers $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{8}$... are called positive fractions.

Negative Fractions: The numbers $-\frac{6}{8}$, $-\frac{7}{19}$, $-\frac{12}{47}$... are called negative fractions.

Note: Between any two different rational numbers a & b there always exists a rational number calculated by taking the average of a and b i.e. $\frac{a+b}{2}$

Irrational Numbers: A non terminating non recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where b $\neq$ 0.

e.g. $\sqrt{2}$, $\sqrt{5}$, π, etc.

Even Numbers: The numbers which are divisible by 2 are called even numbers e.g. -4, 0, 2, 16 etc.

Odd Numbers: The numbers which are not divisible by 2 are odd numbers e.g. -7, -15, 5, 9 etc.

Prime Numbers: Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has exactly two factors –

1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.

2 is the only even prime number.

There are 25 prime numbers up to 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.

Composite Number: A number, which has more than two factors, is called a composite number. e.g. 9, 10, 15, 16, ...

A composite number can be expressed as the product of prime numbers in a unique way. e.g. $100 = 2^2 \times 5^2$

1 is neither a composite number nor a prime number.

Real Numbers: The above sets of natural numbers, integers, whole numbers, rational numbers and irrational numbers constitute the set of real numbers. Every real number can be represented by a point on a number line.

Perfect Numbers: If the sum of all the factors of a number excluding the number itself happens to be equal to the number, then the number is called as perfect number. 6 is the first perfect number. The factors of 6 are 1, 2, 3 & 6. Leaving 6 the sum of other factors of 6 are equal to 6. The next three perfect numbers after 6 are 28, 496 and 8128.

B. Operations on Odd & Even Numbers

(i) Addition or subtraction of any two odd numbers will always result in an even number or zero. E.g. $1 + 3 = 4$; $5 - 3 = 2$.

(ii) Addition or subtraction of any two even numbers will always result in an even number or zero. E.g. $2 + 4 = 6$; $12 - 4 = 8$.

(iii) Addition or subtraction of an odd number from an even number will result in an odd number. E.g. $4 + 3 = 7$; $10 - 3 = 7$.

(iv) Addition or subtraction of an even number from an odd number will result in an odd number. E.g. $3 + 4 = 7$; $5 - 2 = 3$.

(v) Multiplication of two odd numbers will result in an odd number. E.g. $3 \times 3 = 9$.

(vi) Multiplication of two even numbers will result in an even number. E.g. $2 \times 4 = 8$.

(vii) Multiplication of an odd number by an even number or vice versa will result in an even number. E.g. $3 \times 2 = 6$.

(viii) An odd number raised to an odd or an even power is always odd.

(ix) An even number raised to an odd or an even power is always even.

(x) The standard form of writing a number is $m \times 10^n$ where m lies between 1 and 10 and n is an integer. e.g. $0.89713 \Rightarrow 8.9713/10^1 . \Rightarrow 8.9713 \times 10^{-1}$.

(xi) If n is odd. $n(n^2 - 1)$ is divisible by 24. e.g. take $n = 5 \Rightarrow 5(5^2 - 1) = 120$, which is divisible by 24.

(xii) If n is odd prime number except 3, then $n^2 - 1$ is divisible by 24.

(xiii) If n is odd. $2^n + 1$ is divisible by 3.

(xiv) If n is even. $2^n - 1$ is divisible by 3.

(xv) If n is odd, $2^{2n} + 1$ is divisible by 5.

(xvi) If n is even, $2^{2n} - 1$ is divisible by 5.

(xvii) If n is odd, $5^{2n} + 1$ is divisible by 13.

(xviii) If n is even, $5^{2n} - 1$ is divisible by 13

C. Tests of Divisibility

Divisibility rules are very helpful while doing calculations. Following are some important divisibility rules which you should learn from heart.

(i) **By 2** - A number is divisible by 2 when its units place is 0 or divisible by 2. e.g. 120, 138.

(ii) **By 3** – A number is divisible by 3 when the sum of the digits of the number is divisible by 3. e.g. 15834 is divisible by 3 as the sum of its digits is 21 which is divisible by 3. Note that if n is odd, then $2^n + 1$ is divisible by 3 and if n is even, then $2^n - 1$ is divisible by 3.

(iii) **By 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4. As 100 is divisible by 4, it is sufficient if the divisibility test is restricted to the last two digits. e.g. 145896, 128, 18400

(iv) **By 5** - A number is divisible by 5, if its unit's digit is 5 or 0. e.g. 895, 100

(v) **By 6** - A number is divisible by 6, if it is divisible by both 2 and by 3. i.e. the number should be an even number and the sum of its digits should be divisible by 3.

(vi) **By 8** - A number is divisible by 8, if the last three digits of the number are 0s or are divisible by 8. As 1000 is divisible by 8, it is sufficient if the divisibility test is restricted to the last three digits e.g. 135128, 45000

(vii) **By 9** - A number is divisible by 9, if the sum of its digits is divisible by 9. e.g. 810, 92754

(viii) **By 11** - A number is divisible by 11, if the difference between the sum of the digits at odd places and sum of the digits at even places of the number is either 0 or a multiple of 11.

e.g. 121, 65967. In the first case 1+1 - 2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.

(ix) **By 12** - A number is divisible by 12, if it both divisible by 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136.

(x) **By 15 –** A number is divisible by 15, if it is divisible by both 5 and 3.

(xi) **By 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or the last two digits are 0.

(xii) **By 75** - A number is divisible by 75, if it is both divisible by 3 and by 25. i.e. the sum of the digits should be divisible by 3 and the last two digits should be divisible by 25.

(xiii) **By 125** - A number is divisible by 125, if its last three right hand digits are divisible by 125 or the last three digits are 0s. e.g. 1254375, 12000

D. Properties of the numbers

Following are some properties of the numbers which are helpful in solving the questions in the examination.

(i) The sum of 5 successive whole numbers is always divisible by 5.

(ii) The product of 3 consecutive natural numbers is divisible by 6.

(iii) The product of 3 consecutive natural numbers, the first of which is an even number is divisible by 24.

(iv) The sum of a two-digit number and a number formed by reversing its digits is divisible by 11. E.g. 28 + 82 = 110, which is divisible by 11. At the same time, the difference between those numbers will be divisible by 9. e.g. 82 – 28 = 54, which is divisible by 9.

(v) $\Sigma n = \frac{n(n+1)}{2}$, Σn is the sum of first n natural numbers.

(vi) $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$, Σn^2 is the sum of first n perfect squares.

(vii) $\Sigma n^3 = \frac{n^2(n+1)^2}{4} = (\Sigma n)^2$, Σn^3 is the sum of first n perfect cubes.

(viii) $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - ... + y^{n-1})$ when n is odd. Therefore, when n is odd, $x^n + y^n$ is divisible by x + y.
e.g. $3^3 + 2^3 = 35$ and is divisible by 5.

(ix) $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. Therefore, when n is even, $x^n - y^n$ is divisible by x + y.
e.g. $7^2 - 3^2 = 40$, which is divisible by 10.

(x) $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + ... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by x - y.
e.g . $9^4 - 2^4 = 6545$ which is divisible by 7.

E. LCM & HCF

- In case of HCF, if some remainders are given, then first those remainders are subtracted from the numbers given and then their HCF is calculated.

- Sometimes in case of HCF questions, the same remainder is required is given and the remainder is not given. In such questions, the answer is the HCF of the difference of the numbers taken in pairs.

- In case of LCM, if a single remainder is given, then firstly the LCM is calculated and then that single reminder is added in that.

- In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.

LCM and HCF of Fractions

LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$;

e.g. LCM of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{3\text{ (LCM of numerators)}}{2\text{ (HCF of denominators)}}$

HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

e.g. HCF of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{1\text{(HCF of numerators)}}{4\text{(LCM of denominators)}}$

Note that the product of the two fractions is always equal to the product of LCM and HCF of the two fractions.

The product of the two fractions = $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

The product of the LCM and HCF = $\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$.

F. Fractions

Numbers of the form 3/4, 4/5, are called fractions. A fraction can be written as p/q where $q \neq 0$.

- If the numerator and denominator of a fraction are multiplied / divided by the same number then the value of the fraction does not change.
- For any positive proper fraction p/q (p<q), the value of the fraction increases when both the denominator and numerator are added by the same positive number.

  e.g $\frac{3}{4}$ = 0.75, $\frac{3+1}{4+1} = \frac{4}{5}$ = 0.8.
- For any positive proper fraction p/q (p<q), the value of the fraction decreases when both the numerator and denominator are subtracted by the same positive number.

  e.g $\frac{3}{4}$ = 0.75, $\frac{3-1}{4-1} = \frac{2}{3}$ = 0.67

- For any positive improper fraction p/q (p>q), the value of the fraction decreases when both the numerator and the denominator are added to the same positive. E.g. $\frac{5}{4}$ = 1.25, adding 1 to the numerator and the denominator, we get $\frac{5+1}{4+1} = \frac{6}{5}$ = 1.2, which is less than 1.25.
- For any positive improper fraction p/q (p>q), the value of the fraction increases when both the numerator and denominator are subtracted by the same positive number. E.g. $\frac{5}{4}$ = 1.25, by subtracting 1 from both the numerator and denominator we get, $\frac{5-1}{4-1} = \frac{4}{3}$ = 1.33 > 1.25.

Types of fractions

Common Fractions: Fractions such 3/4, 32/43 etc are called common or vulgar fractions.

Decimal Fractions: Fractions whose denominators are 10, 100, 1000, ... are called decimal fractions.

Proper Fraction: A fraction whose numerator is less than its denominator is known as a proper fraction e.g. 3/4

Improper Fraction: A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g. 4/3

Mixed Fractions: Fractions which consists of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g. $1\frac{3}{4}$.

Recurring Decimals: A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal.

e.g. $1/7$ = 0.142857…….. the dots indicate that the figure between 1 and 7 will repeat continuously.

G. Indices

The expression $a^5 = a \times a \times a \times a \times a$

Similarly for any positive integer n, $a^n = a \times a \times a \times$ ..... n times.

In a^n, a is called the base and n is called the index.

Law of Indices

Let m and n be positive integers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

(iv) $(ab)^m = a^m \times b^m$

(v) $a^0 = 1$, where $a \neq 0$

(vi) $a^{-n} = \frac{1}{a^n}$

(vii) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ i.e. the q^{th} root of a raised to the power of p.

(viii) In particular, $a^{\frac{1}{q}} = \sqrt[q]{a}$

H. Remainder Theory

Questions from number system appear regularly in almost all competitive exams. Within number system, the questions on remainders are found to be most tricky. This will help you learn different types of remainder questions and various approaches you can apply to solve these.

The basic remainder formula is:
Dividend = Divisor × Quotient + Remainder

If remainder = 0, then it the number is perfectly divisible by divisor and divisor is a factor of the number e.g. when 8 divides 40, the remainder is 0, it can be said that 8 is a factor of 40.

There are few important results relating to numbers. Those will be covered one by one in the following examples.

(i) Formulas Based Concepts for Remainder:

$(a^n + b^n)$ is divisible by $(a + b)$, when n is odd.
$(a^n - b^n)$ is divisible by $(a + b)$, when n is even.
$(a^n - b^n)$ is always divisible by $(a - b)$, for every n.

(ii) Concept of Negative Remainder:

By definition, remainder cannot be negative. But in certain cases, you can assume that for your convenience. But a negative remainder in real sense means that you need to add the divisor in the negative remainder to find the real remainder.

NOTE: Whenever you are getting a negative number as the remainder, make it positive by adding the divisor to the negative remainder.

(iii) Cyclicity in Remainders:

Cyclicity is the property of remainders, due to which they start repeating themselves after a certain point.

(iv) Role of Euler's Number in Remainders:

Euler's Remainder theorem states that, for co-prime numbers M and N, Remainder $[M^{E(N)} / N] = 1$, i.e. number M raised to Euler number of N will leave a remainder 1 when divided by N. Always check whether the numbers are co-primes are not as Euler's theorem is applicable only for co-prime numbers.

4. Average

Average means Arithmetic mean of the items and it is $= \dfrac{\text{Sum of Items}}{\text{Number of Items}}$

When the difference between all the items is same, then average is equal to $\dfrac{n+1}{2}$ item, where n is the total number of items.

Average speed : If a man covers some journey from A to B at u km/hr. and returns back to A at B uniform speed of v km/hr., then the average speed during the whole journey is $\dfrac{2uv}{u+v}$ km/hr

(i) If x is added in all the items, then average increases by x.

(ii) If x is subtracted from all the items, then average decreases by x.

(iii) If every item is multiplied by x, then average also gets multiplied by x.

(iv) If every item is divided by x, then average also gets divided by x.

(v) Weighted average = $\dfrac{a_1x_1 + a_2x_2 + \dots a_nx_n}{a_1 + a_2 + \dots\dots + a_n}$

5. Basic Algebra

A. Algebraic Formulae

(i) $x^n + y^n = (x+y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - .. + y^{n-1})$ when n is odd. When n is odd, $x^n + y^n$ is divisible by $x + y$

(ii) $x^n - y^n = (x+y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. When n is even, $x^n - y^n$ is divisible by $x + y$.

(iii) $x^n - y^n = (x-y)(x^{n-1} + x^{n-2}.y + ... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by $x-y$.

(iv) $(a+b)^2 = a^2 + b^2 + 2ab$.

(v) $(a-b)^2 = a^2 + b^2 - 2ab$.

(vi) $(a^2 - b^2) = (a+b)(a-b)$.

(vii) $(a+b)^2 - (a-b)^2 = 4ab$.

(viii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$.

(ix) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$.

(x) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$.

(xi) $(a^3 + b^3) = (a+b)(a^2 + b^2 - ab)$.

(xii) $(a^3 - b^3) = (a-b)(a^2 + b^2 + ab)$.

(xiii) $(a+b+c)^2 = [a^2 + b^2 + c^2 + 2(ab + bc + ca)]$.

(xiv) $(a^3 + b^3 + c^3 - 3abc) = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$. If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

(xv) $(x+a)(x+b) = x^2 + (a+b)x + ab$.

B. Linear Equations:

Consider two linear equations:-

$A_1 x + B_1 y = C_1$

$A_2 x + B_2 y = C_2$.

This pair of linear equations has:-

(i) A unique solution if, $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$.

(ii) Infinite solutions if, $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

(iii) No solution if, $\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$.

C. Quadratic Equations:

(i) General form of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$

(ii) The discriminant of a quadratic equation is $D = b^2 - 4ac$

(iii) The roots of the above quadratic equation are $\frac{-b+\sqrt{D}}{2a}$ and $\frac{-b-\sqrt{D}}{2a}$

(iv) Let α and β be the roots of the above quadratic equation. If D > 0, then the roots are real and unequal. The sum of the roots $\alpha + \beta = \frac{-b}{a}$ and the product $\alpha\ \beta = \frac{c}{a}$

(v) If D is a perfect square, then the roots are rational and unequal.

(vi) If D = 0, then the roots are real and equal and is equal to $\frac{-b}{2a}$

(vii) If D < 0, then the roots are complex and unequal. If a, b and c of the quadratic equation are rational, then the roots are conjugates of each other. Ex. if α = p + qi, then β = p - qi

(viii) If $D \geq 0$, then, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.

(ix) If c = a, then the roots are reciprocal.

(x) If b = 0, then the roots are equal in magnitude but opposite in sign.

(xi) If one of the roots of a quadratic equation with rational coefficients is irrational, then the other roots must be irrational conjugate. If $\alpha = p + \sqrt{q}$, then $\beta = p - \sqrt{q}$.

(xii) If α, β are the roots of a quadratic equation, then the equation is $x^2 - (\alpha + \beta)x + \alpha\ \beta = 0$.

D. Inequalities

(i) Inequalities, Interval Notations and Graphs

Inequality	Interval Notation	Graph
a ≤ x ≤ b	[a, b]	
a < x ≤ b	(a, b]	
a ≤ x < b	[a, b)	
a < x < b	(a, b)	
-∞ < x ≤ b, x ≤ b	(-∞, b]	
-∞ < x < b, x < b	(-∞, b)	
a ≤ x < ∞, x ≥ a	[a, ∞)	
a < x < ∞, x > a	(a, ∞)	

(ii) If $a > b$, then $b < a$.

(iii) If $a > b$, then $a - b > 0$ or $b - a < 0$.

(iv) If $a > b$, then $a + c > b + c$.

(v) If $a > b$, then $a - c > b - c$.

(vi) If $a > b$ and $c > d$, then $a + c > b + d$.

(vii) If $a > b$ and $c > d$, then $a - d > b - c$.

(viii) If $a > b$ and $m > 0$, then $ma > mb$.

(ix) If $a > b$ and $m > 0$, then $\frac{a}{m} > \frac{b}{m}$.

(x) If $a > b$ and $m > 0$, then $ma > mb$.

(xi) If $a > b$ and $m < 0$, then $\frac{a}{m} < \frac{b}{m}$.

(xii) If $0 < a < b$ and $n > 0$, then $an < bn$.

(xiii) If $0 < a < b$ and $n < 0$, then $an > bn$.

(xiv) If $0 < a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

(xv) $\sqrt{ab} \le \frac{a+b}{2}$, where $a > 0$, $b > 0$; an equality is valid only if $a = b$.

(xvi) $a + \frac{1}{a} \ge 2$, where $a > 0$; an equality takes place only at $a = 1$.

(xvii) $\sqrt[n]{a_1 a_2 \ldots . a_n} \le \frac{a_1 + a_2 + \ldots . + a_n}{n}$, where $a_1, a_2, \ldots.., a^n > 0$.

(xviii) If $ax + b > 0$ and $a > 0$, then $x > -\frac{b}{a}$.

(xix) If $ax + b > 0$ and $a > 0$, then $x < -\frac{b}{a}$.

(xx) $|a + b| \le |a| + |b|$.

(xxi) If $|x| < a$, then $-a < x < a$, where $a > 0$.

(xxii) If $|x| > a$, then $x < -a$ and $x > a$, where $a > 0$.

(xxiii) If $x^2 < a$, then $|x| < \sqrt{a}$, where $a > 0$.

(xxiv) If $x^2 > a$, then $|x| > \sqrt{a}$, where $a > 0$.

E. Arithmetic and Geometric Progression

Arithmetic Progression:

An arithmetic progression is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference e.g. The sequence 9,6,3,0,-3,.... is an arithmetic progression with –3 as the common difference. The progression –3, 0, 3, 6, 9 is an Arithmetic Progression (AP) with 3 as the common difference.

- The general form of an Arithmetic Progression is a, a + d, a + 2d, a + 3d and so on. Thus n^{th} term of an AP series is $T_n = a + (n – 1) d$. Where $T_n = n^{th}$ term and a = first term. Here d = common difference = $T_n – T_{n-1.}$
- Sometimes the last term is given and either 'd' is asked or 'a' is asked.
  Then formula becomes $l = a + (n – 1) d$
- There is another formula, applied to find the sum of first n terms of an AP: $S_n = n/2[2a+(n-1)d]$
- The sum of n terms is also equal to the formula $S_n = n/2(a + l)$ where is the last term.
- When three quantities are in AP, the middle one is called as the arithmetic mean of the other two. If a, b and c are three terms in AP then b = (a + c)/2.

Geometric Progression:

A geometric progression is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio. The sequence 4, -2, 1, – ½,.. is a Geometric Progression (GP) for which – ½ is the common ratio.

- The general form of a GP is a, ar, ar^2, ar^3 and so on.
- Thus nth term of a GP series is $T_n = ar^{n-1}$, where a = first term and r = common ratio = T_m/T_{m-1}.
- The formula applied to calculate sum of first n terms of a GP: $S_n = a(r^n-1)/r-1$ where $\rightarrow |r| > 1$ and $S_n = a(1-r^n)/1-r$ where $\Rightarrow |r| < 1$.
- When three quantities are in GP, the middle one is called as the geometric mean of the other two. If a, b and c are three quantities in GP and b is the geometric mean of a and c i.e. $b = \sqrt{ac}$
- The sum of infinite terms of a GP series $S_\infty = a/1-r$

6. Percentage & Profit Loss

A. Percentage

To Increase a Number by x %:

- If a number is increased by 10 %, then it becomes 1.1 times of itself.
- If a number is increased by 20 %, then it becomes 1.2 times of itself.
- If a number is increased by 30 %, then it becomes 1.3 times of itself.
- If a number is increased by 40 %, then it becomes 1.4 times of itself.

To Decrease a Number by x %:

- If a number is decreased by 10 %, then it becomes 0.90 times of itself.
- If a number is decreased by 20 %, then it becomes 0.80 times of itself.
- If a number is decreased by 30 %, then it becomes 0.70 times of itself.
- If a number is decreased by 40 %, then it becomes 0.60 times of itself.

Salary/Weight/Income More .

If A's income is R % more than B, then B is income is less than that of A by 100 × $\frac{R}{100+R}$ %. Given below are some of the important results in that context

Salary/Weight/Income Less .

If A's income is R% less than B, then B's income is more than that of A by 100 × $\frac{R}{100-R}$ %. Given below are some important results in that context.

- If A is $16\frac{2}{3}$ % less than B, then B is 20 % more than A.
- If A is 20 % less than B, then B is 25 % more than A.
- If A is 25 % less than B, then B is $33\frac{1}{3}$ % more than A.
- If the price is increased then consumption should be decreased by $\frac{100 \times R}{(100+R)}$ If the price is decreased, then consumption should be increased by $\frac{100 \times R}{(100-R)}$

Increase and Decrease by the same % age.

- If a number is increased by R %, then this number is decreased by R %, then in total there would be a decrease of $\frac{R^2}{100}$ %.

Increase and Decrease by different% age.

- If a number is decreased by X %, then this is again increased by Y %. Then the total increase in the no. will be $X + Y + \frac{XY}{100}$.

B. Simple Interest and Compound Interest

- Simple Interest = (P × R × T)/100, where, P is the amount invested or borrowed, T is the time in years and R is the annual interest rate.
- A = P + SI, where, A is the amount payable or receivable at the end of period T, P is the principal and SI is the simple interest.
- A = P $(1+R/100)^n$, where, A is the amount receivable on compound interest basis, P is the principal invested, R is the annual rate of interest and n is the number of years for which the sum has been invested.
- Compound Interest = A - P, where, C.I is the compound interest, A is the amount receivable with interest and P is the principal invested.

Interest rate on monthly installment plans

- R = 24 × I × 100/N(F+L), where, R is the interest, I is the installment, N is the number of installments, F is the principal left before the first installment and L is the principal left before last installment.

Equated Installment

$$X = \frac{A}{1-\left(\frac{100}{100+R}\right)^n} \times \frac{R}{100} \text{ where,}$$

X is the equated installment, A is the amount borrowed, R is the rate of interest and n is the number of years.

A = $P(1+R/200)^{2N}$, if the interest is calculated half-yearly.
A = $P(1+R/400)^{4N}$, if the interest is calculated quarterly.

Rule of 72 - Time taken to double one's investment can be obtained by dividing 72 by the interest rate. This is only in case of compound interest.

Rule of 69 - This rule is also used to determine the time taken to double one's investment. However, it is a more refined formula. Time taken = 0.35 + (69/Interest rate). This is also true in case of compound interest only.

If D is the difference in CI and SI for 2 years, and R is the rate of interest and P the principal then,
D=R/100 × R/100 × P

The following table lists the values of an initial investment, P = Re. 1 for certain time periods and rates of interest, calculated at both, simple and compound interest. This would be of great help in time management during the paper, if memorized.

Time ⇓ Rate ⇒	5%		10%		20%		30%	
	SI	CI	SI	CI	SI	CI	SI	CI
1 year	1.05	1.05	1.1	1.1	1.2	1.2	1.3	1.3
2 yrs	1.10	1.1025	1.2	1.21	1.4	1.44	1.6	1.69
3 yrs	1.15	1.1575	1.3	1.331	1.6	1.728	1.9	2.197
4 yrs			1.4	1.4641	1.8	2.0736		

C. Profit and Loss

Cost Price (CP): The price paid to buy a particular product is called its cost price. Some overhead expenses such as transportation, taxes etc. are also included in the cost price.

Selling Price (SP): The sum of money received for the product.

Marked Price (MP): The price that is listed or marked on the product. This is also known as printed price/quotation price/invoice price/catalogue price.

Profit: There is a gain in a transaction if the selling price is more than the cost price. The excess of the selling price to the cost price is called profit.

PROFIT = SELLING PRICE – COST PRICE

Loss: When the selling price is less than the cost price there is loss in the transaction. The excess of cost price over the selling price is called loss.

- Loss = Cost Price – Selling Price
- % Profit = 100 × Profit/Cost Price
- % Loss = 100 × Loss/Cost price

Equal % profit & loss on the same selling price of two articles:

If two items are sold each at Rs X, one at a gain of p % and the other at a loss of p %, then the two transactions have resulted in an overall loss of $\frac{p^2}{100}$%, and the absolute value of the loss is = Rs. $\frac{2 \cdot p^2 \cdot X}{100^2 - p^2}$

Equal % profit & loss on the same cost price of two articles:

If the cost price of two items is X, and one is sold at a profit of p % and the other at a loss of p %, then the two transactions have resulted in no gain or no loss.

Trade Discount: To attract customers, it is a common practice to announce discount on the marked price of an article.

Note: The discount is always taken as a % of the marked price, unless otherwise specified.

E.g. Let the list price of an article be Rs. 450. A discount of 5% on its list price is announced. Then, the new selling price = 450 × 95/100 = Rs 422.5.

Cash Discount: In addition to trade discount, the manufacturer may offer an additional discount called the Cash Discount if the buyer makes full payment within a certain specified time. Cash Discount is usually offered on the net price (the price after subtracting discount from the marked price).Therefore, Cash Price = Net Price - Cash Discount

Note: Cash discount is always calculated on net price, unless otherwise specified.

Wrong Weight: When a tradesman claims to sell at cost price, but uses a false weight, then the

percentage profit earned $= \dfrac{100 \times Error}{True\ Weight - Error}$

Successive Discounts: When a tradesman offers more than one discount to the customer, then the total discount offered is calculated by applying the method of decimals learned in the topic of percentage.

- **Example:** A tradesman offers two successive discounts of 20 % and 10 %. Calculate the gross discount offered by the tradesman.
  After a discount of 20%, we are left with 80% or 0.8.Further, after applying a discount of 10%, we are left with 90% or 0.9. The total discount is calculated as: 1- (0.8 × 0.9) = 0.28. Hence, there is a total discount of 28%.

- When the SP of x articles is equal to CP of y articles, what is the profit percent earned?

Profit percent = $\dfrac{100 \times \text{difference in x and y}}{\text{X}}$

7. Ratio, Mixture & Partnership

A. Ratio & Proportion

A **Ratio** is the relation between two quantities of the same kind**.** This relation indicates how many times one quantity is equal to the other. In other words, ratio is a number, which expresses one quantity as a fraction of the other.

Example: Ratio of 12 to 13 is 12/13 or 12 : 13.
The numbers forming the ratio are called terms. The numerator, i.e. "12", is known as the antecedent and the denominator, i.e. "13", in this case, is known as the consequent.

The ratio between two quantities a and b if expressed as a/b, is called fractional form and, a : b is called linear form.

If two different ratios, a : b and c : d are expressed in different units, then the two are compounded to obtain a combined ratio.
Compounding of a : b and c : d yields a × c / b × d .

- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then each of these ratios is equal to $\frac{a+c+e}{b+d+f}$
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ (Invertendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ (Alterendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo dividendo)

Continued Proportion: Four numbers a, b, c and d are said to be in proportion if a : b = c : d. If on the other hand, a : b = b : c = c : d, then the four numbers are said to be in continued proportion.

Let us consider the ratios, a : b = b : c. Here b is called the mean proportional and is equal to the square root of the product of a and c i.e. $b^2 = a \times c \Rightarrow b = \sqrt{ac}$

If a/b = b/c = c/d etc., then a, b, c, d are in geometric progression.
Let a/b = b/c = c/d = k, then, c = dk; b = ck and a = bk

Since c = dk, b = dk × k = dk^2 and a = bk = $dk^2 \times k = dk^3$, implying that they are in geometric progression.

If the three ratios, a : b, b : c, c : d are known, we can find a : d by the multiplying these three ratios a/d = a/b × b/c × c/d

If a, b, c and d are four terms and the ratios a : b, b : c, c : d are known, then one can find the ratio a : b : c : d.

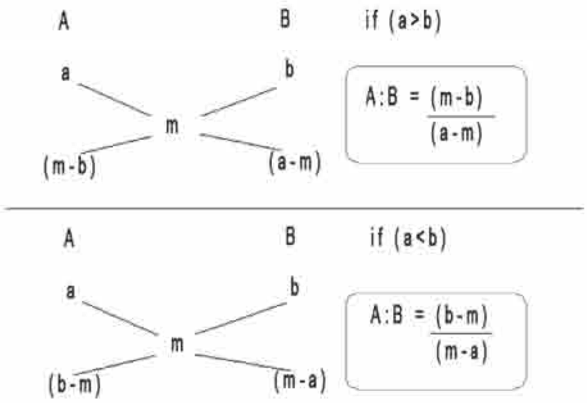
Variation - Direct, Inverse

(i) If a is directly proportional to b, then the ratio of a and b is constant.
(ii) If a is inversely proportional to b, then the product of a and b is constant.
(iii) If a α b and b α c, then a α c.
(iv) If a α b and a α c, then a α (b $\pm$ c).
(v) If a α b and c α d, then ac α bd.

B. Alligation and Mixture

In Mixture problems, different substances are combined, and characteristics of the resulting mixture have to be determined.

In solving mixture problems, we use the Alligation Rule (Alligation means 'linking').The rule states, that "When different quantities of different ingredients are mixed together to produce a mixture of a mean value, the ratio of their quantities is inversely proportional to the differences in their cost from the mean value."



One case in mixtures is, repeated dilution of a mixture with one of the ingredients, by removing, say n litres of the mixture and replacing it with n litres of one of the ingredients. Say there are m litres of water initially. We now remove n litres of the water and replace it by n litres of wine. This operation is done t times. Then,

Quantity of water left in the vessel = $m(1 - n/m)^t$
Where
m = Total quantity
n = Quantity drawn every time
t = No. of times

C. Partnership

Partnership is an association of two or more persons who invest money together in order to carry out a certain business. Partnerships are of two types:

(i) Simple Partnership: When all partners invest in the business at the same time i.e. their capital remains in the business for the same duration it is called simple partnership. In this kind of partnership, the profit is simply distributed amongst the partners, in the ratio of their respective invested capital.

(ii) Compound Partnership: When capital of the partners is invested in the business for different time periods, the partnership is known as compound partnership. In this, the profit sharing ratio is calculated by multiplying the capital invested with the unit of time (mostly months).

The sharing of profit and loss can be better understood with the help of the following illustrations:

Rule 1: In a simple partnership, the loss or profit is distributed amongst the partners in the ratio of their respective investments.
Example: Say, P and Q invested Rs. a and b for one year in a business. Then, the share of profit or loss will be,

P's profit/ loss : Q's profit/ loss = a : b

Rule 2: In a compound partnership, the profit or loss ratio is calculated as capital multiplied by the duration of investment.

Example: P's profit/loss : Q's profit/loss = $a \times t_1 : b \times t_2$
Where, t_1 = P's duration of investment and, t_2 = Q's duration of investment

8. Time and Work

- If a person can do a certain task in t hours, then in 1 hour he would do 1/t portion of the task. A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $\frac{ab}{a+b}$ hours.
- A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.
- A does a particular job in 'a' hours, B does the same job in 'b' hours and C does the same job in 'c' hours, then together they will take $\frac{abc}{ab+bc+ca}$ hours.
- If A does a particular job in 'a' hours and A & B together do the job in 't' hours, the B alone will take $\frac{at}{a-t}$ hours.
- If A does a particular job in 'a' hours, B does the same job in 'b' hours and A, B and C together do the job in 't' hours, then

  C alone can do it in $\frac{abt}{ab-at-bt}$ hours.

  A and C together can do it in $\frac{bt}{b-t}$ hours.

  B and C together can do it in $\frac{at}{a-t}$ hours
- If the objective is to fill the tank, then the Inlet pipes do positive work whereas the Outlet pipes do negative work. If the objective is to empty the tank, then the Outlet pipes do positive work whereas the Inlet Pipes do negative work.

9. Time, Speed & Distance

A. Basics of Time, Speed & Distance

The most important relationship between these three quantities, and possibly the only one which needs to be known is,

Distance = Speed × Time
Speed = Distance traveled/Time
Time = Distance traveled/Speed

- Average speed, if equal distances are covered at a km/hr and b km/hr is 2ab/a+b
- If two bodies are moving in the same direction at a speed of a and b respectively, then their relative speed is the difference of the two speeds.
- If two bodies are moving in the opposite directions at a speed of a and b respectively, then their relative speed is a + b.
- Two objects A and B moving along a circular path in the same direction, having started simultaneously and from the same point traveling at speeds of a and b, will meet again when the faster object has gained one full circle over the slower object, i.e. when the relative speed |a-b| completes one full round. The two objects will again meet at the starting point at a time, which is the LCM of the time taken for each of the objects individually to complete one round.
- If the length of a train is L meters and the speed of the train is S m/s, then the time taken by the train to pass a stationary man/pole is L/S sec.
- If the length of the train is L_1 and its speed is S m/s and the length of a platform (stationary object of comparable length) is L_2, then the time taken by the train to cross the platform is $(L_1 + L_2)/S$ sec.
- If the lengths of 2 trains are L_1 (faster) and L_2 (slower) m, and their speeds are S_1 and S_2 m/s resp., then the time taken by the faster train to overtake the slower train is $L_1+L_2/\ S_1+S_2$ sec, and the time taken for the trains to cross each other is $L_1+L_2/\ S_1+S_2$ sec.
- If the average speed of a train, without stoppages, is S_1 km/hr and the speed with stoppages is S_2, then Stoppage time (in min/hr) = $S_1\text{-}S_2/S_1 \times 60$.

B. Boats and Stream:

If the speed of the boat in still water is say B kmph and if the speed at which the stream is flowing is W kmph,

(i) When the boat is traveling with the stream the speed of the boat = (B + W) kmph
(ii) When the boat is traveling against the stream the speed of the boat = (B – W) kmph.
(iii) If the upstream is denoted as U and downstream is denoted as D then
(iv) B = (D+U) / 2, W= (D – U) / 2

Important Distance and Time Conversions:

1 km = 1000 meter	1 meter = 100 cm
1 hour = 60 min	1 min = 60 sec
1 hour = 3600 sec	1 km/hr = $(1 \times 1000) / (1 \times 3600)$ = 5/18 m/sec.

C. Races

If A beats B by x metres or s seconds, then the speed of B is x/s metres/sec.

If the length of a circular track is L m, and if A and B take x and y sec. respectively, to complete one round, then both of them will meet at the starting point after LCM (x, y) sec.

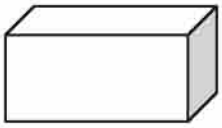
If the length of a circular track is L m, and if the speeds of A and B are x m/sec. and y m/sec respectively, then the time after which both of them will meet at a point other than the starting point is L/x-y sec, if they are running in the same direction and L/x+y, if running in the opposite direction.

10. Geometry & Mensuration

A. Rectangular Solids and Cylinders

Cuboid: A cuboid is a three-dimensional figure formed by six rectangular surfaces, as shown below. Each rectangular surface is a face. Each solid line segment is an edge, and each point at which the edges meet is a vertex. A rectangular solid has six faces, twelve edges, and eight vertices. Edges mean sides and vertices mean corners. Opposite faces are parallel rectangles that have the same dimensions.

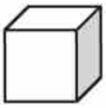
The surface area of a rectangular solid is equal to the sum of the areas of all the faces. The volume is equal to (length) × (width) × (height); in other words, (area of base) × (height).



Body diagonal of a cuboid = Length of the longest rod that can be kept inside a rectangular room is = $\sqrt{L^2 + B^2 + H^2}$.

Cube: A rectangular solid in which all edges are of equal length is a cube. In a cube, just like cuboid, there are six faces, eight vertices & twelve edges.

Volume = a^3.



Surface Area = $6a^2$, where a is the side of a cube.

Body Diagonal = Length of the longest rod inside a cubical room = $a\sqrt{3}$

Right Prism:

A prism is a solid, whose vertical faces are rectangular and whose bases are parallel polygons of equal area.

A prism is said to be triangular prism, pentagonal prism, hexagonal prism, octagonal prism according to the number of sides of the polygon that form the base.

In a prism with a base of n sides, number of vertices = 2n, number of faces = n + 2.

Curved Surface area of vertical faces of a prism = perimeter of base × height.

Total surface area of a prism = perimeter of base × height + 2 × area of base

Volume of a prism = area of base × height

Cylinder: Total Surface area of a right circular cylinder with a base of radius 'r' and height 'h' is equal to $2(\pi r^2) + 2\pi rh$ (the sum of the areas of the two bases plus the area of the curved surface).

The volume of a cylinder is equal to $\pi r^2 h$, that is (area of base) × (height).

Cone: A cone is having one circle on one of its ending & rest is the curved circle part with a corner on the other end.
Volume = 1/3 $\pi r^2 h$. Surface Area (curved) = π r l, where l = slant height.
As per the Pythagoras theorem, $l^2 = r^2 + h^2$.
Surface Area (total) = π r l + πr^2.

Frustum of a cone: A frustum is the lower part of a cone, containing the base, when it is cut by a plane parallel to the base of the cone.

Slant height, L = $\sqrt{h^2 + (R - r)^2}$

Curved Surface area of cone = π (R + r) L.

Total surface area of frustum = Base area + Area of upper circle + Area of lateral surface = π ($R^2 + r^2$ + RL + rL).

Volume of frustum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

Sphere: The set of all points in space, which are at a fixed distance from a fixed point, is called a sphere. The fixed point is the centre of the sphere and the fixed distance is the radius of the sphere.
Volume = $4/3\pi r^3$. Surface Area (curved and total) = 4 πr^2.

Hemisphere: A sphere cut by a plane passing through its centre forms two hemispheres. The upper surface of a hemisphere is a circular region.
Volume = $2/3\pi r^3$. Surface Area (curved) =$2\pi r^2$.
Surface Area (Total) = $2\pi r^2 + \pi r^2 \Rightarrow 3\pi r^2$.

Spherical shell: If R and r are the outer and inner radius of a hollow sphere, then volume of material in a spherical shell = $4/3\pi$ ($R^3 - r^3$).

Pyramid:

A pyramid is a solid, whose lateral faces are triangular with a common vertex and whose base is a polygon. A pyramid is said to be tetrahedron (triangular base), square pyramid, hexagonal pyramid etc, according to the number of sides of the polygon that form the base.

In a pyramid with a base of n sides, number of vertices = n + 1. Number of faces including the base = n + 1.
Surface area of lateral faces
= ½ × perimeter of base × slant height
Total surface area of pyramid
= Base area + ½ × perimeter of base × slant height
Volume of pyramid = 1/3 × Base area × height. A cone is also a pyramid.

B. Lines and Angles

Some important points:

(i) There is one and only one line passing through two distinct points.

(ii) Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non-collinear.

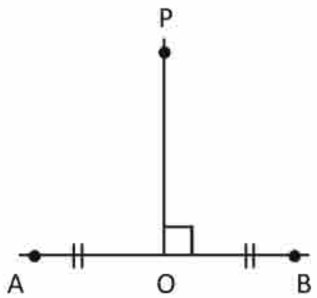
(iii) Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.

(iv) A line, which is perpendicular to a line segment i.e., intersects at 90° and passes through the midpoint of the segment is called he perpendicular bisector of the segment.

(v) Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.
Conversely, if any point is equidistant from the two endpoints of the segment, then it must lie on the perpendicular bisector of the segment.

If PO is the perpendicular bisector of segment AB, then, AP = PB.

Also, if AP = PB, then P lies on the perpendicular bisector of segment AB.

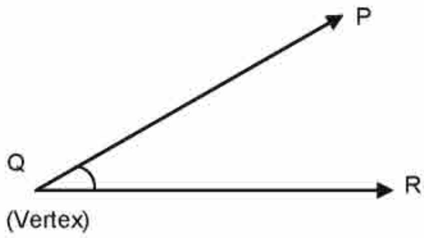


(vi) The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

Angles:

When two rays have the same starting or end points, they form an angle and the common end point is called vertex.

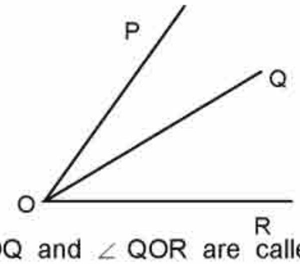
Angle is also defined as the measure of rotation of a ray. For the purpose of Trigonometry, the measure of rotation is termed positive if it is in the anticlockwise direction.



In the figure, ray PQ and QR from angle∠PQR

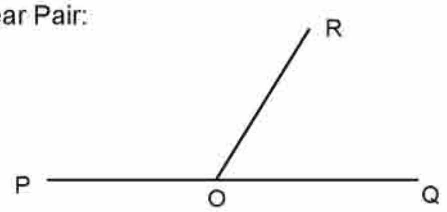
Types of Angles:

(i) Adjacent Angles:



∠ POQ and ∠ QOR are called adjacent angles, because they have a common side and their interiors are disjoint.

(ii) Linear Pair:



∠ POR and ∠ ROQ are said to form a linear pair because they have a common side and other two sides are opposite rays ∠ POR+ ∠ ROQ =180^0

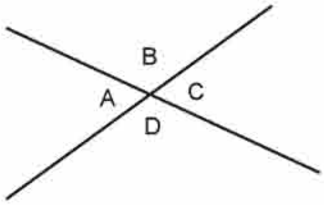
∠ POR and ∠ POQ form a linear pair

(iii) An angle greater than 180°, but less than 360° is called a reflex angle.

(iv) Two angles whose sum is 90° are called complementary angles.

(v) Two angles having a sum of 180° are called supplementary angles.

(vi) When two lines intersect, two pairs of vertically opposite angles are equal. The sum of 2 adjacent angles is 180°.



As given in the above diagram ∠A = ∠C & ∠B = ∠D.
Secondly ∠A + ∠B =180° & ∠C + ∠D = 180°.

Two lines are parallel to each other if

- They are parallel to a 3^{rd} line.
- They are opposite sides of a rectangle/ square/ rhombus/ parallelogram.
- If they are perpendicular to a 3^{rd} line.
- If one of them is a side of the triangle & other joins the midpoints of the remaining two sides.
- If one of them is a side of a triangle & other divides other 2 sides proportionately.

Two lines are perpendicular to each other if

- They are arms of a right-angle triangle.
- If the adjacent angles formed by them are equal and supplementary.
- They are adjacent sides of a rectangle or a square.
- If they are diagonals of a rhombus.
- If one of them is a tangent & other is radius of the circle through the point of contact.
- If the sum of their squares is equal to the square of line joining their ends.

Two angles are said to be equal if

- They are vertically opposite angles.
- Their arms are parallel to each other.
- They are the corresponding angles of two congruent triangles.
- They are the opposite angles of a parallelogram.
- They are the angles of an equilateral triangle.
- They are the angles of a regular polygon.
- They are in same segment of a circle.
- One of them lies between a tangent & a chord thorough the point of contact & other is in the alternate segment, in a circle.

Two sides are equal to each other if

- They are corresponding sides of two congruent triangles.
- They are sides of an equilateral triangle.
- They are opposite sides of a parallelogram.
- They are the sides of a regular polygon.
- They are radii of the same circle.
- They are chords equidistant from centre of circle.
- They are tangents to a circle from an external point.

C. Triangles

Types of Triangles

With regard to their sides, triangles are of three types:

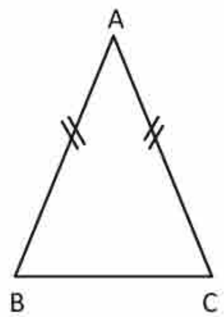
(i) Scalene Triangle: A triangle in which none of the three sides is equal is called a scalene triangle.

(ii) Isosceles Triangle: A triangle in which at least two sides are equal is called an isosceles triangle. In an isosceles triangle, the angles opposite to the congruent sides are congruent.

Conversely, if two angles of a triangle are congruent, then the sides opposite to them are congruent.

In Δ ABC, AB = AC,

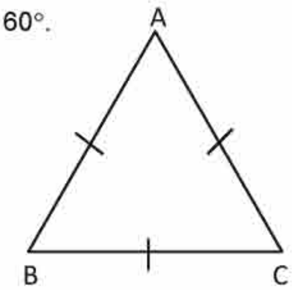
$\angle$ ABC = $\angle$ ACB



(iii) Equilateral Triangle: A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

In Δ ABC, AB = BC = AC.

$\angle$ ABC = $\angle$ BCA = $\angle$ CAB = 60°



With regard to their angles, triangles are of five types:

(i) Acute triangle: If all the three angles of a triangle are acute i.e., less than 90°, then the triangle is an acute-angled triangle.

(ii) Obtuse triangle: If any one angle of a triangle is obtuse i.e., greater than 90°, then the triangle is an obtuse-angled triangle. The other two angles of the obtuse triangle will be acute.

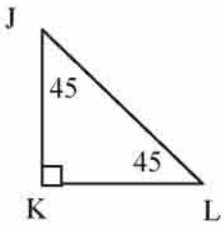
(iii) Right Triangle: A triangle that has a right angle is a right triangle. In a right triangle, the side opposite the right angle is the hypotenuse, and the other two sides are the legs. An important theorem concerning right triangles is the Pythagorean theorem, which states: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

There are some standard Pythagorean triplets, which are repeatedly used in the questions. It is better to remember these triplets by heart.

♠ 3, 4, 5 ♠ 5, 12, 13 ♠ 7, 24, 25 ♠ 8, 15, 17
♠ 9, 40, 41 ♠ 11, 60, 61 ♠ 12, 35, 37 ♠ 16, 63, 65
♠ 20, 21, 29 ♠ 28, 45, 53.

Any multiple of these triplets will also be a triplet i.e. when we say 3, 4, 5 is a triplet, if we multiply all the numbers by 2, it will also be a triplet i.e. 6, 8, 10 will also be a triplet.

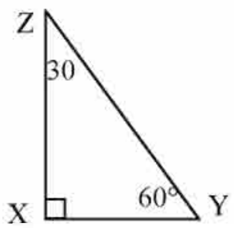
(iv) 45°- 45° - 90° Triangle: If the angles of a triangle are 45°, 45° and 90°, then the perpendicular sides are $1/\sqrt{2}$ times the hypotenuse. In a 45°- 45°- 90° triangle, the lengths of the sides are in the ratio 1 : 1 : √2. For example, in ΔJKL, if JL = 2, then JK = √2 and KL = √2.



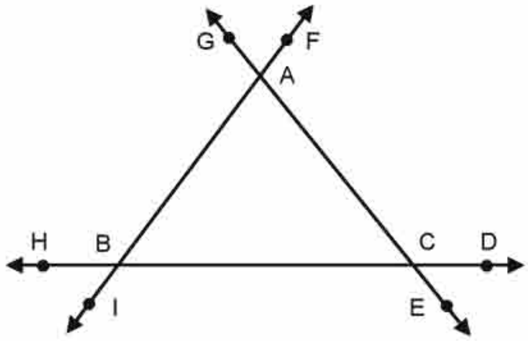
(v) 30°- 60° - 90° Triangle: In 30°- 60° - 90° triangle, the lengths of the sides are in the ratio 1: √3 : 2. For example, in ΔXYZ, if XZ = 3, then XY = 3√3 and YZ = 6. In short, the following formulas can be applied to calculate the two sides of a 30°- 60°-90° triangle, when the third side is given.

Side opposite to 30° = ½ of hypotenuse.

Side opposite to 60° = √3/2 of hypotenuse.



Some important properties of triangles:



(i) In Δ ABC, $\angle$ ABC + $\angle$ BAC + $\angle$ ACB = 180°

(ii) The sum of an interior angle and the adjacent exterior angle is 180°.
In figure, $\angle$ABC + $\angle$ABH = 180°
$\angle$ABC + $\angle$CBI = 180°

(iii) Two exterior angles having the same vertex are congruent.
In figure, $\angle GAB \cong \angle FAC$

(iv) The measure of an exterior angle is equal to the sum of the measures of the two interior angles (called remote interior angles) of the triangle, not adjacent to it.

(v) The sum of any two sides of a triangle is always greater than the third side.
In Δ ABC, AB + BC > AC, also AB + AC > BC and AC + BC > AB.

(vi) The difference of any two sides is always less than the third side.

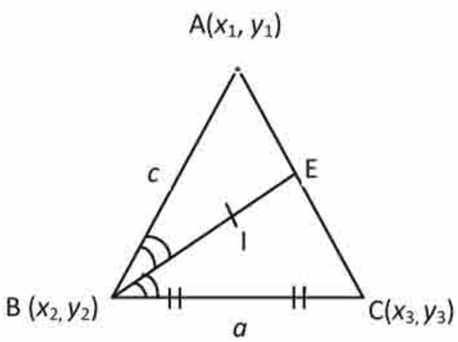
Area of a triangle

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)} = r \times s = \dfrac{abc}{4R}$

where, a, b and c are the sides of the triangle,

s = semi perimeter, r = in-radius, R = circum-radius.

In-centre: Point of intersection of angles bisectors of a triangle is known as the in-centre of triangle. The circle drawn from this point, which touches all the three sides of the triangle, is known as in-circle & its radius is called as in-radius. The in-radius is denoted by the letter 'r'



(i) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the co-ordinates of a triangle and a, b, c are the lengths of sides BC, AC and AB respectively then co-ordinates of the in-centre are:

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

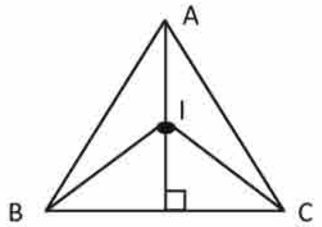
(ii) In centre is equidistant for all the three sides, in-circle can be drawn touching all the three sides.

The radius of this 'circle' is termed as 'in-radius'

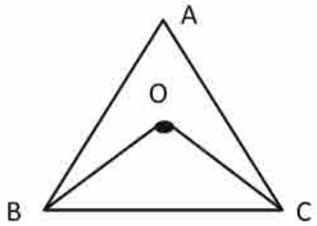
i.e. $r = \dfrac{\Delta}{s} = \dfrac{\text{Area of the triangle}}{\text{Semiperimeter of the triangle}}$

(iii) $\dfrac{AB}{BC}=\dfrac{AE}{EC}$ (Angle Bisector Theorem)

(iv) If I is the in-centre, then $\angle BIC = 90^\circ + \dfrac{1}{2}\angle A$



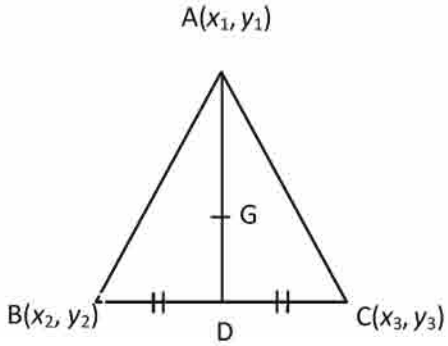
Circumcentre: It is the point of intersection of perpendicular side bisectors of the triangle. The circle drawn with this point as centre and passing through the vertices is known as circumcircle and its radius is called as circumradius. The circumradius is denoted by the letter 'R'.



Circum radius = $\dfrac{abc}{4\Delta}$ R = $\dfrac{\text{Product of three sides}}{4\times\text{Area of the triangle}}$

$\angle BOC = 2\angle A$

Centroid: The segment joining a vertex & midpoint of the opposite side is called median of a triangle. There are three medians & they meet in a single point called, centroid of the triangle. The centroid divides medians in the ratio of 2 : 1.
The following formula is applied to calculate the length of the median. The sum of the squares of two sides = 2[median^2 + (½ 3rd side)^2]

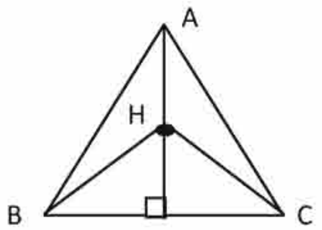


(i) If A(x_1, y_1); B(x_2, y_2); C(x_3, y_3) are the co-ordinates of a triangle , then co-ordinates of centroid G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

(ii) The median divides the triangle in two equal parts (not necessarily congruent)

(iii) The centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. So AG : GD = 2 : 1.

Orthocentre: The line drawn from any vertex, perpendicular to the opposite side is called the altitude/height. The three altitudes meet in a single point called the orthocentre. The angle made by any side at orthocentre is equal to (180° – vertical angle).



- If Δ ABC, is an isosceles triangle with AB $\cong$ AC, then the angle bisector of $\angle$ BAC is the perpendicular bisector of the base BC and is also the median to the base.

  Area of an isosceles triangle $= \frac{c}{4}\sqrt{4a^2 - c^2}$, where c is the unequal side and a is one of the equal sides.

  The altitudes on the congruent sides are equal i.e., BE = CF.

- For an equilateral triangle,

  height $= \frac{\sqrt{3}}{2} \times \text{side};$

  $\text{area} = \frac{\sqrt{3}}{4} \times (\text{side})^2$, $\text{inradius} = \frac{1}{3} \times \text{height};$

  $\text{circumradius} = \frac{2}{3} \times \text{height}$, perimeter = 3 × sides

  Also, the altitude, median, angle bisector, perpendicular bisector of each base is the same and the ortho-centre, centroid, in-centre and circum-centre is the same.

Congruency of triangles: If the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle, then the two triangles are said to be congruent.

Two triangles are congruent if

- Two sides & the included angle of a triangle are respectively equal to two sides & included angle of other triangle (SAS).
- 2 angles & 1 side of a triangle are respectively equal to two angles & the corresponding side of the other triangle (AAS).
- Three sides of a triangle are respectively congruent to three sides of the other triangle (SSS).
- 1 side & hypotenuse of a right-triangle are respectively congruent to 1 side & hypotenuse of other rt. triangle (RHS).

Similarity of triangles:

- Two triangles are similar if they alike in shape only. The corresponding angles are congruent, but corresponding sides are only proportional. All congruent triangles are similar but all similar triangles are not necessarily congruent.

Two triangles are similar if

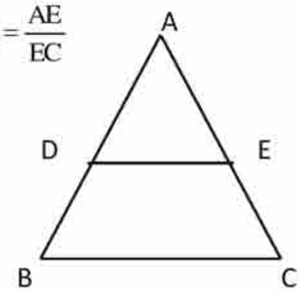
- Three sides of a triangle are proportional to the three sides of the other triangle (SSS).
- Two angles of a triangle are respectively equal to the two angles of the other triangle (AA).
- Two sides of a triangle are proportional to two sides of the other triangle & the included angles are equal (SAS).

Properties of similar triangles:

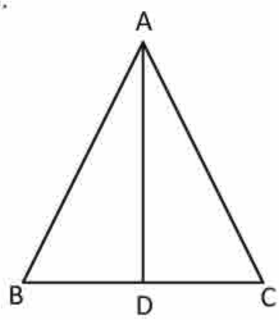
- If two triangles are similar, ratios of sides = ratio of heights = ratio of medians = ratio of angle bisectors = ratio of inradii = ratio of circumradii.
- Ratio of areas = $b_1h_1/b_2h_2 = (s_1)^2/(s_2)^2$, where b_1 & h_1 are the base & height of first triangle and b_2 & h_2 are the base & height of second triangle. s_1 & s_2 are the corresponding sides of first and second triangle respectively.

Some important theorems:

(i) **Basic Proportionality Theorem:** If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points, then the other sides are divided in the same ratio by it. If DE is parallel to BC, then, $\frac{AD}{DB} = \frac{AE}{EC}$



(ii) **The Angle Bisector Theorem:** The angle bisector divides the opposite side in ratio of the lengths of its adjacent arms.

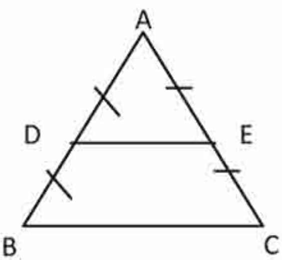


If AD is the angle bisector, then AB/AC = BD/DC.

(iii) **Midpoint Theorem:** The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.

If AD = DB, AE = EC, then DE is parallel to BC and

DE = $\frac{1}{2}$BC .



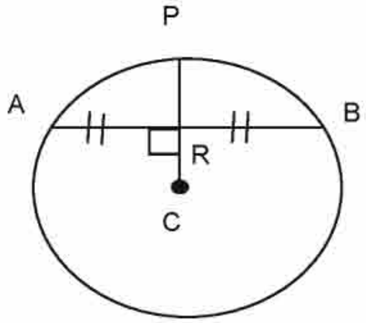
D. Circles

If r is the radius of the circle, then the circumference = $2\pi r$,
The area of a circle of radius r is = πr^2.

Congruent Circles: Circles with equal radii are called congruent circles.
Concentric Circles: Circles lying in the same plane with a common centre are called concentric circles.

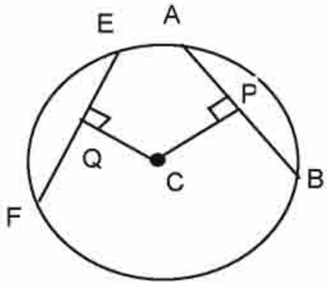
Some Important Properties of Circles

(i) The perpendicular from the centre of a circle to a chord of the circle bisects the chord.

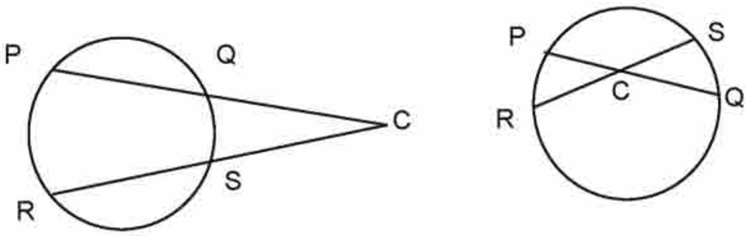


Conversely, the line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

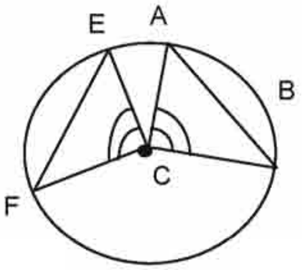
(ii) Equal chords of a circle or congruent circles are equidistant from the centre.



Conversely, two chords of a circle or congruent circles that are equidistant from the centre are equal.
Two chords PQ, RS intersect at a point then
CP × CQ = CR× CS

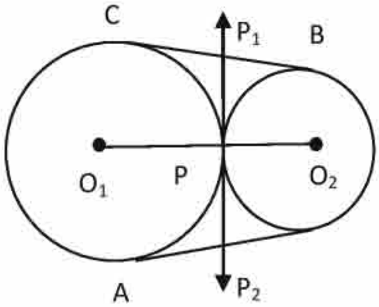


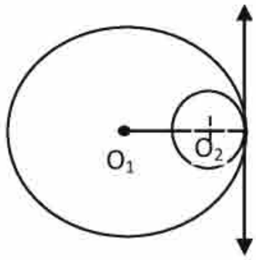
(iii) In a circle or congruent circles, equal chords subtend equal angles at the centre.



Conversely, chords, which subtend equal angles at the centre of the same or congruent circles, are equal.

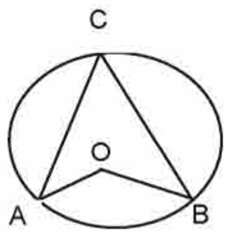
(iv) If the two circles touch each other externally, distance between their centres = sum of their radii.



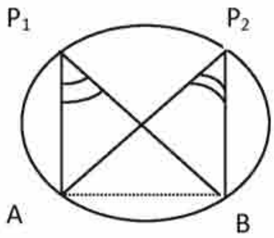


If the two circles touch each other internally, distance between their centres = difference of their radii.

(v) The measure of an inscribed angle is half the measure of its intercepted arc. (Inscribed angle theorem). $\angle ACB = \frac{1}{2} \angle AOB$

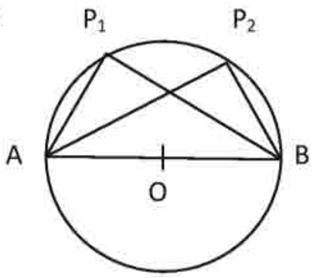


(vi) Angles subtended is the same segment are equal



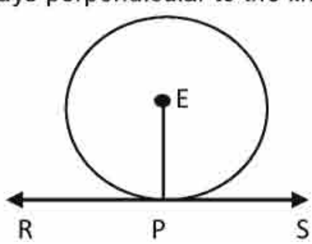
$\angle AP_1B = \angle AP_2B$.

(vii) Angle subtended in a semicircle is 90^0

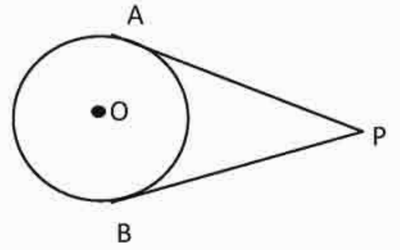


i.e. $\angle AP_1B = 90^0 = \angle AP_2B$

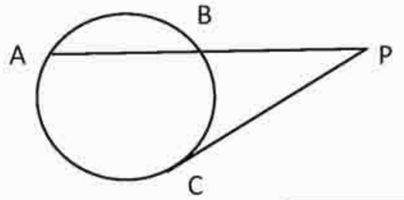
(viii) Tangent is always perpendicular to the line joining the centre and the point of tangency.



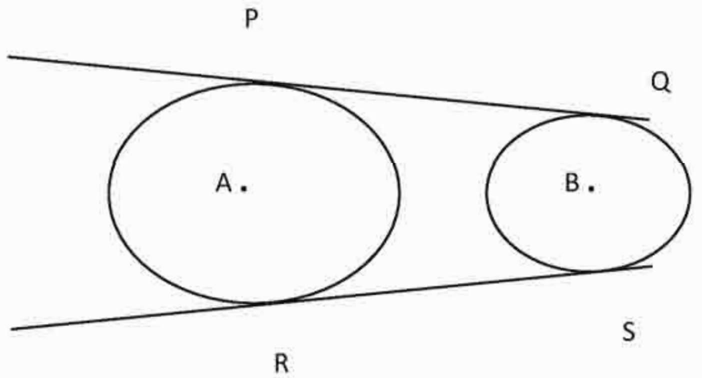
(ix) Two tangents from the same external point are equal in length. PA = PB



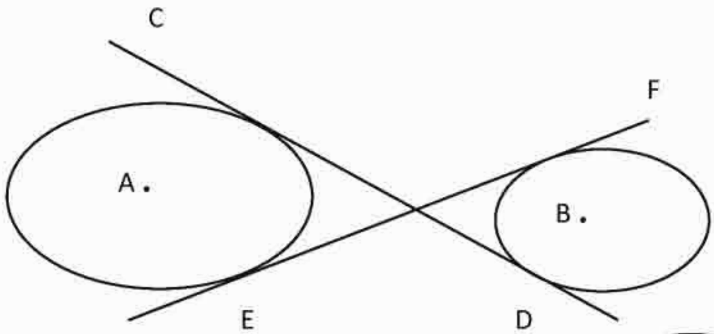
(x) If AB is any chord of a circle and PC is the tangent (both for a external point P) than PA × PB = PC^2



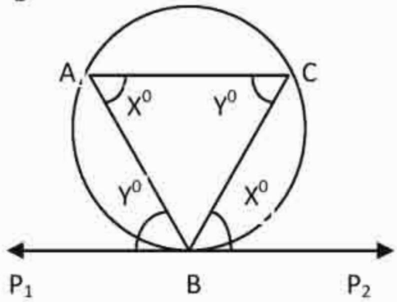
(xi) Length of direct common tangent = $\sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$



(xii) Length of transverse common tangent = $\sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$



(xiii) Alternate Segment Theorem:

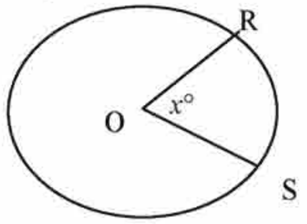


Angle between any chord (at the point of tangency) and the tangent is equal to the angle subtended by the chord to any point on the other side of the segment (alternate segment).

In the fig $\angle CBP_2 = \angle CAB = X^0$ & $\angle P_1BA = \angle ACB = Y^0$

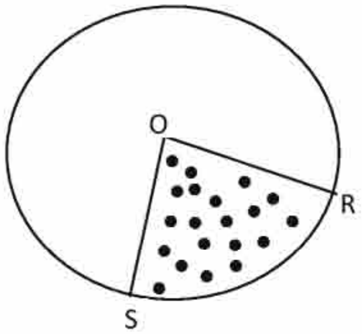
Sectors of a Circle

The number of degrees of arc in a circle (or the number of degrees in a complete revolution) is 360.

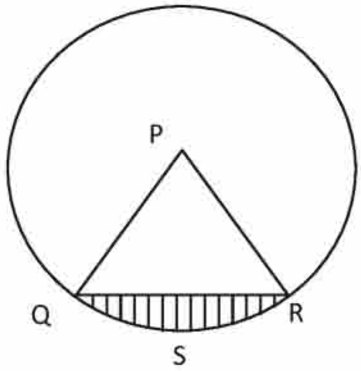


In the circle with center O above, the length of arc RS is x/360 of the circumference of the circle; for example, if x = 60°, then arc RS has length 1/6 of the circumference of the circle. We can remember the following formulas:

- Length of arc RS = $2\pi r \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to x/360.



- Area of Sector ORS = $\pi r^2 \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to x/360.



- Area of the segment of a circle (QSR) = $r^2\left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right]$

Perimeter of the segment = $2\pi\ r\left(\frac{\theta}{360°}\right) + 2r\sin\frac{\theta}{2}$

E. Quadrilaterals & Polygons

Quadrilaterals

(i) Of all the quadrilaterals of the same perimeter, the one with the maximum area is the square.

(ii) The quadrilateral formed by joining the midpoints of the sides of any quadrilateral is always a parallelogram (rhombus in case of a rectangle, rectangle in case of a rhombus and square in case of a square).

(iii) The quadrilateral formed by the angle bisectors of the angles of a parallelogram is a rectangle.

(iv) For a rhombus □ABCD, if the diagonals are AC and BD, then $AC^2 + BD^2 = 4 \times AB^2$.

(v) If a square is formed by joining the midpoints of a square, then the side of the smaller square = side of the bigger square/√2.

(vi) If P is a point inside a rectangle □ABCD, then $AP^2 + CP^2 = BP^2 + DP^2$.

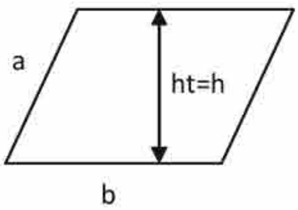
(vii) The segment joining the midpoints of the non-parallel sides of a trapezium is parallel to the two parallel sides and is half the sum of the parallel sides.

(viii) For a trapezium □ABCD, if the diagonals are AC and BD, and AB and CD are the parallel sides, then $AC^2 + BD^2 = AD^2 + BC^2 + 2 \times AB \times BD$.

(ix) If the length of the sides of a cyclic quadrilaterals area, b, c and d, then it's area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is the semi-perimeter.

(x) The opposite angles of a cyclic quadrilateral are supplementary.

Quadrilateral Areas



(i) Square = a^2

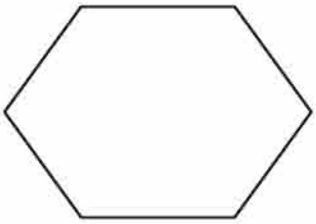
(ii) Rectangle = a × b, where a & b are the length and breadth of the rectangle.

(iii) Parallelogram = b × h, where b is the base and h is the height of the parallelogram.

(iv) For Rhombus = ½ × d_1 × d_2, where d_1 and d_2 are the diagonals of the rhombus.

(v) For Trapezium with a and b parallel sides and height h, Area = $\frac{1}{2}(a+b)h$

Polygons



(i) The sides a_n of regular inscribed polygons, where R is the radius of the circumscribed circle

$= a_n = 2R \sin\frac{180^\circ}{n}$

(ii) Area of a polygon of perimeter P and radius of in-circle r = ½ × p × r

(iii) The sum of the interior angles of a convex POLYGON, having n sides is 180° (n - 2).

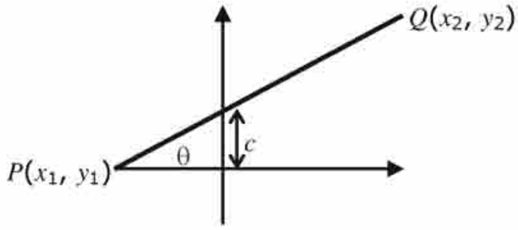
(iv) The sum of the exterior angles of a convex polygon, taken one at each vertex, is 360°.

(v) The measure of an exterior angle of a regular n- sided polygon is $\frac{360^0}{n}$.

(vi) The measure of the interior angle of a regular n-sided polygon is $\frac{(n-2)180^\circ}{n}$.

(vii) The number of diagonals of in an n-sided polygon is $\frac{n(n-3)}{2}$.

11. Co-ordinate Geometry



(i) Distance PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(ii) Slope of PQ = m = y_2 - y_1/x_2 - x_1 = tan θ

(iii) Equation of PQ is y - y_1/y_1 - y_2 = x - x_1/ x_1 - x_2 or y = mx + c

(iv) The product of the slopes of two perpendicular lines is –1

(v) The distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$.

(vi) If point P(x, y) divides the segment AB, where

(vii) A ≡ (x_1, y_1) and B ≡ (x_2, y_2), internally in the ratio m : n, then,

$$x = \frac{mx_2 + nx_1}{m + n} \text{ and } y = \frac{my_2 + ny_1}{m + n}.$$

(viii) A ≡ (x_1, y_1) and B ≡ (x_2, y_2), externally in the ratio m : n, then,

$$x = \frac{mx_2 - nx_1}{m - n} \text{ and } y = \frac{my_2 - ny_1}{m - n}.$$

(ix) If P is the midpoint, then x = x_1 + x_2 / 2 and y = y_1 + y_2 / 2.

(x) If G (x, y) is the centroid of triangle ABC, A ≡ (x_1, y_1), B ≡ (x_2, y_2), C ≡ (x_3, y_3), then x = x_1 + x_2 + x_3 / 3 and y = y_1 + y_2 + y_3 / 3.

(xi) If I (x, y) is the in-centre of triangle ABC,

(xii) A ≡ (x_1,y_1), B ≡ (x_2, y_2), C ≡ (x_3,y_3), then, x = ax_1 + bx_2 + cx_3 / a+b+c and y = ay_1 + by_2 + cy_3 / a+b+c where a, b and c are the lengths of BC, AC and AB respectively.

(xiii) The equation of a straight line is y = mx + c, where m is the slope and c is the y-intercept (tan θ = m, where θ is the angle that the line makes with the positive X-axis)

12. Set Theory

The set which does not contain any element is known as an empty set.

If every point of a set A belongs to B, then A is contained or included in B and is a subset of B, while B is known as the superset of A. This is represented as $A \subset B$ or $B \supset A$.

Two sets are identical if they contain exactly the same points, and is then denoted as A = B.

Another way to represent this is: A = B if and only if $A \subset B$ and $B \subset A$.

Two sets are said to be equivalent if they contain the same number of elements.

The set A – B is the set of all those elements that belong to A, but not B and is called the Difference set.

The set $A \Delta B$ is the set of all those elements that belong to either A or B, but not both. It is called the symmetric difference set.

$A \Delta B = (A \cup B) - (A \cap B)$.

$A \Delta B = (A - B) \cup (B - A)$.

Empty set is a subset of every set.

Every set is a subset of itself.

The set of all subsets of a set is called the power set. It contains 2^n elements, if the original set contains n elements.

In $A \cup B$, the max value of the intersection $A \cap B$ is the min of n(A) and n(B).

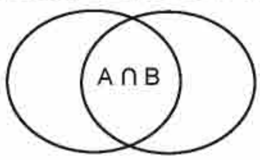
In $A \cup B \cup C$, the maximum value of the intersection $A \cap B \cap C$ is the minimum of the intersections $A \cap B$, $B \cap C$ and $A \cap C$.

De Morgan's laws –

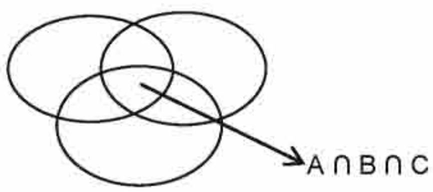
1. $(A \cap B)^C = A^C \cup B^C$.
2. $(A \cup B)^C = A^C \cap B^C$.

Operations with Venn Diagrams

- The union $A \cup B$ of two sets A and B is the set of points, which belong to at least one of them.
- The intersection of sets $A \cap B$ of two sets A and B is the set of points which belong to both of them
- The Union of two sets, $A \cup B = A + B - A \cap B$



The Union of three sets $A \cup B \cup C = A + B + C - (A \cap B + B \cap C + C \cap A) + (A \cap B \cap C)$



It is very important here to understand the meaning of certain terms.

- **At least 1:** means min. 1 i.e. 1 or more than 1.
- **At least 2:** means min. 2 i.e. 2 or more than 2.
- **At the most 2:** means maximum 2 i.e. 2 or less than 2.
- **At the most 3:** means maximum 3 i.e. 3 or less than 3.

13. Logarithms

(i) Definition: $a^x = b$ can be represented in logarithmic form as $\log_a b = x$

(ii) $\log a = x$ means that $10^x = a$.

(iii) $10^{\log a} = a$ (The basic logarithmic identity).

(iv) $\log (ab) = \log a + \log b$, $a > 0$, $b > 0$

(v) $\log \frac{a}{b} = \log a - \log b$, $a > 0$, $b > 0$.

(vi) $\log a^n = n (\log a)$ (Logarithm of a power).

(vii) $\log_x y = \frac{\log_m y}{\log_m x}$ (Change of base rule).

(viii) $\log_x y = \frac{1}{\log_y x}$.

(ix) $\log_x 1 = 0$ ($x \neq 0, 1$).

(x) The natural numbers 1, 2, 3,… are respectively the logarithms of 10, 100, 1000, … to the base 10.

(xi) The logarithm of "0" and negative numbers are not defined.

The logarithm of a number to the base "10" is known as common logarithm and the logarithm of a number to the base "e" is known as natural logarithm.

Characteristic and Mantissa of Common Logarithms:

The integral part of the common logarithm of a number $x > 0$ is called the **Characteristic** and the fractional part is called the **mantissa**.

e.g. the logarithm of 2 to the base 10 is 0.3010, where 0 is the characteristic and 3010 is the mantissa.

Any positive number "x" can be written in the form $x = a10^n$, where $1 < a < 10$ and n is an integer. The number n is called the order of the number x. e.g. 30 can be written as $3*10^1$ and similarly 300 can be written $3*10^2$. The same rule applies to fractions as well where the value of n will be negative.

The characteristic of the logarithm of the given number "x" will be "n" and the mantissa will be the logarithm of "a". Therefore, while log 2 = 0.3010, log 20 will be 1.3010 as "n" in this case is "1".

- Thus, the value of the characteristic of the logarithm of a number will help determine the number of integral digits the number has = characteristic + 1.

14. Permutation and Combination

Factorial, n = n (n – 1) (n – 2) 1

Eg. 5! = 5 × 4 × 3 × 2 × 1

Fundamental Principle of Counting

Multiplication: If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways, then the two jobs in succession can be completed in m × n ways.

Addition: If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in (m + n) ways.

Note: The above principles of counting can be extended to any finite number of jobs.

Permutation of n things taken r at a time, $^nP_r = \frac{n!}{(n-r)!}$ (Includes arrangement)

Combination of n things taken r at a time, $^nC_r = \frac{n!}{r!(n-r)!}$ (Includes only selection)

$^nP_r = {}^nC_r \times r!$

$^nC_r = {}^nC_{n-r}$

The total number of combinations of n distinct things, taken none or some or all at a time = 2^n

The total number of combinations of n things, r taken at a time, where p things always occur = $^{n-p}C_{r-p}$.

The total number of combinations of n things, r taken at a time, where p things will never occur = $^{n-p}C_r$.

The number of ways of dividing n things into various groups, each having p, q, r items = $\frac{n!}{p! \times q! \times r!}$

Permutation of objects not all distinct: The number of mutually distinguishable permutations of 'n' things, taken all at a time, of which p are alike of one kind, q are alike of second such that p + q = n is $\frac{n!}{p!q!}$

If n outcomes can be repeated on r different things, total no. of permutations = r^n

Circular permutation of n things = (n – 1)!.

A deck of cards has 52 cards. There are four suits and each suit has 13 cards.

The total number of possible outcomes from a single throw of a perfect dice is 6.

The possible outcomes, of a single toss, of a fair coin are 2 – H, T.

Permutation of n things taken r at a time, in which one particular thing always occurs, is $r \times {}^{n-1}P_{r-1}$.

Circular permutation of n things, if there is no difference between the clockwise and anti-clockwise arrangement, is $\frac{(n-1)!}{2}$.

$^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$.

${}^{n}C_{r} \times {}^{r}C_{k} = {}^{n}C_{k} \times {}^{n-k}C_{r-k}.$

${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots + {}^{n}C_{n} = 2^{n}.$

${}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + \ldots = {}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{5} + \ldots = 2^{n-1}.$

Division of items into groups of unequal sizes: Number of ways in which (m + n) items can be divided into two unequal groups containing 'm' and 'n' items is $\frac{(m+n)!}{m!n!}$.

Note: The number of ways in which (m + n) items are divided into two groups containing 'm' and 'n' items is same as the number of combinations of (m + n) things. Thus the required number = ${}^{m+n}C_m$ = $\frac{(m+n)!}{m!n!}$.

Note: The number of ways of dividing (m + n + p) items among 3 groups of size m, n and p respectively is = (Number of ways to divide) = $\frac{(m+n+p)!}{m!n!p!}$

Note: The number of ways in which mn different items can be divided equally into m groups each containing n objects and the order of group is important is $\left\{\frac{(mn)!}{(n!)^m} \times \frac{1}{m!}\right\} m! = \frac{(mn)!}{(n!)^m}$.

Note: The number of ways in which (mn) different items can be divided equally into m groups each containing n objects and the order of groups is not important is $\left[\frac{(mn)!}{(n!)^m}\right]\frac{1}{m!}$.

The number of non-negative solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n+r-1}C_{r-1}.$

The number of positive solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n-1}C_{r-1}.$

15. Probability

(i) Probability = $\frac{\text{Favorable Cases}}{\text{Total Cases}}$.

(ii) $0 \leq$ Probability ≤ 1

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iv) If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

(v) Probability of an event A not happening = $P(A^c) = 1 - P(A)$

(vi) If A and B are Independent Events, then $P(A \cap B) = P(A) \times P(B)$

(vii) In case of experiments with only 2 outcomes possible (tossing a coin, passing or failing, hitting or not hitting a target etc.), the probability of getting r successes in n trials ($n \geq r$) is ${}^nC_r \times p^r \times q^{n-r}$, where p is the probability of success and q is the probability of failure.

(viii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

(ix) $P(A \cap B) = P(A) \times P(B)$, if A and B are independent events.

(x) $P(A \cap B) = 0$, if A and B are mutually exclusive events.

Mutually Exclusive Events: Let S be the sample space associated with a random experiment and let A_1 and A_2 be two events. Then A_1 and A_2 are mutually exclusive if $A_1 \cap A_2 = \phi$.

Note–1: If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note–2: If A and B are mutually exclusive events, then $P(A \cap B) = 0$, therefore $P(A \cup B) = P(A) + P(B)$.

Note–3: If A, B, C are three events associated with a random experiment, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

Note–4: If A & B are two events associated with a random experiment, then

(i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ (ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

Let A and B be two events associated with a random experiment. Then the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$ is called conditional probability and it is denoted by $P(\frac{A}{B})$.

Thus $P(\frac{A}{B})$ = Probability of occurrence of A under the condition that B has already occurred.

$P(\frac{B}{A})$ = Probability of occurrence of B under the condition that A has already occurred.

Note–1: If A & B are two events associated with a random experiment, then P(A ∩ B) = P(A) . P($\frac{B}{A}$) if P(A) ≠ 0 or P(A ∩ B) = P(B) . P($\frac{A}{B}$) if P(B) ≠ 0

Note–2: If A_1, A_2, A_3, _ _ _ _ A_n are n events related to a random experiment, then P($A_1 \cap A_2 \cap A_3$ _ _ _ A_n) = P(A_1) . P($\frac{A_2}{A_1}$) . P($\frac{A_3}{A_1 \cap A_2}$ _ _ _ _ P $\left(\frac{A_n}{A_1 \cap A_2 \_\_ A_{n-1}}\right)$

Total Probability: Let S be the sample space and let E_1, E_2, _ _ _ _ E_n be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or _ _ _ _ _ _ _ E_n then P(A) = P(E_1) . P($\frac{A}{E_1}$) + P(E_2) . P($\frac{A}{E_2}$)+ _ _ _ _ _ + P(E_n) . P($\frac{A}{E_n}$).

Baye's Rule: Let S be the sample space and let E_1, E_2, _ _ _ _ _ E_n be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2, _ _ _ _ or E_n, then P($\frac{E_2}{A}$) = $\frac{P(E_i).P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$

1. P (A) + P (A^C) = 1.
2. P ($A^C \cap B^C$) = P [$(A \cup B)^C$] = 1 - P(A ∪ B).
3. P ($A^C \cup B^C$) = P [$(A \cap B)^C$] = 1 - P(A ∩ B).
4. If probability of an event in defined as the odds against or odds in favour then the logic to solve this is as follows. If the odd in favour of happening of an event are given to be in the ratio of p : q. Then the probability then the event will happen is $\frac{p}{p+q}$.
5. If the odds against happening of an event is given to be in the ratio of r :s, then the probability that the event will happen is $\frac{s}{r+s}$. Hence it is to be understood clearly what is being given in the question.

16. Clocks & Calendars

A. Clocks

A clock is a complete circle having 360 degrees. It is divided into 12 equal parts i.e. each part is 360/12 = 30°. As the minute hand takes a complete round in one hour it covers 360° in 60 min. In 1 min. it covers 360/60 = 6°/ minute. Also, as the hour hand covers just one part out of the given 12 parts in one hour, this implies it covers 30° in 60 min. i.e. ½° per minute. This implies that the relative speed of the minute hand is 6 – ½ = 5 ½ degrees.

- Every hour, both the hands coincide once. In 12 hours, they will coincide 11 times. It happens due to only one such incident between 12 and 1'o clock.
- The hands are in the same straight line when they are coincident or opposite to each other.
- When the two hands are at a right angle, they are 15-minute spaces apart. In one hour, they will form two right angles and in 12 hours there are only 22 right angles. It happens due to right angles formed by the minute and hour hand at 3'o clock and 9'o clock.
- When the hands are in opposite directions, they are 30-minute spaces apart.
- If a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast. On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

NOTE: If both the hour hand and minute hand move at their normal speeds, then both the hands meet after $65\frac{5}{11}$ minutes.

B. Calendars

In an ordinary year there are 365 days, which means 52 × 7 + 1, or 52 weeks and one day. This additional day, is called an odd day. Further, every 100^{th}year starting from 1st AD, is a non–leap year, but every 4^{th} century year is a leap year. So any year divisible by 400 will be a leap year e.g.: 1200, 1600 and 2000. And the years 1800, 1900 will be non leap years as they are divisible by 100, but not 400.

The concept of odd days is very important in calendars. In a century – i.e. 100 years, there will be 24 leap years and 76 non–leap years. This means that there will be 24 × 2 + 76 × 1 = 124 odd days. Since, 7 odd days make a week, to find out the net odd days, divide 124 by 7. The remainder is 5. This is the number of odd days in a century. You may memorise the following points related to the concepts of calendars to save time during the paper.

100 years give us 5 odd days as calculated above.

200 years give us $5 \times 2 = 10 - 7$ (one week) – 3 odd days.

300 years give us $5 \times 3 = 15 - 14$ (two weeks) – 1 odd day.

400 years give us {$5 \times 4 + 1$ (leap century)} – 21} – 0 odd days.

Month of January gives us $31 - 28 = 3$ odd days.

Month of February gives us $28 - 28 = 0$ odd day in a normal year and 1 odd day in a leap year and so on for all the other months.

In total first six months i.e. January to June give us 6 odd days in a normal year and $7 - 7 = 0$ odd days in a leap year. This is going to help, when you want to find a day, which is after 30^{th} June.

In total first nine months i.e. January to September give us 0 odd day in a normal year and 1 odd day in a leap year.

Now, if we start from 1st January 0001 AD; for 0 odd day, the day will be Sunday; for 1 odd day, the day will be Monday; for 2 odd days, it will be Tuesday; for 3 odd days, it will be Wednesday and so on.

There are two types of questions, one in which a reference day is given and in the other variety, no reference day is given.

17. Mean and Basic Statistics

Arithmetic mean of the numbers a and b = a + b/2

Geometric mean of the numbers a and b = √ab, if a > 0 and b> 0

Comparing an arithmetic mean and a geometric mean: a + b/2 ≥ √ab, if a > 0 and b> 0

If a given number is split into two parts, then the product of the two parts will be the highest when they are equal.

If a given number is split into two positive multiplicative parts, then the sum of the two parts will be the least when the two parts are the square roots of the number.

Arithmetic mean – The A.M of $x_1, x_2, \dots, x_n$ is $\dfrac{x_1 + x_2 + \dots + x_n}{n}$.

Geometric mean – The G.M of $x_1, x_2, \dots, x_n$ is $\sqrt[n]{x_1 \times x_2 \times \dots \times x_n}$

Harmonic mean – The H.M of $x_1, x_2, \dots, x_n$ is $\dfrac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dots + \dfrac{1}{x_n}}$

Median – To calculate the median of a given set of numbers, arrange the frequencies in the ascending order, if the number of elements is odd, the median is (n +1)/2th term and if even then the average of the two middle terms is the median of the whole group.

Mode – The value occurring with the maximum frequency is called the mode.

A.M ≥ G.M ≥ H. M, and all of them are equal if all the values of x_i are equal.

If $x_1, x_2, \dots, x_n$ are points in the data, and if $\bar{x}$ is the mean, then Variance

$$= \frac{\sum (x_i - \bar{x})^2}{n}$$

Standard Deviation = √Variance

18. Trigonometry

(i) Measurement of Angles: Mainly there are three systems for measuring angles.

Sexagesimal system: In this system, angle is measured in degrees.

I right angle = 90^0, 1^0 = 60' and 1' = 60".

Centesimal system: In this system angle is measured in grades.

I right angle = 100^g, 1^g = 100' and 1' = 100".

Circular Measure: In this system, angle is measured in radius.

The angle subtended at the centre of a circle by an arc of length equal to its radius is 1 radian, written as 1^c.

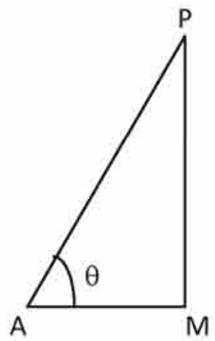
$\pi^c = 180^0 = 200^R$ = 2 right angles

(ii) Trigonometric Ratios:

$$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{PM}{AP}; \cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AM}{AP};$$

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{PM}{AM};$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}; \cot\theta = \frac{\cos\theta}{\sin\theta}.$$



(iii) Identities:

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$ (iii) $\sin^2\theta + \cos^2\theta = 1$

(iv) $1 + \tan^2\theta = \sec^2\theta$ (v) $1 + \cot^2\theta = \text{cosec}^2\theta$

(iv) Values of T-Ratios:

Angle	sin θ	cos θ	tan θ
0^0	0	1	0
$30^0 = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^0 = \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^0 = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^0 = \frac{\pi}{2}$	1	0	∞
$180^0 = \pi$	0	-1	0

(v) Sign of T-Ratios:

1^{st} Quadrant: All positive.

2^{nd} Quadrant: sin θ & cosec θ positive.

3^{rd} Quadrant: tan θ & cot θ positive.

4^{th} Quadrant: cos θ & sec θ positive.

Remember:	**I**	**II**	**III**	**IV**
	All	**sin**	**tan**	**cos**

(vi) Range of T-Ratios:

(i) $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$

Thus, $|\sin\theta| \le 1$ and $|\cos\theta| \le 1$.

The value of sin θ is never greater than 1 and never less than – 1.

The value of cos θ is never greater than 1 and never less than – 1.

(ii) cosec $\theta \ge 1$ and cosec $\theta \le -1$.

(iii) sec $\theta \ge 1$ and sec $\theta \le -1$.

(iv) tan θ may assume any value.

(vii) Increasing & Decreasing Functions:

(i) The value of sin θ increase from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. As θ increases in this interval, then sin θ also increases.

(ii) 1^{st} Quadrant: sin θ increases from 0 to 1 ; cos θ decreases from 1 to 0 and tan θ increases from 0 to ∞.

(iii) 2^{nd} Quadrant: sin θ decreases from 1 to 0 ; cos θ decreases from 0 to – 1; tan θ decreases from ∞ to 0.

(iv) 3^{rd} Quadrant: sin θ decreases from 0 to – 1 ; cos θ increases from – 1 to 0; tan θ increases from 0 to ∞.

(v) 4^{th} Quadrant: sin θ increases from – 1 to 0; cos θ increases from 0 to 1; tan θ decreases from ∞ to 0.

(viii) T-Ratios of Negative, Complementary, Supplementary Angles etc.

(i) $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$.

(ii) $\sin(90^0 - \theta) = \cos\theta$; $\cos(90^0 - \theta) = \sin\theta$; $\tan(90^0 - \theta) = \cot\theta$.

(iii) $\sin(90^0 + \theta) = \cos\theta$; $\cos(90^0 + \theta) = -\sin\theta$; $\tan(90^0 + \theta) = -\cot\theta$.

(iv) $\sin(180^0 - \theta) = \sin\theta$; $\cos(180^0 - \theta) = -\cos\theta$; $\tan(180^0 - \theta) = -\tan\theta$.

(v) $\sin(180^0 + \theta) = -\sin\theta$; $\cos(180^0 + \theta) = -\cos\theta$; $\tan(180^0 + \theta) = \tan\theta$.

(vi) $\sin(360^0 - \theta) = -\sin\theta$, $\cos(360^0 - \theta) = \cos\theta$; $\tan(360^0 - \theta) = -\tan\theta$.

(vii) $\sin(360^0 + \theta) = \sin\theta$, $\cos(360^0 + \theta) = \cos\theta$, $\tan(360^0 + \theta) = \tan\theta$.

(ix) Sum & Difference Formula:

(i) $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

(ii) $\sin(x - y) = \sin x \cos y - \cos x \sin y$.

(iii) $\cos(x + y) = \cos x \cos y - \sin x \sin y$

(iv) $\cos(x - y) = \cos x \cos y + \sin x \sin y$.

(v) $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$.

(vi) $2 \cos x \sin y = \sin(x + y) - \sin(x - y)$.

(vii) $2 \cos x \cos y = \cos(x + y) + \cos(x - y)$.

(viii) $2 \sin x \sin y = \cos(x - y) - \cos(x + y)$.

(ix) $\sin^2 x - \sin^2 y = \sin(x + y). \sin(x - y)$.

(x) $\cos^2 x - \sin^2 y = \cos(x + y). \cos(x - y)$.

(x) Some More Formulae:

(i) $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ii) $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(iii) $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(iv) $\cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(xi) Tangent Formulae:

(i) $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

(ii) $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

(xii) T-Ratios of Multiple Angles:

(i) $\sin 2x = 2 \sin x \cos x$

(ii) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.

(iii) $1 + \cos 2x = 2\cos^2 x$ and $1 - \cos 2x = 2\sin^2 x$.

(iv) $\sin 3x = 3\sin x - 4\sin^3 x$

(v) $\cos 3x = 4\cos^3 x - 3\cos x$

(vi) $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

(vii) $\tan 3x = \left(\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\right)$

(viii) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

(ix) $\sin 2x = \dfrac{2\tan x}{1 + \tan^2 x}$

(xiii) T-Ratios of Sub-multiple Angles:

(i) $\sin x = 2\sin(x/2)\cos(x/2)$

(ii) $\cos x = \cos^2 \dfrac{x}{2} - \sin^2 \dfrac{x}{2} = 2\cos^2 \dfrac{x}{2} - 1 = 1 - 2\sin^2 \dfrac{x}{2}$

(iii) $\tan x = \dfrac{2\tan(x/2)}{1 - \tan^2(x/2)}$

(iv) $1 - \cos x = 2\sin^2 \dfrac{x}{2}$ and $1 + \cos x = 2\cos^2 \dfrac{x}{2}$

(v) $\sin x = \dfrac{2\tan(x/2)}{1 + \tan^2(x/2)}$

(vi) $\cos x = \dfrac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}$

(xiv) T-Ratios of Some Special Angles:

(i) $\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^0$

(ii) $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^0$

(iii) $\sin 18^0 = \frac{\sqrt{5}-1}{4} = \cos 72^0$

(iv) $\cos 18^0 = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^0$

(v) $\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^0$

(vi) $\cos 36^0 = \frac{\sqrt{5}+1}{4} = \sin 54^0$

(vii) $\sin 22\frac{1^0}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

(viii) $\cos 22\frac{1^0}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$