CHAPTER – 9

PERMUTATIONS AND COMBINATIONS

Permutations and Combinations is one of the important areas in many exams because of two reasons. The first is that solving questions in this area is a measure of students' reasoning ability. Secondly, solving problems in areas like Probability requires thorough knowledge of Permutations and Combinations.

Before discussing Permutations and Combinations, let us look at what is called as the "fundamental rule"

"If one operation can be performed in 'm' ways and (when, it has been performed in any one of these ways), a second operation then can be performed in 'n' ways, the number of ways of performing the two operations will be m x n".

This can be extended to any number of operations.

If there are three cities A, B and C such that there are 3 roads connecting A and B and 4 roads connecting B and C, then the number of ways one can travel from A to C is 3 x 4, i.e., 12.

This is a very important principle and we will be using it extensively in Permutations and Combinations. As we use it very extensively, we do not explicitly state every time that the result is obtained by the fundamental rule but directly write down the result.

PERMUTATIONS

Each of the arrangements which can be made by taking some or all of a number of items is called a Permutation. In permutations, "order of the items" is important.

The permutations of three items a, b and c taken two at a time are ab, ba, ac, ca, cb and bc. Since the order in which the items are taken is important, ab and ba are counted as two different permutations. The words "permutation" and "arrangement" are synonymous and can be used interchangeably.

The number of permutations of n things taking r at a time is denoted by ${}^nP_r$ (and read as "nPr")

COMBINATIONS

Each of the groups or selections which can be made by taking some or all of a number of items is called a Combination. In combinations, the order in which the items are taken is not considered as long as the specific things are included.

The combination of three items a, b and c taken two at a time are ab, bc and ca. Here, ab and ba are not considered separately because the order in which a and b are taken is not important but it is only required that a combination including a and b is what is to be counted. The words "combination" and "selection" are synonymous.

The number of combinations of n things taking r at time is denoted by ${}^nC_r$ (and read as "nCr")

When a problem is read, it should first be clear to you as to whether it is a permutation or combination that is being discussed. Some times the problem specifically states whether it is the number of permutations (or arrangements) or the number of combinations (or selections) that you should find out. The questions can be as follows:

For permutations, "Find the number of permutations that can be made ....." OR "Find the number of arrangements that can be made....." OR "Find the number of ways in which you can arrange....."

For combinations,
"Find the number of combinations that can be made ....." OR "Find the number of selections that can be made....." OR "Find the number of ways in which you can select....."

Some times, the problem may not explicitly state whether what you have to find out is a permutation or a combination but the nature of what is to be found out will dictate whether it is the number of permutations or the number of combinations that you have to find out. Let us look at the following two examples to clarify this.

"How many four digit numbers can be made from the digits 1, 2, 3 and 4 using each digit once?"
Here, since we are talking of numbers, the order of the digits matters and hence what we have to find out is permutations.

"Out of a group of five friends that I have, I have to invite two for dinner. In how many different ways can I do this?"

Here, if the five friends are A, B, C, D and E, whether the two friends that I call for dinner on a particular day are A and B **or** B and A, it does not make any difference, i.e., here the order of the "items" does not play any role and hence it is the number of combinations that we have to find out.

Now we will find out the number of permutations and combinations that can be made from a group of given items.

Initially, we impose two constraints (conditions) while looking at the number of permutations. They are

- all the n items are distinct or dissimilar (or no two items are of the same type)
- each item is used at most once (i.e., no item is repeated in any arrangement)

Number of linear permutations of 'n' dissimilar items taken 'r' at a time without repetition (${}^nP_r$)

Consider r boxes each of which can hold one item. When all the r boxes are filled, what we have is an arrangement of r items taken from the given n items. So, each time we fill up the r boxes with items taken from the given n items, we have an arrangement of r items taken from the given n items without repetition. Hence the number of ways in which we can fill up the r boxes by taking things from the given n things is equal to the number of permutations of n things taking r at a time.

Boxes ....
1 2 3 4 r

The first box can be filled in n ways (because any one of the n items can be used to fill this box). Having filled the first box, to fill the second box we now have only (n – 1) items; any one of these items can be used to fill the second box and hence the second box can be filled in (n – 1) ways; similarly, the third box in (n – 2) ways and so on the r^{th} box can be filled in {n – (r – 1)} ways, i.e. [n – r + 1] ways. Hence, from the Fundamental Rule, all the r boxes together can be filled in
$n \cdot (n-1) \cdot (n-2)..................(n-r+1)$ ways

So, ${}^nP_r = n \cdot (n-1) \cdot (n-2).............(n-r+1)$

This can be simplified by multiplying and dividing the right hand side by (n – r) (n – r – 1).........3.2.1 giving us
${}^nP_r = n(n-1)(n-2)\ ....\ [n-(r-1)]$

$$= \frac{(n-1)(n-2).....[n-(r-1)].(n-r).......3.2.1}{(n-r)....................3.2.1}$$

$$= \frac{n!}{(n-r)!}$$

The number of permutations of n distinct items taking r items at a time is

$$\boxed{{}^n\mathbf{P}_r = \frac{\mathbf{n!}}{\mathbf{(n-r)!}}}$$

If we take n items at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the r boxes above, we can find that ${}^nP_n$ is equal to n!

The first box can be filled in n ways, the second one in (n – 1) ways, the third one in (n – 2) ways and so on, then the n^{th} box in 1 way; hence, all the n boxes can be filled in n(n – 1) (n – 2) ....3.2.1 ways, i.e., n! ways. Hence,

$$\boxed{{}^n\mathbf{P}_n = \mathbf{n!}}$$

But if we substitute r = n in the formula for ${}^nP_r$, then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$, we can conclude that 0! = 1

Number of combinations of n dissimilar things taken r at a time

Let the number of combinations ${}^nC_r$ be x. Consider one of these x combinations. Since this is a combination, the order of the r items is not important. If we now impose the condition that order is required for these r items, we can get r! arrangements from this one combination. So each combination can give rise to r! arrangements. x combinations will thus give rise to x . r! arrangements. But since these are all permutations of n things taken r at a time, this must be equal to ${}^nP_r$. So,

$$x.r! = {}^nP_r = \frac{n!}{(n-r)!} \Rightarrow {}^nC_r = \frac{n!}{r!.(n-r)!}$$

The number of combinations of n dissimilar things taken all at a time is 1.

Out of n things lying on a table, if we select r things and remove them from the table, we are left with (n-r) things on the table - that is, whenever r things are selected out of n things, we automatically have another selection of the (n – r) things. Hence, the number of ways of selecting r out of n things is the same as selecting (n – r) things out of n given things, i.e.,

$$\mathbf{{}^nC_r = {}^nC_{n-r}}$$

When we looked at ${}^nP_r$, we imposed two constraints which we will now release one by one and see how to find out the number of permutations.

Number of arrangements of n items of which p are of one type, q are of a second type and the rest are distinct

When the items are all not distinct, then we **cannot** talk of a general formula for $\mathbf{{}^nP_r}$ for any r but we can talk of only ${}^nP_n$ (which is given below). If we want to find out ${}^nP_r$ for a specific value of r in a given problem, we have to work on a case to case basis (this has been explained in one of the solved examples).

The number of ways in which n things may be arranged taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest all distinct is

$$\frac{n!}{p!\,q!\,r!}.$$

Number of arrangements of n distinct items where each item can be used any number of times (i.e., repetition allowed)

You are advised to apply the basic reasoning given while deriving the formula for ${}^nP_r$ to arrive at this result also. The first box can be filled up in n ways; the second box can be filled again in n ways (even though the first box is filled with one item, the same item can be used for filling the second box also because repetition is allowed); the third box can also be filled in n ways and so on ... the r^{th} box can be filled in n ways. Now all the r boxes together can be filled in {n.n.n.n ... r times} ways, i.e., $\mathbf{n^r}$ ways.

The number of permutations of n things, taken r at a time when each item may be repeated once, twice, ... up to r times in any arrangement is $\mathbf{n^r}$

What is important is not this formula by itself but the reasoning involved. So, even while solving problems of this type, you will be better off if you go from the basic reasoning and not just apply this formula.

Total number of combinations:

Out of n given things, the number of ways of selecting **one or more** things is where we can select 1 or 2 or 3 ... and so on up to n things at a time; hence the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \ .....\ {}^nC_n$

This is called "the total number of combinations" and is equal to $2^n - 1$ where n is the number of things.

The same can be reasoned out in the following manner also.

There are n items to select from. Let each of these be represented by a box.

	1	2	3	4		n
No. of ways of dealing with the boxes	2	2	2	2	....	2

The first box can be dealt with in two ways. In any combination that we consider, this box is **either** included **or** not included. These are the two ways of dealing with the first box. Similarly, the second box can be dealt with in two ways, the third one in two ways and so on, the n^{th} box in two ways. By the Fundamental Rule, the number of ways of dealing with all the boxes together in $2 \cdot 2 \cdot 2 \cdot$ .... n times ways, i.e., in 2^n ways. But out of these, there is one combination where we "do not include the first box, do not include the second box, do not include the third box and so on, do not include the n th box." That means, no box is included. But this is not allowed because we have to select **one or more** of the items (i.e., at least one item). Hence this combination of no box being included is to be subtracted from the 2^n ways to give the result of

> **Number of ways of selecting one or more items from n given items is $2^n - 1$**

Dividing given items into groups:

Dividing (p + q) items into two groups of p and q items

Out of (p + q) items, if we select p items (which can be done in ${}^{p+q}C_p$ ways), then we will be left with q items, i.e., we have two groups of p and q items respectively. So, the number of ways of dividing (p + q) items into two groups of p and q items respectively is equal to ${}^{p+q}C_p$ which is equal to $\frac{(p+q)!}{p! \cdot q!}$

> **The number of ways of dividing (p + q) items into two groups of p and q items respectively is $\frac{(p+q)!}{p! \cdot q!}$**

If p = q, i.e., if we have to divide the given items into two EQUAL groups, then two cases arise
(i) when the two groups have distinct identity and
(ii) when the two groups do not have distinct identity.

In the first case, we just have to substitute p = q in the above formula which then becomes

> **The number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{(p!)^2}$ where the two groups have distinct identity.**

In the second case, where the two groups do not have distinct identity, we have to divide the above result by 2!, i.e., it then becomes

> **The number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{2!(p!)^2}$ where the two groups do not have distinct identity.**

Dividing (p + q + r) items into three groups consisting of p, q and r items respectively

> **The number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things respectively is $\frac{(p+q+r)!}{p!q!r!}$**

If p = q = r, i.e., if we have to divide the given items into three EQUAL groups, then we have two cases where no two groups are identical and where the groups are identical.

When no two groups are identical, the number of ways is $\frac{(3p)!}{(p!)^3}$

When the three groups are identical, then the number of ways is $\frac{(3p)!}{3!\,(p!)^3}$

Circular Permutations:

When n distinct things are arranged in a straight line taking all the n items, we get n! permutations. However, if these n items are arranged in a circular manner, then the number of arrangements will not be n! but it will be less than that. This is because in a straight line manner, if we have an arrangement ABCDE and if we move every item one place to the right (in cyclic order), the new arrangement that we get EABCD is not the same as ABCDE and this also is counted in the n! permutations that we talked of. However, if we have an arrangement ABCDE in a circular fashion, by shifting every item by one place in the clockwise direction, we still get the same arrangement ABCDE. So, if we now take n! as the number of permutations, we will be counting the same arrangement more than once.

The number of arrangements in circular fashion can be found out by first fixing the position of one item. Then the remaining (n – 1) items can be arranged in (n – 1)! ways. Now even if we move these (n – 1) items by one place in the clockwise direction, then the arrangement that we get will not be the same as the initial arrangement because one item is fixed and it does not move.

Hence, the number of ways in which n distinct things can be arranged in a circular arrangement is (n – 1)!

If we take the case of five persons A, B, C, D and E sitting around a table, then the two arrangements ABCDE (in clockwise direction) and AEDCB (the same order but in anticlockwise direction) will be different. Here we say that the clockwise and anticlockwise arrangements are different. However, if we consider the circular arrangement of a necklace made of five precious stones A, B, C, D and E, the two arrangements talked of above will be the same because we take one arrangement and turn the necklace around (front to back), then we get the other arrangement. Here, we say that there is no difference between the clockwise and anticlockwise arrangements. In this case the number of arrangements will be half of what it is in the case where the clockwise and anticlockwise arrangements are different.

> The number of **circular arrangements** of **n distinct items** is
> **(n – 1)!** if there is **DIFFERENCE** between clockwise and anticlockwise arrangements and
> **(n – 1)!/2** if there is **NO DIFFERENCE** between clockwise and anticlockwise arrangements

Sum of all numbers formed from given digits:

If n distinct digits are used to make all the possible n-digit numbers, we get n! numbers. We now want to find out the sum if all these n! numbers are added. Let us take an example and understand how it is to be done and then look at it as a formula.

To find the sum of all the four digit numbers formed using the digits 2, 3, 4 and 5 without repetition:

We can form a total of 4! or 24 numbers. When we add all these numbers, let us look at the contribution of the digit 2 to the sum.

When 2 occurs in the thousands place in a particular number, its contribution to the total will be 2000. The number of numbers that can be formed with 2 in the thousands place is 3!, i.e., 6 numbers. Hence, when 2 is in the thousands place, its contribution to the sum is 3! x 2000.

Similarly, when 2 occurs in the hundreds place in a particular number, its contribution to the total will be 200 and since there are 3! numbers with 2 in the hundreds place, the contribution 2 makes to the sum when it comes in the hundreds place is $3! \times 200$.

Similarly, when 2 occurs in the tens and units place respectively, its contribution to the sum is $3! \times 20$ and $3! \times 2$ respectively. Thus the total contribution of 2 to the sum is $3! \times 2000 + 3! \times 200 + 3! \times 20 + 3! \times 2$, i.e., $3! \times 2222$. This takes care of the digit 2 completely.

In a similar manner, the contribution of 3, 4 and 5 to the sum will respectively be $3! \times 3333$, $3! \times 4444$ and $3! \times 5555$ respectively.

The sum can now be obtained by adding the contributions of these four digits. Hence the sum of the numbers formed by using the four digits is $3! \times$ (2222 + + 3333 + 4444 + 5555), i.e.,
$3! \times (2 + 3 + 4 + 5) \times 1111$

We can now generalize the above as

> If all the possible n-digit numbers using n distinct digits are formed, the sum of all the numbers so formed is equal to **(n-1)! × {sum of the n digits} × {11111 ....} n times**

Rank of a word:

Finding the rank of a given word is basically finding out the position of the word when all possible words have been formed using all the letters of this word exactly once and arranged in alphabetical order as in the case of dictionary. Let us understand this by taking an example.

Let us look at the word “POINT”. The letters involved here, when taken in alphabetical order are I, N, O, P, T.

To arrive at the word “POINT”, initially we have to go through the words that begin with I, then all those that begin with N, those that begin with O which are 4! in each case. Then we arrive at words that begin with PI, PN which are 3! in each case. Then we arrive at the word POINT.

There are $3 \times 4! + 2 \times 3! = 84$ words that precede the word POINT i.e., POINT is the 85^{th} word. Hence rank of ‘POINT’ is 85.

The number of diagonals in a n-sided convex polygon

An n-sided convex polygon has n vertices. Joining any two vertices we get a line of the polygon which are ${}^nC_2$ in number. Of these ${}^nC_2$ lines, n of them are sides. Hence number of diagonals is

$${}^nC_2 - n = \frac{n(n-3)}{2}$$

Number of integral solutions of the equation $x_1 + x_2 + \ldots\ldots + x_n = s$

Consider the equation

$x_1 + x_2 + x_3 = 10$

If we consider all possible integral solutions of this equation, there are infinitely many. But the number of positive (or non-negative) integral solutions is finite.

We would like the number of positive integral solutions of this equation, i.e., values of (x_1, x_2, x_3) such that each $x_i > 0$.

We imagine 10 identical objects arranged on a line. There are 9 gaps between these 10 objects. If we choose any two of these gaps, we are effectively splitting the 10 identical objects into 3 parts of distinct identity (x_1 is the first part, x_2 is the second part and x_3 is the third part). Conversely, every split of these 10 objects corresponds to a selection of 2 gaps out of the 9 gaps.

Therefore, the number of positive integral solutions is ${}^9C_2$. In general, if $x_1 + x_2 + \ldots\ldots + x_n = s$ where $s \ge n$, the number of positive integral solutions is ${}^{s-1}C_{n-1}$.

If we need the number of non negative integral solutions, we proceed as follows. Let $a_1, a_2, \ldots\ldots$ be a non-negative integral solution. Than $a_1 + 1, a_2 + 1, \ldots\ldots a_n + 1$ is a positive integral solution of the equation $x_1 + x_2 + \ldots\ldots + x_n = s + n$. Therefore, the number of non-negative integral solutions of the given equation is equal to the number of positive integral solutions of $x_1 + x_2 + \ldots\ldots + x_n = s + n$, which is ${}^{s+n-1}C_{n-1}$.

For $x_1 + x_2 + x_3$ + --------- + x_n = s where s ≥ 0, the number of **positive integral solutions** (when s $\geq$ n) is $^{s-1}C_{n-1}$ and the number of **non-negative integral solutions** is $^{n+s-1}C_{n-1}$

Some additional points:

1. Suppose there are n letters and n corresponding addressed envelopes. The numbers of ways of placing these letters into the envelopes such that no letter is placed in its corresponding envelope is often referred as derangements. The number of derangements of n objects is given by

$$D(n) = n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....+(-1)^n\frac{1}{n!}\right]$$

For example, when n = 3, the number of derangements is

$D(3) = 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = 2$ and when n = 4,

$D(4) = 4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right] = 9$

2. The total number of ways in which a selection can be made by taking some or all out of p + q + r + ... things where p are alike of one kind, q alike of a second kind, r alike of a third kind and so on is [{(p + 1)(q + 1)(r + 1) ....} – 1].

3. $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$ and $^nP_r = r.{}^{n-1}P_{r-1} + {}^{n-1}P_r$

Ex: Consider the word PRECIPITATION. Find the number of ways in which
(i) a selection
(ii) an arrangement
of 4 letters can be made from the letters of this word.

Sol. The word PRECIPITATION has 13 letters I, I, I, P, P, T, T, E, R, C, A, O, N of 9 different sorts.
In taking 4 letters, the following are the possibilities to be considered.
(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

Selections

(a) 4 distinct letters can be selected from 9 distinct letters (I, P, T, E, R, C, A, O, N) in $^9C_4$ = 126 ways.

(b) As 3 letters have to be alike, the only possibility is selecting all the I's. Now the 4th letter can be selected from any of the remaining 8 distinct letters in $^8C_1$ = 8 ways.

(c) Two pairs of two alike letters can be selected from I's, P's and T's in $^3C_2$ = 3 ways.

(d) The two alike letters can be selected in $^3C_1$ = 3 ways and the two distinct letters can now be selected from the 8 distinct letters in $^8C_2$ = 28 ways. Hence required number of ways are $3 \times 28 = 84$
Hence, the total selections are 126 + 8 + 3 + 84 = 221.

Arrangements

For arrangements, we find the arrangements for each of the above selections and add them up.

(a) As the 4 letters are distinct, there are 4! arrangements for each selection. Hence required arrangements are $126 \times 4! = 3024$

(b) Since 3 of the 4 letters are alike, there are $\frac{4!}{3!}$ arrangements for each selection. Hence required arrangements are $8 \times \frac{4!}{3!} = 32$.

(c) The required arrangements here are $3 \times \frac{4!}{2!2!} = 18$

(d) The required arrangements are $84 \times \frac{4!}{2!} = 1008$.
Total number of arrangements are 3024 + 32 + 18 + 1008 = 4082.

Examples

9.01. (a) How many four letter words can be formed using the letters of the word "ROAMING"?

Sol: None of the letters in the word are repeated.
∴ The number of four letter words that can be formed = $^7P_4 = \frac{7!}{3!}$ = (7) (6) (5) (4) = 840.

(b) How many three letter words can be formed using the letters of the word "PRACTICES"?

Sol: PRACTICES
The combinations, the number of combinations and the number of permutations (for each combination) as well as the total are tabulated below.

Combinations	Number of combinations	Number of permutations for each combination	Total number of permutations
x, y, z	$^8C_3 = 56$	6	336
x, x, y i.e. c, c, y	$^7C_1 = 7$	3	21
			----------- 357 -----------

9.02. In a party, each person shook hands with every other person present. The total number of hand shakes was 28. Find the number of people present in the party.

Sol: Let the number of people present in the party be n.

Method 1
The first person shakes hands with a total of (n – 1) persons, the second with (n – 2) other people and so on.

The total number of hand shakes is
(n – 1) + (n – 2) + . . . + 2 + 1

$\frac{n(n-1)}{2} = 28$ (given) $\Rightarrow n = 8$

Method 2
Number of hand shakes = Number of ways of selecting 2 people out of n = ${}^nC_2$.

${}^nC_2 = 28$

$\frac{n(n-1)}{2!} = 28 \Rightarrow n = 8$

Directions for examples 9.03 to 9.07: The following examples are based on the data below.

The letters of NESTLE are permuted in all possible ways.

9.03. How many of these words begin with T?

Sol: NESTLE has 6 letters of which the letter E occurs two times. Therefore the required number of words = Number of ways of filling N, E, S, E and L in the second to sixth positions = $\frac{5!}{2!}$ = 60.

9.04. How many of these words begin and end with E?

Sol: The required number of words = The number of ways of filling N, S, T and L in the second to fifth positions = 4! = 24.

9.05. How many of these words begin with S and end with L?

Sol: The required number of words = The number of ways of filling N, E, T and E in the second to fifth positions = $\frac{4!}{2!}$ = 12.

9.06. How many of these words neither begin with S nor end with L?

Sol: The required number of words = The total number of words which can be formed using the letters N, E, S, T, L and E – (Number of words which begin with S or end with L)

$= \frac{6!}{2!} -$ (Number of words beginning with S + Number of words ending with L – Number of words beginning with S and ending with L)

$= \frac{6!}{2!} - \left(\frac{5!}{2!} + \frac{5!}{2!} - \frac{4!}{2!}\right)$

= 360 – (60 + 60 – 12) = 252.

9.07. How many of these words begin with T and do not end with N?

Sol: The required number of words = The number of words beginning with T – The number of words beginning with T and ending with N

$= \frac{5!}{2!} - \frac{4!}{2!} = 48.$

Directions for examples 9.08 to 9.11: The following examples are based on the data below.

The letters of FAMINE are permuted in all possible ways.

9.08. How many of these words have all the vowels occupying odd places?

Sol: FAMINE has 3 vowels and 3 consonants.
The vowels can be arranged in the odd places in 3! or 6 ways.
The consonants would have to be arranged in even places. This is possible in 3! or 6 ways as well.
∴ The required number of words = 6^2 = 36.

9.09. How many of these words have all the vowels together?

Sol: If all the vowels are together, the vowels can be arranged in 3! ways among themselves.
Considering the vowels as a separate unit and each of the other letters as a unit, we have a total of 4 units which can be arranged in 4! ways.
∴ The required number of words = 4! 3! = 144

9.10. How many of these words have at least two of the vowels separated?

Sol: The required number of words = The total number of words which can be formed using the letters F, A, M, I, N and E – The number of words with all the vowels together = 6! – 4! 3! = 576.

9.11. How many of these words have no two vowels next to each other?

Sol: To ensure that no two vowels are together, we first arrange the 3 consonants say – c_1 – c_2 – c_3 – and place the vowels in the gaps between the consonants or the initial or final position. For each arrangement of the consonants, there are 4 places where the vowels can go. The vowels can be dealt with in 4 (3) (2) ways.
∴ The total number of words is 3! 4! = 144.

Directions for examples 9.12 to 9.14: The following examples are based on the data below.

Raju wrote 7 letters A, B, C, D, E, F and G on a black board.

9.12. How many 4-letter words can be formed using these letters such that at least one letter of the word is a vowel?

Sol: The vowels, the number of ways in which the consonants can be selected and the number of words are tabulated below:

Vowels	No. of ways of selecting consonants	Number of words
A	${}^5C_3 = 10$	10 (24)
E	${}^5C_3 = 10$	10 (24)
AE	${}^5C_2 = 10$	10 (24)
		10 (72) = 720

9.13. How many 7 letter words can be formed using these letters such that the last two letters of the word are adjacent consonants?

Sol: B and C, C and D as well as F and G are adjacent consonants. The letters in each of these pairs can be arranged in 2! = 2 ways.
The remaining letters can be arranged in 5! = 120 ways.
$\therefore$ The required number of words = (3) (2) (120) = 720.

9.14. How many 7 letter words can be formed using these letters such that the letter at one of its ends is a vowel and that at the other end is a consonant?

Sol: If the first letter is a vowel, it has 2 possibilities.
In this case, the last letter has 5 possibilities.
The remaining 5 letters can be arranged in 5! = 120 ways.
$\therefore$The total number of words = (2) (5) (120)
If the last letter is a vowel, it similarly follows that the number of words = (2) (5) (120)
So the total number of words = (2) (2) (5) (120) = 2400.

Directions for examples 9.15 and 9.16: The following examples are based on the data below.

All possible four-digit numbers are formed using the digits 1, 2, 3 and 4 without repeating any digit.

9.15. How many of these numbers have the even digits in even places?

Sol: There are two even places.
$\therefore$ The even digits can be arranged in 2! = 2 ways.
$\therefore$ The required number = (2) (2) = 4.

9.16. If all the numbers are arranged in an ascending order of magnitude, find the position of the number 3241.

Sol: If the first digit is 1 or 2, in each case, the remaining digits can be arranged in 3! or 6 ways.
$\therefore$ a total of (2) (6) = 12 numbers have their first digit as 1 or 2.
If the first digit is 3, the possible numbers in ascending order are 3124, 3142, 3214, 3241,
$\therefore$ The position of 3241 = 16.

Directions for examples 9.17 and 9.18: The following examples are based on the data below.

A committee of 5 is to be formed from 4 women and 6 men.

9.17. In how many ways can it be formed if it consists of exactly 2 women?

Sol: The committee must have 2 women and 3 men.
$\therefore$ The required number of ways = ${}^4C_2\ {}^6C_3 = 120$

9.18. In how many ways can it be formed if it consists of more women than men?

Sol: The committee must have either 4 women and 1 man or 3 women and 2 men.
$\therefore$ The required number of ways
$= {}^4C_4\ {}^6C_1 + {}^4C_3\ {}^6C_2 = 6 + 60 = 66.$

9.19. Find the number of four-digit numbers which can be formed using four of the digits 0, 1, 2, 3 and 4 without repetition.

Sol: The first digit has 4 possibilities (1, 2, 3 and 4).
The second digit has 4 possibilities (0 and any of the three digits not used as the first digit).
The third digit has 3 possibilities.
The last digit has 2 possibilities.
$\therefore$ The required number of numbers = (4) (4) 3 (2) = 96.

9.20. The number of diagonals of a regular polygon is four times the number of its sides. How many sides does it have?

Sol: Let the number of sides in the polygon be n.
$\frac{n(n-3)}{2} = 4n$
$n(n - 11) = 0$
$n > 0$
$\therefore n - 11 = 0$
$\therefore n = 11$

Concept Review Questions

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. **(a)** The value of $^8P_2$ is
(A) 28 (B) 56 (C) 48 (D) 36

(b) The value of $^{10}C_2$ is
(A) 90 (B) 20 (C) 45 (D) 50

(c) The value of $^{45}C_{42}$ is
(A) $^{45}C_1$ (B) $^{45}C_{41}$ (C) $^{45}C_2$ (D) $^{45}C_3$

(d) The value of $^{2009}C_0$ is
(A) 0 (B) 1 (C) 2009 (D) 2008

(e) The value of $^{2009}C_1$ is
(A) 0 (B) 1 (C) 2009 (D) 2008

(f) The value of $^{2009}C_{2008}$ is
(A) 2009 (B) 2008 (C) 1 (D) 0

2. If $^nC_2 = {}^nC_{10}$, then the value of n is $\boxed{}$

3. If $^8C_3 + {}^8C_4 = {}^nC_4$, then n = $\boxed{}$

4. The relation between ^nP_r and ^nC_r is
(A) $^nP_r = {}^nC_r$ (B) $r.{}^nP_r = {}^nC_r$
(C) $^nP_r = r!\,{}^nC_r$ (D) $^nP_r \times r! = {}^nC_r$

5. If $^nP_4 = 7920$, then find $^nC_4$.
(A) 165 (B) 330 (C) 495 (D) 660

6. A man has 12 blazers, 10 shirts and 5 ties. Find the number of different possible combinations in which he can wear the blazers, shirts and ties. $\boxed{}$

7. The number of ways of arranging 6 people in a row is
(A) 6 (B) 30 (C) 120 (D) 720

Directions for questions 8 to 10: These questions are based on the data given below.

All the letters of the word "RAINBOW" are arranged in all possible ways.

8. Find the number of 7-letter words possible, such that each letter is used at most once.
(A) 1 (B) 24 (C) 120 (D) 7!

9. The number of 7-letter words that begin with R when each letter occurs only once is
(A) 6 (6!) (B) 7! 2! (C) 6! (D) 2(7!)

10. If each letter is used exactly once, the number of seven-letter words which begin with R and end with W is
(A) 6! (B) 5! (C) 5! 2! (D) 4!

11. Find the number of ways of posting 4 letters in 5 letter boxes.
(A) 5^4 (B) 4^5 (C) 2^5 (D) 5^2

12. Find the number of passwords of length 5 that can be formed using all the vowels of the alphabet.
$\boxed{}$

13. A five lettered word is formed using some of the letters {a, b, h, i, p, r, s}. How many of them will be palindromes?
(A) 125 (B) 225 (C) 343 (D) 729

14. Using all the letters of the word MOBILE, how many words can be formed, which begin with M and end with E?
(A) 6 (B) 24 (C) 120 (D) 84

15. How many three letter words can be formed using the letters of the word RELATION? $\boxed{}$

16. In how many ways can nine books be distributed among six students such that each student is eligible to receive any number of books?
(A) 6^9 (B) 9^6 (C) 54 (D) 15

17. How many different words can be formed by using all the letters of the word INSTITUTE?
(A) $\frac{9!}{2!}$ (B) 9 ! (C) $\frac{9!}{3!}$ (D) $\frac{9!}{3!2!}$

18. In how many ways can a cricket team of 11 members be selected from 15 players, so that a particular player is included and another particular player is left out?
(A) 216 (B) 826 (C) 286 (D) 386

19. There are 8 letters and 8 corresponding envelopes. If 8 letters are placed into 8 envelopes randomly, find the number of ways in which exactly one letter is placed into a wrong envelope.
(A) 1 (B) 0 (C) 8 ! (D) 7 !

20. A group contains n persons. If the number of ways of selecting 6 persons is equal to the number of ways of selecting 9 persons, then the number of ways of selecting four persons from the group is
$\boxed{}$

21. The number of ways of arranging 10 books on a shelf such that two particular books are always together is
(A) 9! 2! (B) 9! (C) 10! (D) 8

22. The number of 3-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 such that each digit occurs at most once in every number is $\boxed{}$

23. Find the number of four-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 when each digit can occur any number of times in each number.
(A) 4^6 (B) $^6P_4$ (C) $^6P_6$ (D) 6^4

24. Find the number of even numbers that can be formed using all the digits 1, 2, 3, 4, 5 when each digit occurs only once in a number.
(A) $^5P_4$ (B) 4! 2 (C) 5! (D) 4!

25. Find the number of ways of arranging 6 people around a circular table.

(A) 6! (B) $\frac{6!}{2!}$ (C) 5! (D) $\frac{5!}{2!}$

26. Find the number of ways of selecting a team of 5 people from a group of 8.

(A) $^8C_3$ (B) $^8P_5$ (C) 8! (D) 5!

27. (a) Find the number of ways of selecting a team of 6 people from a group of 10 people such that a particular person is always included in the team.

(A) $^9C_5$ (B) $^9C_6$ (C) $^{10}C_5$ (D) $^{10}C_6$

(b) Find the number of ways of selecting a team of 4 people from a group of 7 persons such that a particular person is not included in the team.

(A) $^7C_4$ (B) $^6P_4$ (C) $^6C_4$ (D) $^6C_3$

28. (a) Find the number of ways of studding 10 beads to form a necklace.

(A) $\frac{9!}{2!}$ (B) 9! (C) 9! 2! (D) $\frac{10!}{2}$

(b) Find the number of ways of inviting at least one among 6 people to a party.

(A) 2^6 (B) $2^6 - 1$ (C) 6^2 (D) $6^2 - 1$

29. The number of ways of forming a committee of six members from a group of 4 men and 6 women is $\boxed{}$

30. The number of distinct lines that can be formed by joining 20 points on a plane, of which no three points are collinear is

(A) 190 (B) 380 (C) 360 (D) 120

31. Find the number of triangles that can be formed by joining 24 points on a plane, no three of which are collinear.

(A) 2024 (B) 2026 (C) 2023 (D) 2025

32. (a) The number of rectangles that can be formed on a 8×8 chessboard is $\boxed{}$

(b) The number of squares that can be formed on a 8×8 chessboard is $\boxed{}$

33. (a) The number of diagonals of a convex decagon is

(A) 53 (B) 35 (C) 45 (D) 60

(b) If the number of diagonals of a convex 'n' sided polygon is 77, then the value of 'n' is

(A) 10 (B) 14 (C) 15 (D) 18

34. In how many ways can seven persons be selected from 6 men, 2 women, 3 boys and 4 girls? $\boxed{}$

35. In how many ways can four consonants and three vowels be selected from the letters of the word VALEDICTORY?

(A) 140 (B) 200 (C) 180 (D) 320

36. Find the number of ways of drawing four cards, all of different suits from a pack of 52 playing cards. (Diamonds, spades, hearts and clubs are the suits in a pack)

(A) $^{52}C_4$ (B) $^{52}P_4$
(C) 13^4 (D) None of these

37. An eight-letter word is formed by using all the letters of the word "EQUATION". How many of these words begin with a consonant and end with a vowel? $\boxed{}$

38. In how many ways can 6 boys and 5 girls be arranged in a row so that boys and girls sit alternately?

(A) $(6!)^2$ (B) $(5!)^2$ (C) 6! 5! (D) 2.5! 6!

39. A committee of 5 members is to be formed from a group of 6 men and 4 women. In how many ways can the committee be formed such that it contains more men than women?

(A) 180 (B) 186 (C) 126 (D) 66

40. How many four-letter words can be formed by using the letters of the word PREVIOUS?

$\boxed{}$

41. In how many ways can ten students be seated around a circular table so that three students always sit together?

(A) 7! (B) 7! 3! (C) 2.7! (D) 3 (7!)

42. In how many ways can 7 boys and 6 girls be arranged in a row so that no two girls sit together?

(A) 13! (B) $^8P_6 \times 7!$
(C) $6!\,^8P_7$ (D) 12!

43. A man has 7 friends. In how many ways can he invite at least one of his friends for a dinner?

$\boxed{}$

44. In how many ways can 13 beads be arranged in a necklace?

(A) $\frac{13!}{2!}$ (B) 12! (C) $12! \times 2!$ (D) $\frac{12!}{2}$

45. In how many ways can 10 boys and 10 girls be arranged in a row so that all the girls sit together?

(A) 10! (B) 11! (C) 20! (D) 10! 11!

Exercise – 9(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. How many words can be formed using all the letters of the word QUESTION without repetition so that the vowels occupy the even places? ☐

2. In how many ways can the letters of the word HEPTAGON be permuted so that the vowels are never separated?
(A) 720 (B) 1440 (C) 5040 (D) 4320

3. In how many ways can the letters of the word COMBINATION be permuted?
(A) 11! (B) $\frac{11!}{2!\,2!\,2!}$
(C) $\frac{11!}{5!\,6!}$ (D) $\frac{11!}{2!\,2!\,2!\,5!}$

4. How many four-digit numbers having distinct digits can be formed using the digits 0 to 9?
(A) 5040 (B) 2526 (C) 3656 (D) 4536

5. In the above problem, how many of the numbers are divisible by 5?
(A) 342 (B) 504 (C) 448 (D) 952

6. How many four-digit numbers that are divisible by 3 can be formed using the digits 0, 2, 3, 5, 8, if no digit occurs more than once in each number? ☐

7. How many even numbers between 20,000 and 40,000 (excluding the extremes) can be formed using the digits 0, 2, 3, 4, 6, 8 if any digit can occur any number of times?
(A) 2160 (B) 2593 (C) 2161 (D) 2159

8. A boat is to be manned by eight men, of whom, one cannot row on the bow side and two cannot row on the stroke side. In how many ways can the crew be arranged? ☐

9. In how many ways can 6 boys and 6 girls sit around a circular table so that no two boys sit next to each other?
(A) $(5!)^2$ (B) $(6!)^2$ (C) 5! 6! (D) 11!

10. Sheetal invites 10 of her friends for lunch and seats 5 of them around a round table and the remaining 5 around another round table. Find the total number of ways in which she can seat all her 10 friends.
(A) $(4!)^2$ (B) $\frac{10!\,(4!)^2}{(5!)^2}$
(C) $10!\,(5!)^2$ (D) 10!

11. A double-decker bus can accommodate 100 passengers, 60 in the lower deck and 40 in the upper deck. In how many ways can 100 passengers be accommodated, if 15 of them want to be in the lower deck only and 10 want to be in the upper deck only?
(A) $\frac{75!\,60!\,40!}{45!\,30!}$ (B) $\frac{75!}{45!\,30!}$
(C) $\frac{100!}{60!\,40!}$ (D) $\frac{75!\,60!\,40!}{50!\,25!}$

12. In how many ways can a panel of 6 doctors be formed from 5 surgeons and 6 physicians if the panel has to include more surgeons than physicians?
(A) 82 (B) 81 (C) 65 (D) 135

13. In how many ways can a delegation of 4 professors and 3 students be constituted from 8 professors and 5 students, if Balamurli an Arts student refuses to be in the delegation when Prof. Siddharth, the Science professor is included in it? ☐

14. Prahaas attempts a question paper that has 3 sections with 6 questions in each section. If Prahaas has to attempt any 8 questions, choosing at least two questions from each section, then in how many ways can he attempt the paper?
(A) 18000 (B) 10125
(C) 28125 (D) 9375

15. In how many ways can 20 different books be divided equally
(i) among 4 boys?
(A) 4^5 (B) 5^4
(C) $\frac{20!}{(4!)^4}$ (D) $\frac{20!}{(5!)^4}$
(ii) into 4 parcels?
(A) $\frac{20!}{5!(4!)^4}$ (B) $\frac{20!}{4!(5!)^4}$
(C) $\frac{20!}{(5!\,4!)^4}$ (D) $\frac{20!}{5!\,4!}$

16. A certain group of friends met on a new year eve party and each person shook hands with everybody else in the group exactly once and the number of handshakes turned out to be 66. On the occasion of Pongal (harvest festival), if each person in this group sends a greeting card to every other person in the group, then how many cards are exchanged?
☐

17. If Mr. Kapil, one of the members of the group referred to in the previous question, wants to invite home one or more of his friends (from that group) for dinner, then in how many ways can the invitation be extended?
(A) 1024 (B) 2048 (C) 1023 (D) 2047

18. Neha has 12 chocolates with her; 4 similar Kit Kats, 5 similar Perks, and 3 similar Milky Bars, which she wants to distribute among her friends.
In how many ways can Neha give away one or more chocolates?
(A) 120 (B) 119 (C) 60 (D) 59

19. From 3 green dyes, 4 red dyes and 2 yellow dyes, the number of ways in which the dyes can be selected so that at least one green dye and at least one yellow dye is selected is _____ .
(A) 336 (B) 335 (C) 60 (D) 59

20. P is an integer whose digits are zeros and ones. The sum of the digits of P is 4 and $10^5 < P < 10^7$.
How many values can P take? ☐

21. (a) Find the number of selections that can be made by taking 4 letters from the word INKLING.

(b) For the word in the previous question, find the number of arrangements by taking 4 letters.

22. If the letters of the word NOTES are permuted in all possible ways and the words thus obtained are arranged alphabetically as in a dictionary, then what is the rank of the word STONE?
(A) 95 (B) 96 (C) 105 (D) 106

23. Find the sum of all four-digit numbers formed by taking all the digits 2, 4, 6, 8.
(A) 133320 (B) 533280
(C) 244420 (D) 335240

24. There are 12 points in a plane. If 4 of them are on a straight line and no other three points are on a straight line, then find the difference between the number of triangles and the number of straight lines that can be formed using these points.

25. For a convex octagon what is the maximum number of points of intersection of the diagonals inside the octagon?
(A) 35 (B) 126 (C) 70 (D) 21

26. There are seven letters and corresponding seven addressed envelopes. All the letters are placed randomly into the envelopes – one in each envelope. In how many ways can exactly two letters be placed into their corresponding envelopes?

27. In how many ways can 5 prizes be given away to 3 boys if each boy is eligible for one or more prizes?
(A) 5^3 (B) 3^5 (C) ${}^5P_3$ (D) ${}^5C_3$

28. Find the number of non-negative integral solutions for the equation $x_1 + x_2 + x_3 + x_4 = 15$.
(A) 216 (B) 165 (C) 364 (D) 816

29. The number of positive integral solutions to the equation $x + y + z = 20$ is
(A) 131 (B) 110 (C) 55 (D) 171

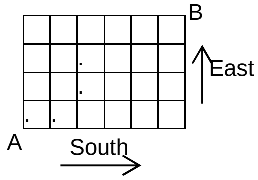
30. A man has (2n + 1) friends.The number of ways in which he can invite at least n + 1 friends for a dinner is 4096. Find the number of friends of the man.

31. A set 'A' has 6 elements. In how many ways can the elements be divided into 2 groups?
(A) 62 (B) 31 (C) 52 (D) 32

32. How many different 5-letter words can be formed which contain 2 consonants and 3 vowels using the letters of the word COURAGE?
(A) 1240 (B) 1440 (C) 1640 (D) 1140

33. There are six doors in a row. In how many ways can these doors be painted with four different colours, such that no two adjacent doors are painted with the same colour?

34. In the figure, the lines represent the one - way roads allowing cars to travel only eastwards or southwards. In how many ways can a car travel from A to B?



35. If there are 10 lines in a plane such that no two lines are parallel and no three lines are concurrent, how many regions are formed with these 10 lines?
(A) 47 (B) 45 (C) 76 (D) 56

Exercise – 9(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Consider the set A = {a, b, c, d, e, f, g, h} Find the number of subsets of A which contain at least 6 elements and include 'c' and 'e'.

2. Find the number of ways of arranging the letters of the word CALENDAR in such a way that exactly two letters are present in between L and D.
(A) 2640 (B) 3600 (C) 2600 (D) 7200

3. In how many ways, can the letters of the word EUROPE be arranged so that no two vowels are together?
(A) 12 (B) 24
(C) 360 (D) Not possible

4. Raju has forgotten his six-digit ID number. He remembers the following: the first two digits are either 1, 5 or 2, 6, the number is even and 6 appears twice. If Raju uses a trial and error process to find his ID number at the most, how many trials does he need to succeed?
(A) 972 (B) 2052 (C) 729 (D) 2051

5. A four-digit number is formed using the digits 0, 2, 4, 6, 8 without repeating any one of them. What is the sum of all such possible numbers?

6. How many four-digit odd numbers can be formed, such that every 3 in the number is followed by a 6?
(A) 108 (B) 2592 (C) 2696 (D) 2700

7. How many four-digit numbers are there between 3200 and 7300, in which 6, 8 and 9 together or separately do not appear?
(A) 1421 (B) 1420 (C) 1422 (D) 1077

8. How many times does the digit 5 appear in the numbers from 9 to 1000? $\boxed{}$

9. A matrix with four rows and three columns is to be formed with entries 0, 1 or 2. How many such distinct matrices are possible?
(A) 12 (B) 36 (C) 3^{12} (D) 2^{12}

10. There are 5 bowls numbered 1 to 5, 5 identical green balls and 6 identical black balls. Each bowl is to be filled by either a green or a black ball and no two adjacent bowls can be filled by green balls. The number of possible arrangements is _____ .
(A) 8 (B) 7 (C) 13 (D) 256

11. How many 4-digit numbers can be formed such that the digit in the hundreds place is greater than that in the tens place? $\boxed{}$

12. In how many ways can 4 postcards be dropped into 8 letter boxes?
(A) ${}^{8}P_{4}$ (B) 4^8 (C) 8^4 (D) 24

13. The number of positive integral solutions of the equation a + b + c + d = 20 is $\boxed{}$

14. There are 4 identical oranges, 3 identical mangoes and 2 identical apples in a basket. The number of ways in which we can select one or more fruits from the basket is
(A) 60 (B) 59 (C) 57 (D) 55

15. In how many ways can 5 boys and 3 girls sit around a table in such a way that no two girls sit together?
(A) 480 (B) 960 (C) 320 (D) 1440

16. Find the number of ways in which the letters of the word MATHEMATICS can be arranged so that all Ms are together and all Ts are together.
(A) 11! (B) $\frac{11!}{2!2!2!}$ (C) $\frac{9!}{2!2!2!}$ (D) $\frac{9!}{2!}$

17. In how many arrangements of the word MATHEMATICS, the two A's are separated?
(A) $\frac{10!}{2!2!2!}$ (B) $\frac{9!}{2!2!2!}$
(C) $9 \times 10\,!$ (D) $\frac{9 \times 10\,!}{2!2!2!}$

18. Consider the word INSTITUTE.
(i) In how many ways can 5 letters be selected from the word?
(A) 41 (B) 30 C) 36 (D) 40

(ii) How many arrangements can be made by taking 5 letters from the word?
(A) 2790 (B) 2250 (C) 4320 (D) 7200

19. The letters of the word AGAIN are permuted in all possible ways and are arranged as in a dictionary. What is the 28^{th} word?
(A) GAIAN (B) GAINA (C) GANIA (D) NGAIA

20. If all possible five-digit numbers that can be formed using the digits 4, 3, 8, 6 and 9 without repetition are arranged in the ascending order, then the position of the number 89634 is $\boxed{}$

21. In which regular polygon, is the number of diagonals equal to two and half times the number of sides?
(A) Heptagon (B) Pentagon
(C) Decagon (D) Octagon

22. In how many ways can 12 distinct pens be divided equally
(i) among 3 children?
(A) $\frac{12!}{(3!)^4}$ (B) $\frac{12!}{(4!)^3\,3!}$ (C) $\frac{12!}{3!\,4!}$ (D) $\frac{12!}{(4!)^3}$

(ii) into 3 parcels?
(A) $\frac{12!}{(4!)^4}$ (B) $\frac{12!}{(4!)^3}$
(C) $\frac{12!}{3!\,4!}$ (D) $\frac{12!}{(4!)^3\,3!}$

23. In a certain question paper, a candidate is required to answer 5 out of 8 questions, which are divided into two parts containing 4 questions each. He is permitted to attempt not more than 3 from any group. The number of ways in which he can answer the paper is
(A) 24 (B) 96 (C) 48 (D) 84

24. The sides PQ, QR and RS of ΔPQR have 4, 5 and 6 points (not the end points) respectively on them. The number of triangles that can be constructed using these points as vertices is $\boxed{}$

25. There are 8 different books and 2 identical copies of each in a library. The number of ways in which one or more books can be selected is
(A) 2^8 (B) $3^8 - 1$ (C) $2^8 - 1$ (D) 3^8

26. The number of four-digit telephone numbers that have at least one of their digits repeated is
$\boxed{}$

27. There are five balls, each of a different colour- Blue, Green, Red, Pink and Black. There are five boxes of the same five colours. In how many ways can one ball be placed in each box such that no ball is placed in a box of the same colour?
(A) 40 (B) 44 (C) 42 (D) 36

28. A question paper consists of 5 problems, each problem having 3 internal choices. In how many ways can a candidate attempt one or more problems?
(A) 63 (B) 511 (C) 1023 (D) 15

29. Six points are marked on a straight line and five points are marked on another line which is parallel to the first line. How many straight lines, including the first two, can be formed with these points?
(A) 29 (B) 33 (C) 55 (D) 32

30. The number of sequences in which 7 players can throw a ball, so that the youngest player may not be the last is
(A) 4000 (B) 2160 (C) 4320 (D) 5300

31. Sixteen guests have to be seated around two circular tables, each accommodating 8 members. 3 particular guests desire to sit at one particular table and 4 others at the other table. The number of ways of arranging these guests is

(A) ${}^{9}C_5$ (B) $\frac{9!(7!)}{4!5!}$

(C) $\frac{9!(7!)^2}{4!5!}$ (D) $(7!)^2$

32. In how many ways is it possible to choose two white squares so that they lie in the same row or same column on an 8×8 chessboard?

33. The number of non-negative integral solutions to the equation a + b + c = 14 is
(A) 78 (B) 45 (C) 120 (D) 110

34. In how many ways can the letters of the word RESULT be arranged so that the vowels appear in the even places only?

35. In how many ways can one or more of 5 letters be posted into 4 mail boxes, if any letter can be posted into any of the boxes?
(A) 5^4 (B) 4^5 (C) $5^5 - 1$ (D) $4^5 - 1$

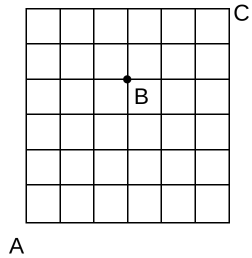
36. From a group of people, the number of ways of selecting 8 people is same as the number of ways of selecting 12 people. In how many ways can 18 people be selected from this group?

37. In how many ways can 12 differently coloured beads be strung on a necklace?

(A) 12! (B) $\frac{12!}{2}$ (C) 11! (D) $\frac{11!}{2}$

38. Find the number of ways in which the letters of the word INCLUDE can be permuted so that no two vowels appear together.
(A) 7! – 5! 3! (B) 7! – 4! 2!
(C) $4! \times 3!$ (D) $\frac{4! \times 5!}{2}$

39. In the figure, in how many ways can a person walk from A to C via B, if he walks either rightwards or upwards?



40. Manavseva, a voluntary organisation, has 50 members who plan to visit 3 slums in an area. They decide to divide themselves into groups of 25, 15 and 10. In how many ways can the group division be made?

(A) 25! 15! 10! (B) $\frac{50!}{25!\,15!\,10!}$

(C) 50! (D) 25! + 15! + 10!

Key

Concept Review Questions

1. (a) B
 (b) C
 (c) D
 (d) B
 (e) C
 (f) A
2. 12
3. 9
4. C
5. B
6. 600
7. D
8. D
9. C
10. B
11. A
12. 3125
13. C
14. B
15. 336
16. A
17. D
18. C
19. B
20. 1365
21. A
22. 120
23. D
24. B
25. C
26. A
27. (a) A
 (b) C
28. (a) A
 (b) B
29. 210
30. A
31. A
32. (a) 1296
 (b) 204
33. (a) B
 (b) B
34. 6435
35. A
36. C
37. 10800
38. C
39. B
40. 1680
41. B
42. B
43. 127
44. D
45. D

Exercise – 9(a)

1. 576
2. D
3. B
4. D
5. D
6. 42
7. D
8. 5760
9. C
10. B
11. B
12. B
13. 490
14. C
15. (i) D
    (ii) B
16. 132
17. D
18. B
19. A
20. 30
21. (a) 18
    (b) 270
22. B
23. A
24. 155
25. C
26. 924
27. B
28. D
29. D
30. 13
31. B
32. B
33. 972
34. 210
35. D

Exercise – 9(b)

1. 22
2. B
3. D
4. B
5. 519960
6. C
7. D
8. 299
9. C
10. C
11. 4050
12. C
13. 969
14. B
15. D
16. D
17. D
18. (i) A
    (ii) B
19. B
20. 89634
21. D
22. (i) D
    (ii) D
23. C
24. 421
25. B
26. 4464
27. B
28. C
29. D
30. C
31. C
32. 96
33. C
34. 144
35. C
36. 190
37. D
38. D
39. 350
40. B