CHAPTER – 8
LOGARITHMS

In the equation $a^x = N$, we are expressing N in terms of a and x. The same equation can be re-written as, $a = N^{1/x}$. Here we are expressing a in terms of N and x. But, among a, x and N, by normal algebraic methods known to us, we cannot express x in terms of the other two parameters a and N. This is where logarithms come into the picture. When $a^x = N$, then we say x = logarithm of N to the base a, and write it as $x = \log_a N$. The definition of logarithm is given as: "the logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number."
Thus,

$$\text{if } a^x = N \text{ then } x = \log_a N$$

This is read as "log N to the base a".

In the above equation, N is a **POSITIVE NUMBER** and a is a **POSITIVE NUMBER OTHER THAN 1**.

This basic definition of logarithm is very useful in solving a number of problems on logarithms.

Example of a logarithm : $216 = 6^3$ can be expressed as $\log_6 216 = 3$.

Since logarithm of a number is a value, it will have an "integral" part and a "decimal" part. The integral part of the logarithm of a number is called the CHARACTERISTIC and the decimal part of the logarithm is called the MANTISSA.

Logarithms can be expressed to any base (positive number other than 1.) Logarithms in one base can be converted to logarithms in any other base. (One of the formulae given below will help do this conversion). However, there are two types of logarithms that are commonly used.

i) Natural Logarithms or Napierian Logarithms: These are logarithms expressed to the base of a number called "e."

ii) Common Logarithms: These are logarithms expressed to the base 10. For most of the problems under LOGARITHMS, it is common logarithms that we deal with. In examinations also, if logarithms are given without mentioning any base, it can be taken to be logarithms to the base 10.

The following should be remembered by the student regarding Common Logarithms.

I. The characteristic of the common logarithm of a number greater than unity is positive and is one less than the number of digits in its integral part.

For example, the characteristic of log245 will be 2 (because the number has 3 digits and the characteristic should be one less than the number of digits in the number).

Similarly, the characteristic of log4758 will be 3.

II. The characteristic of the common logarithm of a number between 0 and 1 is negative and its magnitude is one more than the number of zeroes immediately after the decimal point.

For example, the characteristic of log0.0034 will be –3 or $\bar{3}$.

III. The mantissas are the same for the logarithms of all numbers which have the same significant digits in the same order. The values of mantissas, which are necessary to solve a problem, are usually given in the problem itself, as part of data.

Let us look at the value of log0.02 given that the value of log2 is 0.3010.

The characteristic of log 0.02 will be $\bar{2}$. The mantissa will be the same as that for log2. Hence the value of log0.02 is $\bar{2}.3010$. Here the mantissa 0.3010 is positive while the characteristic is negative. But the same can be written with a negative mantissa, in which case, the characteristic will be –1. Let us see how to do this conversion.

$\bar{2}.3010 = -2 + 0.3010$
$= -2 + 1 - 1 + 0.3010$ (by adding and subtracting 1)
$= \{-2 + 1\} + \{-1 + 0.3010\}$
$= -1 + (-0.6990) = -1.6990$

So, the value of log 0.02 can be written as $\bar{2}.3010$ or as –1.6990 and both are the same.

Similarly, given that the value of log3 is 0.4771, we can find out the value of log 0.003. Since there are two zeroes in this number immediately after the decimal point, the characteristic is $\bar{3}$ and the mantissa is positive and the same as that for log3. So the value of log 0.003 is $\bar{3}.4771$. This can also be written as –2.5229 (You should do this conversion of $\bar{3}.4771$ into –2.5229 in the same way we did for log 0.02).
Given below are some **important rules** in logarithms:

i) $\log_a a = 1$ (logarithm of any number to the same base is 1)
ii) $\log_a 1 = 0$ (log of 1 to any base other than 1 is 0)
iii) $\log_a (mn) = \log_a m + \log_a n$
iv) $\log_a (m/n) = \log_a m - \log_a n$
v) $\log_a m^p = p \times \log_a m$
vi) $\log_a b = \dfrac{1}{\log_b a}$
vii) $\log_a m = \dfrac{\log_b m}{\log_b a}$
viii) $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$
ix) $a^{\log_a N} = N$
x) $a^{\log b} = b^{\log a}$

You should memorize these rules because they are very helpful in solving problems.
Like in the chapter on INDICES, in LOGARITHMS also there will be problems on

i) Simplification using the rules listed above and
ii) Solving for the value of an unknown given in an equation

The following rules also should be remembered while solving problems on logarithms:

Given an equation $\log_a M = \log_b N$,

(i) if M = N, then a will be equal to b; if $M \neq 1$ and $N \neq 1$.
(ii) if a = b, then M will be equal to N.

Please note that unless otherwise specified, all the logarithms are taken to the base 10.

Examples

8.01. Simplify : log 315 + 4 log 25 – 6 log 9 – 3 log 49.

Sol: log 315 + 4 log 25 – 6 log 9 – 3 log 49
= log ((9) (7) (5)) + 4 log 5^2 – 6 log 9 – 3 log7^2
= log 9 + log 7 + log 5 + 8 log 5 – 6 log 9 – 6 log 7
= 9 log 5 – 5 log 7 – 5 log 3^2
= 9 log 5 – 5 log 7 – 10 log 3

8.02. Simplify : log 700 + log 1280 + 3 log 25.

Sol: log 700 + log 1280 + 3 log 25
= log (7) $(10)^2$ + log (2^7) (10) + 3 log 5^2
= log 7 + 2 + 7 log2 + 1 + 6 log 5
= 3 + log 7 + log 2 + 6 (log 2 + log 5)
= 9 + log 14

8.03. Solve for x : $\log_{10} 20x = 4$

Sol: Given that $\log_{10} 20x = 4$
$\Rightarrow 20x = 10^4 = 10000$
$\therefore x = 500$

8.04. Solve for x : log 3x – log 6 = log 12

Sol: log 3x – log 6 = log 12
log 3x = log 12 + log 6 = log 72
3x = 72
x = 24.

8.05. Solve for x : log (x + 3) + log (x – 3) = log 72

Sol: log (x + 3) + log (x – 3) = log 72
log (x + 3) (x – 3) = log 72
(x + 3) (x – 3) = 72
$x^2 = 81$
x = 9 (If x = – 9, log (x – 3) would be undefined)

8.06. Express $\log \frac{\sqrt{a^3}}{b^6 c^4}$ in terms of log a, log b and log c.

Sol: $\log \frac{\sqrt{a^3}}{b^6 c^4}$

$= \log \sqrt{a^3} - \log b^6 - \log c^4$

$= \frac{3}{2} \log a - 6 \log b - 4 \log c$

8.07. Find the number of digits in 294^{20} given that log 6 = 0·778 and log 7 = 0·845

Sol: log 294^{20} = 20 log (7^2 6)
= 20 (2 log 7 + log 6)
= 20 (2 (0·845) + 0·778) = 20 (1.69 + 0.778)
= 49·36
Characteristic = 49.
$\therefore$ 294^{20} has 50 digits.

8.08. Obtain an equation between x and y, without involving logarithms, if 3 log x = 4 log y + 5.

Sol: 3 log x = 4 log y + 5
log x^3 = logy^4 + log 10^5
log x^3 = log $y^4 10^5$
$\therefore$ $x^3 = 10^5 y^4$

8.09. Find the value of $\log_{\sqrt[3]{2}} 32 \sqrt[3]{16}$.

Sol: $\log_{\sqrt[3]{2}} 32 \sqrt[3]{16} = \log_{2^{1/3}} 2^5 \left(2^4\right)^{\frac{1}{3}}$

$= \log_{2^{1/3}} \left(2^{\frac{1}{3}}\right)^{19} = 19$

8.10. Find the number of zeros after the decimal point in $\left(\frac{3}{4}\right)^{500}$, given that log 3 = 0·4771 and log 2 = 0·3010.

Sol: $\log \left(\frac{3}{4}\right)^{500} = 500 \left(\log \frac{3}{4}\right)$
= 500 (log 3 – 2 log 2)
= 500 (0·4771 – 2 (0·3010)) = – 62.4500.
$\therefore$ Number of zeros after the decimal point is 62.

8.11. If log 2 = 0·301, find the values of log 1250, log 0·001250 and log 125000.

Sol: $\log 1250 = \log \frac{10000}{8}$
= 4 log 10 – 3 log 2 = 4 – 3 (0·3010) = 3·097
$\log 0{\cdot}001250 = \log \frac{1250}{10^6} = 3{\cdot}097 - 6 = -2{\cdot}903$
log 125000 = log (1250) (100)
= log 1250 + 2 = 5·097

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify $\log_{(32)(18)}(48)(12)$.

2. Simplify the following:

(i) $\log_{9\sqrt{3}} 243$.

(A) 2 (B) 5 (C) 25/2 (D) 1/4

(ii) $\log_{\sqrt{3125}} 625$

(A) 1/10 (B) 8/5 (C) 4/5 (D) 2/5

(iii) $\log_{3125}5^{25} + \log_{125}(25)^{60}$

(A) 85 (B) 145 (C) 45 (D) 145/4

(iv) $\log_{\sqrt{32}} (1/1024)$

(A) 1 (B) –1 (C) –1/4 (D) –4

3. Simplify the following:

(i) log7 + 3 log2 – log14 – log4

(A) 0 (B) log14 (C) 2log56 (D) 1

(ii) 2log7 – log81 + log189 – log343 =

(A) 0 (B) log21 (C) log7 (D) –log3

(iii) $5 + \log_{13}(1/2197)$

(A) 8 (B) 2 (C) 1 (D) 3

(iv) log25 + log49 + log175 + 7log2 – 3log14

(A) 2log7 (B) 3 (C) 4 (D) 4log5

(v) 2log25 + 8log16 – 5log32 + 3log5

(A) 16 (B) 9 (C) 25 (D) 7

4. Simplify: $\log_2(\log_2(\log_2(\log_{11}(14641)^4)))$

5. Simplify:

2log169 – 3log143 + log1100 – log1300 + log121

(A) 0 (B) 1

(C) 10 (D) None of these

6. Simplify:

$\log_3 81 + \log_9(1/243) + \log_{27}6561 + \log_{729}(81)^{5/4}$.

(A) 35/6 (B) 20/3 (C) 5 (D) 6

7. Simplify:

$$\frac{\log_5 3125 \times \log_3 2187 \times \log_4 1024}{\log_6(7776) \times \log_{11^3}(14641)^5}$$

8. Simplify: $\log_{(343)^4}(2401)^3$.

(A) 2/3 (B) 3/4 (C) 1 (D) 0

9. Simplify: $729^{\log_{9^3}\sqrt{512}}$.

(A) $2^{9/2}$ (B) 2^9 (C) 2^{18} (D) 2^{27}

10. What is the value of $\log_3 x^0$ where $x \neq 0$?

11. $\log_9 27^2$ =

(A) 3 (B) 6 (C) 1 (D) 2

12. $\dfrac{\log_{11} 64}{\log_{11} 81}$ =

(A) $\log_3 2$ (B) $\log_2 3$

(C) $\dfrac{3\log 2}{\log 3}$ (D) $\log_9 8$

13. What is the value of $\log_{(1/5)}$ 0.0000128?

(A) –7 (B) –5 (C) 5 (D) 7

14. $\log_{0.0625} 2$ =

(A) –1/4 (B) 1/4 (C) 4 (D) –4

15. $\log_3 4 + \log_3 16 = \log_3 x$. Find x.

16. $\log_2 72 - \log_2 3 = \log_2 x$. Find x.

(A) 69 (B) 75 (C) 24 (D) 216

17. If $(\log_{27} 8)(\log_x 3) = 1$, find x.

(A) 2 (B) 4 (C) 8 (D) 16

18. If $4^{\log_4 5^2} = x$, find x.

19. Solve for x:

(i) $\dfrac{\log 729}{\log 81} = \log\sqrt{x}$

(A) $\sqrt[3]{10000}$ (B) 1000

(C) $\sqrt{1000}$ (D) 100

(ii) logx + log4 + log50 = 3

(A) 5 (B) 10 (C) 1/2 (D) 2

(iii) $\log(x - 1) + \log(x^2 + x + 1) = \log 7$

(A) 8 (B) 4 (C) 2 (D) 0

(iv) $\log(x^2 - 4) - \log(x + 2) = \log 3$

(A) 1 (B) 5 (C) 8 (D) 10

(v) $\log_{\sqrt{3}}(x^3 - 18) = 4$

(A) 4 (B) 9 (C) 3 (D) $\sqrt[3]{21}$

(vi) log(3x + 4) + log4 – log7 = log(x + 3)

(A) 3 (B) 2 (C) 5 (D) 1

(vii) log50 + log(5x + 1) = log(5x – 7) + $2\log_{10}10$

(A) 1 (B) 15 (C) 5 (D) 3

(viii) $\dfrac{\log 81}{\log 3} = \log_7(7x)$

(A) 49 (B) 343 (C) 7 (D) 2187

20. If log (7x + 8) – 2 = log (x + 5) – log25, then find x.

21. If $\log_7(x-7) + \log_7(x^2+7x+49) = 4$, then x =
(A) 196 (B) 7 (C) 49 (D) 14

22. Find the value of $x^2 - y^2$, if $\log_y(x-1) + \log_y(x+1) = 2$.
(A) 2 (B) 2y
(C) 1 (D) 2xy

23. Find x if $\frac{2\log 6561}{\log 243} = \log_{32}x + 2$. ☐

24. Find the value of p if
$\log_{3125}p \times \log_9 25 \times \log_{343}243 \times \log_2 49 = 4$
(A) 256 (B) 64 (C) 512 (D) 128

25. If $a > 1$, $\log_a a + \log_{a^{\frac{1}{2}}} a + \log_{a^{\frac{1}{3}}} a + \ldots\ldots + \log_{a^{\frac{1}{20}}} a =$ ☐

26. If $\frac{\log a}{5} = \frac{\log b}{6} = \frac{\log c}{7}$, then b^2 =
(A) ac (B) a^2 (C) bc (D) ab

27. If $\log_b a = \log_c a$ and both b and c exceed a, which of the following is true?
(A) b is equal to c.
(B) b is not equal to c.
(C) b need not be equal to c.

28. If x is the product of the logarithms of the first 10 natural numbers, which of the following is true?
(A) $x = 1$ (B) $x > 1$
(C) $x < 1$ (D) None of these

29. Find the integral part of $\log_2 20000$.
(A) 4 (B) 5 (C) 14 (D) 15

30. If N is an 18-digit number, find the integral part of $\log_{10} N$. ☐

Exercise – 8(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. What is the value of x, if $y = \log_7 \log_7 x$ and $5^{\left(\frac{y}{\log_7 5}\right)} = 2$? ☐

2. If $\log_a x = \log_b x$, what is $\log_a b$ if $x \neq 1$?
(A) x (B) x^2 (C) 1 (D) 1 – x

3. If $\log_{10} x = y$ and $\log_e 10 = 1/m$, what is x (in terms of y, e and m)?
(A) y/m (B) $\log_m y$ (C) $e^{y/m}$ (D) $m \log_e y$

4. What is the value of
$\log\left(\sqrt{b\sqrt{b\sqrt{b\sqrt{b.....\infty}}}}\right)\left(\sqrt{a\sqrt{a\sqrt{a\sqrt{a.....\infty}}}}\right)$?
(A) (log a)/(log b) (B) ∞
(C) 0 (D) $1/2 \log_{10} a$

5. If (log tan5°) (log tan10°) (log tan 15°)…….....(l og tan 60°) = x, what is the value of x?
(A) $\log(\sin 5°)^{12}$ (B) 1
(C) 0 (D) log(cos60°)

6. Solve for x if $\log_2 \log_2 \log_x 6561 = 2$ and $x > 0$.
(A) 3 (B) 2 (C) $\sqrt{7}$ (D) $\sqrt{3}$

7. Which of the following is true?
(A) $1/4 > \log_{10} 2 > 1/8$ (B) $1/2 > \log_{20} 3 > 1/3$
(C) $1/9 < \log_{10} 3 < 1/3$ (D) $1/2 > \log_{10} 4 > 1/4$

8. If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$ what is the value of xy + yz + zx?
(A) $\log_{xyz} abc$ (B) xyz
(C) $\log_{abc} xyz$ (D) 1

9. If $a^x = b^y = c^z$ and $y^2 = xz$ what is the value of $\frac{(\log_b a)}{(\log_c b)}$?
(A) 1 (B) $\log_y x/\log_z y$
(C) xz/y (D) xyz

10. If a is a perfect square, what is the value of a given that $a^x = \frac{1}{8}$ and $x = \frac{3}{2}[(\log_2 a) - 3]$? ☐

11. If $f(x) = \frac{e^{2x} + e^{-2x}}{2}$, what is the value of $f(\log_e 3)$?
(A) e^6 (B) $9/e^{6z}$ (C) 41/9 (D) 1

12. If a, b, c are distinct, what is the value of abc if $(\log_b a)(\log_c a) + (\log_a b)(\log_c b) + (\log_a c)(\log_b c) - 3 = 0$? ☐

13. If $\log_{12} 27 = a$, what is the value of $\log_6 16$?
(A) $\frac{4(3-a)}{3+a}$ (B) $\frac{3(9-4a)}{3-2a}$
(C) $a^4 - 9$ (D) $\frac{3+a}{3-a}$

14. Simplify: $\log\left\{\frac{\sqrt[3]{a^4}}{\sqrt{b^3c}} \Big/ \left(\frac{a^2b^3}{c^3}\right)^{1/6}\right\}$
(A) loga – logb
(B) $\frac{1}{2}$ loga + $\frac{1}{3}$ logb – logc
(C) $\frac{1}{3}$ loga + $\frac{1}{2}$ logc
(D) loga – 2 logb

15. If $z \log y + \log(\log x) = \log[\log x + \log y + \log z]$, which of the following is true, given that x, y and z are distinct?
(A) $x^{z^y} = 1$ (B) $x^{y^z} = xyz$
(C) $x^y = z$ (D) $y^2 = z$

16. (a) If $\log_6 161 = a$, $\log_6 23 = b$, what is the value of $\log_7 6$ in terms of a and b?
(A) a/b (B) a + b
(C) 1/(a – b) (D) b/a

(b) If $\log_4 3 = 1/a$, what is
$\log_{\sqrt{3}} 36 + \log_{\sqrt{3}} 36 + ...20$ times ?
(A) 9a/4 (B) 80 + 40a
(C) $\frac{\sqrt{3}a}{36}$ (D) $\frac{9a+4}{\sqrt{3}}$

17. If $\frac{1}{\frac{\log 2}{\log x} + \frac{\log 4}{\log x} + \frac{\log 8}{\log x}} = \log_2 y$, then what is the relation between x and y?
(A) $x = y^6$ (B) $x = y^3$ (C) $x = y^2$ (D) x = y

18. Solve for x, if $\log_2(1 - 1/2^x) = x - 2$.
☐

19. If $\log_x 2 . \log_{(x/16)} 2 = \log_{(x/64)} 2$, then the value of x is
☐ or ☐

20. If $\log_2[5(2^x) + 1)]$, $\log_4(2^{1-x} + 1)$, and 1 are in arithmetic progression, then what is the value of x?
(A) 1 (B) $\log_2 \sqrt{5}$
(C) $\log_2(2/5)$ (D) $\log_{\sqrt{2}} 5$

21. The value of

$$\frac{1}{\log_a x}+\frac{1}{\log_{\sqrt{a}} x^{1/4}}+\frac{1}{\log_{\sqrt[3]{a}} x^{1/9}}+\ldots\ldots+\frac{1}{\log_{\left(\sqrt[20]{a}\right)}\left(x^{1/400}\right)}$$ is

(A) $\log_x a^{210}$ (B) $x^{20}+a$
(C) $\log_e x$ (D) $\log_a x^{44}$

22. If $49^{\log_7 5}+\log_2(2^{2^{2^2}})=31+\log_{10}x$, find x.

(A) 0 (B) 1
(C) 10 (D) 10^{10}

23. If $\log_{10}1234=3.0913$, $\log_{10}769874=5.88642$, find the value of $\sqrt[8]{0.000000001234}$.

(A) 0.0769874 (B) 0.030913
(C) 0.00769874 (D) 0.0030913

24. The logarithm of 7623 is 3.8821. Find the number whose logarithm is –0.1179.

(A) 0.08821 (B) 0.7623
(C) 0.07623 (D) 0.8821

25. (a) How many digits are there in $(441)^{50}$ if log3 = 0.4771 and log7 = 0.845? ☐

(b) The number of zeros immediately after the decimal point of N is 5. If 0 < N < 1, what is the least integer greater than or equal to $\log_{10}$ N?

(A) –5 (B) –4
(C) –6 (D) Cannot be determined

26. If $0 < a \le b < 1$, then the value of $\left(\log_a\frac{a^3}{b^2}+\log_b\frac{b^3}{a^2}\right)$ can be

(A) 1 (B) 3
(C) Both A and B (D) Neither A nor B

27. If $\log_5\log_2\log_3\left(\sqrt{x+14}+\sqrt{x-13}\right)=0$, then find the value of x. ☐

28. If $\log|x^3+y^3|-\log|x^2-xy+y^2|+\log|x^3-y^3|-\log|x^2+xy+y^2|=\log 247$, then how many integral values can (x, y) have? ☐

29. $\log_6(x+18)>\log_6 x+\log_6 1.06$ only when x is less than

(A) 250 (B) 290 (C) 300 (D) 150

30. If $\log_7\left(\frac{9}{4}\right)+3\log_{343}\left(\frac{16}{x}\right)\ge 2$, then the range of x is _____ .

(A) $\left(0,\frac{36}{49}\right]$ (B) $\left(-\infty,\frac{36}{49}\right]$
(C) $\left[\frac{36}{49},\infty\right)$ (D) $(2,\infty)$

Exercise – 8(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. What is $\log_{25}125-\log_{125}25$?

(A) 1
(B) $\log_{25}100$
(C) 3/25
(D) 5/6

2. What is the relation between a and b, if $a\log_{0.2}x=b\log_{0.008}x$ and $x\ne 1$?

(A) $a=b^3$ (B) $a=b/3$
(C) $a=100b$ (D) $\sqrt{a}=(b)^{1/3}$

3. If x, y, z are consecutive positive integers, which of the following is the value of log(xz + 1)?

(A) log(x + y + z) (B) 0
(C) 1 – logy (D) 2 logy

4. If the logarithm of x to the base equal to square root of y is 2, what is the logarithm of the cube of x to the base equal to cube root of y? ☐

5. If $5^y=(0.5)^x=1000$, what is the value of $\left(\frac{1}{y}-\frac{1}{x}\right)$?

(A) 1/3 (B) 0.95 (C) 10 (D) 1

6. Simplify:

$15\log\left(\frac{48}{35}\right)+9\log\left(\frac{80}{243}\right)-15\log\left(\frac{64}{63}\right)+6\log 2.5$.

(A) 1 (B) 0
(C) –log3 (D) log2 + log7

7. If $\log_{(x+y)}(x-y)=3$, then what is the value of $\log_{\left(x^2-y^2\right)}\left(x^2+2xy+y^2\right)$?

(A) 0 (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) 1/2

8. If $\log_r 6=a$, $\log_r 3=b$, what is the value of $\log_r(r/2)$ in terms of a and b?

(A) 1 – a + b (B) ab – a – b
(C) a + b (D) 1

9. If $a^2+4b^2=12ab$, what is the value of log(a + 2b)?

(A) log(a/2) + log(b/2) + log 2
(B) (log a + log b – log 2) 1/2
(C) 1/2 (log a + log b + 4 log 2)
(D) 1/2(log a – log b + 4 log 2)

10. If $\log_2 x = \dfrac{\log_{10} 512}{\log_{10} 64}$ then find x.

(A) $2\sqrt{2}$ (B) 3/2
(C) 8 (D) 4

11. If $\log_{10}(2x + 3) - 1 = \log_{10} x$, then find x.

(A) $\dfrac{2}{7}$ (B) $\dfrac{3}{4}$
(C) $\dfrac{7}{8}$ (D) $\dfrac{3}{8}$

12. Which of the following can never be a value of $\log_{2x}\dfrac{2x}{3y} + \log_{3y}\dfrac{3y}{2x}$ given that $2x \geq 3y$ and $y > \dfrac{1}{3}$?

(A) –2·5 (B) –2 (C) 1 (D) –1·5

13. If $\dfrac{1}{x-1} = \log_{bc} a$, $\dfrac{1}{y-1} = \log_{ac} b$ and $\dfrac{1}{z-1} = \log_{ab} c$, then which of the following is true?

(A) $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$ (B) $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$
(C) x + y + z = 1 (D) xy + yz + zx = 1

14. If $a = \sqrt{b} = \sqrt[3]{c} = \sqrt[4]{d} = \sqrt[5]{e}$, then find the value of $\log_a (abcde)$. ☐

15. If $\dfrac{1}{3}\log_7 x - 3\log_7 y = 1 + \log_{0.125} 2$, then the relation between x and y is

(A) $y = \dfrac{7}{x^3}$ (B) $y = \dfrac{49}{x^3}$
(C) $x = 49y^9$ (D) $x = \dfrac{49}{y^9}$

16. If $\log_4 3$, $\log_4 (3^m - 2)$ and $\log_4\left(3^m - \dfrac{8}{3}\right)$ are in arithmetic progression, then the number of possible values of m is ☐

17. If log2 = 0·3010, then find the value of log6250.
☐

18. If $\log_x 3 \log_{\frac{x}{81}} 3 = \log_{\frac{x}{729}} 3$, then find the value(s) of x.

(A) 9 (B) 27
(C) 243 (D) Both (A) and (B)

19. If x > 0 and x ≠ 1, then the number of values of $\log_k x$ satisfying the equation $\log_{kx} x^2 + \log_{kx^2} x^3 = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

20. If $x^y = y^z = z^x$ and (x, y, z) > 0, then $\dfrac{1}{x}\log_z xyz + \dfrac{1}{y}\log_x xyz + \dfrac{1}{z}\log_y xyz =$

(A) $3\left[\dfrac{xy + yz + zx}{xyz}\right]$ (B) $\dfrac{xy + yz + zx}{xyz}$
(C) $\dfrac{x + y + z}{xyz}$ (D) $\dfrac{xy + yz + zx}{x + y + z}$

21. If $\log_8 [\log_5 (\log_3 x)] = 0$, then find x. ☐

22. Which of the following is a possible value of x if $\log_3 x^2 - \log_3 x\sqrt{x} = 8\log_x 3$?

(A) $\dfrac{1}{81}$ (B) $\dfrac{1}{243}$
(C) 243 (D) 9

23. If $\log_x 162 = m$ and $\log_x 72 = n$, then what is the value of $\log_x 7776$ in terms of m and n?

(A) $\dfrac{m + 3n}{m + 5n}$ (B) $\dfrac{3m - 5n}{m + 2n}$
(C) $\dfrac{m + 3n}{2}$ (D) $\dfrac{3m - 5n}{2}$

24. For how many positive integer values of x less than 100, is $\log_{5x}(4x - 15) > \dfrac{1}{2}$? ☐

25. If $49^{\left\{\log_7 \frac{1}{3} + 2\log_x \sqrt{3}\right\}} = \dfrac{1}{3}$, then x =

(A) $\dfrac{1}{49}$ (B) $\dfrac{1}{7}$ (C) 49 (D) 7

26. Simplify: $\dfrac{\log_m p . \log_n p}{\log_m p + \log_n p}$.

(A) 1 (B) $\log_p (m + n)$
(C) $\log_p mn$ (D) $\log_{mn} p$

27. Find the sum of all the possible values of m, given that $m^n = \dfrac{1}{5^6}$ and $n = \dfrac{6}{5}\left[(\log_5 m) - 6\right]$. ☐

28. If x > 1, then find the minimum value of $3\log_{20} x - \log_x 0{\cdot}125 + 3\log_x 10$.

(A) 6 (B) 4 (C) 2 (D) 3

29. Find the sum of all the possible values of x, given that $\dfrac{\log 10 + \log(x^2 + 5x)}{\log\sqrt{60}} = 2$. ☐

30. Find the number of digits in 1764^{50}, if log 2 = 0·301, log 3 = 0·4771 and log 7 = 0·845.

(A) 161 (B) 163 (C) 162 (C) 164

31. Find the number of zeros immediately after the decimal point in $(5/6)^{400}$, given that log2 = 0.3010 and log3 = 0.4771. ☐

32. Given that log3 = 0.4771 and log2 = 0.3010. Find x, if x > 0 and $5^x \cdot 27^{1-x} = 0.1$.

(A) 1.84 (B) 1.32 (C) 0.84 (D) 3.32

33. Solve for x, if $\log_x [\log_5 (\sqrt{x+5} + \sqrt{x})] = 0$.

34. If a, b, c, d are integers satisfying $\log_b a = \frac{2}{3}$, $\log_d c = \frac{4}{5}$ and c – a = 7, then b – d equals _____.

(A) –5 (B) –3 (C) 5 (D) 3

35. If a =$\log_4 31$, then _____ .

(A) a < 2 (B) 2 < a < 2.5

(C) 2.5 < a < 2.8 (D) 2.8 < a

Key

Concept Review Questions

1. 1
2. (i) A
   (ii) B
   (iii) C
   (iv) D
3. (i) A
   (ii) D
   (iii) B
   (iv) C
   (v) D
4. 1
5. A
6. C
7. 5.25
8. C
9. A
10. 0
11. A
12. D
13. D
14. A
15. 64
16. C
17. A
18. 25
19. (i) B
    (ii) A
    (iii) C
    (iv) B
    (v) C
    (vi) D
    (vii) D
    (viii)B
20. 4
21. D
22. C
23. 64
24. B
25. 210
26. A
27. C
28. C
29. C
30. 17

Exercise – 8(a)

1. 49
2. C
3. C
4. A
5. C
6. D
7. B
8. B
9. A
10. 4
11. C
12. 1
13. A
14. D
15. B
16. (a) C
    (b) B
17. A
18. 1
19. 4 or 8
20. C
21. A
22. D
23. A
24. B
25. (a) 133
    (b) D
26. A
27. 22
28. 16
29. C
30. A

Exercise – 8(b)

1. D
2. B
3. D
4. 9
5. A
6. B
7. D
8. A
9. C
10. A
11. D
12. C
13. B
14. 15
15. C
16. 2
17. 3.796
18. D
19. B
20. A
21. 243
22. A
23. C
24. 94
25. C
26. D
27. 3130
28. A
29. –5
30. B
31. 31
32. D
33. 6
34. A
35. B