CHAPTER – 10
PROBABILITY

PROBABILITY

This is an important topic for MBA entrance tests. Besides the entrance tests, the topic is an important part of the management courses themselves. Therefore, students aspiring to be future managers need a sound understanding of the basics.

Natural phenomena are of two types - deterministic and probabilistic. For example, the direction in which or the time at which the sun rises everyday is a deterministic phenomenon, while, where or when it may rain is a probabilistic phenomenon. Similarly, **experiments** (the operations of doing or observing something resulting in some final outcomes) are of two types - those in which the outcome is definite and others in which the actual outcome may be any one of all the possible outcomes. For example, hydrogen is allowed to react with oxygen. They react in a certain ratio and produce water. The outcome is definite-while if a coin is tossed, it may turn up showing either heads or tails, the outcome may be one of the two possible outcomes. Experiments of the second kind are called **random experiments** (RE). One instance of such an experiment is a **trial**. The set of all possible outcomes for a particular random experiment is its **sample space** S. This corresponds to the concept of the universal set in set theory. Each outcome is said to be a point in S. Thus for the experiment of tossing a coin, the sample space is the set of the two outcomes - heads and tails. For the experiment of rolling a die, it is {1, 2, 3, 4, 5, 6}. For drawing a card from a deck of cards it is the set of all the 52 possible outcomes - corresponding to the 52 cards. Any subset of the sample space is a **simple event**. Two (or more) events which occur for two (or more) different experiments - or for two (or more) trials of the same experiment are called **compound events**. Thus if a coin is tossed and a die is rolled, the event of getting a head (in the case of the coin) and say 5 (in the case of the die) is a compound event. Similarly, if a die is rolled twice, the event of getting an even number on the first roll and an odd on the second is a compound event.

Equally likely outcomes:

If a normal coin is tossed, it may come up either heads or tails. Both the outcomes are equally likely. We can accept this intuitively, even though at the moment, we do not know how to compute (or measure) the probability of either outcome. For the purpose of this discussion, we start with this assumption - that we can recognise intuitively whether all the possible outcomes are equally likely or not. We know from experience that all coins are not unbiased. Sometimes, the mass in the coin is so distributed that it shows up one side more than the other. Such coins (or dice) are said to be biased.

We can now consider one definition of probability. This is the only one that we need for the questions that we shall face. For a random experiment with n "equally likely" outcomes, if E is an event which can be considered to have occurred for m of the outcomes, the probability (mathematical probability or a priori probability) of E is $\frac{m}{n}$, i.e. $P(E) = \frac{m}{n}$

The **complement** of an event is the event of the non-occurrence of E. It is denoted by $\bar{E}$ and $P(\bar{E}) = 1 - \frac{m}{n}$.

For example, if the RE is tossing a coin and E is the event of getting heads, $P(E) = \frac{1}{2}$. Also $P(\bar{E}) = 1 - \frac{1}{2} = \frac{1}{2}$. The complement of getting a head is getting a tail. If the RE is rolling a die and E = {2, 3}, $\bar{E}$ = {1, 4, 5, 6}. In this case, $P(E) = \frac{1}{3}$, $P(\bar{E}) = \frac{2}{3}$

With this definition, we can consider the two extreme cases. An event is any subset of S. If E is the null set P(E) = 0 (an impossible event) and if E = S, P(E) = 1 (a certain event)
For example, let the RE be rolling a die and consider the "event" of getting a 0. In our notation, E would be the null set. For no element of S, can it be said that E has occurred. $\bar{E}$ would be the event of not getting a 0 and $P(\bar{E}) = 1$

Instead of E saying that the probability of an event is m/n, we can also say that the odds in favour of the event are m to n – m i.e $P(E)/P(\bar{E})$. Similarly, the odds against the event are n – m to m i.e. $P(\bar{E})/P(E)$.

Mutually Exclusive Events:

If there is a set of events, such that if any one of them occurs, none of the others can occur, the events are said to be mutually exclusive. Consider the RE of rolling a die and the following events.
$E_1 = \{1\}$
$E_2 = \{2, 3\}$
$E_3 = \{4, 5\}$
The three events are mutually exclusive.

Collectively Exhaustive Events:

If there is a set of events such that at least one of them is bound to occur, the events are said to be collectively exhaustive. For the RE considered above, if $E_4 = \{1, 2, 3, 4\}$ and $E_5 = \{3, 4, 5, 6\}$, E_4 and E_5 are collectively exhaustive.

If a set of events are both mutually exclusive and collectively exhaustive, the sum of their probabilities is 1.

Addition Theorem of Probability:

If A and B are two events, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
This result follows from the corresponding result in set theory. If n (X) represents the number of elements in set X, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Example: If a die is rolled, what is the probability that the number that comes up is either even or prime?
A = The event of getting an even number = {2, 4, 6}

B = The event of getting a prime = {2, 3, 5}
$A \cup B$ = {2, 3, 4, 5, 6}
$A \cap B$ = {2}
$P(A) = \frac{3}{6}$, $P(B) = \frac{3}{6}$, $P(A \cup B) = \frac{5}{6}$ and $P(A \cap B) = \frac{1}{6}$.
We can verify that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability and the Multiplication Theorem of Probability

The events that we have considered so far are without reference to other events (or conditions). But, very often, we need to consider events, in relation to other events (or conditions). We can continue with the same RE. Let A be the event of getting a prime and B be the event of getting an even number, i.e. A = {2, 3, 5} and B = {2, 4, 6}, $P(A) = \frac{3}{6}$

But if B is known to have occurred, then $P(A) = \frac{1}{3}$.
We write this as follows:
$P\left(\frac{A}{B}\right) = \frac{1}{3}$, This is read as follows:

The probability of A, given B is $\frac{1}{3}$.
In general,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ or } P(A \cap B) = P(B).P\left(\frac{A}{B}\right) \rightarrow (1)$$

Also, as $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, it follows that

$$P(B \cap A) = P(A).P\left(\frac{B}{A}\right). \quad \rightarrow \quad (2)$$

This result (1) or (2) is known as the multiplication theorem of probability.
If $P\left(\frac{A}{B}\right) = P(A)$, then A and B are said to be independent.
A is independent of B, because whether B occurs or does not occur, the probability of A does not change.
Example, A number is selected at random from the integers 1 to 50. A is the event of getting a multiple of 5 and B is the event of getting an even number.

Then A = {5, 10, 15....45, 50}, B = {2, 4, ..., 48, 50}
$P(A) = \frac{10}{50}$
$P\left(\frac{A}{B}\right) = \frac{5}{25}$, also $P\left(\frac{A}{\overline{B}}\right) = \frac{5}{25}$
$\therefore$ A and B are independent events.
If A and B are independent, $P\left(\frac{A}{B}\right) = P(A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \qquad \text{or } P(A \cap B) = P(A).P(B)$$

If A and B are independent, so are A, $\overline{B}$; $\overline{A}$, B and $\overline{A}, \overline{B}$

i.e. if $P(A) = P\left(\frac{A}{B}\right)$ then each of the following is true

$P(A) = P\left(\frac{A}{\overline{B}}\right)$

$P(B) = P\left(\frac{B}{A}\right) = P\left(\frac{B}{\overline{A}}\right)$

We can show the following results
If $P(A) = P\left(\frac{A}{B}\right)$, then $P(A) = \left(\frac{A}{\overline{B}}\right)$.
Also $P\left(\frac{B}{A}\right) = P(B) = P\left(\frac{B}{\overline{A}}\right)$

Pairwise independence and mutual independence:

If A, B, C are three events such that each of the 3 pairs A, B; B, C and C, A are independent, A, B, C are said to be pairwise independent. Let P(A), P(B), P(C) be a, b, c respectively.
As A, B are independent, $P(A \cap B) = ab$,
As B, C are independent, $P(B \cap C) = bc$,
As A, C are independent, $P(C \cap A) = ca$

Even if these three conditions are true, $P(A \cap B \cap C)$ is not necessarily equal to abc. In case it is, the events are said to be mutually independent. We note that mutual independence is a stronger condition. It implies pairwise independence (while pairwise independence does not necessarily mean mutual independence)

For more events, we can generalise the concept, we can talk of pairwise (or 2-wise), tripletwise (or 3-wise), quadrapletwise (or 4-wise) independence and so on. Mutual independence of n events would mean i-wise independence for i = 2, 3, 4....n

Example:

A positive integer from 1 to 60 is selected at random.
A is the event of selecting a multiple of 3.
B is the event of selecting a multiple of 4.
C is the event of selecting a multiple of 5.
The following results are true.

$a = P(A) = \frac{20}{60} = \frac{1}{3}$	$P(A \cap B) = \frac{5}{60}$	$P(A \cap B \cap C) = \frac{1}{60}$
$b = P(B) = \frac{15}{60} = \frac{1}{4}$	$P(A \cap C) = \frac{4}{60}$	
$c = P(C) = \frac{12}{60} = \frac{1}{5}$	$P(B \cap C) = \frac{3}{60}$	

Thus, $P(A \cap B) = ab$, $P(A \cap C) = ac$, $P(B \cap C) = bc$ and $P(A \cap B \cap C) = abc$,
$\therefore$ A, B, C are not merely pairwise independent but also mutually independent

Example:

There are four three-digit numbers;
1 1 2; 1 2 1; 2 1 2; 2 2 1
One of these is selected at random.
A_1 is the event that the first digit is 1

A_2 is the event that the second digit is 1
A_3 is the event that the third digit is 1
What can be said about the 3 pairs, and the triplets?

P (A_1) = $\frac{1}{2}$, P (A_2) = $\frac{1}{2}$, P (A_3) = $\frac{1}{2}$, P $(A_1 \cap A_2)$ = $\frac{1}{4}$,

P $(A_2 \cap A_3)$ = 0, P $(A_3 \cap A_1)$ = $\frac{1}{4}$

P $(A_1 \cap A_2 \cap A_3)$ = 0
$\therefore$ A_1, A_2 are independent;
A_2, A_3 are not independent;
A_3, A_1 are independent;

The question of mutual independence need not be considered as A_2, A_3 are not independent, i.e. A_1, A_2, A_3 are not mutually independent because they are not pairwise independent.

Bayes' Theorem:

Let A_1, A_2 .... A_n be n mutually exclusive and collectively exhaustive events with respective probabilities of p_1, p_2...... p_n. Let B be an event such that P (B) $\neq$ 0 and P $\left(\frac{B}{A_i}\right)$ for i = 1 to n be q_1, q_2.....q_n. Then the conditional probability of A_i given B is $\frac{p_i q_i}{p_1 q_1 + p_2 q_2 + ... + p_n q_n}$

Example:

Box 1 contains 3 white and 2 black balls. Box 2 contains 1 white and 4 black balls. A ball is picked from one of the two boxes. It turns out to be black. Find the probability that it was drawn from box 1.

The data is tabulated below

	Box 1	Box 2
White	3	1
Black	2	4

The event that box 1 is selected is say A_1.
The event that box 2 is selected is A_2

$\therefore$ p_1 = P (A_1) = $\frac{1}{2}$ and p_2 = P (A_2) = $\frac{1}{2}$

Let B be the event that a black ball is selected

$\therefore P\left(\frac{B}{A_1}\right) = \frac{2}{5}$ and P $\left(\frac{B}{A_2}\right) = \frac{4}{5}$

Now it is given that a black ball has been drawn and we need to find P (A_1) i.e. we need P $\left(\frac{A_1}{B}\right)$. We use the result above

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)P\left(\frac{B}{A_1}\right)}{P(A_1)P\left(\frac{B}{A_1}\right) + P(A_2)P\left(\frac{B}{A_2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)}{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)} = \frac{2}{2+4} = \frac{1}{3}$$

Expected value

The concept of "expected value" is very important in the Theory of Probability. This concept is very useful in managerial decision-making.

The theory of probability has its origin in gambling. When people went to gambling houses or casinos they used to get certain amounts of money if they achieved a certain result in the game. Mathematicians wanted to find out as to how much a person will earn if the game is played a large number of times.

Let us say that a man is playing a game of "throwing a die". He is given ₹6 if he throws a "four" and ₹9 if he throws a "six" on the die and not paid anything if he throws any other number (of course, he will have to pay some amount to the gambling house owner each time he wants to throw the die and this aspect will be considered later). Suppose he throws the die a large number of times - say 6,00,000 times. As the number of times the experiment is repeated becomes very large, we know that the number of times each event will occur is given by probability.

A "four" will appear with a probability of 1/6, i.e., it is expected to appear 1,00,000 times out of a total of 6,00,000 times the die is thrown. Similarly, a "six" will appear 1,00,000 times (because the probability is 1/6). Hence, the amount he will get in the long run will be 1,00,000 $\times$ 6 + 1,00,000 $\times$ 9 = 15,00,000. The amount he gets per throw will be 1500000/600000 = ₹2.5.

We say that the person's **expected value** for this game per throw in the long run is ₹2.5.

This can be calculated without the number of throws coming into the picture. Once the events are defined, we should have the probabilities of all the events and the monetary value associated with each event (i.e., how much money is earned or given away if that particular event occurs). Then,

Expected Value = $\sum_i$ [Probability (E_i)]
$\times$ [Monetary value associated with event E_i]

Example:

A person tosses a coin. If it comes up heads, he gets ₹10. If it comes up tails, he has to pay ₹5. What is his expected value?

Event	Heads	Tails
Probability	$\frac{1}{2}$	$\frac{1}{2}$
Expectation	10	–5

$\therefore$ E = $\dfrac{\left(\frac{1}{2}\right)10+\left(\frac{1}{2}\right)(-5)}{\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)}$ = 2.5

Examples:

10.01. Two dice are rolled simultaneously. Find the probability that one of them shows a number which is at least 4 and the other shows a number which is at most 3.

Sol: When two dice are rolled simultaneously, total number of outcomes = (6) (6) = 36.
Out of these, favourable cases are (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2) and (6, 3). i.e 9

$\therefore$ Required probability = $9\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) \times 2 = \frac{1}{2}$

10.02. Two dice are rolled simultaneously. Find the probability that the sum of the numbers on them is not 8.

Sol: Probability of sum of the numbers is not 8 = 1 – (Probability of sum of the numbers on them is 8)
Favourable cases for sum of the numbers being 8 are (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2)
Probability (sum of the numbers on them is 8) $= 5\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{5}{36}$

Required probability = $1-\frac{5}{36}=\frac{31}{36}$

10.03. Three fair dice are rolled simultaneously. Find the probability that the first die shows a composite number, second die shows an odd number and third die shows an odd prime number.

Sol: Required probability = probability of first die shows a composite number x probability of second die shows an odd number x probability of third die shows an odd prime number = $\frac{2}{6}\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)=\frac{1}{18}$.

10.04. Five fair coins are tossed together. Find the probability of getting exactly 4 heads.

Sol: The event of getting exactly 4 heads will be the combination of 4 heads and 1 tail.
Number of arrangements possible with this combination

$^5C_4 = \frac{5!}{4!\,1!} = 5$

$\therefore$ Required probability = $\frac{5}{2^5}=\frac{5}{32}$.

10.05. Five fair coins are tossed together. Find the probability of getting at most 3 heads.

Sol: Probability of getting at most 3 heads
= 1 – probability of getting at least 4 heads
= 1 – [(probability of getting 4 heads) + (probability of getting 5 heads)]
$= 1-\left[{}^5C_4\left(\frac{1}{32}\right)+\frac{1}{32}\right] = 1-\frac{6}{32}=\frac{13}{16}$.

10.06. A card is drawn from a well shuffled pack of cards. Find the probability of it being
(i) a jack.
(ii) a black numbered card.
(iii) a diamond.
(iv) a red king.

Sol: Number of ways of selecting one card out of 52 cards = $^{52}C_1$ = 52.

(i) One jack can be drawn from 4 jacks in $^4C_1$ = 4 ways.
$\therefore$ The required probability = $\frac{^4C_1}{^{52}C_1}=\frac{4}{52}=\frac{1}{13}$

(ii) A pack of cards has 26 black cards of which 18 are numbered. A black numbered card can be drawn in $^{18}C_1$ = 18 ways.
$\therefore$ The required probability
$= \frac{^{18}C_1}{^{52}C_1}=\frac{18}{52}=\frac{9}{26}$

(iii) A pack of cards has 13 diamonds.
A diamond can be drawn in $^{13}C_1$ = 13 ways.
$\therefore$ The required probability = $\frac{^{13}C_1}{^{52}C_1}=\frac{13}{52}=\frac{1}{4}$

(iv) A pack of cards has 2 red kings.
A red king can be drawn in $^2C_1$ = 2 ways.
$\therefore$ The required probability
$= \frac{^2C_1}{^{52}C_1}=\frac{2}{52}=\frac{1}{26}$

10.07. Two cards are drawn simultaneously from a well shuffled pack of cards. Find the probability of both being
(i) kings
(ii) honours
(iii) black honours

Sol: Two cards can be drawn from a pack of cards in $^{52}C_2$ ways.
(i) There are 4 kings in a pack of cards.
Two kings can be drawn in $^4C_2$ ways.
The required probability = $\frac{^4C_2}{^{52}C_2}$

(ii) There are 16 honours in a pack of cards
The required probability = $\frac{^{16}C_2}{^{52}C_2}$.

(iii) There are 8 black honours in a pack of cards.
$\therefore$The required probability = $\frac{^8C_2}{^{52}C_2}$

10.08. Two cards are drawn simultaneously from a pack of cards. Find the probability that both are hearts or both are diamonds.

Sol: Both cards should be hearts or diamonds.
These are mutually exclusive events.
Let X denote the event of getting both hearts and Y denote the event of getting both diamonds.

$P(X) = \frac{{}^{13}C_2}{{}^{52}C_2}$ and $P(Y) = \frac{{}^{13}C_2}{{}^{52}C_2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$2\left(\frac{{}^{13}C_2}{{}^{52}C_2}\right)$ [$\because P(X \cap Y) = 0$]

10.09. Two cards are drawn simultaneously from a pack of cards. Find the probability that both are black or both are queens.

Sol: Here there are two events which are not mutually exclusive.
If both cards are black it is possible for them to be queens also.
Let P and Q be the events of selecting both black and both queens respectively.
The required probability
$= P(P \cup Q) = P(P) + P(Q) - P(P \cap Q)$

$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2}$

10.10. Three cards are drawn in succession from a pack of cards with replacement. Find the probability of the first card drawn being a club, the second card drawn being black and the third card being a non-honour card.

Sol: Let X, Y and Z be the events of drawing a club, drawing a black card and drawing a non honoured cards respectively.
The cards are replaced.
$\therefore$ X, Y and Z are independent.
$\therefore$ required probability = $P(X \cap Y \cap Z)$

$= P(X). P(Y). P(Z) = \frac{{}^{13}C_1}{{}^{52}C_1}\frac{{}^{26}C_1}{{}^{52}C_1}\frac{{}^{36}C_1}{{}^{52}C_1}$

$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{9}{13}\right) = \frac{9}{104}$.

10.11. Three cards are drawn simultaneously from a pack of cards. Find the probability that one of them is red, another is a spade and the third is a club.

Sol: There are 26 red cards, 13 spades and 13 clubs in a pack of cards.

$\therefore$ The required probability = $\frac{{}^{26}C_1\,{}^{13}C_1\,{}^{13}C_1}{{}^{52}C_3}$.

10.12. Two balls are drawn in succession without replacement from a box containing 3 black balls and 6 blue balls. Find the probability that
(i) the first ball is black and the second ball is blue.
(ii) both are blue.

Sol: The first ball can be drawn in ${}^{9}C_1 = 9$ ways and as it is not replaced, the second ball can be drawn in ${}^{8}C_1 = 8$ ways.
(i) A black ball can be drawn in ${}^{3}C_1 = 3$ ways in the first draw. A blue ball can be drawn in ${}^{6}C_1 = 6$ ways in the second draw.

$\therefore$ The required probability = $\left(\frac{3}{9}\right)\left(\frac{6}{8}\right) = \frac{1}{4}$

(ii) A blue ball can be drawn in ${}^{6}C_1 = 6$ ways in the first draw.
Since this ball is not replaced, a blue ball can be drawn in ${}^{5}C_1 = 5$ ways in the second draw.

$\therefore$ The required probability = $\left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \frac{5}{12}$

10.13. A bag contains 3 blue balls, 4 green balls and 5 red balls. A ball is drawn at random. Find the probability that it is
(i) not a green ball.
(ii) a red ball.

Sol: A ball can be drawn from 12 balls in ${}^{12}C_1$ = 12ways.
(i) A ball other than a green ball (3 + 5 = 8) can be drawn in ${}^{8}C_1$ ways.
Hence, the probability of not drawing a green ball = $\frac{{}^{8}C_1}{{}^{12}C_1} = \frac{2}{3}$.

(ii) A red ball out of the 5 red balls present in the bag can be drawn in ${}^{5}C_1 = 5$ ways.
Hence, the probability of drawing a red ball $= \frac{{}^{5}C_1}{{}^{12}C_1} = \frac{5}{12}$.

10.14. A bag contains 3 blue balls, 4 green balls and 5 red balls. Three balls are drawn simultaneously at random. Find the probability that
(i) the balls are of different colour.
(ii) two are blue and one is green.
(iii) the balls are of the same colour.

Sol: Three balls can be drawn from 12 balls in ${}^{12}C_3$ ways.
(i) We can draw a blue ball out of 3 blue balls in ${}^{3}C_1$ ways, a green ball out of 4 green balls in ${}^{4}C_1$ ways and a red ball out of 5 red balls in ${}^{5}C_1$ ways.
The required probability = $\frac{{}^{3}C_1\,{}^{4}C_1\,{}^{5}C_1}{{}^{12}C_3}$

$= \frac{3}{11}$

(ii) Two blue balls and one green ball can be drawn in ${}^{3}C_2.{}^{4}C_1$ ways
$\therefore$ The required probability = $\frac{{}^{3}C_2.{}^{4}C_1}{{}^{12}C_3} = \frac{3}{55}$

(iii) 3 blue balls or 3 green balls or 3 red balls can be drawn in ${}^{3}C_3$ or ${}^{4}C_3$ or ${}^{5}C_3$ ways
Required probability = $\frac{{}^{3}C_3 + {}^{4}C_3 + {}^{5}C_3}{{}^{12}C_3}$

$= \frac{3}{44}$

10.15. Mohan had 5 fifty rupee notes, 3 hundred rupee notes and 4 five hundred rupee notes in his pocket. Two notes were drawn at random. Find the odds in favour of both being fifty rupee notes.

Sol: Total number of ways of drawing two notes from the pocket containing 12 notes is ${}^{12}C_2$ ways. The number of ways in which two fifty rupee notes can be drawn = ${}^{5}C_2$
The probability of choosing two fifty rupee notes $= \frac{{}^{5}C_2}{{}^{12}C_2} = \frac{5}{33}$

$\therefore$ Odds in favour of the event = favourable ways : unfavourable ways = 5 : 28

10.16. There are 4 red, 6 blue and 8 white balls in a bag. Raju drew 2 balls from it one after another. Find the probability of the second ball drawn being blue if the first ball is
(i) replaced.
(ii) not replaced.

Sol: (i) The required probability = (Probability of drawing any ball from the bag as the first ball). (Probability of drawing a blue ball from it as the second ball) = 1 $\left(\frac{6}{18}\right) = \frac{1}{3}$

(ii) There are two cases. The first ball is blue or it is not blue.
$\therefore$The probability that the second ball is blue is
$$\frac{6}{18}\left(\frac{5}{17}\right)+\frac{12}{18}\left(\frac{6}{17}\right)=\frac{6}{18}\left(\frac{5}{17}+\frac{12}{17}\right)=\frac{6}{18}=\frac{1}{3}$$

10.17. Rohit participated in a game involving rolling an unbiased die. Each participant was given thrice as many rupees as the number which came up if it was prime and twice as many rupees as the number which came up, if it was composite. If 1 turned up, instead of getting money, the participant would have to pay ₹48. Find the expected value per roll in the long run for Rohit.

Sol: The outcomes and the corresponding money that Rohit would get are tabulated below.

Outcome	1	2	3	4	5	6
Money	–48	6	9	8	15	12

$\therefore$ The expected value = Σ P_i x Monetary value $= \frac{1}{6}(-48+6+9+8+15+12)=\frac{1}{3}$

10.18. A game involved a biased die. Each time the die shows up a score of 2, the participant was paid ₹4.50. For every other score he was paid ₹8. The dice was such that the score of 2 occurred twice as frequently as any other score. Find the amount that a person should be willing to pay as entry fee for each time he plays, if he neither wishes to gain nor lose in the long run.

Sol: In order for him to neither gain nor lose in the long run, the entry fee should be equal to the expected value of the game.
Let the probability of getting any number other than 2 be P.
Total probability = P + 2P + P + P + P + P = 1
$\Rightarrow$ 7P = 1
$P = \frac{1}{7}$, $2P = \frac{2}{7}$

expected value = $\frac{2}{7}(4.50) + \frac{1}{7}(8+8+8+8+8)$
= ₹7.

10.19. Sneha is known to speak the truth in 75% of the cases. She selects a natural number from 1 to 10 at random and says that it is a prime number. What is the probability that the number selected is actually a prime number?

Sol: Let E denote the event that the number selected is a prime number and A denote the event that Sneha says that the number is a prime number.
Then, $P(E) = \frac{4}{10}$ and $P(\bar{E}) = \frac{6}{10}$

$P\left(\frac{A}{E}\right) = \frac{3}{4}$ and $P\left(\frac{A}{\bar{E}}\right) = \frac{1}{4}$

From Bayes' rule,
$$P\left(\frac{E}{A}\right) = \frac{P(E)P\left(\frac{A}{E}\right)}{P(E)P\left(\frac{A}{E}\right) + P(\bar{E})P\left(\frac{A}{\bar{E}}\right)}$$
$$= \frac{\frac{4}{10}\left(\frac{3}{4}\right)}{\frac{4}{10}\left(\frac{3}{4}\right) + \frac{6}{10}\left(\frac{1}{4}\right)} = \frac{12}{18} = \frac{2}{3}$$

10.20. Akshay speaks the truth in 45% of the cases. In the rainy season, on each day there is a 75% chance of raining. On a certain day in the rainy season, Akshay tells his mother that it is raining outside. What is the probability that it is actually raining?

Sol: Let E denote the event that it is raining and A denote the event that Akshay tells his mother that it is raining outside.
Then, $P(E) = \frac{3}{4}$,

$P(\bar{E}) = \frac{1}{4}$

$P\left(\frac{A}{E}\right) = \frac{45}{100} = \frac{9}{20}$ and $P\left(\frac{A}{\bar{E}}\right) = \frac{11}{20}$

From Bayes' Rule,
$$P\left(\frac{E}{A}\right) = \frac{P(E)P\left(\frac{A}{E}\right)}{P(E)P\left(\frac{A}{E}\right) + P(\bar{E})P\left(\frac{A}{\bar{E}}\right)}$$
$$= \frac{\frac{3}{4}\left(\frac{9}{20}\right)}{\frac{3}{4}\left(\frac{9}{20}\right) + \frac{1}{4}\left(\frac{11}{20}\right)} = \frac{27}{38}$$

10.21. One number is selected at random from the set. S = {2222, 4422, 6622, 2244, 4444, 6644, 2266, 4466, 6666}

A is the event that the number starts with 22.
B is the event that the number ends with 44.
C is the event that the units digit and thousands digit are equal.

Which of the following is true?
(A) A and B are dependent.
(B) B and C are dependent.
(C) A, B, C are mutually independent.
(D) A, B, C are pair-wise independent.

Sol: $P(A) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap B) = \frac{1}{9}$

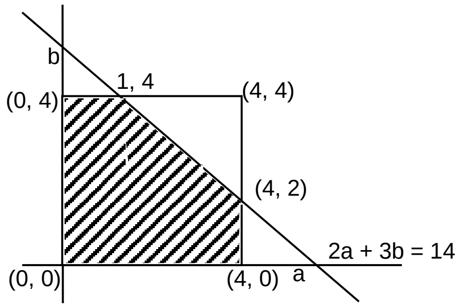
$P(B) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap C) = \frac{1}{9}$

$P(C) = \frac{3}{9} = \frac{1}{3}$ $P(B \cap C) = \frac{1}{9}$

$P(A \cap B) = P(A)\,P(B)$
$P(B \cap C) = P(B)\,P(C)$
$P(C \cap A) = P(C)\,P(A)$
but $P(A)P(B)P(C) \neq P(ABC)$
They are pairwise independent but not mutually independent. Choice (D)

10.22. If a, b are two positive numbers such that a<4 and b<4, find the probability that 2a + 3b<14.

Sol:



If we have a on the x-axis, and b-on the y-axis, 2a + 3b = 14 is a line as shown above.
In the shaded region 2a + 3b < 14
Required probability

$$= \frac{4(4) - \frac{1}{2}(3)(2)}{4(4)} = \frac{13}{16}$$

Concept Review Questions

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. **(a)** Probability of any event is always
(A) non negative but less than or equal to one
(B) a real number less than 1
(C) lies between –1 and 1
(D) lies between 0 and 10

(b) Probability of an impossible event is
(A) 1 (B) 0
(C) $\frac{1}{3}$ (D) not defined

(c) Probability of a sure or certain event is
(A) 1
(B) $\frac{1}{2}$
(C) 0
(D) any real number less than 1

2. **(a)** If P(E) = 0.2, find $P(\overline{E})$.

$\boxed{}$

(b) For any event E, $P(E) + P(\overline{E}) =$
(A) 0 (B) 1
(C) $\frac{1}{2}$ (D) None of these

3. If E_1 and E_2 are two mutually exclusive events, then $P(E_1 \cap E_2) =$
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 0

4. **(a)** If a coin is tossed n times, then the total number of possible outcomes is
(A) 2^{n+1} (B) 2^{n-1} (C) 2^n (D) 2^{2n}

(b) If a coin is tossed three times, then the number of outcomes in which heads occurs exactly two times is
(A) 2 (B) 3 (C) 1 (D) 6

5. A coin is tossed 10 times. Find the probability of obtaining heads at least once.
(A) $\frac{1}{2^{10}}$ (B) $\frac{10}{2^{10}}$ (C) $\frac{9}{2^{10}}$ (D) $\frac{2^{10}-1}{2^{10}}$

6. **(a)** The probability of getting at least 4 tails when 6 coins are tossed is
(A) $\frac{3}{4}$ (B) $\frac{15}{16}$ (C) $\frac{7}{16}$ (D) $\frac{11}{32}$

(b) The probability of getting no tail when 4 coins are tossed is
(A) $\frac{1}{16}$ (B) $\frac{15}{16}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

7. If a dice is rolled 3 times, then the total number of possible outcomes is

$\boxed{}$

8. A dice is rolled twice. In how many outcomes is the sum of the numbers shown on the two dice a prime number?

$\boxed{}$

9. A dice is rolled thrice. In how many outcomes is the sum of the numbers shown on them is at least 17?

$\boxed{}$

10. When two dice are rolled together, what is the probability that the sum of the numbers on the two dice is 9?
(A) $\frac{1}{8}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$

11. When three dice are rolled together, what is the probability of getting the sum as 10?
(A) $\frac{1}{36}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) $\frac{1}{9}$

12. If three dice are rolled together, the probability of getting composite numbers on all the three dice is
(A) $\frac{1}{8}$ (B) $\frac{1}{27}$ (C) $\frac{1}{54}$ (D) $\frac{1}{64}$

13. The probability of getting at least one 6 when three unbiased dice are thrown together is
(A) $\frac{71}{216}$ (B) $\frac{91}{216}$ (C) $\frac{1}{72}$ (D) $\frac{5}{216}$

14. If a card is drawn from a pack of cards, then the number of favourable cases for that card being a king is

$\boxed{}$

15. If a card is drawn from a pack of cards, find the probability that the card drawn is an honour.
(A) $\frac{4}{13}$ (B) $\frac{10}{13}$ (C) $\frac{1}{13}$ (D) $\frac{12}{13}$

16. If a card is drawn from a pack of cards, then find the probability that the card drawn is a numbered card.
(A) $\frac{6}{13}$ (B) $\frac{39}{52}$ (C) $\frac{40}{52}$ (D) $\frac{9}{13}$

17. If a card is drawn from a pack of cards, then find the probability that the card drawn is a spade or an honour card.
(A) $\frac{25}{52}$ (B) $\frac{24}{52}$ (C) $\frac{11}{26}$ (D) $\frac{23}{52}$

18. If three cards are drawn at random from a pack of cards, what is the probability that each card is a queen?

$\frac{1}{\boxed{}}$

19. If two cards are picked at random from a pack of cards, what is the probability that both the cards picked are queens or diamonds?

(A) $\frac{1}{221}$ (B) $\frac{14}{221}$ (C) $\frac{1}{13}$ (D) $\frac{1}{18}$

20. If E_1 and E_2 are two mutually exclusive events and $P(E_1) = 0.75$ and $P(E_2) = 0.15$, then $P(E_1 \cup E_2) =$

21. The probability that a number selected at random from the first 25 natural numbers is a prime number is

22. If a number is chosen from the set {1, 2, 3…56}, then the probability that the chosen number is a multiple of 9 is

(A) $\frac{1}{28}$ (B) $\frac{3}{28}$ (C) $\frac{1}{27}$ (D) $\frac{9}{17}$

23. The probability that a number chosen at random from the set B = {1, 3, 5, 7, 9, 11, 13, 15} being a multiple of 3 is

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{3}{8}$

24. If two distinct numbers are picked at random from the set {1, 3, 6, 8, 9, 10}, then what is the probability that the sum of the numbers picked is odd?

25. When twenty boys are to be seated in a row, the probability that two particular students always sit together is

26. A number P is chosen from {1, 3, 5, ……47}. The probability that P satisfies the equation $(x - 2)(x - 7)(x - 5)(x - 47) = 0$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$

27. If A and B are two independent events such that $P(A \cup B) = 0.71$ and $P(B) = 0.19$, then $P(A) =$

(A) $\frac{29}{81}$ (B) $\frac{52}{81}$ (C) $\frac{67}{81}$ (D) $\frac{74}{81}$

28. Given that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{5}$ and events A and B are independent. Find $P(A \cap B)$.

(A) $\frac{3}{7}$ (B) $\frac{1}{5}$

(C) $\frac{14}{15}$ (D) None of these

29. The odds against an event E are 3 : 4. The probability of occurrence of the event E is

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{1}{7}$ (D) $\frac{2}{7}$

30. A and B are two mutually exclusive and exhaustive events associated with a random experiment. If odds in favour of A are 2 : 3, then find the probability of the event B.

31. The probability that a given problem will be solved by A, B and C are $\frac{2}{3}$, $\frac{5}{7}$ and $\frac{4}{5}$ respectively. Find the probability of the problem being solved.

(A) $\frac{40}{105}$ (B) $\frac{101}{105}$

(C) $\frac{103}{105}$ (D) None of the above

32. If A and B are two equally likely events and the probability of the event B happening is $\frac{3}{4}$, then the probability that event A does not happen is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

33. In a basket that contains 10 fruits, 4 are rotten. If three fruits are drawn from the basket, find the probability that none of the fruits drawn is rotten.

(A) $\frac{6}{10}$ (B) $\frac{^6C_3}{^{10}C_3}$

(C) $\frac{6}{^{10}C_3}$ (D) $\frac{3}{^{10}C_3}$

34. Find the expected value of x, from the data given in the following table

x	0	1	3	5	6
P(x)	1/17	5/17	4/17	4/17	3/17

(A) $3\frac{2}{17}$ (B) $3\frac{4}{17}$

(C) $3\frac{13}{17}$ (D) $3\frac{15}{17}$

35. A person is asked to roll a dice. In case the dice shows an even number, he will be paid ₹50 and for an odd number he has to pay ₹30. Find the amount the person gets per turn, in the long run. (in ₹)

Exercise – 10(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. From a collection of ten books of which six are biographies and rest are autobiographies, two books are selected simultaneously at random. What is the probability that

(i) one is a biography and the other an autobiography?
(A) 8/15 (B) 7/15
(C) 2/15 (D) 13/15

(ii) both are autobiographies?
(A) 1/3 (B) 2/15
(C) 2/3 (D) 7/15

2. A box contains ten cards. Seven of these cards have the letter 'I' printed on them, and the others have the letter 'M' printed on them. If three cards are picked up one after the other at random, and placed on a table in that order, then what is the probability that the word formed is 'IIM'?

$\boxed{}$

3. When a fair coin is tossed nine times, find the probability of getting heads at least once.
(A) 1/512 (B) 511/512
(C) 9/512 (D) 503/512

4. An unbiased coin is tossed until it shows up the same face in two consecutive throws. What is the probability that the number of tosses is not more than 4?

$\boxed{}$

5. A bag contains 4 five rupee coins, 3 two rupee coins and 3 one rupee coins. If 6 coins are drawn from the bag at random, what are the odds in favour of the draw yielding maximum amount?
(A) 1 : 70 (B) 1 : 69 (C) 69 : 70 (D) 70 : 1

6. What is the probability that a quadratic equation $ax^2 + bx + c = 0$ has equal roots if a, b and c are distinct and are taken from {1, 2, 3, 4, 6, 8, 9}?
(A) 1/35 (B) 2/35 (C) 1/105 (D) 2/105

7. Varun throws two unbiased dice together and gets a sum of 7. If his friend Tarun, now throws the same two dice, what is the probability that the sum is lesser than that?
(A) 1/6 (B) 7/12 (C) 1/2 (D) 5/12

8. Find the odds against throwing six at least once with a single die in three trials.
(A) 125 : 216 (B) 91 : 216
(C) 125 : 91 (D) 91 : 125

9. Four fair dice are thrown together. If p and q respectively are the probabilities of the sum of the scores on the dice being 20 and 21, then p : q is
(A) 2 : 3 (B) 5 : 7 (C) 9 : 7 (D) 7 : 4

10. A cube has four of its faces blank, one face is marked 5 and the other is marked 6. In a game involving throwing this cube, a person is said to have a success, if he throws a numbered face. Two persons A and B participate in this game. A throws the cube thrice while B throws it once. Find the ratio of A's chance of success to that of B.
(A) 19 : 9 (B) 9 : 19 (C) 8 : 9 (D) 9 : 8

11. Kids and Toys factory is transporting balls of 5 different colours – yellow, blue, red, green and white. Mr. Bholeram, a worker in the factory has to separate these balls as per their colours into different boxes and label them with the corresponding coloured labels. Mr. Bholeram, after separating the balls, sealed the boxes and then labelled the boxes at random. What is the probability that

(i) all the boxes are labelled correctly?

$\dfrac{\boxed{}}{120}$

(ii) at least one box is labelled incorrectly?
(A) 1 (B) 0
(C) 1/120 (D) 119/120

(iii) exactly one box is labelled incorrectly?
(A) 1 (B) 0
(C) 11/120 (D) 44/120

(iv) all the boxes are incorrectly labelled?

$\dfrac{\boxed{}}{30}$

12. From a pack of cards, if three cards are drawn in succession without replacement, what is the probability that the first one is an ace, the second is a king and the third is a jack?
(A) 1/5525 (B) 7/16575
(C) 16/5525 (D) 8/16575

13. If 2 cards are selected at random from a pack of cards, what is the probability that the cards are

(i) both numbered cards (2 to 10) or both honours?
(A) 125/221 (B) 1/221
(C) 55/221 (D) 32/221

(ii) both honours or both hearts?
(A) 125/221 (B) 1/221
(C) 55/221 (D) 32/221

14. While shuffling a pack of cards, 4 cards are accidentally dropped. The probability that

(i) all of them are numbered cards (2 to 10) of the same suit is

(A) $\dfrac{4\times {}^9C_4}{{}^{52}C_4}$ (B) $\dfrac{\left({}^9C_4\right)^4}{{}^{52}C_4}$

(C) $\dfrac{4\times {}^9C_1}{{}^{52}C_4}$ (D) $\dfrac{\left({}^9C_1\right)^4}{{}^{52}C_4}$

(ii) all of them are numbered cards of different suits is

(A) $\frac{4 \times {}^{9}C_4}{{}^{52}C_4}$ (B) $\frac{\left({}^{9}C_4\right)^4}{{}^{52}C_4}$

(C) $\frac{4 \times {}^{9}C_1}{{}^{52}C_4}$ (D) $\frac{\left({}^{9}C_1\right)^4}{{}^{52}C_4}$

15. A bag contains 6 blue and 8 yellow balls. Find the probability of drawing two yellow balls in succession, when the ball that is drawn first is

(i) replaced.
(A) 4/49 (B) 16/49
(C) 24/91 (D) 45/91

(ii) not replaced.
(A) 4/49 (B) 4/13
(C) 24/91 (D) 45/91

16. A bag contains 6 red and 4 white balls and another bag contains 5 red and 5 white balls. If one of the bags is selected at random and a draw of two balls is made at random from the bag thus selected, what is the probability that both the balls are white?
(A) 51/90 (B) 8/45 (C) 45/49 (D) 4/49

17. From a bag containing 2 Pears, 3 Peaches and 4 Figs, three fruits are drawn at random. What is the probability that

(i) all the three fruits are of the same variety?
(A) 5/84 (B) 1/9
(C) 1/84 (D) 79/84

(ii) exactly two of them are of the same variety?
(A) 10/84 (B) 55/84
(C) 36/84 (D) 48/84

(iii) the three fruits are of different varieties?
(A) 3/28 (B) 5/7
(C) 2/7 (D) 1/9

18. The odds in favour of Saurabh getting a final selection in a B-school are 5 to 7 and the odds against Sweth getting it are 4 to 3. Find the probability that

(i) at least one of them gets a final selection in the B-school.
(A) 2/3 (B) 23/28
(C) 1/3 (D) 29/49

(ii) exactly one of them gets a final selection in the B-school.
(A) 41/84 (B) 71/84
(C) 23/49 (D) 26/49

19. If a square is selected at random from an 8×8 chessboard, what is the probability that it is a 3×3 square?
(A) 3/8 (B) 3/17 (C) 1/64 (D) 5/17

20. Arpit and Bipin pick up a ball at random from a bag containing 5 violet, 2 red and 3 orange balls one after the other, replacing it every time till one of them gets an orange ball and the one who first gets an orange ball is declared a winner. If Arpit begins the game, then the probability of Bipin winning the game is
(A) 10/17 (B) 7/17 (C) 7/10 (D) 3/10

21. Two biased dice are thrown together. On one of them, 6 appears twice as often as any other number while on the other, an odd number appears thrice as frequently as an even number. What is the probability that the sum of the scores on them is 11 or 12?
(A) 1/12 (B) 9/28 (C) 3/28 (D) 5/12

22. Hrithik draws a card from a well shuffled pack of cards. If the card is a red king, he is paid ₹39 while if it is a black honour, he is paid ₹26 and in all the other cases, he has to pay ₹13. In the long run per draw of a card, Hrithik makes an average
(A) profit of ₹5
(B) loss of ₹5
(C) profit of ₹3.50
(D) loss of ₹3.50

23. Shreya participates in a game involving throwing two coins together, being promised ₹35 if the coins show the same face value, else ₹25. The coins are biased in such a way that on one of them head appears twice as frequently as tail, while on the other, tail appears 1½ times as frequently as head. What is the maximum amount Shreya will be willing to pay as an entry fee, if in the long run, she wants to make an average profit of ₹15? (in ₹)

$\frac{\boxed{\quad}}{3}$

24. Kiran rotates a roulette wheel which has markings form 201 to 300. If the wheel stops at a multiple of 7, he wins ₹7,000. If the wheel stops at a multiple of 13, he wins ₹13,000 and if the wheel stops at a number which is a multiple of both 7 and 13, he wins ₹91,000. If Kiran has to pay an amount of ₹2,700 every time he rotates the wheel as a participation fee, then, in the long run what is the average profit he makes per game? (in ₹)

25. Three bulb holders are fitted in a room. From a box containing 20 bulbs of which 25% are fused, 3 bulbs are taken at random and fitted into these bulb holders. What is the probability that the room is lighted?
(A) 91/228 (B) 113/114
(C) 1/114 (D) 137/228

26. A and B are two events of an experiment with respective probabilities 0.6 and 0.25. What is the probability that either of the events happen, if

(i) A and B are independent?

(ii) A and B are mutually exclusive?

27. A number is selected at random from all possible four-digit numbers that are formed using the digits 0, 2, 5, 7. Given that the number is even, what is the probability that it is divisible by 5?

28. Avinash picks a number from the numbers 1 to 25 and found it to be an even number. What is the probability that the number is 12?

(A) $\frac{1}{13}$ (B) $\frac{12}{25}$ (C) $\frac{1}{12}$ (D) $\frac{1}{6}$

29. An urn A contains 6 white balls and 7 black balls. and urn B contains 8 white balls and 6 black balls. A person draws a ball at random from one of the two urns. It turns out to be black. What is the probability that the ball was drawn from urn A?

(A) $\frac{7}{14}$ (B) $\frac{49}{88}$

(C) $\frac{39}{88}$ (D) None of the above

30. A team of four members is to be formed from a group of 3 boys and 4 girls. What is the probability that at least two girls are included in the team?

(A) $\frac{1}{35}$ (B) $\frac{4}{35}$ (C) $\frac{31}{35}$ (D) $\frac{2}{35}$

31. There are two routes that a bus can take from Dapodi to Bopodi. On the first route, it takes 20 minutes while on the second it takes 30 minutes. The probability that it takes the second route is 0.4 Find the expected time in which the bus covers the distance. (in min)

$\boxed{}$

Directions for questions 32 to 35: These questions are based on the following information.

A bag contains 11 fruits of which 3 are spoilt. If 4 fruits are chosen at random, find the probability that

32. all the fruits are spoilt.

(A) 0 (B) 1 (C) $\frac{14}{55}$ (D) $\frac{4}{165}$

33. exactly two of them are spoilt.

(A) $\frac{13}{55}$ (B) $\frac{11}{54}$ (C) $\frac{7}{54}$ (D) $\frac{14}{55}$

34. one is good and three are spoilt.

(A) $\frac{3}{173}$ (B) $\frac{4}{165}$ (C) $\frac{3}{125}$ (D) $\frac{4}{173}$

35. at least one of the fruits is good.

(A) 0 (B) 1 (C) $\frac{32}{113}$ (D) $\frac{4}{173}$

Exercise – 10(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If 4 consecutive letters are selected at random from the English alphabet, then the probability that one of the letters is a vowel is

(A) $\frac{13}{23}$ (B) $\frac{16}{23}$ (C) $\frac{5}{12}$ (D) $\frac{17}{23}$

2. Box A contains 4 red and 6 green balls. Box B contains 7 red and 3 green balls. A ball is drawn from box A and without seeing its colour, it is put into box B. If a ball is now drawn from box B, then the probability that it is green is

(A) $\frac{5}{11}$ (B) $\frac{12}{55}$ (C) $\frac{18}{55}$ (D) $\frac{6}{55}$

3. The letters of the word 'ANSWER' are arranged at random. Find the probability of having exactly two letters in between A and E.

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{10}$ (D) $\frac{1}{2}$

4. 400 tickets are numbered as 000, 001, 002, 003,……., 399. If a ticket is drawn at random from them and M is the event that the sum of the digits of the number is 6, then P(M) = $\boxed{}$

5. An unbiased dice is rolled 4 times. Out of 4 numbers that are shown up, the probability that the least is greater than 3 and the greatest is less than 6 is

(A) $\frac{1}{3}$ (B) $\frac{1}{243}$

(C) $\frac{1}{81}$ (D) $\frac{1}{9}$

6. Six balls numbered 1 to 6 are to be placed in six boxes numbered 1 to 6 (one in each). If the balls are placed at random into the boxes, then the probability that

(i) all the balls are placed in corresponding numbered boxes is

(A) $\frac{1}{120}$ (B) $\frac{1}{60}$

(C) $\frac{1}{720}$ (D) $\frac{1}{30}$

(ii) at least two balls are placed in corresponding numbered boxes is

(A) $\frac{191}{720}$ (B) $\frac{529}{720}$

(C) $\frac{1}{720}$ (D) $\frac{1}{60}$

(iii) none of the balls are placed in their respective boxes is

(A) $\frac{111}{720}$ (B) $\frac{5}{37}$

(C) $\frac{1}{40}$ (D) $\frac{53}{144}$

7. If a coin is tossed 7 times, then what is the probability of exactly one pair of consecutive tosses turning up the same face?

$\frac{\boxed{}}{32}$

8. The probability that a square selected at random from an 8×8 chessboard is of size 4×4 is

(A) $\frac{1}{17}$ (B) $\frac{5}{17}$ (C) $\frac{25}{204}$ (D) $\frac{1}{16}$

9. If five dice are rolled together, then find the probability that the total score on the five dice is 28.

(A) $\frac{5}{108}$ (B) $\frac{5}{6^5}$ (C) $\frac{3}{6^5}$ (D) $\frac{5}{2\left(6^4\right)}$

10. If the letters of the word "GRAPHICS" are arranged at random, what is the probability that the vowels are never together?

$\boxed{}$

11. If four squares of size 1×1 are chosen at random on a chessboard of size 8×8, what is the probability that they are on the same diagonal?

(A) $\frac{182}{64^4}$ (B) $\frac{126}{64^4}$ (C) $\frac{126}{{}^{64}C_4}$ (D) $\frac{364}{{}^{64}C_4}$

12. A fair coin is tossed 10 times. What is the probability of a tail occurring as many times in the first six tosses, as in the last four?

(A) $\frac{105}{512}$ (B) $\frac{5}{12}$ (C) $\frac{1}{32}$ (D) $\frac{112}{210}$

13. In a non-leap year, what is the probability that in a group of 10 people, (none of whom was born on 29 February) at least two have the same birthday?

(A) $\frac{{}^{365}P_{10}}{365^{10}}$ (B) $\frac{{}^{364}P_9}{365^{10}}$

(C) $\frac{365^9 - {}^{364}P_9}{365^9}$ (D) $\frac{{}^{365}P_3}{365^{10}}$

14. If the integers m and n are chosen at random between 1 and 50, then the probability that a number of the form $4^m + 4^n$ is divisible by 5 is

(A) $\frac{2}{3}$ (B) $\frac{5}{26}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

15. Three men shot at the same target. The probability of the first, second and third man hitting the target respectively is 0.3, 0.5 and 0.6. Find the probability that exactly one of them hits the target.

$\boxed{}$

16. A and B are two events of an experiment such that $P(\overline{A}) = 0.7$ and $P(\overline{A} \cap \overline{B}) = 0.2$. Find P(B) given that

(i) A and B are mutually exclusive.

$\boxed{}$

(ii) A and B are independent.

(A) $\frac{5}{7}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{3}{8}$

17. Two cards are drawn from a pack of cards one after another. What is the probability that the first drawn card is red and the second is an ace, when the card that is drawn first is

(i) replaced?

(A) $\frac{11}{51}$ (B) $\frac{3}{26}$

(C) $\frac{1}{26}$ (D) $\frac{1}{13}$

(ii) not replaced?

(A) $\frac{29}{442}$ (B) $\frac{3}{104}$

(C) $\frac{6}{169}$ (D) $\frac{1}{26}$

18. The odds in favour of a player being selected for the national team with three independent selectors are 4:3, 2:1 and 1:4 respectively. What is the probability that of the three selectors a majority will be favourable?

(A) $\frac{2}{5}$ (B) $\frac{10}{21}$ (C) $\frac{6}{35}$ (D) $\frac{2}{21}$

19. Sadikh, Akhil and Afroz start a game of dice. They throw the dice by turns, first Sadikh, Akhil then Afroz, then once again Sadikh, Akhil and Afroz and so on until one of them wins. Sadikh is considered to have won if he throws an odd number, Akhil if he throws a prime and Afroz if he throws an even number. What is the probability that sadikh wins?

$\frac{\boxed{}}{7}$

20. A biased die is loaded so that the probability of showing a composite number is thrice that of showing a prime number and also twice that of showing of number 1. If the die is thrown twice, what is the probability that the product of numbers on the die is 1?

(A) $\frac{1}{49}$ (B) $\frac{4}{441}$ (C) $\frac{2}{49}$ (D) $\frac{2}{21}$

21. There are 3 urns containing 3 white, and 4 green balls; 2 white and 4 green balls; 1 white and 1 green ball. One urn is chosen at random and a ball is drawn from it. What is the probability that it is a green ball?

(A) $\frac{73}{126}$ (B) $\frac{1}{6}$ (C) $\frac{4}{21}$ (D) $\frac{7}{12}$

22. In a box containing 1000 bags, 100 are defective. What is the probability that out of a sample of 8 bags, none are defective?

(A) $(0.1)^8$ (B) $(0.9)^8$ (C) $(0.6)^8$ (D) $8(0.1)^8$

23. A bag contains 3 white, 5 blue and 4 green balls. If two balls are drawn at random from the bag, then find the probability that both the balls are of the same colour.

$\frac{\boxed{}}{66}$

24. Ravi rolls a fair die. He is promised an amount (in rupees) thrice the number showing up if the number is odd and an amount 4 times the number showing up if it is even. What is the maximum amount Ravi should be willing to pay each time, to throw the dice, if in the long run he wants to make an average profit of ₹10 per throw? (in ₹)

$\boxed{}$

25. Kiran selects a card numbered from 101 to 200. If the number on the card is a multiple of 8 (but not of 12) he wins ₹40. If the number on the card is a multiple of 12 (and not of 8) he wins ₹65. If the number on the card is a multiple of both 8 and 12 he wins ₹80. In the long run, what is the amount that Kiran will gain on an average if he has to pay ₹3.60 as participation fee for each draw? (in ₹)

$\boxed{}$

26. Ameer tossed a biased coin in which heads appears on 80% of the occasions. In a game involving this coin, if Ameer is paid ₹25 per head and he has to pay ₹30 for a tail, in the long run, per game, Ameer makes an average

(A) loss of ₹8 (B) profit of ₹12
(C) loss of ₹20 (D) profit of ₹14

27. Raghavender draws a card from a pack of cards. If the card is a red honour, he is paid ₹65 while if it is a black Jack, he is paid ₹52. In all other cases, he has to pay ₹26. In the long run, per draw, Raghavender makes an average

(A) profit of ₹9 (B) profit of ₹10.50
(C) loss of ₹9 (D) profit of ₹3.50

28. Neelu picked up a letter from the English alphabet and found it to be a consonant. What is the probability that it is letter C?

(A) $\frac{1}{21}$ (B) $\frac{1}{26}$ (C) $\frac{21}{26}$ (D) $\frac{5}{7}$

29. Two letters arrive – one from Hyderabad and the other from Adilabad. The name of the originating post office is marked on the envelopes with a rubber stamp. One of the envelops is put into a paper shredder. On one of the shreds, just two consecutive letters are visible. Assume that every pair of consecutive letters has an equal chance of appearing on such a shred. What is the probability that the envelope that is put into the shredder is the one from Adilabad given that the consecutive letters visible are 'AD'?

(A) $\frac{7}{23}$ (B) $\frac{2}{3}$ (C) $\frac{16}{23}$ (D) $\frac{3}{4}$

30. A committee of 3 members is to be formed from a group of 4 men and 5 women. What is the probability that at least two women are included in the committee?

$\frac{\boxed{}}{42}$

31. The following table shows the values of a discrete random variable X with the corresponding probabilities.

Find the expected value of X. $\boxed{}$

X	1	2	3	4	5
P (X)	0.3	0.2	0.2	0.2	0.1

Directions for questions 32 to 35: These questions are based on the following information.

A bag contains 10 mobiles, of which 4 are damaged. If 3 mobiles are chosen at random, find the probability that

32. all the mobiles are damaged.

(A) $\frac{1}{25}$ (B) $\frac{1}{18}$ (C) $\frac{1}{30}$ (D) $\frac{1}{15}$

33. two mobiles are good and one is damaged.

(A) $\frac{1}{4}$ (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

34. one is good and the other two are damaged.

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{1}{5}$ (D) $\frac{3}{5}$

35. at least one mobile is damaged.

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{5}{6}$ (D) $\frac{4}{5}$

Key

Concept Review Questions

1. (a) A
   (b) B
   (c) A
2. (a) 0.8
   (b) B
3. D
4. (a) C
   (b) B
5. D
6. (a) D
   (b) A
7. 216
8. 15
9. 4
10. D
11. C
12. B
13. B
14. 4
15. A
16. D
17. A
18. 5525
19. B
20. 0.9
21. 0.36
22. B
23. D
24. 0.6
25. 0.1
26. A
27. B
28. B
29. B
30. 0.6
31. C
32. B
33. B
34. B
35. 10

Exercise – 10(a)

1. (i) A
   (ii) B
2. 0.175
3. B
4. 0.875
5. B
6. C
7. D
8. C
9. D
10. A
11. (i) 1
    (ii) D
    (iii) B
    (iv) 11
12. D
13. (i) A
    (ii) D
14. (i) A
    (ii) D
15. (i) B
    (ii) B
16. B
17. (i) A
    (ii) B
    (iii) C
18. (i) A
    (ii) A
19. B
20. B
21. C
22. B
23. 44
24. 30
25. B
26. (i) 0.7
    (ii) 0.85
27. 0.5
28. C
29. B
30. C
31. 24
32. A
33. D
34. B
35. B

Exercise – 10(b)

1. D
2. C
3. 0.2
4. 0.055
5. C
6. (i) C
   (ii) A
   (iii) D
7. 3
8. C
9. D
10. 0.75
11. D
12. A
13. C
14. C
15. 0.41
16. (i) 0.5
    (ii) A
17. (i) C
    (ii) D
18. B
19. 4
20. A
21. A
22. B
23. 19
24. 2.50
25. 5.80
26. D
27. C
28. A
29. C
30. 25
31. 2.6
32. C
33. D
34. B
35. C