# Quant Mastery B

## Answers and Explanations

## ANSWERS AND EXPLANATIONS

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | 9. | B | 17. | D |
| 2. | D | 10. | C | 18. | B |
| 3. | B | 11. | D | 19. | E |
| 4. | B | 12. | D | 20. | E |
| 5. | A | 13. | C | 21. | B |
| 6. | D | 14. | A | 22. | D |
| 7. | D | 15. | D | 23. | E |
| 8. | C | 16. | D | 24. | C |

**1. (A)**

Car X began traveling at an average speed of 35 miles per hour. After 72 minutes, car Y began traveling at an average speed of 49 miles per hour. When both cars had traveled the same distance, both cars stopped. How many miles did car X travel from the time car Y began traveling until both cars stopped?

- ○ 105
- ○ 120
- ○ 140
- ○ 147
- ○ 168

**Step 1: Analyze the Question**

Let's start by paraphrasing the given information: Car X gets a head start, and then car Y starts driving at a faster rate until it catches up with car X. The cars' speeds are given in miles per hour, but we're given a time in minutes, so we'll need to convert all of the information into the same units (probably hours).

We can be sure we're going to use the rate formula in this question. The most important rate in question, however, is not the rate of car X or of car Y alone, but rather the rate at which car Y catches up with car X.

**Step 2: State the Task**

The question asks us for the distance that X travels starting from the time that Y begins to move. We can first use the rate formula and the information we're given to determine how much of a head start car X got. Then, we'll calculate the speed at which car Y closed the gap and use the rate formula a second time to determine how long it took car Y to catch up with car X. Finally, we'll use the rate formula a third time to determine how far car X traveled in that time.

**Step 3: Approach Strategically**

72 minutes is $\frac{6}{5}$ hours. In that time, car X traveled $\left(\frac{35 \text{ miles}}{\text{hour}}\right)\left(\frac{6}{5} \text{ hours}\right) = 42$ miles. Therefore, 42 miles is the distance of the head start that car X got.

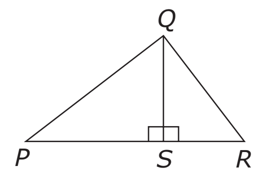
Since car Y went 14 miles per hour faster than car X, it closed the gap at a rate of $\frac{14 \text{ miles}}{\text{hour}}$. The 42-mile head start divided by $\frac{14 \text{ miles}}{\text{hour}}$ gives us 3 hours—the amount of time it took car Y to catch up with car X.

During those 3 hours, car X traveled $\left(\frac{35 \text{ miles}}{\text{hour}}\right)(3 \text{ hours}) = 105$ miles. The correct answer is **(A)**.

**Step 4: Confirm your Answer**

If you doubted your answer, you could confirm it by comparing the total distances that car X and car Y traveled, since they ended up traveling the same distance. We calculated the distances car X traveled for both legs of the trip, 42 miles and 105 miles, so car X traveled a total of 147 miles. Car Y traveled 49 miles per hour, and it took 3 hours to catch car X, so it went $\left(\frac{49 \text{ miles}}{\text{hour}}\right)(3 \text{ hours}) =$ 147 miles. The distances match, confirming that **(A)** is correct.

**2. (D)**




In the figure above, the measure of angle *PQR* is 90 degrees, and the measure of angle *QSP* is 90 degrees. What is the area of triangle *PQR*?

(1) The length of *PS* is 16, and the length of *RS* is 9.

(2) The length of *PR* is 25, and the ratio of the length of *PQ* to the length of *QR* is 4 to 3.

**Step 1: Analyze the Question Stem**

This Value Data Sufficiency question asks us to determine the area of triangle *PQR*. To do this, we will need to know the lengths of the base and height of triangle *PQR*. Since we are told that angle *PQR* = 90 degrees, we have two options available for values to plug into the area formula: We could use *PR* for the base and *QS* for the height, or we could use *QR* for the base and *PQ* for the height.

It's helpful on a problem like this to sketch the shape on your noteboard, filling in as much information about the figures as you can. Start by labeling angle *QPR* with a degree measure of *x*. Label angle *QRP* with a degree measure of *y*. Make a note that angle *PQR* is 90 degrees.

These labels make it easier to see a crucial piece of information: All three of these triangles are similar. That is, they have the same degree measures. Triangles *PQS* and *PQR* share an angle with a degree measure of *x* and another angle with a degree measure of 90. If triangles have two angle measures in common, then they must have all three angle measures in common. Notice, too, that triangles *QRS* and *PQR* share angles measuring 90 degrees and *y*. Again, *QRS* and *PQR* must share all three angles in common.

Recognizing that these triangles are similar will allow us to use information about one of them to make important deductions about the others. With that in mind, let's move to the statements.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1) provides the length of the base *PR*. This statement is sufficient if we can use it to find the height *QS*. Indeed, we can do so: Because triangles *PQS* and *QRS* are similar, the ratio of the corresponding sides is always the same.

Side *PS* in triangle *PQS* corresponds to side *QS* in triangle *QRS* because the sides are opposite the same interior angles. Similarly, side *QS* in triangle *PQS* corresponds to side *RS* in triangle *QRS*. So $\frac{PS}{QS} = \frac{QS}{RS}$. We know that *PS* = 16 and *RS* = 9, so we can plug these values into our proportion: $\frac{16}{QS} = \frac{QS}{9}$.

Cross-multiplying this equation, we have

$$(16)(9) = (QS)(QS)$$

$$144 = (QS)^2$$

$$QS = \sqrt{144} = 12$$

Thus, $QS = 12$. Since we know triangle $PQR$'s base ($PS + RS = 25$) and height ($QS = 12$), we can find its area. Statement (1) is sufficient. Eliminate choices **(B)**, **(C)**, and **(E)**.

Statement (2) provides the length of $PR$, rather than the length of the two sides that combine to form $PR$. However, Statement (2) also gives us information about the ratio of the length of $PQ$ to the length of $QR$. This information will allow us to circumvent finding $QS$, since we can use $QR$ as the base and $PQ$ as the height of right triangle $PQR$.

Since in right triangle $PQR$, the ratio of the length of $PQ$ to the length of $QR$ is 4 to 3, triangle $PQR$ is a 3:4:5 right triangle. Since the length of hypotenuse $PR$ is 25, which equals $5 \times 5$, triangle $PQR$ is a 3:4:5 right triangle with each member of the 3 to 4 to 5 ratio multiplied by 5. So $PQ = 5 \times 4 = 20$, and $QR = 5 \times 3 = 15$. Because we know the lengths of $PQ$ and $QR$, we've got everything we need to calculate the triangle's area. Statement (2) is also sufficient, and choice **(D)** is correct.

**3. (B)**

The ratio of the number of sophomores to juniors to seniors in a room is 3 to 5 to 34. If the number of seniors is 104 more than the sum of the sophomores and juniors, then what is the sum of the number of juniors and seniors in the room?

- ○ 168
- ○ 156
- ○ 148
- ○ 136
- ○ 117

**Step 1: Analyze the Question**

It is important to remember that a ratio can be the relationship between a part and another part or between a part and the whole. This question provides a three-part ratio, in other words, a part-to-part-to-part ratio.

**Step 2: State the Task**

The question provides the ratio of sophomores to juniors to seniors in a room and relates these three quantities with an equation. We are asked to solve for the sum of the juniors and seniors in the room.

**Step 3: Approach Strategically**

The ratio of sophomores to juniors to seniors in the room is 3 to 5 to 34. Let's say that the number of sophomores is $3x$, the number of juniors is $5x$, and the number of seniors is $34x$. The question is asking for the number of juniors and seniors, which means it's asking for the value of $5x + 34x$, or $39x$. The correct answer must therefore be a multiple of 39. We can therefore eliminate answer choices that are not divisible by 39 (we can use mental math to determine the next highest multiple of 40 and subtract the number of times 40 goes into that multiple to find the nearest multiple of 39). $\frac{168}{39}$ is not an integer (since $200 - 5 = 195$, which is $39 \times 5$). $\frac{148}{39}$ also is not an integer (since $160 - 4 = 156$, which is $39 \times 4$). And $\frac{136}{39}$ is not an integer (since $160 - 4 = 156$, which is $39 \times 4$). Eliminate **(A)**, **(C)**, and **(E)**.

Now we are left with only **(B)** and **(D)**, so we can Backsolve to identify the correct answer. Try choice **(B)** first—and remember that whether it works or fails, we'll have our answer, since only two options remain.

Testing **(B)**, we get $\frac{156}{39} = 4$. Therefore, the number of sophomores, $3x$, equals 12. The number of juniors, $5x$, equals 20. The number of seniors, $34x$, equals 136. According to the stem, the number of seniors should be 104 greater than the number of sophomores and juniors combined. And sure enough, $12 + 20 + 104$ does equal 136, the number of seniors we calculated earlier. **(B)** is correct.

We could also have solved this question algebraically. First, translate the equation provided in the question stem: $34x = (3x + 5x) + 104$.

Solving for $x$, we have $34x = 8x + 104$, so $26x = 104$, and $x = \frac{104}{26} = 4$. The number of juniors in the room is $5x = 5(4) = 20$, and the number of seniors in the room is $34x = 34(4) = 136$. The sum of the number of juniors in the room and the number of seniors in the room is $20 + 136 = 156$. Choice **(B)** is correct.

**Step 4: Confirm Your Answer**

Take a moment to make sure that you've answered the right question. It would be a shame to do all the math correctly, and then misinterpret what the correct answer choice represents.

**4. (B)**

Darcy, Gina, Ray, and Susan will be the only participants at a meeting. There will be three soft chairs in the room where the meeting will be held and one hard chair. No one can bring more chairs into the room. Darcy and Ray will arrive simultaneously, but Gina and Susan will arrive individually. The probability that Gina will arrive first is $\frac{1}{3}$, and the probability that Susan will arrive first is $\frac{1}{3}$. The probability that Gina will arrive last is $\frac{1}{3}$, and the probability that Susan will arrive last is $\frac{1}{3}$. Upon arriving at the meeting, each of the participants will select a soft chair, if one is available. If Darcy and Ray arrive and see only one unoccupied soft chair, they will flip a fair coin to determine who will sit in that chair. By what percent is the probability that Darcy will sit in a soft chair greater than the probability that Gina will sit in a soft chair?

- ○ 50%
- ○ 25%
- ○ $16\frac{2}{3}\%$
- ○ $12\frac{1}{2}\%$
- ○ 0%

**Step 1: Analyze the Question**

There are many components to this challenging question. We must systematically translate each sentence into the appropriate math terms, then solve for the desired probabilities.

**Step 2: State the Task**

Determine the probabilities of each possible desired outcome, then calculate the percent difference between them.

**Step 3: Approach Strategically**

Because the probability that Gina will arrive first is $\frac{1}{3}$ and the probability that Gina will arrive last is $\frac{1}{3}$, the probability that Gina will arrive second is $1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$.

Similarly, because the probability that Susan will arrive first is $\frac{1}{3}$, and the probability that Susan will arrive last is $\frac{1}{3}$, the probability that Susan will arrive second is $1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$.

We therefore know that the probability that Darcy and Ray arrive first is $\frac{1}{3}$, the probability that Darcy and Ray arrive second is $\frac{1}{3}$, and the probability that Darcy and Ray arrive last is $\frac{1}{3}$.

Let's find the probability that Darcy sits in a soft chair. The probability that Darcy and Ray arrive first is $\frac{1}{3}$. If Darcy and Ray arrive first, then they will sit in soft chairs because there are three soft chairs and they are the first two people there. The probability that Darcy and Ray arrive first and Darcy sits in a soft chair is $\frac{1}{3}$.

The probability that Darcy and Ray arrive second is $\frac{1}{3}$.

If Darcy and Ray arrive second, then they will sit in soft chairs because there are three soft chairs and only one soft chair is already occupied, leaving two soft chairs for Darcy and Ray. The probability that Darcy and Ray arrive second and Darcy sits in a soft chair is $\frac{1}{3}$.

The probability that Darcy and Ray arrive last is $\frac{1}{3}$. If Darcy and Ray arrive last, then two soft chairs will already be occupied. At this point, Darcy has a $\frac{1}{2}$ chance of getting a soft chair.

The probability that Darcy and Ray arrive last and Darcy sits in a soft chair is $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

The overall probability that Darcy will sit in a soft chair is then $\frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6} = \frac{5}{6}$.

Now let's find the probability that Gina will sit in a soft chair. Note that if Gina arrives either first or second, she is guaranteed to get a soft chair. If she arrives last, on the other hand, she is guaranteed *not* to get a soft chair, since three people will have shown up before her. Therefore, her chances of getting a soft chair are equal to her chances of arriving first or second: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3} = \frac{4}{6}$.

The question asks us by what percentage Darcy's chances are better than Gina's. $\frac{5}{6}$ is 25% greater than $\frac{4}{6}$. **(B)** is the correct answer.

**Step 4: Confirm Your Answer**

This problem illustrates how important it is to be sure that you're answering the right question. $\frac{1}{6}$ is the difference between Darcy's odds of getting a soft chair and Gina's—and that fraction is equal to $16\frac{2}{3}$%, which shows up as answer choice **(C)**. But the question asks for the *percent increase* from Gina's odds to Darcy's, which is 25%. Reread the stem and confirm that you understand what the question asks before you commit to an answer.

**5. (A)**

Each of the 8 numbers $s, t, u, v, w, x, y$, and $z$ is positive. Is the average (arithmetic mean) of $s, t, u, v, w, x, y$, and $z$ greater than 46?

(1) The average (arithmetic mean) of $s, t, u, v$, and $w$ is greater than 74.

(2) The average (arithmetic mean) of $x, y$, and $z$ is greater than 120.

**Step 1: Analyze the Question Stem**

Note that this is a Yes/No Data Sufficiency question.

We can translate the inequality in the question stem as $\frac{s+t+u+v+w+x+y+z}{8} > 46$, or $s + t + u + v + w + x + y + z > 8(46)$. The question is then equivalent to: "Is $s + t + u + v + w + x + y + z > 368$?"

Notice also that each of the variables represents a *positive* number. This will likely prove to be important information. Now let's look at the statements.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1), translated, says that $\frac{s+t+u+v+w}{5} > 74$, or $s + t + u + v + w > 370$. Using this information about this subset $(s + t + u + v + w)$ in conjunction with the information from the question stem, we have a revised question: "Is (a value greater than 370) + $(x + y + z) > 368$?" Since we are told that all of the variables are positive numbers, we know with certainty that $(s + t + u + v + w) + (x + y + z) > 368$. The answer is "always yes," and therefore Statement (1) is sufficient, even though it only tells us about five of the eight variables. Eliminate choices **(B)**, **(C)**, and **(E)**.

Statement (2), translated, says that $\frac{x+y+z}{3} > 120$, or $x + y + z > 360$. Using this information about the subset $(x + y + z)$ in conjunction with the information from the question stem, we have a revised question: "Is $(s + t + u + v + w)$ + (a value greater than 360) $> 368$?" It is possible to answer this question "yes," and it is possible to answer this question "no," depending on the values of the variables $s$, $t$, $u$, $v$, and $w$. Since more than one answer to the question is possible, Statement (2) is insufficient. Choice **(A)** is correct.

**6. (D)**

Jim and Tina begin running toward each other on a straight avenue that is intersected every quarter mile by a cross-street. As one proceeds up the avenue from south to north, its cross-streets are numbered sequentially from 1st Street to 100th Street, without skipping an integer. Jim begins running north from 16th Street at a speed of 38 blocks per hour. At the same time, Tina begins running south from 56th Street at a speed of 42 blocks per hour. At which street do they pass each other?

- ○ 19th
- ○ 21st
- ○ 34th
- ○ 35th
- ○ 37th

**Step 1: Analyze the Question**

This is a rate question that requires us to determine the point at which two runners cross paths. We're given the details of each runner's respective speed and starting point.

**Step 2: State the Task**

Use the information given to think critically through the scenario in order to determine the street at which the runners cross paths.

**Step 3: Approach Strategically**

Let's start by considering how far apart Jim and Tina are to start. We know that $56 - 16 = 40$, so that's 40 blocks they have to cover between the two of them. For now, we can disregard the exact starting points (16th or 56th Street). We can come back to that later. The next critical question is, "How long it will take for them to cover a total of 40 blocks?"

Between them, using their respective rates, they would cover 80 blocks $(38 + 42)$ in one hour. So they'll need $\frac{1}{2}$ hour to cover the 40 blocks between 16th Street and 56th Street. To see where they'll meet after running for a half hour, we just need to consider how far either one of them will travel in this time. Let's pick Jim. In $\frac{1}{2}$ hour, he'll travel $\frac{1}{2}\text{ hour} \times \frac{38\text{ blocks}}{\text{hour}} = 19$ blocks. Starting from 16th Street, that will get him to $16 + 19 = 35$th Street. Choice **(D)** is correct.

**Step 4: Confirm Your Answer**

This answer choice makes sense since we know that Tina, running at a faster rate, will travel farther than Jim. With 40 blocks between them, we know Tina will cover slightly more than half the distance, putting her at a street south of 36th Street, which is the midway point (56th Street − 20 streets). Since her rate is not that much faster than Jim's, we're left with either **(C)** or **(D)**. We can confirm our work by applying the same arithmetic to Tina.

Tina starts at 56th Street and travels $\frac{1}{2}\text{ hour} \times \frac{42\text{ blocks}}{\text{hour}} = 21$ blocks. Since Tina starts at 56th Street and travels 21 blocks, she'll meet Jim at $56 - 21 = 35$th Street.

**7. (D)**

A train traveled the first $d$ miles of its journey at an average speed of 60 miles per hour, the next $d$ miles of its journey at an average speed of $y$ miles per hour, and the final $d$ miles of its journey at an average speed of 160 miles per hour. If the train's average speed over the total distance was 96 miles per hour, what is the value of $y$?

- ○ 68
- ○ 84
- ○ 90
- ○ 120
- ○ 135

**Step 1: Analyze the Question**

This is an average rate question. We're given the average rate, 96 miles per hour, of a train's entire journey and asked to find the rate, $y$, of one portion of the train's journey. There are three legs of the journey, each with a different rate. We know the time traveled during each leg will be different, because we're told that the train travels the same distance, $d$, during each of the three legs of the train's journey.

**Step 2: State the Task**

We need to use our rate formula for each leg of the journey and our average rate formula for the entire journey.

Rate formula: Rate $= \frac{\text{Distance}}{\text{Time}}$ or $r = \frac{d}{t}$.

Average rate formula: Average rate $= \frac{\text{Total distance}}{\text{Total time}}$.

To calculate the average speed of the journey, we need to determine the total distance and total time of the journey. To do this, we need to utilize our rate formula for each leg of the journey.

**Step 3: Approach Strategically**

The total distance for the journey is $3d$, since the train traveled $d$ miles for each of the three legs of the journey. Now, let's find the time components of each leg in order to determine total time for our average rate equation.

Since the train traveled the first $d$ miles of its journey at an average speed of 60 miles per hour, the time that it took to travel the first $d$ miles was $\frac{d \text{ miles}}{60 \text{ miles per hours}}$, or $\frac{d}{60}$ hours. Similarly, on the second leg of the journey, the time that it took to travel the next $d$ miles was $\frac{d \text{ miles}}{y \text{ miles per hour}}$, or $\frac{d}{y}$ hours, and on the third leg, $\frac{d \text{ miles}}{160 \text{ miles per hour}}$, or $\frac{d}{160}$ hours. The total time that it took the train to travel the $3d$ miles was $\left(\frac{d}{60} + \frac{d}{y} + \frac{d}{160}\right)$ hours. The train's average speed was then $\frac{3d}{\frac{d}{60} + \frac{d}{y} + \frac{d}{160}}$ miles per hour.

Setting this expression equal to the average speed for the entire journey, we have the equation $\frac{3d}{\frac{d}{60} + \frac{d}{y} + \frac{d}{160}} = 96$.

Solve for $y$:

$$\frac{3d}{d\left(\frac{1}{60}+\frac{1}{y}+\frac{1}{160}\right)} = 96$$

$$\frac{3}{\left(\frac{1}{60}+\frac{1}{y}+\frac{1}{160}\right)} = 96$$

$$3 = 96\left(\frac{1}{60}+\frac{1}{y}+\frac{1}{160}\right)$$

$$3 = 96\left(\frac{1}{60}\right)+96\left(\frac{1}{y}\right)+96\left(\frac{1}{160}\right)$$

$$3 = 1.6+\frac{96}{y}+0.6$$

$$3 = 2.2+\frac{96}{y}$$

$$0.8 = \frac{96}{y}$$

$$y = \frac{96}{0.8} = \frac{960}{8} = 120$$

Choice **(D)** is correct.

**Step 4: Confirm Your Answer**

We can confirm our math by Picking Numbers, or we could have solved the question using Picking Numbers in the first place. A manageable value to pick for $d$ would be the least common multiple of 60 and 160, the rates of speed for the first and third legs of the journey. Prime factorization can help us determine that the least common multiple of 60 and 160 is 480. We can use a chart to keep track of the given information and the numbers we pick:

| | Rate | Time | Distance |
|---|---|---|---|
| **Leg 1** | 60 | | 480 |
| **Leg 2** | $y$ | | 480 |
| **Leg 3** | 160 | | 480 |
| **Total** | 96 | | 1,440 |

Let's use the values in our chart, in conjunction with the rate formula, to solve for the amount of time each leg took:

Leg 1: $R_1 = \frac{d}{T_1}$ or $60 = \frac{480}{T_1}$; so $T_1 = 8$ hours.

Leg 2: $R_2 = y = \frac{d}{T_2}$ (we'll come back to this below).

Leg 3: $R_3 = \frac{d}{T_3}$ or $160 = \frac{480}{T_3}$; so $T_3 = 3$ hours.

Total: $R_{Avg} = \frac{D_T}{T_T}$ or $96 = \frac{1{,}440}{T_T}$; so $T_T = 15$ hours.

Our updated chart now looks like this:

| | Rate | Time | Distance |
|---|---|---|---|
| **Leg 1** | 60 | 8 | 480 |
| **Leg 2** | $y$ | $T_2$ | 480 |
| **Leg 3** | 160 | 3 | 480 |
| **Total** | 96 | 15 | 1,440 |

Adding the values in the Time column to solve for $T_2$ gives us $8 + T_2 + 3 = 15$. Then $T_2 = 15 - 11 = 4$ hours.

| | Rate | Time | Distance |
|---|---|---|---|
| **Leg 1** | 60 | 8 | 480 |
| **Leg 2** | $y$ | 4 | 480 |
| **Leg 3** | 160 | 3 | 480 |
| **Total** | 96 | 15 | 1,440 |

Finally, we can solve for $y$: $y = \frac{480}{4} = 120$ miles per hour, the same rate we determined through our algebraic approach.

**8. (C)**

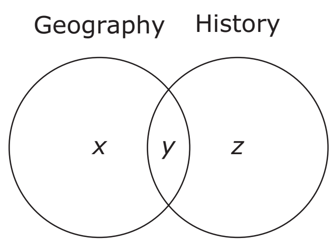
How many students in a room are currently taking either geography or history, but not both?

(1) The number of students in the room who are taking at least one of the subjects history and geography is 127.

(2) The number of students in the room who are taking both geography and history is 46.

**Step 1: Analyze the Question Stem**

For this Value Data Sufficiency question, we need to determine how many students are enrolled in either or both of two subjects. While there's not much information given in the stem, it can be helpful to draw a Venn diagram, which will make this question easier to visualize.




The circle on the left represents students who are taking geography, while the circle on the right represents students who are taking history. The $x$ in the circle on the left is the number of students in the room who are taking geography but not history. The $y$ in both circles is the number of students who are taking both geography and history. The $z$ in the circle on the right is the number of students in the room who are taking history but not geography.

Therefore, the question stem can be considered to be asking, "What is the value of $x + z$?"

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1), using our Venn diagram variables, says that $x + y + z = 127$. Since we have no way of determining the value of $y$, this statement is insufficient. We can eliminate **(A)** and **(D)**.

Statement (2), using our Venn diagram variables, says that $y = 46$. Since we have no way of determining the value of $x$, the value of $z$, or the value of $x + z$, this statement is insufficient. We can eliminate **(B)**.

Our possible answer choices are then **(C)** or **(E)**. We must combine the statements and evaluate them together.

Again, using our Venn diagram variables, Statement (1) says that $x + y + z = 127$, while Statement (2) says that $y = 46$. Substituting 46 for $y$ in the equation $x + y + z = 127$, we have $x + 46 + z = 127$, and then $x + z = 81$. This answers our question from the stem. The statements taken together are sufficient. Choice **(C)** is correct.

**9. (B)**

What is the average (arithmetic mean) of $x$, $y$, and $z$?

(1) $3x - 2y + 7z = 23$

(2) $4x - 3y + 5z = 5$ and $-x + 6y - 2z = 58$

**Step 1: Analyze the Question Stem**

This is a Value question. We are asked for the average of three variables. While having the individual values of all three numbers would, of course, be sufficient, it's important for us to remember that when the GMAT asks for the value of an expression, we may be able to solve without knowing the value of each element within the expression.

It is not necessary to set up the average formula here. If we can find the values of the individual variables or of the expression $x + y + z$, we will know we have sufficient information.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1) is simpler, so let's evaluate it first. The equation $3x - 2y + 7z = 23$ is one linear equation with three variables. We cannot solve for the value of any of the variables $x$, $y$, and $z$, and we cannot solve the equation for the value of $x + y + z$. Statement (1) is insufficient. We can eliminate **(A)** and **(D)**.

Now, let's consider Statement (2). We have two linear equations with three variables. In this case, there is no way to solve for the value of any individual variable. But—and here is a deduction that separates expert test takers from those stuck in the classroom math world—if we add the equations, we have $(4x - 3y + 5z) + (-x + 6y - 2z) = 5 + 58$. We can simplify this to $4x - 3y + 5z - x + 6y - 2z = 63$ and, further, by combining the like terms among the variables, to $3x + 3y + 3z = 63$. If we divide both sides of this equation by 3, we have $x + y + z = 21$. That is sufficient to determine the average of $x$, $y$, and $z$, which is 7. Statement (2) is sufficient to answer the question. There is no need to combine statements. Choice **(B)** is correct.

**10. (C)**

A new exhibit at a zoo is rectangular and measures 60 feet by 80 feet. An observation walkway around the whole exhibit will be added that is 10 feet wide. If it takes 1 ton of concrete to pave 10 square feet of walkway, how many tons of concrete will be needed to pave the entire walkway?

- ○ 150
- ○ 240
- ○ 320
- ○ 480
- ○ 2,400

**Step 1: Analyze the Question**

This question asks us to determine how many tons of concrete will be needed to pave the walkway around a new exhibit at the zoo.

We are given the dimensions of the new exhibit: 60 feet $\times$ 80 feet. We are also told that the walkway is 10 feet wide and extends around the entire exhibit. Finally, we are told that it will take 1 ton of concrete to pave 10 square feet of walkway.

**Step 2: State the Task**

We need to determine the area (square footage) of the walkway and multiply by $\frac{1 \text{ ton}}{10 \text{ square feet}}$ to determine the amount of concrete needed to pave the walkway.

**Step 3: Approach Strategically**

We can find the area of the walkway by subtracting the smaller rectangle, the dimensions of which we are given in the question as 60 by 80, from the area of the big rectangle, the area that will include the proposed walkway.

Since the 10-foot-wide walkway will completely surround the exhibit, we will have to add $2 \times 10 = 20$ to both the length and the width given in the question. The area of the large rectangle is therefore $(20 + 60)(20 + 80) = 80 \times 100 = 8{,}000$.

The area of the small rectangle is $60 \times 80 = 4{,}800$.

The area of the walkway $= 8{,}000 - 4{,}800 = 3{,}200$ square feet.

One ton of concrete is required for every 10 square feet, so $\frac{3{,}200}{10} = 320$ tons of concrete. Answer choice **(C)** is correct.

**Step 4: Confirm Your Answer**

Reread the question stem, making sure that you didn't miss any important information about the problem. Make sure that you're answering the right question, which is about the amount of concrete needed, not about the area of the walkway.

**11. (D)**

In each of the years 2005, 2006, and 2007, the profits of company X were 10 percent greater than in the previous year. What were the profits of company X in the year 2004?

(1) The sum of the profits of company X in 2006 and 2007 were $10,164,000.

(2) The profits of company X in 2007 were $924,000 greater than the profits of company X in 2005.

**Step 1: Analyze the Question Stem**

This Value question asks what the profits of company X were in 2004. The question states that in each of the years 2005, 2006, and 2007, the profits were 10% greater than in the previous year. Assign a variable ($P$) to represent the profit in 2004:

Profits in 2004 $= P$

To calculate the profit in 2005, find 10% of $P = 0.10P$ and then add to the original 2004 profits:

Profits in 2005 $= P + 0.10P = 1.10P$

To calculate the profit in 2006, find 10% of $1.10P = 0.11P$ and then add to the 2005 profits:

Profit in 2006 $= 1.10P + 0.11P = 1.21P$

Finally, to calculate the profit in 2007, find 10% of $1.21P = 0.121P$ and then add to the 2006 profits:

Profit in 2007 $= 1.21P + 0.121P = 1.331P$

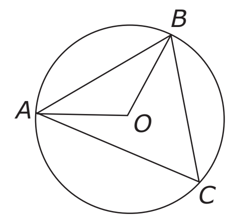
But luckily for us, we wouldn't need to do these calculations to know whether we have sufficiency. Since we know the exact growth rate for each year and can express it algebraically, knowing—or being able to calculate—the actual profit for any of the years 2005, 2006, or 2007 would be sufficient to calculate the 2004 profits, or $P$.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1): Sufficient. This statement says that $1.21P + 1.331P = 10{,}164{,}000$. This is a linear equation with the single variable $P$, and we could solve for $P$. Statement (1) is sufficient. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): Sufficient. This statement says that $1.331P = 1.1P + 924{,}000$. This is a linear equation with the single variable $P$, and we could solve for $P$. Statement (2) is sufficient. **(D)** is correct.

**12. (D)**




In the figure above, *O* is the center of the circle. What is the measure of angle *AOB*?

(1) The measure of angle *ACB* is 59 degrees.

(2) The measure of angle *OAB* is 31 degrees.

**Step 1: Analyze the Question Stem**

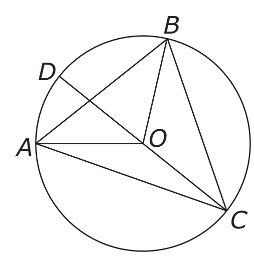
This Value Data Sufficiency question asks us to determine the measure of angle *AOB*.

We are given some important information. The center of the circle is given to be *O*, which tells us that *OA* and *OB* are radii of the circle. This means that triangle *OAB* is isosceles.

Now that we've garnered the information from the stem, let's move to the statements.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1): Let's start by drawing a diameter of the circle that has point *C* as one of its endpoints:




Statement (1): We are given the angle *ACB*. The Central Angle Theorem tells us that when we have two angles that share endpoints on the edge of a circle, one of which has a vertex at the center of that circle and the other on the edge, the central angle will have an angle measure exactly twice that of the inscribed angle. Since angle *ACB* is 59°, angle *AOB* is 118°. Statement (1) is sufficient. Eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): We can evaluate Statement (2) by working with the figure in the question stem. Triangle *OAB* is an isosceles triangle because *OA* and *OB* are both radii of the circle, so angles *OAB* and *OBA* are equal. Since angle $OAB = 31°$, angle $OBA = 31°$. The sum of the measures of the interior angles in any triangle is 180°, so in triangle *OAB*, the measure of angle *AOB* is $180° - 31° - 31° = 118°$. Statement (2) is also sufficient. Choice **(D)** is correct.

**13. (C)**

If $wxyz \neq 0$, is $\frac{w}{x} > \frac{y}{z}$?

(1) $wz > xy$

(2) $xz > 0$

**Step 1: Analyze the Question Stem**

This is a Yes/No question. According to the question stem, $wxyz \neq 0$, which means that each of $w$, $x$, $y$, and $z$ is nonzero. The question asks whether $\frac{w}{x} > \frac{y}{z}$. When an inequality question involves unknown values, it is always important to determine whether the values are positive or negative.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1): To evaluate this statement, we want to see whether we can manipulate the given inequality to look like the inequality from the question stem. If $xz > 0$, then dividing both sides of the inequality $wz > xy$ by the positive quantity $xz$ keeps the direction of the inequality sign the same, and we have $\frac{wz}{xz} > \frac{xy}{xz}$ and then $\frac{w}{x} > \frac{y}{z}$. In this case, the answer to the question is "yes."

If $xz < 0$, then dividing both sides of the inequality $wz > xy$ by the negative quantity $xz$ reverses the direction of the inequality sign, and we have $\frac{wz}{xz} < \frac{xy}{xz}$ and then $\frac{w}{x} < \frac{y}{z}$. In this case, the answer to the question is "no." Since different answers to the question are possible, Statement (1) is insufficient. We can eliminate **(A)** and **(D)**.

We can also Pick Numbers to show that Statement (1) is insufficient. If $w = 4$, $x = 1$, $y = 1$, and $z = 1$, then we have $wz = (4)(1) = 4$ and $xy = (1)(1) = 1$. Thus, $wz > xy$, so Statement (1) is true. We have $\frac{w}{x} = \frac{4}{1} = 4$, while $\frac{y}{z} = \frac{1}{1} = 1$. In this case, $\frac{w}{x} > \frac{y}{z}$, so the answer to the question is "yes."

If $w = 4$, $x = -1$, $y = 3$, and $z = 1$, then we have $wz = (4)(1) = 4$ and $xy = (-1)(3) = -3$. Thus, $wz > xy$, so Statement (1) is true. When we plug these values into the question stem, we have $\frac{w}{x} = \frac{4}{-1} = -4$, while $\frac{y}{z} = \frac{3}{1} = 3$. In this case, $\frac{w}{x} < \frac{y}{z}$, so the answer to the question is "no." Since different answers to the question are possible, Statement (1) is insufficient.

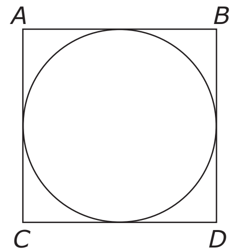
Statement (2): This statement provides no information about $w$ or $y$, so it is insufficient. We can eliminate **(B)**.

We can also Pick Numbers to show that Statement (2) is insufficient. If $w = 4$, $x = 1$, $y = 1$, and $z = 1$, then we have $xz = (1)(1) = 1$. Thus, $xz > 0$, so Statement (2) is true. We have $\frac{w}{x} = \frac{4}{1} = 4$, while $\frac{y}{z} = \frac{1}{1} = 1$. In this case, $\frac{w}{x} > \frac{y}{z}$, so the answer to the question is "yes."

If $w = 4$, $x = -1$, $y = -3$, and $z = -1$, then we have $xz = (-1)(-1) = 1$. Thus, $xz > 0$, so Statement (2) is true. We have $\frac{w}{x} = \frac{4}{-1} = -4$, while $\frac{y}{z} > \frac{-3}{-1} = 3$. In this case, $\frac{w}{x} < \frac{y}{z}$, so the answer to the question is "no." Since different answers to the question are possible, Statement (2) is insufficient.

Combined: Statement (1) says that $wz > xy$, and Statement (2) says that $xz > 0$. Let's divide both sides of the inequality $wz > xy$ by the positive quantity $xz$. When we divide both sides of an inequality by a positive quantity, the direction of the inequality sign remains the same, and we have $\frac{wz}{xz} > \frac{xy}{xz}$ and then $\frac{w}{x} > \frac{y}{z}$. The statements taken together lead to an answer of "always yes," and **(C)** is correct.

**14. (A)**




In the figure above, *ABCD* is a square, and exactly one point of each side of square *ABCD* touches the circumference of the circle. Is the area of the circle greater than $25\pi$?

(1) The perimeter of square *ABCD* is greater than 40.

(2) The diagonal of square *ABCD* is greater than $8\sqrt{3}$.

**Step 1: Analyze the Question Stem**

This Yes/No Data Sufficiency question asks whether the area of the circle is greater than $25\pi$. In other words, "Is $\pi r^2 > 25\pi$?" We can also rephrase this question to ask, "Is $r > 5$?"

We are given important information. *ABCD* is a square, and exactly one point of each side of this square touches the circumference of the circle. We therefore know that a side of the square *ABCD* is a diameter of the inscribed circle.

Now that we've noted the information from the stem, let's move to the statements.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1) says that the perimeter of the square is greater than 40. The perimeter of a square is four times the length of a side. So $4s > 40$. Therefore, the length of a side, $s$, of the square is greater than 10.

Furthermore, the length of each side of the square is equal to the diameter of the circle. So the length of each side of the square is $2r$, using $r$ for the radius. Then, $2r > 10$, and $r > 5$. This answers our question definitively. Statement (1) is sufficient to answer the question. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2) provides us with the diagonal of square *ABCD*. The diagonal of a square divides the square into two identical isosceles right triangles, where the diagonal is the hypotenuse and each side of the square is a leg of the right triangle. The leg length to leg length to hypotenuse length ratio in an isosceles right triangle is $x:x:x\sqrt{2}$.

A side of the square, $s$, is equal to the diameter of the circle, $s = 2r$. Then the diagonal of the square has a length of $2r\sqrt{2}$. Therefore, the statement tells us that $2r\sqrt{2} > 8\sqrt{3}$, or $r\sqrt{2} > 4\sqrt{3}$. Squaring both sides of this inequality, we have $(r\sqrt{2})^2 > (4\sqrt{3})^2$. Simplifying this inequality, we can solve for $r^2$:

$$(r\sqrt{2})(r\sqrt{2}) > (4\sqrt{3})(4\sqrt{3})$$

$$2r^2 > (16)(3)$$

$$r^2 > (8)(3)$$

$$r^2 > 24$$

If $r^2 > 24$, then $\pi r^2 > 24\pi$. Therefore, Statement (2) says that the area of the circle is greater than $24\pi$. Since different answers to the question "Is $\pi r^2 > 25\pi$?" are possible—for instance, $\pi r^2$ could equal $24.5\pi$ or $26\pi$—Statement (2) is insufficient. Choice **(A)** is correct.

**15. (D)**

A magazine survey of its subscribers finds that 20 percent are male. If 70 percent of the subscribers are married and 10 percent of these are male, what percent of the male subscribers are not married?

- ○ 7%
- ○ 13%
- ○ 35%
- ○ 65%
- ○ 90%

**Step 1: Analyze the Question**

This is an overlapping sets question, but we're given percentages, not values. We'll Pick Numbers to make the question more concrete.

**Step 2: State the Task**

We are asked to find the percentage of male subscribers who are unmarried, so we will solve for the information needed to plug into this version of the percent formula: $\frac{\text{Unmarried male subscribers}}{\text{Total male subscribers}} \times 100\%$.

**Step 3: Approach Strategically**

Since this question involves percentages, let's choose 100 for the total number of subscribers and apply the information in the question stem.

20% of our 100 subscribers are male, so we have 20 males.

70% of our 100 subscribers are married, so we have 70 married subscribers.

10% of the 70 married subscribers are male, so we have 7 married male subscribers.

Of our 20 male subscribers, 7 are married, so we have $20 - 7 = 13$ unmarried male subscribers. Plugging these numbers into our percent formula yields $\frac{\text{Unmarried male subscribers}}{\text{Total male subscribers}} \times 100\% = \frac{13}{20} \times 100\% = 65\%$.

Answer choice **(D)** is correct.

**Step 4: Confirm Your Answer**

Check your calculations to confirm your answer is correct. It's especially important on this question to make sure you have solved for the exact percentage that was asked for: Identifying the desired numerator and denominator of the percent formula is an essential part of Step 2 of the Method. Note that all the other answer choices are trap answers that misuse numbers you would encounter in solving the problem.

**16. (D)**

The length of a rectangular solid is 5, the width is 3, and the greatest possible distance between any two points on the surface of the rectangular solid is $7\sqrt{2}$. What is the volume of the rectangular solid?

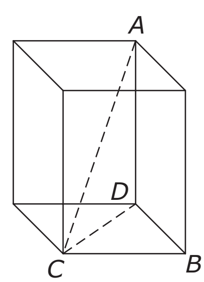
- ○ 84
- ○ 90
- ○ 105
- ○ 120
- ○ $15\sqrt{73}$

**Step 1: Analyze the Question**

We are asked to find the volume of a rectangular solid. We are given two of the solid's dimensions, length and width, which are 5 and 3, respectively. We are also told that the greatest possible distance between any two points on the surface of the rectangular solid is $7\sqrt{2}$.

**Step 2: State the Task**

We know the volume of a rectangular solid is length times width times height. Since we know the length and width of this rectangular solid, we just need to find its height. Let's draw a picture of this rectangular solid to assist us.




Let's say that $BC$ is the length of the rectangular solid, which is 5, and $BD$ is the width of the rectangular solid, which is 3. We need to determine the height $AD$ to calculate the volume.

**Step 3: Approach Strategically**

$ACD$ is a right triangle, with angle $ADC$ being the right angle. The diagonal $AC$ has a length that is the greatest possible distance between any two points on the surface of the rectangular solid and is equal to $7\sqrt{2}$. To calculate the height $AD$, we also need the length of $CD$, which is also the hypotenuse of triangle $BCD$. Triangle $BCD$ is a right triangle, with angle $CBD$ being the right angle. We know the lengths of legs $BC$ and $BD$ in this right triangle.

We can use the Pythagorean Theorem in this right triangle to find the length of hypotenuse $CD$ of right triangle $BCD$:

$$(CD)^2 = (BC)^2 + (BD)^2$$

$$(CD)^2 = 5^2 + 3^2$$

$$(CD)^2 = 25 + 9$$

$$(CD)^2 = 34$$

$$CD = \sqrt{34}$$

Using the Pythagorean Theorem in right triangle $ACD$, we have

$$(AC)^2 = (AD)^2 + (CD)^2$$
$$(7\sqrt{2})^2 = (AD)^2 + (\sqrt{34})^2$$
$$(7\sqrt{2})(7\sqrt{2}) = (AD)^2 + 34$$
$$(7)(7)(\sqrt{2})(\sqrt{2}) = (AD)^2 + 34$$
$$49(2) = (AD)^2 + 34$$
$$98 = (AD)^2 + 34$$
$$64 = (AD)^2$$
$$AD = \sqrt{64}$$
$$AD = 8$$

Thus $AD$, which is the height of the rectangular solid, is 8. Plugging in the values for the length, width, and height of the rectangular solid, the volume is $5 \times 3 \times 8 = 15 \times 8 = 120$. Choice **(D)** is correct.

**Step 4: Confirm Your Answer**

Reread the question stem, making sure you didn't miss any important information about the problem. Also, check your calculations.

**17. (D)**

List $L$: $v$, $w$, $x$, $y$, $z$

List $M$: $v + 12$, $w + 12$, $x + 12$, $y + 12$, $z + 12$

List $N$: $8v$, $8w$, $8x$, $8y$, $8z$

What is the standard deviation of the numbers in list $L$?

(1) The standard deviation of the numbers in list $M$ is 3.5.

(2) The standard deviation of the numbers in list $N$ is 28.

**Step 1: Analyze the Question Stem**

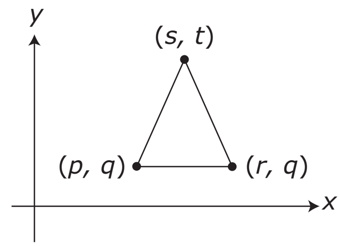
This is a Value question that asks for the standard deviation of the numbers in List $L$. Since this is a Data Sufficiency question, we don't actually need to calculate the standard deviation; rather, we just need to determine whether we have all the information needed to calculate the value.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1): Sufficient. The standard deviation of a set of numbers is a description of the amount of dispersion of those numbers. Each number in list $M$ is 12 more than the corresponding number in list $L$. So the numbers of list $L$ and the numbers of list $M$ are dispersed in exactly the same way. Therefore, the standard deviations of the numbers in lists $L$ and $M$ are equal. Since the standard deviation of the numbers in list $M$ is 3.5, the standard deviation of the numbers in list $L$ is also 3.5. Statement (1) is sufficient. We can eliminate **(B)**, **(C)**, and **(E)**.

Statement (2): Sufficient. Each number in list $N$ is 8 times the corresponding number in list $L$. Therefore, the standard deviation of the numbers in list $N$ is 8 times the standard deviation of the numbers in list $L$. Since the standard deviation of the numbers in list $N$ is 28, the standard deviation of the numbers in list $L$ is $\frac{28}{8} = \frac{7}{2} = 3.5$. Statement (2) is sufficient. Choice **(D)** is correct.

**18. (B)**




In the figure above, the area of the triangle is $a$. Which the following equations describing $t$ is correct?

- ○ $t = \dfrac{2a}{r - p}$
- ○ $t = q + \dfrac{2a}{r - p}$
- ○ $t = q + \dfrac{2a}{p - r}$
- ○ $t = q + \dfrac{r - p}{2a}$
- ○ $t = q + \dfrac{2a}{q - p}$

**Step 1: Analyze the Question**

We are asked to find an equation from the choices that correctly describes the $y$-coordinate $t$. We are also told that the area of the triangle in the figure is $a$.

**Step 2: State the Task**

We need to find the $y$-coordinate of the top point of the triangle. Because the answer choices all include variables, we know that it will be possible to Pick Numbers.

**Step 3: Approach Strategically**

Pick permissible, manageable numbers. Let's try $t = 5$, $r = 6$, $p = 1$, and $q = 1$. The base is therefore 5, and the height is 4. One-half of their product is 10. So $a = 10$ for the purpose of evaluating the answer choices.

Now plug those numbers into the answer choices to see which one gives us 5 for the value of $t$.

**(A):** $\dfrac{(2)(10)}{6 - 1} = \dfrac{20}{5} = 4$. Incorrect.

**(B):** $1 + \dfrac{(2)(10)}{6 - 1} = 1 + \dfrac{20}{5} = 5$. Possibly correct.

**(C):** $1 + \dfrac{(2)(10)}{1 - 6} = 1 + \dfrac{20}{-5} = -3$. Incorrect.

**(D):** $1 + \dfrac{6 - 1}{(2)(10)} = 1 + \dfrac{5}{20} = \dfrac{5}{4}$. Incorrect.

**(E):** $1 + \dfrac{(2)(10)}{1 - 1} = 1 + \dfrac{20}{0} =$ undefined. Incorrect.

The correct answer is **(B)**.

We could also have approached this problem algebraically.

The area of a triangle is area $= \dfrac{1}{2}$(base)(height). Let's say that the base of the triangle is the side whose endpoints are $(p, q)$ and $(r, q)$. Since both of the points $(p, q)$ and $(r, q)$ have the same

$y$-coordinate, $q$, the length of the base is the positive difference of the $x$-coordinates. So the length of the base is $r - p$.

The height drawn from the point ($s$, $t$) to the base will be parallel to the $y$-axis because the base is parallel to the $x$-axis. So the height will be the positive difference between the $y$-coordinate $t$ of the point ($s$, $t$) and the $y$-coordinate $q$ of any point on the base whose endpoints are ($p$, $q$) and ($r$, $q$). Therefore, the height is $t - q$.

Thus, we now know that the base is $r - p$ and the height is $t - q$. The area of the triangle is then $a = \frac{1}{2}(r - p)(t - q)$. Let's solve this equation for $t$:

$$\frac{1}{2}(r - p)(t - q) = a$$

$$(r - p)(t - q) = 2a$$

$$(t - q) = \frac{2a}{r - p}$$

$$t = \frac{2a}{r - p} + q$$

Choice **(B)** is correct.

**Step 4: Confirm Your Answer**

Reread the question stem, making sure you didn't miss any important information about the problem. Also, check your work. If you solved algebraically, you could Pick Numbers to check your work as well.

**19. (E)**

The events $A$ and $B$ are independent. The probability that event $A$ occurs is 0.6, and the probability that at least one of the events $A$ and $B$ occurs is 0.94. What is the probability that event $B$ occurs?

- ○ 0.34
- ○ 0.65
- ○ 0.72
- ○ 0.76
- ○ 0.85

**Step 1: Analyze the Question**

This question involves the calculation of the probability of a series of events. Note that the question stem does not tell us what these events are, but merely that $A$ and $B$ are independent.

**Step 2: State the Task**

The task is to calculate the probability that event $B$ occurs.

**Step 3: Approach Strategically**

Let's say that the probability that event $A$ occurs is $P(A)$, the probability that event $B$ occurs is $P(B)$, the probability that at least one of the events $A$ and $B$ occurs is $P(A \text{ or } B)$, and the probability that both of the events $A$ and $B$ occur is $P(A \text{ and } B)$. We can jot down the information given by the question stem as follows:

$P(A) = 0.6$

$P(B) = ?$

$P(A \text{ or } B) = 0.94$

To find the probability that $B$ occurs, we will need to use the information provided regarding the probability that $A$ or $B$ occurs. This probability is calculated using the following formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Since the events $A$ and $B$ are independent, $P(A \text{ and } B) = P(A)P(B)$.

So we can rewrite our formula as $P(A \text{ or } B) = P(A) + P(B) - P(A)P(B)$.

Substituting the given values, $P(A \text{ or } B) = 0.94$ and $P(A) = 0.6$, into the formula, we get

$0.94 = 0.6 + P(B) - 0.6P(B)$

$0.94 = 0.6 + 0.4P(B)$

$0.34 = 0.4P(B)$

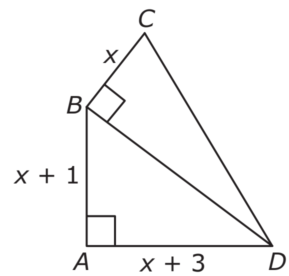
$P(B) = \frac{0.34}{0.4} = \frac{34}{40} = \frac{17}{20} = \frac{85}{100}$

Thus, $P(B) = 0.85$. Choice **(E)** is correct.

**Step 4: Confirm Your Answer**

To check our work, we can plug the value $P(B) = 0.85$ back into the equation for calculating the $P(A \text{ or } B)$ to verify that it equals 0.94, as stated in the question stem.

**20. (E)**




In the figure above, the length of *CD* is $5\sqrt{5}$. What is the value of $x$?

- ○ $2\frac{1}{2}$
- ○ 3
- ○ 4
- ○ $4\frac{2}{3}$
- ○ 5

**Step 1: Analyze the Question**

For a complex-looking figure such as the one in this question, we should first break down the figure into smaller ones. We're given a lot of information here: Both triangles are right triangles. The hypotenuse of triangle *ABD* is also the long leg of triangle *BCD*. We're also told the relationship among three of the sides, in terms of the variable $x$. We also know that the hypotenuse of triangle $CDB = 5\sqrt{5}$.

Because $5\sqrt{5}$ squares to 125, the sum of $x^2 + (BD)^2$ will equal 125.

**Step 2: State the Task**

We're looking for the value of $x$, the variable that relates three of the sides of the two triangles.

**Step 3: Approach Strategically**

Because of the complexity of the algebraic approach to this question, Backsolving is a smart strategy. The answer choices with fractions will be tougher to plug in, so we should start with the integers in **(B)**, **(C)**, or **(E)**.

**(E)** makes triangle *ABD* a 3:4:5 triangle, so it's easy to test: $AB = 6$, $AD = 8$, so *BD* must equal 10. Therefore, triangle *CDB* has legs of length 5 and 10. The sum of the squares of the two legs equals the square of the hypotenuse: $25 + 100 = 125$. **(E)** is consistent with the given information and is therefore correct.

Alternatively, we could have applied the Pythagorean Theorem to triangles *ABD* and *BCD*. Solve for $x$ by substituting one equation into the other.

Triangle *ABD*:

$$(BD)^2 = (AB)^2 + (AD)^2$$

$$(BD)^2 = (x + 1)^2 + (x + 3)^2$$

Triangle *BCD:*

$$(CD)^2 = (BC)^2 + (BD)^2$$
$$(5\sqrt{5})^2 = x^2 + (BD)^2$$

We can now plug in the value for $(BD)^2$ from triangle *ABD* into the equation for triangle *BCD:*

$$(5\sqrt{5})^2 = x^2 + (x + 1)^2 + (x + 3)^2$$

We now have one equation with one variable, so we can simplify the equation:

$$(5\sqrt{5})^2 = x^2 + (x + 1)^2 + (x + 3)^2$$
$$(5)^2(5) = x^2 + (x + 1)(x + 1) + (x + 3)(x + 3)$$
$$(25)(5) = x^2 + (x^2 + x + x + 1) + (x^2 + 3x + 3x + 9)$$
$$125 = 3x^2 + 8x + 10$$
$$0 = 3x^2 + 8x - 115$$

To solve for $x$, we must factor the right side of the equation. The only factors of 115 are $5 \times 23$, so our binomials will be in this form: $(\quad 5)(\quad 23)$. Because the 115 is negative, the factors 5 and 23 must be opposite in sign.

Looking at the coefficient on $x^2$, we know we must have an $x$ and a $3x$ term in our factored equation. We can test out the possibilities to determine which combination of signs and coefficients is correct:

$$0 = (x - 5)(3x + 23)$$
$$0 = 3x^2 + 23x - 15x - 115$$
$$0 = 3x^2 + 8x - 115$$

We now have our binomials: $(x - 5)$ and $(3x + 23)$. To solve for $x$, we must set each binomial equal to zero: $x - 5 = 0$, so $x = 5$; or $3x + 23 = 0$, so $x = -\frac{23}{3}$. Remember that we are solving for the length of a side in a figure, and since lengths are never negative, the correct answer must be $x = 5$. **(E)** is correct.

**Step 4: Confirm Your Answer**

When factoring or FOILing an equation, you can check your work by FOILing or factoring the equation back to its original state. This will help confirm that your answer is correct.

**21. (B)**

In how many different ways can 3 sophomores, 3 juniors, and 4 seniors be standing in a line if the 3 sophomores are in 3 consecutive locations, the 3 juniors are in 3 consecutive locations, and the 4 seniors are in 4 consecutive locations?

- ○ 864
- ○ 5,184
- ○ 6,048
- ○ 7,560
- ○ 8,640

**Step 1: Analyze the Question**

This question involves use of the formula $n!$ to calculate the number of different ways to arrange $n$ different objects, where $n!$ is the product of the first $n$ positive integers.

**Step 2: State the Task**

Calculate the number of different arrangements for 3 sophomores, 3 juniors, and 4 seniors in a line, if the 3 sophomores stand consecutively, the 3 juniors stand consecutively, and the 4 seniors stand consecutively.

**Step 3: Approach Strategically**

Because each group (sophomores, juniors, and seniors) stands consecutively, we will first calculate the number of ways to arrange the 3 blocks. There are 3 blocks, so $n = 3$, and the number of arrangements of the 3 blocks is $3! = 3 \times 2 \times 1 = 6$.

Now we need to calculate the arrangements of the individuals within each block. The number of different ways for the 3 sophomores to stand in line is $3! = 3 \times 2 \times 1 = 6$.

Similarly, the number of different ways for the 3 juniors to stand in line is $3! = 3 \times 2 \times 1 = 6$.

The number of different ways for the 4 seniors to stand in line is $4! = 4 \times 3 \times 2 \times 1 = 24$.

Therefore, the number of different ways that the 3 sophomores, 3 juniors, and 4 seniors can be standing in a line is $6 \times 6 \times 6 \times 24 = 36 \times 144 = 5{,}184$. Choice **(B)** is correct.

**Step 4: Confirm Your Answer**

In advanced combination or permutation questions, check to see if you need to calculate the number of combinations/arrangements of a block of items in addition to the combinations/arrangements of the individual items within the block.

**22. (D)**

If the perimeter of square *A* is 12 units and the volume of cube *B* is 8 cubic units, what is the ratio of the area of square *A* to the area of any face on cube *B*?

- ○ 2:27
- ○ 2:3
- ○ 3:2
- ○ 9:4
- ○ 9:2

**Step 1: Analyze the Question**

This is a geometry question that requires us to use the perimeter, area, and volume formulas.

**Step 2: State the Task**

Using the provided information, we are asked to find the ratio of the area of square *A* to the area of any face on cube *B*.

**Step 3: Approach Strategically**

The first piece of information is that the perimeter (the sum of all four sides) of square *A* is 12, which means that each side of square *A* has a length of 3. Next, if the volume (side $\times$ side $\times$ side, or side$^3$) of cube *B* is 8, then each side of cube *B* is 2.

Using that information, we can calculate the area of square *A*, Area $= 3 \times 3 = 9$, and the area of any face on cube *B*, Area $= 2 \times 2 = 4$.

Now expressing (as a ratio) the area of square *A* to the area of any face of cube *B* gives you 9:4. Choice **(D)** is correct.

**Step 4: Confirm Your Answer**

Check to make sure that your answer expresses the correct ratio.

**23. (E)**

For a certain exam, was the standard deviation of the scores for students U, V, W, X, Y, and Z less than the standard deviation of the scores for students A, B, and C?

(1) The standard deviation of the scores of students U, V, and W was less than the standard deviation of the scores of students A, B, and C on the exam.

(2) The standard deviation of the scores of students X, Y, and Z was less than the standard deviation of the scores of students A, B, and C on the exam.

**Step 1: Analyze the Question Stem**

We're asked to compare the standard deviations of the scores of two groups of students. Remember, standard deviation is usually only tested as a concept on the GMAT, so we are not going to be concerned with the lack of numerical values of the scores.

**Step 2: Evaluate the Statements Using 12TEN**

Statement (1) tells us that the standard deviation of the scores of half of the students of one group was smaller than the standard deviation of the scores of the entire second group, but we don't know anything about the remaining members of the first group. Statement (1) is insufficient. Eliminate **(A)** and **(D)**.

Statement (2) gives us the same information but for the other half of the students in the first group, so it is also insufficient. Eliminate **(B)**.

Now let's consider the statements combined. Even though the two halves of the first group, {U, V, W} and {X, Y, Z}, individually have standard deviations smaller than those of the second group {A, B, C}, the groups themselves can be widely or tightly spread around their mean. **(E)** is correct.

To illustrate this, we can quickly Pick Numbers. Let's say U, V, and W all have the same score of 10; X, Y, and Z all have the same score of 100; and A, B, and C have scores of 40, 45, and 50, respectively. Because U, V, and W all have the same score, the standard deviation of the group is 0; likewise, the standard deviation of the group of X, Y, and Z is also 0. The standard deviation of A, B, and C will be larger than each of these groups individually because the scores of A, B, and C are spread around their mean of 45. The conditions of our two statements have been met. But when we consider the entire group of {U, V, W, X, Y, Z}, our chosen values of {10, 10, 10, 100, 100, 100} are much more widely spread around their mean of 45 than our chosen values for A, B, and C {40, 45, 50}. So the answer to the question stem is "no." We could choose other numbers that would have {U, V, W, X, Y, Z} grouped more tightly than {A, B, C}—for example, {48, 49, 50, 50, 51, 52} and {10, 50, 100}—and the answer to our question stem would be "yes." Choice **(E)** is correct.

**24. (C)**

The only contents of a bag are 4 pens that write blue and 3 pens that write green. If 4 of these pens are chosen at random, what is the probability that 2 of the pens write blue and 2 of the pens write green?

- ○ $\frac{2}{7}$
- ○ $\frac{1}{2}$
- ○ $\frac{18}{35}$
- ○ $\frac{4}{7}$
- ○ $\frac{9}{14}$

**Step 1: Analyze the Question**

For this question, we must use our understanding of both combinations and probabilities. First, we need to determine what our desired outcomes are. This question asks us to find the probability of pulling out two blue pens and two green pens from a bag of seven pens.

If we think of each pen as distinct, there are a number of combinations of individual pens we can select from each color subset in the bag, so we need to apply the combinations formula to each color set to determine how many ways we can pull out two pens of each color.

**Step 2: State the Task**

Apply the combinations formula to determine how many ways there are to select two blue pens from the set of four and two green pens from the set of three. Then, determine the probability of getting these desired outcomes out of the total possible outcomes.

**Step 3: Approach Strategically**

The number of ways to choose $k$ objects from a group of $n$ different objects is defined using the combinations formula: $_{n}C_{k} = \frac{n!}{k!(n-k)!}$, where $n$ is a positive integer, and $k$ is an integer such that $0 \leq k \leq n$.

The number of ways to choose two pens that write blue from four pens that write blue is $_{4}C_{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = \frac{12}{2} = 6$.

The number of ways to choose two pens that write green from three pens that write green is $_{3}C_{2} = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = 3$.

So the number of ways to choose two pens that write blue and two pens that write green is $_{4}C_{2} \times {}_{3}C_{2} = 6 \times 3 = 18$. This is the number of desired outcomes, which will be the numerator of our probability fraction.

To find the denominator of our probability fraction, we must find the number of ways to choose four pens from seven pens. Again we use the combinations formula: $_{7}C_{7} = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1} = 35$.

So the probability of choosing two pens that write blue and two pens that write green is $\frac{18}{35}$. **(C)** is correct.

**Step 4: Confirm Your Answer**

Review your calculations to confirm that your answer makes sense logically.