Path Wala

3 Boolean Logic

In This Chapter

3.1 DEVELOPMENT OF BOOLEAN LOGIC

Long ago Aristotle constructed a complete system of formal logic and wrote six famous works on the subject, contributing greatly to the organization of man's reasoning. For centuries afterward, mathematicians kept on trying to solve these logic problems using conventional algebra but only *George Boole* could manipulate these symbols successfully to arrive at a solution with his own mathematical system of logic. Boole's revolutionary paper *'An Investigation of the laws of the thought'* was published in 1854 which led to the development of new system, the *algebra of logic*, 'BOOLEAN ALGEBRA' or 'BOOLEAN LOGIC'.

Boole's work remained confined to papers only until 1938 when *Claude E. Shannon* wrote a paper titled *'A Symbolic Analysis of Relay Switching Circuits'*. In this paper he applied Boolean Logic to solve relay logic problems. As logic problems are binary decisions and Boolean logic effectively deals with these binary values. Thus it is also called *'Switching Algebra'*.

3.2 BINARY VALUED QUANTITIES

Everyday we have to make logic decisions : "Should I carry the book or not ?" "Should I use calculator or not ?" ; "Should I miss TV Programme or not ?". Each of these questions requires a YES or NO answer as there are only these two possible answers.

BINARY DECISION

The decision which results into either YES (TRUE) or NO (FALSE) is called a Binary Decision.

Therefore, each of the above mentioned is a binary decision. Binary decision making also applies to formal logic.

For example, let us consider the following :

1. Indira Gandhi was the only woman Prime Minister of India.
2. 13 – 2 = 11.
3. Delhi is the biggest state in India.
4. What do you say ?
5. What did I tell you yesterday ?

Values *true* and *false* are called Truth values.

1st and 2nd sentences are TRUE but 3rd is FALSE; 4th and 5th are questions which cannot be answered in TRUE and FALSE.

Thus, sentences which can be determined to be *true* or *false* are called *logical statements or truth functions* and the results TRUE or FALSE are called *truth values*. The truth values are depicted by *logical constants* TRUE and FALSE or 1 and 0. 1 means TRUE and 0 means FALSE. And the variables which can store these truth values are called *logical variables* or ***binary valued variables*** as these can store one of the two values TRUE or FALSE.

NOTE

Boolean variables can have value either as 1 (True) or as 0 (zero)

3.3 LOGICAL OPERATIONS

There are some specific operations that can be applied on truth functions. Before learning about these operations, you must know about compound logical functions and logical operators.

3.3.1 Logical Function or Compound Statement

Algebraic variables like *a, b, c* or *x, y, z* etc. are combined with the help of *mathematical operators* like +, –, ×, / to form algebraic expressions *e.g.,*

$$2 \times A + 3 \times B - 6 \times C = (10 \times Z)/2 \times Y \quad i.e., \quad 2A + 3B - 6C = 10Z/2Y$$

Similarly, logic statements or truth functions are combined with the help of *Logical Operators* like AND, OR and NOT to form a *Compound statement or Logical function. e.g.,*

He prefers tea *not* coffee.

He plays guitar *and* she plays sitar.

I watch TV on Sundays *or* I go for swimming.

These logical operators are also used to combine logical variables and logical constants to form *logical expressions e.g.,* assuming *x, y* are logical variables

X NOT Y OR Z

Y AND X OR Z

3.3.2 Logical Operators

Before we start discussion about logical operators, let us first understand *what a Truth Table is.*

For example, following logical statements can have only one of the *two* values (TRUE (YES) or FALSE (NO))

1. I want to have tea. 2. Tea is readily available.

Let us represent all the possible combinations of values these statements can have in the tabular form :

I want to have tea	T	T	F	F
Tea is readily available	T	F	T	F
(Result) I'll have tea	T	F	F	F

T represents True
F represents False

TRUTH TABLE

A Truth Table is a table which represents all the possible values of logical variables / statements along with all the possible results of the given combinations of values.

Or If we represent first statement as X and second statement as Y and result as R then the above table can also be written as follows :

Table 3.1

X	Y	R
1	1	1
1	0	0
0	1	0
0	0	0

1 represents TRUE value and 0 represents FALSE value

This is a truth table *i.e., table of truth values of truth functions.*

Now let us proceed with our discussion about logical operators *i.e.,*

- NOT Operator
- AND Operator
- OR Operator

TAUTOLOGY

If the result of any logical statement or expression is always TRUE or 1 for all input combinations, it is called Tautology.

FALLACY

If the result of any logical statement or expression is always FALSE or 0 for all input combinations, it is called Fallacy.

NOT Operator

This operator **operates on single variable** and operation performed by NOT operator is called *complementation* and the symbol we use for it is ‾ (bar). Thus $\overline{X}$ means complement of X and $\overline{YZ}$ means complement of YZ. As we know, the variables used in boolean equations have a unique characteristic that they may assume only one of two possible values 0 and 1, where 0 denotes FALSE and 1 denotes TRUE value. Thus the complement operation can be defined quite simply.

$$\overline{0} = 1$$
$$\overline{1} = 0$$

Table 3.2 *Truth Table for NOT Operators*

X	$\overline{X}$ (i.e., NOT X)
0	1
1	0

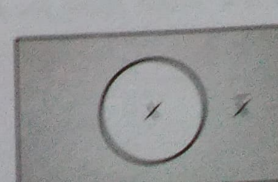


Figure 3.1 Venn diagram for $\overline{X}$

Several other symbols *e.g.* ~, ′ are also used for the complementation symbol. If ~ is used then ~ X is read is 'negation of X' and if ′ is used then X′ is read as complement of X. NOT operation is singular or unary operation as it operates on single variable. Venn diagram for $\overline{X}$ is given in Fig. 3.1 where shaded area depicts $\overline{X}$.

OR Operator

A second important operator in boolean algebra is OR operator which denotes operation called *logical addition* and the symbol we use for it is +. The + symbol, therefore, does not have the 'normal' meaning, but is a *logical addition* or logical OR symbol. Thus X + Y can be read as **X OR Y**. For OR operation the possible input and output combinations are as follows :

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

Shaded Portion shows *X* + *Y*

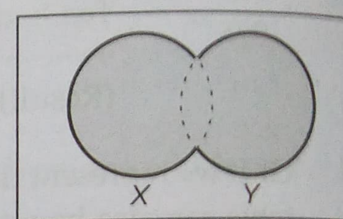


Figure 3.2 Venn diagram for X + Y.

And the truth table of OR operator is given below :

Table 3.3 *Truth Table for OR Operator*

X	Y	X + Y (i.e., X OR Y)
0	0	0
0	1	1
1	0	1
1	1	1

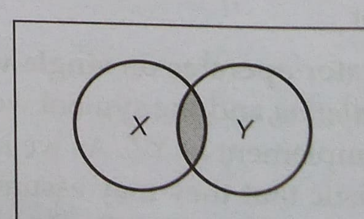
Note that when any one of X and Y is 1, X + Y is 1.

To avoid ambiguity, there are other symbols *e.g.*, *U*, *v*, and *V* have been recommended as replacements for the + sign. Computer people still use the + sign, however, which was the symbol originally proposed by Boole. Venn diagram for X + Y is given (Fig. 3.2), where shaded area depicts X + Y.

AND Operator

AND operator performs another important operation of boolean algebra called *logical multiplication* and the symbol for AND operation is (.) dot. Thus X . Y will be read as X AND Y. The rules for AND operation are :

$$0 . 0 = 0$$
$$0 . 1 = 0$$
$$1 . 0 = 0$$
$$1 . 1 = 1$$



Shaded Portion shows *X* · *Y*

X.Y

Figure 3.3 Venn diagram for (*X* . *Y*).

and the truth table for AND is as follows :

Table 3.4 *Truth Table for AND Operator*

X	Y	X . Y (i.e., X AND Y)
0	0	0
0	1	0
1	0	0
1	1	1

Note that only when both X and Y are 1's, then XY has the result 1.

Venn diagram for X . Y is given in Fig. 3.3, where shaded area depicts (X . Y).

3.3.3 Evaluation of Boolean Expressions using Truth Table

Logical variables are combined by means of logical operators (AND, OR, NOT) to form a boolean expression.

For example, $X + \overline{Y.Z} + \overline{Z}$ is a boolean expression.

It is often convenient to shorten X . Y . Z to XYZ, and using this convention, above expression can be written as $X + \overline{YZ} + \overline{Z}$

To study a boolean expression, it is very useful to construct a table of values for the variables and then to evaluate the expression for each of the possible combinations of variables in turn.

Consider the expression $X + \overline{YZ}$. Here three variables X, Y, Z are forming the expression, each of the variables can assume the value 0 or 1. The possible combinations of values may be arranged in ascending order as in Table 3.5.

NOTE

A truth table of *n* input variables with have 2^n input combinations *i.e.*, 2^n rows *e.g.*, a 4-variable truth table will have 2^4 *i.e.*, 16 rows in it.

To see
Truth Table Formation
in action

Scan
QR Code

Table 3.5 *Possible Combinations of X, Y and Z*

X	Y	Z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Since X, Y, Z are three (3) variables in total a truth table involving 3 input variables will have 2^3 i.e., 8 rows in total. The left most column will have half of total entries (i.e., 4 entries) as zeros and half as 1's (in total 8).

The next column will have no of zero's and 1's halved than first column completing 8 rows and so on. That is why, first column has 4 0's and 4 1's, next column has two 0's followed by two 1's completing 8 rows in total and the last column has one 0's followed by one 1's completing 8 rows

So a column is added to list Y . Z (Table 3.6)

Table 3.6 *Truth Table for (Y . Z)*

X	Y	Z	Y . Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Here AND operation is applied only on columns Y and Z

One more column is now added to list the values of $\overline{YZ}$ (Table 3.7).

Table 3.7 *Truth Table for Y . Z and $\overline{YZ}$.*

X	Y	Z	Y . Z	$\overline{YZ}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

Note that $\overline{YZ}$ contains complemented values of YZ

Now values of X are ORed (*logical addition*) to the values of $\overline{YZ}$ and the resultant values are contained in the last column (Table 3.8).

Table 3.8 *Truth Table for $X + \overline{YZ}$.*

X	Y	Z	Y . Z	$\overline{YZ}$	$X + \overline{YZ}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	1

Now observe the expression $X + \overline{YZ}$, after ANDing Y and Z, the result has been complemented and then ORed with X.

Here the result is 0 only when both the columns X and $\overline{YZ}$ have 0, otherwise if there is 1 in any of the two columns X and $\overline{YZ}$, the result is 1.

Please note here, while evaluating boolean expression there is a precedence order which is to be taken care of always. Always the order of evaluation of logical operators is firstly NOT then AND and then OR. If there are parenthesis, then the expression in parenthesis is evaluated first.

EXAMPLE 1 *Using Boolean logic, verify using truth table that $X + XY = X$ for each X, Y in {0, 1}.*

SOLUTION

As the expression $X + XY = X$ is a two-variable expression, so we require possible combination of values of X, Y. Truth Table will be as follows :

X	Y	XY	X + XY
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Comparing the columns $X + XY$ and X, we find, contents of both the columns are identical, hence verified.

Check Point 3.1

1. Name the person who developed boolean logic.
2. What is the other name of boolean logic ? In which year was the boolean logic/algebra developed ?
3. What is a binary decision ? What do you mean by a binary valued variable ?
4. What do you mean by tautology and fallacy ?
5. What is a logic gate ? Name the three basic logic gates.

EXAMPLE 2 *Using Boolean logic, verify using truth table that $(X+Y)' = X'Y'$ for each X, Y in {0, 1}.*

SOLUTION As it is a 2-variable expression, truth table will be as follows :

X	Y	X + Y	(X + Y)′	X′	Y′	X′ Y′
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Comparing the columns $(X+Y)'$ and $X'Y'$, both the columns are identical, hence verified.

EXAMPLE 3 *Prepare a table of combinations for the following boolean logic expressions :*

(a) $\overline{X}\,\overline{Y} + \overline{X}Y$ (b) $XY\overline{Z} + \overline{X}\,\overline{Y}Z$ (c) $\overline{X}Y\overline{Z} + X\overline{Y}$

SOLUTION (a) As $\overline{X}\overline{Y} + \overline{X}Y$ is a 2-variable expression, its truth table is as follows :

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X}\,\overline{Y}$	$\overline{X}\,Y$	$\overline{X}\,\overline{Y} + \overline{X}\,Y$
0	0	1	1	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

(b) Truth table for this 3 variable expression is as follows :

X	Y	Z	$\overline{X}$	$\overline{Y}$	$\overline{Z}$	$XY\overline{Z}$	$\overline{X}\,\overline{Y}Z$	$XY\overline{Z} + \overline{X}\,\overline{Y}Z$
0	0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	1	1
0	1	0	1	0	1	0	0	0
0	1	1	1	0	0	0	0	0
1	0	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0	0
1	1	0	0	0	1	1	0	1
1	1	1	0	0	0	0	0	0

Check Point 3.2

1. Which gates implement logical addition, logical multiplication and complementation ?
2. What is the other name of NOT gate ?
3. What is a truth table ? What is the other name of truth table ?
   (i) $A + 0 = ?$ (ii) $A + 1 = ?$
   (iii) $A \, . \, 0 = ?$ (iv) $A \, . \, 1 = ?$
4. How many input combinations can be there in the truth table of a logic system having (N) input binary variables ?

(c) Truth table for $\overline{X}Y\overline{Z} + X\overline{Y}$ is as follows :

X	Y	Z	$\overline{X}$	$\overline{Y}$	$\overline{Z}$	$\overline{X}Y\overline{Z}$	$X\overline{Y}$	$\overline{X}Y\overline{Z} + X\overline{Y}$
0	0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0	0
0	1	0	1	0	1	1	0	1
0	1	1	1	0	0	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	0	1	1
	1	0	0	0	1	0	0	0
	1	1	0	0	0	0	0	0

EXAMPLE 4 *Prepare truth table for the following boolean expressions :*

(a) $X(\overline{Y}+\overline{Z})+X\overline{Y}$ (b) $X\overline{Y}(Z+Y\overline{Z})+\overline{Z}$ (c) $A[(\overline{B}+C)+\overline{C}]$

SOLUTION

(a) Truth table for $X(\overline{Y}+\overline{Z})+X\overline{Y}$ is as follows :

X	Y	Z	$\overline{Y}$	$\overline{Z}$	$(\overline{Y}+\overline{Z})$	$X(\overline{Y}+\overline{Z})$	$X\overline{Y}$	$X(\overline{Y}+\overline{Z})+X\overline{Y}$
0	0	0	1	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	1	0	0	1	1	0	0	0
0	1	1	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1
1	0	1	1	0	1	1	1	1
1	1	0	0	1	1	1	0	1
1	1	1	0	0	0	0	0	0

(b) Truth Table for $X\overline{Y}(Z+Y\overline{Z})+\overline{Z}$ is as follows :

X	Y	Z	$\overline{Y}$	$\overline{Z}$	$Y\overline{Z}$	$Z+Y\overline{Z}$	$X\overline{Y}$	$X\overline{Y}(Z+Y\overline{Z})$	$X\overline{Y}(Z+Y\overline{Z})+\overline{Z}$
0	0	0	1	1	0	0	0	0	1
0	0	1	1	0	0	1	0	0	0
0	1	0	0	1	1	1	0	0	1
0	1	1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1	0	1
1	0	1	1	0	0	1	1	1	1
1	1	0	0	1	1	1	0	0	1
1	1	1	0	0	0	1	0	0	0

(c) Truth Table for $A[(\overline{B}+C)+\overline{C}]$ is as follows :

A	B	C	$\overline{B}$	$\overline{C}$	$(\overline{B}+C)$	$(\overline{B}+C)+\overline{C}$	$A[(\overline{B}+C)+\overline{C}]$
0	0	0	1	1	1	1	0
0	0	1	1	0	1	1	0
0	1	0	0	1	0	1	0
0	1	1	0	0	1	1	0
1	0	0	1	1	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	1	1
1	1	1	0	0	1	1	1

3.4 BASIC LOGIC GATES

After *Shannon* applied boolean logic in telephone switching circuits, engineers realized that boolean algebra could be applied to computer electronics as well.

In the computers, these boolean operations are performed by logic gates.

What is a Logic Gate ?

A Gate is a basic electronic circuit which operates on one or more signals to produce an output signal.

Gates are digital (two-state) circuits because the input and output signals are either low voltage (denotes 0) or high voltage (denotes 1). Gates are often called *logic circuits* because they can be analyzed with boolean logic. There are *three* types of logic gates :

- Inverter (NOT gate)
- OR gate
- AND gate

GATE

A Gate is a basic electronic circuit which operates on one or more signals to produce an output signal.

3.4.1 Inverter (NOT Gate)

An **Inverter (Not Gate)** is a gate with only one input signal and one output signal; the output state is always the opposite of the input state.

An inverter is also called a NOT gate because the output is not the same as the input. The output is sometimes called the *complement* (opposite) of the input.

INVERTER (NOT GATE)

An Inverter (Not Gate) is a gate with only one input signal and one output signal; the output state is always the opposite of the input state.

Following tables summarise the operation :

Table 3.9 *Truth Table for NOT gate*

X	$\overline{X}$
Low	High
High	Low

Table 3.10 *Alternative Truth Table for NOT Gate*

X	$\overline{X}$
0	1
1	0

A low input *i.e.*, 0 produces high output *i.e.*, 1, and vice versa. The symbol for inverter is given in adjacent Fig. 3.4.

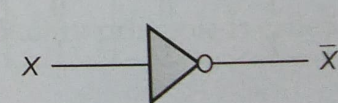


Figure 3.4 NOT gate symbol

3.4.2 OR Gate

The **OR Gate** has two or more input signals but only one output signal. If any of the input signals is 1 (high), the output signal is 1 (high).

If all inputs are 0 then output is also 0. If one or more inputs are 1, the output is 1.

An OR gate can have as many inputs (2 or more inputs) as desired. No matter how many inputs are there, the action of OR gate is the same : one or more 1 (high) inputs produce output as 1.

OR GATE

The **OR Gate** has two or more input signals but only one output signal. If any of the input signals is 1 (high), the output signal is 1 (high).

Following tables show OR action.

Table 3.11 *Two Input OR Gate*

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

F = X + Y

Table 3.12 *Three Input OR Gate*

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

F = X + Y + Z

The symbol for OR gate is given below :

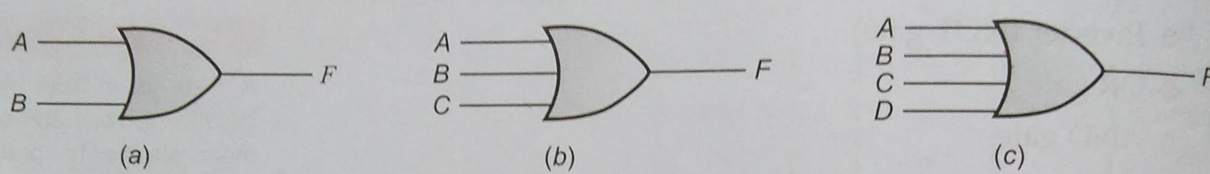


Figure 3.5 (*a*) Two input OR gate (*b*) Three input OR gate (*c*) Four input OR gate.

3.4.3 AND gate

The **AND Gate** can have two or more than two input signals and produce an output signal. When all the inputs are 1 *i.e.*, high then the output is 1 otherwise output is 0 only.

If any of the inputs is 0, the output is 0. To obtain output as 1, all inputs must be 1.

An AND gate can have as many inputs (2 or more inputs) as desired. Following tables illustrate AND action.

> **AND GATE**
>
> The **AND Gate** can have two or more than two input signals and produce one output signal. When all the inputs are 1 *i.e.*, high then the output is 1 otherwise output is 0 only.

Table 3.13 *Two Input AND Gate*

X	Y	F
0	0	0
0	1	0
1	0	0
1	1	1

Here, F = X . Y

Table 3.14 *Three Input AND Gate*

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Here, F = X . Y . Z

The symbol for AND is

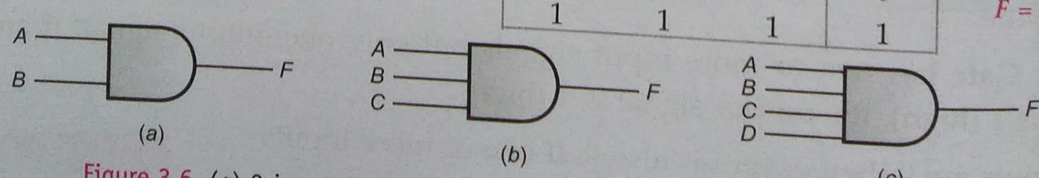


Figure 3.6 (*a*) 2-input AND gate (*b*) 3-input AND gate (*c*) 4-input AND gate

EXAMPLE 5 *From the expressions given below, identify the logic gates that can implement such expressions.*

(*i*) XYZ (*ii*) $\overline{Y}Z$ (*iii*) $\overline{X}+Y$ (*iv*) $P+Q$ (*v*) $P+\overline{Q}+\overline{R}$

(*vi*) $\overline{A}B+\overline{C}D$ (*vii*) $A\overline{B}\,\overline{C}$

SOLUTION

(*i*) $X.Y.Z \rightarrow$. signifies AND gate, hence only AND gates required.

(*ii*) $\overline{Y}.Z \rightarrow$ $\overline{\ }$ signifies NOT and . signifies AND, hence NOT and AND gates.

(*iii*) $\overline{X}+Y \rightarrow$ $\overline{\ }$ signifies NOT and + signifies OR, hence NOT and OR gates.

(*iv*) $P+Q \rightarrow$ + signifies OR, hence OR gate.

(*v*) $P+\overline{Q}+\overline{R} \rightarrow$ $\overline{\ }$ signifies NOT and + signifies OR, hence NOT and OR gates.

(*vi*) $\overline{A}.B+\overline{C}.D \rightarrow$ $\overline{\ }$ signifies NOT, . signifies AND and + signifies OR, hence NOT, AND and OR gates.

(*vii*) $A.\overline{B}.\overline{C} \rightarrow$ $\overline{\ }$ signifies NOT, . signifies AND, hence NOT and AND gates.

3.5 BASIC POSTULATES OF BOOLEAN LOGIC

Boolean logic algebra, being a system of mathematics, consists of *fundamental laws* that are used to build a workable, cohesive framework upon which are based the theorems of boolean algebra. These fundamental laws are known as *Basic postulates of boolean logic algebra.* These postulates state basic relations in boolean algebra, that follow :

I. If $X \neq 0$ then $X = 1$; and If $X \neq 1$ then $X = 0$

II. OR Relations (*Logical Addition*)

$0+0=0$; $0+1=1$; $1+0=1$; $1+1=1$

III. AND Relations (*Logical Multiplication*)

$0.0=0$; $0.1=0$; $1.0=0$; $1.1=1$

IV. Complement Rules : $\overline{0}=1$; $\overline{1}=0$

3.6 PRINCIPLE OF DUALITY

This is a very important principle used in boolean logic. This states that *starting with a boolean relation, another boolean relation can be derived by* :

1. changing each OR sign (+) to an AND sign (.)
2. changing each AND sign (.) to an OR sign (+)
3. replacing each 0 by 1 and each 1 by 0.

The derived relation using duality principle is called *dual of original expression.*

For instance, we take postulate II related to logical addition, which states :

(*a*) $0+0=0$ (*b*) $0+1=1$ (*c*) $1+0=1$ (*d*) $1+1=1$

Now working according to above guidelines, + is changed to (.) and 0's are replaced by 1's, these become

(*i*) $1.1=1$ (*ii*) $1.0=0$ (*iii*) $0.1=0$ (*iv*) $0.0=0$

which are nothing but *same as that of postulate III* related to logical multiplication. So *i, ii, iii, iv* are the duals of *a, b, c* & *d*. We'll be applying this duality principle in the theorems of boolean algebra which is our next topic.

Check Point 3.3

1. Write the dual of : $1+1=1$
2. Give the dual of the following in Boolean algebra :
   (*i*) $X . X' = 0$ for each X
   (*ii*) $X + 0 = X$ for each X.
3. What is the significance of Principle of Duality ?
4. Write dual of the following Boolean Expression :
   (*a*) $(x + y')$ (*b*) $xy + xy' + x'y$
   (*c*) $a + a'b + b'$ (*d*) $(x + y' + z)(x + y)$

3.7 BASIC THEOREMS OF BOOLEAN ALGEBRA/LOGIC

Basic postulates of boolean algebra are used to define *basic theorems of boolean algebra* that provide all the tools necessary for manipulating boolean expressions. Although simple in appearance, these theorems may be used to construct the boolean algebra.

3.7.1 Properties of 0 and 1

(*a*) $0 + X = X$ (*b*) $1 + X = 1$

(*c*) $0 . X = 0$ (*d*) $1 . X = X$

Proof. (*a*) $0 + X = X$

Truth table for above expression is given below in Table 3.15, where *R* signifies the output

Table 3.15 *Truth Table for* $0 + X = X$.

0	X	R
0	0	0
0	1	1

as *X* can have values either 0 or 1 (*postulate I*) both the values ORed with 0 produce the *same output as that of X*.

Hence proved.

(*b*) $1 + X = 1$

Truth table for this expression is given below in Table 3.16, where *R* signifies the output.

Table 3.16 *Truth Table for* $1 + X = 1$

1	X	R
1	0	1
1	1	1

Again *X* can have values 0 or 1. Both the values (0 and 1) ORed with 1 produce the output as 1. Hence proved.

Therefore $1 + X = 1$ *is a* ***tautology***.

(*c*) $0 . X = 0$

As both the possible values of *X* (0 and 1) are to be ANDed with 0, so, the truth table for this expression is as follows where (*R* signifies the output)

Table 3.17 *Truth Table for* $0 . X = 0$.

0	X	R
0	0	0
0	1	0

Both the values of X(0 and 1) when ANDed with produce the *output as* 0. Hence proved.

Therefore, $0 . X = 0$ is a ***fallacy***.

(*d*) $1 . X = X$

Now both the possible values of X (0 and 1) are to be ANDed with 1. Thus the truth table for it will be as follows :

Table 3.18 *Truth Table for* $1 . X = X$

1	X	R
1	0	0
1	1	1

Now observe both the values (0 and 1) when ANDed with 1 produce the *same output as that of* X. Hence proved. Here properties *b* and *c* are duals of each other and properties *a* and *d* are duals of each other.

3.7.2 Indempotence Law

This law states that

(*a*) X + X = X (*b*) X . X = X

Proof. (*a*) **X + X = X**

To prove this law, we will make truth table for above expression. As X is to be ORed with itself only, we will prepare truth table with the two possible values of X (*i.e.*, 0 and 1).

Table 3.19 *Truth Table for X + X = X*

X	X	R
0	0	0
1	1	1

$0 + 0 = 0$ (*ref. postulate II*)

and $1 + 1 = 1$ (*ref. postulate II*)

⇒ X + X = X, as it holds true for both values of X. Hence proved.

(*b*) **X . X = X**

Here X is ANDed with itself. Again we will prepare truth table for this expression taking 2 possible values of X (0 and 1).

Table 3.20 *Truth Table for X . X = X*

X	X	R
0	0	0
1	1	1

$0 . 0 = 0$ (*ref. postulate III*)

and $1 . 1 = 1$ (*ref. postulate III*)

⇒ X . X = X, as it holds true for both values of X. Hence proved.

(*a*) and (*b*) are duals of each other.

3.7.3 Involution

This law states that : $(\overline{\overline{X}}) = X$

To prove this, again we'll prepare truth table which is given below.

Table 3.21 *Truth Table for* $\overline{\overline{X}} = X$

X	$\overline{X}$	$\overline{\overline{X}}$
0	1	0
1	0	1

In Boolean Algebra, if an expression holds *true* then its dual is also *true* and vice-versa.

First column represents possible values of X, second column represents complement of X (*i.e.*, $\overline{X}$) and the third column represents complement of $\overline{X}$ (*i.e.*, $\overline{\overline{X}}$) which is same as that of X. Hence proved.

This law is also called *double-inversion rule.*

3.7.4 Complementarity Law

These laws state that

(*a*) $X + \overline{X} = 1$

(*b*) $X . \overline{X} = 0$

Proof. (*a*) $X + \overline{X} = 1$

We will prove $X + \overline{X} = 1$ with the help of truth table which is given below :

Table 3.22 *Truth Table for* $X + \overline{X} = 1$

X	$\overline{X}$	$X + \overline{X}$
0	1	1
1	0	1

Here, in the first column possible values of X have been taken, second column consists of $\overline{X}$ values (*complement values of* X), X and $\overline{X}$ values are ORed and the output is shown in third column as

$0 + 1 = 1,$ (*ref. postulate II*)

$1 + 0 = 1$ (*ref. postulate II*)

$\Rightarrow X + \overline{X} = 1$, as it holds true for both possible values of X.

Hence proved. It is a *tautology*.

(*b*) $X . \overline{X} = 0$

Truth table for this expression is as follows :

Table 3.23 *Truth Table for* $X . \overline{X} = 0$

X	$\overline{X}$	$X . \overline{X}$
0	1	0
1	0	0

as $0 . 1 = 0$ (*ref. postulate III*)

and $1 . 0 = 0$ (*ref. postulate III*)

$\Rightarrow X . \overline{X} = 0$, as it holds true for both the values of X. Hence proved. *Observe here* $X . \overline{X} = 0$ *is dual* of $X + \overline{X} = 1$.

Changing (+) to (.) and 1 to 0, and we get $X . \overline{X} = 0$.

It is a *fallacy*.

3.7.5 Commutative Law

These laws state that

(*a*) $X + Y = Y + X$ (*b*) $X . Y = Y . X$

Proof.

(*a*) $X + Y = Y + X$

Truth Table for $X + Y = Y + X$ is given below :

Table 3.24 *Truth Table for* $X + Y = Y + X$

X	Y	X + Y	Y + X
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Compare the columns X + Y and Y + X, both of these are identical. Hence proved.

(*b*) $X . Y = Y . X$

Truth table for $X . Y = Y . X$ is given below :

Table 3.25 *Truth Table for* $X . Y = Y . X$

X	Y	X . Y	Y . X
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Both of the columns X . Y and Y . X are identical, hence proved.

3.7.6 Associative Law

These laws state that

(*a*) $X + (Y + Z) = (X + Y) + Z$ (*b*) $X (YZ) = (XY) Z$

Proof. (*a*) Truth table for $X + (Y + Z) = (X + Y) + Z$ is given on next page.

Table 3.26 *Truth Table for* $X + (Y + Z) = (X + Y) + Z$

X	Y	Z	Y + Z	X + Y	X + (Y + Z)	(X + Y) + Z
0	0	0	0	0	0	0
0	0	1	1	0	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Compare the columns $X + (Y + Z)$ and $(X + Y) + Z$, both of these are identical. Hence proved[1].
Since rule (*b*) is dual of rule (*a*), hence it is also proved.

3.7.7 Distributive Law

This law states that : (*a*) $X(Y + Z) = XY + XZ$ (*b*) $X + YZ = (X + Y)(X + Z)$

Proof. (*a*) Truth table for $X(Y + Z) = XY + XZ$ is given below :

Table 3.27 *Truth Table for* $X(Y + Z) = XY + XZ$

X	Y	Z	Y + Z	XY	XZ	X(Y + Z)	XY + XZ
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Both the columns $X(Y + Z)$ and $XY + YZ$ are identical, **hence proved.**

(*b*) Since rule (*b*) is dual of rule (*a*), hence it is also proved.

However, we are giving the algebraic proof of law $X + YZ = (X + Y)(X + Z)$

R.H.S. $= (X + Y)(X + Z) = XX + XZ + XY + YZ$

$= X + XZ + XY + YZ$ ($XX = X$, *Indempotence law*)

$= X + XY + XZ + YZ = X(1 + Y) + XZ + YZ$

$= X.1 + XZ + YZ$ ($1 + Y = 1$, *property of 0 and 1*)

$= X + XZ + YZ$ ($X.1 = X$, *property of 0 and 1*)

$= X(1 + Z) + YZ = X.1 + YZ$ ($1 + Z = 1$, *property of 0 and 1*)

$= X + YZ$ ($X.1 = X$, *property of 0 and 1*)

$=$ L.H.S.

Hence proved.

NOTE

$X + YZ$ expression is **sum of two product**-terms ($X.1, YZ$) and $(X + Y)(X + Z)$ is product of sum-terms $(X + Y, X + Z)$. So, this law is a useful one to convert a sum-of-product type expression to product-of-sum type expression and vice-versa.

1. Recall that if a boolean expression is true then its dual is also true.

3.7.8 Absorption Law

According to this law : (*a*) $X + XY = X$ (*b*) $X(X + Y) = X$

Proof.

(*a*) $X + XY = X$

Truth table for $X + XY = X$ is given below :

Table 3.28 *Truth Table for* $X + XY = X$

X	Y	XY	X + XY
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Column X and X + XY are identical. Hence proved. Also it can be proved algebraically as

$$\text{L.H.S.} = X + XY$$
$$= X(1 + Y)$$

Putting $1 + Y = 1$ (*ref. properties of 0, 1 Theorem 1*)

$X.1 = X$ = R.H.S. (*ref. properties of 0, 1*)

Hence proved.

(*b*) $X(X + Y) = X$

Since rule (*b*) is dual of rule (*a*), it is also proved. However, we are giving the algebraic proof of this law.

$$\text{L.H.S.} = X(X + Y) = X.X + XY$$
$$= X.X + XY$$
$$= X + XY$$

($X.X = X$: *Indempotence Law*)

$$= X(1 + Y)$$
$$= X.1$$

(*using* $1 + Y = 1$, *properties of 0, 1*)

$= X$ ($X.1 = X$, *using property of 0, 1*)

$= \text{R.H.S}$

Hence proved.

3.7.9 Some Other Rules of Boolean Logic Algebra

There are some more rules of Boolean algebra which are given below :

$X + \overline{X}Y = X + Y$ (Sometimes also referred to as the *third distributive law*)

This rule can easily be proved by truth tables. As you are quite familiar with truth tables now, truth table proof is left for you as an exercise, the other proofs of these rules are being given here :

$$X + \overline{X}Y = X + Y$$

Proof.

$\text{L.H.S} = X + \overline{X}Y$

$= X.1 + \overline{X}Y$ (Putting $X = X.1$, *property of 0 and 1*)

$= X(1 + Y) + \overline{X}Y$ (Putting 1 as $1 + Y$, $\because 1 + Y = 1$, *property of 0 and 1*)

$= X + XY + \overline{X}Y$

$= X + Y(X + \overline{X})$

$= X + Y.1$ ($X + \overline{X} = 1$, *complementarity law*)

$= X + Y$ ($Y.1 = Y$, *property of 0 and 1*)

$= \text{R.H.S.}$

Hence proved.

All the theorems of Boolean algebra, which we have covered so far, are summarised in the following table :

1.	*Properties of 0*	$0 + X = X$; $0 . X = 0$
2.	*Properties of 1*	$1 + X = 1$; $1 . X = X$
3.	*Indempotence law*	$X + X = X$; $X . X = X$
4.	*Involution*	$\overline{\overline{X}} = X$
5.	*Complementarity law*	$X + \overline{X} = 1$; $X . \overline{X} = 0$
6.	*Commutative law*	$X + Y = Y + X$; $X . Y = Y . X$
7.	*Associative law*	$X + (Y + Z) = (X + Y) + Z$; $X(YZ) = (XY)Z$
8.	*Distributive law*	$X(Y + Z) = XY + XZ$; $X + YZ = (X + Y)(X + Z)$
9.	*Absorption law*	$X + XY = X$; $X . (X + Y) = X$
10.	*Other (3rd distributive law)*	$X + \overline{X}Y = X + Y$

3.8 DEMORGAN'S THEOREMS

One of the most powerful identities used in Boolean logic is DeMorgan's theorem. *Augustus DeMorgan* had paved the way to Boolean logic by discovering these two important theorems. This section introduces these two theorems of DeMorgan.

3.8.1 DeMorgan's First Theorem

It states that $\overline{X + Y} = \overline{X}\,\overline{Y}$

Proof. To prove this theorem, we need to recall complementarity laws, which state that

$$X + \overline{X} = 1 \text{ and } X . \overline{X} = 0$$

i.e., a logical variable/expression when added with its complement produces the output as 1 and when multiplied with its complement produces the output as 0.

Now to prove DeMorgan's first theorem, we will use complementarity laws.

Let us assume that $P = X + Y$ where, P, X, Y are Logical/Boolean variables. Then, according to complementation law : $P + \overline{P} = 1$ and $P . \overline{P} = 0$.

That means, if P, X, Y are Boolean variables then this complementarity law must hold for variable P too. In other words, if $\overline{P}$ *i.e.*, if $\overline{X + Y} = \overline{X}\,\overline{Y}$ then

$(X + Y) + \overline{X}\,\overline{Y}$ must be equal to 1. (as $X + \overline{X} = 1$)

and $(X + Y) . \overline{X}\,\overline{Y}$ must be equal to 0. (as $X . \overline{X} = 0$)

Let us first prove the first part, *i.e.*,

$$(X + Y) + (\overline{X}\,\overline{Y}) = 1$$

$(X + Y) + \overline{X}\,\overline{Y} = ((X + Y) + \overline{X}).((X + Y) + \overline{Y})$ (ref. $X + YZ = (X + Y)(X + Z)$)

$= (X + \overline{X} + Y).(X + Y + \overline{Y})$ (ref. $X + \overline{X} = 1$)

$= (1 + Y).(X + 1)$ (ref. $1 + X = 1$)

$= 1.1$

$= 1$ So first part is proved.

Now let us prove the second part *i.e.*,

$$(X+Y).\overline{X}\,\overline{Y}=0$$

$$\begin{aligned}(X+Y).\overline{X}\,\overline{Y} &= \overline{X}\,\overline{Y}.(X+Y) && (ref.\ X(YZ)=(XY)Z)\\ &= \overline{X}\,\overline{Y}\,X+\overline{X}\,\overline{Y}\,Y && (ref.\ X(Y+Z)=XY+XZ)\\ &= X\,\overline{X}\,\overline{Y}+\overline{X}\,Y\,\overline{Y}\\ &= 0.\overline{Y}+\overline{X}.0 && (ref.\ X.\overline{X}=0)\\ &= 0+0=\mathbf{0}\end{aligned}$$

So, second part is also proved, thus : $\overline{X+Y}=\overline{X}\,\overline{Y}$

3.8.2 DeMorgan's Second Theorem

This theorem states that : $\overline{X.Y}=\overline{X}+\overline{Y}$

Proof. Again to prove this theorem, we will make use of complementarity law *i.e.*,

$$X+\overline{X}=1 \quad \text{and} \quad X.\overline{X}=0.$$

If XY's complement is $\overline{X}+\overline{Y}$ then it must be true that

(*a*) $XY+(\overline{X}+\overline{Y})=1$ and (*b*) $XY(\overline{X}+\overline{Y})=0$

To prove the *first part*

$$\begin{aligned}L.H.S &= XY+(\overline{X}+\overline{Y})\\ &= (\overline{X}+\overline{Y})+XY && (ref.\ X+Y=Y+X)\\ &= (\overline{X}+\overline{Y}+X).(\overline{X}+\overline{Y}+Y) && (ref.\ X+YZ=(X+Y)(X+Z))\\ &= (X+\overline{X}+\overline{Y}).(\overline{X}+Y+\overline{Y})\\ &= (1+\overline{Y}).(\overline{X}+1) && (ref.\ X+\overline{X}=1)\\ &= 1.1 && (ref.\ 1+X=1)\\ &= 1=R.H.S\end{aligned}$$

Now the *second part i.e.*,

$$XY.(\overline{X}+\overline{Y})=0$$

$$\begin{aligned}L.H.S &= XY.(\overline{X}+\overline{Y})\\ &= XY\overline{X}+XY\overline{Y} && (ref.\ X(Y+Z)=XY+XZ)\\ &= X\overline{X}Y+XY\overline{Y}\\ &= 0.Y+X.0 && (ref.\ X.\overline{X}=0)\\ &= 0+0=0=R.H.S.\end{aligned}$$

$$XY.(\overline{X}+\overline{Y})=0$$

and $$XY+(\overline{X}+\overline{Y})=1$$

$\Rightarrow$ $\overline{XY}=\overline{X}+\overline{Y}$. Hence the theorem.

Although the identities above represent DeMorgan's theorem, the transformation is more easily performed by following these steps :

(*i*) Complement the entire function

(*ii*) Change all the ANDs (.) to ORs (+) and all the ORs (+) to ANDs (.)

(*iii*) Complement each of the individual variables.

> **NOTE**
> 'Break the line, change the sign" to demorganize a boolean expression.

This process is called *demorganization*. For example,

$$\begin{aligned}\overline{\overline{AB}+\overline{A}+AB} &= \overline{\overline{AB}}.\overline{\overline{A}}.\overline{AB} && \text{[Changed + to . and complemented individual expressions]}\\ &= AB\,.\,A\,.\,\overline{AB} && [\because \overline{\overline{AB}}=AB \text{ and } \overline{\overline{A}}=A]\\ &= AB.\overline{AB}.A\\ &= 0.A && [AB.\overline{AB}=0]\\ &= 0 && [\because 0.A=0]\end{aligned}$$

Alternatively, you may solve it as follows :

$$\overline{\overline{AB}+\overline{A}+AB} = \overline{[\overline{A}+\overline{B}]+\overline{A}+AB]} \qquad (\because \overline{AB}=\overline{A}+\overline{B}\text{ ; DeMorgan's 2}^{nd}\text{ theorem})$$

$$=\overline{(\overline{A}+\overline{B})+(\overline{A}+AB)} = \overline{(\overline{A}+\overline{B})}\cdot\overline{(\overline{A}+AB)} \qquad (\because \overline{(X+Y)}=\overline{X}.\overline{Y}$$

$$=\overline{\overline{A}}.\overline{\overline{B}}.(\overline{\overline{A}}.\overline{AB}) \qquad \text{DeMorgan's 2}^{nd}\text{ theorem})$$

$$=A.B(A.(\overline{A}+\overline{B}))=AB(A\overline{A}+A\overline{B})=AB(0+A\overline{B}) \qquad (A\overline{A}=0)$$

$$=AB.0+ABA\overline{B}=0+AAB\overline{B}$$

$$=0+A\,A.0 \qquad (B\overline{B}=0)$$

$$=0+0=0$$

EXAMPLE 6 *Find the complement of expression : $A+\overline{BC}$*

SOLUTION Complement of given expression

$$=\overline{A+\overline{BC}}.$$

$$=\overline{A}.\overline{\overline{BC}} \qquad \text{(Using DeMorgan's 1st theorem)}$$

$$=\overline{A}+BC.$$

EXAMPLE 7 *Find the complement of expression : $A\overline{B}+B\overline{C}\,\overline{D}$*

SOLUTION Complement of given expression

$$=\overline{A\overline{B}+B\overline{C}\,\overline{D}}$$

$$=\overline{A\overline{B}}.\overline{B\overline{C}\,\overline{D}} \qquad \text{(Using DeMorgan's 1st law)}$$

$$=(\overline{A}+\overline{\overline{B}}).(\overline{B}+\overline{\overline{C}}+\overline{\overline{D}}) \qquad \text{(Using DeMorgan's 2nd law)}$$

$$=(\overline{A}+B).(\overline{B}+C+D)$$

EXAMPLE 8 *Find the complement of expression : $(\overline{X}+\overline{Y})(X\overline{Y}Z+\overline{X}\overline{Z})$.*

SOLUTION Complement of given expression

$$=\overline{(\overline{X}+\overline{Y})(X\overline{Y}Z+\overline{X}\overline{Z})}$$

$$=\overline{(\overline{X}+\overline{Y})}\,\overline{(X\overline{Y}Z+\overline{X}\overline{Z})} \qquad \text{(Using DeMorgan's 2nd law)}$$

$$=(\overline{\overline{X}}+\overline{\overline{Y}}).(\overline{X\overline{Y}\,Z}.\overline{\overline{X}}\,\overline{\overline{Z}}) \qquad \text{(Using DeMorgan's 1st law)}$$

$$=(XY)+((\overline{X}+\overline{\overline{Y}}+\overline{Z}).(\overline{\overline{X}}+\overline{\overline{Z}})) \qquad \text{(Using DeMorgan's 2nd law)}$$

$$=\mathbf{XY+(\overline{X}+Y+\overline{Z}).(X+Z)}$$

EXAMPLE 9 *Find the complement of expression : $(A+BC+D)EF$*

SOLUTION The complement of given expression is

$$=\overline{(A+BC+D)EF}$$

$$=\overline{(A+BC+D)}+\overline{EF} \qquad \text{(Using DeMorgan's 2nd law)}$$

$$=(\overline{A}.\overline{BC}.\overline{D})+(\overline{E}+\overline{F}) \qquad \text{(Using DeMorgan's 1st and 2nd laws)}$$

$$=(\overline{A}.(\overline{B}+\overline{C}).\overline{D})+(\overline{E}+\overline{F}) \qquad \text{(Using DeMorgan's 2nd law)}$$

$$=\mathbf{\overline{A}(\overline{B}+\overline{C})\overline{D}+(\overline{E}+\overline{F})}$$

EXAMPLE 10 *Find the complement of expression :* $A\overline{B} + C\overline{D} + EF$.

SOLUTION The complement of given expression is

$$= \overline{A\overline{B} + C\overline{D} + EF}$$

$$= \overline{A\overline{B}}\,.\,\overline{C\overline{D}}\,.\,\overline{EF} \qquad \text{(Using DeMorgan's 1st law)}$$

$$= (\overline{A} + \overline{\overline{B}}).(\overline{C} + \overline{\overline{D}}).(\overline{E} + \overline{F}) \qquad \text{(Using DeMorgan's 2nd law)}$$

$$= (\overline{A} + B)(\overline{C} + D)(\overline{E} + \overline{F})$$

Basic Duality of Boolean Logic

We already have talked about the duality principle. If you observe all the theorems and rules covered so far, you'll find a basic duality which underlies all Boolean algebra. The postulates and theorems which have been presented can all be divided into pairs.

For example, $X + X\,.\,Y = X$

Its dual will be $X\,.(X + Y) = X$ *(Remember change . to + and vice-versa ; complement 0 and 1.)*

Similarly, $(X + Y) + Z = X + (Y + Z)$ is the dual of $(X\,.\,Y)\,.\,Z = X\,.(Y\,.\,Z)$

and $X + 0 = X$ is dual of $X\,.1 = X$

In proving the theorems or rules of Boolean algebra, it is then necessary to prove only one theorem, and the dual of the theorem follows necessarily. In effect, *all Boolean algebra is predicated on this two-for-one basis.*

EXAMPLE 11 *Give the dual of the following result in Boolean algebra :* $X\,.\,X' = 0$ *for each X.*

SOLUTION Using duality principle, dual of $X\,.\,X' = 0$ is $X + X' = 1$ (By changing (.) to (+) and vice-versa and by replacing 1's by 0's and vice-versa).

Check Point 3.4

1. Which of the following Boolean equations is/are incorrect ? Write the correct forms of the incorrect ones :
   (a) $A + A' = 1$ (b) $A + 0 = A$
   (c) $A.1 = A$ (d) $AA' = 1$
   (e) $A + AB = A$ (f) $A(A + B)' = A$
   (g) $(A + B)' = A' + B$ (h) $(AB)' = A'B'$
   (i) $A + 1 = 1$ (j) $A + A = A$
   (k) $A + A'B = A + B$
   (l) $X + YZ = (X + Y)(X + A)$
2. Find the complement of the following functions applying De'Morgan's theorem
   (a) $F(x, y, z) = x'yz' + x'y'z$
   (b) $F(x, y, z) = x(y'z + yz)$
3. What is the logical product of several variables called ? What is the logical sum of several variables called ?
4. What is the procedure "Break the line, change the sign" ?

EXAMPLE 12 *Give the dual of* $X + 0 = X$.

SOLUTION

Using duality principle, dual of $X + 0 = X$ is $X.1 = X$.

EXAMPLE 13 *State the principle of duality in boolean algebra and give the dual of the boolean expression :* $(X + Y).(\overline{X} + \overline{Z}).(Y + Z)$

SOLUTION

Principle of duality states that from every boolean relation, another boolean relation can be derived by

(i) changing each OR sign (+) to an AND (.) sign
(ii) changing each AND (.) sign to an OR (+) sign
(iii) replacing each 1 by 0 and each 0 by 1.

The new derived relation is known as the dual of the original relation.

Dual of $(X + Y).(\overline{X} + \overline{Z}).(Y + Z)$ will be :

$$(X\,.\,Y) + (\overline{X}\,.\,\overline{Z}) + (Y\,.\,Z) = XY + \overline{X}\,\overline{Z} + YZ$$

Do you know that you can find the complement of a Boolean expression using its Dual also ? It is very simple :

1. Write the dual form of given Boolean expression.
2. Swap variables and complemented variables.

For example, to get the complement of expression $(A + BC' + DA')$

1. Write its Dual : $A.(B + C').(D + A')$
2. Replace variables with their complemented form : $A'.(B' + C).(D' + A)$ (the result)

Let us find the complement using the DeMoragan Laws and compare the result.

$$F = (A + BC' + DA')$$
$$F = (A + BC' + DA')' = (A'.(BC')'.(DA')')$$
$$= A'.(B' + C'').(D' + A'') = A'.(B' + C).(D' + A)$$

It is the same as the complement determined using Duality principle.

3.9 MORE ABOUT LOGIC GATES

We have covered three basic logic gates NOT, OR, AND so far. But there are some more logic gates also which are derived from three basic gates (*i.e.,* AND, OR and NOT). These gates are more popular than NOT, OR and AND and are widely used in industry. This section introduces NOR, NAND, XOR, XNOR gates.

NOR GATE

The Nor Gate has two or more input signals but only one output signal. If all the inputs are 0 (*i.e.,* low), then the output signal is 1 (high).

3.9.1 NOR Gate

The **Nor Gate** has two or more input signals but only one output signal. If all the inputs are 0 (*i.e.,* low), then the output signal is 1 (high). If either of the two inputs is 1 (*high*), the output will be 0 (*low*). NOR *gate is nothing but inverted* OR *gate.*

The NOR gate can have as many inputs (2 or more inputs) as desired. No matter how many inputs are there, the action of NOR gate is the same *i.e.,* All 0 (*low*) inputs produce output as 1.

Following truth Tables (3.29 and 3.30) illustrate NOR action.

Table 3.29 *2-input NOR gate*

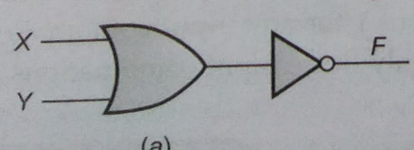
X	Y	F
0	0	1
0	1	0
1	0	0
1	1	0

$F = \overline{X + Y}$

Table 3.30 *3-input NOR gate*

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$F = \overline{X + Y + Z}$



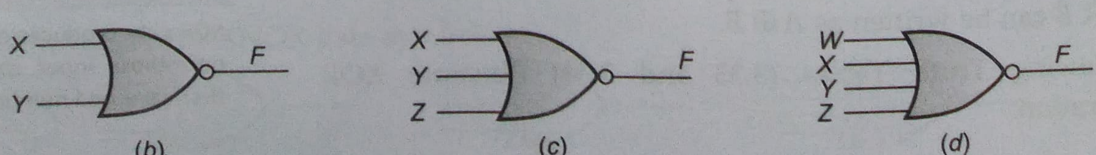


Figure 3.7 (*a*) Logical meaning of NOR gate (*b*) 2 input NOT gate (*c*) 3 input NOR gate (*d*) 4 input NOR gate

3.9.2 NAND Gate

The **NAND Gate** has two or more input signals but only one output signal. If all of the inputs are 1 (high), then the output produced is 0 (low).

NAND *gate is inverted* AND *gate*. Thus, for all 1 (*high*) inputs, it produces 0 (*low*) output, otherwise for any other input combination, it produces a 1 (*high*) output. NAND gate can also have as many inputs as desired.

NAND GATE

The NAND Gate has two or more input signals but only one output signal. If all of the inputs are 1 (high), then the output produced is 0 (low).

NAND action is illustrated in following Truth Tables (3.31 and 3.32).

Table 3.31 *2-input NAND gate*

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	0

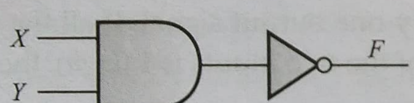
$F = \overline{XY}$

Table 3.32 *3-input NAND gate*

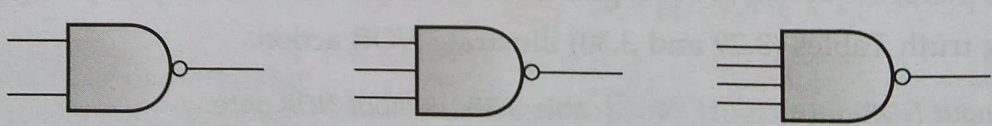
X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$F = \overline{XYZ}$

The logical meaning of NAND gate can be shown as follows :



The symbols of 2, 3, 4 input NAND gates are given below :



(a) (b) (c)

Figure 3.8 (*a*) 2-input NAND gate (*b*) 3-input NAND gate (*c*) 4-input NAND gate.

3.9.3 XOR Gate (Exclusive OR Gate)

The **XOR Gate** can also have two or more inputs but produces one output signal. Exclusive-OR gate is different from OR gate. OR gate produces output 1 for any input combination having one or more 1's, but XOR gate produces output 1 for only those input combinations that have odd number of 1's.

In boolean algebra ⊕ sign stands for XOR operation. Thus *A* XOR *B* can be written as $A \oplus B$.

Following Truth Tables (3.33 and 3.34) illustrate XOR operation.

XOR GATE

XOR gate produces output 1 for only those input combinations that have odd number of 1's.

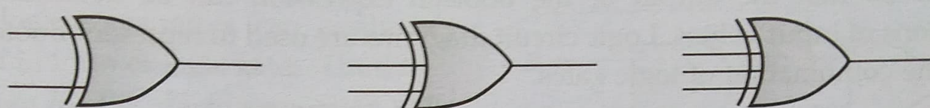
Table 3.33 *2-input XOR gate*

No. of 1's even/odd	X	Y	F
Even	0	0	0
Odd	0	1	1
Odd	1	0	1
Even	1	1	0

Table 3.34 *3-input XOR gate*

No. of 1's	X	Y	Z	F
Even	0	0	0	0
Odd	0	0	1	1
Odd	0	1	0	1
Even	0	1	1	0
Odd	1	0	0	1
Even	1	0	1	0
Even	1	1	0	0
Odd	1	1	1	1

The symbols of XOR gates are given below :



(a) (b) (c)

Figure 3.9 (*a*) 2-input XOR gate (*b*) 3-input XOR gate (*c*) 4-input XOR gate.

XOR addition can be summarised as follows :

$$0 \oplus 0 = 0, \quad 0 \oplus 1 = 1, \quad 1 \oplus 0 = 1, \quad 1 \oplus 1 = 0$$

NOTE

Remember odd number of 1's produce output 1.

3.9.4 XNOR Gate (Exclusive NOR gate)

The **XNOR Gate** is logically equivalent to an inverted XOR *i.e.*, XOR gate followed by a NOT gate (inventor). Thus XNOR produces 1 (high) output when the input combination has even number of 1's. Following tables (3.35 and 3.36) illustrate XNOR action.

XNOR GATE

XNOR gate produces output 1 for only those input combinations that have even number of 1's.

Table 3.35 *2-input XNOR gate*

No. of 1's	X	Y	F
Even	0	0	1
Odd	0	1	0
Odd	1	0	0
Even	1	1	1

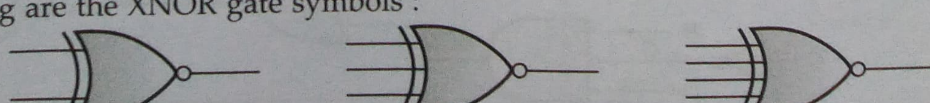
NOTE

The XNOR Gate is logically equivalent to an inverted XOR *i.e.*, XOR gate followed by a NOT gate (inventor).

Table 3.36 *3-input XNOR gate*

No. of 1's	X	Y	Z	F
Even	0	0	0	1
Odd	0	0	1	0
Odd	0	1	0	0
Even	0	1	1	1
Odd	1	0	0	0
Even	1	0	1	1
Even	1	1	0	1
Odd	1	1	1	0

Following are the XNOR gate symbols :



(a) (b) (c)

Figure 3.10 (*a*) 2-input XNOR gate (*b*) 3-input XNOR gate (*c*) 4-input XNOR gate.

The bubble (small circle), on the outputs of NAND, NOR, XNOR gates represents complementation.

Now that we are familiar with logic gates, we can use them in designing logic circuits.

3.10 LOGIC CIRCUITS

A logic circuit is a circuit that carries out a set of logic actions based on an expression. To execute a Boolean expression, you require a logic circuit and the input values for the variables of Boolean expression.

LOGIC CIRCUIT

A logic circuit is a circuit that carries out a set of logic actions based on an expression.

You can represent a Boolean expression in the form of a logic ***circuit using a combination of logic gates*** so that the output of the Boolean expression can be determined for various combinations of input values. Logic circuit diagrams are used to represent Boolean expressions through the combination of logic gates.

The rules to create a logic circuit are :

1. Break the Boolean expression in smaller sub expressions,

   e.g.,

   for a Boolean expression : $AB + \overline{B}CD$, there are two sub-expressions : AB and $\overline{B}CD$.

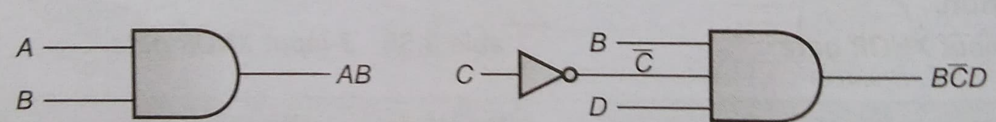
2. For each sub-expression, determine the logic gates that can implement them. (*Refer to Example 3.5 to recall this*), *e.g.,*

   for sub-expressions determined in previous step,

   AB : requires *AND* gate

   $B\overline{C}D$: requires *NOT* and *AND* gates

3. Implement the sub-expressions using the gates determined in previous steps :

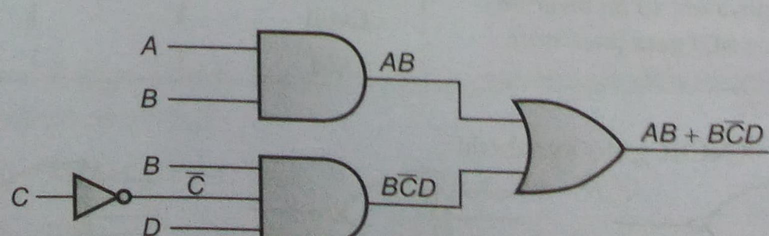


4. Determine the Logic gate for the symbol joining the sub-expressions, *e.g.,*

   for the given expression,

   symbol + joins the sub-expressions (AB and $B\overline{C}D$) ; the symbol + signifies OR gate.

5. Using the logic gate for the joining symbol (from step 4), connect the sub-expressions' gate implementation (from step 3)



6. Yay! Our logic circuit diagram is ready.

Let us consider some examples now.

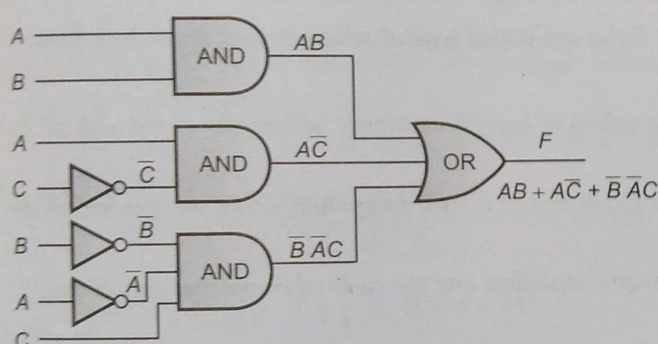
EXAMPLE 14 *Design a circuit to realise the following :* $F(a, b, c) = AB + A\overline{C} + \overline{B}\,\overline{A}C$.

SOLUTION The given boolean expression can also be written as follows :

$$F(a, b, c) = A.B + A.\overline{C} + \overline{B}.\overline{A}.C$$

or $F(a, b, c)$ = (A AND B) OR
(A AND (NOT C)) OR
((NOT B) AND (NOT B) AND C)

Now these logical operators can easily be implemented in form of logic gates. Thus, the circuit diagram for the above expression will be as follows :



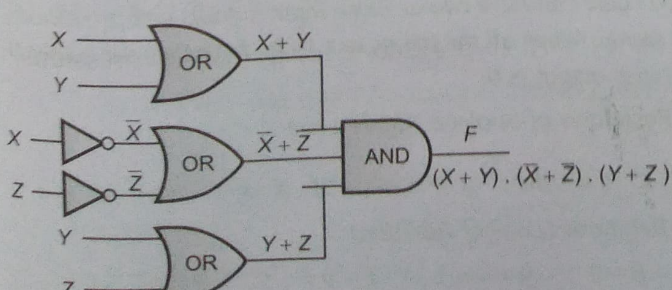
EXAMPLE 15 *Draw the diagram of digital circuit for the function :*

$$F(X, Y, Z) = (X + Y).(\overline{X} + \overline{Z}).(Y + Z).$$

SOLUTION Above expression can also be written as :

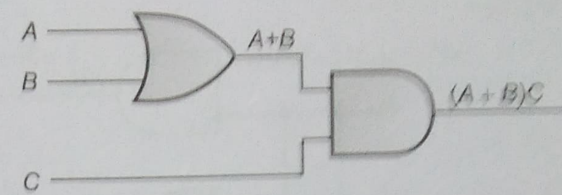
$F(X, Y, Z)$ = (X OR Y) AND ((NOT X) OR
(NOT Z)) AND (Y OR Z)

Thus circuit diagram will be



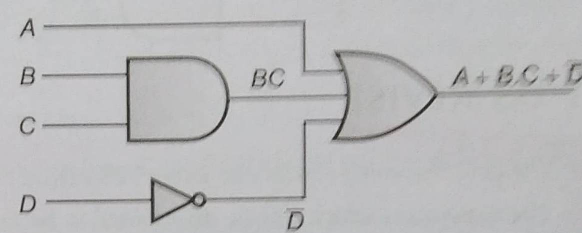
EXAMPLE 16 *Design a logic circuit and draw its diagram for the Boolean expression :* $(A + B)C$.

SOLUTION



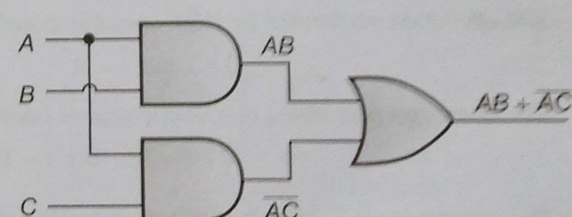
EXAMPLE 17 *Design a logic circuit and draw its diagram for the Boolean expression :* $A + BC + \overline{D}$.

SOLUTION



EXAMPLE 18 *Design a logic circuit and draw its diagram for the Boolean expression :* $AB + \overline{AC}$.

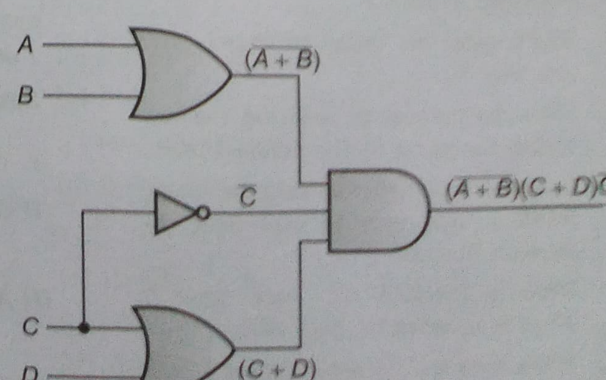
SOLUTION



EXAMPLE 19 *Design a logic circuit and draw its diagram for the Boolean expression :*

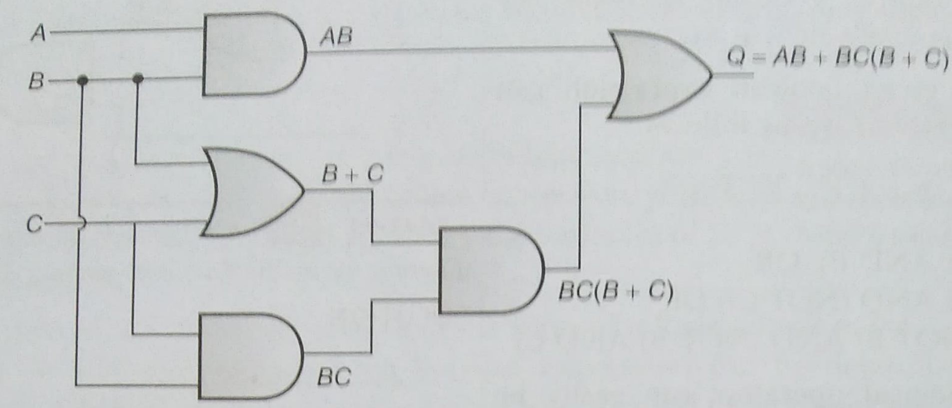
$$\overline{(A + B)}(C + D)\overline{C}.$$

SOLUTION



EXAMPLE 20 *Design a logic circuit and draw its diagram for the Boolean expression : $Q = AB + BC(B + C)$.*

SOLUTION



LET US REVISE

- *The decision which results into either YES (TRUE) or NO (FALSE) is called a binary decision.*
- *The statements which can be determined to be True or False are called* ***logical statements*** *or* ***truth functions.***
- *Truth values are TRUE and FALSE or 1 and 0.*
- *Truth table is a table which represents all the possible values of logical variables/ statements along with all the possible results of the given combinations of values.*
- *If result of any logical statement or expression is always TRUE or 1, it is called* ***tautology*** *and if the result is always FALSE or 0 it is called* ***fallacy.***
- *The operation performed by NOT operator is called* ***complementation*** *and the rules of complementation are :*

  $$\overline{0} = 1 \text{ and } \overline{1} = 0.$$

- *The operation performed by OR operator is called* ***logical addition*** *and the rules are :*

  $$0 + 0 = 0; \quad 0 + 1 = 1; \quad 1 + 0 = 1; \quad 1 + 1 = 1$$

- *The operation performed by AND operator is called logical multiplication and the rules are :*

  $$0.0 = 0; \quad 0.1 = 0; \quad 1.0 = 0; \quad 1.1 = 1.$$

- *The NOT gate or inverter is a gate with only one input signal and one output signal; the output state is always the opposite of the input state.*
- *The OR gate has two or more input signals but only one output signal. If any of the input signals is 1 (high), the output signal is 1 (high).*
- *The AND gate can have two or more input signals and produce an output signal. When all the inputs are 1 i.e., high then the output is 1 otherwise output is 0.*
- *Basic Postulates of Boolean Algebra are :*

  I If $X \neq 0$ then $X \neq 1$ and if $X \neq 1$ then $X = 0$

  II OR Relations *(Logical Addition) :*

  $$0 + 0 = 0 \;; \quad 0 + 1 = 1 \;; \quad 1 + 0 = 1 \;; \quad 1 + 1 = 1$$

  III AND Relations *(Logical Multiplication) :*

  $$0.0 = 0 \;; \quad 0.1 = 0 \;; \quad 1.0 = 0 \;; \quad 1.1 = 1$$

  IV Complementation Rules : $\overline{0} = 1$; $\overline{1} = 0$

Check Point 3.5

1. Why are NAND and NOR gates called Universal gates ?
2. Which gates are called Universal gates and why ?
3. State the purpose of reducing the switching functions to the minimal form.
4. Draw a logic circuit diagram using NAND or NOR only to implement the Boolean function $F(a, b) = a'b' + ab$.
5. What is inverted AND gate called ? What is inverted OR gate called ?
6. When does an XOR gate produce a high output ? When does an XNOR gate produce a high output ?

- The principle of duality sates that starting with a boolean relation, another boolean relation can be derived by changing OR (+) to AND (.) and vice versa and by replacing each 0 by 1 and vice versa.
- Basic theorems of Boolean Algebra are :
  - (i) Properties of 0 and 1 : $0+X=X;\ 1+X=1;\ 0.X=0;\ 1.X=X$
  - (ii) Indempotence Law : $X+X=X;\ X.X=X$
  - (iii) Involution (double-inversion rule) : $(\overline{\overline{X}})=X$
  - (iv) Complementarity Law : $X+\overline{X}=1;\ X.\overline{X}=0$
  - (v) Commutative Law : $X+Y=Y+X;\ X.Y=Y.X$
  - (vi) Associative Law : $X+(Y+Z)=(X+Y)+Z\ ;\ X(YZ)=(XY)Z$
  - (vii) Distributive Law : $X(Y+Z)=XY+XZ\ ;\ X+YZ=(X+Y)(X+Z)$
  - (viii) Absorption Law : $X+XY=X;\ X(X+Y)=X$
  - (ix) $X+\overline{X}Y=X+Y;\quad XY+YZ+\overline{Y}Z=XY+Z$
- DeMorgan's theorems are :

  First theorem : $\overline{X+Y}=\overline{X}\,\overline{Y}$; Second Theorem : $\overline{X.Y}=\overline{X}+\overline{Y}$
- The NOR gate has two or more input signals but only one output signal. If all the inputs are 0 (i.e., low), then the output is 1 (high).
- The NAND gate has two or more input signals but only one output signal. If all of the inputs are 1 (high), then the output produced is 0 (low).
- XOR gate has two or more inputs but one output. It produces output 1 for only those input combinations that have odd number of 1's.
- XNOR gate is inverted XOR gate. It produces 1 (high) output when the input combination has even number of 1's.

Objective Type Questions

OTQs

Multiple Choice Questions

1. According to Boolean laws : $A+1=?$

   (a) 1 (b) A (c) 0 (d) A'

2. According to Boolean laws : $A+0=?$

   (a) 1 (b) A (c) 0 (d) A'

3. According to Boolean laws : $A\,.\,1=?$

   (a) 1 (b) A (c) 0 (d) A'

4. According to Boolean laws : $A\,.\,0=?$

   (a) 1 (b) A c) 0 (d) A'

5. The expression for the Absorption law is given by ______ .

   (a) $A+AB=A$ (b) $A+AB=B$
   (c) $AB+AA'=A$ (d) $A+B=B+A$

6. The involution of A is equal to ______ .

   (a) A (b) A' (c) 1 (d) 0

7. If an input A is given to an inverter gate, the output will be :

   (a) $1/A$ (b) 1
   (c) A (d) $\overline{A}$

8. The output of a two-input OR gate is high when

   (a) both inputs are low
   (b) both inputs are high
   (c) any one input is high
   (d) only one input is high

9. The output of a two-input AND gate is high when

   (a) both inputs are low
   (b) both inputs are high
   (c) any one input is high
   (d) only one input is high

10. According to the associative law :

    (a) $A+B=B+A$ (b) $A=A+A$
    (c) $(A+B)+C=A+(B+C)$
    (d) $A+0=A$

11. According to the commutative law
    (*a*) $AB = BA$ (*b*) $A = AA$
    (*c*) $(AB)C = A(BC)$ (*d*) $A.0 = A$
12. According to the distributive law $A(B+C) = ?$
    (*a*) ABC (*b*) $AB + AC$
    (*c*) $A + B + C$ (*d*) $A + BC$
13. According to the DeMorgan laws, the complement of a product of variables is equal to
    (*a*) the complement of the sum
    (*b*) the sum of the complements
    (*c*) the product of the complement
    (*d*) none of these
14. Which of the following is/are the universal logic gates ?
    (*a*) OR and NOR (*b*) AND
    (*c*) NAND and NOR (*d*) NOT
15. The logic gate that provides high output for same inputs for all variables in a 2-variable truth table is ______ gate.
    (*a*) NOT (*b*) XNOR (*c*) AND (*d*) XOR
16. Which gate produces 0 when all inputs are high ?
    (*a*) NOT (*b*) NAND (*c*) AND (*d*) NOR
17. Which gate produces 1 when all inputs are low ?
    (*a*) NOT (*b*) NAND (*c*) AND (*d*) NOR
18. Gate NAND can be simplified as :
    (*a*) AND followed by OR
    (*b*) AND followed by NOT
    (*c*) NOT followed by AND
    (*d*) OR followed by AND
19. Gate XNOR can be simplified as :
    (*a*) XOR followed by OR
    (*b*) XOR followed by NOT
    (*c*) NOT followed by XOR
    (*d*) XOR followed by AND
20. NAND cannot implement ______ gate.
    (*a*) AND (*b*) OR
    (*c*) NOT (*d*) none of these

Fill in the Blanks

1. The statements that can be determined as TRUE/FALSE are called ______ statements.
2. Truth values are ______ and ______ .
3. A logical expression, which is always TRUE for all inputs, is termed as ______ .
4. A logical expression, which is always FALSE for all inputs, is termed as ______ .
5. The operation performed by a NOT gate is called ______ .
6. Dual of a Boolean expression is obtained by swapping ______ with ______ and ______ with ______ .
7. The NOT gate takes only ______ input.
8. The NOR gate produces 1 when all inputs are ______ .
9. The XOR gate produces ______ when odd number of 1's are there in the input combination.
10. The XOR gate produces ______ when even number of 1's are there in the input combination.
11. The XNOR gate produces ______ when odd number of 1's are there in the input combination.
12. The XNOR gate produces ______ when even number of 1's are there in the input combination.

True/False Questions

1. Anything ANDed with a 0 is equal to 0.
2. Anything ANDed with a 1 is equal to 1.
3. Anything ANDed with a 1 is equal to itself.
4. Anything ORed with a 0 is equal to 0.
5. Anything ORed with a 0 is equal to itself.
6. Anything ORed with a 1 is equal to itself.
7. Anything ORed with a 1 is equal to 1.
8. Anything ANDed with itself is equal to 1.
9. Anything ANDed with itself is equal to itself, *e.g.*, $A.A = A$.
10. Anything ORed with itself is equal to 0.
11. Anything ORed with itself is equal to itself, *e.g.*, $A + A = A$.
12. Anything ANDed with its own complement equals 1.
13. Anything ANDed with its own complement equals 0.
14. Anything ORed with its own complement equals 0.
15. Anything ORed with its own complement equals 1.
16. Anything complemented twice is equal to its complement.

17. Anything complemented twice is equal to the original.
18. The dual of a Boolean expression is also a Boolean expression.
19. If a Boolean expression is True, its dual will be False.
20. The dual of the dual is the complement of the Boolean expression.
21. If a Boolean expression is True, its complement is also True.
22. If a Boolean expression is True, its dual is also True.
23. NAND and NOR are called universal gates.
24. XOR and XNOR are called universal gates.
25. Inverted XOR is called the OR gate.

NOTE : Answers for OTQs are given at the end of the book.

Solved Problems

1. *State which of the following are logical statements :*

   (i) *NOT gate is also called inverter.*
   (ii) *Try to run fast.*
   (iii) *Do not tell a lie.*
   (iv) *Australia is the world cricket champion, 2003.*
   (v) *Why did you come late ?*

   Solution. Statements (*i*) and (*iv*) are logical statements since these statements result into either *true* or *false* values.

2. *Prove that* $X.(X+Y) = X$ *by truth table method.*

   Solution.

X	Y	X + Y	X . (X + Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

From the above table, it is obvious that $X.(X+Y) = X$ because both these columns are identical.

3. *Give duals for the following :*

   (*i*) $X + \overline{X}Y$ (*ii*) $XY + X\overline{Y} + \overline{X}Y$ (*iii*) $(A+0).(A.1.\overline{A})$
   (*iv*) $AB + \overline{A}B$ (*v*) $ABC + A\overline{B}C + \overline{A}B\overline{C}$

   Solution. Using duality principle, changing (+) to (.) and vice-versa and by replacing 0's with 1's and 1's with 0's, the duals for the given expressions are as follows :

   (*i*) $X.(\overline{X}+Y)$ (*ii*) $(X+Y).(X+\overline{Y}).(\overline{X}+Y)$
   (*iii*) $(A.1)+(A+0+\overline{A})$ (*iv*) $(A+B).(\overline{A}+B)$
   (*v*) $(A+B+C).(A+\overline{B}+C).(\overline{A}+B+\overline{C})$

4. *State the distributive laws of boolean algebra. How do they differ from the distributive laws of ordinary algebra ?*

   Solution. Distributive laws of boolean algebra state that

   (*i*) $X(Y+Z) = XY + XZ$ (*ii*) $X + YZ = (X+Y)(X+Z)$

   I[st] law $X(Y+Z) = XY + XZ$ holds good for all values of X, Y and Z in ordinary algebra whereas $X + YZ = (X+Y)(X+Z)$ holds good only for two values (0, 1) of X, Y and Z.

5. *Name the law shown below and verify it using a truth table :* $A + BC = (A + B)(A + C)$ (Delhi 2014)

Solution. Distributive Law

A	B	C	A + B	A + C	BC	A + BC	(A + B) . (A + C)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

From columns $(A+B)(A+C)$ and $A+BC$, we observe that

$A+BC=(A+B)(A+C)$. Hence verified.

6. *State and verify Involution law.*

Solution. **Involution law** states : $\overline{\overline{X}} = X$

Truth Table for $\overline{\overline{X}} = X$

X	$\overline{X}$	$\overline{\overline{X}}$
0	1	0
1	0	1

7. *Verify the following using truth table :* (*i*) $X.X' = 0$ (*ii*) $X + 1 = 1$

Solution.

X	X′	X + 1	X·X′
0	1	1	0
1	0	1	0

8. *State and verify Absorption law in Boolean Algebra.*

Solution. (*a*) **Absorption Law** states (*i*) $X + XY = X$ (*ii*) $X(X+Y) = X$

(*i*) Truth table for $X + XY = X$ is given below :

Truth Table for $X + XY = X$.

Input			Output
X	Y	XY	X + XY
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Both the columns $X + XY$ and X are identical.
Hence proved.

(*ii*) Truth table for $X.(X+Y) = X$ is given below :

Truth Table for $X.(X+Y) = X$.

Input			Output
X	Y	X + Y	X(X + Y)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Column X and $X(X+Y)$ are identical.
Hence proved.

9. *State DeMorgan's Law of Boolean Algebra and verify them using truth table.* (Outside Delhi 2017)

Solution. DeMorgan's First theorem states that : $\overline{X+Y} = \overline{X}\,.\,\overline{Y}$

DeMorgan's Second theorem states that : $\overline{X\,.\,Y} = \overline{X} + \overline{Y}$

Truth Table for DeMorgan's second theorem.

X	Y	X . Y	$\overline{X\,.\,Y}$	$\overline{X}$	$\overline{Y}$	$\overline{X} + \overline{Y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Comparing the columns $\overline{X\,.\,Y}$ and $\overline{X} + \overline{Y}$, we obtain that $\overline{X\,.\,Y}$ and $\overline{X} + \overline{Y}$ are identical.

Hence proved that $\overline{X\,.\,Y} = \overline{X} + \overline{Y}$.

10. *How many input combinations can be there in the truth table of a logic system having (N) input binary variables ?*

Solution. 2^N

11. *Perform the following : (a) State and prove the absorption law algebraically. (b) State DeMorgan's Laws. Verify one of the DeMorgan's Laws using a truth table.*

Solution. (*a*) Absorption law states that (*i*) $X + XY = X$ and (*ii*) $X(X + Y) = X$

Proof. (*i*) $X + XY = X$

$$LHS = X + XY = X(1 + Y)$$
$$= X\,.1 \qquad [\because 1 + Y = 1]$$
$$= X = RHS.$$

Hence proved.

(*ii*) $X(X + Y) = X$

$$LHS\ X(X + Y) = X\,.\,X + XY$$
$$= X + XY \qquad [\because X\,.\,X = X]$$
$$= X(1 + Y)$$
$$= X\,.1 \qquad [\because 1 + Y = 1]$$
$$= X = RHS.$$

Hence proved.

Since both parts (*i*) and (*ii*) are proved, absorption law is proved.

(*b*) Refer to solved problem 9.

12. *How do you obtain a dual of a Boolean expression ?*

Solution. By swapping AND(.) with OR(+) and vice versa and 1's with 0's and vice versa, we get the dual of a Boolean expression.

13. *What is the dual of :*

(*i*) $A+(BC)+(0(D+1))$ (*ii*) $F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$

(*iii*) $(A+C)(\overline{A}+B) = AB + A\overline{C}$ (*iv*) $\overline{AB} + \overline{A} + AB = 0$

Solution. The dual is :

(*i*) $A\,.\,(B+C)\,.\,(1+(D\,.\,0))$ (*ii*) $A+B+(C(D+L'+G+(B'AE)))+(H(J'+A'+B))$.

(*iii*) $AC + \overline{A}B = (A+B)\,.\,(A+\overline{C})$ (*iv*) $((\overline{A+B}))\,.\,\overline{A}\,.\,(A+B) = 1$

14. *Find the complement of Boolean expression :* $\overline{A} + (\overline{B} + C)$.

Solution. $\overline{\overline{A} + (\overline{B} + C)} = \overline{\overline{A}}\,.\,(\overline{\overline{B}\,.\,C})$ (*DeMorgan's theorem*)

$= A\,.\,(\overline{\overline{B}} + \overline{C})$ (*DeMorgan's theorem*)

$= A\,.\,(B\,.\,\overline{C})$

15. *Determine the complement of Boolean Expression :*

$$(a+ac+b'c+ab')(a+b+c').$$

Solution.

$$=((a+ac+b'c+ab')+(a+b+c'))'$$
$$=(a+ac+b'c+ab')'(a+b+c')'$$
$$=a'.(ac)'.(b'c)'.(ab')'+(a'.b'.c'')$$
$$=a'(a'+c')(b+c')(a'+b)+a'b'c$$
$$=\mathbf{a'(a'+c')(b+c')(a'+b)+a'b'c}$$

16. *Find the complement of : $F=ac'+abd'+acd$.*

Solution. Given

$$F=ac'+abd'+acd$$
$$F'=(ac'+abd'+acd)'=(ac')'.(abd')'.(acd)'$$

(DeMorgan's 1st Law)

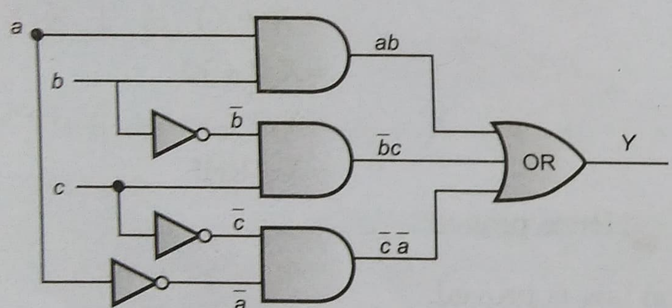
$$=(a'+c'').(a'+b'+d'').(a'+c'+d')$$

(DeMorgan's 2nd Law)

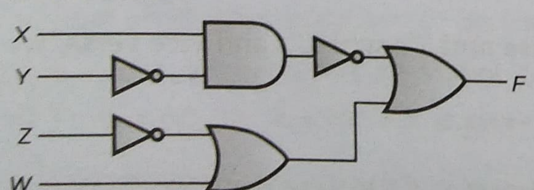
$$=\mathbf{(a'+c).(a'+b'+d).(a'+c'+d')}$$

17. *Draw logic circuit diagram for the following expression :* $Y=ab+\bar{b}c+\bar{c}\,\bar{a}$.

Solution.

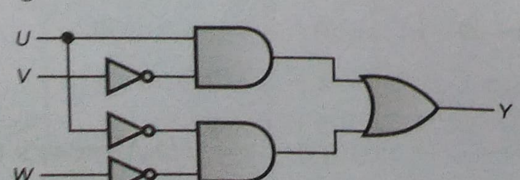


18. *Obtain the Boolean Expression for the logic circuit shown below :*



Solution. $F=\overline{X\overline{Y}}+(\overline{Z}+W)$

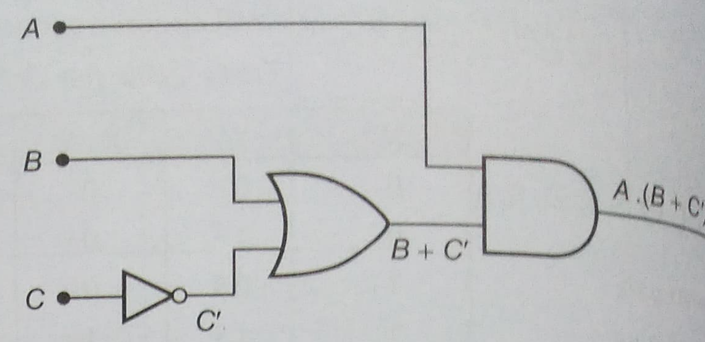
19. *Write the equivalent Boolean expression for the Logic Circuit :*



Solution. $Y=UV'+U'W'$

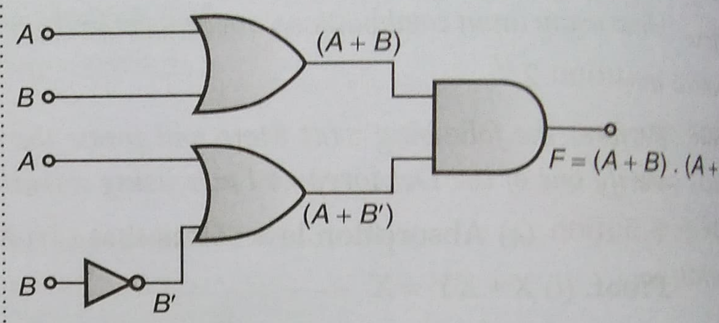
20. *Draw a Logical Circuit Diagram for the following Boolean Expression :* $A.(B+C')$

Solution.



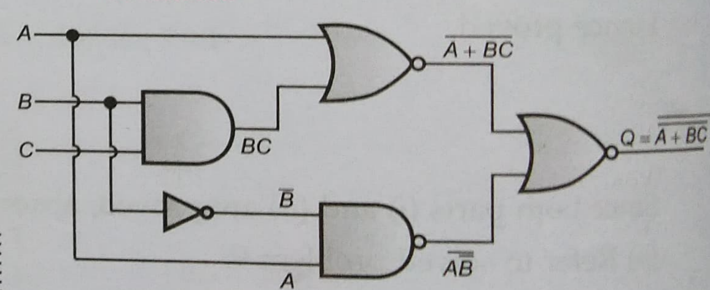
21. *Draw a logic circuit diagram for the Boolean expression :* $F=(A+B).(A+B')$.

Solution.



22. *Draw a logic circuit diagram for the Boolean expression* $Q=\overline{\overline{A+BC}+\overline{\overline{A}\,\overline{B}}}$:

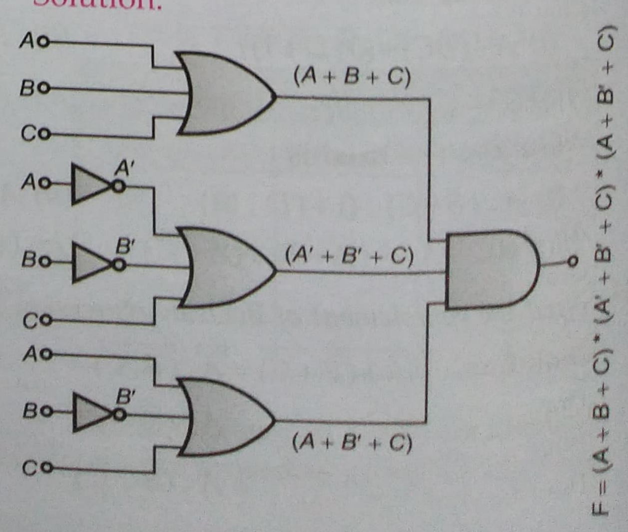
Solution.



23. *Draw a logic circuit diagram for the Boolean expression :*

$$F=(A+B+C).(A'+B'+C).(A+B'+C)$$

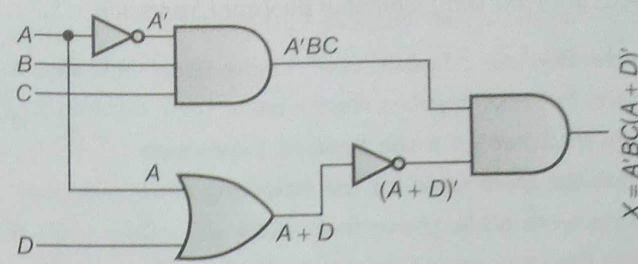
Solution.



24. *Draw a logic circuit diagram for the Boolean expression :*

$X = A'BC(A+D)'$.

Solution.



GLOSSARY

AND gate	A logic circuit whose output is 1 (*high*) only when all inputs are 1 (*high*).
Boolean algebra	A modern algebra which uses the set of numbers 0 and 1, and consists of three basic operations OR, AND and NOT.
Complement	A word in which all 1's have been changed to 0's and all 0's to 1's. The output of an inverter.
DeMorgan's theorems	The Boolean formulae $\overline{A+B} = \overline{A}.\overline{B}$ and $\overline{A.B} = \overline{A} + \overline{B}$
Gate	A logic circuit with one or more input signals but only one output signal.
NAND gate	Logically means an AND gate followed by an inverter. All inputs must be 1 (*high*) to get a 0 (*low*) output.
NOR gate	Logically means an OR gate followed by an inverter. All inputs must be 0 (*low*) to get a 1 (*high*) output.
NOT gate	A gate with one input and one output signal, output being complement of the input.
OR gate	A logic circuit whose output is 1 (*high*) only when one or more inputs are 1 (*high*).
XNOR gate	Logically means an XOR gate followed by an inverter. Inputs must have even number of 1's to get a 1 (high) output.
XOR gate	A gate which produceses 1 (*high*) output only when inputs have even number of 1's.

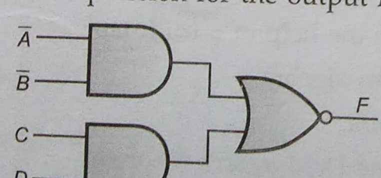
Assignments

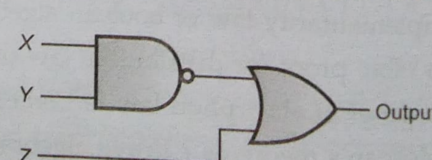
Type A : Short Answer Questions/Conceptual Questions

1. What do you understand by 'truth value' and 'truth function' ? How are these related ?
2. What do you understand by 'logical function' ? What is its alternative name ? Give examples for logical functions.
3. What is meant by tautology and fallacy ? Prove that $1+Y$ is a tautology and $0.Y$ is a fallacy.
4. What is a truth table ? What is its significance ?
5. What are the basic postulates of boolean algebra ?
6. What does duality principle state ? What is its usage in boolean algebra ?
7. State the distributive laws of boolean algebra. How do they differ from the distributive laws of ordinary algebra ?
8. Prove the complementarity law of boolean algebra with the help of a truth table.
9. Give the truth table proof for distributive law of boolean algebra.
10. Give algebraic proof of absorption law of boolean algebra.
11. What are DeMorgan's theorems ? Prove algebraically the DeMorgan's theorem.
12. Which Boolean Law/theorem do you use to determine complement of a Boolean expression ?
13. Are dual and complement of a Boolean expression the same ?
14. Are dual and complement of a Boolean expression related ?
15. Draw a truth table of 2 input NAND and 3 input NAND.
16. Draw a truth table of 2 variable and 3 variable NOR.
17. Draw a truth table of 2 variable and 3 variable XOR.
18. Draw a truth table of 2 variable and 3 variable XNOR.
19. What are logic circuits ?
20. How do you draw logic circuit diagrams ?

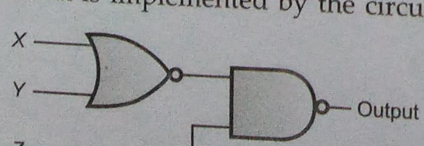
Type B : Application Based Questions

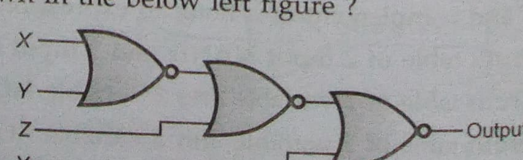
1. In the Boolean Algebra, verify using truth table that $X + XY = X$ for each X, Y in {0, 1}.
2. In the Boolean Algebra, verify using truth table that $(X + Y)' = X'Y'$ for each X, Y in {0, 1}.
3. Give truth table for the Boolean Expression $(X + Y')'$.
4. Draw the truth table for the following equations : (a) $M = N(P + R)$ (b) $M = N + P + N\overline{P}$
5. Using truth table, prove that : $AB + BC + C\overline{A} = AB + C\overline{A}$.
6. State the principle of duality in boolean algebra and give the dual of the boolean expression :
$$(X + Y).(\overline{X} + \overline{Z}).(Y + Z)$$
7. Prove the idempotence law of boolean algebra with the help of a truth table.
8. Use the duality theorem to derive another boolean relation from : $A + \overline{A}B = A + B$
9. What would be the complement of the following : (a) $\overline{A}(B\overline{C} + \overline{B}C)$ (b) $xy + \overline{y}z + \overline{z}z$?
10. Find the complement of the following Boolean function : $F_1 = AB' + C'D'$
11. Find the complement of Boolean expression $(A + \overline{B} + C)(A + \overline{B}C)$.
12. Find the complement of Boolean expression $\overline{A}D + \overline{C}D + \overline{A}B$.
13. Find the complement of Boolean expression $B + \overline{A}C + \overline{B}A$.
14. Find the complement of Boolean expression : $X\overline{Y}Z + \overline{X}\,\overline{Y}Z$.
15. Find the dual of Boolean expression $(A + \overline{B} + C)(A + \overline{B}C)$.
16. Find the dual of Boolean expression $\overline{A}D + \overline{C}D + \overline{A}B$.
17. Find the dual of Boolean expression $B + \overline{A}C + \overline{B}A$.
18. Find the dual of Boolean expression : $X\overline{Y}Z + \overline{X}\,\overline{Y}Z$.
19. Design a logic circuit to realize the Boolean function $f(x, y) = x . y + x' . y'$
20. Draw the logic circuit for this boolean equation : $y = \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}D + AB\overline{C}D + ABC\overline{D}$
21. Draw the AND-OR circuit for : $y = A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + ABCD$
22. Given the Boolean function $\overline{A}D + \overline{C}D + A\overline{B}$
   (i) Obtain the truth table of the function. (ii) Draw the logic circuit diagram.
23. Given the Boolean function $F = \overline{w}xy + w\overline{x}y + wxy$
   (i) Obtain the truth table of the function. (ii) Draw the logic circuit diagram.
24. Given the Boolean function $(ZX + \overline{Y})(XY + \overline{Z})$
   (i) Obtain the truth table of the function. (ii) Draw the logic circuit diagram.
25. Given the Boolean function $(A + \overline{B} + \overline{C})(\overline{A}\,\overline{B} + B\overline{C})$
   (i) Obtain the truth table of the function. (ii) Draw the logic circuit diagram.
26. Derive a boolean expression for the output F at the network shown in the below left figure.





27. What function is implemented by the circuit shown in the above right figure ?
28. What function is implemented by the circuit shown in the below left figure ?





29. What function is implemented by the circuit shown in the above right figure
30. Draw the logic circuit diagram for expressions : (a) $(A' + BC)(B' + C'A)$ (b) $AB' + B'C' + ABC$