

- o Arrays
  - o Matrices
  - o Linked list
  - o Stack
  - o Queues
  - o Tree
    - o LV.1
    - o LV.2
    - ≤ o W.3
  - o Graph
  - o Hashing
  - o Recursion
  - o Sorting
- "Arrangement of data" and how to utilize

Pointer used for :

1. Accessing Heap
2. Accessing resources
3. Parameter passing

Heap Storage

`p = new a[10]`

Reference

`int main()`

```
{   a=10;           r=11
    int &r=a; }     a=11   [a=r]
```

## Pointer to a Structure

```
Struct Rectangle{ int length, breadth; }  
Int main(){ Struct Rectangle *p;  
Struct Rectangle r;  
*p = r;  
p->length = 50; }
```

Heap Storage allocation      dynamic object

```
Int main(){ Struct rectangle *p;  
p = malloc(sizeof(Struct Rectangle));  
(Struct rectangle*)  
p->length = 200; }
```

Function

## » Parameter passing

1. Pass by value
  2. Call by address
  3. Call by reference. [ $a = \&x$ ]
- o Structure as parameter
  - o Array as parameter

## # Template Class

### Generic Class

Template <class T>

Class Arithmetic

{ Private:

    T a;

    T b;

Public:

    Arithmetic(T a, T b)

    int add();

    int Sub();

};

Template <class T>

Arithmetic::Arithmetic(T a, T b)  
    <T>

Template <class T>

    T Arithmetic<T> :: add()

    {

        T c;

        c = a + b;

        return c; }

Int main()

{

    Arithmetic <int> A(10, 5);

    cout << a.add();

Template <class T>  
Class Arithmetic  
{ private:  
    T a;  
    T b;

Public:  
    Arithmetic(T a, T b)  
    T add()  
    T sub() };

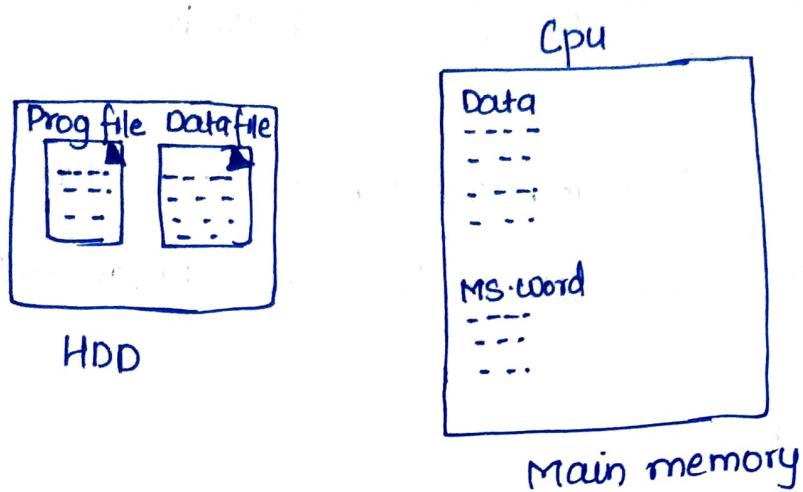
template <class T>  
Arithmetic<T>; Arithmetic(T a, T b)

{  
    This  $\rightarrow$  a = a;  
    This  $\rightarrow$  b = b; }

② T: Arithmetic <T> !! add()  
{  
    T c;  
    return c };

## Data Structure :

"Arrangement of data"  
To utilised it efficiently.



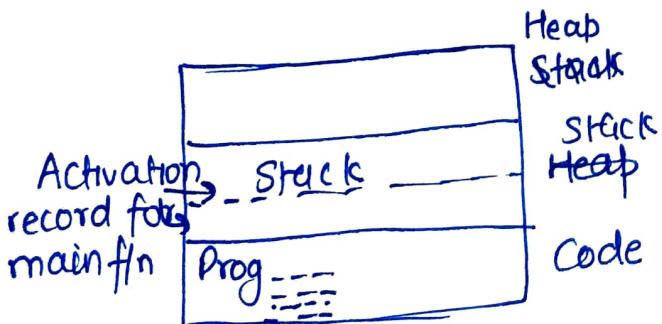
"Commerical data is stored in the arrangement of table"

"Data is stored in form of relational model is known as Data base".

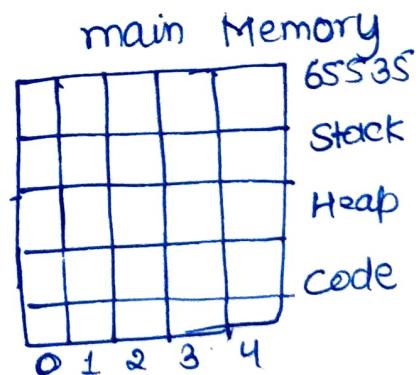
'Data warehouse': Storing data in table disk

algorithm used are Data-mining.

## # Static vs Dynamic Memory Allocation



Segment 64kb



C++      `P = new int[5];`

delocate the memory

`delete[] P;`

## Types of Data Structure.

1. Array  
Linklist } Physical datastructure .

Array : when you know how much data is used.

Linklist : Size can be increased dynamically.

Memory [ Stack  
Queues ]

Memory [ Tree  
Graph ] } Logical data structure .

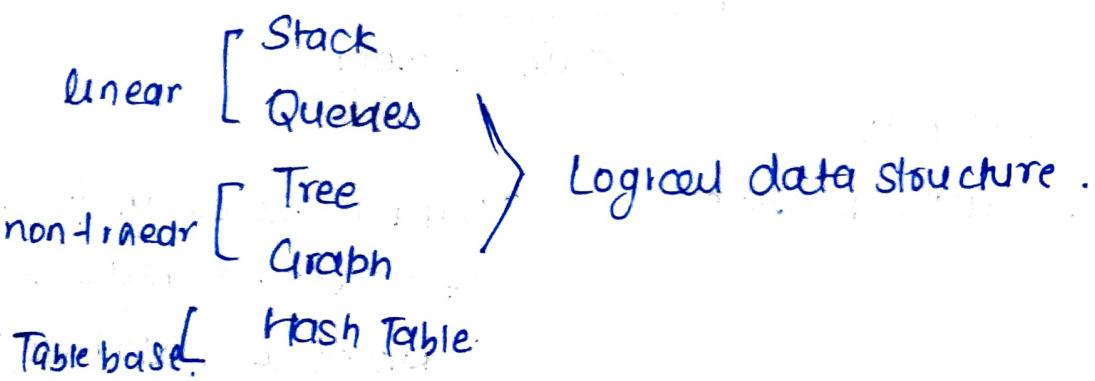
Processor, Bus, Memory

# Types of Data Structure.

1. Array > Physical datastructure.  
Linklist

Array: when you know how much data is used.

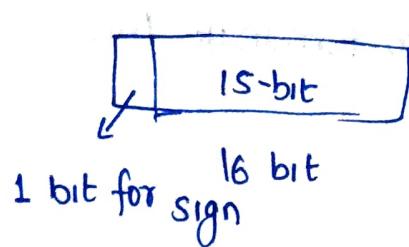
Linklist: Size can be increased dynamically.



## Abstract Data Type

1. Representation of data

2. Operation on Data



data & operation together

known as Abstract Data Type.

list:   
A diagram showing a horizontal sequence of five small squares connected by vertical lines, representing a linked list structure.

- Data:
  - Space for storing elem
  - Capacity
  - Size.

Operations:

- add
- remove
- search

## Time and Space Complexity

$\log_2 n$ : dividing all the element by 2.

for loop used for increment by 1.

void Add(int n)

{ int i, j;

for(i=0; i<n; i++)

    for(j=0; j<n; j++)

$[n+1]$

$n*(n+1)$

$$c[i][j] = A[i][j] + B[i][j] \quad (n)^*(n)$$

$$f(n) = 2n^2 + 2n + 1$$

$O(n^2)$

## Recursion

"function calling itself known as Recursion"

- Some condition be there to terminate the statement.

void fun1(int n)

{ if(n>0)

{ printf("%d",n)

fun1(n-1) }

}

fun1(3)

3

fun1(2)

2

fun1(1)

1

fun1(0)

O/P : 3 2 1

Void fun2(int n)

{ if(n>0)

{ fun2(n-1);

printf("%d",n);

O/P

fun2(3)

fun2(2)

fun2(1)

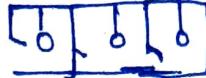
fun2(0)

1

X

O/P : 1 2 3

1. Switch on Bulb
2. goto next room
1. goto next room
2. Switch on Bulb.



`void fun(int n)`

```
{
    1. Calling
    2. fun(n-1)*2
    3. Returning;
}
```

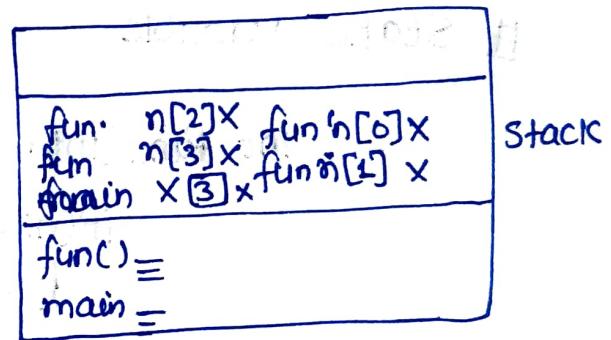
## # How Recursion is used

`Void main()`

```
{
    Int x=3;
    fun(x)
}
```

`Void fun1(int n)`

```
1 - if(n>0)
    cout<<n;
    call = n+1; O(n)
    fun1(n-1);
```



"Recursive fn is memory consuming fn"

`Void fun(int n) — T(n)`

```
{ if(n>0) — + 1
    { printf(n); — 1
        fun(n+1); — T(n+1)}
```

$$T(n) = T(n-1) + 2$$

$$T(n) \begin{cases} 1 & n=0 \\ T(n-1)+2 & n>0 \end{cases}$$

$$T(n) = T(n-1) + 1 \quad \text{--- (1)}$$

Assume  $n-k=0$

$$\therefore T(n) = T(n-1) + 1$$

$$\therefore n=k$$

$$T(n-1) = T(n-2) + 1$$

$$T(n) = T(n-k) + k$$

$$T(n) = n+1$$

$$T(n) = T(n-2) + 2$$

$$O(n)$$

$$T(n) = T(n-3) + 1 + 1$$

$$T(n) = T(n-k) + k$$

## # Static Variable

```
int func(int n)
{
    if(n>0)
        { return func(n-1)+n; }
    return 0;
}
```

```
main()
{
    int a=5
    printf("%d", func(5))
}
```

$$\text{func}(5) =$$

$$\text{func}(4) \swarrow \searrow 15$$

$$\text{func}(3) \swarrow \searrow 10$$

$$\text{func}(2) \swarrow \searrow 6$$

$$\text{func}(1) \swarrow \searrow 3$$

$$\text{func}(0) \swarrow \searrow 1$$

```

int func(int n)
{
    static int x=0;
    if(n>0)
    {
        x++;
        return func(n-1)+x;
    }
    return 0;
}

```

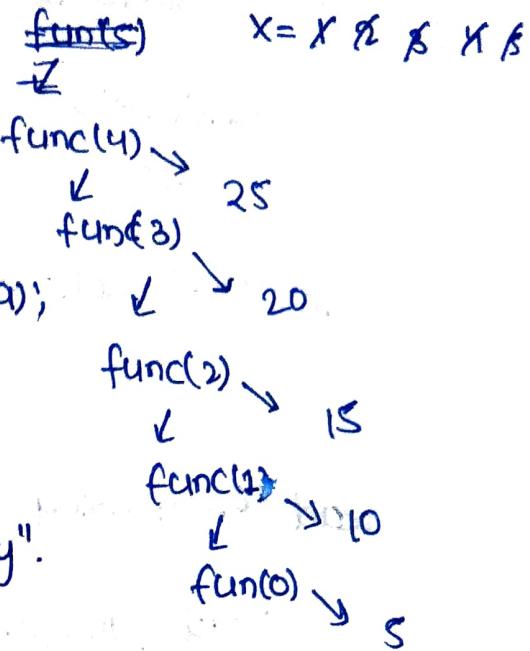
```

int main()
{
    int a=5;
    cout<<func(a);
}

```

"Global variable also have single copy".

Since static variable don't copy  
so, value is get changed every time.



## Types of Recursion

1. Tail Recursion

2. Head Recursion

3. Tree Recursion

4. Indirect recursion

5. Nested Recursion

## Tail Recursion

"If func is called recursively in the last statement then it is called Tail Recursion".

func(n-1);      func(n-1)+n;  
    ✓                  X

In tail recursion

loop is better than Tail recursion.

## Head Recursion

Void func(int n)

{    If(n>0)  
      func(n-1);

====  
}

"no processing during calling time".

## Tree Recursion

"func calling itself more than one time is known as Tree function recursion".

func(n)

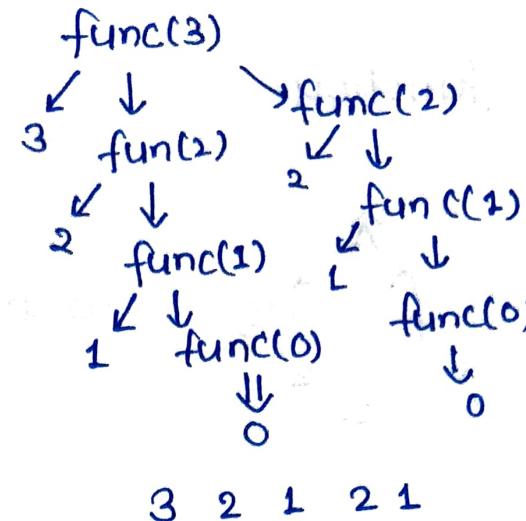
{    IF(n>0)  
      {    =  
          func(n-1)  
      =  
          func(n-1);

```

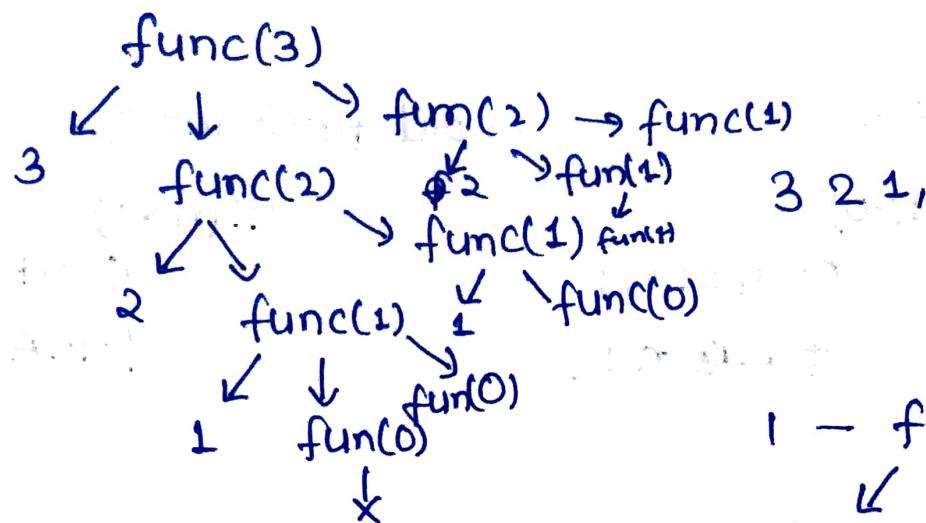
Void fun(int n)
{
    If(n > 0)
    {
        Printf("%d", n)
        fun(n-1);
        fun(n-1);
    }
}

```

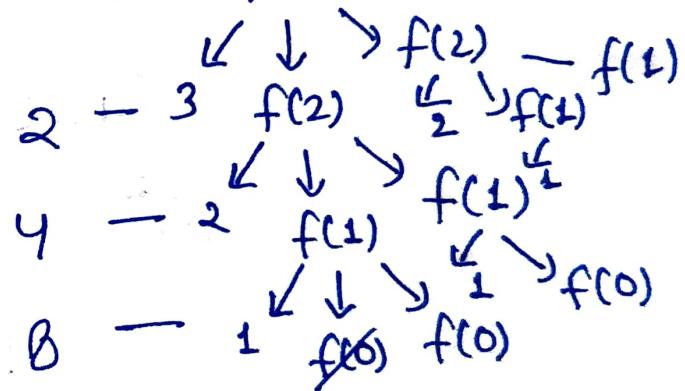
$n = 3$



3 2 1 2 1



1 - f(3)



Time :  $O(2^n)$

Space :

## Indirect function



```
void A(int n)
```

```
{ if(c-->)
```

```
{ ==
```

```
B(n-1)
```

```
}
```

```
void funA(int n)
```

```
if(n>0)
```

```
cout<<n;
```

```
funB(n-1);
```

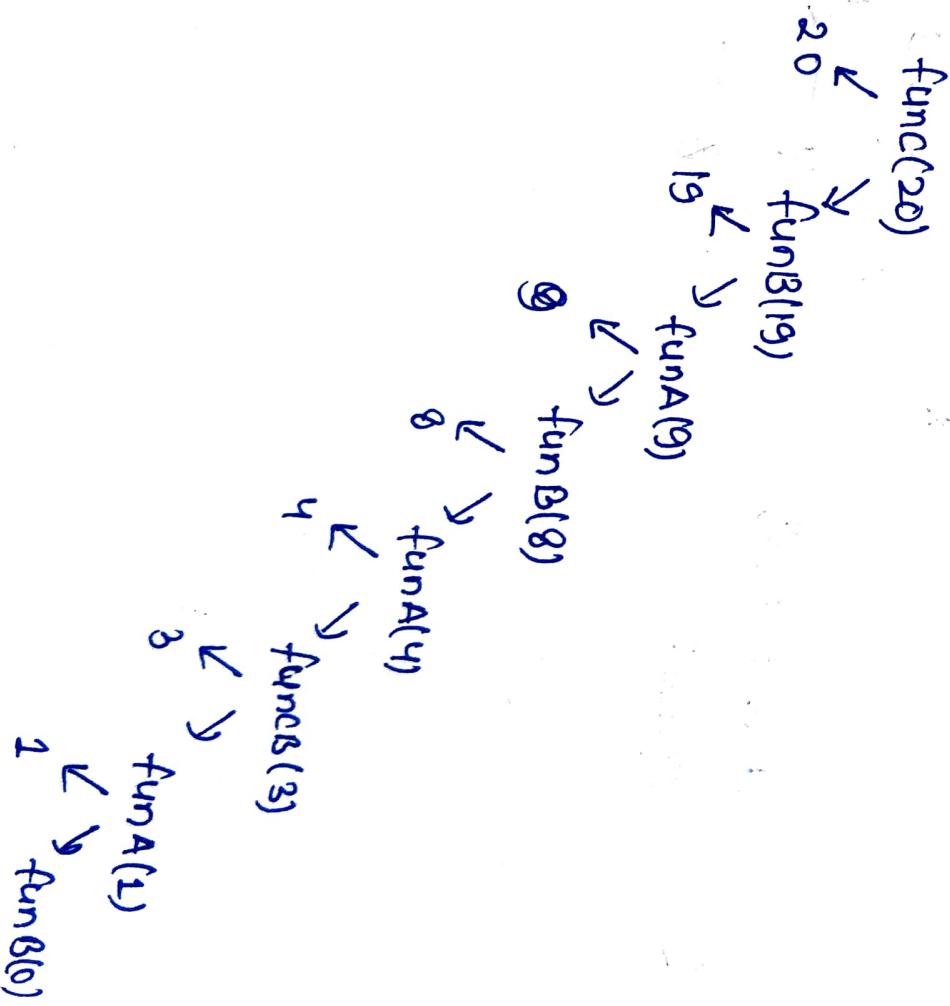
```
funA(n/2)
```

```
void funB(int n)
```

```
if(n>1)
```

```
cout<<n
```

```
funA(n/2)
```



## Nested Recursion

```
int fun(int n)
{
    if(n>100)
        return n-100;
    else
        return fun(fun(n+1));
}

# Sum of n natural numbers

recursion(int n)
{
    if(n>0)
        recursion(n-1);
    sum+=n;
}

int Sum(int n)
{
    if(n==0)
        return 0;
    else
        return sum(n-1)+n;
}

int power(int m, int n)
{
    if(m==0)
        return 0;
    else if(m>0)
        return 1;
    else
        return power(m,n-1)*m;
```

fun(95)  
↓  
fun(fun(106))  
fun(96)  
↓  
fun(97)  
fun(fun(97))  
fun(98)  
fun(fun(108))  
fun(99)  
fun(fun(109))  
fun(100)

```
int pow(int m, int n)
```

```
{  
    if(n==0)  
        return 0;
```

```
    if(n%2==0)  
        return pow(m*m, n/2);  
    else  
        return pow(m*m, n-1/2);
```

## # Taylor Series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} - \dots \quad (3)$$

$$\frac{(n)}{\text{fac}(n)}$$

$$\frac{\text{pow}(4, n)}{\text{fac}(4)}$$

$$\frac{\text{pow}(3)}{\text{fac}(3)} -$$

$$[\text{taylor}(n-1) * n]$$

taylor(n-1) + n

$$1 + 3 + \frac{9}{2} + \frac{27}{6}$$

if(n>0)

$$\frac{[\text{pow}(3)]}{[\text{fac}(3)]} * \text{taylor}(3-2)$$

else if(n==0)

$$1 + 3 + 4.5 + \frac{27}{6}$$

return t;

$$f = f * f$$

Taylor Series

$$e = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$$

$e(x, n)$

$$\begin{aligned} e(x, 3) &\rightarrow 1 + \frac{x^3}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \\ e(x, 2) &\rightarrow 1 + \frac{x^2}{1!} + \frac{x^2}{2!} \\ e(x, 1) &\rightarrow 1 + \frac{x}{1!} \\ \Phi &\rightarrow 1 + \frac{x}{1!} \end{aligned}$$

`Int e(Int x, Int n)`

`Static Int p=1; f=1;`

`r = e(x, n-1);`

`p = p*x;`

`f = f*n;`

`return r + p/f;`

$$1 + x \left[ 1 + \frac{x}{2} \left[ 1 + \frac{x}{3} \left[ \frac{x+x}{4} \right] \right] \right] \quad e(x, n)$$

$r = e(x, n-1);$   
 $\downarrow$   
 $e(x, n) \quad f(x)$   
 $\downarrow$   
 $e(x, 3)$   
 $\downarrow$   
 $e(x, 2)$   
 $\downarrow$   
 $p=x \quad f=1$   
 $\downarrow$   
 $e(x, 1)$

$\left\{ 2 \left\{ f \left( \Phi [f(x)] \right) \right\} \right\}$

$$\begin{aligned} \left[ 1 + \frac{x}{4} \right] &\quad e(x, 0) \rightarrow 1 \\ f(n-1) &\quad \left[ \begin{array}{l} e(x, 4) \\ 1 + e(x, n-1) \end{array} \right] \quad \left( 1 + \frac{x}{4} \right) + \frac{x^2}{2} + \dots \end{aligned}$$

`function f(n)`

$$\left\{ \begin{array}{l} e(x, 4) \rightarrow 1 + 2 + \frac{x^2}{2} + \frac{4x^3}{3} + \frac{4x^4}{4!} \\ e(x, 3) \rightarrow 1 + 2 + \frac{x^2}{2} + \frac{4x^3}{3} + \frac{16x^4}{24} + \frac{16x^5}{120} \end{array} \right.$$

$e(x, 4)$

$$r = 1 + e(x, n-1); \quad e(x, n-1); \quad e(x, 2) \rightarrow 1 + x \left[ 2 + \frac{x}{2} \right]$$

`return tressm(e(x, 1))`

$$\Phi \rightarrow 1 + x$$

$$x + e(x, 1) \rightarrow 1 + x$$

$$e(x, 0) \rightarrow 1 + 0$$

int e(int x, int n)

int s=1;

for(n>0; n--)

{ s = 1 + x \* s }

return s;

int e(int x, int n)

{ static s=1

if(n==0)

return s;

s = 1 + (x/n) \* s

return e(x, n-1) }

Sin, Cos Series

# Fibonacci Series.

e(7)

e(6)

e(5)

e(4)

e(3)

e(2)

e(1)

e(0)

s = e(n-1) + n;

courses  
e(n-1);

e(7)

e(6)

e(5)

e(4)

e(3)

e(2)

e(1)

e(0)

s = 0

s = e(n-1) + n;

$f(n) \quad 0 \quad 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \quad 12$

$$f(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ f(n-2) + f(n-1) & n>1 \end{cases}$$

$\text{fib(int } n)$

if ( $n \leq 1$ )

return  $n$ ; ; Excessive Recursion

return  $\text{fib}(n-2) + \text{fib}(n-1)$ ;

$O(2^n)$

int fib(int  $n$ )

int  $t_0=0, t_1=1, s=1$

if ( $n \leq 1$ ) return  $n$ ;

for ( $i=2; i < n; i++$ )

$s = t_0 + t_1$ ;

$t_0 = t_1$ ;

$t_1 = s$ ;

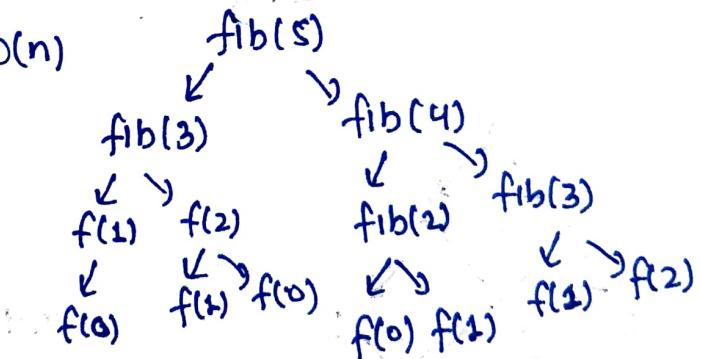
return  $s$ ;

memoization

F [ ]  $O(n)$

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   |   |   |   |   |   |
| 0 | 1 | 2 | 3 | 4 | 5 |

"storing the result".



int F[10];

int fib(int  $n$ )

if ( $n \leq 1$ )  $F[n] = n$   
return  $n$

else

if ( $F[n-2] == -1$ )

$f(n-2) = \text{fib}(n-2) \oplus$

Int F[10];

Int fib(int n)

if (n <= 1)

F[n] = n;

return n;

else if (F[n-2] == -1)

F[n-2] = fib(n-2);

if (F[n-1] == -1)

F[n-1] = fib(n-1);

return F[n-2] + F[n-1];

## Combination formula

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

$${}^5 C_2 = \frac{5!}{2!3!}$$

Comb(int n, int r)

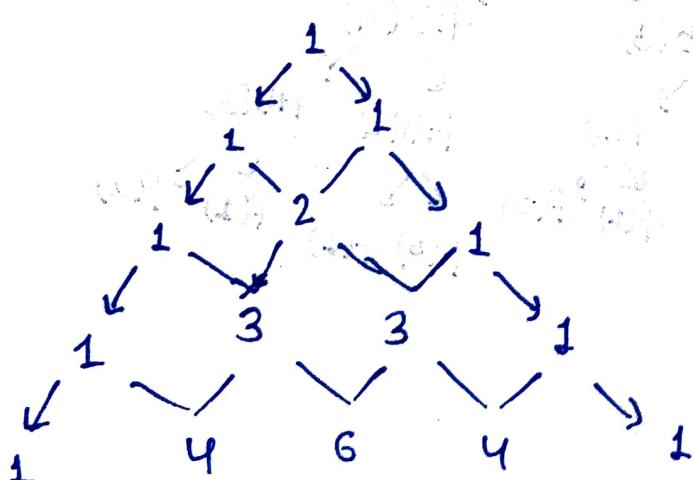
{ Int t1, t2, t3

t1 = fact(n)

t2 = fact(r)

t3 = fact(n-r) }

return [t1 / (t2 \* t3)]; }



$${}^{n-1} C_{r-1} + {}^{n-1} C_r = {}^n C_r$$

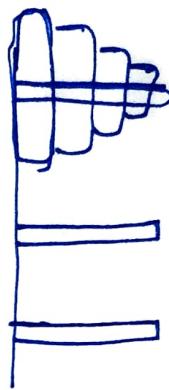
```
if(r==0 || n==r)
```

```
    return 1
```

```
else
```

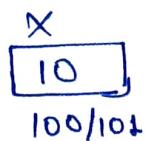
```
    return  $n-1$  C14 + C( $n-1, r$ );
```

» Tower of Hanoi

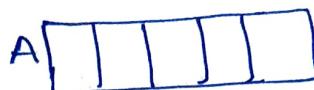


## Arrays

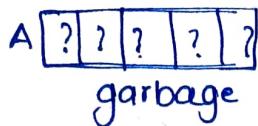
scalar  $\Rightarrow$  int  $x = 10$ ,



"Collection of similar element".



int A[5]



In C++, array can be created at run-time  
C, at compile time.

int \*P

C++  $P = \text{new int } A[5];$

$P = (\text{int } *)\text{malloc}(5 * \text{sizeof(int)})$

C++  $\text{delete } [] P;$

C  $\text{free}(P)$

stack =  $P$



Heap

\* Stack array <sup>size</sup> can't be increased or decreased

But heap array can be.

int \*P = new int [5]

int \*q = new int [10]

for( $i=0; i<5; i++$ )

$q[i] = P[i];$

memcpy

$\text{delete } [] P;$

$P = q;$

$q = \text{NULL};$

int \*A[3];

int \*A[3];

Int \*A[3];

A[0] = new Int[4];

A[1] = new Int[4];

A[2] = new Int[4];

Int \*\*A

A  $\square$

A = new Int[3]

A[0] = new Int[4]

A[1]

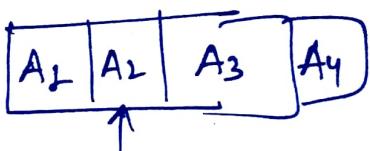
A[2]

[10][15]

A[i][j]

100

A[i][j] = [n\*i + j]/L



$a[1] = a[2];$

$a_n$

$[A_4 = A_3]$

$[A_3 = A_2]$

$a[2] = a[1]$

# Arrays in compiler

Int A[3][4]

1. Row-major mapping
2. Column-major mapping

|   |   |   |   |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
|   |   |   |   |
|   |   |   |   |

A[1][2]

$$\text{Addr}_r([i][j]) = \text{loc} + [i \cdot n + j] \cdot w$$

Add + [i \* 4 + j] \* 2

$$A[1][2] \equiv 200 + [2 * 3 + 1] * 2$$

## 1. Array ADT

Operations:

```
class Array {
    int *A;
    int size;
    int length; }
```

Struct Array {

int A[10];

int size;

int length; };

cin >> &arr.size

Void display( Struct Array A)

```
{ int i: for(i=0; i<arr.length; i++)
        cout << arr.A[i]; }
```

Void Append( Struct Array \*arr, int x)

```
int (arr->length < arr->size)
    arr->A[arr->length + 1] = x
```

Void Insert( Struct Array \*arr, int index, int x)

```
{ int i:
    if(index > 0 && index <= arr->length)
        for(i = arr->length; i > index; i--)
            arr->A[i] = arr->A[i-1];
        arr->A[index] = x;
        arr->length++; }
```