Homework 1

CSCI 2897 - Calculating Biological Quantities - Larremore - Fall 2021

Notes: Remember to (1) familiarize yourself with the collaboration policies posted on the Syllabus, and (2) turn in your homework to Canvas as a **single PDF**. Hand-writing some or most of your solutions is fine, but be sure to scan and PDF everything into a single document. Unsure how? Ask on Slack!

Pushups

Calculate these derivatives.

1. $\frac{d}{dx}x^3 =$
2. $\frac{d}{dx}x^{-3} =$
3. $\frac{d}{dx}e^{\alpha x} =$
4. $\frac{d}{dx}e^{\pi x^{-2}} =$
5. $\frac{d}{dx}\ln 2x =$

Squats

Find solutions to each of these differential equations.[1]

6. $\frac{dy(t)}{dt} = 0$
7. $\frac{dy(t)}{dt} = t$
8. $\frac{dy(t)}{dt} = y(t)$

[1] Hint: ask yourself, "What function, if I were to take its derivative, would satisfy this equation?"

Cooldown

You may remember that there are infinitely many solutions to so-called *indefinite integrals* (integrals without limits of integration). For example,

$$\int x\, dx = x^2/2 + c,$$

where c is an arbitrary constant. If we take a derivative of both sides of that equation, we see that indeed, $x = \frac{d}{dx}\left(x^2/2 + c\right)$, no matter what c is.

9. When solving the equations in the Squats section above, did arbitrary constants factor into your solutions? If not, why not? If so, how?

10. In a couple sentences, in your own words, can you explain what it means for a differential equation to have a *family of solutions*?

Going viral

A new semester is starting, and, feeling a sense of renewal, you turn on your smartphone, and log into your TikTok app to create some Day 1 content for your followers. Whatever you do, it works, and your content gets viewed a great many times. It hits you: you are going viral.

Naturally, you are the CEO of *Calculating Biological Quantities*, so you being to wonder: *could I write down a* **dynamical model** *to predict and understand my* **unread notifications**? You make the following observations in your Influencer Lab Notebook:

- You only check your phone every 60 minutes.
- Each time you check your phone, you see that there are new unread notifications. You notice that these new unread notifications are following a *growing* pattern: 0 when you first post, 20 in the first hour, 40 in the second hour, 60 in the third hour, 80 in the fourth . . . , at which point you decide it's time to write down a model.
- Before putting your phone away, you address 50% of the total unread notifications.

Due to the fact that mathematical modeling lives in your head rent free, you decide to write down a model to track the number of total unread notifications that you'll see each time you check your phone.

11. For this model, would you consider continuous or discrete time? Explain.
12. What is the variable of this model? Jot down two questions about the variable that you could use your model to answer.
13. What would be an appropriate timescale?
14. Draw a Life Cycle Diagram for this model.
15. Use the Life Cycle Diagram to write down (a) a recursion equation and (b) a difference equation.
16. Using Python, *code up your model*. How many unread notifications will there be after a week, assuming that you start with zero unread notifications? (Assume that you have hired a social media manager who deals with your inbox, exactly like you would, while you sleep. Or assume that you don't sleep.)
17. Make a plot of unread notifications vs time during that week. Be sure to label your axes and include a legend.

Extra Credit: To consider more generic scenarios of going viral, identify constants in the model above and replace them with generic parameters (i.e., letters). What are the units of those parameters? Produce a plot of the first week of unread notifications using different parameters, and write 2-3 sentences describing how the parameters affected the plot, as compared with the baseline scenario.

Trust but verify

18. What is the order of this equation? Is it linear or nonlinear?

$$(1-x)y'' - 4xy' + 5y = cos(x)$$

19. What is the order of this equation? Is it linear or nonlinear?

$$(\sin\theta)y''' - (\cos\theta)y' = 2$$

20. Verify that the function solves the ODE or show that it does not.

$$2y' + y = 0; \qquad y = e^{-x/2}$$

21. Verify that the function solves the ODE or show that it does not.

$$y'' - y = \cos 2x; \qquad y = -\tfrac{1}{5}\cos 2x$$

22. Verify that the function solves the ODE or show that it does not.

$$y'' + y = \sin 2x; \qquad y = \tfrac{1}{5}\sin 2x$$

23. Find a value of the parameter a so that the function $y(t) = e^{at}$ is a solution to the given differential equation.

$$5y' = 4y$$

24. Find a value of the parameter r so that the function $n(t) = ar^t$ is a solution to the given recursion, assuming that $a \neq 0$.

$$n(t+1) = n(t) + n(t-1)$$

Last but not least

25. Describe a time-varying phenomenon in the biological or physical world around you that makes you curious. Tell me a bit about it, pose a quantitative question about it, and write down the processes or steps in a model with a corresponding diagram (of any of the types we have discussed in class). Identify parameters and their units, choose a timescale, and if possible, write some equations for your model. This is a *creative* exercise so you will not be evaluated on whether the model is correct or plausible. No need to solve or analyze the equations.