10/7/2021
Exam Review

- Open Note
- Open Book
- Not open friend
- Not ok to post exam to Q&A forum.

- Tuesday, 10/12
- Live, In-Class
  - if extra time → email re logistics
  - In-person: take in person.
    - Submit on paper.
  - Zoom: take live on Zoom.
    - Submit to Canvas at end of class
    - Post PDF of test to Github.
  - NO coding on exam.
- OH Monday 9-11 as usual.

Topics

- Exam Review Outline (Github)
- Will post old exam. (too long!) (this time: shorter.)

Survey

- ✓ I.F. example.
- ✓ Like the tank problem.
- ✓ Diagram to equation.
- Cons. Resource ?

1) Integrating Factor

- Is it separable ?

yes → S.o.V.

no → I.F. method.

$$\frac{dy}{dx} + P(x)\,y = f(x)$$

separable for sure if $f(x) = 0$.

$$2\frac{dy}{dx} = x - xy$$

$$2\frac{dy}{dx} = x - xy$$

$$\frac{2\frac{dy}{dx}}{2} + \frac{xy}{2} = \frac{x}{2}$$

$$\frac{dy}{dx} + \left(\frac{x}{2}\right)y = \frac{x}{2}$$

$$\text{I.F.} = e^{\int \frac{x}{2}dx}$$

$$= e^{x^2/4} \quad \boxed{}$$

no const of integration needed for I.F.

mult both sides of ODE by IF:

$$e^{\frac{x^2}{4}}\left[\frac{dy}{dx} + \frac{x}{2}y\right] = \frac{x}{2}e^{\frac{x^2}{4}}$$

$$\int \frac{d}{dx}\left[e^{\frac{x^2}{4}} \cdot y\right]dx = \int \frac{x}{2}e^{\frac{x^2}{4}}\,dx$$

$$e^{x^2/4}\,y = \underbrace{\int \frac{x}{2}e^{x^2/4}\,dx}$$

let $u = \frac{x^2}{4}$ $\quad du = \underbrace{\frac{2x}{4}dx} = \frac{x}{2}dx$

$$dx = \frac{2}{x}du$$

$$\int \frac{x}{2}e^{x^2/4}dx = \int \frac{\cancel{x}}{\cancel{2}}e^u\frac{\cancel{2}}{\cancel{x}}du = \int e^u du$$

$$= e^u + c$$

$$= e^{\frac{x^2}{4}} + c$$

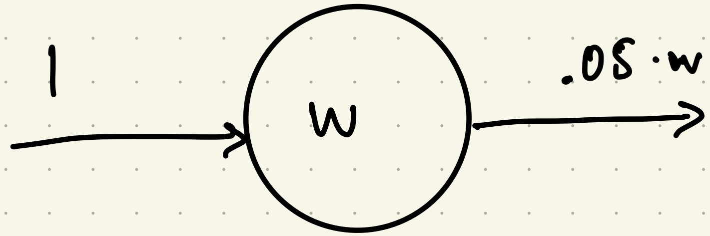
$$e^{\frac{x^2}{4}}y = e^{\frac{x^2}{4}} + c$$

$$\boxed{y = 1 + ce^{-x^2/4}}$$

↑ 1 kg/day

↓ 5% /day

today: $w = 5$ kg



w in kg

timescale: 1 day

$$\frac{dw}{dt} = \oplus \text{ inflow } \ominus \text{ outflow}$$

$$\boxed{\frac{dw}{dt} = 1 - 0.05\,w \qquad w(0) = 5}$$

Equilibrium

$$\frac{dw}{dt} = 0$$

$$0 = 1 - 0.05\,w \quad ✓$$

$$0.05\,w = 1$$

$$\frac{w}{20} = 1$$

$\boxed{w = 20}$ at steady state.

$\frac{dw}{dt} = 1 - 0.05w, \quad w(0) = 5$

I.F. $\rightarrow$

$$\frac{dw}{dt} + 0.05w = 1 \qquad \mu(t) = e^{\int 0.05\,dt} = e^{t/20}$$

$$\int \frac{d}{dx}\left[e^{t/20} \cdot w\right] = \int e^{t/20} \rightarrow e^{t/20} w = 20e^{t/20} + c$$

$$\boxed{w = 20 + ce^{-t/20}}$$

$$5 = 20 + c$$

$$\Rightarrow c = -15$$

$$\boxed{w = 20 - 15e^{-t/20}}$$

$\updownarrow$ ✓

$$\frac{dw}{dt} = 1 - 0.05w$$

S.o.V.

$$\frac{dw}{1 - 0.05w} = dt$$

$$\int \frac{dw}{1-0.05w} = \int dt$$

$$\frac{\ln(1-0.05w)}{-0.05} = t + c$$

$$-20\ln(1-0.05w) = t + c$$

$$\ln(1-0.05w) = -\frac{t}{20} + k$$

$$1 - 0.05w = me^{-t/20}$$

$$1 - me^{-t/20} = \frac{w}{20}$$

$$\boxed{20 - 20 \cdot me^{-t/20} = w} \rightarrow \boxed{20 - 15e^{-t/20} = w}$$

Apply I.C. $w(0) = 5$

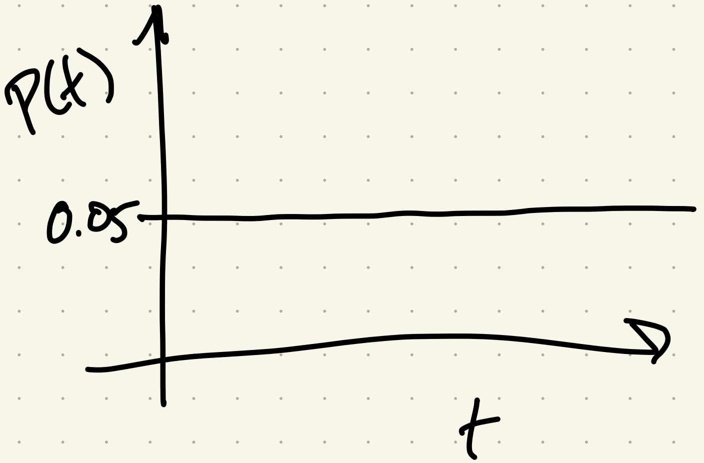
$$20 - 20 \cdot m \cdot 1 = 5$$

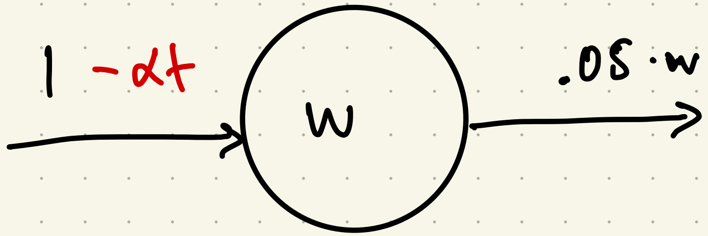
$$20 - 20m = 5$$

$$-20m = -15$$

$$m = \frac{3}{4}$$

$$P(t) = 0.05$$





$$\frac{dw}{dt} = (1-\alpha t) - \frac{w}{20}$$

growth loss

$$\frac{dw}{dt} = 1-\alpha t - \frac{w}{20}$$

$$\frac{dw}{dt} + \frac{w}{20} = 1-\alpha t \qquad \mu = e^{t/20}$$

$$\int \frac{d}{dt}\left[e^{t/20} \cdot w \right] dt = \int e^{t/20}(1-\alpha t)\,dt$$

$$e^{t/20} \cdot w = \underbrace{\int e^{t/20}\,dt}_{\text{easy}} - \alpha \underbrace{\int t e^{t/20}\,dt}_{\text{I.B.P.}}$$

$$\int x e^{kx} = e^{kx} \frac{kx-1}{k^2} + c$$

$$e^{t/20} \cdot w = 20 e^{t/20} - \alpha \left[e^{t/20} \frac{\frac{t}{20} - 1}{\left(\frac{1}{20}\right)^2} + C \right]$$

$$w = 20 - \alpha \left[\frac{\frac{t}{20} - 1}{\left(\frac{1}{20}\right)^2} + C e^{-t/20} \right]$$

Consumer/Resource Model

exchange rate between C and R *

loss of consumers in the absence of R.

$$\frac{dC}{dt} = \varepsilon g(R,C) - h(C)$$

* ε = how much growth does the C get per R consumed?

$$\frac{dR}{dt} = f(R) - g(R,C)$$

resource grows, irresp. of consumer's influence, in this term $f(R)$

minus because depletes R.

consumption of R by C

- Equilibria?
- Interpretation?

- Post these notes
- Post Spring '21 exam