Calculating Biological Quantities

CSCI 2897

Prof. Daniel Larremore

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@danlarremore

2021, Lecture 1

Lecture 1 Plan

1. **Course mechanics & setup**
    1. Website
    2. Syllabus
    3. Canvas
2. **Schedule & Syllabus Review**
3. **Textbooks**
4. **A tour of mathematical models and linear algebra**
5. **About me**

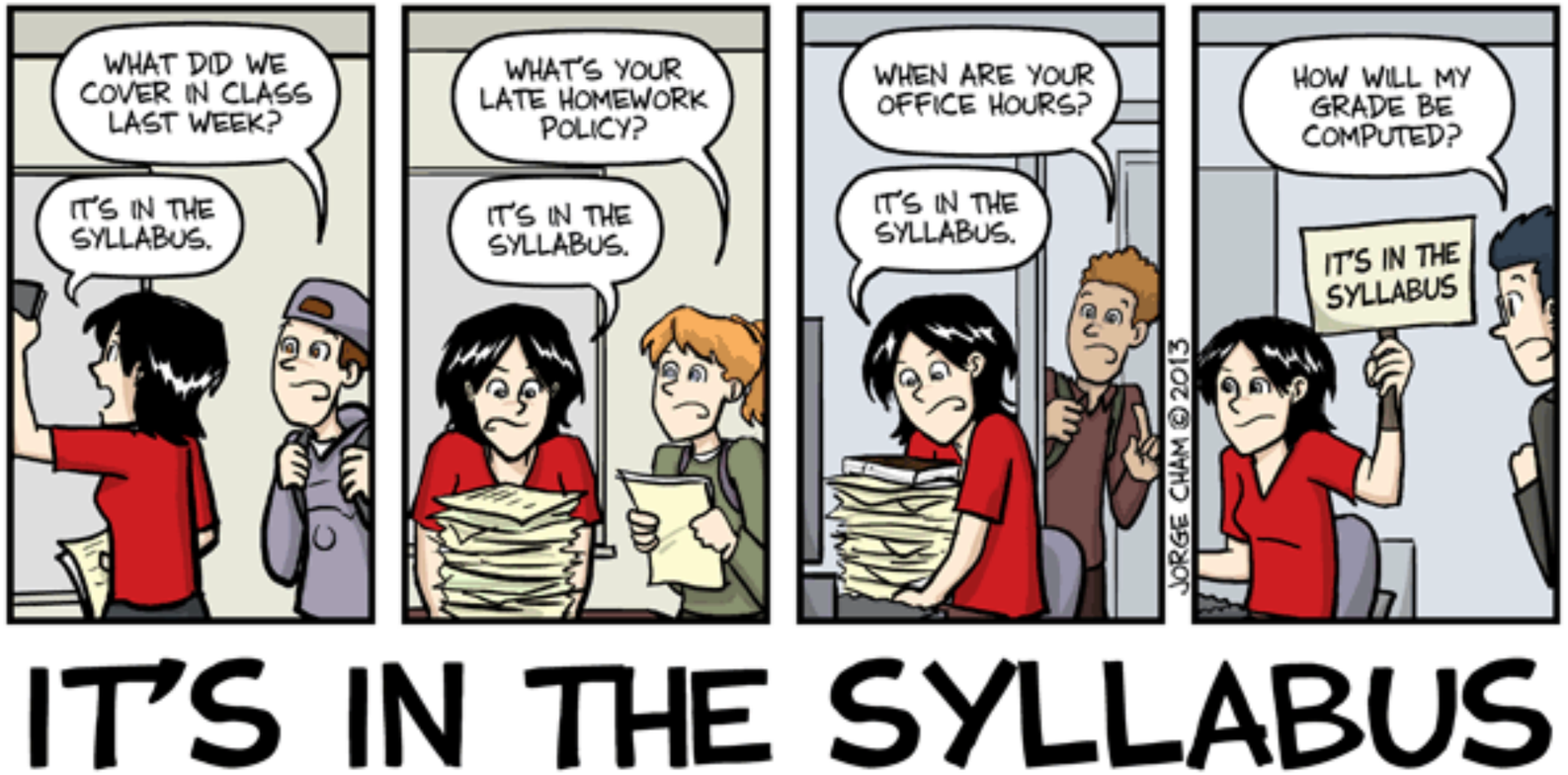
Course Mechanics

- Website: https://github.com/dblarremore/CSCI2897
  - Homework & reading posted
  - Code examples
  - Class notes
- Syllabus: https://github.com/dblarremore/CSCI2897#syllabus
- Canvas:
  - Turn in homeworks ✓
  - Lecture links ✓
  - Check grades ✓
- Slack:
  - Q&A
  - Swap files

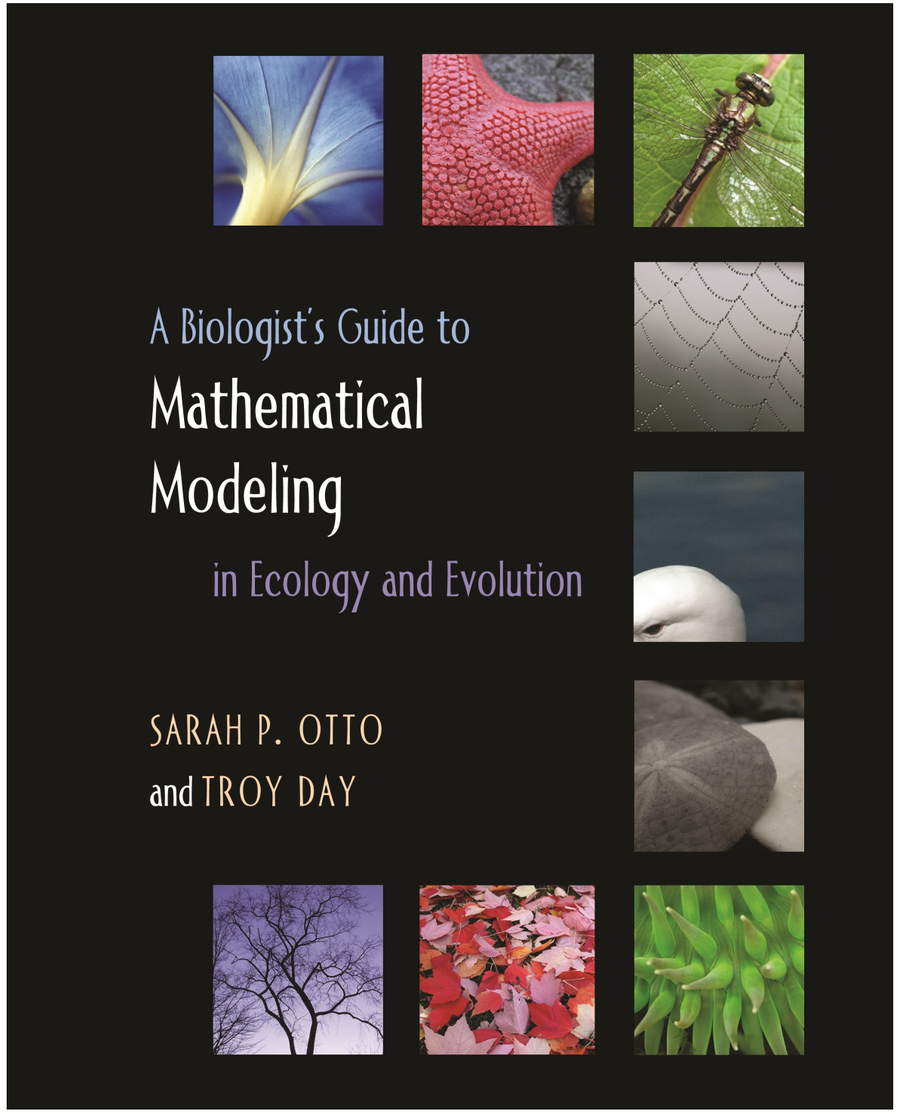
Schedule & Syllabus Review

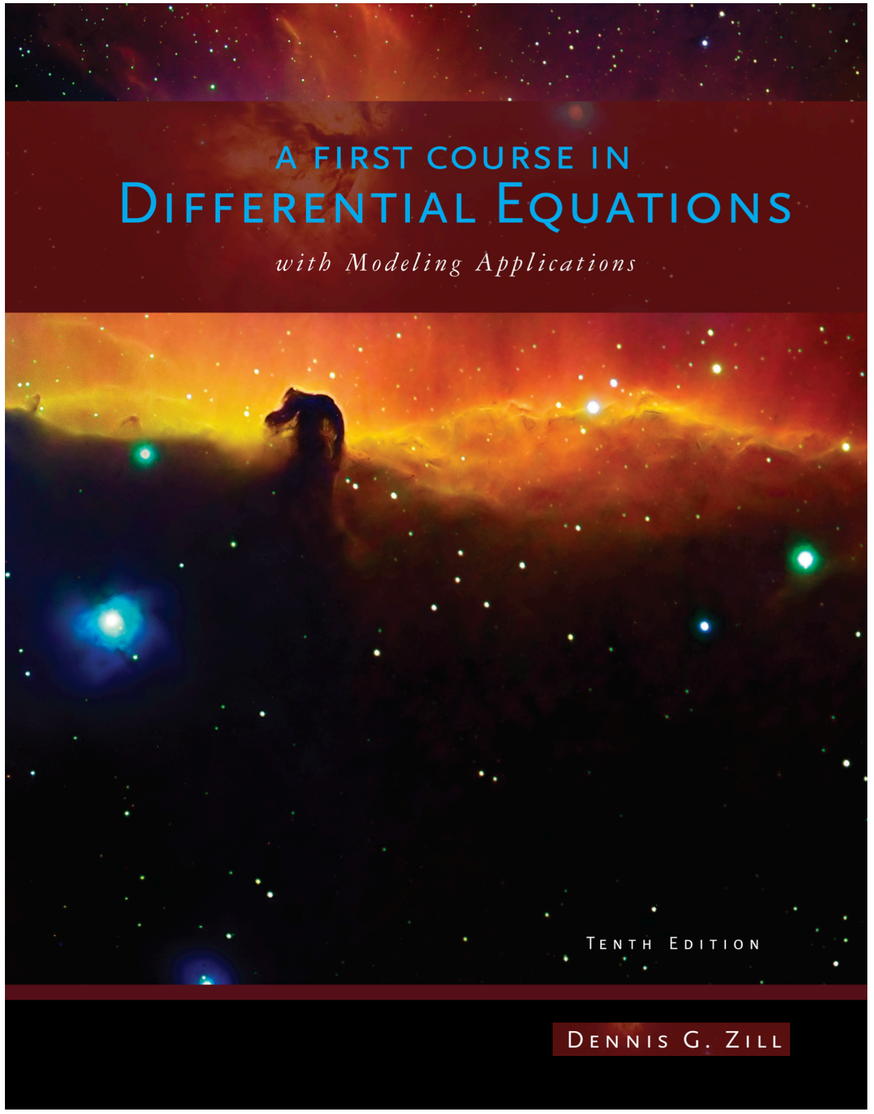
- https://github.com/dblarremore/CSCI2897

Piled Higher and Deeper ***by Jorge Cham*** **www.phdcomics.com**



Textbooks

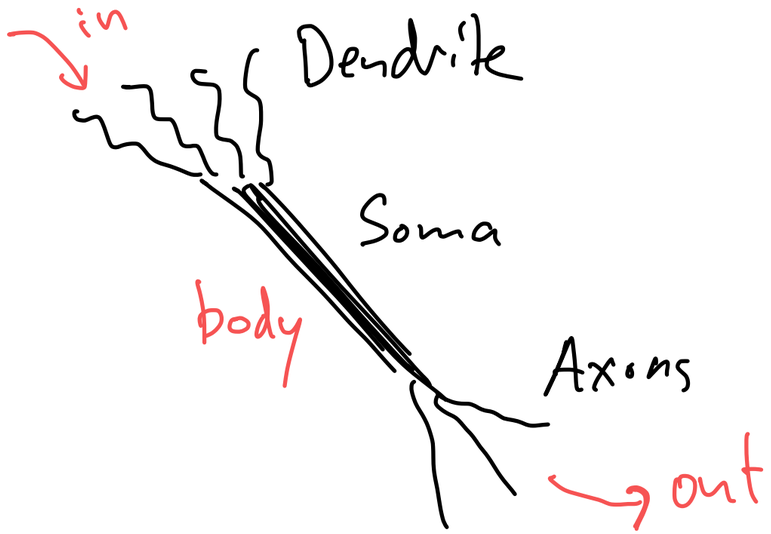

A Biologist's Guide to
Mathematical
Modeling
in Ecology and Evolution
SARAH P. OTTO
and TROY DAY


A FIRST COURSE IN
DIFFERENTIAL EQUATIONS
with Modeling Applications
TENTH EDITION
DENNIS G. ZILL

Mathematical models & linear algebra

Point 1: Our understanding of biology *is* a model—an abstraction or simplification.

- Neuron



Model

- Structure of RNA
  - guess/model structure of pairing combinations
- Structure of proteins

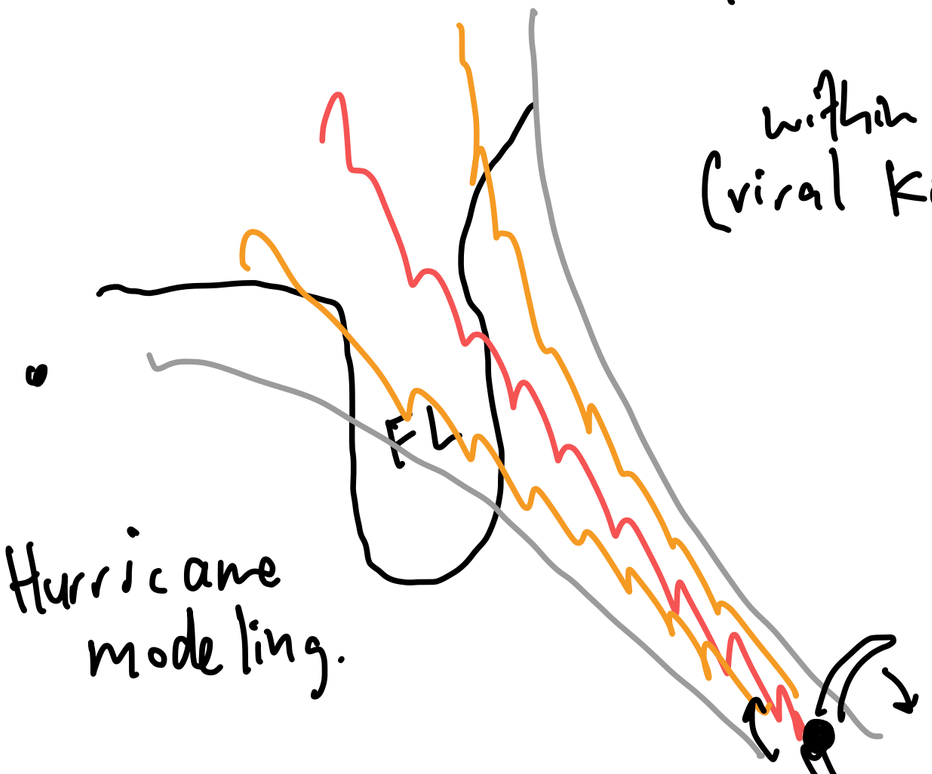
- Abstractions of abstractions

Subpoint: not all models are mathematical.

Mathematical models & linear algebra

Point 2: Mathematical models are better. We get *quantitative predictions* and *mechanism*.

- Population Models
  - birth, death, emigration (out), immigration (in).
    (people, fishery, infections disease)
    - within host (viral kinetics)
    - between hosts (epidemiology)



Hurricane modeling.

Value in math. models which transcends the conceptual models.

Key modeling vocabulary

Variable: a quantity that changes (e.g. over time)

What we want to track.

Dynamics: the patterns of changes that occur over time.

in the variables

Parameter: a quantity in a model that remains constant over time.

Climate — Air pressure
Temperature

COVID-19 — Hospitalizations
Testing per day

Hurricane — Wind Speed
(lat, lon)

Some of the classics:

Variables

Parameters

Discrete time

Population growth:

$$n(t+1) = Rn(t)$$

# of animals at time t+1

# of animals at time t

change this only to alter growth/non-growth rate.

Logistic growth:

$$n(t+1) = n(t) + rn(t)\left(1 - \frac{n(t)}{K}\right)$$

growth rate

carrying capacity

Disease dynamics:

Continuous Time

$$\frac{dS}{dt} = \dot{S} = -\beta SI$$

$$\frac{dI}{dt} = \dot{I} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \dot{R} = \gamma I$$

infectiousness

recovery rate

R_0 "basic reproduction #"

$$R_0 = \frac{\beta}{\gamma}$$

FitzHugh Nagumo Neuron:

$$\dot{v} = v - \frac{v^3}{3} - w + RI_{\text{ext}}$$

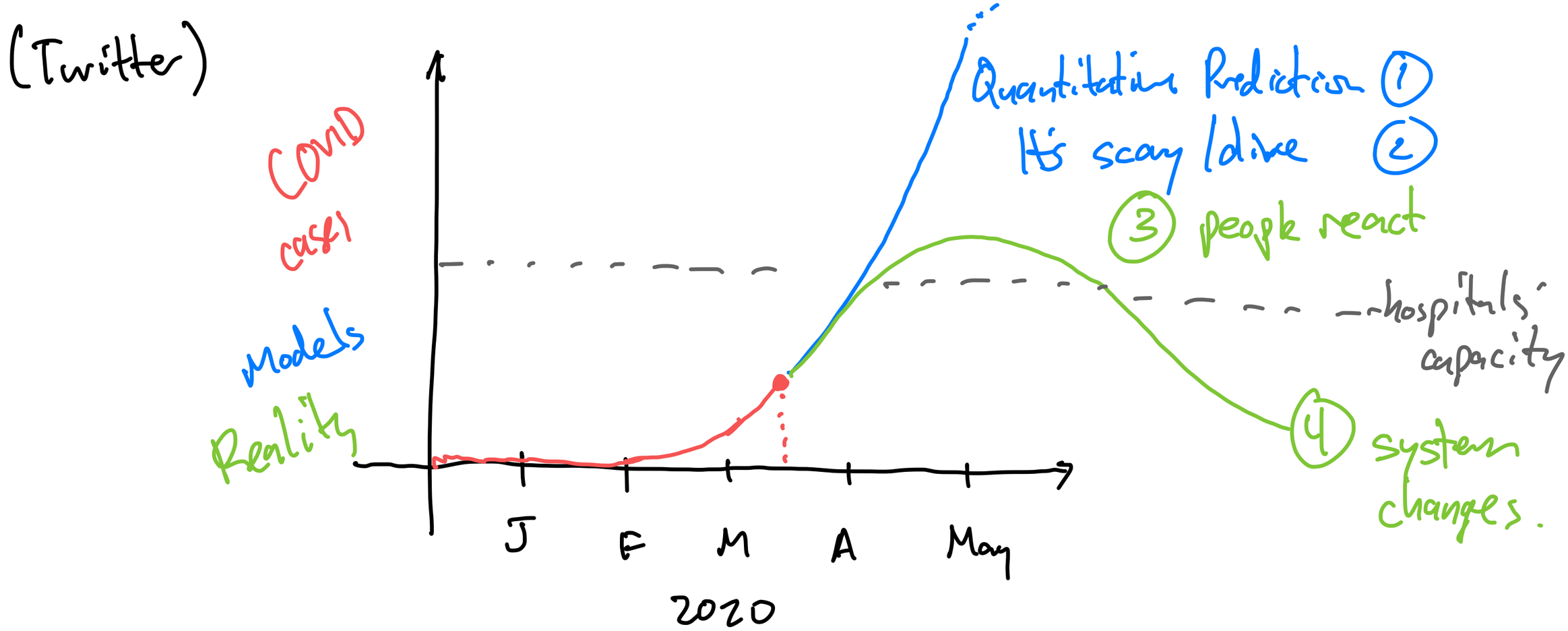
$$\tau\dot{w} = v + a - bw$$

voltage

non-physical

Mathematical models & linear algebra

Point 3: Mathematical models are stories we tell about how the future comes about.



Subpoint: sometimes *being wrong is the point!*

Models. Great. So…why linear algebra too?

Linear algebra is…algebra.

$\frac{1}{6} 6x = 12 \frac{1}{6}$

$x = \frac{12}{6} = 2$

But simplified (or lazy? or elegant?)

$3x + 5y = 8$

$x + 2y = 11$

solve for x.

$5y = 8 - 3x$

$y = \frac{8}{5} - \frac{3}{5}x$

matrix vecto vecta

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$M^{-1} M \quad v = M^{-1} w$

$v = M^{-1} w$

$x_n = x_{n-1} + x_{n-2}$

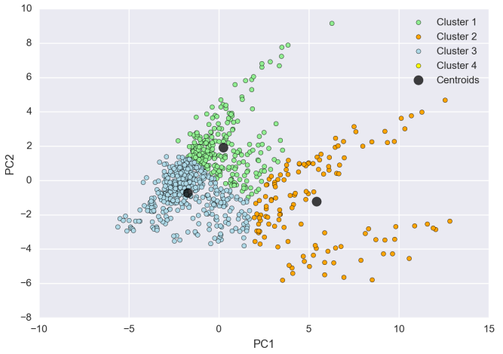
Fibonacci

↳ Matrix → Analyze → Golden Ratio

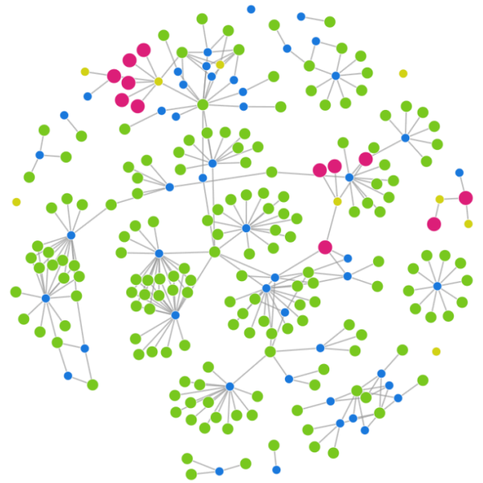
Linear algebra comes up *everywhere*

Computer Graphics

AI & Machine Learning



PCA & Clustering

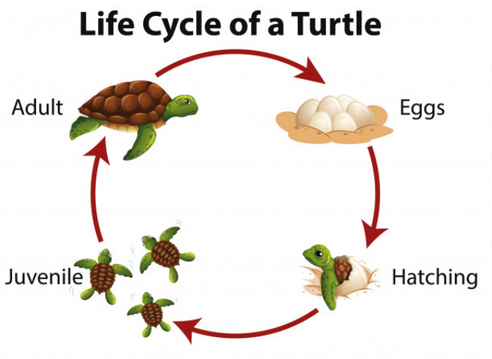


Networks

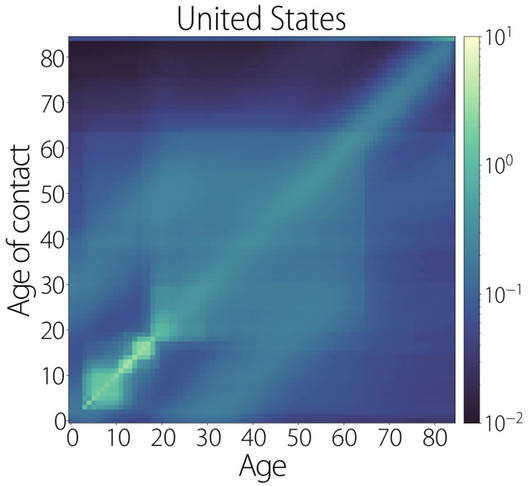
Google

Technology company

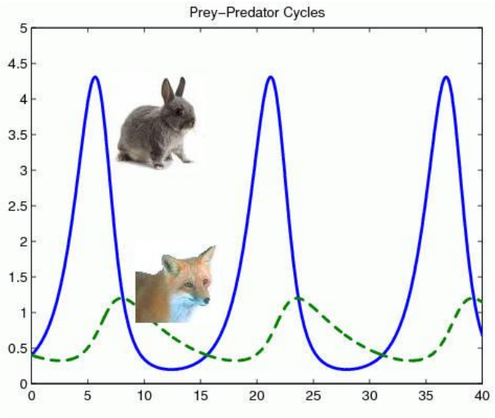
Web Search



Population Structure



Age-structured models



Predator-prey dynamics

In this course…

Dynamic Models
in Biology

Mathematical Approaches
to Analysis

Demystifying Jargon
and Vocabulary

Coding to Explore
and Understand

Concept, Application, Practice

Questions & Answers

When textbooks? → Slack.

How would you define a matrix:

- 2D table of numbers. rows, columns. (Excel, Sheets)

3D matrix??

- Tensor!

40/40/20

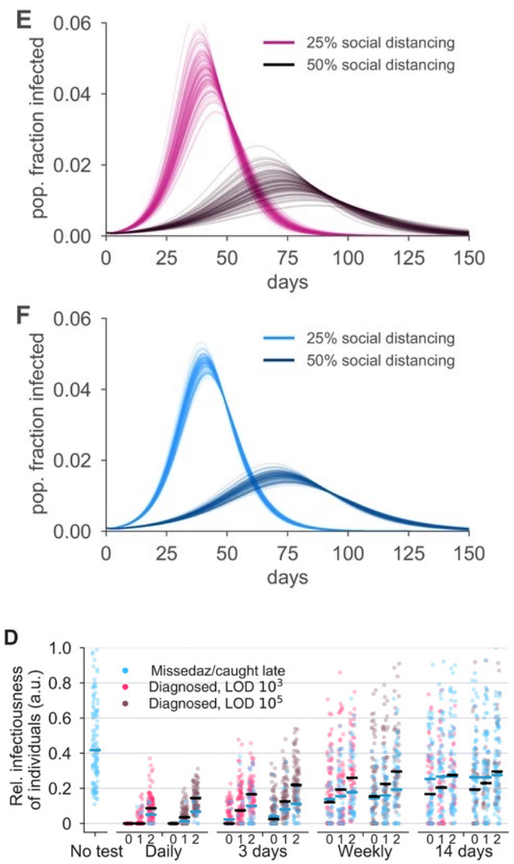
Math/Bio / CS

About me

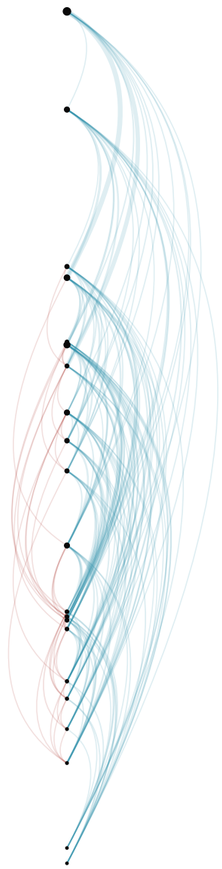
Assistant Professor, BioFrontiers Institute & Department of Computer Science

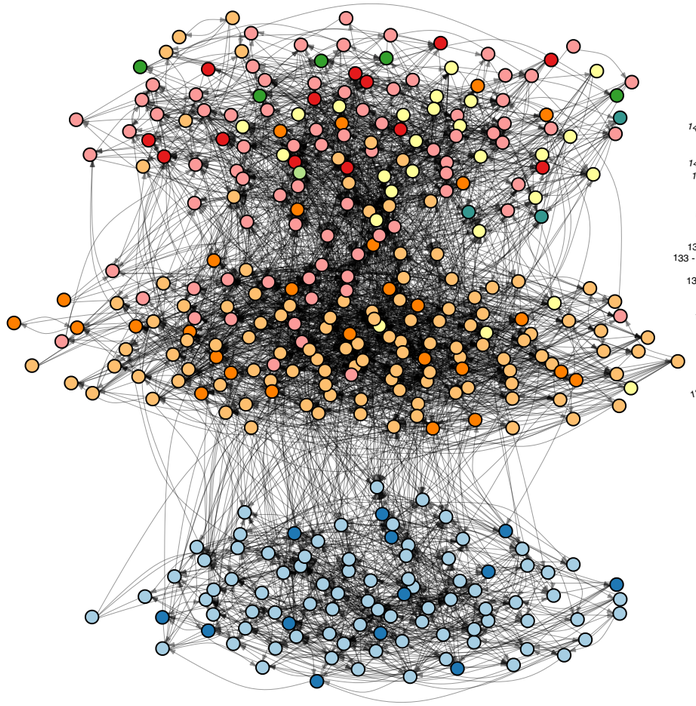
External Faculty: Harvard School of Public Health — Center for Communicable Disease Dynamics

infectious disease epidemiology



mathematical methods for statistical inference/analysis





the scientific ecosystem & the science of science

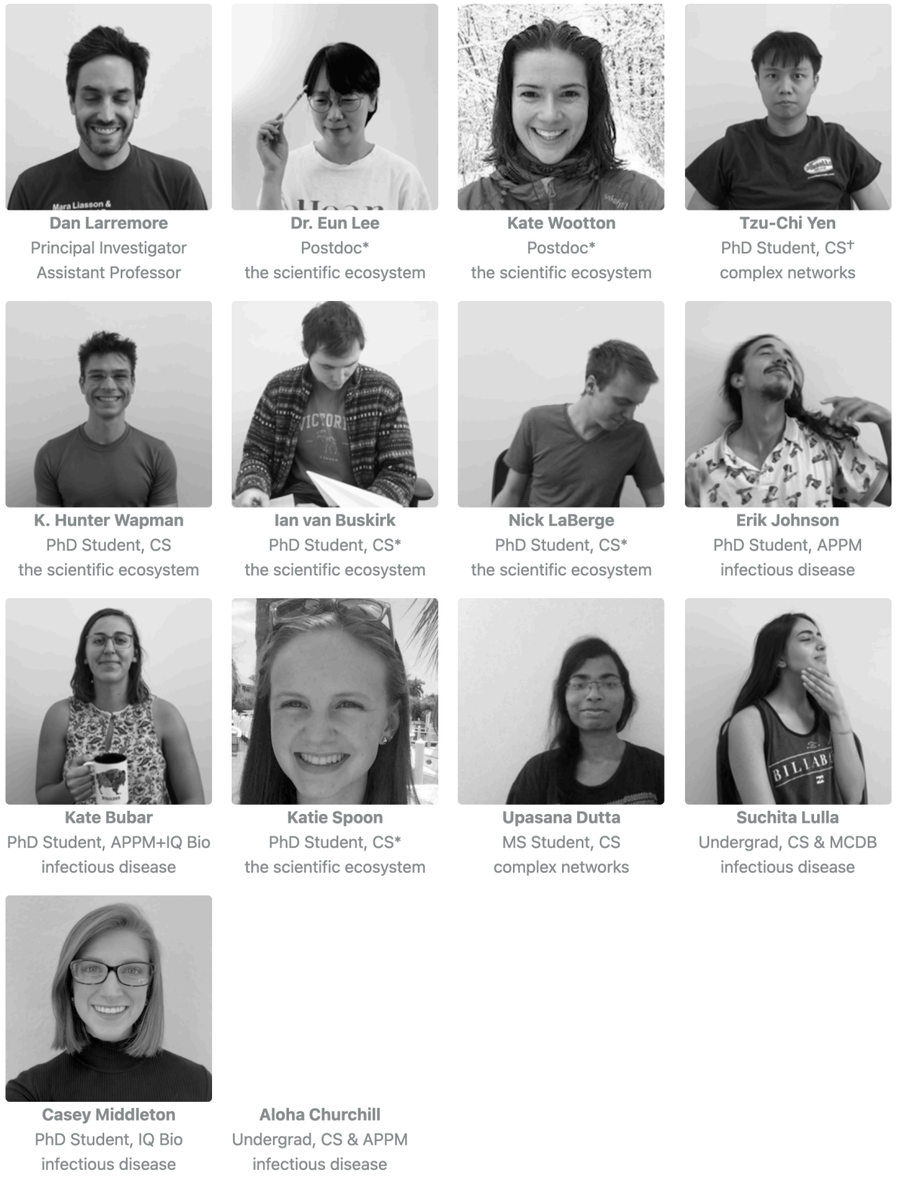
research: LarremoreLab.github.io

Research should be **fun**

malaria parasite evolution
and epidemiology

optimal vaccine prioritization for
SARS-CoV-2 vaccine rollout

prestige & gender inequality in
academic faculty hiring



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Casey Middleton
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Aloha Churchill
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infectious disease

structured hierarchy in
online dating markets

provably optimal play in
generalized *misere* Connect 4

the dominance hierarchy
in the network of hockey fighters

Homework

Before next class:

1. Join the CBQ Slack. (Email)
2. Go to the github and bookmark it. github.com/dblarremore/CSCI2897
3. Get the textbooks. (Slack)
4. Take the survey (Canvas).
5. Do the reading (literally just a 10-tweet thread) (calendar, github)