Homework 5

CSCI 2897 - Calculating Biological Quantities - Larremore - Fall 2021

Notes: Remember to (1) familiarize yourself with the collaboration policies posted on the Syllabus, and (2) turn in your homework to Canvas as a **single PDF**. Hand-writing some or most of your solutions is fine, but be sure to scan and PDF everything into a single document. Unsure how? Ask on Slack!

Hamstring curls

Compute the following, supposing that $a = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$**,** $b = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$**, and** $c = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$**.**

1. $a + b + c =$
2. $a^T b =$
3. $a + 2b + 3c =$
4. $ab^T =$

Calf raises

Using the same a**,** b**, and** c **as above, and with** $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$**, solve the following, or explain why they cannot be solved.** [1]

5. $Da + c =$
6. $a^T D + c =$
7. $D^2 =$
8. $D^9 a - D^9 b + D^9 c =$
9. $a^T b c^T =$
10. $ab^T c =$

[1] Note: just like with regular multiplication, squaring a matrix means multiplying the matrix by itself!

Reasoning about matrices

11. Suppose you know that A is a symmetric $n \times n$ matrix. Let the matrix $B = A - A^T$. Let x be a $n \times 1$ vector. Let $y = Bx$. What is the **dimension** of y? What is y?

12. Let matrix P have dimensions 5×2, Q have dimensions 3×2, and R have dimensions 3×5. What is the dimension of RPQ^T? What about $QQ^T R$?

13. In reference to the previous problem, write out *three* different ways that you could multiply the matrices P, Q, R or their transposes to creates a 5×5 matrix. You may use each matrix at most two times in each multiplication.

14. Suppose that I hand you two square $n \times n$ matrices, X and Y. You multiply them and find that $XY = I$, where I is the identity matrix. How and X and Y related?

15. For the same matrices X and Y from the previous problem, compute the *trace* of $(XYX)(YX)(YXYXY)$.

Computing with matrices

16. Let $A = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$. Knowing that $Ax = b$, solve for the vector x.
17. What is the determinant of A?
18. Let $M = \begin{pmatrix} -2 & 1 \\ \alpha & -1 \end{pmatrix}$. Knowing that $Mx = d$, where $d = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, solve for the vector x.
19. The solution to the previous question fails to exist at a particular value of α. What is this value? Explain why the solution ceases to exist at that value.
20. Extra credit: Make up a 2×2 matrix in Python, call it L. Make up a 2×1 vector in Python too, and call it x. Now do the following: (i) Compute $y = Lx$. (ii) Compute $x = y/||y||$, where $||y||$ is the same as `numpy.linalg.norm(y)`. (iii) Repeatedly do (i) and (ii) 100 times by using x to compute y and then using y to get a new x, over and over. What do you notice? Start with a few new initial x vectors and repeat this process. What do you find? Explore changing L and x and write up some of your findings in under one page with some examples.