Calculating Biological Quantities

CSCI 2897

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2021, Lecture 12

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Last time on CSCI 2987: The SIR Model

$$\dot{S} = -\beta SI$$

$$\dot{I} = \beta SI - \gamma I$$

$$\dot{R} = \gamma I$$

where $S + I + R = 1$

$$\implies \dot{S} + \dot{I} + \dot{R} = 0$$

Equilibrium when:

$$I = 0$$

Epidemic peak:

$$S^* = \frac{\gamma}{\beta}$$

Herd Immunity (vaccination)

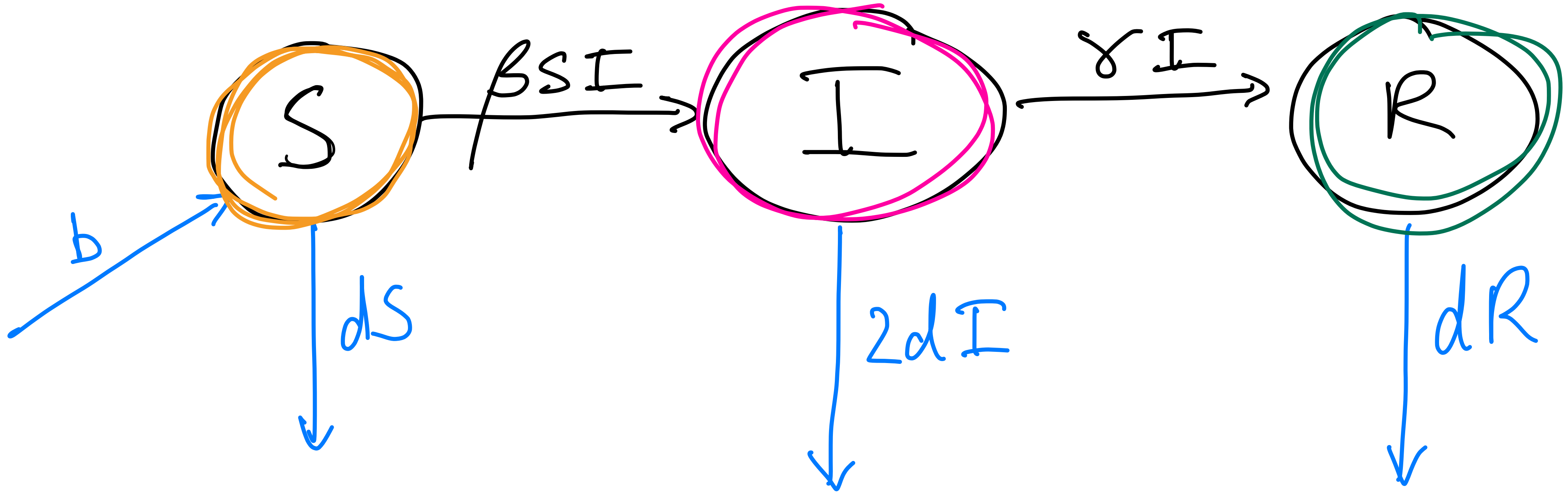
$$v > 1 - \frac{\gamma}{\beta}$$

Basic Reproduction Number

$$R_0 = \frac{\beta}{\gamma}$$

Midterm: SIR model with birth and death

$$\frac{dN}{dt} = b - dS - 2dI - dR = b - dI - d(S+I+R) = b - d(I+N)$$



$$\frac{dS}{dt} = b - dS - \beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I - 2dI$$

$$\frac{dR}{dt} = \gamma I - dR$$

The S**E**IR model—exposure without infectiousness

Some diseases have a **latent period** in which a person is infected but not yet infectious to others.

$$\dot{S} = -\beta S I$$
$$\dot{E} = \beta S I - \alpha E$$
$$\dot{I} = \alpha E - \gamma I$$
$$\dot{R} = \gamma I$$

Let’s consider a **new** compartment: **E**xposed, with a transition rate α.

$$S \xrightarrow{\beta S I} E \xrightarrow{\alpha E} I \xrightarrow{\gamma I} R$$

E puts a delay in the model!

$$\dot{S} + \dot{E} + \dot{I} + \dot{R} = 0$$
(constant pop. size)

SEIR Model — Equilibrium

$\dot{S} = -\beta S I = 0$

④ $-\beta S \cancel{I}^{\,0} = 0$ (by ①)

$\dot{E} = \beta S I - \alpha E = 0$

③ $\beta S \cancel{I}^{\,0} - \alpha \cancel{E}^{\,0} = 0$ (by ①) (by ②)

satisfied

$\dot{I} = \alpha E - \gamma I = 0 \longrightarrow$ ② $\alpha E - \gamma \cancel{I}^{\,0} = 0 \Rightarrow E = 0$

$\dot{R} = \gamma I = 0 \longrightarrow$ ① $I = 0$

where $S + E + I + R = 1$

equilibrium $= (S, 0, 0, 1-S)$

SEIR Model — Out of Equilibrium?

$\dot{S} = -\beta SI$

$\dot{E} = \beta SI - \alpha E$

$\dot{I} = \alpha E - \gamma I$

Let's hop into Jupyter Notebooks to explore a bit.

$\dot{R} = \gamma I$

where $S + E + I + R = 1$

SEIR Model — Out of Equilibrium?

With Social Distancing? $0 \leq \rho \leq 1$

$$\dot{S} = -\beta S I \rho$$

$$S \xrightarrow{\beta S I \cdot \rho} E \xrightarrow{\alpha E} I \xrightarrow{\gamma I} R$$

$$\dot{E} = \rho \beta S I - \alpha E$$

$$\dot{I} = \alpha E - \gamma I$$

Let's hop into Jupyter Notebooks to explore a bit.

ρ = fraction of contacts that are still allowed

$$\dot{R} = \gamma I$$

where $S + E + I + R = 1$

$\rho = 1 \longrightarrow$ No Soc. Dist.

$\rho = 0 \longrightarrow$ total isolation.

As you change ρ from 1 downward, you "flatten the curve."

How should we model vaccination in the SEIR model?

$$\dot{S} = -\beta SI$$

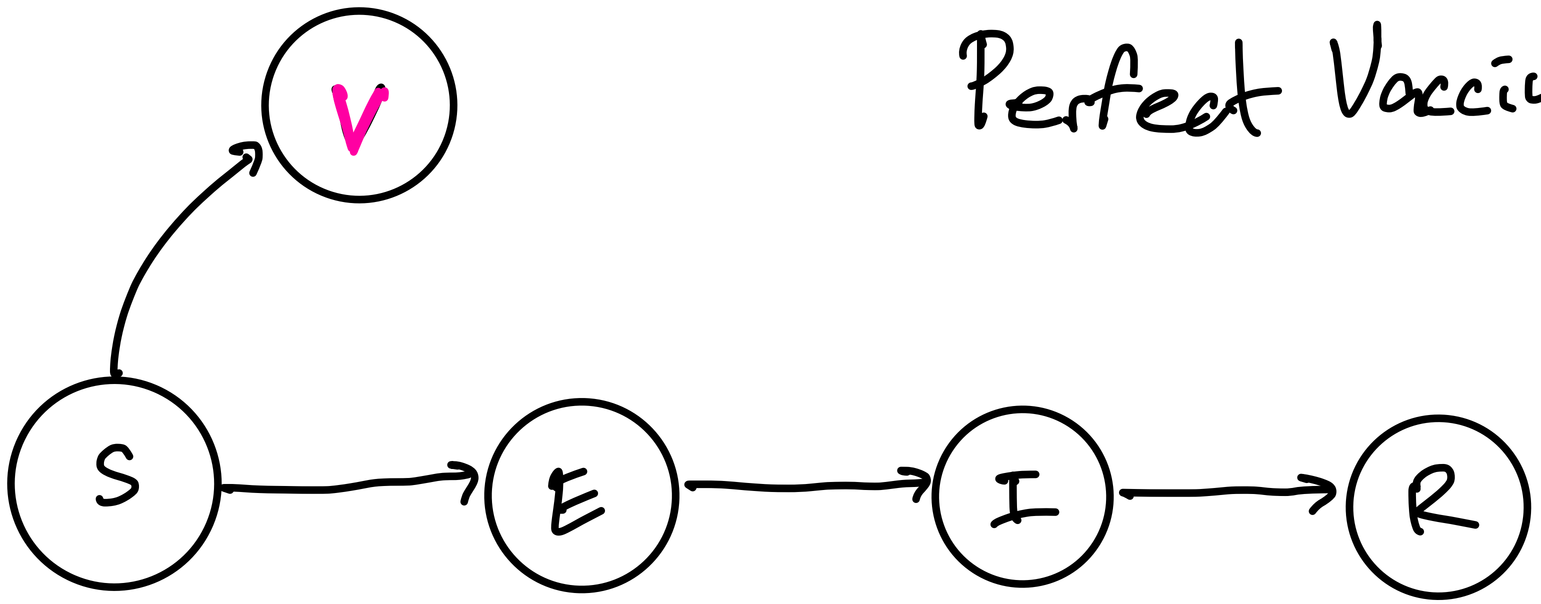
$$\dot{E} = \beta SI - \alpha E$$

$$\dot{I} = \alpha E - \gamma I$$

$$\dot{R} = \gamma I$$

where $S + E + I + R \overset{+V}{=} 1$

Perfect Vaccine Model

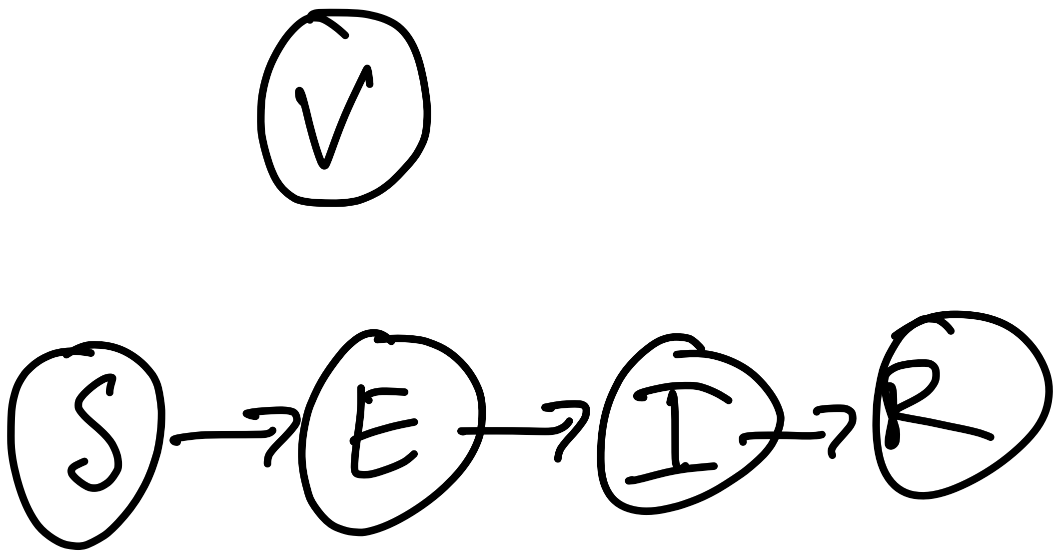


vaccination makes people not-susceptible.

1. Prevents infection – protects you.
2. Prevents you from spreading virus – protects others.
3. Prevents morbidity + mortality – protects you.

Model 1: The Perfect Vaccine model

A **perfect vaccine** provides complete protection against infection.



$$\dot{S} = -\beta S I$$

$$\dot{E} = \beta S I - \alpha E$$

$$\dot{I} = \alpha E - \gamma I$$

$$\dot{R} = \gamma I$$

$$\dot{V} = 0$$

$$S + E + I + R + V = 1$$

Model 1: The Perfect Vaccine model

A **perfect vaccine** provides complete protection against infection.

$$\dot{S} = -\beta SI$$

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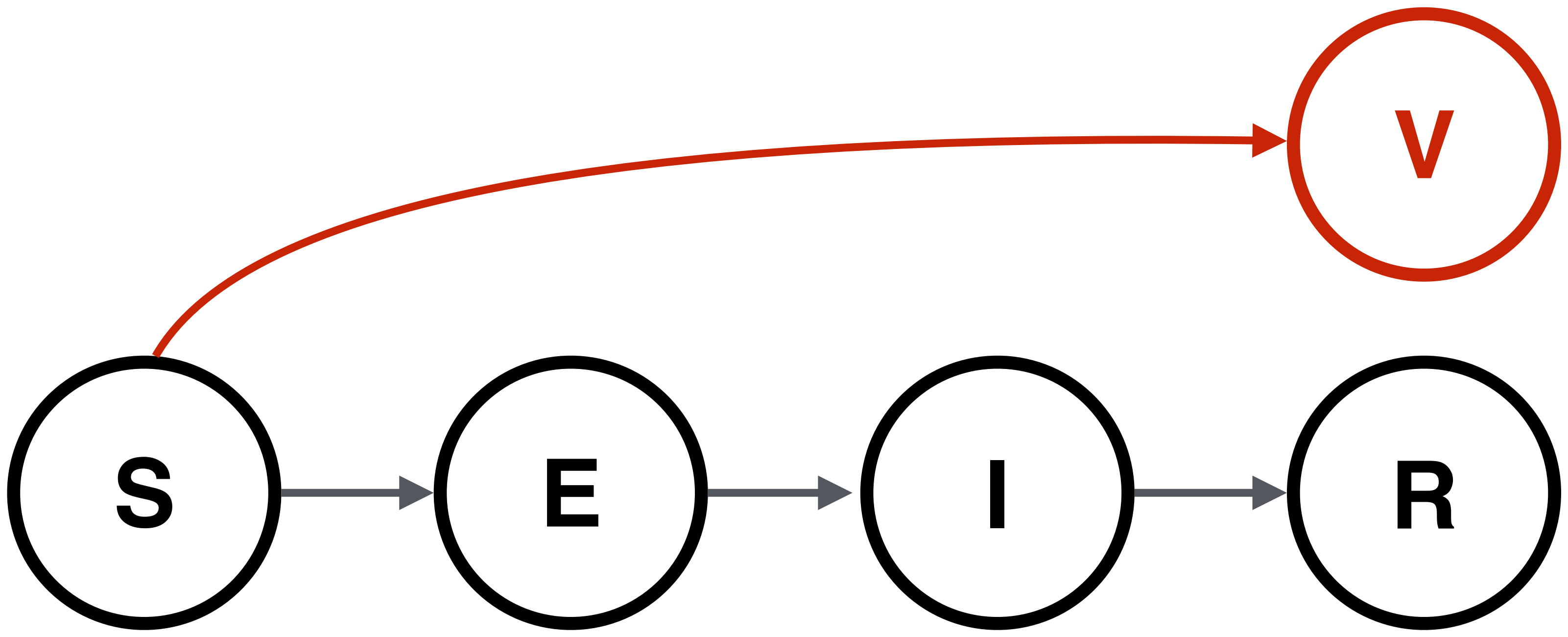


This is a model for a vaccine with $VE = 1$.
What does this mean?

What about vaccines with imperfect protection?

Vaccine efficacy (VE) is the reduction in disease outcomes in a vaccinated group compared to an unvaccinated group under trial conditions.

Vaccine effectiveness is the ability of the vaccine to prevent those disease outcomes in the real world.

1 J+J → J+J ~4x
→ Pfizer ~17x
→ Moderna ~36x

↑ in antibody levels after vax #2.

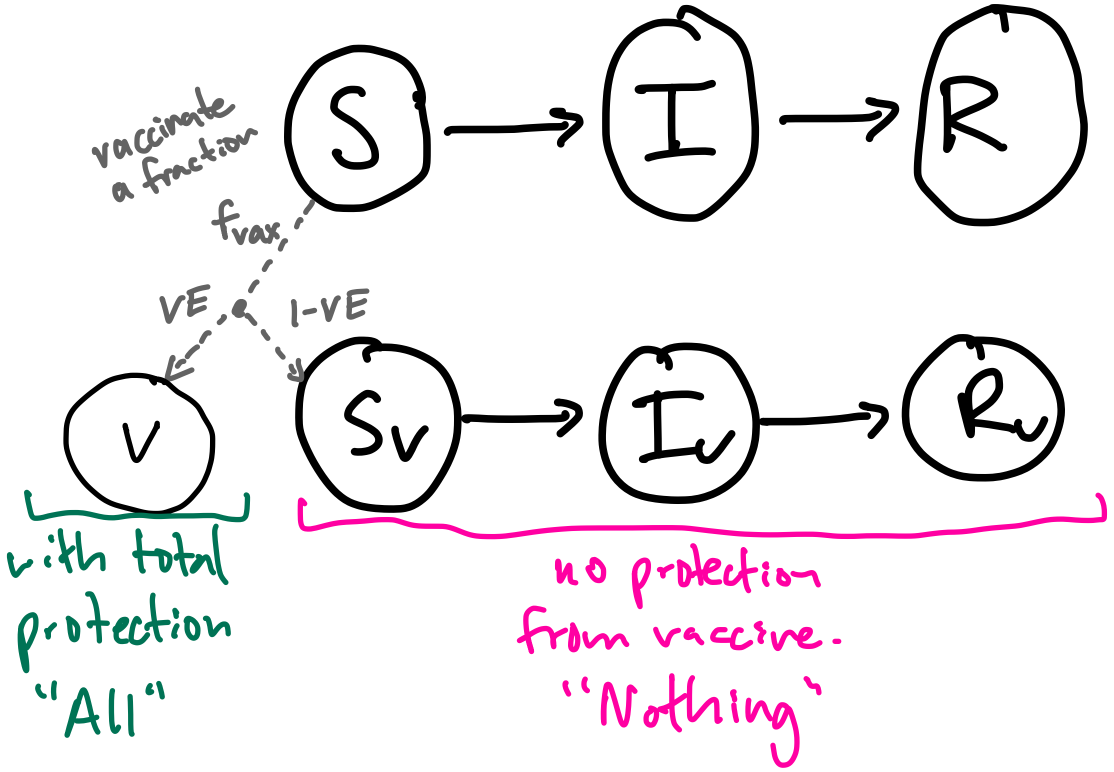
What are some of the determinants of VE?

- strength of immune response to the vaccine (antibody titers, neutralization assays)
- specificity of immune response. (vax is against WildType SARS-CoV2 spike, but circulating variant is different.)
- individual effects – e.g. age – immunosenescence (immune sleepiness)

Model 2: The All-or-Nothing vaccine model

new SIR

An **all-or-nothing** vaccine completely protects VE and leaves $1 - VE$ unprotected.



$$\dot{S} = -\beta S I - \beta S I_v = -\beta S(I + I_v)$$

$$\dot{I} = \beta S(I + I_v) - \gamma I$$

$$\dot{R} = \gamma I$$

$$\dot{S}_v = -\beta S_v I_v - \beta S_v I = -\beta S_v(I + I_v)$$

$$\dot{I}_v = \beta S_v(I + I_v) - \gamma I_v$$

$$\dot{R}_v = \gamma I_v$$

$$\dot{V} = 0$$

[All of the vaccination takes place as an initial condition (Assumed).]