Homework 2

CSCI 2897 - Calculating Biological Quantities - Larremore - Fall 2021

Notes: Remember to (1) familiarize yourself with the collaboration policies posted on the Syllabus, and (2) turn in your homework to Canvas as a **single PDF**. Hand-writing some or most of your solutions is fine, but be sure to scan and PDF everything into a single document. Unsure how? Ask on Slack!

Pushups

Calculate these indefinite integrals. Don't forget your constant!

1. $\int x^2 \, dx =$

2. $\int x^{-2} \, dx =$

3. $\int e^{2\pi x} \, dx =$

4. $\int \sin x \, dx =$

5. $\int \sin x \, \csc x \, dx =$

Squats

For each family of solutions below, use the initial condition to solve for the unknown constant α, and then write the solution with the solved-for constant plugged in and simplified.

6. $y(t) = \alpha e^{3t}, \quad y(0) = 10$

7. $y(t) = \alpha e^{t/2}, \quad y(6) = e$

8. $n(t) = \dfrac{K}{1 + \alpha K e^{-rt}}, \quad n(0) = 1$

9. $n(t) = \dfrac{K}{1 + \alpha K e^{-rt}}, \quad n(0) = K$

10. $y(x) = \dfrac{1}{x^2 + \alpha}, \quad y(2) = \dfrac{1}{3}$

Stay Hydrated

Torricelli's Law explains how fast water flows out of a tank. Imagine a cylindrical tank, oriented with the flat parts of the cylinder on bottom and top, in which the water is filled up to height h meters (m), and suppose there is a hole in the bottom of the tank. Suppose that the tank has cross sectional area A_T and the hole has cross sectional area A_H, both in units m^2. The top of the tank is open. Torricelli's law tells us that the *speed of the water* leaving the tank is $v = \sqrt{2gh}$ m/s, where g is a constant and h is, again, the height of the water in the tank.

But there is also an open faucet above the tank, and water is flowing into the top of the tank at a constant rate of $\frac{1}{10}$ m³/s. In other words, water is flowing in at a constant rate, and water is flowing out at a rate that depends on Torricelli's law.

Our goal is to be able to describe (1) the volume of water $V(t)$ in the tank, and (2) the height of the water $h(t)$ in the tank.

11. If water flows out according to Torricelli's law, and the area of the hole is A_H, what is the *volume* of water flowing out of the tank per second?[1]

12. If the tank is cylindrical, the height of the water is h, and the tank's cross sectional area is A_T, what is the volume of water in the tank as a function of the height of water h?

13. Draw a picture of the tank, the faucet above, and the flow out the bottom. Label the flows in and out, the height of the water in the tank, and anything else you know. *Next to* your drawing, draw a *flow diagram* for the volume of water in the tank V.

14. Write a differential equation for the rate of change of the volume of water in the tank with respect to time, dV/dt.

15. Use this to write a differential equation for the rate of change of the *height* of water in the tank with respect to time, dh/dt.

16. Find an equilibrium solution for the height of water in the tank and explain your results in words.

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[1]Hint: volumetric flow has units of m³/s. Area has units m². Speed has units m/s.

Separation of Variables

Classify each equation as separable or not separable. Then, for separable equations, separate the variables *but do not integrate.*

17. $t\dfrac{dy}{dt} = 4y$

18. $t\dfrac{dy}{dt} = 4 + t$

19. $y \ln t \dfrac{dy}{dt} = \left(\dfrac{y+1}{t}\right)^2$

20. $e^t + e^y = \dfrac{dy}{dt}$

21. $e^{t+y} = \dfrac{dy}{dt}$

22. Your equation from Question 15.

Solve these initial value problems by using Separation of Variables.[2]

23. $\dfrac{dy}{dt} = \dfrac{y-1}{t-1}, \quad y(5) = 2$

24. $t^2 \dfrac{dy}{dt} = y - ty, \quad y(-1) = -1$

25. $\dfrac{dy}{dt} = \dfrac{2t+1}{2y}, \quad y(-2) = 1$

Extra Credit. Find a solution of $t\dfrac{dy}{dt} = y^2 - y$ that passes through $(t, y) = (\frac{1}{2}, \frac{1}{2})$

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[2]Hint: (1) separate, (2) integrate, (3) don't forget that constant of integration, (4) plug in the initial values to solve for the constant, (5) put it all together.