Midterm Exam

CSCI 2897 - Calculating Biological Quantities - Larremore - Fall 2021

Instructions

- This exam needs to be taken live, while you're on Zoom for class *or* sitting in the classroom.
- This exam is open note and open textbook.
- Collaboration with others in the class *or* posting this exam to any online service during the exam will be considered cheating. Complete this exam entirely on your own.
- If you have questions, please DM me on Zoom or raise your hand..
- Your completed exam should be submitted at the end of class via Canvas or handed in on paper.
- The Canvas submission window will close at the end of class +10 minutes, i.e. 1:05 P.M.
- Please show your work for the math. One way to do this would be to write on a tablet, for Zoomers. Another way would be to write on paper and then snap a photo. You can drag the photo into MS Word, for instance.

Name (Printed) :

Honor Code: "On my honor, as a University of Colorado Boulder student I have neither given nor received unauthorized assistance."

By signing my name here, I commit to the Honor Code above:

Efrain's of Boulder [50]

Efrain's of Boulder is undoubtedly the best restaurant in Boulder. Why? Because the food is delicious, the prices are low, the margaritas are strong, and the green chile is *hot*. We like Efrain's.

You are, for the purposes of this exam problem, a chile farmer from Pueblo, Colorado. It is your sacred duty to provide green chiles to the team at Efrain's, and you take your job very seriously. In addition to being a chile farmer, you have taken CSCI 2897, and you set about modeling the growth of your chile crop.

In your first season farming, you conduct numerous experiments and find that, without any harvesting, the growth of your chile crop's mass $\dot{m}(t)$, measured in kilograms per week, follows this differential equation:

$$\dot{m}(t) = \alpha m \left(1 - \frac{m}{\beta}\right)$$

1. (5 points) Name this differential equation and explain what the constants α and β mean.

2. (5 points) Suppose that you were trying to decide between (a) expanding your farm, (b) buying fertilizer, (c) planting a faster-growing variety of chile, or (d) planting a variety of chile that uses fewer resources. Which of these potential changes might be modeled as a change to α and which might be modeled as a change to β? Explain your reasoning for each of the four options.

3. (5 points) You measure $\alpha = 2$ and $\beta = 100$, and plant $m(0) = 1$ kg to start with. According to your model, what will be the steady-state amount of chiles on your farm? Show your work.

Your second season as a farmer has arrived, and with it, great opportunities to use your CSCI 2897 skills for good. With your model for crop growth established, it's time to make sure Efrain's has the chiles they need to keep being Boulder's #1 restaurant. That means it's time to include *harvesting* in your model.

4. (5 points) Write down a new ODE (based on the old one) that includes harvesting chiles at a constant rate of 42 kg per week.

5. (5 points) Choose three of the following options to describe your new ODE with harvesting.
   - A. first order
   - B. second order
   - C. linear
   - D. nonlinear
   - E. separable
   - F. not separable

6. (10 points) What are the two equilibria for your ODE with harvesting? How can there be two equilibria like this, biologically speaking?

7. (5 points) Critique this model. List at least two reasons why it might be a poor description of an actual agricultural crop. Give an example of a different biological system that might be better modeled using an equation like the one you wrote down in problem 4.

8. (10 points) Using the sign of $\dot{m}$ for various choices of m, explain whether each of your equilibria from problem 6 is stable or unstable. What does this tell you about when you should begin harvesting?

Short Answers [20]

Answer the following in complete sentences.

9. (10 points) Consider the Lotka-Volterra model for competition between species 1 and 2 that we discussed in class.

$$\frac{dn_1}{dt} = n_1\left(1 - \frac{n_1 + \alpha_{12}n_2}{K_1}\right)$$
$$\frac{dn_2}{dt} = n_2\left(1 - \frac{n_2 + \alpha_{21}n_1}{K_2}\right)$$

Explain what α_{21} means. How would setting $\alpha_{21} = 10$ affect the population of species 2?

10. (10 points) Suppose that you are studying levels of lipids (fats) in blood samples from humans after they eat a meal. Would a discrete-time or continuous-time model be more appropriate, and why? What might be an appropriate timescale for your model?

Math and Modeling [30+]

11. (10 points) When modeling the spread of an infectious disease in class, we assumed a constant population. What if the population could change? Draw a flow diagram for the spread of an infectious disease in a population where people are born at a constant rate b, susceptible and recovered people die at a per-capita rate d, and infected people die at a per-capita rate $2d$. Assume that babies are born susceptible. Then use your diagram to write differential equations for $\dot{S}$, $\dot{I}$, and $\dot{R}$.

12. (10 points) Showing all your work, solve the following initial value problem using the integrating factor method.

$$3\frac{dy}{dx} = x^2y + x^2, \qquad y(0) = 2.$$

13. (10 points) Explain what it means for a function to "solve" a differential equation.

14. (Extra Credit) Show that your answer in Problem 12 solves the differential equation you set out to solve.