Calculating Biological Quantities

CSCI 2897

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2021, Lecture 13

- Dan is slow @ grading → Exams Thursday
- Next HW due 2 weeks from this Thursday.

Model 1: The Perfect Vaccine model

A **perfect vaccine** provides complete protection against infection.

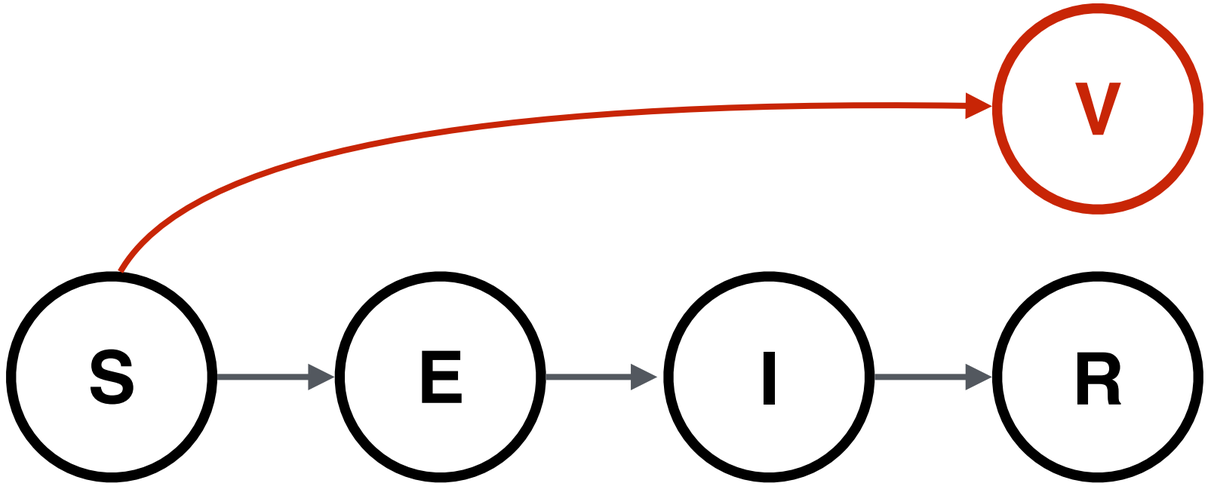
$$\dot{S} = -\beta SI$$

$$\dot{E} = \beta SI - \alpha E$$

$$\dot{I} = \alpha E - \gamma I$$

$$\dot{R} = \gamma I$$

where $S + E + I + R + V = 1$



This is a model for a vaccine with $VE = 1$.
What does this mean?

What about vaccines with imperfect protection?

Vaccine efficacy (VE) is the reduction in disease outcomes in a vaccinated group compared to an unvaccinated group under trial conditions.

Vaccine effectiveness is the ability of the vaccine to prevent those disease outcomes in the real world.

1 J+J → J+J ~4x
→ Pfizer ~17x
→ Moderna ~36x

↑ in antibody levels after vax #2.

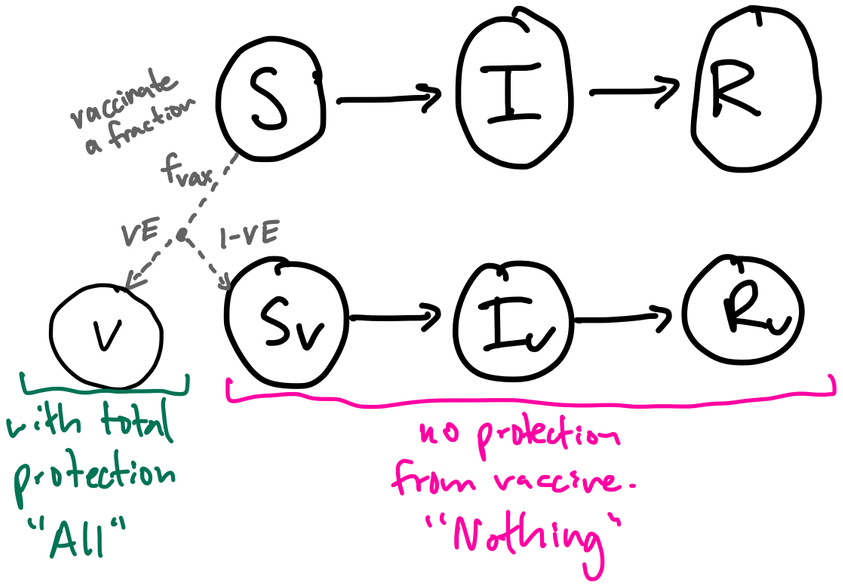
What are some of the determinants of VE?

- strength of immune response to the vaccine (antibody titers, neutralization assays)
- specificity of immune response. (vax is against WildType SARS-CoV2 spike, but circulating variant is different.)
- individual effects – e.g. age – immunosenescence (immune sleepiness)

Model 2: The All-or-Nothing vaccine model

new SIR

An **all-or-nothing** vaccine completely protects VE and leaves $1 - VE$ unprotected.



[All of the vaccination takes place as an initial condition (Assumed).]

$$\dot{S} = -\beta S I - \beta S I_v = -\beta S(I + I_v)$$

$$\dot{I} = \beta S(I + I_v) - \gamma I$$

$$\dot{R} = \gamma I$$

$$\dot{S}_v = -\beta S_v I_v - \beta S_v I = -\beta S_v(I + I_v)$$

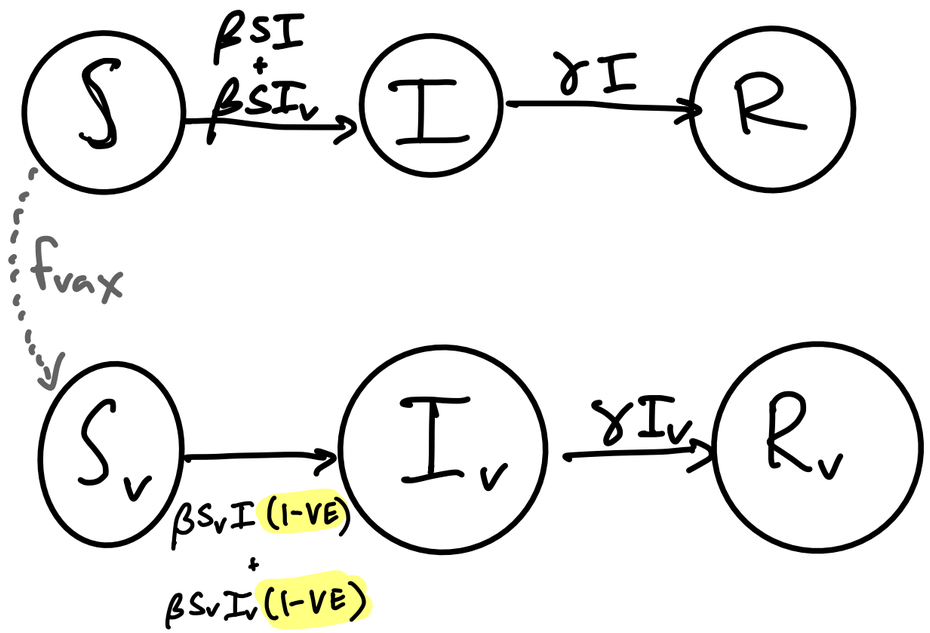
$$\dot{I}_v = \beta S_v(I + I_v) - \gamma I_v$$

$$\dot{R}_v = \gamma I_v$$

$$\dot{V} = 0$$

Model 3: The Leaky Vaccine model

A **leaky** vaccine provides *ve* partial protection to everyone.



$$\dot{S} = -\beta S(I_v + I)$$

$$\dot{I} = \beta S(I_v + I) - \gamma I$$

$$\dot{R} = \gamma I$$

$$\dot{S}_v = -\beta S_v(I_v + I)(1-VE)$$

$$\dot{I}_v = \beta S_v(I_v + I)(1-VE) - \gamma I_v$$

$$\dot{R}_v = \gamma I_v$$

Test: If you add a model feature w/ parameter value (here: VE), setting parameter to 0 or 1 (or whatever is appropriate) should recover "known" simpler behavior.

$0 \le VE \le 1$

↑ no protection ↑ complete protection

Model 4: The Three-Factor Vaccine model

IFR = Infection Fatality Rate

≠

CFR = Case Fatality Rate

A three-factor vaccine considers ve_s, ve_I and ve_p (ve_D)...

- ve_s: reduced susceptibility
- ve_I: reduced infectiousness (to others)
- ve_p: reduced risk of disease (protective effects)

$$\dot{S} = -\beta S\left(I + I_v[1-VE_I]\right)$$

$$\dot{I} = \beta S\left(I + I_v[1-VE_I]\right) - \gamma I$$

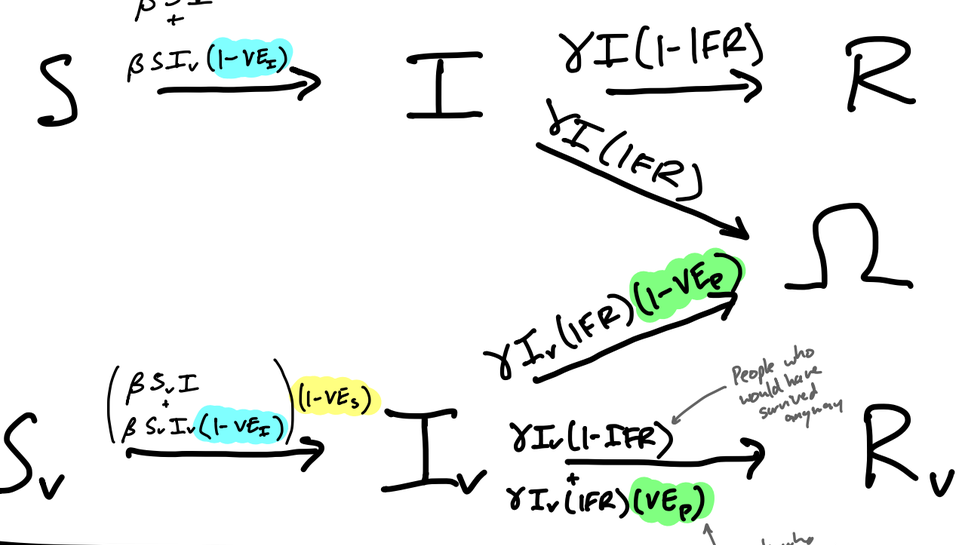
$$\dot{R} = \gamma I(1-IFR)$$

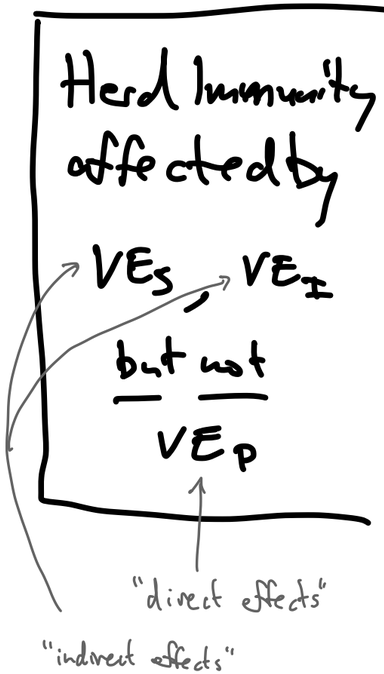
$$\dot{S}_v = -\beta S_v\left(I + I_v[1-VE_I]\right)(1-VE_S)$$

$$\dot{I}_v = \beta S_v\left(I + I_v[1-VE_I]\right)(1-VE_S) - \gamma I_v$$

$$\dot{R}_v = \gamma I_v\left[(1-IFR) + (IFR)VE_p\right]$$

$$\dot{\Omega} = \gamma I(IFR) + \gamma I_v(IFR)(1-VE_p)$$





$$VE_{trial} = 1 - (1-VE_S)(1-VE_p)$$

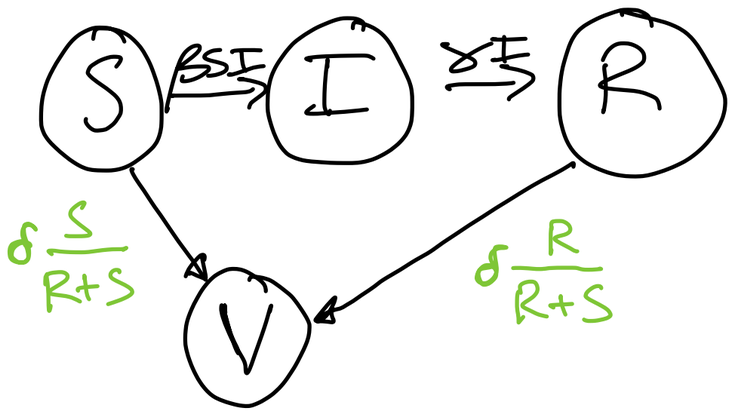
$Pr(\text{protected}) = Pr(\text{not not protected against both infection } (1-VE_S) \text{ AND severe disease } (1-VE_p))$

Initial conditions or vaccine rollout?

Key Question: Is vaccination happening at the same time as transmission?

Yes ↓

No ↓



Continuous Rollout

- COVID-19 Vax in U.S.
- Polio vaccine.
- Reactive Vax Campaign (outbreak → vaccinate)
- Ebola
- Flu shots (mid flu season)

Initial Condition

- childhood vaccines
- COVID-19 Vax in New Zealand.
- Flu shots (before flu season)

$$\dot{S} = -\beta SI - \delta \frac{S}{R+S}$$

$$\dot{I} = \beta SI - \gamma I$$

$$\dot{R} = \gamma I - \delta \frac{R}{R+S}$$

$$\dot{V} = \delta \frac{R}{R+S} + \delta \frac{S}{R+S}$$

$$= \delta \frac{1}{R+S}(R+S)$$

$$= \delta$$

The durability of immunity: SIRS model

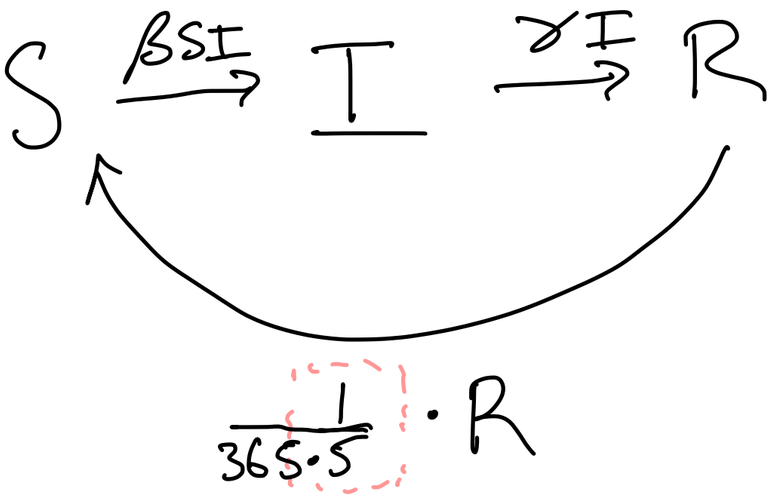
Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

\xi

$$\dot{S} = -\beta SI + \xi R$$

$$\dot{I} = \beta SI - \gamma I$$

$$\dot{R} = \gamma I - \xi R$$



If rate = ξ

typical time to make transition = $\frac{1}{\xi}$

I want typical time till R → S to be 5 years.

5 · 365 days

$5 = \frac{1}{\xi}$ so $\xi = \frac{1}{5}$

$\xi = \frac{1}{5 \cdot 365}$

The durability of immunity: SIRS model

Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

Point #1: Constant per-capita outflows are exponential.

If no I.

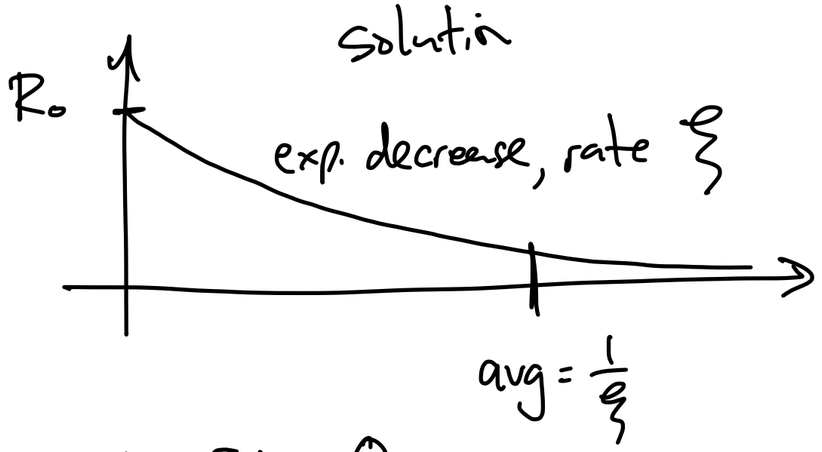
$$R(0) = R_0$$

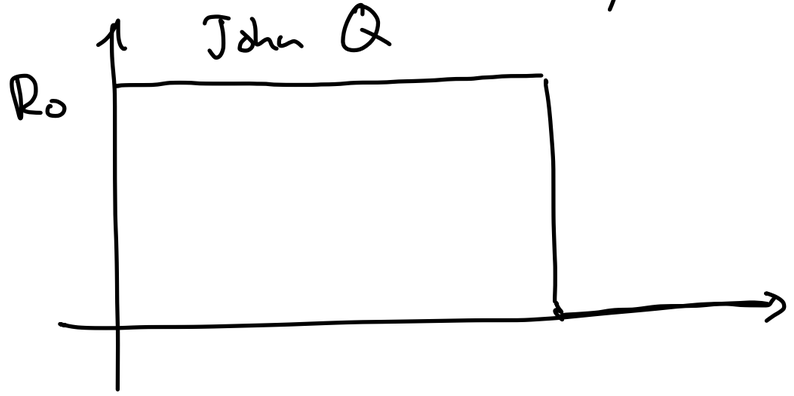
$$\dot{R} = \gamma I - \xi R \qquad (I \to 0)$$

$$\dot{R} = -\xi R$$

s.v

$$R(t) = R_0 e^{-\xi t}$$





The durability of immunity: SIRS model

Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

Point #1: Constant per-capita outflows are exponential.
Point #2: Typical waiting time = 1 / exponential rate.

The durability of immunity: SIRS model

Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

$$\dot{S} = -\beta SI + \frac{1}{5 \cdot 365} R$$

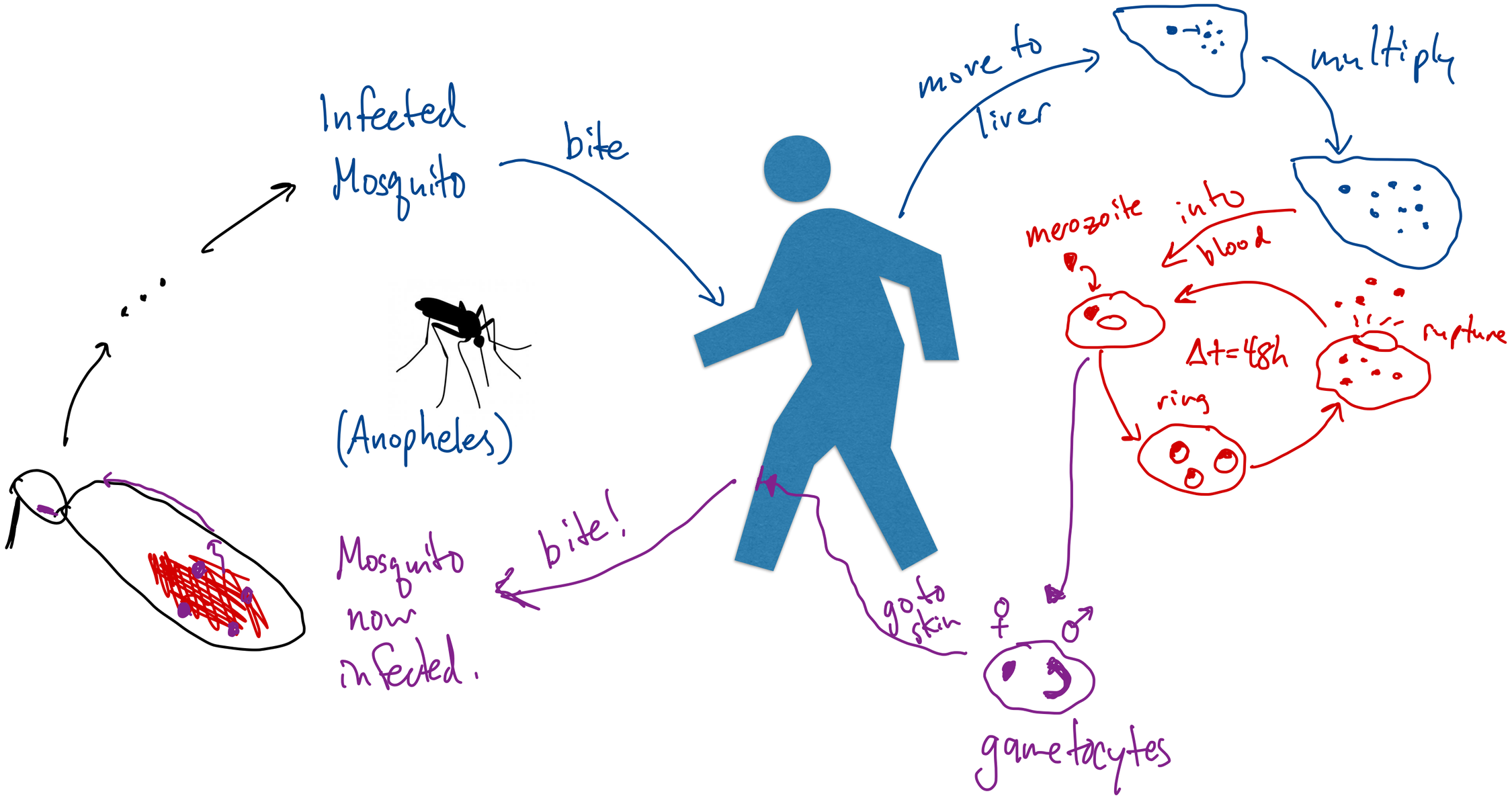
$$\dot{I} = \alpha E - \gamma I$$

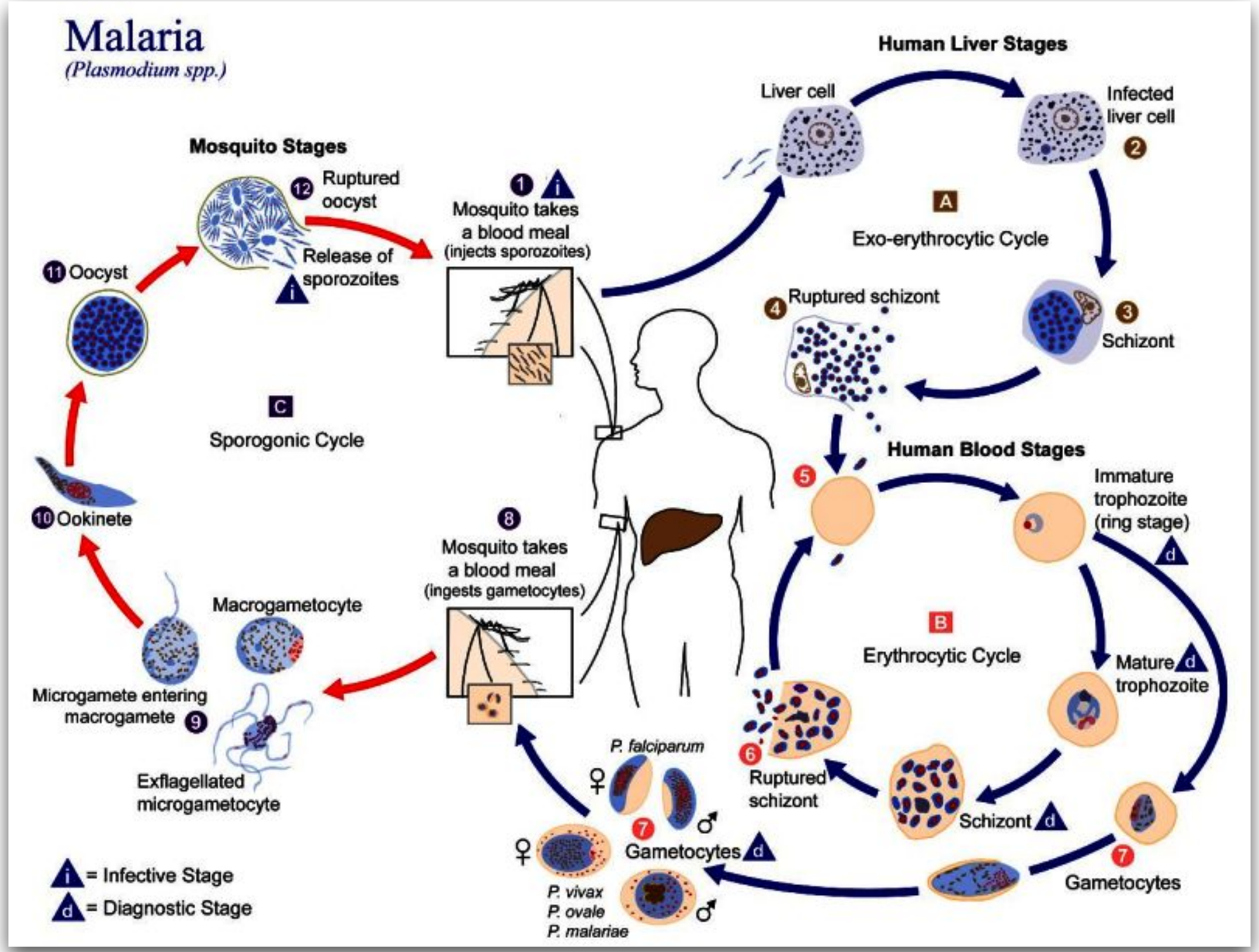
$$\dot{R} = \gamma I - \frac{1}{5 \cdot 365} R$$

where $S + I + R = 1$

The Malaria Parasite Life Cycle

vector: other party that facilitates transmission.




Malaria
(Plasmodium spp.)
Human Liver Stages
Liver cell
Infected liver cell
2
A
Exo-erythrocytic Cycle
3
Schizont
4
Ruptured schizont
Human Blood Stages
5
Immature trophozoite (ring stage)
d
Mature trophozoite
B
Erythrocytic Cycle
6
Ruptured schizont
Schizont
d
7
Gametocytes
P. falciparum
P. vivax
P. ovale
P. malariae
Gametocytes
d
8
Mosquito takes a blood meal (ingests gametocytes)
Macrogametocyte
Microgamete entering macrogamete
9
Exflagellated microgametocyte
10
Ookinete
11
Oocyst
Mosquito Stages
12
Ruptured oocyst
Release of sporozoites
1
i
Mosquito takes a blood meal (injects sporozoites)
C
Sporogonic Cycle
i = Infective Stage
d = Diagnostic Stage

NIH

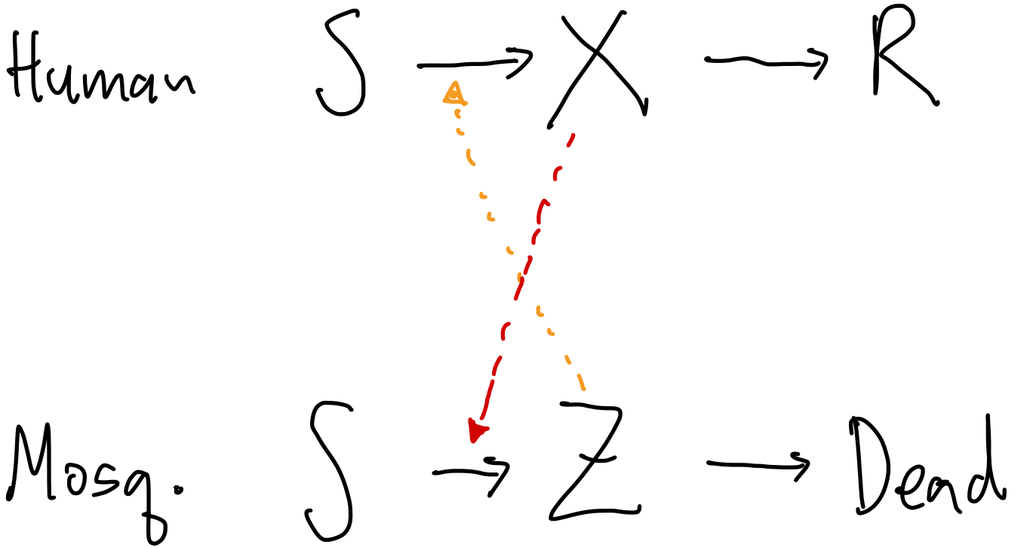
Ross & MacDonald - Malaria

contemporaries of Lotka

Factors that affect the rates of transition from $S \to X$, $S \to Z$

Two Infected Populations!

field survey → $x = \frac{X}{H}$, $z = \frac{Z}{M}$



- What fraction of humans are in X?
- What fraction of mosquitoes are in Z?
- Biting rate a – proportion of mosquitoes that bite a human each day
- Effectiveness of each infectious bite

lab! **c** · to acquire parasites (infect mosquito)

b · to unload parasites (infect human)

$m = \frac{M}{H}$ mosquito to human ratio

Let H and M be the population densities of humans, mosquitoes, respectively.

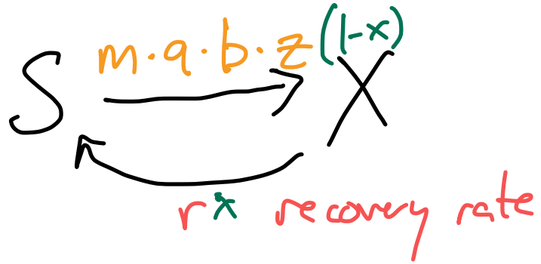
Ross & MacDonald - Malaria

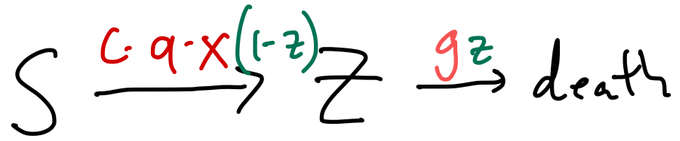
EIR = entomological inoculate rate
$= m \cdot a \cdot z$

prevalence x (humans)

z (mosquitoes)

m (mosquito-to-human ratio)





$\Pr(\text{mosquito infected} \mid \text{bit an infected human}) = c$

$\Pr(\text{human infected} \mid \text{bitten by infected mosq.}) = b$

Proportion of mosq. that bite per day $= a$

$$\dot{x} = mabz(1-x) - rx$$

$$\dot{z} = cax(1-z) - gz$$

- system of coupled ODEs.
- tracking only infected animals.

Ross & MacDonald - Malaria

$$\dot{x} = mabz(1-x) - rx - qx$$

$$\dot{z} = acx(1-z) - gz$$

$x = \frac{X}{H}$ prevalence in humans

$z = \frac{Z}{M}$ prevalence in mosquitoes

$m = \frac{M}{H}$ mosquito-to-human ratio

a = Proportion of mosquitoes that feed on humans per day

b = Proportion of infectious mosquito bites that infect a human

c = Probability that a mosquito becomes infected after biting an infected human.

g = mosquito death rate

r = human recovery rate

Bednets: reduce contact rate. ↓ a

Mass Drug Administration: recovery at rate q.