Calculating Biological Quantities

CSCI 2897

Prof. Daniel Larremore

2021, Lecture 13

daniel.larremore@colorado.edu

@danlarremore

- Dan is slow @ grading → Exams Thursday
- Next HW due 2 weeks from <u>this</u> Thursday.

Model 1: The Perfect Vaccine model

A **perfect vaccine** provides complete protection against infection.

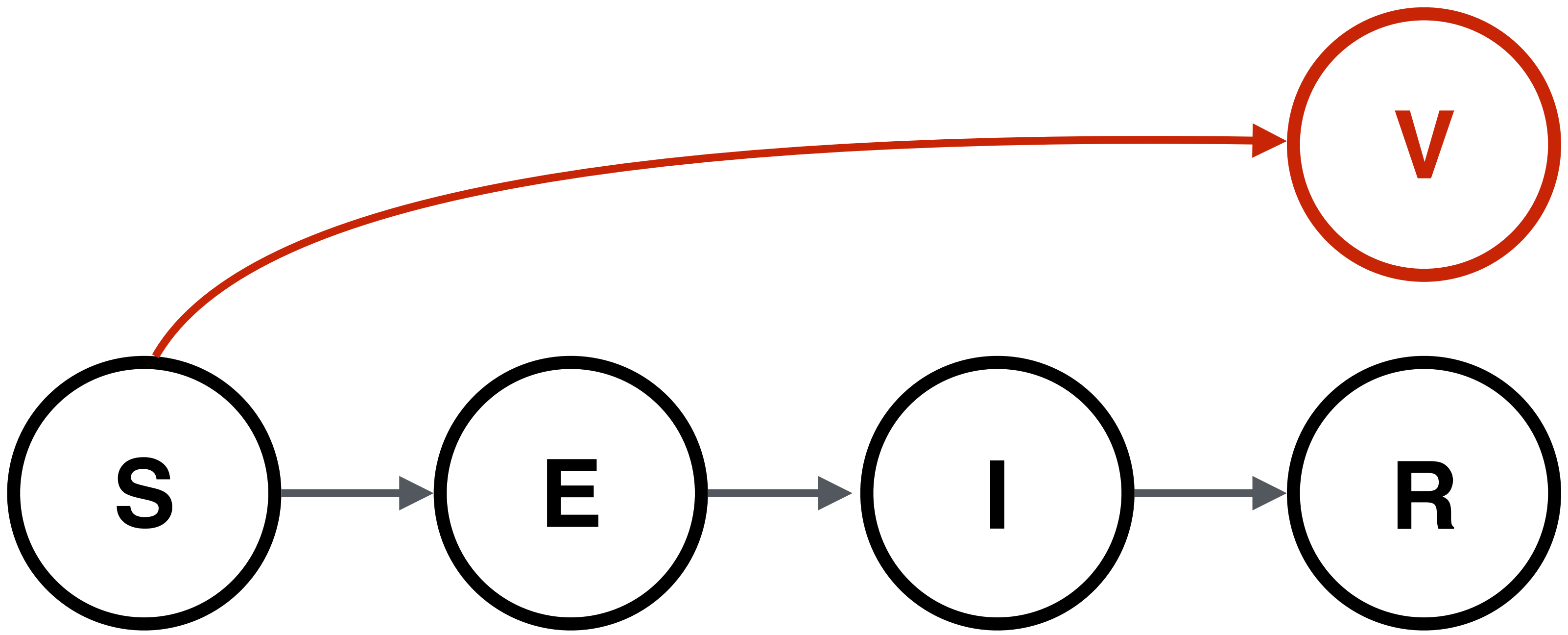
$$\dot{S} = -\beta S I$$

$$\dot{E} = \beta S I - \alpha E$$

$$\dot{I} = \alpha E - \gamma I$$

$$\dot{R} = \gamma I$$

where $S + E + I + R + V = 1$



This is a model for a vaccine with $VE = 1$.
What does this mean?

What about vaccines with imperfect protection?

Vaccine efficacy (VE) is the reduction in disease outcomes in a vaccinated group compared to an unvaccinated group under trial conditions.

Vaccine effectiveness is the ability of the vaccine to prevent those disease outcomes in the real world.

1 J+J → J+J ~4x
→ Pfizer ~17x
→ Moderna ~36x

↑ in antibody levels after vax #2.

What are some of the determinants of VE?

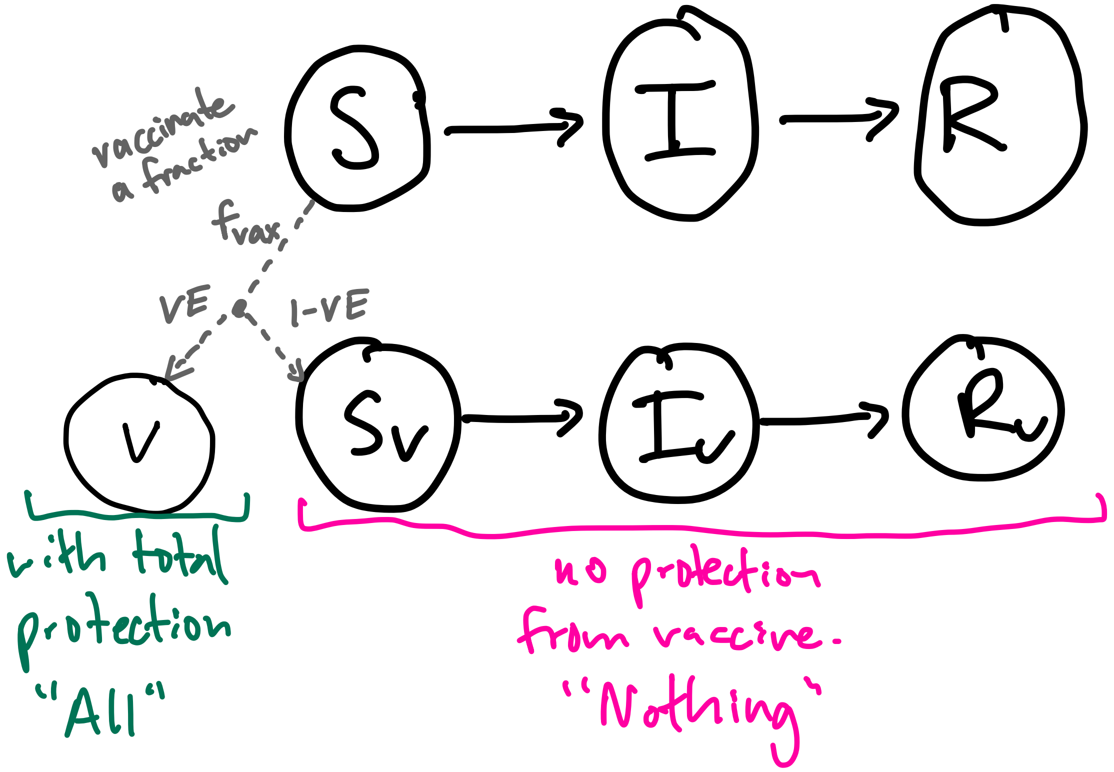
- strength of immune response to the vaccine (antibody titers, neutralization assays)
- specificity of immune response. (vax is against WildType SARS-CoV2 spike, but circulating variant is different.)
- individual effects – e.g. age – immunosenescence

immune sleepiness

Model 2: The All-or-Nothing vaccine model

new SIR

An **all-or-nothing** vaccine completely protects VE and leaves $1 - VE$ unprotected.



$$\dot S = -\beta S I - \beta S I_v = -\beta S(I+I_v)$$

$$\dot I = \beta S(I+I_v) - \gamma I$$

$$\dot R = \gamma I$$

$$\dot S_v = -\beta S_v I_v - \beta S_v I = -\beta S_v(I+I_v)$$

$$\dot I_v = \beta S_v(I+I_v) - \gamma I_v$$

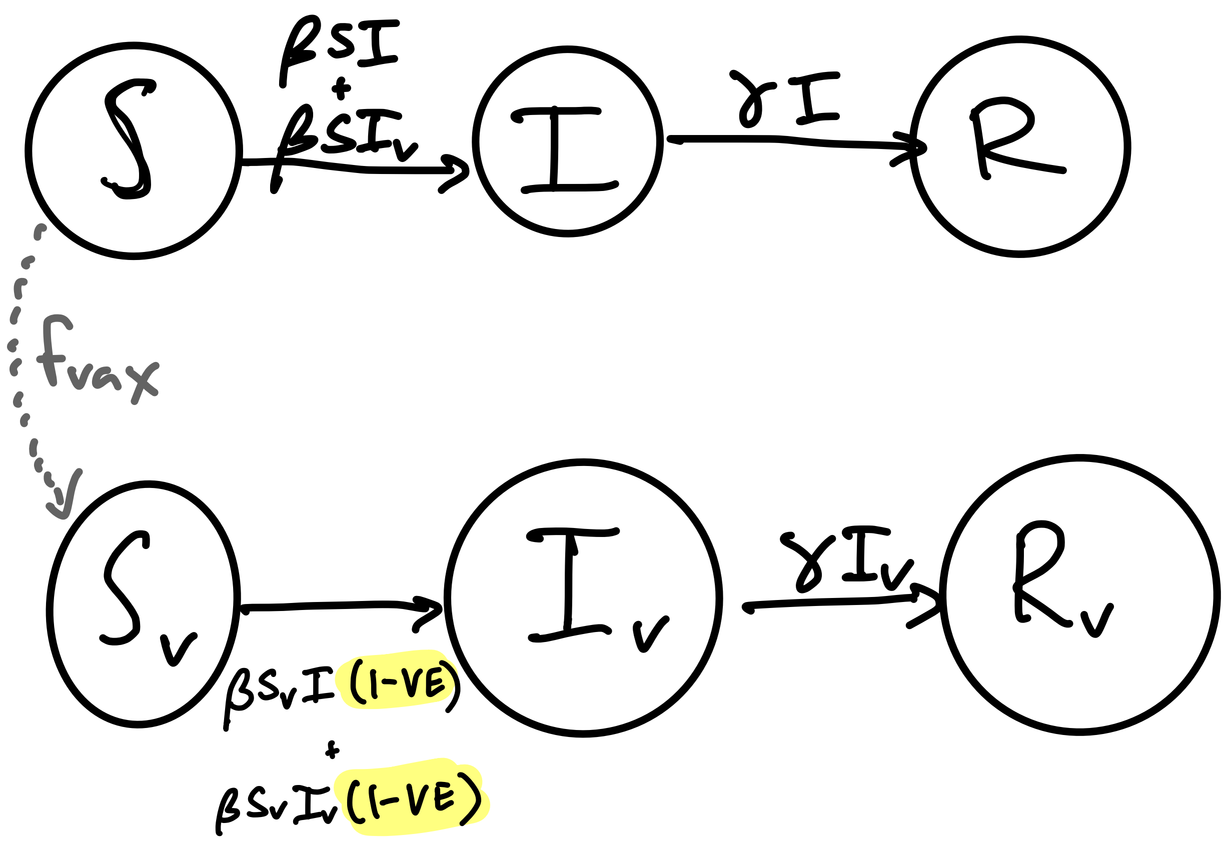
$$\dot R_v = \gamma I_v$$

$$\dot V = 0$$

[All of the vaccination takes place as an initial condition (Assumed).]

Model 3: The Leaky Vaccine model

A **leaky** vaccine provides *ve* partial protection to everyone.



$$\dot{S} = -\beta S(I_v + I)$$

$$\dot{I} = \beta S(I_v + I) - \gamma I$$

$$\dot{R} = \gamma I$$

$$\dot{S}_v = -\beta S_v(I_v + I)(1 - VE)$$

$$\dot{I}_v = \beta S_v(I_v + I)(1 - VE) - \gamma I_v$$

$$\dot{R}_v = \gamma I_v$$

$$0 \leq VE \leq 1$$

no protection

complete protection

Test: If you add a model feature w/ parameter value (here: VE), setting parameter to 0 or 1 (or whatever is appropriate) should recover "known" simple behavior.

Model 4: The Three-Factor Vaccine model

A three-factor vaccine considers ve_S, ve_I and ve_p... (ve_D)

reduced susceptibility

reduced infectiousness (to others)

reduced risk of disease (protective effects)

IFR = Infection Fatality Rate ≠ CFR = Case Fatality Rate

$$\dot S = -\beta S(I + I_v[1-VE_I])$$

$$\dot I = \beta S(I + I_v[1-VE_I]) - \gamma I$$

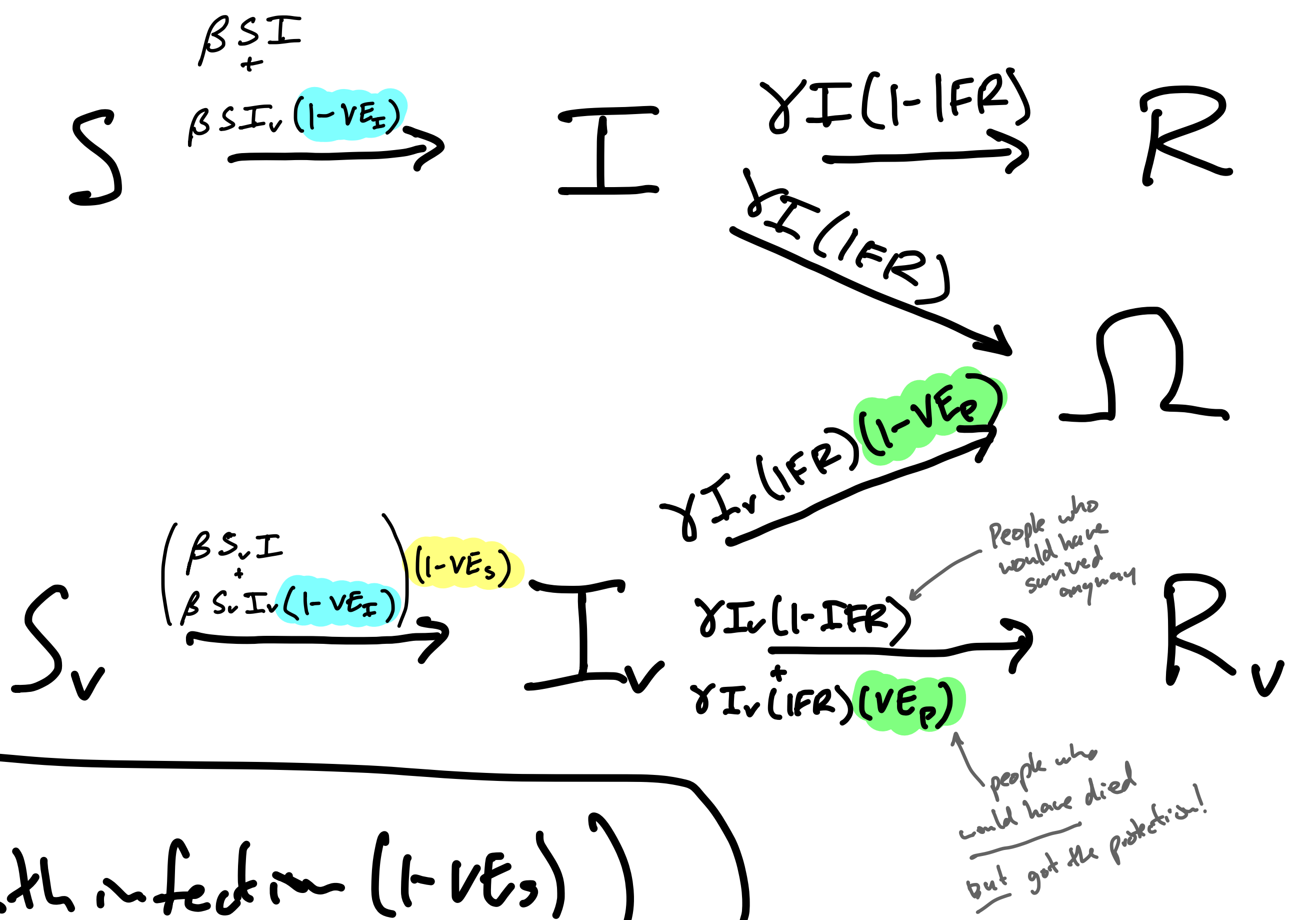
$$\dot R = \gamma I(1-IFR)$$

$$\dot S_v = -\beta S_v(I + I_v[1-VE_I])(1-VE_S)$$

$$\dot I_v = \beta S_v(I + I_v[1-VE_I])(1-VE_S) - \gamma I_v$$

$$\dot R_v = \gamma I_v[(1-IFR) + (IFR)VE_P]$$

$$\dot \Omega = \gamma I(IFR) + \gamma I_v(IFR)(1-VE_P)$$



Herd Immunity affected by VE_S, VE_I but not VE_P

"indirect effects"

"direct effects"

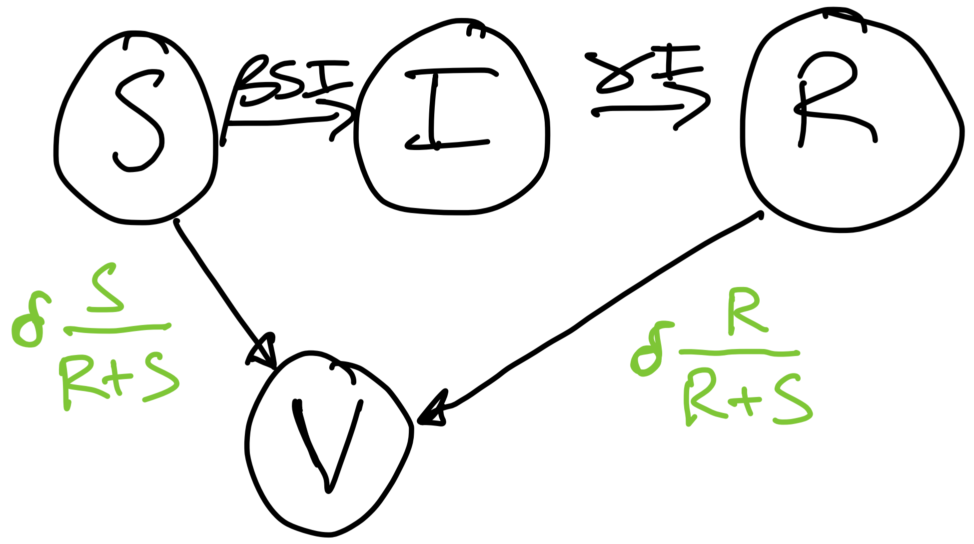
$$VE_{trial} = 1-(1-VE_S)(1-VE_P)$$

Pr(protected) = Pr(not not protected against both infection $(1-VE_S)$ AND severe disease $(1-VE_P)$)

Initial conditions or vaccine rollout?

Key Question: Is vaccination happening at the same time as transmission?

Yes ↙ ↘ No



Continuous Rollout

- COVID-19 Vax in U.S.
- Polio vaccine.
- Reactive Vax Campaign (outbreak → vaccinate)
- Ebola
- Flu shots (mid flu season)

Initial Condition

- childhood vaccines
- COVID-19 Vax in New Zealand.
- Flu shots (before flu season)

$$\dot{S} = -\beta S I - \delta \frac{S}{R+S}$$

$$\dot{I} = \beta S I - \gamma I$$

$$\dot{R} = \gamma I - \delta \frac{R}{R+S}$$

$$\dot{V} = \delta \frac{R}{R+S} + \delta \frac{S}{R+S}$$

$$= \delta \frac{1}{R+S} (R+S)$$

$$= \delta$$

The durability of immunity: SIRS model

Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

\xi

$$\dot{S} = -\beta S I + \xi R$$

$$\dot{I} = \beta S I - \gamma I$$

$$\dot{R} = \gamma I - \xi R$$

$$\begin{tikzcd} S \arrow[r, "\beta S I"] & I \arrow[r, "\gamma I"] & R \arrow[ll, bend left, "\frac{1}{365\cdot 5}\cdot R"] \end{tikzcd}$$

If rate $= \xi$

typical time to make transition $= \frac{1}{\xi}$

I want typical time till $R \to S$ to be 5 years. $5 = \frac{1}{\xi}$ so $\xi = \frac{1}{5}$

$5 \cdot 365$ days $\quad \xi = \frac{1}{5\cdot 365}$

The durability of immunity: SIRS model

Suppose that immunity lasts only 5 years, on average.
How can we model this scenario?

Point #1: Constant per-capita outflows are exponential.

If no I.

$$R(0) = R_0$$

$$\dot{R} = \gamma \cancel{I}^{\to 0} - \xi R$$

$$\dot{R} = -\xi R$$

S.V

$$\boxed{R(t) = R_0 e^{-\xi t}}$$