Homework 3

CSCI 2897 - Calculating Biological Quantities - Larremore - Fall 2021

Notes: Remember to (1) familiarize yourself with the collaboration policies posted on the Syllabus, and (2) turn in your homework to Canvas as a **single PDF**. Hand-writing some or most of your solutions is fine, but be sure to scan and PDF everything into a single document. Unsure how? Ask on Slack!

Lat pulldowns

1. Find the general solution[1] of
$$\frac{dy}{dx} = 3x^2 e^{-y}$$
2. Find a particular solution for problem 1 that satisfies $y(0) = 1$.
3. Find the general solution of
$$\frac{tz}{t+1} = \frac{dz}{dt}$$
4. Find the particular solution of
$$\frac{dn}{dt} = e^{2t+n}$$
that has $n = 0$ when $t = 0$.

Tricep extensions

For each 1st order ODE below, find the equilibrium solution or solutions! You can assume that any Greek letters, like α, β, γ are constants.

5. $\dot{p} = \alpha(1 - \frac{p}{\beta})p$
6. $\dot{q} = q^2 - q - \gamma$
7. $\dot{r_1} = \alpha\, r_1 r_2 + \beta\, r_1$
$\dot{r_2} = \gamma(1 - r_2)r_2$

[1] Recall that the general solution will contain a constant of integration.

SARS-CoV-2 Delta Variant and Selection

In these problems, our goal will be to learn more about our haploid model of natural selection. You may want to revisit Lecture 6. However, here, instead of talking about *two alleles* of a gene, we will instead be talking about *two groups of variants* of SARS-CoV-2: the delta variant and the non-delta variants.

Recall that the equation governing the *proportion* of the population of the variant is predicted by the model

$$\dot{p} = s_c p(t) \left(1 - p(t)\right) \ .$$

8. Solve the differential equation above to find a particular solution with the "generic" initial condition $p(0) = p_0$. (We often use a subscript 0 in this way, so that later, when we have an actual initial condition, we can just pop it right in! We pronounce the 0 subscript as "naught".)

9. Create a Jupyter notebook. Create a new function for your **solution** from the previous problem. Use this function to create two plots:

   (a) Plot these four curves for p (y-axis) vs t (x-axis). Let t range from 0 weeks to 32 weeks.

   - In black: $s_c = 1$, $p_0 = 0.01$
   - In purple: $s_c = 1$, $p_0 = 0.02$
   - In red: $s_c = 1$, $p_0 = 0.04$
   - In orange: $s_c = 1$, $p_0 = 0.08$

   (b) Plot these four curves for p (y-axis) vs t (x-axis). Let t range from 0 weeks to 32 weeks.

   - In grey: $s_c = 0.5$, $p_0 = 0.02$
   - In purple: $s_c = 1$, $p_0 = 0.02$
   - In blue: $s_c = 2$, $p_0 = 0.02$
   - In green: $s_c = 4$, $p_0 = 0.02$

10. Comment on how values of s_c and values of p_0 affect the shape of these curves.

11. Navigate to https://ourworldindata.org/grapher/covid-cases-delta. Look familiar? These are the shares of SARS-CoV-2 sequences that are the delta variant from various countries. If you want you may click the DOWNLOAD tab and then the blue button to get the full covid-cases-delta.csv datafile. However, I have already extracted the data from the USA for you into a separate CSV file, which you can find on the course GitHub. [This step is important, but there's nothing to grade here.]

12. Load the data from the CSV file into your Jupyter notebook.[2] Now create a **scatter plot**[3] of column Delta (p) vs column Week (t). Again, let p be on the $y-axis$ and t on the x-axis, with t ranging from 0 to 32 weeks. Be sure to label your axes.

13. Using your knowledge from problem 9, experiment with different values of p_0 and s_c to find values that fit your scattered data well. What are those values of p_0 and s_c that you found to fit the data best?

[2] There are many ways to do this, but a common way that data scientists use is through a package called pandas. Here is a tutorial: https://datatofish.com/import-csv-file-python-using-pandas/.

[3] You can find some examples at https://pythonspot.com/matplotlib-scatterplot/.

14. Produce a plot of (1) your scattered data in black, as in problem 12, with your best-fitting curve from problem 13 in red. Be sure to include a legend and axis labels.

15. **Extra credit.** Using the raw data file provided in Our World In Data's download link (problem 11), redo the process above for another country of your choice. The data file will have more than 32 weeks of data, so you may find it useful to cut and paste a 32-week selection into a separate CSV file, for convenience. Comment on similarities or differences between the USA's values of p_0 and s_c and this second country's values.