Problem Solving Workshop #6 Tech Interviews and Competitive Programming Meetup
May 7, 2016 https://www.meetup.com/tech-interviews-and-competitive-programming/
Instructor: Eugene Yarovoi (can be contacted through the group Meetup page above under Organizers)

More practice questions: leetcode.com, glassdoor.com, geeksforgeeks.org
Books: Elements of Programming Interviews, Cracking the Coding Interview
Have questions you want answered? Contact the instructor, or ask on Quora. You can post questions and follow the instructor and other people who write about algorithms.

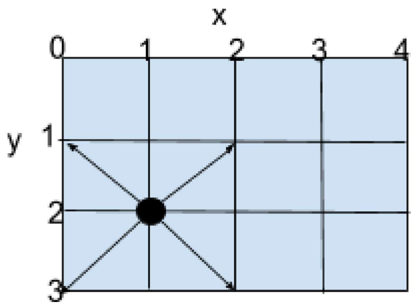
Try to find optimized solutions, and provide a time and space complexity analysis with every solution for the algorithms questions.

"Pinball"

The goal of this strategy is to play optimally a certain kind of "pinball" game. There is an MxN rectangular grid, the four sides of which are walls. The top left corner of the grid has coordinates (0, 0), the bottom left (0, N), the top right (M, 0), and the bottom right (M, N).

Inside the grid, there are lights positioned at integer coordinates. The ball, after it is shot, can roll over lights (they do not interrupt the ball in any way), and you score points equal to the number of distinct lights hit (you only get a point once for hitting the same light). You pick an initial launch position and direction for the ball, and the ball moves according to the rules described below to hit lights. Your goal is to determine the optimal ball starting position and direction that you should choose to maximize your score.

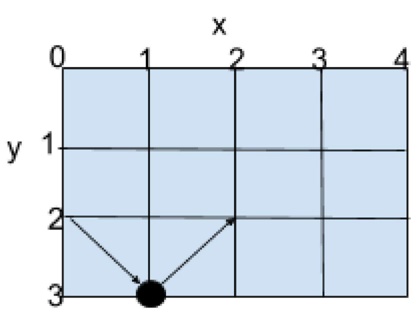
Ball positioning rules
For shooting the ball, you're allowed to select any position having integer coordinates (X, Y) inside the grid or on its boundary (subject to 0 <= X <= M and 0 <= Y <= N), and a ball will be placed at that location. Then, you must select one of the four standard diagonal directions for the ball: up-right, up-left, down-right, or down-left. (If the ball is being deployed from a boundary of the grid, you can only shoot into the grid, you can't choose a direction that would shoot it out.)



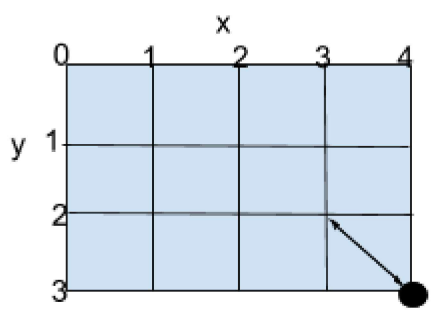
If we were to pick location (1, 2) for the starting position of the ball, the diagram shows this position and the four directions we can choose from.

Ball movement

The ball travels in a straight line in the chosen direction, never losing speed (we assume a frictionless surface). When the ball hits a wall, it reflects off of it. A collision with the right or left wall inverts the sign of the x-component of the ball's direction while keeping the y-component the same, and a collision with the top or bottom wall inverts the sign of the y-component while keeping the x-component the same. A collision with a corner counts as a collision with both of the walls forming the corner and results in both the x- and y- components being inverted, meaning that the ball will travel back in the direction it came from. Because the ball never loses speed, it rolls around forever, but at some point no further lights will be hit and at that point we can view the score as final.



Ball bounces when hitting a wall



Ball bounces in direction it came from when hitting a corner.

During its motion, the ball may roll over some of the lights. A light is hit when the ball passes through the position of the light. The ball and the lights are treated as points, meaning the ball has to pass through the coordinates of the light exactly to register a collision.

Inputs:

Integers M and N specifying the size of the MxN grid

A list of L coordinate (x, y) pairs specifying the coordinates of the lights

Output: the starting ball location and direction that maximizes your final score, and the score.

Subproblems:
(i) Give any solution to this problem. A naive solution should have complexity $O(M^2N^2)$ or so. Note that L (the number of lights) does not appear in this expression because $L <= (M+1)(N+1)$.
(ii) Give some optimization that improves the solution to $O(M^2N)$.
(iii) Add another optimization to give a solution that runs in $O(MN)$.
(iv) Solve the problem in $O(M + N + K)$.

Consider the following addition to the problem: just *once* over the course of the game, at some point in the ball's trajectory, we are allowed to "nudge" the table, changing the ball's direction. This can happen at any position (not necessarily just integer coordinates). The ball's new direction must be changed to one of the other 4 diagonal directions.

(v) Solve this new version in $O(MN)$
(vi)... and in $O(M + N + K)$